

1 **Observation of a large lee wave in the Drake Passage**

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ABSTRACT

11 Lee waves are thought to play a prominent role in Southern Ocean dynamics,
12 facilitating a transfer of energy from the jets of the Antarctic Circumpolar Cur-
13 rent to microscale turbulent motions important in water mass transformations.
14 Two EM-APEX profiling floats deployed in the Drake Passage during the Di-
15 apycnal and Isopycnal Mixing Experiment (DIMES) independently measured
16 a 120 ± 20 m vertical amplitude lee wave over the Shackleton Fracture Zone.
17 A model for steady EM-APEX motion is developed to calculate absolute ver-
18 tical water velocity, augmenting the horizontal velocity measurements made
19 by the floats. The wave exhibits fluctuations in all three velocity components
20 of over 15 cm s^{-1} , and an intrinsic frequency close to the local buoyancy
21 frequency. The wave is observed to transport energy and horizontal momen-
22 tum vertically at respective peak rates of $1.3 \pm 0.2 \text{ W m}^{-2}$ and $8 \pm 1 \text{ N m}^{-2}$.
23 The rate of turbulent kinetic energy dissipation is estimated using both Thorpe
24 scales and a method that isolates high-frequency vertical kinetic energy and is
25 found to be enhanced within the wave to values of order $10^{-7} \text{ W kg}^{-1}$. The
26 observed vertical flux of energy is significantly larger than expected from ide-
27 alised numerical simulations, and also larger than observed depth integrated
28 dissipation rates. These results provide the first unambiguous observation of
29 a lee wave in the Southern Ocean with simultaneous measurements of its en-
30 ergetics and dynamics.

31 **1. Introduction**

32 Lee waves can be generally defined as internal gravity waves generated by the interaction of a
33 quasi-steady stratified flow with topography. Observations of such phenomena in the ocean are
34 rare, with notable examples including high frequency, tidally forced waves in the lee of ridges
35 (e.g. Pinkel et al. 2012; Alford et al. 2014). Propagating waves must have a frequency between
36 the local inertial frequency, f , and buoyancy frequency, N , which precludes their generation in
37 many regions of the ocean where bottom flows are not sufficiently strong and topography is not
38 of the correct scale to excite such a frequency. Global maps of energy input to lee waves from
39 geostrophic flows (Scott et al. 2011; Nikurashin and Ferrari 2011) highlight the importance of the
40 Southern Ocean because it contains many regions that meet the dynamical requirements, usually
41 centred on ridges and fracture zones such as Phoenix Ridge and the Shackleton Fracture Zone in
42 Drake Passage. Lee waves extract energy and horizontal momentum from the forcing flow, and can
43 transport them both vertically and horizontally, redistributing them throughout the water column
44 via nonlinear interactions with other waves, the large-scale flow or instabilities that result in wave
45 breaking (e.g. Munk 1980). Lee waves have garnered growing interest in recent years, as efforts
46 have been made to understand the origins of small-scale turbulence and its role in returning dense
47 waters to the upper layers of the ocean as part of the global overturning circulation (Talley 2013;
48 Waterhouse et al. 2014).

49 Turbulent kinetic energy dissipation and mixing are consistently found to be enhanced over re-
50 gions of rough bathymetry, using a variety of measurement techniques including tracer releases
51 and microstructure profiles (Ledwell et al. 2000; Watson et al. 2013). The presence of lee waves
52 in these regions is usually inferred from finescale (order 100 m) measurements of variance in ve-
53 locity shear and isopycnal strain, which show a predominance of upward-travelling wave energy

54 (Naveira Garabato et al. 2004; Kunze et al. 2006; Waterman et al. 2013; Sheen et al. 2013) indica-
55 tive of bottom generation. In addition, a more limited number of microstructure profiles indicates
56 that turbulent kinetic energy dissipation is enhanced within ~ 1 km of the ocean floor over to-
57 pography (St. Laurent et al. 2012; Sheen et al. 2013). Shear and strain based parameterisation
58 methods (e.g. Polzin et al. 2014) are also used to estimate dissipation rates, and while there is cur-
59 rently an unresolved quantitative discrepancy between these results and those from microstructure
60 (Hibiya et al. 2012; Waterman et al. 2014), the qualitative picture of bottom-enhanced dissipation
61 is robust. The inference from this range of observations is that lee waves are generated over rough
62 bathymetry and eventually break, causing turbulence in the vicinity of the topography. However,
63 this picture remains open to alternative interpretations, as the unambiguous observation of lee
64 waves in the Southern Ocean has remained elusive.

65 It has been appreciated in the atmospheric literature that lee waves, or mountain waves, play
66 an important role in the momentum budget and influence aspects of the general circulation (e.g.
67 Fritts 2003) and that the results of general circulation models are improved when their effects are
68 accounted for (McFarlane 1987). The dominant momentum balance in the Antarctic Circumpolar
69 Current (ACC) is between wind stress at the surface and form stress across large bathymetric
70 features, such as ridges, on scales of 1000 km (Vallis 2006). Further, recent work estimating the
71 lee wave drag on the geostrophic flow from an application of wave radiation theory suggests that
72 regions of the ACC with rough bathymetry of the required lateral scale to excite waves (1 – 10
73 km) may add a non-negligible wave drag to the momentum balance (Naveira Garabato et al. 2013).
74 Direct measurements of lee wave momentum fluxes and convergence in the Southern Ocean are
75 required to test this hypothesis. The results would have implications for numerical models that
76 do not resolve small-scale topography and internal waves, since their effect on the momentum
77 balance would need to be parameterised.

78 In this paper, we document the first observations of a lee wave in the Southern Ocean and de-
79 termine its properties, fluxes of energy and horizontal momentum, and turbulent kinetic energy
80 dissipation levels. The observations were obtained with two Electromagnetic Autonomous Profil-
81 ing Explorer (EM-APEX) floats deployed in Drake Passage under the auspices of the Diapycnal
82 and Isopycnal Mixing Experiment in the Southern Ocean (DIMES), a U.S. - U.K. program to
83 investigate mixing processes in the ACC (Gille et al. 2007). Previous investigations of internal
84 waves using EM-APEX floats have focussed on diagnosing near-inertial waves, which oscillate
85 with a time period of approximately 14 hours at 57° S, significantly longer than the time it takes to
86 profile (Kilbourne and Garton 2015; Meyer et al. 2016). Here, we focus on the measurement of a
87 near-buoyancy frequency wave with a period close to 1 hour in a frame of reference moving with
88 the mean flow. This has presented new challenges in analysis because time-dependence cannot be
89 neglected. Several methods for estimating vertical water velocity and turbulent kinetic energy dis-
90 sipation are adapted and applied to the measurements, allowing almost complete characterisation
91 of the wave in terms of frequency, wavelength, momentum flux, energy flux and dissipation rate.

92 A description of the floats and data sampling strategy is provided in Section 2, which also in-
93 cludes an assessment of a theoretical model of profiling float motion used to calculate absolute
94 vertical water velocity. In Section 3, the float measurements are used to characterise the observed
95 lee wave, and estimate its associated fluxes of energy and momentum and turbulent dissipation
96 rates. A discussion of the significance of our findings for the emerging picture of the role of lee
97 waves in the Southern Ocean circulation is offered in Section 4, followed by concluding remarks.

98 **2. Data and Methods**

99 *a. Instrumentation and sampling strategy*

100 The primary observations of this work were obtained by two EM-APEX floats, numbered 4976
101 and 4977, deployed at the same time and position in the Drake Passage from the *RSS James*
102 *Cook* (68° 11' 1.4" W, 57° 34' 14.9" S) on 31 December 2010 at 12:18 UTC. Float trajectories
103 are displayed in Figure 1. EM-APEX floats, described in greater detail by Sanford et al. (2005),
104 are modified APEX floats that were developed at the Applied Physics Laboratory, University of
105 Washington, in collaboration with Teledyne Webb Research Corporation. Electrodes on the outer
106 casing measure the potential difference across the instrument induced by the motion of the ocean
107 through the vertical component of the Earth's magnetic field (Sanford 1971). This information,
108 along with measurements of instrument tilt and magnetic compass heading, is used to calculate
109 relative horizontal water velocity with a characteristic precision of 1 cm s⁻¹. Relative velocity
110 is converted to absolute velocity by using surface GPS positions to estimate a depth-independent
111 constant offset. The floats are also equipped with a Seabird Electronics SBE-41 pumped CTD.

112 Using a piston to pump oil into and out of an external bladder, the floats were programmed to
113 change their buoyancy in such a way as to maintain an approximately constant vertical speed of
114 12 cm s⁻¹. The position of the piston was recorded and transmitted along with measurements
115 from the EM system, CTD and GPS position via Iridium telecommunication satellites while at the
116 surface. The sampling frequency varied but on average CTD measurements were made every 20 s
117 or 2.5 m, while EM measurements were made every 25 s or 3 m. Both floats analysed here were
118 programmed to profile continuously to 1500 dbar, taking about 3.5 hours to complete an ascent or
119 descent, pausing only while at the surface for an average of 30 minutes to transmit data.

120 *b. Derived variables*

121 Analysis was performed on several variables not directly observed by the floats, and their deriva-
122 tion is described here briefly. Relative horizontal velocity measurements were converted to abso-
123 lute horizontal velocity using the method described by Phillips and Bindoff (2014). In summary,
124 the relative horizontal velocity measured from a descent / ascent profile pair is integrated with
125 respect to time, providing a displacement estimate. The difference between this displacement and
126 the measured GPS displacement at the surface is then divided by the time taken to profile and
127 constitutes a constant depth-independent velocity that is added back to the relative velocity. This
128 method also provides an estimate for subsurface float position (x,y) , in metres, in the zonal and
129 meridional direction from the point of descent.

