Secondary Market Liquidity and Security Design: Theory and Evidence from ABS Markets

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Abstract

We develop a theory of primary market discounts demanded by ex ante identical strategic uninformed investors facing heterogeneous carrying cost realizations. Such investors demand primary market discounts equaling expected secondary market trading losses plus carrying costs. Security design is shown to complement strategic trading ability, as repackaging cash flow gives uninformed investors flexible exit options. Issuers minimize discounts by splitting cash flow into tranched debt claims, with secondary market liquidity increasing in seniority. The optimal number of tranches increases with cash flow information-sensitivity and decreases with carrying costs. Deadweight loss is socially excessive due to excessively thin tranches. Consistent with the model, empirical tests confirm ABS trading costs decrease and trading volume increases with seniority, while the number of tranches increases with information-sensitivity.

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The financial crisis of 2007/8 illustrated the hazards of investing in assets with illiquid secondary markets. Amihud and Mendelson (1986) show rational investors should anticipate illiquidity and demand primary market discounts. However, Garleanu and Pedersen (2004) caution against overstating illiquidity costs, showing that primary market discounts must be understood as being attenuated by investors' ability to trade strategically. Indeed, strategic trading is an important weapon in investors' arsenal allowing them to reduce the effective costs of secondary market liquidity. For example, if expected underpricing is severe, an investor facing a liquidity shock has the option to hold onto his claim. Conversely, if carrying costs are high, an investor can liquidate despite getting a less-than-fair price.

In this paper we argue the ability to trade strategically represents incomplete protection against the costs of secondary market liquidity. In particular, we show that in order for uninformed investors to reap the full benefits of strategic trading, underlying cash flow streams should be repackaged into a more diverse set of claims, allowing investors to make refined bespoke liquidation decisions. In other words, security design and strategic trading must be understood as complements. Issuers who exploit this complementarity benefit from a reduction in primary market discounts.

To illustrate the argument, suppose, as we do, that uninformed investors face identically distributed heterogeneous carrying costs shocks, but are reluctant to sell due to adverse selection (underpricing) arising from the presence of an informed speculator who trades strategically in secondary markets. An uninformed investor holding a claim on the entire cash flow faces a crude all-or-nothing trading decision when hit with a liquidity shock: incur trading losses on the entire cash flow or bear carrying costs on the entire cash flow. If instead the issuer splits the cash flow into multiple securities with varying degrees of information-sensitivity, the same investor hit with a liquidity shock can make refined trading decisions, retaining (selling) those claims for which expected trading losses are greater (less) than his carrying cost, resulting in higher surplus.

We flush out the implications of the preceding exit option argument using a tractable strategic trading framework in the spirit of Spiegel and Subrahmanyam (1992) which may be of independent interest given its tractability. The model considers a continuum of uninformed investors facing carrying cost shocks that are identical ex ante but heterogeneous ex post, taking on J possible values. An uninformed competitive liquidity provider stands ready to buy securities in the secondary market.[1] Liquidity provision is hindered by the existence of a speculator endowed with a private signal of the true cumulative distribution function (good or bad) governing cash flow. The good ($\overline{F}$) and bad ($\underline{F}$) distribution functions have common supports, with the former dominant according to the hazard rate order.

We show uninformed investors demand primary market discounts equal to the sum of expected trading losses incurred by those uninformed investors who choose to sell plus expected carrying costs borne by those who choose to retain, with investors making the sell/retain decision on a security-by-security basis. The issuer's objective is to package the cash flow in the form that minimizes the

[1] In 2012, Market Axess reported 80 broker-dealers as active on its ABS E-trading platform.

total primary market discount demanded by uninformed investors accounting for their strategic trading.[2] Payoff mappings on admissible securities must be monotone in realized cash flow and non-negative.

We begin by showing financial structure is irrelevant if all uninformed investors face a fully absorbing carrying cost shock. Intuitively, nuance is lost on such investors since they liquidate all claim-holdings in the secondary market, leaving the total trading base and primary market discount unchanged. We next show that in selecting the optimal packaging, attention can be confined to no more than $J + 1$ securities. Intuitively, the issuer should consider creating a claim tailored to each ex post liquidity clientele as defined by realized carrying cost. Consistent with observed practices in ABS markets, the optimal structuring features tranched debt claims. Intuitively, a sequence of prioritized debt claims minimizes expected underpricing per unit of funding raised by each liquidity clientele.

Marginal increases in cumulative face value ("detachment point") result in lower carrying costs, as more funds are raised. However, this results in higher expected trading losses as the traded basis increases. At interior optima, marginal reductions in carrying costs are equated with marginal increases in uninformed trading losses. Therefore, optimal detachment points are decreasing in cash flow information-sensitivity, but increasing in carrying costs. Since higher detachment points (thicker tranches) imply a reduction in the total number of tranches, it follows that the optimal number of tranches is increasing in cash flow information-sensitivity and decreasing in carrying costs. For example, pass-through securities are optimal for extremely safe cash flows, while high information-sensitivity cash flows are optimally packaged into J tranched debt claims plus a fully illiquid equity claim.

Post-crisis, commentators criticized the "slicing-and-dicing" pervasive in ABS markets. However, our analysis reveals that tranching actually minimizes the expected costs of secondary market illiquidity for uninformed investors, the very same illiquidity that commentators bemoaned at the height of the crisis. Nevertheless, our analysis reveals a conflict between issuer incentives and social welfare. In particular, the issuer minimizes the sum of expected carrying costs and uninformed trading losses. A social planner seeking to minimize deadweight loss would ignore the latter, since trading losses are simply a transfer across investors. We show expected carrying costs (deadweight loss) decrease monotonically in tranche detachment point cutoffs. It follows that such a social planner would choose higher detachment points, resulting in a reduction in the number of tranches. In this subtle sense, the critics of thinly sliced tranches have a point.

The models of Allen and Gale (1988) and DeMarzo (2005) can also be understood as predicting the optimality of multiple tranched claims. The former rationalizes alternative security designs as arising from market incompleteness. The latter rationalizes an infinite number of tranched debt claims as maximizing an issuer's payoff in separating equilibria, with tranche granularity due to granularity of the support for cash flow, which is privately observed by the issuer. Both models

[2]Hennessy (2013) shows an issuer with the discretion to retain will hold either residual equity or nothing.

are silent on the issue of secondary market liquidity, the central mechanism in our model. A key distinguishing prediction of our proposed theory is that secondary market liquidity should increase with seniority. A second subtle distinguishing prediction is that there should be a positive relationship between the priority position of the most junior traded claim in a pool and that same claim's illiquidity.

We take these distinguishing predictions to the data using a unique set of data covering all transactions on the ABS market provided by the Financial Industry Regulatory Authority (FINRA). Dealers and brokers are mandated to report virtually all ABS transactions to the Trade Reporting and Compliance Engine (TRACE) starting on May 16, 2011.[3]

Our comprehensive data set covers the whole ABS market and allows, for the first time, a clean test of the relationship between ABS security design and secondary market liquidity. Given the previous opacity of the ABS market such an analysis has not been possible before. In our empirical analysis we use trading volume and round-trip costs as proxy for liquidity. We emphasize that our round-trip cost measure reflects the cost of trading more accurately compared to other well established measures in the academic literature because it relies on dealer-specific information, which our data comprises. More importantly, in contrast to earlier studies, we employ granular measures of seniority based on the detailed description of each security for the whole ABS market. We compute the exact numerical priority position and the detachment point of each tranche within its pool. Existing work on ABS instead relies on either credit ratings that could be interpreted as crude proxies for seniority or use coarse seniority levels by classifying tranches into junior, mezzanine, and senior claims, see e.g., Hollifield et al. (2014) and Friewald et al. (2015). Both papers provide evidence that better rated securities have indeed lower transaction costs. However, credit ratings do not provide a clean test for the relation of tranche seniority and liquidity as ratings primarily reflect credit risk. Empirically, we observe that tranches that do differ in their tranche priority often exhibit exactly the same credit rating. Indeed, our results still hold true even after controlling for credit ratings. Consequently, such crude measures of seniority hinder a proper analysis of the relation between tranche priority and liquidity. Thus, the mixed evidence of a positive relation between tranche seniority and liquidity uncovered in Friewald et al. (2015) may follow from their usage of coarse seniority levels. This is not surprising given that many tranches with in fact different tranche priorities are classified into the same seniority level. Moreover, their results may also be distorted by the absence of controlling for pooled-fixed effects. In our empirical analysis we thus first assign each tranche to its underlying pool permitting us to compute detachment points of each tranche, crucial information about tranche priority.

Consistent with model predictions, we find that liquidity increases monotonically in seniority. For example, in pair-wise comparisons between all possible neighboring tranches within each pool, the more senior of the two tranches is more liquid for 64.89% of the observations, with an average

[3] This data set has been also used in other papers, albeit with a different research focus. See, e.g., Atanasov and Merrick (2013), Bessembinder et al. (2013), Hollifield et al. (2014) and Friewald et al. (2015).

difference in round-trip costs of 11.35 basis points. For tranches with a priority position difference of 15, the more senior claim is more liquid for 80.00% of the observations, with an average difference in round-trip costs of 114.77 basis points. A regression analysis confirms that these results are statistically and economically significant. Consistent with the model, we also document a positive relationship between the numerical priority position of the most junior traded claim in a pool and that same claim's illiquidity.

These empirical results should be of general interest to those seeking a better understanding of the relationship between seniority and liquidity. Spiegel (2008) cites as puzzling the fact that senior corporate debt is less liquid than junior corporate equity. Such evidence is not a controlled test for the effect of priority on liquidity given that corporate debt and equity are traded on different platforms. Further, in contrast to ABS, corporate securities are not equivalent to the passive cash flow waterfalls assumed by existing theories, discussed below, of security design under hidden information. As shown by Hennessy and Zechner (2011), moral hazard (hidden action) can cause senior corporate debt to become less liquid than equity.

Gorton and Pennacchi (1990) analyze security design when uninformed investors face informed speculators and have a demand for safe storage. Uninformed investors carve out a safe debt claim which they use to carry funds across periods. In their model, the marketing of a single riskless debt claim eliminates adverse selection costs. Nachman and Noe (1994) analyze the optimal design of a single marketed security when the issuer is privately informed at the time the security is designed. With a fixed issuer funding target, the equilibrium necessarily entails pooling. They show that under technical conditions, including monotonicity, debt is the optimal funding source. Intuitively, debt minimizes cross-subsidies from high to low types as informally argued by Myers and Majluf (1984). DeMarzo and Duffie (1999) and Biais and Mariotti (2005) show debt is the optimal marketed claim for a liquidity-motivated issuer designing one security before observing private information and before choosing the quantity to be marketed.

Axelson (2007) determines the one optimal marketed security for a price-impacting uninformed issuer facing informed bidders. In contrast, we analyze the optimal set of marketed securities to be sold to atomistic uninformed investors who trade non-cooperatively, while facing an informed speculator. As in Axelson (2007), there is a tradeoff between carrying costs and trading loss discounts, but one that is less mechanical. In his model, the issuer incurs a carrying cost on his retained claim and trading losses on the marketed claim. In our model, carrying costs and trading losses hit multiple marketed claims with the extent depending on endogenous secondary market trading in each security.

