Modelling Financial Volatility Using Bayesian and Conventional methods

Submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy

by

Xuguang Li

Department of Economics

Lancaster University

September 2016

Acknowledgements

I am indebted to many people for their help and support. First I would like to thank my supervisor Professor Mike Tsionas for his encouragement, support and guidance. I must also thank Dr Marwan Izzeldin who has given me a lot of help during my period of study. Without their consistent and illuminating instruction, this thesis could not have reached its present form.

I would like to thank Gerald Steele for his kindness in helping go through mistakes in my English. I am grateful to the staff at the faculty and staff of the Department of Economics for their help to support my research in many different ways.

I gratefully acknowledge the funding from the Economic and Social Research Council (ESRC) that made my PhD work possible.

Lastly and most importantly, I would like to thank my beloved family for all their love, encouragement and understanding through all these years. To my parents, who raised me with a love of science and have been a constant source of support, and to my girlfriend, who always stands by me and encourages me with her love. I dedicate this thesis.

Declaration of Authorship

I hereby declare that this thesis is my own work and has not been submitted for the award of a higher degree elsewhere. This thesis contains no material previously published or written by any other person except where references have been made in the thesis.

Xuguang Li

September 2016

Abstract

This thesis investigates different volatility measures and models, including parametric and non-parametric volatility measurement. Both conventional and Bayesian methods are used to estimate volatility models.

Chapter 1: We model and forecast intraday return volatility based on an extended stochastic volatility (SV) specification. Compared with the standard SV, we incorporate the trading duration information which includes both actual and expected durations. We use the Autoregressive Conditional Duration (ACD) model to calculate the expected duration that can be used to measure the surprise in durations. We find that the effect of surprise in durations on intraday volatility is highly significant. If there is an unexpected increase for the lag actual duration, the current volatility tends to decrease, and vice versa. We also take into account the duration and volatility intraday patterns. Our empirical results is based on the SPDR S&P 500 (SPY) and Microsoft Corporation (MSFT) data. According to the in-sample and out-of-sample empirical results, the extend SV model outperforms the GARCH and GARCH augmented with duration information.

Chapter 2: We examine contagion effects resulting from the US subprime crisis on a sample of EU countries (UK, Switzerland, Netherlands, Germany and France) using a Multivariate Stochastic Volatility (MSV) framework augmented with implied volatilities. The MSV framework is estimated using Bayesian techniques. We compare the the MSV framework with the Multivariate GARCH (M-GARCH) framework and find the contagion effect is more significant under MSV framework. Moreover, augmenting the MSV framework with implied volatilities further increases model fit. Compared with the original MSV framework, we find that the contagion effect becomes more significant when we incorporate implied volatilities. Therefore, implied volatility information is useful for detecting financial contagion, or double checking some cases

of market interdependence (strong linkages but insignificant increase in correlations).

Chapter 3: We extend the Heterogeneous AR (HAR) model to allow the autoregressive parameter of daily realized volatility (RV) to be time varying (TV-HAR). The daily lag weights are adjusted according to the fluctuations of RV around its longer time average level (monthly RV). We compare the TV-HAR model with the HAR model and the recently introduced HARQ model. We observe a regular pattern of RV which the HAR and HARQ models do not fully capture: if there is an increase in the lag daily RV compared with its longer-term average level (monthly RV), the current RV tends to decrease rapidly to its long term level; conversely, if there is a decrease in the lag daily RV compared with its longer-term average level (monthly RV), that reversion takes longer. The TV-HAR model can capture this RV pattern. We find that the TV-HAR model performs better than the benchmark HAR model and the HARQ model for both simulated and empirical data. Our empirical analysis is based on the S&P 500 equity index, SPY index and ten series of stocks data from 2000 to 2010.