130 In situ and potential density as well as buoyancy frequency were calculated from CTD tem-
131 perature, salinity and pressure measurements using the 2010 equation of state for seawater (IOC
132 et al. 2010). Smooth reference potential density profiles referenced to 1000 dbar, ρ_{ref} , were com-
133 puted by averaging 5 profiles before and after the target profile. Density perturbations, ρ' , were
134 calculated by subtracting reference density from measured density. Smooth ‘reference’ buoyancy
135 frequency profiles were generated using the adiabatic levelling method (Bray and Fofonoff 1981;
136 Millard et al. 1990). Pressure perturbation was estimated by integrating buoyancy perturbation,
137 $b' = -g\rho'/\rho_0$, with depth assuming hydrostatic balance before subtracting the depth average, us-
138 ing a method described by Kunze et al. (2002) and further analysed by Nash et al. (2005).

139 *c. Estimation of vertical velocity*

140 1) DERIVATION

141 Following previous work on the estimation of oceanic vertical flow from gliders (Merckelbach
142 et al. 2010; Frajka-Williams et al. 2011), we have developed a theoretical model describing the
143 vertical motion of EM-APEX floats in a stratified, stationary fluid. After optimisation of the
144 model parameters, absolute vertical water velocity, w , is estimated as the difference between the
145 measured float vertical velocity, w_m , and the steady vertical velocity that it is predicted to have in
146 still water, w_s ,

$$w = w_m - w_s, \quad (1)$$

147 where $w_m = \frac{dz_m}{dt}$. Float height, z_m , is determined from pressure and latitude using the TEOS-10
148 package. In order to determine w_s , it is necessary to solve the steady equation of motion of the
149 float,

$$M \frac{dw_s}{dt} = g(M - \rho V) - \rho C_D A |w_s| w_s, \quad (2)$$

150 with

$$\frac{dz_s}{dt} = w_s, \quad (3)$$

151 where z_s is the float height in still water, the first term on the right of Equation (2) is the buoyancy
152 force, and the second term is a quadratic drag force suitable for an object fully immersed in a
153 high Reynolds number flow (Batchelor 2000). The variables are gravitational acceleration, g , float
154 mass, M , water density, ρ , float volume, V , float cross sectional area A and a non-dimensional
155 drag coefficient C_D . The float volume is a function of pressure and the volume of oil pumped

156 into the external bladder. In principle, it is necessary to solve the system of differential equations
 157 described by Equations (2) and (3) to fully diagnose w_s . However, if a steady force balance is
 158 assumed, setting $\frac{dw_s}{dt} = 0$, the equations can be simplified.

159 Given a steady-state assumption, Equation (2) can be rearranged for w_s as

$$w_s = \text{sgn}(\rho V - M) \sqrt{\frac{|g(M - \rho V)|}{\rho C_D A}}. \quad (4)$$

160 Float volume is assumed to change linearly with pressure, p and piston position k ,

$$V = V_0(1 + \alpha_p[p - p_0]) + \alpha_k(k - k_0), \quad (5)$$

161 where V_0 , p_0 and k_0 denote the volume, pressure and piston position at the ballast point. Variables
 162 α_p and α_k are the coefficient of compressibility and the change in volume with piston position,
 163 respectively. We have neglected the effects of thermal expansion because they are difficult to
 164 separate from those of pressure, since in this area of the ocean both sets of effects cause a decrease
 165 in volume with depth. Variations in temperature during profiles do not typically exceed 5 °C, and
 166 if a thermal expansion coefficient of $3.6 \times 10^{-5} \text{ °C}^{-1}$ (as quoted in the technical specifications
 167 for EM-APEX floats) is assumed, then thermal changes in volume over a profile are typically one
 168 order of magnitude smaller than compressive changes, and thus can justifiably be neglected.

169 2) OPTIMISATION

170 The steady model contains 7 parameters, of which mass, ballast piston position and ballast
 171 pressure are known, having been measured or set prior to deployment. The float diameter is 16.5
 172 cm, giving a cross-sectional area of 0.02 m² that is assumed to remain constant with depth. In
 173 subsequent calculations the area is combined with the drag coefficient into a single parameter, C_D^* ,

174 the value of which is not initially known. The remaining parameters are optimised by minimising
175 the following cost function for vertical water velocity variance over many profiles,

$$\sum_t w(t)^2 \quad (6)$$

176 where $w(t)$ denotes any absolute water velocity measurement at time t regardless of depth. This
177 cost function follows from conservation of volume in an incompressible fluid, which is a very good
178 approximation for the entire ocean, but is also assumed to hold over the smaller spatial and time
179 scales covered by a float. We defer to Frajka-Williams et al. (2011) for a more thorough discussion
180 of cost functions. In summary, they assessed four and found that one was as effective as (6), while
181 two were worse and did not produce physically consistent results.

182 Standard least squares methods were used to perform the optimisation separately for each float,
183 using 150 profiles shortly after the lee wave was observed. Parameter estimates from technical
184 specifications were used as initial values. It is possible that parameter values may change over the
185 lifetime of a float, for example the drag coefficient can change as a result of biofouling (Merckel-
186 bach et al. 2010). Profiles to optimise to were chosen so that the model would be reliable at the
187 time of the lee wave observation, while also keeping the observations independent from the model
188 parameters. The resulting parameters and their uncertainties are summarised in Table 1, along
189 with values expected from technical specifications. Uncertainties were estimated by repeating the
190 optimisation many times on random sub-samples of the chosen profiles, to build a distribution
191 of possible parameters from which the standard deviation was calculated. Over a small range of
192 parameter values close to the optimum, C_D^* and α_k co-vary with compensating effect on vertical
193 velocity. This may have resulted in a somewhat unrealistic, albeit small, difference between these
194 parameters for the two floats.

195 3) VALIDATION AND UNCERTAINTIES

196 Without independent measurements of vertical velocity with which to compare, only a limited
197 validation of the model is possible. The first check is the distribution of vertical velocities, which
198 should be centred on zero, as constrained by the optimisation procedure. Figure 2 shows the
199 distribution of measurements. This closely approximates a Gaussian distribution with a mean of
200 0.0 mm s^{-1} and a standard deviation of 9 mm s^{-1} . In total, 51% of velocities are less than 1 cm
201 s^{-1} .

202 The Garrett - Munk (GM) spectrum (e.g. Gregg and Kunze 1991) provides an estimate of the
203 expected internal wave induced variance of several physical quantities, including vertical velocity
204 or vertical kinetic energy (VKE) (Thurnherr et al. 2015) as a function of vertical wavenumber.

$$\text{VKE}(m) = \pi E_0 b N f j_* \frac{1}{(m + m_*)^2}, \quad (7)$$

205 where the nondimensional spectral energy level $E_0 = 6.3 \times 10^{-5}$; b is the stratification e -folding
206 scale taken as 1000 m in the Drake Passage (Thurnherr et al. 2015); j_* is the peak wave number,
207 which quantifies the bandwidth of the internal wave field; $m_* = j_* \frac{\pi N}{b N_0}$; and $N_0 = 5.3 \times 10^3 \text{ rad s}^{-1}$.

208 Analysis of vertical velocity from LADCP measurements (Thurnherr et al. 2015) find that such
209 a spectrum holds in many regions of the ocean, spanning a range of latitudes, up to a limiting
210 wavenumber. The average VKE spectrum from the two floats, computed from 100 profiles distant
211 from the observed wave, is compared to the GM spectrum in Figure 3. In general the GM spectrum
212 with default parameter values is about a factor of 2 more energetic than the measured average
213 spectrum but is still encompassed by the spread of individual profile spectra, denoted in the figure
214 by faint grey lines. Measured energy levels decline from large to small vertical scales at a rate
215 that is consistent with the power law proportional to m^{-2} over the wavenumber range 0.03 to 0.2

216 rad m^{-1} . A notable deviation from this power law includes a broad peak at 0.02 rad m^{-1} . This is
217 likely caused by processes with a time scale of $2\pi/N$ aliasing the spatial signal, since for a float
218 travelling at $w_f \approx 0.12 \text{ m s}^{-1}$, and $N \approx 2 \times 10^{-3} \text{ rad s}^{-1}$, $N/w_f \approx 0.02 \text{ rad m}^{-1}$.

219 The standard deviation in vertical velocity from different choices in model parameter, estimated
220 from the distributions generated when optimising the model, is 1 mm s^{-1} . This is an uncertainty
221 that manifests as a constant bias in the profile velocity. An additional uncertainty of 1 mm s^{-1} at
222 high frequencies is caused by random noise the pressure sensor. The final source of uncertainty
223 is introduced by a systematic bias in the model as a result of necessary simplification of float
224 dynamics. A test on the accuracy of the steady model was performed by solving the fully time-
225 dependent equations of motion and comparing to the time-independent solution (not shown). The
226 difference between solutions was found to be greatest where the float was undergoing acceleration,
227 such as at the beginning and end of profiles, and when the piston was moved to alter buoyancy.
228 Synthetic profiles of density and pressure were generated, and the time response of the equations
229 to a step change in piston position was assessed. It was found that the float reached 99% of the
230 new terminal velocity after 15 s, corresponding to a vertical distance of less than 1.5 m, which is
231 smaller than the characteristic sampling distance. Thus, for measurements of processes changing
232 on time scales longer than this adjustment time or over larger vertical distances, the no acceleration
233 assumption is justifiable.