Boot and Thakor (1993) and Fulghieri and Lukin (2001) consider the optimal bifurcation (splitting) of marketed cash flows by a privately informed issuer who wants to stimulate informed speculation in a pooling equilibrium. In these models, tranching marketed cash flows promotes information production by relaxing speculator wealth constraints. Essentially, these models rationalize tranching as a device for helping the informed to better exploit non-strategic uninformed investors.

Conversely, in our model, granular tranching is a device for helping strategic uninformed investors help themselves.

While the security design theories discussed above offer a partial explanation of observed ABS structures, they also have limitations in this regard. After all, the marketed portion of ABS rarely comes in the form of a single debt claim. Rather, the marketed portion of ABS is generally split into *multiple* tranches. A more complete theory of ABS design must explain granular prioritization amongst marketed claims, in addition to the empirical regularities we document.

The remainder of the paper is as follows. Section 1 describes the economic setting. Section 2 presents a theory of discounts associated with adverse selection and carrying costs. Section 3 analyzes the optimal packaging of cash flow to minimize the total discount. Section 4 contains the empirical analysis.

1 Assumptions

There is one asset. It generates a single cash payment $x \in \mathcal{X} \equiv [\underline{x}, \overline{x}] \subseteq \mathbb{R}_{++}$ accruing in the final period. The c.d.f. governing cash flow is determined by the latent profitability state $\omega \in \{\underline{\omega}, \overline{\omega}\}$. The two profitability states are equiprobable. If the state is $\underline{\omega}$ ($\overline{\omega}$), cash flow is distributed according to the c.d.f. $\underline{F}$ ($\overline{F}$). Each c.d.f. is continuously differentiable, and the respective densities $\underline{f}$ and $\overline{f}$ are strictly positive on $\mathcal{X}$.

It is assumed $\overline{F}$ stochastically dominates $\underline{F}$ according to the hazard rate order.[4]

$$\textbf{Assumption 1}: \quad \frac{\overline{f}(x)}{1-\overline{F}(x)} \leq \frac{\underline{f}(x)}{1-\underline{F}(x)} \quad \forall \quad x \in \mathcal{X}.$$

We recall hazard rate dominance implies $\overline{F}$ stochastically dominates $\underline{F}$ in the first-order sense:

$$\overline{F}(x) \leq \underline{F}(x) \quad \forall \quad x \in \mathcal{X}.$$

There are three periods: 1, 2, and 3. Long-term investors are present at all dates and are uninformed regarding the profitability state. Short-term investors enter in period 2 and consist of an informed speculator and an uninformed liquidity provider.[5] All investors are risk-neutral, value consumption equally at all dates, and have access to a riskless storage technology delivering zero interest.

At the start of period 1 the rights to the cash flow x are held by the issuer. The issuer only values consumption in period 1 and so wants to sell all rights to the cash flow in the primary market. Alternatively, one may think of the issuer as being forced to sell all claims on the asset

[4]We thank an anonymous referee for pointing out the feasibility of replacing likelihood ratio dominance with weaker hazard rate dominance.

[5]Equivalently, one could assume short-term investors are present in period 1 but have limited wealth or high discount rates from period 1 to 2.

by a regulator in order to comply with capital requirements. The issuer's objective is to maximize the amount the long-term investors are willing to pay for claims backed by the cash flow. That is, the issuer minimizes the illiquidity discount demanded by uninformed investors. That uninformed investors determine primary market prices follows Holmström and Tirole (1993) and Maug (1998), for example.

The key model feature is that structuring anticipates non-cooperative liquidations of multiple securities held by price-taking agents (the long-term investors) in competitive secondary markets. This may be contrasted with models in which a single price-impacting decisionmaker decides how much to sell of one pre-designated marketable security, e.g. DeMarzo and Duffie (1999) and Axelson (2007).

Let $\mathcal{A}$ denote the set of admissible security payoff mappings. All payoff functions $a \in \mathcal{A}$ must satisfy the following limited liability and monotonicity constraints:

Assumption 2 : $0 \leq a(x) \leq x \quad \forall \quad x \in \mathcal{X}.$

Assumption 3 : *a is non-decreasing on $\mathcal{X}$.*

An extant literature including Innes (1990), Nachman and Noe (1994), DeMarzo and Duffie (1999), DeMarzo (2005), and Axelson (2007) considers that decreasing securities may be inadmissible. This requirement can be understood as a response to two ex post moral hazard problems that arise if some securities are decreasing. First, if an agent's claim is decreasing, he can gain from destroying cash flow through sabotage. Second, if one agent's claim is decreasing, the other agent can gain from making a clandestine contribution to the cash pool.

For an arbitrary measurable mapping $a : \mathcal{X} \to \mathbb{R}$, let:

$$\overline{\mu}_a \equiv \int_{\underline{x}}^{\overline{x}} a(x)\overline{f}(x)dx$$

$$\underline{\mu}_a \equiv \int_{\underline{x}}^{\overline{x}} a(x)\underline{f}(x)dx.$$

It is readily verified that Assumptions 1 and 3 imply:[6]

$$\overline{\mu}_a \geq \underline{\mu}_a \quad \forall \quad a \in \mathcal{A}. \tag{1}$$

Long-term investors are identical ex ante and thus have identical willingness to pay in the primary market. Prior to secondary market trading, long-term investors face exogenous shocks that bias them toward selling or holding securities. The shocks are identically distributed and correlated. In particular, at the start of period 2 each long-term investor privately learns about his

[6] In fact, the result follows from first-order stochastic dominance and monotonicity.

own cost to selling securities in the secondary market or holding them until period 3.

A subset of the long-term investors face carrying cost shocks that bias them against holding securities until the cash flow accrues in period 3. An investor falling into carrying cost category j faces a cost that will absorb a fraction c_j of period 3 payoffs. There are $J \geq 1$ possible carrying cost categories, with

$$0 < c_1 < \ldots < c_J \leq 1.$$

An equal fraction γ of long-term investors experience costs falling into each of the J carrying cost categories. This fraction is a binary random variable drawn from $\{\underline{\gamma}, \overline{\gamma}\}$. It is assumed that $0 < \underline{\gamma} < \overline{\gamma} \leq 1$, and $J\overline{\gamma} \leq 1$. The random variable γ is labeled the *liquidity demand state* since, as shown below, the volume of securities sold by the long-term investors will be scaled by γ. That is, the state variable γ will serve to shift the supply of securities tendered by uninformed investors. The liquidity demand state is not observed by any agent and is uncorrelated with the profitability state ω.[7] The high liquidity demand state $\overline{\gamma}$ occurs with probability ψ, where $0 < \psi < 1$.

The remaining measure $1 - J\gamma$ of long-term investors do not experience carrying costs. Instead, they face a fully absorbing selling cost if they liquidate their holdings in the secondary market. As real-world examples, one can think of the carrying costs as taxes on terminal dividends and selling costs as pecuniary (e.g. tax) or non-pecuniary (hassle) costs of financial transactions. Alternatively, one can think of the carrying cost as capturing outside investment opportunities or pressing consumption needs. Similarly, the selling cost can be thought of as a reduced-form agency cost of holding excess cash at the interim date. For example, it may be easy for managers to divert surplus cash from outside investors and so investors may prohibit premature liquidations.

For further motivation of carrying and selling costs, the reader is referred to the discussion in Garleanu and Pedersen (2004). Their model features shocks that are analogous to the carrying and selling costs posited here. In particular, their model features positive and negative carrying costs with positive carrying costs biasing agents toward selling and negative costs biasing them toward holding.

At the start of period 2 short-term investors enter. There are two short-term investors: a competitive liquidity provider and a speculator. At the start of period 2, the speculator observes a noisy signal of the profitability state ω which is correct with probability $\sigma \in (1/2, 1]$. At the end of period 2, the liquidity provider stands ready to buy securities on competitive terms. As in Kyle (1985), the speculator and the long-term investors submit market orders. There is no market segmentation and the liquidity provider observes orders in all markets. After observing the aggregate sell orders across markets, the liquidity provider buys securities at prices equal to their respective conditional expected payoffs. Note, the adverse selection problem would vanish trivially if the liquidity provider could simply buy and hold the entire cash flow stream from period 1 to period 3. In reality, broker-dealers face legal restrictions and costs limiting their ability and willingness to carry inventory over long horizons.

[7] If the two were correlated then the so-called uninformed investors would have information regarding fundamentals.

In period 3 cash flow is verified and investors are paid.

2 A Theory of Illiquidity Discounts

This section considers secondary market trading and pricing if the entire cash flow were to be marketed as one security. This allows us to exposit the process of price formation before tackling the issue of optimal structuring in light of the pricing process. The analysis may be of independent interest even to those uninterested in security design since we offer a tractable theory of illiquidity discounts.

The equilibrium concept is perfect Bayesian equilibrium (PBE) in pure strategies. For a security k with payoff function a_k and for each carrying cost category j, let Φ_k^j be an indicator function equal to 1 if a long-term investor with carrying cost c_j will find it optimal to sell the security in the secondary market.

For an arbitrary security k, aggregate secondary market selling by uninformed investors, denoted u_k, is a binary random variable:

$$\begin{aligned} u_k &\in \{\underline{u}_k, \overline{u}_k\}. \\ \underline{u}_k &\equiv \underline{\gamma}\sum_{j=1}^{J}\Phi_k^j \\ \overline{u}_k &\equiv \overline{\gamma}\sum_{j=1}^{J}\Phi_k^j. \end{aligned} \tag{2}$$

The variable β denotes the updated belief of the liquidity provider regarding the probability of the profitability state being $\overline{\omega}$. This belief is based on observed sell orders across all markets. The liquidity provider sets the secondary market price of each security according to:

$$P_k = \beta\overline{\mu}_k + (1-\beta)\underline{\mu}_k \quad \forall \quad a_k \in \mathcal{A}. \tag{3}$$

Table 1 depicts potential order flow configurations. The third column of Table 1 can be understood as follows. In light of the trading pattern of the uninformed investors, the only profitable trading strategy for the informed speculator is to short-sell $(\overline{u} - \underline{u})$ units if she observes a negative signal. This order size has the potential to confound the liquidity provider regarding the speculator's signal. To see this, note that the aggregate sell order $-\overline{u}$ can arise from either: negative speculator signal and low uninformed investor selling volume or positive signal and high uninformed investor selling volume.