Table of Contents

Abstract 4

Introduction 8

1 A Stochastic Volatility Model for Modelling the Impact of Duration Information on Volatility 11

1.1 Introduction . . . 12

1.2 Literature Review . . . 15

1.2.1 Intraday Return Volatility . . . 15

1.2.2 Volatility and Duration . . . 17

1.3 Data Description and Analysis . . . 24

1.3.1 Trade Duration . . . 25

1.3.2 Intraday Returns . . . 27

1.4 Intraday SV Model with Duration . . . 28

1.4.1 Intraday Pattern Adjustment for Duration and Volatility . . . 29

1.4.2 Model Structure . . . 30

1.5 Intraday SV Model Estimation . . . 33

1.6 Empirical Illustration . . . 37

1.6.1 Estimation Results . . . 38

1.6.2 Competing Models . . . 40

1.6.3 The In-sample Fit . . . 41

1.6.4 Forecasting Results . . . 44

1.7 Conclusion . . . 47

1.8 Appendix . . . 64

2 Measuring financial contagion: A multivariate stochastic volatility approach 68

2.1 Introduction . . . 69

2.2 Literature Review . . . 73

2.2.1 Financial Contagion . . . 73

2.2.2 Detecting Financial Contagion . . . 75

2.2.3 Dynamic Correlation-Multivariate Models . . . 77

2.2.4 Crisis Period Identification . . . 79

2.3 Data and Descriptive Statistics 81
2.4 Methodology 82
2.4.1 The Models 83
2.4.2 Bayesian Estimation 85
2.4.3 Regime changes and Markov-Switching models 88
2.4.4 Testing for Contagion 90
2.5 Empirical Results 92
2.5.1 Dynamic Correlations with Different Models 92
2.5.2 DC-MSV augmented with Implied Volatility 93
2.5.3 Dynamic Correlations and Contagion Analysis 95
2.6 Conclusion 98
2.7 Appendix 112

3 A time varying HAR model for realized volatility forecasting 114

3.1 Introduction 115
3.2 Literature Review 117
3.3 Theoretical Framework 121
3.4 Models 122
3.4.1 The HAR 122
3.4.2 The HARQ 124
3.4.3 The TV-HAR 126
3.5 Simulation study 129
3.5.1 Design and settings 129
3.5.2 Monte Carlo Results 132
3.6 Data Description 133
3.7 Empirical Results 135
3.7.1 In-sample estimation results 135
3.7.2 Out-of-sample forecasting results 138
3.8 Conclusions 140
3.9 Appendix 157

Concluding Remarks 179

References 181

Introduction

Volatility is a way of measuring the dispersion of returns of a certain security, asset class or market index. Volatility plays a crucial role in asset pricing, risk management, and portfolio allocations, and has been one of the most active areas of research in empirical finance and time series econometrics during the past decade. This thesis focus on financial return volatility measures and models.

Volatility measurement can be classified under parametric approaches and nonparametric approaches. The parametric approaches are based on explicit functional form assumptions regarding the expected and/or instantaneous volatility. The nonparametric approaches are generally free from such functional form assumptions and treat volatility as an ex post observable variable. In this case we can model and forecast volatility directly. In Chapter 1 and Chapter 2, we use parametric approach and model volatility based on both stochastic volatility (SV) and GARCH specifications. In Chapter 3, we rely on non-parametric approach and introduce a Time Varying Heterogeneous Auto-Regressive (TV-HAR) to forecast volatility.

There are different frequencies of financial time series. The early financial studies mainly investigate the daily data. With the rapid development in computing power, storage capacity and trading recoding technology, data now are available at higher frequencies. Various financial practitioners prefer different frequencies. For example, high-frequency traders and risk managers need analysis intraday return volatility rather than only focus on the daily volatility. On the other hand, long term investors adjust their positions infrequently. In Chapter 1, we use the high-frequency data. We directly model and forecast intraday return volatility rather than aggregate into daily realized volatility. In Chapter 2, we employ the lower frequency, daily financial data, to investigate the financial contagion based on multivariate volatility models. In Chapter 3, we aggregate the intraday returns to generate realized volatility.