234 *d. Estimation of internal wave properties*

235 Internal wave properties are estimated by application of linear internal wave theory, summarised
236 in the Appendix, to the measurements. Properties that can be deduced without knowledge of the
237 wavenumber components are aspect ratio, α , intrinsic frequency, ω_0 , energy density, E , and the
238 vertical fluxes and of energy and horizontal momentum, denoted $\overline{w'p'}$ and $(\overline{w'u'}, \overline{w'v'})$ respectively.

239 To estimate the wave perturbation of horizontal velocity, (u', v') , a linear background shear is
240 removed from absolute horizontal velocity measurements.

241 To estimate the aspect ratio and intrinsic frequency, fourteen sets of coherent velocity and buoy-
242 ancy maxima/minima were identified from profiles using a peak detection algorithm, and con-
243 firmed by eye. The amplitudes at the maxima/minima were then applied in Equations (A10) and
244 (A11). By isolating maxima in this way we assume that the variability is dominated by a single
245 monochromatic wave. Energy density was calculated by isolating segments of velocity and buoy-
246 ancy profiles that contained an integer number of wave oscillations, identified from subsequent
247 maxima by eye, before computing the time average over those isolated sections following Equa-
248 tion (A12). The sections used are those depicted in Figure 4. The vertical fluxes of energy and
249 horizontal momentum were also estimated for the isolated segments following the same approach.
250 The above quantities, deduced without attempting to estimate any wavenumber components, are
251 referred to as the ‘observed’ quantities.

252 The impact of background oceanographic variability (which is significantly larger in magnitude
253 than instrumental noise) on the energy and momentum flux diagnostics was investigated by repeat-
254 ing the calculation with the addition of red noise with spectral properties, such as slope and energy
255 level, given by a background spectrum. The background spectrum was computed by averaging the
256 absolute velocity spectra from 100 profiles in the far field. The standard deviation of results after
257 many repetition is the error, quoted in subsequent analysis. Ultimately, the results are found to be
258 insensitive to choices of the type and energy level of background variability used.

259 To deduce the wavenumber, we fit monochromatic plane waves to observations of velocity,
260 buoyancy and pressure perturbation. Once deduced, the wavenumber implies, following linear
261 theory, values for all the quantities discussed above. The quantities deduced from this fitting are
262 referred to as ‘plane wave’ estimates. Two illustrative profiles are presented in Section 3. The fits

263 take into account the combination of spatial and temporal variability present in the observations
264 by using the depth measurement from the float's pressure sensor, the horizontal position estimated
265 from time-integrated horizontal velocity, and time from the internal clock. In this way, it was
266 possible to account for advection of the float by the local flow field.

267 The fitting procedure optimises five parameters: the three wavenumber components, the pressure
268 perturbation amplitude induced by the wave, and an arbitrary phase shift. Doppler shifting was
269 accounted for by using the mean horizontal velocity of each profile, and a background shear was
270 subtracted from the horizontal velocity. Markov Chain Monte Carlo methods were used to conduct
271 the fitting and produce likelihood distributions for the parameter values. Likelihood distributions
272 are proportional to the posterior probability distribution, which describes the probability that the
273 model fits the data with given parameter values. The most likely parameter set is the best estimate
274 of the parameter value and the width of the distribution is a measure of the confidence interval of
275 that parameter set.

276 *e. Estimation of the turbulent kinetic energy dissipation rate*

277 To estimate the rate of turbulent kinetic energy dissipation rate, ε , we employ the large eddy
278 method of Beaird et al. (2012), which has previously been applied to vertical velocity measure-
279 ments from gliders. We also use the more established Thorpe scale method (Thorpe 1977; Dillon
280 1982) for comparison.

281 1) LARGE EDDY METHOD

282 The large eddy method can be derived from simple scaling of turbulent motions, specifically, the
283 turbulent kinetic energy relation (Taylor 1935),

$$\varepsilon \sim \frac{q'^3}{l} \quad (8)$$

284 where q' is the turbulent velocity scale and l a length scale associated with the largest overturning
 285 eddies. The choice of an appropriate length scale is subject to certain arbitrariness (Kantha and
 286 Clayson 2000). However, if one chooses the buoyancy length, defined as the vertical displacement
 287 over which a water parcel will convert its kinetic energy to potential energy in a stratified fluid and
 288 given non-rigorously as $q'N^{-1}$, then one arrives at the following equation,

$$\varepsilon = c\langle q'^2 \rangle N \quad (9)$$

289 where c is a constant of proportionality. A complementary interpretation is that turbulent eddies
 290 are dissipated over a time proportional to N^{-1} , known as the eddy turnover time. An assumption
 291 of the method is that the largest turbulent scales are isotropic, and that it is sufficient to measure
 292 the kinetic energy of one (in this case, the vertical) velocity component, equal to the mean square
 293 velocity $\langle q'^2 \rangle$, to estimate the energy of an overturn. Tests of the scaling (Beaird et al. 2012,
 294 and references therein) indicate that it is valid for a range of oceanic conditions, including weak
 295 dissipation regimes, down to $q' \sim 0.2 \text{ mm s}^{-1}$ (Peters et al. 1995).

296 The constant of proportionality also corrects implicitly for limitations of the float vertical veloc-
 297 ity model, and for measurements that may not fully isolate turbulent motions and include small-
 298 scale internal waves. The vertical microstructure profile measurements made shortly before de-
 299 ployment of the floats (Sheen et al. 2013), marked as stars in Figure 1, provide the best available
 300 calibration data. The statistics of ε from the large eddy method and microstructure match for
 301 $c = 0.146$ (float 4976) and $c = 0.123$ (float 4977).

302 To isolate the vertical eddy velocity signal, first a temporal low-pass filter was applied to vertical
303 velocity profiles with a cut-off period of 100 s. This was necessary to remove signals associated
304 with internal electronic noise with an approximate length scale of 9 m resulting from a suspected
305 time-stamp recording error, exhibited by both floats. The narrow bandwidth of the noise allowed
306 for its complete removal. A spatial high-pass filter was then applied with a cut-off wavelength of
307 40 m. Steady height, $z_s = \int w_s dt$, rather than measured height, z , was used as the spatial variable
308 so as to reduced aliasing caused by changes in float profiling speed and advection by vertical flows.
309 Root-mean-square vertical velocity and mean buoyancy frequency were calculated in a sliding 20
310 m window. Comparison of vertical kinetic energy spectra between profiles with high and low
311 average ε values (not shown) indicate that energy is most enhanced at scales less than 100 m. The
312 filter cut-off length scale is chosen pragmatically to capture this variance.

313 The vertical kinetic energy content at scales less than 40 m is likely to be dominated by internal
314 waves for all but the most turbulent conditions and as noted by Beaird et al. (2012), the lack of
315 a separation of scales between turbulence and waves makes it impossible to remove the wave
316 signal. This might be expected to cause an overestimation of the dissipation rate, however, since
317 the method is calibrated against microstructure measurements, the coefficient c , is proportionally
318 smaller to account for wave energy. The fact that the method theoretically relies on measuring the
319 eddy energy, rather than the wave energy remains a cause of concern. Some reassurance can be
320 taken from the documented, albeit poorly understood relationship between wave vertical kinetic
321 energy and dissipation found by Thurnherr et al. (2015) in a variety of regions, including the
322 Drake Passage. Thurnherr et al. (2015) use their findings as the basis for a new parametrisation of
323 dissipation in terms of VKE alone, which appears to provide more accurate results than shear-strain
324 based parametrisations. This is relevant because it implies that internal wave VKE is strongly

325 connected to dissipation. We accept that some readers may not be convinced by the large eddy
326 method and so we also estimate dissipation using the more established Thorpe scale method.

327 2) THORPE SCALE METHOD

328 The theoretical basis of the Thorpe scale method is that, in a stratified fluid with buoyancy
329 frequency N , the dissipation rate is related to the largest isotropic turbulent scales, defined by the
330 Ozmidov scale L_O ,

$$\varepsilon = L_O^2 N^3. \quad (10)$$

331 At scales larger than the Ozmidov scale, stratification suppresses vertical motion and turbulent
332 eddies become anisotropic. At smaller scales, there exists an inertial subrange where energy cas-
333 cades to the dissipation scale. By comparing a profile of density with the same data monotonically
334 sorted, such that it forms a stable profile, it is possible to estimate the vertical displacement of
335 density parcels in overturning regions. The Thorpe scale, L_T , is defined as the root mean square
336 displacement of data in an overturn and empirically related to the Ozmidov scale by the relation
337 $L_O = (0.8 \pm 0.4)L_T$ (Dillon 1982).

338 The method is sensitive to spurious density measurements, especially in weakly stratified regions
339 of the water column, which may occur due to salinity spiking. To counter this problem we use
340 the intermediate profile method of Ferron et al. (1998) and reject overturns using an overturn ratio
341 criteria (Gargett and Garner 2008).