Consider now the liquidity provider's updated beliefs (final column in Table 1). There are three possible aggregate order flows. Upon observing the aggregate sell order $-\underline{u}$, the liquidity provider knows the speculator observed the signal $\overline{\omega}$ and so forms the belief $\beta = \sigma$. Upon observing the aggregate sell order $-(2\overline{u}-\underline{u})$, the liquidity provider knows the speculator observed the signal $\underline{\omega}$ and so forms the belief $\beta = 1-\sigma$. In these two cases, order flow reveals the signal observed by the speculator and there is zero expected profit from informed trading. In contrast, the liquidity provider is confounded upon observing the aggregate sell order $-\overline{u}$. Here he uses Bayes' rule to form the updated belief:[8]

$$\beta_n \equiv 1-\sigma-\psi+2\sigma\psi. \tag{4}$$

Consider now the liquidation decision of one of our atomistic long-term investors hit by a carrying cost shock. We conjecture he liquidates if and only if the realized carrying cost shock is above some threshold. In particular, it is optimal to sell if his expectation of the secondary market price exceeds the expected terminal payoff net of his carrying cost c_j. His expectation of the secondary market price is conditioned on the fact that he has drawn a positive carrying cost. This conditional price expectation can be computed as the sum over conditional probabilities of each liquidity demand state times the expected price in each liquidity demand state. From Table 1 the expected price is:

$$\begin{aligned}\mathbb{E}[P_x|c_j &> 0] = \left[\frac{\psi\overline{\gamma}}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\right]\frac{1}{2}\left[\overline{\mu}_x+\underline{\mu}_x-(1-\psi)(2\sigma-1)(\overline{\mu}_x-\underline{\mu}_x)\right] \\ &+\left[\frac{(1-\psi)\underline{\gamma}}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\right]\frac{1}{2}\left[\overline{\mu}_x+\underline{\mu}_x+\psi(2\sigma-1)(\overline{\mu}_x-\underline{\mu}_x)\right] \\ &= \frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)-\frac{1}{2}\frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}(\overline{\mu}_x-\underline{\mu}_x).\end{aligned} \tag{5}$$

Equation (5) shows that uninformed investors facing carrying cost shocks are vulnerable to underpricing since the conditional expectation of the secondary market price is less than the expected terminal payoff. Intuitively, each long-term investor views his own carrying cost shock as suggestive of high uninformed liquidity demand ($\gamma=\overline{\gamma}$). And with high uninformed selling volume, he expects a lower secondary market price given that the liquidity provider views high selling volume as suggesting the speculator received a negative signal regarding the profitability state ω. Consistent with this intuition, equation (5) shows the conditional expectation of underpricing is increasing in σ, with zero underpricing had the speculator been endowed with a completely uninformative signal ($\sigma = 1/2$).

A long-term investor experiencing a carrying cost finds it optimal to sell if the conditional expectation of the secondary market price is greater than the expected terminal payoff net of

[8] In response to off-equilibrium sell orders, LP believes speculator observed the negative signal.

carrying costs, or:

$$\frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)-\frac{1}{2}\frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}(\overline{\mu}_x-\underline{\mu}_x) \geq \frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)(1-c_j) \quad (6)$$
$$\Updownarrow$$
$$c_j \geq \lambda_x.$$

where

$$\lambda_k \equiv \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{\mu}_k-\underline{\mu}_k}{\overline{\mu}_k+\underline{\mu}_k} \quad \forall \quad a_k \in \mathcal{A}. \quad (7)$$

The variable λ_k measures the conditional expectation of security underpricing as a percentage of the expected terminal payoff on security k. A long-term investor prefers to liquidate his position in security k if and only if his carrying cost exceeds expected percentage underpricing ($c_j \geq \lambda_k$).

Consider now the expected trading gain of the speculator if the entire cash flow is wrapped as one security. From Table 1 it follows the speculator's expected trading gain is:

$$\begin{aligned} G &= \frac{\overline{u}_x-\underline{u}_x}{2}\left[\begin{array}{c}(1-\sigma)(1-\psi)[\beta_n\overline{\mu}_x+(1-\beta_n)\underline{\mu}_x-\overline{\mu}_x]+(1-\sigma)\psi[(1-\sigma)\overline{\mu}_x+\sigma\underline{\mu}_x-\overline{\mu}_x]\\ +\sigma(1-\psi)[\beta_n\overline{\mu}_x+(1-\beta_n)\underline{\mu}_x-\underline{\mu}_x]+\sigma\psi[(1-\sigma)\overline{\mu}_x+\sigma\underline{\mu}_x-\underline{\mu}_x]\end{array}\right] \quad (8)\\ &= \frac{1}{2}(2\sigma-1)\psi(1-\psi)(\overline{\mu}_x-\underline{\mu}_x)(\overline{\gamma}-\underline{\gamma})\sum_{j=1}^{J}\Phi_x^j. \end{aligned}$$

Holding fixed the trading decisions of the uninformed investors (Φ_x^j), the speculator's expected trading gain G is increasing in her signal precision, increasing in the wedge between expected cash flow across the two profitability states, and increasing in the variability of shocks hitting the uninformed investors as captured by the wedge between the two liquidity demand states. Of course, this ceteris paribus reasoning is subject to caveat since variation in the underlying parameters affects the willingness-to-trade of uninformed investors.

The *primary market valuation* of the total securitized cash flow is denoted V. The willingness-to-pay of long-term investors is equal to expected cash flow less expected trading losses and carrying costs. Since the liquidity provider makes zero expected profit, the speculator's expected trading gain must be just equal to the expected trading losses of uninformed investors.

Let V_{PT} denote the primary market valuation attained under a pass-through structure in which the entire cash flow is sold off as an equity claim paying x. Using the prior expression for speculator trading gains and accounting for expected carrying costs we have:

$$\begin{aligned} V_{PT} &= \frac{1}{2}(\overline{\mu}_x+\underline{\mu}_x)-\frac{1}{2}(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})(\overline{\mu}_x-\underline{\mu}_x)\sum_{j=1}^{J}\Phi_x^j \quad (9)\\ &\quad -\frac{1}{2}[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}](\overline{\mu}_x+\underline{\mu}_x)\sum_{j=1}^{J}(1-\Phi_x^j)c_j. \end{aligned}$$

Equation (9) illustrates the role endogenous trading plays in mitigating illiquidity discounts. To illustrate, consider the effect of a discrete increase in speculator signal precision. This would give rise to a discrete increase in expected uninformed investor trading losses in the absence of a change in their trading. However, it is apparent that marginal uninformed investors, with c_j just above the initial $\lambda_x(\sigma)$, would respond to a discrete increase in expected trading losses by retaining rather than selling. It follows that the effect of changes in σ on informed trading gains (equation (8)) and uninformed trading losses is ambiguous. However, the net effect of changes in σ on the primary market discount is unambiguous. To see this, note that we can rewrite the primary market valuation of a pass-through security (9) as follows:

$$V_{PT} = \frac{1}{2}(\overline{\mu}_x + \underline{\mu}_x) - \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}](\overline{\mu}_x + \underline{\mu}_x)c_j + \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}](\overline{\mu}_x + \underline{\mu}_x)\sum_{j=1}^{J}(c_j - \lambda_x)^+. \quad (10)$$

Equation (10) illustrates the effect of exit option value on primary market discounts. The second term in the equation captures the primary market discount that would arise in the absence of the option to sell in the secondary market. The final term captures uninformed seller surplus arising from the optimal exercise of exit options. Since expected fractional trading loss (λ_x) is increasing σ, as shown in equation (7), it is apparent that primary market value is decreasing in σ.

Returning to equation (9) it is apparent that the effect of a marginal increase in some carrying cost parameter c_j on the primary market discount of a pass-through security depends on whether it is above or below λ_x. In particular, an uninformed investor experiencing a carrying cost $c_j < \lambda_x$ finds it optimal to hold rather than sell, implying this carrying cost will be capitalized into the primary market discount. Conversely, an uninformed investor experiencing $c_{j'} > \lambda_x$ finds it optimal to sell rather than hold, implying any increase in the carrying cost $c_{j'}$ will have no effect on the primary market discount. Rational trading by uninformed investors truncates the effect of carrying costs on primary market discounts.

3 Optimal Structuring

The previous section considered the primary market discount on a pass-through equity security in light of rational trading by uninformed investors. We now consider whether and how an issuer can increase the total amount raised in the primary market by repackaging the total cash flow into separate securities. We begin by showing that tranching of claims is indeed optimal. We then move on to a detailed discussion of the optimal tranche detachment points, followed by a discussion of microstructure implications.

3.1 The Optimality of Tranched Claims

The objective of the issuer is to minimize the primary market discount demanded by the uninformed investors. This discount is given by:

$$D = \begin{array}{l} \frac{1}{2}(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) \sum_{k=1}^{K} (\overline{\mu}_k - \underline{\mu}_k) \sum_{j=1}^{J} \Phi_k^j \\ + \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}] \sum_{k=1}^{K} (\overline{\mu}_k + \underline{\mu}_k) \sum_{j=1}^{J} (1 - \Phi_k^j) c_j . \end{array} \tag{11}$$

The first term in the discount expression represents expected trading losses of those uninformed investors who liquidate, while the second term captures carrying costs borne by those who do not liquidate.

Repeating the same argument as that applied in the context of the pass-through security, it is readily verified that the trading rule for uninformed investors takes the form:

$$c_j \geq \lambda_k \Rightarrow \Phi_k^j = 1. \tag{12}$$

That is, each uninformed investor evaluates trading security-by-security, only liquidating those claims for which the expected percentage uninformed trading loss is less than their idiosyncratic carrying cost. The next proposition follows directly from the preceding inequality and equation (11).

Proposition 1. *If uninformed investors face a homogeneous carrying cost $c = 1$, all structurings of the cash flow result in the same primary market valuation as that arising from a pass-through structure.*

The preceding proposition is useful in showing a technical difficulty that may arise in predicating theories of security design on investors hit with extreme forms of shocks. In particular, if $c = 1$ then trading loss discounts will hit the entire cash flow regardless of how it is packaged, leaving the primary market discount unchanged.

The following identity is useful in pinning down the optimal structuring:

$$\sum_{k=1}^{K} a_k = a \Rightarrow \sum_{k=1}^{K} \left(\frac{\overline{\mu}_k + \underline{\mu}_k}{\overline{\mu}_k + \underline{\mu}_k} \right) \lambda_k = \lambda_a; \quad \{a_k \in \mathcal{A}\}. \tag{13}$$

Identity (13) states that the weighted average of the expected percentage trading losses of a set of securities is just equal to the expected percentage trading loss on a security that wraps them into one security. The identity leads to the following lemma.

Lemma 1. *Given J possible interim-date carrying cost realizations, any primary market valuation attainable with more than $J + 1$ securities is attainable with no more than $J + 1$ securities.*

It is useful to provide here the proof of the preceding lemma since it anticipates the construction of the optimal structuring. Suppose the issuer were to market the cash flows as more than $J+1$ securities. One could sort these securities into $J+1$ mutually exclusive baskets, with basket j consisting of those securities k satisfying:

$$c_{j-1} < \lambda_k \leq c_j$$

with any remaining fully illiquid securities grouped into basket 0. That is, for $j \in \{1, 2, \ldots, J\}$, each of the original securities in basket j would have been liquidated by those long-term investors experiencing carrying costs c_j and higher. The securities in basket 0 would not have been liquidated by any long-term investor. From identity (13) it follows that bundling together the securities in basket j results in a new security, call it Security j, with

$$\begin{aligned} c_{j-1} &< \lambda_j \leq c_j \\ \lambda_0 &> c_J. \end{aligned} \tag{14}$$

By construction, for each $j \in \{1, 2, \ldots, J\}$, Security j will be liquidated by those long-term investors experiencing carrying costs c_j and higher. Security 0 will not be liquidated by any long-term investor. It follows from equation (11) that the primary market discount is left unchanged by such a repackaging of securities.