We use both the conventional and Bayesian methods to estimate models: although GARCH models can be easily estimated by the maximum likelihood estimation method, SV models belong to the family of nonlinear non-Gaussian state space models and the maximum likelihood estimation method cannot be used directly. So we consider the quasi-maximum likelihood method (Chapter 1) and Bayesian Markov chain Monte Carlo method (Chapter 2) to estimate SV models. In Chapter 3, we extend the standard Heterogeneous Auto-Regressive (HAR) model. The HAR model is popular because it is simpler to estimate than fractionally integrated processes. Our extension retains this advantage and we can use the OLS method to estimate the model.

In Chapter 1, we extend the standard SV model by incorporating the duration information to model and forecast the intraday return volatility. The duration information is calculated from the Autoregressive Conditional Duration (ACD) model. We first generate the expected duration from the ACD model. Then we transform the irregular space duration to a regular space duration. The effect of surprise in durations can be measured by combining the actual and expected durations. According to the empirical results for SPY and MSFT data, the duration information is highly significant for modeling intraday return volatility. An unexpected increase in duration tends to decrease the intraday volatility, whereas an unexpected decrease in duration tends to increase the intraday volatility. The extended intraday SV model outperforms the GARCH and GARCH augmented with duration information.

In Chapter 2, we use the multivariate volatility models to investigate the contagion effects resulting from the US subprime crisis on a sample of EU countries (UK, Switzerland, Netherlands, Germany and France). The existence of financial contagion can be supported by a significant change of cross-market correlation. In financial contagion topic, the assumption of constant correlation should be relaxed, so the dynamic correlation models are widely used by literature. Unlike most of the existing studies, we use the Multivariate SV (MSV) rather than the Multivariate GARCH specification to obtain correlation estimates. We directly compare the contagion effects detected by the Dynamic Correlation-MSV (DC-MSV) and Dynamic Conditional Correlation-GARCH (DCC-GARCH) models. The contagion effect is more significant under the DC-MSV model. We also extend the DC-MSV model by incorporating implied volatility information into the volatility equations (DC-MSV-IV). The DC-MSV-IV fits the data better than the DC-MSV model so that we can get more accurate estimations for the dynamic correlations. Compared with the DC-MSV model, the contagion effect under the DC-MSV-IV model is more significant. We provide the evidence of contagion effects from USA to the investigated EU countries.

In Chapter 3, we model and forecast the realized volatility using an extended HAR model. Long-range dependence is a well documented stylized fact of RV. Fractionally integrated ARFIMA models are widely used to characterize this strong dependency. However, recent studies treat the simple and easy-to-estimate approximate long-memory HAR model as the preferred specification for RV based forecasting. We extend the HAR model to allow the autocorrelation parameter of daily lags to be time varying(TV-HAR). We observe a regular pattern of RV which is captured by the TV-HAR model: if there is an increase in the lag daily RV compared with its longer-term average level (monthly RV), the current RV tends to decrease rapidly to its long term level; conversely, if there is a decrease in the lag daily RV compared with its longer-term average level (monthly RV), that reversion takes longer. We compare the TV-HAR model with the standard HAR and HARQ models. The better performance of the TV-HAR model can be supported by both the simulation and empirical data.

Chapter 1

A Stochastic Volatility Model for Modelling the Impact of Duration Information on Volatility

1.1 Introduction

During recent decades, the rapid development of algorithmic trading systems has boosted the high-frequency trading. How fast an order can be sent to the market and how volatile the market is at that time, are important factors in capturing price and managing risk. By implication, the daily variance and daily volatility model is unable to meet the increasing demand by high-frequency traders and risk managers. Therefore, it is meaningful to study intraday volatility model. Owing to the technological process in trade recording and the growing dominance of electronic trading, it is possible to obtain higher frequencies with fewer recorded errors. For example, the identification problem in the matching process of trade data and quote data has been alleviated. For the data during the 1990s and early 2000s, the Lee and Ready (1991) 'five-seconds rules' is applicable (but not for the later years' data). A trade is linked to the quote posted at least 5s before the corresponding transaction. Because of the development of trade recording, Henker and Wang (2006) argue that the time delay is 1s rather than 5s. After that, the most recent study of Hautsch (2012) find that perfect matching is available nowadays. This is another reason that has inspired the recent high-frequency research.