342 **3. Results**

343 *a. Observed wave properties*

344 1) LARGE-SCALE OBSERVATIONS

345 Between the 2nd and 4th of January 2011, two EM-APEX floats were advected eastwards over
346 the northern segment of the Shackleton Fracture Zone (SFZ), a chain of sea mounts and large
347 bathymetric features that extends between the Antarctic Peninsula and the South American conti-
348 nental shelf. They maintained a horizontal separation of approximately 4 km during this period.
349 The boxed area in Figure 1 marks this region, and all subsequent analysis is concentrated within
350 it. The upper-ocean buoyancy frequency and velocity upstream of the SFZ are shown in Figure
351 5 as an average of 20 profiles. The mean zonal flow speed between 100 m and 1500 m was 33
352 cm s^{-1} , with a vertical shear of $1.35 \times 10^{-4} \text{ s}^{-1}$. The mean meridional flow over the same depth
353 range was 2 cm s^{-1} , with some variability between profiles and no significant shear. There also
354 exists a minimum in buoyancy frequency at 350 m depth, which may reflect upward propagating
355 internal waves with a frequency greater than $1.4 \times 10^{-3} \text{ rad s}^{-1}$.

356 Figure 6a displays the measured depth-averaged horizontal flow vectors around the SFZ, as well
357 as the standard deviation of vertical water velocity measured below 100 m depth, shown by the
358 vector shading. In the lee of a large topographic ridge, oscillatory vertical velocity perturbations
359 with an amplitude exceeding 20 cm s^{-1} were measured by both floats, resulting in large values of
360 vertical velocity standard deviation. Away from this region, vertical velocity measurements were
361 typically less than 2 cm s^{-1} . Figure 6b displays a section of vertical velocity as a function of height
362 and distance from the ridge crest. The largest vertical velocities were measured within 20 km of
363 the crest. The sawtooth-like trajectory is typical of a profiling float being advected by a strong
364 mean flow. All the topographic data used originate from version 17.1 of the Smith and Sandwell

365 (1997) global bathymetric database, since high-resolution multibeam bathymetric measurements
366 were not available.

367 Figure 4 shows velocity and buoyancy perturbations from a sequence of profiles centred on the
368 largest vertical velocity signal. Vertical velocity from these profiles were binned and displayed
369 as a histogram in Figure 2, from which it can be seen that the distribution of velocity differs
370 greatly from the far field mean. The greatest vertical displacement of density surfaces, estimated
371 as b_0/N^2 , was observed to be (120 ± 20) m (profile 32 float 4976). The shaded segments indicate
372 measurements where vertical velocity amplitude exceeds 10 cm s^{-1} and also varies coherently
373 with at least one other component of velocity. Profiles 31 and 32 from float 4976, and profiles
374 26 and 27 from float 4977, contain such segments. These four profiles are used in the following
375 analysis to quantify the wave properties. While Figure 4 shows several other profiles that contain
376 less conspicuous wave-like signals, noise in the horizontal velocity and buoyancy components
377 makes it difficult to confidently assess wave properties from those profiles.

378 2) FREQUENCY AND ASPECT RATIO

379 Figure 7a amalgamates the observational estimates of aspect ratio and frequency from fourteen
380 sets of maxima from four profiles (those shaded in Figure 4) into box and whisker diagrams. The
381 mean aspect ratio is 1.0 ± 0.6 . Using Equation (A9), the mean frequency is $(1.8 \pm 1) \times 10^{-3} \text{ rad}$
382 s^{-1} , and using Equation (A10) it is $(1.4 \pm 0.4) \times 10^{-3} \text{ rad s}^{-1}$. Both values are close to the local
383 mean buoyancy frequency $N \approx 2.2 \times 10^{-3} \text{ rad s}^{-1}$ and one order of magnitude larger than the local
384 inertial frequency $f \approx 1.2 \times 10^{-4} \text{ rad s}^{-1}$. The period associated with the estimated frequency is
385 approximately 1 hour. The spread of results is a consequence of the limited profiling speed, which
386 is likely capturing the gradually changing characteristics of a wave propagating through a vertical
387 shear and non-uniform stratification.

388 3) ENERGY AND MOMENTUM FLUXES

389 The shaded regions in Figure 4 indicate the isolated sections for which energy density and ver-
390 tical fluxes of energy and horizontal momentum were calculated. The peak energy density was
391 found to be $26 \pm 4 \text{ J m}^{-3}$ in profile 32 float 4976. Results from the four main profiles are dis-
392 played in Figure 8a as box and whisker plots, and range in magnitude from 10 to 26 J m^{-3} .

393 Observational estimates for the time-mean quantities $\overline{w'p'}$ and $F_M^{(z)}$, respectively representing the
394 vertical fluxes of energy and horizontal momentum are displayed in Figures 8b and c. The peak
395 energy flux was $1.3 \pm 0.2 \text{ W m}^{-2}$. All fluxes are positive, indicating upward wave propagation.
396 The smallest value was found for profile 27 from float 4977, where the wave signal occurs higher
397 in the water column, consistent with the group velocity diminishing as the depth of minimum N
398 is approached. The average vertical group velocity corresponding to the observed flux and energy
399 density is, following Equation (A15), found to be $4 \pm 1 \text{ cm s}^{-1}$. These energy flux diagnostics are
400 likely to be underestimates, due to limitations in the method for estimating p' . For a wave with
401 $\alpha \sim 1$, the hydrostatic approximation on which estimation of p' relies (Nash et al. 2005) holds only
402 weakly. However, tests performed on a series of synthetic waves with α in the range 0.5 to 1.5
403 indicate that the method is typically in error by less than a factor of two. So while the uncertainty
404 on the measured energy flux is substantial, the order of magnitude is correct and the real peak
405 value is likely to be closer to 2 W m^{-2} .

406 Estimates of the vertical flux of horizontal momentum range from 1 to 8 N m^{-2} in magnitude.
407 The uncertainty on individual measurements is larger than in the energy flux case because the
408 quantity is more sensitive to oceanographic variability in the horizontal velocity. Momentum flux
409 vectors are displayed in Figure 9, and are oriented predominantly in the northwest - southwest
410 quadrant. The scatter in vector direction is likely indicative of the three-dimensional nature of

411 the wave generation process, occurring off a complex topographic feature that does not lie per-
412 pendicular to the mean flow, but could also be spatial variability. In the classic textbook lee wave
413 problem, the momentum flux vector would be orientated in direct opposition to the mean flow. The
414 mean zonal momentum flux was $-3.1 \pm 0.4 \text{ N m}^{-2}$, and the mean meridional momentum flux was
415 $0.5 \pm 0.4 \text{ N m}^{-2}$. In comparison, mean flow velocity vectors are orientated eastward (Figure 6a) in
416 the opposite direction to the mean momentum flux. The limitations of the floats' spatio-temporal
417 sampling of the wave mean that we cannot definitively establish whether the wave is imparting a
418 drag on the mean flow, or radiating horizontal momentum elsewhere .

419 *b. Wave characterisation with plane wave fits*

420 EM-APEX floats profile slowly compared to the observed wave period of 1 hour, and this will
421 have caused temporal aliasing of the measurements. The apparent vertical wavelength observed
422 from subsequent maxima in vertical velocity from Figure 4 is approximately 400 m. If the wave
423 is stationary, its horizontal wavelength can be deduced from the Doppler relation (Equation (A3)),
424 as $\omega_0 = -kU$. For the observed frequency and mean flow speed, this results in an approximate
425 zonal wavelength of 1200 m, which will be the same as the vertical wavelength for $\alpha = 1$. The
426 conclusion from this estimate is that the intrinsic wavelength could be significantly larger than the
427 apparent wavelength.

428 Fits of Equation (A8) to measurements from two profiles chosen for having the cleanest wave
429 signal (profile 32 from float 4976 and profile 26 from float 4977) were conducted to compare the
430 observations to the simplest possible theoretical explanation, a monochromatic plane wave. Doing
431 so also provides a separate determination of the vertical fluxes of energy and momentum. The
432 resulting parameter estimates (wavenumbers and pressure perturbation) from this fitting procedure
433 were inserted into the linear internal wave equations (Equation (A8)) to produce the red curves in

434 Figure 10. The fit to profile 26 shows good agreement with observations for all variables, with the
435 exception of u , which is not of the correct amplitude. Profile 32 contains small scale fluctuations
436 in velocity which are not explained by a monochromatic plane wave, however, the large scale
437 variation is captured. The quantities are plotted as a function of time, rather than height, to remove
438 the temporal-aliasing that causes cusping, visible in Figure 4. Cusping occurs as alternating phases
439 of wave motion force the floats against their direction of motion, in some cases causing a complete
440 reversal of direction, and then propel them in the same direction of motion, greatly increasing the
441 profiling speed. Such forcing aliases the observations away from an expected sinusoidal shape.

442 Figure 11a shows the likelihood distributions of the plane wave derived wavenumber compo-
443 nents as box and whisker plots. It should be noted that the range of the distributions is typically
444 less than 1% of the parameter value and so uncertainties are not quoted. For both profiles the
445 fitting method finds the optimal zonal wavenumber, k , to be $-0.002 \text{ rad m}^{-1}$, which corresponds
446 to a zonal wavelength of 4000 m. This is likely to be an underestimate of the real wavenum-
447 ber, because the fits do not reproduce the observed zonal velocity amplitude which is related to
448 the wavenumber by the polarisation relation in Equation (A4), and we would therefore expect a
449 smaller zonal wavelength. There is a difference in sign between profiles as to the direction of the
450 meridional wavenumber, likely due to the different time and position at which the profiles were
451 taken; however it is of similar magnitude to the zonal wavenumber. The negative sign on the zonal
452 wavenumber is significant, because it indicates that the wave phase velocity opposes the mean
453 flow. The non-negligible magnitude of the meridional wavenumber means that the total horizontal
454 wave vector is not directed exactly westward against the predominantly eastward mean flow, as
455 was also found in observational estimates of the momentum flux vectors. The vertical wavenum-
456 ber is negative, indicating upward propagation, and the vertical wavelength is 1800 m for profile
457 32 and 1000 m for profile 26.