Following the proof of Lemma 1 we now pin down the optimal design of each Security j for $j \in \{0, 1, 2, \ldots, J\}$ as defined above. That is, each security will be indexed by the lowest carrying cost category that will sell it in the secondary market. Security 0 is not sold by any long-term investor. Letting k index the security and j index the investor carrying cost category, the primary market discount is:

$$\begin{aligned} D &= \frac{1}{2}(2\sigma - 1)\psi(1-\psi)(\overline{\gamma} - \underline{\gamma}) \sum_{k=1}^{J} \left[(\overline{\mu}_k - \underline{\mu}_k) \left(\sum_{j=1}^{J} \Phi_k^j \right) \right] \\ &\quad + \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}] \sum_{k=1}^{J} \left[(\overline{\mu}_k + \underline{\mu}_k) \left(\sum_{j=1}^{J} (1 - \Phi_k^j) c_j \right) \right] \\ &\quad + \frac{1}{2}[\psi\overline{\gamma} + (1-\psi)\underline{\gamma}](\overline{\mu}_0 + \underline{\mu}_0) \sum_{j=1}^{J} c_j. \end{aligned} \tag{15}$$

As shown in the appendix, rearranging terms in equation (15) and dropping irrelevant constants, minimizing the primary market discount is equivalent to maximizing the following objective function:

$$\sum_{k=1}^{J}\sum_{j=1}^{J}\Phi_k^j[(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j(\overline{\mu}_k+\underline{\mu}_k)-(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})(\overline{\mu}_k-\underline{\mu}_k)]. \tag{16}$$

Rearranging terms in the preceding equation allows us to rewrite the maximand from the preceding equation as a weighted sum of the expected security payoffs:

$$\begin{aligned}
&\sum_{k=1}^{J}\sum_{j=k}^{J}[\underline{\kappa}^j\underline{\mu}_k-\overline{\kappa}^j\overline{\mu}_k] \qquad (17)\\
\underline{\kappa}^j &\equiv (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})+(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j\\
\overline{\kappa}^j &\equiv (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j.
\end{aligned}$$

In turn, as shown in the appendix, expected security payoffs can be expressed as:

$$\begin{aligned}
\mu_k &= \int_0^{\overline{x}}(1-F(x))\delta_k(x)dx \qquad (18)\\
\delta_k &\equiv a_k'.
\end{aligned}$$

Substituting equation (18) into equation (17), and letting

$$\Omega_k(x)\equiv\left(\sum_{j=k}^{J}\underline{\kappa}^j\right)(1-\underline{F}(x))-\left(\sum_{j=k}^{J}\overline{\kappa}^j\right)(1-\overline{F}(x))$$

the program can be written as an infinite-dimensional linear-programming problem:[9]

$$\begin{aligned}
&\max_{\{\delta_k\}_{k=1}^{k=J}}\int_0^{\overline{x}}\sum_{k=1}^{J}\Omega_k(x)\delta_k(x)dx \qquad (19)\\
&\text{subject to}\\
\sum_{k=1}^{J}\delta_k(x) &\leq 1 \quad \forall \quad x\in\mathcal{X}\\
\delta_k(x) &\geq 0 \quad \forall \quad x\in\mathcal{X}.
\end{aligned}$$

Notice, in the preceding program we have ignored the incentive constraints (14) ensuring that the uninformed investors will liquidate the respective securities as posited. This is valid since, as shown in the appendix, the neglected constraints are slack at the unconstrained optimum.

Inspecting the program it is apparent the optimal control policy can be pinned down in two steps. In the first step, at each point x in the state space we must identify amongst securities

[9] We thank an anonymous referee for suggesting this simpler linear-programming approach as opposed to an optimal control approach.

$k \in \{1, \ldots, J\}$ the security, call it h, with the highest value of $\Omega_k(x)$. For the second step, if $\Omega_h(x)$ is positive then it is optimal to assign security h the cash flow increment, with $\delta_h^*(x) = 1$. If instead $\Omega_h(x)$ is negative then it is optimal to assign incremental cash flow to the fully illiquid security, Security 0, with $\delta_0^*(x) = 1$.

To begin our characterization of the optimal structuring, we begin by noting that:

$$\Omega_1(\underline{x}) > \ldots > \Omega_J(\underline{x}) > 0. \tag{20}$$

From the preceding equation it follows there is some right neighborhood of $\underline{x}$ over which it is optimal to assign all cash flow to Security 1, which is the security that will be sold by all investors hit with carrying cost shocks. We have the following proposition.

Proposition 2. *An optimal structuring always features a super-senior claim that will be liquidated by each investor hit with a carrying cost shock. If a pass-through structure is optimal, it will be liquidated by each investor hit with a carrying cost shock.*

As shown in the appendix, the following proposition follows directly from comparison of the various Ω_k values.

Proposition 3. *Packaging the entire cash flow as a pass-through security is optimal if*

$$\begin{aligned} & \left[\frac{\overline{f}(\overline{x})}{\underline{f}(\overline{x})}\right][(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) - (\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_1] \\ \leq \; & (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) + (\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j. \end{aligned}$$

Otherwise the optimal structuring consists of $k+1$ tranched claims with k being maximal amongst $j \in \{1, \ldots, J\}$ satisfying

$$\begin{aligned} & \left[\frac{\overline{f}(\overline{x})}{\underline{f}(\overline{x})}\right][(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) - (\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j] \\ > \; & (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) + (\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j. \end{aligned} \tag{21}$$

The face value on each tranche j is θ_j defined implicitly by

$$\begin{aligned} \frac{1-\overline{F}(\Theta_j)}{1-\underline{F}(\Theta_j)} &= \frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) + (\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j}{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma}) - (\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j} \\ \text{where } \Theta_j &\equiv \sum_{t=1}^{j} \theta_t. \end{aligned} \tag{22}$$

Liquidity is decreasing in the tranche index, with the expected percentage uninformed trading loss on each tranche j satisfying

$$c_{j-1} < \lambda_j < c_j. \tag{23}$$

Proposition 2 and 3 offer a number of testable implications. We begin with a discussion of Proposition 2. First, the proposition predicts that all securitizations should feature an ultra-senior claim that is highly liquid. Second, the proposition shows that a pass-through structure should only be utilized in those instances where the underlying cash flow stream has very low information-sensitivity. The third implication, related to the first and second, is that pass-through securities should be very liquid.

Consider next Proposition 3. The proposition shows that the optimal set of exit options to provide to uninformed investors is a basket of tranched debt claims. Uninformed investors hit with low carrying cost shocks will liquidate only the more senior tranches. Those hit with higher carrying cost shocks will work their way down the menu and also sell more junior tranches. Intuitively, giving uninformed investors such a menu increases their expected seller surplus. We leave to the next subsection a discussion of the determinants of the optimal detachment points (Θ_j) for the various tranches.

The final statement of Proposition 3 offers an important empirical prediction, distinguishing our theory of multiple tranches from that of Allen and Gale (1988) and DeMarzo (2005). While their theories are silent on the question of secondary market liquidity, we predict that illiquidity as measured by expected percentage uninformed trading losses, is increasing in the tranche index. As discussed below, this implies lower trading volume and higher transactions costs.

3.2 Optimal Detachment Points

The derivation of the optimal contract in Proposition 3 may obscure the tradeoffs determining the optimal detachment points (Θ_j) for the various tranches. In order to illustrate the tradeoffs in the simplest possible way, consider a marginal increase in some detachment point Θ_n holding fixed all other detachment points. This is implemented via a marginal increase in θ_n while holding fixed all other detachment points. That is, the tranche n is allowed to eat into tranche $n+1$ at the margin. Using the fact that only tranches n and $n+1$ are affected by the contemplated zero-sum perturbation, from equation (15) it follows:

$$D'(\theta_n) = \underbrace{\frac{1}{2}(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})[\overline{\mu}_n'(\theta_n)-\underline{\mu}_n'(\theta_n)]}_{\text{Marginal Expected Trading Loss}} - \underbrace{\frac{1}{2}[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}][\overline{\mu}_n'(\theta_n)+\underline{\mu}_n'(\theta_n)]c_n}_{\text{Marginal Expected Carrying Costs}}. \tag{24}$$

The first term in the preceding equation captures the increase in expected uninformed trading losses incurred on the senior tranche. The second term captures the reduction in expected carrying costs, as investors with carrying costs c_n raise a bit more funding in the secondary market. Using the fact that

$$\mu_n'(\theta_n) = 1 - F(\Theta_n) \tag{25}$$

we have the following first-order condition:

$$\begin{aligned} & (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})[(1-\overline{F}(\Theta_n))-(1-\underline{F}(\Theta_n))] \\ = \; & (\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_n[(1-\overline{F}(\Theta_n))+(1-\underline{F}(\Theta_n))]. \end{aligned} \tag{26}$$

The first-order condition states that the optimal detachment points equates marginal trading losses with marginal carrying costs. Rearranging terms in the preceding equation one arrives at the optimality condition in equation (22) of Proposition 3.

It is instructive to contrast the issuer's choice of detachment point with that preferred by a social planner seeking to minimize deadweight losses (DWL). Given that uninformed investor trading losses constitute a transfer to the informed speculator, it follows from equation (24) that the marginal effect of an increase in detachment point Θ_n is given by:

$$DWL'(\theta_n) = \underbrace{-\frac{1}{2}[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}][\overline{\mu}'_n(\theta_n)+\underline{\mu}'_n(\theta_n)]c_n}_{\text{Marginal Expected Carrying Costs}} < 0. \tag{27}$$

The next result follows directly from the preceding inequality and Proposition 3.

Proposition 4. *In order to minimize expected deadweight carrying costs, each detachment Θ_n should be increased to the point that the incentive constraint $\lambda_n \leq c_n$ binds. These constraints are slack at the detachment points chosen by the issuer, as defined by equation (22).*

Consider next the comparative statics properties of the detachment points that will be chosen by the issuer. It is readily verified that the right-side of equation (22) is decreasing in σ and increasing in c_j. And we recall that hazard rate dominance implies the left-side of the equation is increasing in Θ_j. It follows that an optimal structuring entails the following comparative statics:

$$\begin{aligned} \frac{\partial\Theta_j}{\partial\sigma} &< 0 \\ \frac{\partial\Theta_j}{\partial c_j} &> 0. \end{aligned} \tag{28}$$

The intuition for these comparative statics is best understood by returning to equation (24) capturing the effect of a marginal increase in a detachment point. Here, a marginal increase in σ increases marginal trading losses and so it is optimal to lower the detachment point. Conversely, an increase in c_j raises the marginal benefit to reducing carrying costs and so it is optimal to increase Θ_j. Thus, it is apparent that optimal spacing between detachment points is dictated by the configuration of $(c_1, \ldots, c_J)$.

It is apparent from Proposition 3 that an optimal structuring may very well consist of less than $J+1$ claims. From condition (21) it follows the optimal number of claims is increasing in information-sensitivity, as measured by the ratio $\overline{f}(\overline{x})$ to $\underline{f}(\overline{x})$. Intuitively, as shown above, the position of each detachment point reflects the tradeoff between trading losses and carrying costs. Higher information-sensitivity increases expected trading losses which induces a leftward shift of tranche detachment points. This implies an increase in the total number of securities.