Most studies tend to aggregate high frequency data into a daily 'realized volatility' (RV) measure to avoid directly modeling intraday returns and volatility (for reviews, see Andersen and Teräsvirta, 2009 and McAleer and Medeiros, 2008). RV methods are a popular volatility forecasting approach. ARFIMA and HAR processes tend to be used for RV as they capture the long memory of RV. However, RV is not appropriate for studying higher frequency variance than daily level. More detail and the relevant literature for studying the intraday volatility model rather than only to focus on RV can be found in Section 1.2.1: Intraday Return Volatility.

Compared with RV, the persistence in autocorrelation on an intraday level is lower than realized volatility, so ARFIMA processes are not appropriate (Hautsch, 2012). GARCH and SV models have been used for modeling intraday volatility. This chapter introduces a new SV model for intraday return volatility. We consider two sampling frequencies: 5 minutes and 10 minutes. Compared with traditional SV model, the intraday SV model incorporates duration information in the variance equation. Duration is defined as the difference between successive transaction times. The relationship between duration and intraday volatility has been identified in the literature, such as Gerhard and Pohlmeier (2002) and Renault and Werker (2011). They illustrate that a considerable proportion of intraday price volatility is caused by duration dynamics. We fit duration with Autoregressive Conditional Duration (ACD) model to obtain expected duration, we then find the average of every 5-minute and 10-minute expected duration as an input for the intraday SV model. We are not the first to use the expected duration for intraday volatility model, but there is no study that attempts to link the SV model with expected duration. With the actual duration and expected duration, we can obtain unexpected duration as a explanation variable of the volatility equation.

Based on the empirical results for the SPY and MSFT intraday data, our main findings and contributions can be summarized as follows: We offer a new model for the intraday return volatility. Unlike Engle (2000)'s ACD-GARCH model, we use the SV specification rather than GARCH to link the duration and volatility, and we transform the irregular space of ACD expected duration to a regular space duration information in order to model the regular space intraday return volatility. Many studies try to model the regular space intraday return volatility but most focus on GARCH specification and do not link duration and volatility. Some recent examples are Darrat *et al.* (2007), McMillan and Garcia (2009) and Engle and Sokalska (2012). We extend the traditional volatility models by incorporating duration as one factor of the volatility equation. It is important to consider duration because it is directly linked with traders' activity and habits, which can influence return volatility. The relationship between duration and volatility has been discussed by literature (see Section 1.2.2). From the estimation results for SPY and MSFT data, we find that the duration information is highly significant for modeling intraday return volatility. In order to see whether or not the intraday SV model offers a better forecasting result, we compare its out-of-the-sample forecasting performance with both the GARCH model and the GARCH duration model based on Mean Absolute Error (MAE). We find that the MAE of intraday SV model is smaller than other models. The different forecasting accuracy is also highlighted by the Diebold-Mariano test.

The rest of the chapter is organized in the following way. Section 1.2 reviews relevant literature. Section 1.3 presents the data description and analysis. Section 1.4 gives details of the intraday SV model. Section 1.5 discusses the estimation method for the intraday SV model. Section 1.6 shows the empirical results, including the in-sample empirical results of the intraday SV model based on different horizons for SPY, S& P 500 and MSFT data, and the out-sample forecasting performance compared with GARCH and GARCH duration models. Section 1.7 concludes.

1.2 Literature Review

In Section 1.2.1, we discuss the reasons for our focus directly the intraday return volatility rather than on the aggregated RV. Section 1.2.2 discusses the intraday return volatility and durations. We first review the SV models for volatility and ACD models for duration, then discuss the links between the volatility and duration based on existing literature.