458 The frequency determined from the fits is displayed in Figure 11b. It overlaps with the observa-
459 tional estimate (grey box and whisker), and is $0.3N$ for profile 32 and $0.8N$ for profile 26. Eulerian
460 frequencies are $3 \times 10^{-4} \text{ rad s}^{-1}$ and $7 \times 10^{-4} \text{ rad s}^{-1}$, corresponding to periods of 3 to 6 hours.
461 If the horizontal wavevector has been underestimated, then so have these periods following from
462 Equations (A10) and (A11). Thus, the wave is not perfectly stationary, but a fixed observer would
463 notice a significant Doppler shift.

464 Energy density and the vertical fluxes of energy and horizontal momentum estimates are dis-
465 played in Figures 11c, d and e. The energy density of the best fits are 12 J m^{-3} and 15 J m^{-3} ,
466 smaller than the direct estimates from observations because the model has some difficulty in re-
467 producing the full measured velocity amplitude. Energy fluxes are slightly larger than the direct
468 estimates, at 1.2 W m^{-2} and 2.5 W m^{-2} , but within a factor of 2. Momentum fluxes are within the
469 bounds of the direct estimates, with values of 3.5 N m^{-2} and 7 N m^{-2} .

470 In summary, while not providing a precise description, monochromatic plane waves do give a
471 reasonable characterisation of the observed lee wave. This is estimated to have horizontal and
472 vertical wavelengths in the range of 1 to 4 km; to propagate upward and against the eastward mean
473 flow; to be quasi-stationary; and to transport energy and horizontal momentum vertically at large
474 rates that are within a factor of 2 to 3 of the direct estimates.

475 *c. Turbulent kinetic energy dissipation*

476 A section of the rate of turbulent kinetic energy dissipation is displayed on a logarithmic colour
477 scale in Figure 12. Results from Thorpe scale analysis are shown as large circles in Figure 12b
478 and results from the large eddy method are displayed as small circles in Figure 12c. Background
479 levels of dissipation in Drake Passage are typically of order $10^{-10} \text{ W kg}^{-1}$, less than the detection
480 level of either method, and are blanked out over the majority of the section. Both methods indicate

481 a patch of high dissipation above and in the lee of the ridge crest, coincident with profiles of large
482 vertical velocity. Notably large overturns of order 10 m in scale are detectable using the Thorpe
483 scale method, with dissipation rates in such patches approaching $10^{-6} \text{ W kg}^{-1}$, while the majority
484 of overturns are smaller than this. The depth-integrated dissipation rate, $P = \int_{-Z}^0 \rho \epsilon dz$, peaks at
485 20 mW m^{-2} .

486 Using the large eddy method, dissipation rates are found to be largest in the profiles containing
487 the strongest wave signal, and peak at $10^{-7} \text{ W kg}^{-1}$ at roughly 1000 m depth. The depth-integrated
488 dissipation rate peaks at 6 mW m^{-2} , significantly less than the estimated vertical flux of energy
489 associated with the wave. The sensitivity of these results to method parameter choices was as-
490 sessed by systematically varying parameters, such as filter cut-off scale and window length, over
491 plausible ranges. The spatial distribution of dissipation did not change, but the magnitude of the
492 integrated dissipation rate varied by up to 20%.

493 **4. Discussion and conclusions**

494 In this paper, observations of a wave-like feature in the vicinity of a sharp ridge made by two EM-
495 APEX floats have been analysed to document the feature's physical characteristics. The limited
496 number of profiles and the necessity of considering their time-dependent nature made analysis and
497 interpretation of some properties challenging. Nonetheless, linear internal wave theory provides
498 a good description of the dominant mode of variability, which has a positive vertical energy flux
499 and negative vertical wavenumber, indicating upward propagation. The zonal phase velocity is
500 directed westward, in opposition to the mean flow, resulting in a quasi-stationary pattern, while the
501 meridional structure of the wave appears variable. This result, deduced from coherent oscillations
502 of velocity and buoyancy over several wave periods, leads to the conclusion that the floats observed
503 a lee wave, likely generated at the ridge and forced by the flow of the ACC. However, naive

504 application of infinitesimal linear wave generation theory (Bell 1975) for a near bottom flow speed
505 of order 20 cm s^{-1} , near bottom stratification of $1 \times 10^{-3} \text{ rad s}^{-1}$ and topographic wavelength of 40
506 km imply that the resulting wave would be evanescent. This is in contradiction to the observations,
507 which indicate a wave of frequency near N and wavelength closer to 4 km in the upper most 1500
508 m of the water column.

509 This contradiction may be resolved by considering the steepness parameter s . The steepness
510 parameter is defined as the ratio of topographic height, h , to characteristic wave height, U/N ,
511 giving Nh/U , for a given near-bottom flow speed and stratification (Nikurashin and Ferrari 2010).
512 Large values of s imply that the flow does not have sufficient kinetic energy to fully mount the
513 topography, such that a deeper portion of the water column may be blocked or diverted making the
514 wave generation process highly nonlinear. The value at which this transition occurs is in the range
515 of 0.4 to 0.7, depending on topographic configuration (Aguilar and Sutherland 2006; Nikurashin
516 et al. 2014). Infinitesimal linear theory requires that the steepness parameter be much less than
517 this value range. Given that the ridge height is roughly 1500 m, and that near-bottom stratification,
518 as measured from ship-based CTD casts, is $0.8 \times 10^{-3} \text{ rad s}^{-1}$, flow speeds in excess of 3 m s^{-1}
519 would be required for a sufficiently small steepness parameter. This is not a physically reasonable
520 speed for a near-bottom oceanic flow, and we conclude that the flow is highly likely to be blocked
521 below some depth.

522 High-resolution modelling efforts in two and three dimensions using a domain analogous to the
523 Drake Passage (Nikurashin et al. 2014) show that, for large values of the steepness parameter, the
524 time-mean energy flux into lee waves saturates at 10 mW m^{-2} . For very long ridges in which the
525 flow configuration is largely two-dimensional, the energy flux at generation saturates at 100 mW
526 m^{-2} . These values are smaller than the energy fluxes estimated from our observations, of order
527 1 W m^{-2} , which are in good agreement with those for a propagating monochromatic plane wave

528 constrained by linear theory. It is possible to estimate the expected energy flux from linear theory
529 (Bell 1975) for the portion of the water column above which blocking occurs. Doing so reduces
530 the height of the topography to an effective height h_e . Taking $h_e = 200$ m, for which the topo-
531 graphic wavelength is roughly equal to the observed zonal wavelength of 4000 m, extrapolating
532 the observed mean flow speed to be 0.2 m s^{-1} near ridge top, and using the ship based CTD esti-
533 mate of stratification, we get a linear energy flux value of 0.5 W m^{-2} . This value is within a factor
534 3 of the observed value. We conclude that our observations are consistent with linear generation
535 above a blocking level. However, we also acknowledge that important small scale bathymetric
536 features may exist that are not resolved by the database used (Smith and Sandwell 1997).

537 Observed integrated dissipation rates in the Southern Ocean (St. Laurent et al. 2012; Sheen et al.
538 2013) are typically less than 5 mW m^{-2} . Our estimated values are similar to this, however, there
539 is some uncertainty in this result due to quantitative limitations of the Thorpe scale and large eddy
540 methods. A significant finding of our work is that the diagnosed vertical energy flux is almost two
541 orders of magnitude larger than the depth-integrated dissipation rate. This result lends support to
542 the idea that not all lee wave energy is dissipated locally (Waterman et al. 2014), however, we are
543 not able to deduce the fate of the wave energy from the limited observations available.

544 It is possible to make a basic assessment of the wave's propensity to shear instability, using the
545 Richardson number, $\text{Ri} = N^2 / (\frac{\partial u}{\partial z})^2$. A necessary condition for shear instability is that $\text{Ri} < \frac{1}{4}$
546 (Miles 1961; Howard 1961). For a single wave, the induced vertical shear $\frac{\partial u}{\partial z} = u_0 m$, where u_0 is
547 the horizontal velocity amplitude and m the vertical wavenumber. For the criterion to be satisfied,
548 we find that $m > 0.01 \text{ rad m}^{-1}$. The observations indicate that m is less than this value by a factor of
549 2 to 4. In a process distinct from shear instability, a wave will become statically unstable when the
550 ratio of the horizontal velocity amplitude to the horizontal phase speed, $u_0 \omega / k > 1$ (Orlanski and
551 Bryan 1969), and evidence from numerical models suggests that this can occur at slightly less than

552 1 (Liu et al. 2010). Our estimate for the static stability is in the range 0.1 to 0.25, within a factor
553 of 4 to 10 of the condition. These estimates indicate that the wave, at its point of observation, is on
554 the verge of undergoing shear and / or static instability. Interaction with the mean flow, changing
555 stratification or other waves may play a role in inducing or amplifying such instabilities.

556 A significant fraction of the diagnosed vertical flux of horizontal momentum associated with
557 the wave was oriented in opposition to the mean flow, which is approximately zonal. Signif-
558 icant non-zonal components of the momentum flux are likely a consequence of the nonlinear,
559 three-dimensional nature of the generation process but could also be a result of spatio-temporal
560 variability or advection. It was not possible to deduce the divergence of the momentum flux, and
561 therefore, the implied drag force. However, the magnitude of the flux is more than two orders of
562 magnitude greater than the time-mean wind stress on the ACC (Wunsch 1998), suggesting that
563 lee waves have the potential to be a significant term in the local momentum budget of ACC jets
564 as suggested by Naveira Garabato et al. (2013). Further work will be needed to understand the
565 temporal and spatial occurrence of such wave events and a targeted observational campaign will
566 be required to conclusively test this hypothesis.