Another empirical prediction we take to the data is that there should be a positive relationship between the number of tranches and the underlying cash flow information-sensitivity (λ_x). For example, the first statement in Proposition 3 indicates that a cash flow stream with very low information-sensitivity is optimally packaged as a fully liquid pass-through security. Conversely, if the underlying cash flow has very high information-sensitivity, it is optimally packaged as J debt claims and a fully illiquid residual equity claim (our Security 0). Here, the liquidity of the debt claims decreases monotonically in priority position with $\lambda_1 < c_1$ and $\lambda_J \in (c_{J-1}, c_J)$. Cash flow streams with intermediate information-sensitivity are optimally packaged as k^* claims, with $k^* \in \{2, \ldots, J\}$. Here claim k^* will be a residual equity claim with intermediate liquidity, specifically $\lambda_{k^*} \in (c_{k^*-1}, c_{k^*})$.

The preceding analysis implies the following novel prediction: There should be a positive relationship between the most junior traded security's priority position and the illiquidity of that same security. To see this, consider again a cash flow stream with very high information-sensitivity. As discussed above, the most junior traded security will have priority position J and will have low liquidity since $\lambda_J \in (c_{J-1}, c_J)$. And note, this relationship is not mechanical. For example, if our theory did not hold, an issuer could package a cash flow stream with very low information-sensitivity into many tranches with even the most junior traded security being very liquid. Then one would observe a claim sitting low in the priority structure but retaining high liquidity. In contrast, we would here predict a junior-most priority position equal to 1 and maximum liquidity.

Consider next the effect of an increase in speculator signal precision. It was shown above, equation (28), that each of the optimal detachment points is decreasing in σ. This implies an increase in the number of tranches, consistent with condition (21).

Conversely, consider the effect of increasing each of the carrying costs by a discrete amount. It follows from condition (21) that the optimal number of tranches will fall. Intuitively, we have seen that in response to higher carrying costs it is optimal to increase each of the detachment points. This implies a decrease in the total number of tranches.

We summarize these results in the following proposition.

Proposition 5. *Each of the optimal detachment points Θ_n is decreasing in speculator signal precision, increasing in c_n and invariant to c_m for all $m \neq n$. The optimal number of tranches is decreasing in carrying costs, and increasing in both speculator signal precision and the information-sensitivity of cash flow. Across pools, there is a positive relationship between the priority position of the most junior traded claim and that claim's information-sensitivity (λ).*

3.3 Volume and Price Impact

This subsection considers the model's implications in terms of liquidity measures of interest to market microstructure researchers.

Consider first trading volume. Recall from Table 1 that total trading volume is increasing in uninformed trading volume u_n. Respectively, expected uninformed trading on tranche n is given by:

$$\mathbb{E}[u_n] \quad = \quad [\psi\overline{\gamma} + (1-\psi)\underline{\gamma}] \sum_{j=1}^{J} \chi_{c_j \geq \lambda_n}. \tag{29}$$

It follows from equation (23) that expected trading volume is predicted to increase monotonically with seniority. Intuitively, expected percentage uninformed trading losses are lower on more senior claims, so uninformed trading volume increases endogenously for claims high in the priority structure.

Consider next the predicted relationship between seniority and secondary market price impact. One proxy for liquidity in the model is λ, expected percentage underpricing as computed by an uninformed investor. As discussed above, it follows from equation (23) that expected percentage underpricing is lower for more senior claims.

Another measure of liquidity is the price-order flow relationship. We recall from Table 1 that for each security there are three possible order flows. From Table 1 it follows that the respective price of tranche n under, call it P_n, under high, medium and low order flows is given by:

$$\begin{aligned} P_n^H &= \sigma\overline{\mu}_n + (1-\sigma)\underline{\mu}_n \\ P_n^M &= (1-\sigma - \psi + 2\sigma\psi)\overline{\mu}_a + (\sigma + \psi - 2\sigma\psi)\underline{\mu}_a \\ P_n^L &= (1-\sigma)\overline{\mu}_a + \sigma\underline{\mu}_a. \end{aligned}$$

The price change associated with a move from low to medium order flow, normalized by the change in order flow is:

$$\frac{P_n^M - P_n^L}{\overline{u}_n - \underline{u}_n} = \frac{\psi(2\sigma-1)(\overline{\mu}_n - \underline{\mu}_n)}{\overline{u}_n - \underline{u}_n} = \frac{\psi(2\sigma-1)(\overline{\mu}_n - \underline{\mu}_n)}{(\overline{\gamma} - \underline{\gamma}) \sum_{j=1}^{J} \chi_{c_j \geq \lambda_n}}. \tag{30}$$

A normalized measure of price impact can be obtained by dividing the preceding price-flow slope by the average price. We obtain:

$$\frac{(P_n^M - P_n^L)/(\overline{u}_n - \underline{u}_n)}{\mathbb{E}[P]} \quad = \quad 2\left[\frac{\psi\overline{\gamma} + (1-\psi)\underline{\gamma}}{(1-\psi)(\overline{\gamma} - \underline{\gamma})}\right]\left[\sum_{j=1}^{J} \chi_{c_j \geq \lambda_n}\right]^{-1} \lambda_n. \tag{31}$$

It follow from the preceding equation that normalized price impact is predicted to increase monotonically with the detachment point. This is due to two effects. First, the expected percentage uninformed trading loss is higher for lower tranches, since the incentive constraints are slack with $c_{n-1} < \lambda_n < c_n$. Second, as captured by the penultimate term, trading volume is declining with the tranche detachment point since uninformed investors are less willing to liquidate their positions in light of such high expected trading losses.

Consider next the price-flow relationship associated with a move from medium to higher order flow. Again, a normalized measure of price impact can be obtained by dividing the price-flow slope by the average price. We obtain:

$$\frac{(P_n^H - P_n^M)/(\overline{u}_n - \underline{u}_n)}{\mathbb{E}[P]} = 2\left[\frac{\psi\overline{\gamma} + (1-\psi)\underline{\gamma}}{\psi(\overline{\gamma} - \underline{\gamma})}\right]\left[\sum_{j=1}^{J} \chi_{c_j \geq \lambda_n}\right]^{-1} \lambda_n. \tag{32}$$

Once again we see that normalized price impact is predicted to increase monotonically with the tranche detachment point.

4 Empirical Tests

This section provides an empirical analysis that tests various implications of the presented model. We make use of a unique data set, covering all transactions of the ABS market provided to us by FINRA. This sample allows quantifying the liquidity for a wide range of structured securities and, thus, provides an ideal setting to compare the difference in liquidity of securities within and across various underlying asset pools. In particular, we first test whether senior tranches are more liquid than junior tranches. In a further analysis we test for a positive relationship between the numerical priority position of the most junior traded claim in a pool and that same claim's illiquidity.

4.1 Data Description

The opacity in the over-the-counter market of structured products certainly played a key role during the financial crisis. FINRA, thus, launched a project to improve the transparency of the US fixed-income structured product market. Since May 16, 2011 dealers and brokers have to report virtually all transactions to the Trade Reporting and Compliance Engine (TRACE). Our data set covers the initial phase of this transparency project and comprises all transactions and product specific information from May 2011 to June 2012. FINRA classifies the structured product market into three subsegments: Asset-Backed Securities (ABS), Collateralized Mortgage Obligations (CMO) and Mortgage-Back Securities (MBS). We focus on the ABS subsegment as the structure of these securities are most closely linked to the presented model.[10]

[10]The subsegment of MBS cover only pass-through securities with either implicit or explicit guarantees from government-sponsored enterprises and, thus, are not directly comparable to more risky securities without such guarantees. The CMO market consists of tranched securities that are often based on complex underlying pools, e.g. they may have more than just one loan group. See, for example, Friewald et al. (2015) and Ghent et al. (2014) for a detailed description of the structured product market.

ABS are created by bundling loans, such as automobile loans, credit card debt or real estate loans. In most cases, multiple securities are offered on a given portfolio. These tranches are all based on a single pool of underlying loans, but have different levels of risk. Consistent with the model payments are first distributed in a "waterfall" structure to the holders of the lowest-risk securities, and then sequentially to the holders of higher-risk securities, in order of seniority.

For these securities, we have all transactions available, comprising the price, volume, buy/sell side indicator and a coded identity of the dealer that stands behind the trade. Our security-related data are on a monthly basis and comprise the coupon, maturity, issuer, loan type and updated outstanding amount which reflects incurred losses and/or prepayments (i.e., it represents the current tranche size). For each tranche we know its *seniority class* (priority position) which is given as an integer number where 1 refers to the most senior tranche and higher numbers reflect more junior tranches. Given the detailed security description we are able to assign each security to its underlying pool and compute the total number of tranches within that pool. We obtain credit ratings from Standard & Poor's, Moodys and Fitch from Bloomberg and assign integer numbers to these ratings, e.g., in the notation of Standard & Poor's AAA=1, AA+=2, ..., C=21. For the empirical analysis we compute for each tranche its *detachment point* which we define as the ratio of the total size of all relatively more senior tranches to the total pool size.[11]

We apply various cleaning and filtering procedures on our transaction data set. For a detailed description of the filters we refer to Friewald et al. (2015). Based on our final data set, we estimate liquidity on a security-level using two different measures: transaction costs and trading volume. To determine the transaction costs, we use the round-trip cost measure by following Goldstein et al. (2007) and define it as the relative price difference (to the mid-price and given in basis points) for a given dealer, between buying a certain amount of a security and selling the same amount of this security within a particular time period. Note that a round-trip can consist of a either a single or a sequence of trades as long as the overall buy and sell amounts match. For our empirical analysis we include round-trips taking place within a time period of at most 30 days.[12] We emphasize that compared to other liquidity measures our round-trip cost measure reflects the cost of trading more accurately, since our data allow us to assign each single trade to a particular dealer. For a further discussion on the round-trip cost measure we refer the reader to, e.g., Friewald et al. (2015). The second liquidity measure that we use is the average trading volume per round-trip trade. We use monthly averages of both measures in our empirical analysis to ensure that we have the same sampling frequency as for the security-related information.

Table 2 shows the total number of monthly observations for the round-trip cost measure and the trading volume, the underlying tranches and pools for the time period between May 2011 and

[11]We use this definition in our empirical analysis for the sake of consistency with the seniority class. In our notation, the seniority class 1 has detachment point zero and represent the most senior security. Higher detachment points reflect more junior tranches.

[12]Thus, according to our definition, the observation, e.g., of a single buy- or sell transaction within the specified time period would not be sufficient to compute a round-trip cost.

June 2012. Overall, the sample covers 2193 pools for which we observe 5816 traded securities (tranches) with 18015 security/month observations. The pools are based on auto, credit card, mortgage/commercial real estate and other loans (e.g., student loans or boat leasing).

Table 3 provides descriptive statistics for our liquidity measures (round-trip cost measure and trading volume) and the basic characteristics of the traded ABS securities. On average, we observe round-trip costs of 45.8 bp and a trading volume per round-trip trade of $5.9 million. Both liquidity measures exhibit substantial variation, i.e., for the round-trip cost the 10%-quantile is around 1 bp and the 90%-quantile 115 bp. The corresponding figures for the trade volume are $0.1 and $13.6 million, respectively. The average pool has around 16 tranches with an inter-quartile range from 4 to 27. The average seniority class is 4.6, indicating that traded tranches tend to be more senior. Note that the most subordinated tranches are frequently retained and, thus, not offered to investors. Similarly, we find that the mean detachment point is 26.5%, i.e., only a quarter of the pool size, on average, is more senior compared to the traded tranche. The average credit rating corresponds to A+ (in the Standard and Poor's jargon) but for less than half of the observations credit information is available. The average tranche size is around $300 million with a coupon of 3.7% and maturity of 19.4 years, again showing considerable variations across observations.