1.2.1 Intraday Return Volatility

Theoretically supported by Andersen and Bollerslev (1998), Barndorff-Nielsen and Shephard (2001) and Comte and Renault (1998), ex-post nonparametric RV has been widely used in the high-frequency finance area. They show that in a frictionless market the sum of intraday squared returns over a fixed time interval achieves consistency for the underlying squared volatility for that period, when returns are sampled at increasingly higher frequency. RV that incorporates the intraday return information has proven to be extremely useful for studying the daily level volatility, and it has become the natural benchmark against which to gauge daily volatility forecasts (This point is further supported by Andersen *et al.* (1999), Andersen *et al.* (2003) and Andersen *et al.* (2004)).

However, if we want to study and forecast intraday level volatility directly, RV is not appropriate. As mentioned by Andreou and Ghysels (2002) and Oomen (2004), if the fixed time interval of RV changes from daily to higher intraday level, the contamination by microstructure noise causes RV to be a biased and inconsistent estimator of the integrated volatlity. Therefore, Tran (2006) argues that the only way to study the intraday level volatility is to model volatility directly by a parametric method. Daily volatility has been studied in the financial area for a long time but as mentioned by Bauwens and Giot (2001), Beltratti and Morana (1999) and Giot (2005), active market participants are more interested in higher frequency volatility than daily level and the daily volatility cannot meet the demand by the high-frequency traders and risk managers. Hautsch (2012) points out that the development of trading systems together with technology has speeded up trade execution and allows traders to automatize trading strategies, so the ability to capture the best price and manage the risk now strongly depends on how fast you can send your order to the market and how volatile the market is at that specific time. The intraday volatility should be carefully considered rather than only focus on daily volatility.

Some studies try to model the intraday return volatility directly by GARCH or SV models rather than aggregate to realized volatility. Chan *et al.* (1991) use GARCH model to fit 5-minute return volatility and to incorporate dummy variables for the initial 5-minute of every trading days to catch the intraday pattern of volatility. Ederington and Lee (2001) use the GARCH model to fit the intraday return volatility and study the impact of recent past volatilities on predicting intraday volatility. They also use dummy variables to model the intraday volatility pattern. Martens *et al.* (2002) model the intraday return volatility by the GARCH model and investigate the relationship between the intraday seasonal pattern and forecasting performance. Darrat *et al.* (2003) model the 5-minute intraday return volatility by the exponential GARCH (EGARCH) model and try to find the relationship between trading volume and intraday volatility. Similarly, Darrat *et al.* (2007) also use the EGARCH model to study the intraday return volatility.

Giot (2005) applies GARCH, student GARCH for intraday return volatility and focus on the market risk. Engle and Sokalska (2012) specify the conditional variance of intraday return to be a multiplicative product of daily, diurnal, and stochastic intraday volatility, then they use GARCH model to fit the intraday volatility. Tran (2006) model the intraday return volatility by SV model and take account of market microstructure noise. Stroud and Johannes (2014) model the 5-minute intraday return by a two-factor SV model and incorporate the component of intraday pattern in the volatility specification.

1.2.2 Volatility and Duration

A. SV models for Volatility

There are two main streams of modeling the volatility in the financial area: GARCH and SV models. Both can explain the major stylized facts of asset returns. Unlike GARCH models, maximum likelihood estimation (MLE) is difficult for SV models, which is a reason for that the GARCH model is more commonly used by the literature.