567 This paper documents the first unambiguous observation of a lee wave in the ACC. A thorough
568 analysis of sparse of observations was conducted to produce optimal estimates of wave proper-
569 ties, which are broadly consistent with inferences from previous, spatially incoherent finescale
570 measurements. The extremely energetic nature of the wave is conducive to large vertical fluxes
571 of energy and momentum and to the generation of significant amounts of turbulence, reinforcing
572 current appreciation for the dynamically important role that lee waves likely play in the circulation
573 of the Southern Ocean.

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580 APPENDIX

581 **Linear internal wave theory**

582 We summarise here the results of linear internal wave theory that are used in the analysis of
583 observations, following Gill (1982). The linearised, Boussinesq, momentum equations for an
584 incompressible fluid assuming a constant stratification, N , constant Coriolis parameter, f , and
585 constant mean flow $\mathbf{U} = (U, V, 0)$, can be combined into the following equation for vertical velocity
586 perturbations, w' ,

$$\left[\left(\frac{\partial}{\partial t} + \mathbf{U} \cdot \nabla \right)^2 \nabla^2 + f^2 \frac{\partial^2}{\partial z^2} + N^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] w' = 0. \quad (\text{A1})$$

587 Plane wave solutions are assumed such that,

$$w' = w_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}, \quad (\text{A2})$$

588 where w_0 is the velocity amplitude, $\mathbf{k} = (k, l, m)$ is the wavevector, $\mathbf{x} = (x, y, z)$ is the position
589 vector, and ω the Eulerian frequency as would be measured in a frame of reference stationary with
590 respect to the Earth. Substituting this solution into Equation (A1) gives the familiar internal wave
591 dispersion relation,

$$(\omega - \mathbf{k} \cdot \mathbf{U})^2 = \omega_0^2 = \frac{f^2 m^2 + (k^2 + l^2) N^2}{k^2 + l^2 + m^2}, \quad (\text{A3})$$

592 where ω_0 is the intrinsic wave frequency. It can be seen that the intrinsic frequency of a propa-
 593 gating wave measured by an observer travelling with the flow must lie between f and N , else the
 594 frequency would be imaginary and the solution evanescent. In the presence of a mean flow, \mathbf{U} ,
 595 a Doppler shifted (Eulerian) frequency, ω , would be measured by a stationary observer and the
 596 relationship between the two frequencies is $\omega = \mathbf{k} \cdot \mathbf{U} + \omega_0$.

597 An internal wave generates fluctuations in all components of velocity, $\mathbf{u}' = (u', v', w')$ as well as
 598 pressure, p' , and buoyancy, b' . Here we have divided pressure by mean density, $p' = P'/\rho_0$, and
 599 define buoyancy as, $b' = -g\rho'/\rho_0$. The relative amplitude of these fluctuations are related to the
 600 wave length scales by the polarisation relations,

$$u_0 = \frac{k\omega_0 + ilf}{\omega_0^2 - f^2} p_0 \quad (\text{A4})$$

$$v_0 = \frac{l\omega_0 - ikf}{\omega_0^2 - f^2} p_0 \quad (\text{A5})$$

$$w_0 = \frac{-m\omega_0}{N^2 - \omega_0^2} p_0 \quad (\text{A6})$$

$$b_0 = \frac{imN^2}{N^2 - \omega_0^2} p_0. \quad (\text{A7})$$

601 The final plane wave solutions for velocity, buoyancy and pressure are then given by,

$$(u', v', w', b', p') = (u_0, v_0, w_0, b_0, p_0) e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}. \quad (\text{A8})$$

602 Thus, for a given mean flow speed, stratification and Coriolis parameter, linear waves are com-
603 pletely described by a few key parameters: the components of wavenumber (inverse wavelength)
604 in all three directions, and the amplitude of the pressure perturbation. Frequency is fixed by the
605 ratio of horizontal to vertical wavenumber, or aspect ratio, $\alpha^2 = (k^2 + l^2)/m^2$. The amplitude of
606 velocity fluctuations is set by the pressure perturbation amplitude and wavenumber. Much infor-
607 mation can therefore be deduced from limited observations of a few key variables.

608 By dividing the Equations (A6) and (A7), one gets a succinct measure of the wave frequency
609 from the amplitude of buoyancy and vertical velocity perturbations,

$$\left| \frac{w_0}{b_0} \right| N^2 = \omega_0. \quad (\text{A9})$$

610 The dispersion relation can be re-cast in terms of the aspect ratio,

$$\omega_0^2 = \frac{f^2 + \alpha^2 N^2}{1 + \alpha^2}. \quad (\text{A10})$$

611 Equations (A9) and (A10) provide two methods for deducing internal wave frequency from
612 measurements of velocity and buoyancy amplitude made by EM-APEX floats, both of which are
613 used in subsequent analysis. For a nonhydrostatic wave, where $N \geq \omega_0 \gg f$, it can be shown
614 using Equations (A4), (A5) and (A6), that the aspect ratio is related to the velocity amplitudes as
615 follows,

$$\frac{w_0^2}{u_0^2 + v_0^2} \approx \alpha^2, \quad (\text{A11})$$

616 and this result can be substituted into Equation (A10) to deduce the intrinsic frequency from
617 velocity amplitude alone.

618 *a. Energy flux*

619 Internal waves have an energy density, E , consisting of a kinetic part relating to the motion of
 620 water parcels, and a potential part relating to the displacement of density surfaces from equilib-
 621 rium,

$$E = \frac{1}{2}\rho_0(\overline{u'^2 + v'^2 + w'^2}) + \frac{1}{2}\rho_0 N^{-2} \overline{b'^2}. \quad (\text{A12})$$

622 Here an over-bar denotes an average over one wave period. Linear internal waves flux energy in
 623 the direction of the group velocity, \mathbf{c}_g , so that the energy flux vector is given by

$$\mathbf{F}_E = E \mathbf{c}_g, \quad (\text{A13})$$

624 which is also defined more generally as the average covariance of pressure and velocity pertur-
 625 bations,

$$\mathbf{F}_E = \rho_0 \overline{p' \mathbf{u}'}. \quad (\text{A14})$$

626 Often one is interested in the vertical energy flux, $F_E^{(z)}$, which is simply the energy density
 627 multiplied by the vertical component of the group velocity,

$$F_E^{(z)} = E c_g^{(z)}, \quad (\text{A15})$$

628 or alternatively

$$F_E^{(z)} = \rho_0 \overline{p' w'}. \quad (\text{A16})$$

629 The equation for the vertical component of the group velocity can be derived by taking the
 630 derivative of the dispersion relation (Equation (A10)) with respect to vertical wavenumber, $\frac{\partial \omega_0}{\partial m}$,
 631 giving the result

$$c_g^{(z)} = \frac{-(N^2 - f^2)\alpha^2}{m(1 + \alpha^2)^{\frac{3}{2}}(f^2 + \alpha N^2)^{\frac{1}{2}}}. \quad (\text{A17})$$

632 It can be seen that, for fixed α , the vertical group velocity increases with wavelength (inverse
 633 wavenumber) and has opposite sign to the wavenumber, such that negative vertical wavenumber
 634 indicates upward group velocity and upward energy flux. To estimate vertical energy fluxes from
 635 observations requires knowledge of energy density, aspect ratio and wavelength before applying
 636 these in Equations (A15) and (A17) (e.g. Kunze and Sanford 1984). Alternatively, it can be esti-
 637 mated from measurements of pressure perturbation and vertical velocity, applying Equation (A16)
 638 (e.g. Nash et al. 2005).

639 *b. Momentum flux*

640 The absolute vertical flux of horizontal momentum is defined as

$$F_M^{(z)} = \rho_0 [(\overline{u'w'})^2 + (\overline{v'w'})^2]^{\frac{1}{2}}, \quad (\text{A18})$$

641 where the covariance of velocities are summed in quadrature to account for transport of both
 642 zonal and meridional momentum. In the case of linear lee wave generation by infinitesimal topog-
 643 raphy (e.g. Gill 1982), the vertical flux of horizontal momentum is equal in magnitude to the drag
 644 force exerted on the mean flow. If finite-amplitude effects are taken into account, including flow
 645 blocking and splitting, the drag becomes a nonlinear function of the steepness parameter (Welch
 646 et al. 2001).