4.2 Results

Using our liquidity measures we now test various implications that follow from the theoretical model. An important prediction of the model is that tranching – as part of the ABS design – is used to offer securities with differing levels of liquidity to the investors to serve their particular ex-post liquidity needs. The model predicts that a security's liquidity is increasing in its seniority. We first test this empirical prediction by comparing transaction costs and trading volumes across tranches of all underlying asset pools. In a second step, we control for the underlying asset pool by analyzing the liquidity differences of securities within the same pool.

Figure 1 shows the average round-trip costs for the different seniority classes (priority position) in Panel (a). We do not provide results beyond seniority class 18 as the corresponding tranches represent less than 1% of the total observations. In Panel (b) we plot round-trip costs for 20 equidistant buckets based on the detachment points where bucket 1 refers to the most senior tranches with detachment points between 0% and 5%. Both panels clearly show that more senior tranches tend to be more liquid, e.g., seniority class 1 in Panel (a) exhibits round-trip costs of around 30 bp whereas seniority classes beyond 15 have costs of around 150 bp. Note, that a seniority class of 1 also reflects pass-through securities, i.e., a security with just one single tranche. However, pass-through securities are a real exception in the ABS subsegment as we only observe 11 pass-throughs with around 40 observations in our sample. The average round-trip cost measure of these instruments is 32.8 bp and, thus, the liquidity of these securities is comparable to the level of liquidity of the most senior tranches, consistent with our theoretical predictions.

Figure 2 provides similar results when using the trading volume as a liquidity measure. Again, the results are consistent with the underlying theory. In particular, we find that the higher the tranche seniority (based either on the seniority class or detachment point) the higher the trading volume, which confirms our previous results. For example, for the detachment point bucket 1 we observe an average trading volume of $7 million whereas for bucket 20 the trading volume is roughly $3 million. Since both the round-trip cost measure and trading volumes share similar properties we only focus on the round-trip measure in the remaining part of our analysis.

In the next step of the analysis, we hold fix the underlying asset pool and, thus, compare the liquidity of ABS securities within the same pool. In doing so, we consider all possible combinations of tranche pairs of a given pool and month and compute the difference in their seniority class. For example, assume for a given pool in a given month that two tranches are traded, one having seniority class 3 and the other 7. Thus, the senior class difference of this pair is 4. We then compute for each such pair the difference of the round-trip costs between the more junior and the more senior tranche. Given our previous example, we would compute the difference in the round-trip costs of seniority class 7 and 3. Based on all observed pairs with a given seniority class difference we then compute the relative frequency with the more senior tranche being more liquid.

Table 4 reports the results for seniority class differences between 1 and 15 (again, higher differences represent less than 1% of the observations). The results show that within a given pool the more senior tranches tend to be more liquid, i.e., for a given pair the likelihood of observing the more senior tranche to be more liquid is always greater than 50%. For example, considering pairs with a difference in their seniority classes of 1 which refer to directly consecutive tranches, the frequency that the more senior tranche is more liquid amounts to 65%.[13] The average difference in their round-trip costs is approximately 11 bp. Moreover, we observe that the greater the difference in the seniority classes of a given pair the more likely it is that the more senior tranche is more liquid and the larger the difference in their round-trip cost measure. For example, given a difference in the seniority class of 15 we find that in 80% of the observations the more senior tranche is more liquid, with a round-trip cost difference of 115 bp.

As a final test of the model prediction regarding the relation between seniority and liquidity, we employ regression-based models using fixed pool effects throughout our analysis and report the results for different specifications in Table 5. We regress the round-trip cost measure on the seniority class in specification (1) and on the detachment point in specification (2). We find that liquidity significantly decreases with seniority class, i.e., on average, round-trip costs increase by 7.1 bp, going from one tranche to the next subordinated tranche. Similarly, increasing the detachment point by 10% (i.e., making the tranche more junior) increases round-trip costs by about 7.2 bp, on average. In specification (3) and (4) we use the security's coupon, maturity and tranche size as further explanatory variables, while additionally controlling for the securities' credit ratings. The relation between liquidity and tranche seniority remains statistically and economically significant.

[13]We expect this figure to be lower than the model's prediction of 100% due to market microstructure noise, such as asynchronous trading.

We also find that security characteristics are related to liquidity. For example, securities with a higher time-to-maturity tend to be less liquid while the larger the tranche size the lower the round-trip costs.

We close our empirical analysis by testing a novel model prediction. Recall, the model predicts that more information-sensitive underlying cash flow streams should be packaged into more tranches. Consistent with this prediction, we have seen that pass-through securities in our sample have high liquidity. That is, issuers apparently package low information-sensitivity cash flow streams as pass-throughs.

We would like to conduct a more general test of the prediction that high information-sensitivity of the underlying cash flow causes issuers to choose a higher number of tranches. Unfortunately, information-sensitivity is not directly observable. However, from Proposition 5 we know that across pools there should be a positive relationship between the illiquidity of the most junior traded tranche and its numerical seniority class. For example, if the underlying cash pool has high information-sensitivity, the most junior traded tranche will be highly illiquid and this same tranche will have a high numerical seniority class. That is, high information-sensitivity causes both high illiquidity of claims and a large number of tranches. And we recall that if our theory did not hold, an issuer could package a cash flow stream with very low information-sensitivity into many tranches with even the most junior traded security being very liquid, in which case one would observe the conjunction of low illiquidity but a high number of traded tranches.

In Figure 3, the round-trip cost of the most junior traded tranche is on the horizontal axis, and the numerical seniority class of that same security is on the vertical axis. Consistent with the theory, the number of tranches does indeed appear to increase with our proxy for underlying cash flow information-sensitivity. Table 6 contains the corresponding regression analysis. The dependent variable is the numerical seniority class of the most junior traded claim, with the key explanatory variable being the illiquidity (round-trip cost) of this same claim. We here see that controlling for coupon, maturity, tranche size and credit rating, the information-sensitivity proxy helps predict the number of tranches.

5 Conclusion

This paper presents a tractable theoretical framework for analyzing adverse selection discounts in competitive securities markets, accounting for strategic secondary market trading by uninformed investors. With discretionary liquidation decisions, the adverse selection cost borne by an uninformed investor is equal to the minimum of expected trading losses and carrying costs. We consider how an issuer can package securities in order to minimize adverse selection discounts, accounting for strategic trading. Security design is shown to complement strategic trading. An issuer minimizes adverse selection discounts by splitting cash flow into tranched debt claims. This offers the uninformed investors a rich set of exit options. In the proposed theory, liquidity is increasing in seniority. In contrast with existing empirical work that focuses on corporate debt and equity, we present empirical evidence from the ABS market consistent with notion that liquidity increases in seniority.

Our paper can be understood as a natural extension of a research program initiated by Spiegel and Subrahmanyam (1992), who considered strategic trading in a Kyle-type framework, but ignored the issue of optimal structuring. The analysis suggests it may be productive to move microstructure-based models of corporate finance away from assuming the uninformed do not trade strategically. Indeed, novel and interesting theories of corporate finance may emerge if one considers that all agents optimize.

A Appendix: Proofs and Derivations

Derivation of Objective Function

The objective is to minimize:

$$(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})\sum_{k=1}^{J}(\overline{\mu}_k-\underline{\mu}_k)\sum_{j=1}^{J}\Phi_k^j+[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]\sum_{k=1}^{J}(\overline{\mu}_k+\underline{\mu}_k)\sum_{j=1}^{J}(1-\Phi_k^j)c_j \tag{33}$$
$$+[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}](\overline{\mu}_0+\underline{\mu}_0)\sum_{j=1}^{J}c_j.$$

Expanding the second term in the preceding equation and grouping terms, we can say that the objective is to minimize:

$$(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})\sum_{k=1}^{J}(\overline{\mu}_k-\underline{\mu}_k)\sum_{j=1}^{J}\Phi_k^j-[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]\sum_{k=1}^{J}(\overline{\mu}_k+\underline{\mu}_k)\sum_{j=1}^{J}\Phi_k^jc_j \tag{34}$$
$$+[\psi\overline{\gamma}+(1-\psi)\underline{\gamma}](\overline{\mu}_x+\underline{\mu}_x)\sum_{j=1}^{J}c_j.$$

Dropping the final term in the preceding equation, which is a constant, and multiplying by -1, the objective is to maximize:

$$\sum_{k=1}^{J}\sum_{j=1}^{J}\Phi_k^j[(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j(\overline{\mu}_k+\underline{\mu}_k)-(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})(\overline{\mu}_k-\underline{\mu}_k)] \tag{35}$$
$$=\ [\psi\overline{\gamma}+(1-\psi)\underline{\gamma}]\sum_{k=1}^{J}\sum_{j=1}^{J}(\overline{\mu}_k+\underline{\mu}_k)(c_j-\lambda_k)^+.$$

We express the maximand concisely as:

$$\sum_{k=1}^{J}\left(\sum_{j=k}^{J}\underline{\kappa}^j\right)\underline{\mu}_k-\sum_{k=1}^{J}\left(\sum_{j=k}^{J}\overline{\kappa}^j\right)\overline{\mu}_k \tag{36}$$
$$\underline{\kappa}^j\ \equiv\ (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})+(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j$$
$$\overline{\kappa}^j\ \equiv\ (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})-(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j.$$

Next consider that we can rewrite the expected payoff on any security as follows using integration by parts:

$$\begin{aligned}
& \int_{\underline{x}}^{\overline{x}} a(x)f(x)dx && (37)\\
= & \int_0^{\overline{x}} a(x)f(x)dx \\
= & \; a(\overline{x}) - \int_0^{\overline{x}} \delta(x)[F(x)-1+1]dx \\
= & \; a(\overline{x}) + \int_0^{\overline{x}} \delta(x)[1-F(x)]dx - \int_0^{\overline{x}} \delta(x)dx \\
= & \; a(0) + \int_0^{\overline{x}} \delta(x)[1-F(x)]dx \\
= & \int_0^{\overline{x}} \delta(x)[1-F(x)]dx.
\end{aligned}$$

Thus, we can rewrite the maximand as:

$$\begin{aligned}
& \sum_{k=1}^{J}\left(\sum_{j=k}^{J}\underline{\kappa}^j\right)\int_0^{\overline{x}}\delta_k(x)[1-\underline{F}(x)]dx - \sum_{k=1}^{J}\left(\sum_{j=k}^{J}\overline{\kappa}^j\right)\int_0^{\overline{x}}\delta_k(x)[1-\overline{F}(x)]dx && (38)\\
= & \sum_{k=1}^{J}\int_0^{\overline{x}}\left[\left(\sum_{j=k}^{J}\underline{\kappa}^j\right)(1-\underline{F}(x)) - \left(\sum_{j=k}^{J}\overline{\kappa}^j\right)(1-\overline{F}(x))\right]\delta_k(x)dx \\
= & \sum_{k=1}^{J}\int_0^{\overline{x}}\Omega_k(x)\delta_k(x)dx.
\end{aligned}$$

Lemma 2. *Necessary and sufficient conditions for $\Omega_h(x)$ to be positive and maximal amongst all $\Omega_j(x)$ for $j \in \{1,\dots,J\}$ are*

$$\begin{aligned}
[1-\overline{F}(x)]\overline{\kappa}^{h-1} &\geq [1-\underline{F}(x)]\underline{\kappa}^{h-1} \\
& \textit{and} \\
[1-\overline{F}(x)]\overline{\kappa}^{h} &\leq [1-\underline{F}(x)]\underline{\kappa}^{h}.
\end{aligned}$$

Proof: To begin, we note the following equalities.

$$\begin{aligned}
A1 &: \Omega_j(x) = \Omega_{j-1}(x) + [(1-\overline{F}(x))\overline{\kappa}^{j-1} - (1-\underline{F}(x))\underline{\kappa}^{j-1}]; \quad j \in \{2,\dots,J\} \\
A2 &: \Omega_{j+1}(x) = \Omega_j(x) + [(1-\overline{F}(x))\overline{\kappa}^{j} - (1-\underline{F}(x))\underline{\kappa}^{j}]; \quad j \in \{1,\dots,J-1\} \\
A3 &: \Omega_J(x) = -[(1-\overline{F}(x))\overline{\kappa}^{J} - (1-\underline{F}(x))\underline{\kappa}^{J}].
\end{aligned}$$

Further, note that the term in square-brackets in A1 and A2 decreases strictly with the index j.