The earliest SV model dealing with volatility clustering is introduced by Taylor (1982). It catches unscheduled news by an unpredictable component in volatility terms. Hull and White (1987) is a well known paper in the use of continuous-time SV models for option pricing. The smiles and skews in option implied volatilities can be caught by SV models. It is further confirmed by Renault (1997) who find that smiles and smirks emerge naturally from SV models via leverage effects. Harvey *et al.* (1994) change the distribution of return error term in the standard SV model from normal distribution to t-distribution (t-SV model). The main motivation for the t-SV model is that the kurtosis in many daily financial series is greater than the kurtosis which results from incorporating conditional heteroscedasticity into a Gaussian process. Harvey and Shephard (1996) offer an asymmetric SV model that incorporates the leverage effect in the standard SV model. The leverage effect is first introduced by Black (1976): the volatility of stocks tends to increase as the price drops. Based on the asymmetric SV model Harvey and Shephard (1996) find that the leverage effect is significant for equity markets but is less significant for currency market. Eraker *et al.* (2003) extend the standard SV models by adding jumps to the price process, which can catch significant discontinuities of the price process. The two factor stochastic volatility has also been considered in models of return volatility (e.g. Bollerslev and Zhou, 2002; Alizadeh *et al.*, 2002). One of the two volatility factors is strongly mean-reverting, and close to independent; the other is highly persistent, and close to non-stationary. The two factors can capture both the jump process and the persistence of the volatility. The model structure is shown in Section 3.5.1. As mentioned by Shephard and Andersen (2009), specifying the (log) volatility process via a sum of first-order autoregressive components, leading to multi-factor SV models, can approximate the long memory feature of volatility. It is also possible to directly incorporate the longer run volatility dependencies. For example, Harvey (2002) introduce a long memory SV model and model the log volatility as a fractionally integrated process. Jacquier and Miller (2010) incorporate RV in the standard SV model, where they find the technically simple addition of exogenous variables to the volatility equation is potentially very useful extension of the SV model.

Due to the difficulty of using MLE to estimate the SV model, the early SV paper by Taylor (1982) calibrated the discrete-time model using the method of moments (MM). Harvey *et al.* (1994) suggest a quasi-maximum likelihood (QML) method that is based on the Kalman-filtering approach. Jacquier *et al.* (1994) introduce a likelihood-based

Bayesian Markov Chain Monte Carlo (MCMC) method for SV model. Jacquier *et al.* (2004) further extend the SV model by incorporating fat-tails and correlated errors. Kim *et al.* (1998) introduce a mixture sampler method for the SV model. It is a more efficient algorithm that overcomes the slow convergence of the Jacquier *et al.* (1994) MCMC algorithm. Chib *et al.* (2002) extend the method for SV-jumps and SV-t models, and Omori *et al.* (2007) extend the method for the SV-leverage model.

Similar to GARCH models, SV models can be applied to intraday level volatilities (e.g. Tran, 2006; Stroud and Johannes, 2014). But the impact of duration information on volatility has not previously been incorporated into the volatility equation of SV models. So in this chapter we incorporate the duration information to study the intraday return volatility. We use the unexpected duration as a component of the volatility equation.

B. ACD Models for Duration

In order to model the time between every two successive trades, Engle and Russell (1998) introduce the ACD model. Similar to the GARCH model for volatility, the ACD model catches duration clustering and is widely used for calculating expected duration. As mentioned by Hautsch (2012), the model can be directly applied to any other positive valued (continuous) process, such as trading volumes (Manganelli, 2005) , market depth, bid-ask spreads or the number of trades (if they are sufficiently continuous). The basic idea is to (dynamically) parameterize the conditional duration mean rather than the intensity function itself.