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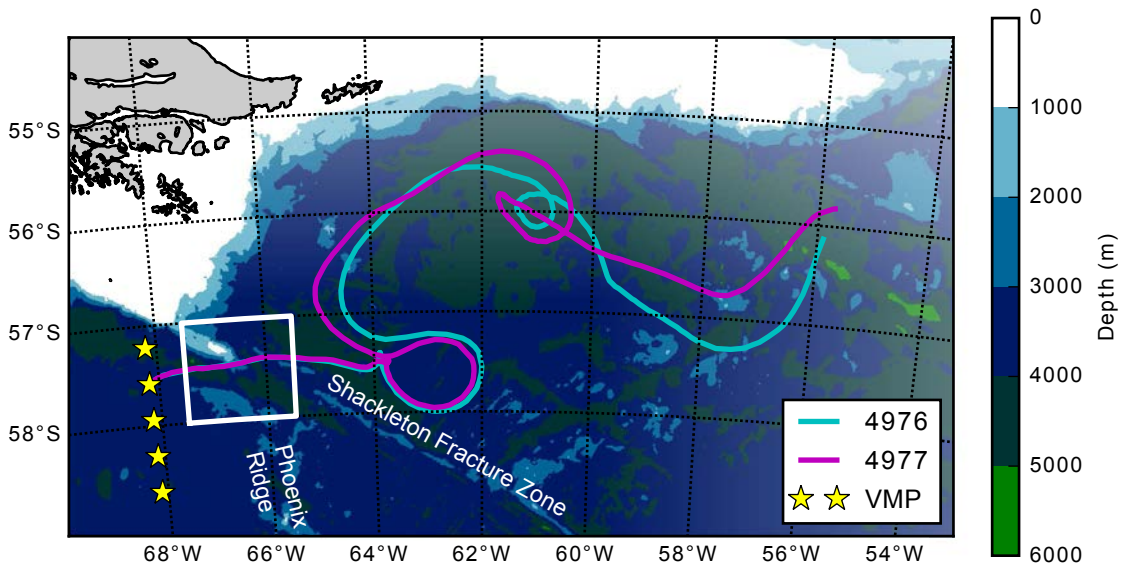
Parameter	Units	Expected	Float 4976	Float 4977
V_0	10^{-2} m^3	2.62	2.62 ± 0.0	2.62 ± 0.0
C_D^*	10^{-2} m^2	2.9	3.5 ± 0.6	2.2 ± 0.4
α_p	$10^{-6} \text{ dbar}^{-1}$	3.67	3.6 ± 0.3	3.8 ± 0.2
α_k	10^{-6} m^3	1.156	1.5 ± 0.3	1.0 ± 0.2