Consider first the case in which the stated conditions are met for $h \in \{2,\dots,J-1\}$. From A1 we have:

$$\begin{aligned} [1-\overline{F}(x)]\overline{\kappa}^{h-1} &\geq [1-\underline{F}(x)]\underline{\kappa}^{h-1} \\ &\Updownarrow \\ \Omega_h(x) &\geq \Omega_{h-1}(x) \\ &\Downarrow \\ \Omega_j(x) &> \Omega_{j-1}(x) : j < h \end{aligned} \tag{39}$$

From A2 we have:

$$\begin{aligned} [1-\overline{F}(x)]\overline{\kappa}^{h} &\leq [1-\underline{F}(x)]\underline{\kappa}^{h} \\ &\Updownarrow \\ \Omega_h(x) &\geq \Omega_{h+1}(x) \\ &\Downarrow \\ \Omega_j(x) &> \Omega_{j+1}(x) : j > h. \end{aligned} \tag{40}$$

And finally, from A3 we have:

$$[1-\overline{F}(x)]\overline{\kappa}^{h} \leq [1-\underline{F}(x)]\underline{\kappa}^{h} \Rightarrow [1-\overline{F}(x)]\overline{\kappa}^{J} < [1-\underline{F}(x)]\underline{\kappa}^{J} \Leftrightarrow \Omega_J(x) > 0. \tag{41}$$

Consider next the case in which the stated conditions are met for $h = 1$. From A2 we have:

$$\begin{aligned} [1-\overline{F}(x)]\overline{\kappa}^{1} &\leq [1-\underline{F}(x)]\underline{\kappa}^{1} \\ &\Updownarrow \\ \Omega_1(x) &\geq \Omega_2(x) \\ &\Downarrow \\ \Omega_j(x) &> \Omega_{j+1}(x) : j > 1. \end{aligned} \tag{42}$$

And from A3 we have:

$$[1-\overline{F}(x)]\overline{\kappa}^{1} \leq [1-\underline{F}(x)]\underline{\kappa}^{1} \Rightarrow [1-\overline{F}(x)]\overline{\kappa}^{J} < [1-\underline{F}(x)]\underline{\kappa}^{J} \Leftrightarrow \Omega_J(x) > 0. \tag{43}$$

Consider finally the case in which the stated conditions are met for $h = J$. From A1 we have:

$$\begin{aligned} [1-\overline{F}(x)]\overline{\kappa}^{J-1} &\geq [1-\underline{F}(x)]\underline{\kappa}^{J-1} \\ &\Updownarrow \\ \Omega_J(x) &\geq \Omega_{J-1}(x) \\ &\Downarrow \\ \Omega_j(x) &> \Omega_{j-1}(x) : j < J. \end{aligned} \tag{44}$$

And from A3 we have:

$$[1-\overline{F}(x)]\overline{\kappa}^{J} \leq [1-\underline{F}(x)]\underline{\kappa}^{J} \Leftrightarrow \Omega_J(x) \geq 0.\blacksquare \tag{45}$$

Proposition 3

Under the first condition stipulated in the proposition, it follows from Lemma **2** that $\Omega_1(x)$ is maximal and positive for all x and so it is optimal to assign all cash flow to Security 1. For the second part of the proposition, suppose k is maximal amongst $j \in \{1, \ldots, J\}$ satisfying:

$$\left[\frac{\overline{f}(\overline{x})}{\underline{f}(\overline{x})}\right]\overline{\kappa}^{j} > \underline{\kappa}^{j}.$$

Then it follows from Lemma **2** that for each $j \in \{1, .., k\}$ it is optimal to cease assigning incremental cash flows to Security j and to begin assigning them to Security $j+1$ at θ_j such that:

$$[1-\overline{F}(\theta_j)]\overline{\kappa}^{j} = [1-\underline{F}(\theta_j)]\underline{\kappa}^{j}.$$

If $k = J$, the final residual security is the fully illiquid Security 0.

Incentive Compatibility

As a final step, we must verify that each security is sold and retained by the posited categories of investors. The first incentive condition is that investors with carrying cost c_j (and higher) are willing to sell security j. We demand:

$$\begin{aligned}
\underbrace{\frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{\mu}_j-\underline{\mu}_j}{\overline{\mu}_j+\underline{\mu}_j}}_{\lambda_j} &\leq c_j \qquad (46)\\
(\psi\overline{\gamma}+(1-\psi)\underline{\gamma})c_j(\overline{\mu}_j+\underline{\mu}_j) &\geq (2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})(\overline{\mu}_j-\underline{\mu}_j)\\
\underline{\kappa}^j\underline{\mu}_j &\geq \overline{\kappa}^j\overline{\mu}_j\\
\underline{\kappa}^j\int_0^{\overline{x}}\delta_j(x)(1-\underline{F}(x))dx &\geq \overline{\kappa}^j\int_0^{\overline{x}}\delta_j(x)(1-\overline{F}(x))dx\\
\int_0^{\overline{x}}[\underline{\kappa}^j(1-\underline{F}(x))-\overline{\kappa}^j(1-\overline{F}(x))]\delta_j(x)dx &\geq 0
\end{aligned}$$

From Lemma 2 the term in square brackets in the preceding integrand is positive if $\delta_j(x)\neq 0$.

The second incentive is that investors with carrying cost c_{j-1} (and lower) do not want to sell security j. Following the same steps as above, we demand:

$$\begin{aligned}
\frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{\mu}_j-\underline{\mu}_j}{\overline{\mu}_j+\underline{\mu}_j} &\geq c_{j-1}\\
\int_0^{\overline{x}}\left[\underline{\kappa}^{j-1}(1-\underline{F}(x))-\overline{\kappa}^{j-1}(1-\overline{F}(x))\right]\delta_j(x) &\leq 0.
\end{aligned}$$

From Lemma 2 the term in square brackets in the preceding integrand is negative if $\delta_j(x)\neq 0$.

The final incentive condition is that none of the investors want to sell security 0 should that security be issued. To verify that this is the case, it is sufficient to check that even the investor with the highest carrying cost does not want to sell that security. We then demand:

$$\begin{aligned}
\frac{(2\sigma-1)\psi(1-\psi)(\overline{\gamma}-\underline{\gamma})}{\psi\overline{\gamma}+(1-\psi)\underline{\gamma}}\frac{\overline{\mu}_0-\underline{\mu}_0}{\overline{\mu}_0+\underline{\mu}_0} &\geq c_J\\
\int_0^{\overline{x}}\left[\underline{\kappa}^{J}(1-\underline{F}(x))-\overline{\kappa}^{J}(1-\overline{F}(x))\right]\delta_0(x) &\leq 0.
\end{aligned}$$

And we recall, Security 0 gets incremental cash flow only if the preceding integrand is negative, since:

$$\Omega_J(x)\leq 0 \Leftrightarrow 1\leq \frac{1-\overline{F}(x)}{1-\underline{F}(x)}\frac{\overline{\kappa}^J}{\underline{\kappa}^J}.\blacksquare$$

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State	Signal	S Order	UI Order	Total Order	Probability	LP Belief
$\overline{\omega}$	$\overline{\omega}$	0	$-\underline{u}$	$-\underline{u}$	$\frac{\sigma(1-\psi)}{2}$	σ
$\overline{\omega}$	$\overline{\omega}$	0	$-\overline{u}$	$-\overline{u}$	$\frac{\sigma\psi}{2}$	$1-\sigma-\psi+2\sigma\psi$
$\overline{\omega}$	$\underline{\omega}$	$-(\overline{u}-\underline{u})$	$-\underline{u}$	$-\overline{u}$	$\frac{(1-\sigma)(1-\psi)}{2}$	$1-\sigma-\psi+2\sigma\psi$
$\overline{\omega}$	$\underline{\omega}$	$-(\overline{u}-\underline{u})$	$-\overline{u}$	$-(2\overline{u}-\underline{u})$	$\frac{(1-\sigma)\psi}{2}$	$1-\sigma$
$\underline{\omega}$	$\underline{\omega}$	$-(\overline{u}-\underline{u})$	$-\underline{u}$	$-\overline{u}$	$\frac{\sigma(1-\psi)}{2}$	$1-\sigma-\psi+2\sigma\psi$
$\underline{\omega}$	$\underline{\omega}$	$-(\overline{u}-\underline{u})$	$-\overline{u}$	$-(2\overline{u}-\underline{u})$	$\frac{\sigma\psi}{2}$	$1-\sigma$
$\underline{\omega}$	$\overline{\omega}$	0	$-\underline{u}$	$-\underline{u}$	$\frac{(1-\sigma)(1-\psi)}{2}$	σ
$\underline{\omega}$	$\overline{\omega}$	0	$-\overline{u}$	$-\overline{u}$	$\frac{(1-\sigma)\psi}{2}$	$1-\sigma-\psi+2\sigma\psi$

Table 1: Order Flow Possibilities. This table lists secondary market order flow possibilities and liquidity provider beliefs in responds to those order flows.

	ABS All	Auto	Card	Real Estate	Other
Observations	18015	3050	396	9520	5049
Tranches	5816	633	85	2798	2300
Pools	2193	236	58	751	1148

Table 2: Sample Characteristics. This table reports the number of monthly observations of round-trip cost measures and trading volumes for ABS securities, the underlying number of traded tranches (securities) and pools for the time period between May 2011 and June 2012.

	$Q_{0.10}$	$Q_{0.25}$	Mean	$Q_{0.75}$	$Q_{0.90}$	N
Round-trip costs (bp)	0.78	4.37	45.84	46.29	114.99	18015
Trading volume (millions)	0.11	0.48	5.87	5.80	13.57	18015
Number of tranches in pool	4.00	6.00	16.40	27.00	30.00	18015
Seniority class	1.00	2.00	4.58	6.00	8.00	18015
Detachment point (%)	0.00	0.00	26.46	44.76	83.09	18015
Coupon (%)	0.64	1.25	3.65	5.45	5.92	18015
Maturity (years)	2.85	5.55	19.35	30.96	35.85	18015
Tranche size (millions)	22.32	59.26	311.90	408.10	783.74	18015
Credit rating	1.00	1.00	4.97	7.00	15.50	8341

Table 3: Descriptive Statistics. This table reports the 10%, 25%, 75%, 90% quantiles and the mean of the round-trip cost measure, trading volume and basic characteristics of ABS securities in our sample. The characteristics provide the total number of tranches within the corresponding pool, seniority class (where 1 refers to the most senior tranche and higher numbers to more junior tranches), detachment point (defined as the ratio of the total size of the relatively more senior tranches to the total pool size), coupon, maturity, tranche size and the security's credit rating (AAA=1,..., C=21).

Difference in seniority class	Observations	Frequency of more senior tranche to be more liquid (in %)	Difference in round-trip costs (bp)
1	3500	64.89	11.35
2	2181	66.44	18.28
3	1602	67.67	28.40
4	975	67.38	31.92
5	698	67.19	46.33
6	472	72.03	56.29
7	294	70.41	69.96
8	192	73.96	101.72
9	140	72.14	85.65
10	106	66.98	112.70
11	77	74.03	102.07
12	58	74.14	130.32
13	34	67.65	107.22
14	31	83.87	140.09
15	15	80.00	114.77

Table 4: Pairwise Liquidity Comparisons. This table reports for all possible combinations of tranche pairs of a given pool and month the difference in their seniority class, the total number of pairs, the average frequency of observing the more senior tranche to be more liquid and the average difference in their round-trip costs.

	(1)	(2)	(3)	(4)
Seniority class	7.056***		4.332***	
	(17.554)		(9.647)	
Detachment point		71.965***		45.050***
		(24.599)		(12.010)
Coupon			−0.550	−3.064***
			(−0.547)	(−2.824)
Maturity			3.297***	2.240***
			(5.608)	(3.624)
Tranche size			−0.011***	−0.003
			(−4.627)	(−1.296)
Fixed effect (Pool)	*Y*	*Y*	*Y*	*Y*
Credit ratings	*N*	*N*	*Y*	*Y*
Adj. R^2	0.052	0.050	0.102	0.100
Num. obs.	18015	18015	18015	18015

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

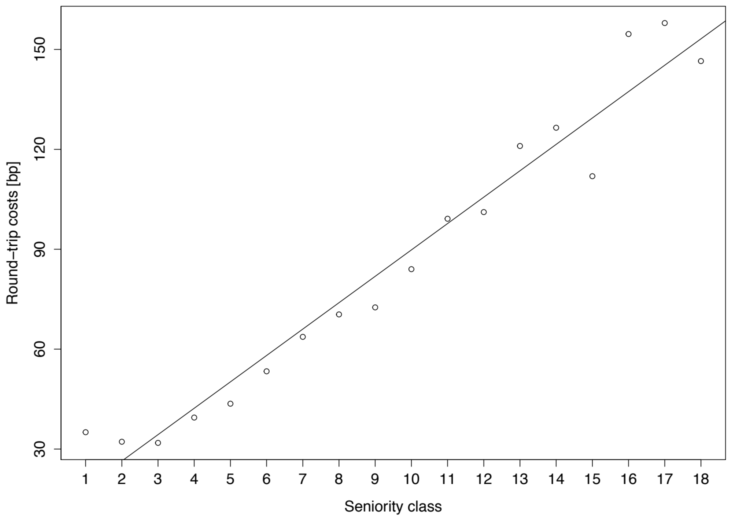
Table 5: Regression Analysis of Liquidity Determinants. This table reports the results of regressing the round-trip cost measure on the seniority class in specification (1) and the detachment point in (2), respectively. In specification (3) and (4) we use the security's coupon, maturity and tranche size as further explanatory variables, while additionally controlling for credit rating. We report t-statistics based on robust standard errors in parenthesis, correcting for heteroskedasticity.

	(1)	(2)
Intercept	4.752***	2.191***
	(66.688)	(15.891)
Round-trip costs	0.005***	0.002***
	(8.305)	(3.679)
Coupon		0.281***
		(10.492)
Maturity		0.099***
		(12.040)
Tranche size		0.001***
		(4.485)
Credit ratings	*N*	*Y*
Adj. R^2	0.021	0.182
Num. obs.	4782	4782

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Table 6: Information-Sensitivity and Number of Tranches. This table reports the results of regressing the seniority class of the most subordinated tranche of each pool on the round-trip cost measure in specification (1). In specification (2) we use the security's coupon, maturity and tranche size as further explanatory variables, while additionally controlling for credit rating. We report t-statistics based on robust standard errors in parenthesis, correcting for heteroskedasticity.

(a) Transaction costs and seniority classes.



(b) Transaction costs and detachment points.

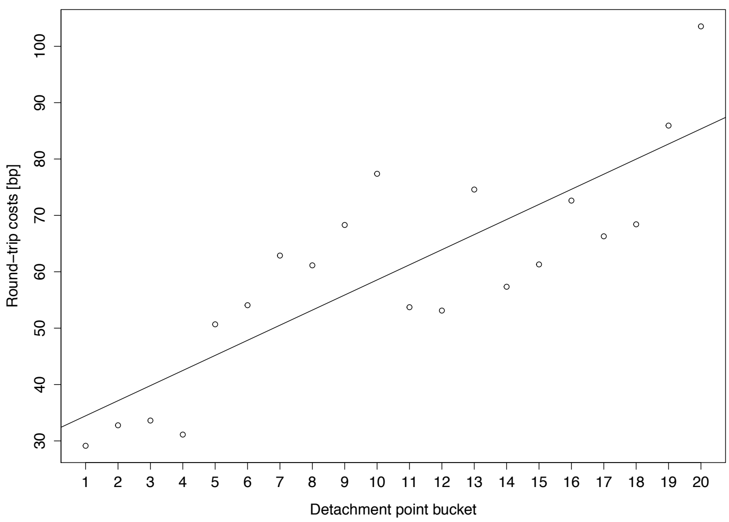
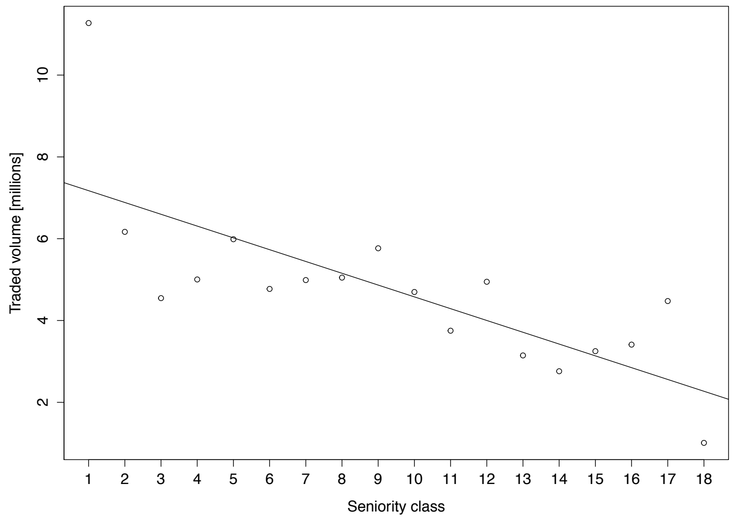


Figure 1: Round-trip Costs and Seniority. This figure shows the average round-trip cost measure for different seniority classes in Panel (a) and for equidistant buckets based on the detachment points in Panel (b) for ABS securities. A seniority class of 1 refers to the most senior tranche and higher numbers to more junior tranches. We define the detachment point for each tranche as the ratio of the total size of all relatively more senior tranches to the overall size of the pool, e.g., the detachment point of the most senior tranche is zero.

(a) Trading volume and seniority classes.



(b) Trading volume and detachment points.

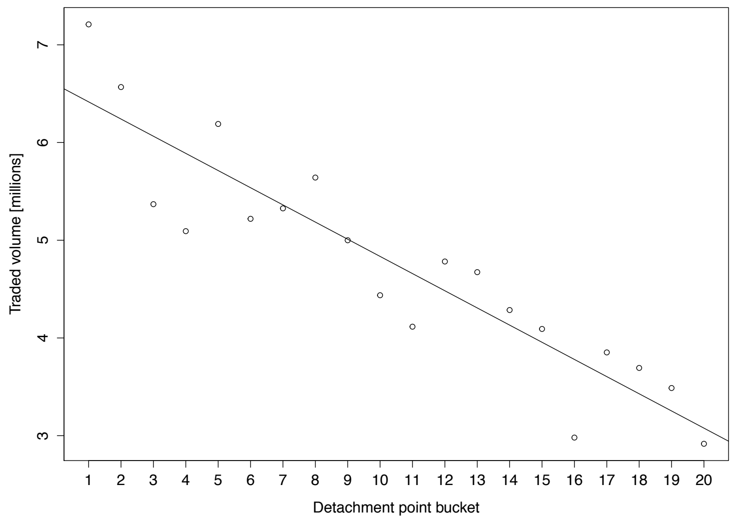


Figure 2: Trading Volumes and Seniority. This figure shows the average trading volume per round-trip for different seniority classes in Panel (a) and for equidistant buckets based on the detachment points in Panel (b) for ABS securities. A seniority class of 1 refers to the most senior tranche and higher numbers to more junior tranches. We define the detachment point for each tranche as the ratio of the total size of all relatively more senior tranches to the overall size of the pool, e.g., the detachment point of the most senior tranche is zero.

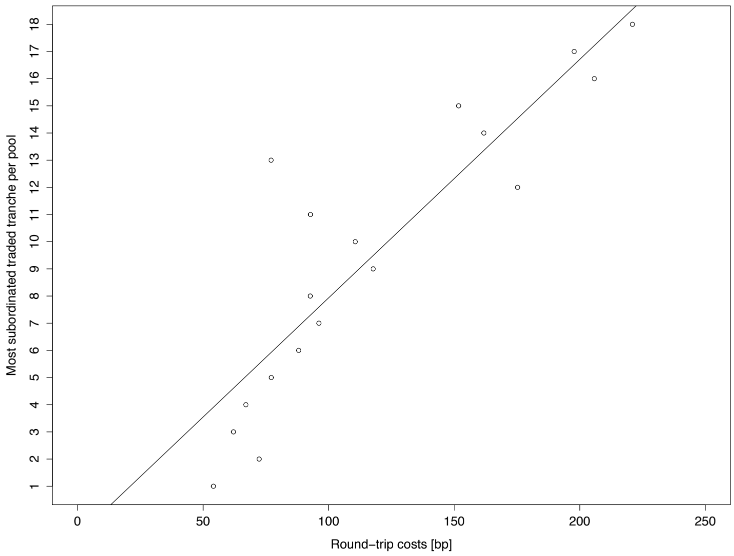


Figure 3: Information-Sensitivity and Number of Tranches. This figure shows the seniority class (priority position) of the most junior traded tranche of each pool and the average round-trip cost of these tranches.