LIST OF FIGURES

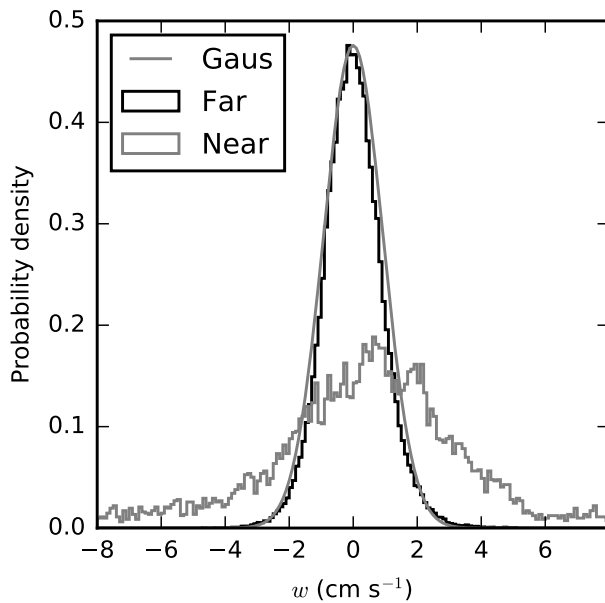
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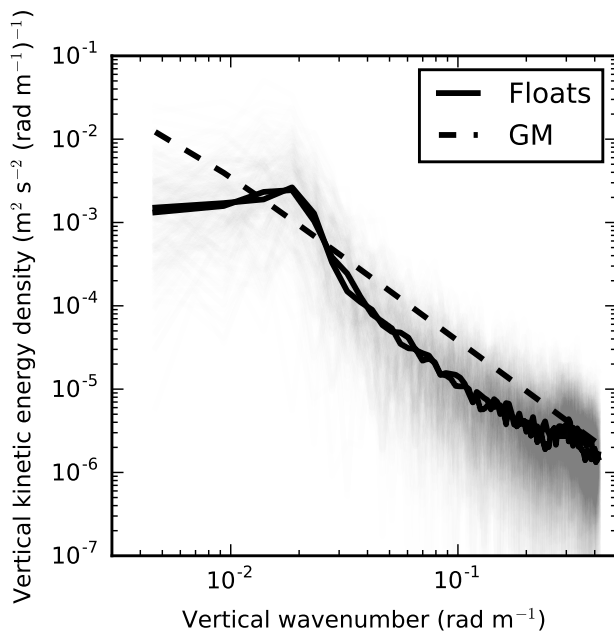
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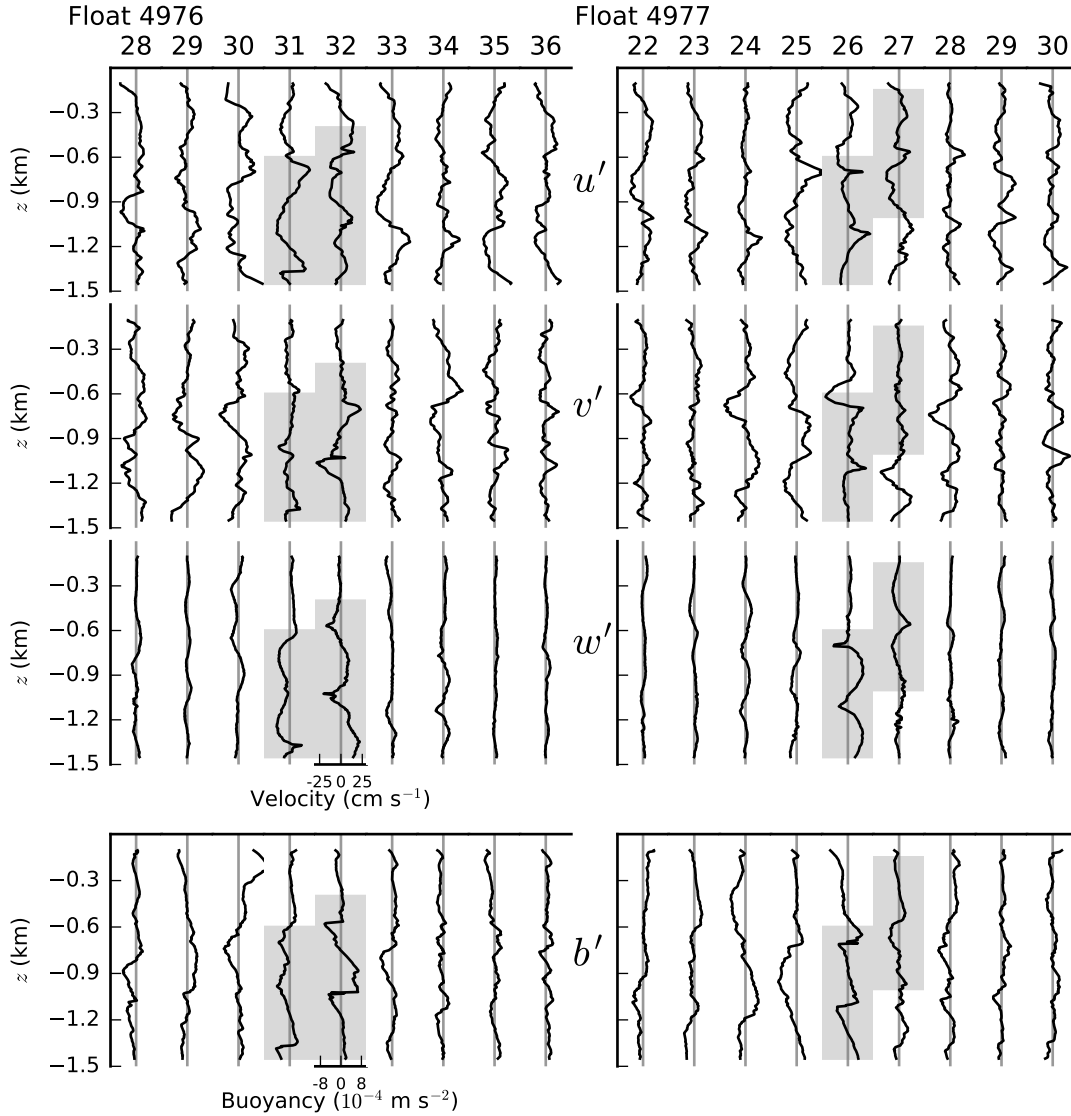
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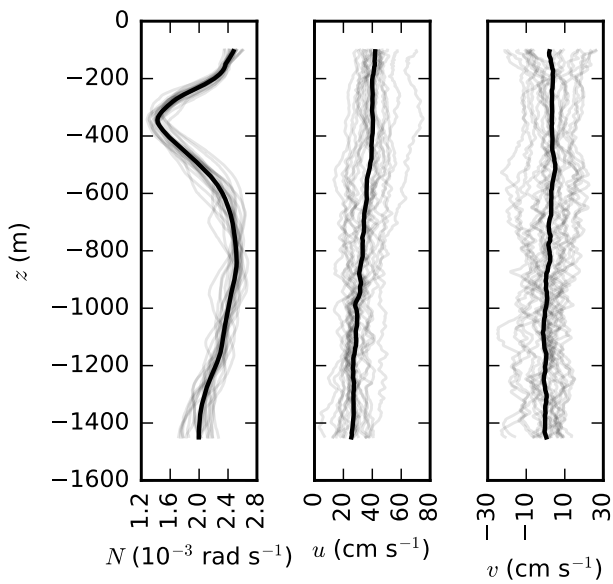
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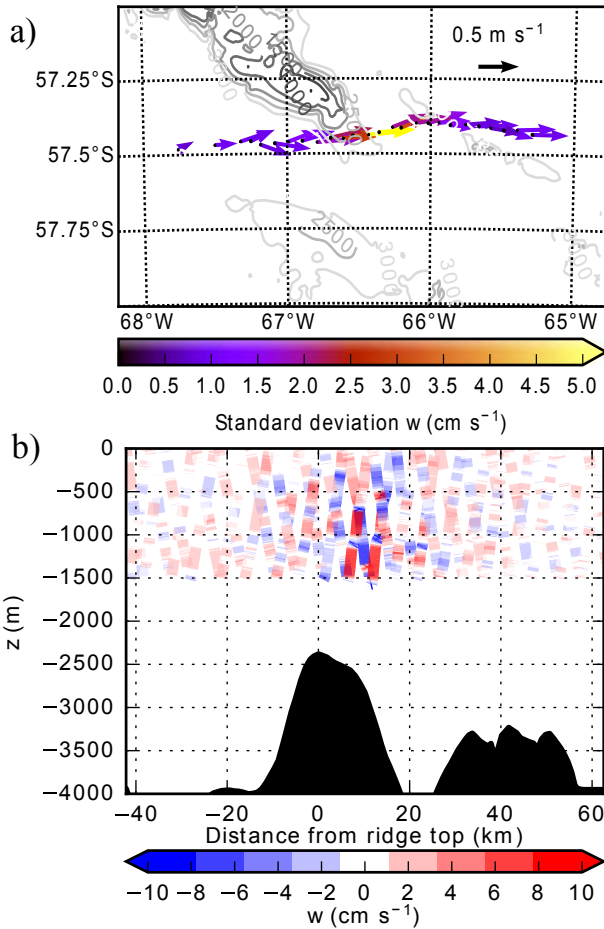
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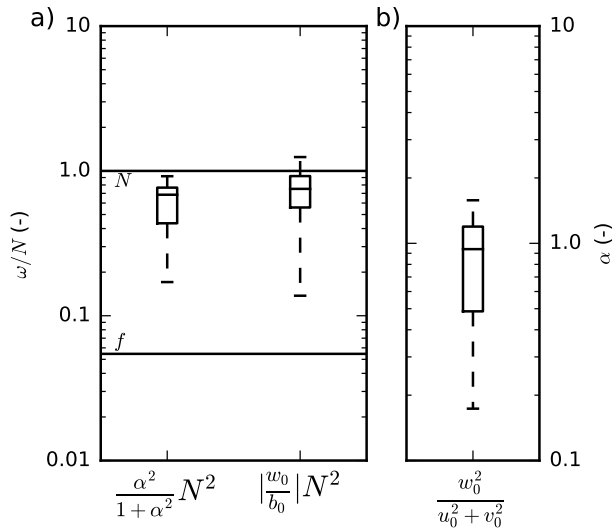
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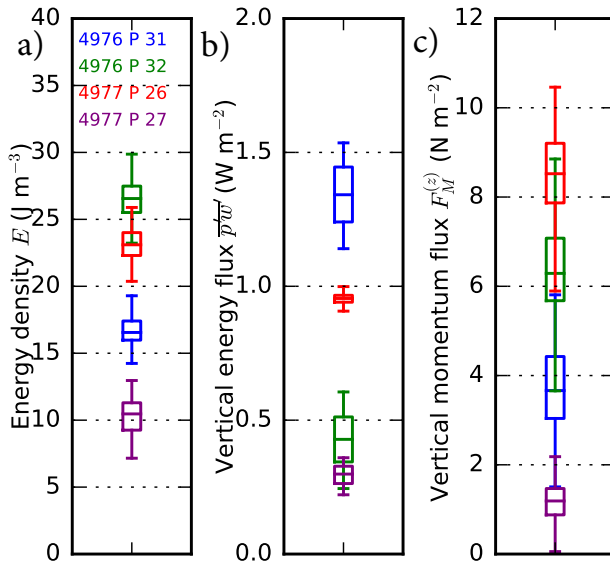
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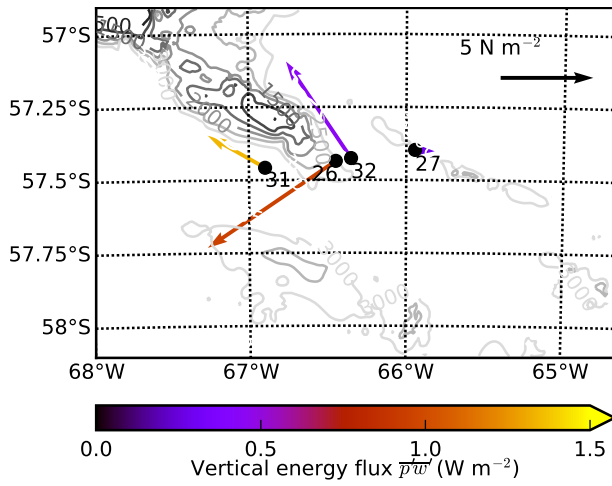
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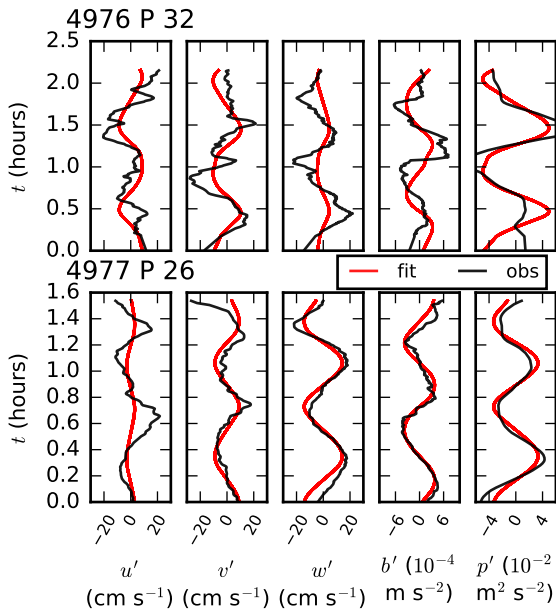
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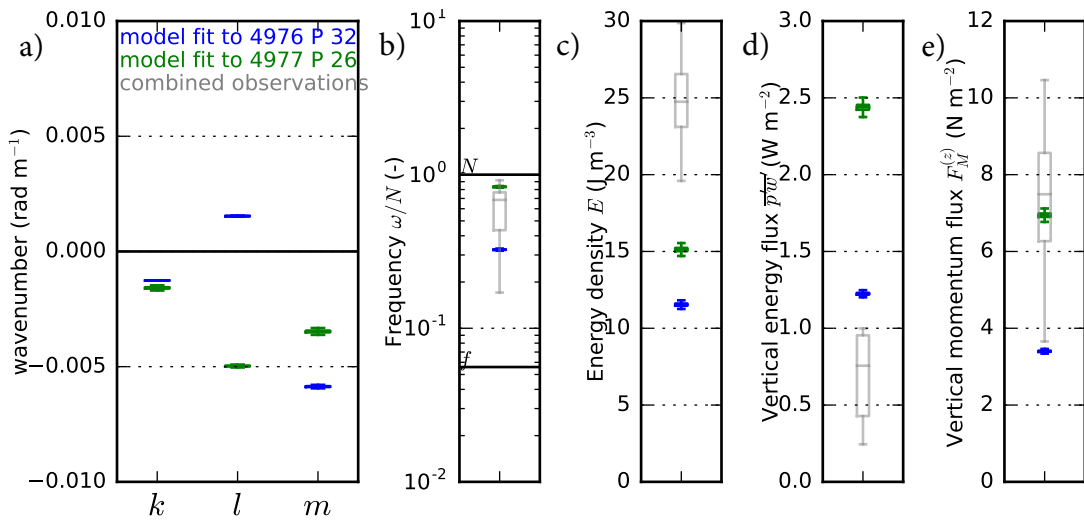
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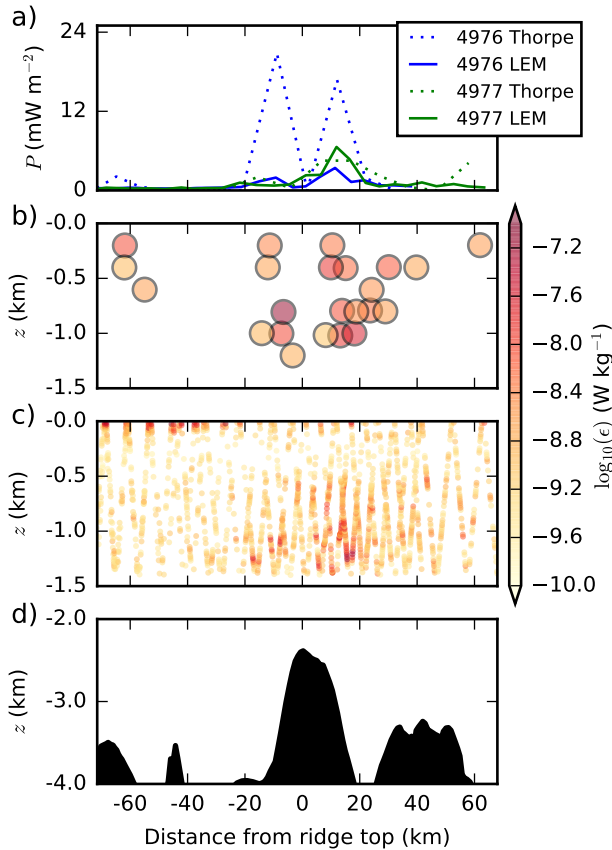
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