Engle and Russell (1998) introduce the most popular ACD model that assumes that the error term follows the standard exponential distribution; it is also called EACD. They also use the standard Weibull distribution as the error term of ACD model and the model is so called WACD model. The generalized gamma distribution (Lunde, 1999) and the Burr distribution (Grammig and Maurer, 2000) also have been used for ACD model. Bauwens and Giot (2000) propose a logarithmic ACD (LACD) model that allows the introduction of additional variables without sign restrictions on their coefficients, as the LACD ensures the non-negativity of durations. Fernandes and Grammig (2006) develop a family of augmented ACD (AACD) models that encompasses the standard ACD model, the Log-ACD model and other ACD models inspired by the GARCH literature. Some extended ACD models allow for regime-dependence of the conditional mean function. Zhang *et al.* (2001) propose a threshold ACD (TACD) model to allow the expected duration to depend nonlinearly on past information variables. Unlike the TACD model, where the transition between states follows a jump process, Meitz and Teräsvirta (2006) introduce a smooth transition ACD (STACD) model. Based on the strong persistence of the trading duration, some long memory ACD models have been introduced. Based on the Ding and Granger (1996) two-component model for volatility, Engle (2000) applies the two-component model for duration. This allows for a slower decay autocorrelation function compared to the corresponding standard model. Jasiak (1999) introduces a fractionally integrated ACD (FIACD) model which is based on a fractionally integrated process for the expected duration. The FIACD model is closely linked with the fractionally integrated GARCH model proposed by Baillie *et al.* (1996). The FIACD model is not covariance stationary and implies infinite first and second unconditional moments of the duration. Karanasos (2004) provides an alternative long memory ACD model which is analogous to the long-memory GARCH introduced by Robinson and Henry (1999). Drost and Werker (2004) develop a semiparametric

ACD model that can relax the assumption of independently, identically distributed innovations of the standard ACD model. Like the similarity between the ACD and GARCH models, and based on the idea of SV model, Bauwens and Veredas (2004) propose the stochastic conditional duration (SCD) model for duration. The SCD model is based on the assumption that the durations are generated by a dynamic stochastic latent variable.

The estimation methods for ACD models are dependent upon the assumptions of the error terms. A nature choice is that the error terms follow the exponential distribution, because it is the central distribution for stochastic processes defined on positive support and can be seen as the counterpart to the normal distribution for random variables defined on $\mathbb{R}$ (Hautsch, 2012; Pacurar, 2008). A considerable advantage of error terms in this form is that it allows QML estimator for the ACD parameters (Engle and Russell, 1998). The estimators are consistent and asymptotically normal as discussed by Engle (2002b), building on results by Lee and Hansen (1994). Drost and Werker (2004) also discuss that consistent estimates are obtained when the QML estimation is based on the standard gamma family, hence including the exponential. For more general distributions of the error terms, the ACD model is not estimated by QML but by standard ML. Allen *et al.* (2008) point out that the exponential QML properties of the linear ACD model cannot be straightforwardly carried over to the Log-ACD model. They propose estimating the Log-ACD model based on the log-normal distribution.

Most literature relating to ACD models focus on modelling the duration itself. Few studies try to incorporate the duration information calculated from the ACD models into the intraday volatility models, and even fewer consider SV models. In this chapter, we calculate the expected duration based on the ACD model, and then transform them to regularly 5-minute duration information as a component of volatility equation. In the next subsection we discuss the links between intraday volatility and duration that have been identified in the literature.

C. The Links between Volatility and Duration

An intuitive sense of the links between intraday volatility and duration is gained from the intraday pattern of duration and volatility. We first discuss intraday patterns for financial modelling, before showing show the links behind the intraday patterns. Within a trading day financial markets are subject to significant seasonality patterns. Before the appearance of ACD models, intraday financial studies were mainly focused on the behavior of volatility. According to Andersen and Bollerslev (1997), the intraday volatility pattern should be considered when we model the intraday return volatility. Similarly, in the context of duration data, Engle and Russell (1998) also remove the intraday pattern of duration before estimate ACD models.

We can find the links between the intraday patterns of volatility and duration from literature. The intraday volatility has a clear U-shape that has been reported by many studies, such as Wood *et al.* (1985), Andersen and Bollerslev (1997), Areal and Taylor (2002) and Taylor (2005). The intraday pattern of duration has an inverted-U-shape, see Bauwens and Giot (2003), Bauwens and Veredas (2004) and Giot (2005). The regular patterns of intraday financial markets can be explained by the habits of traders. According to Taylor (2005), the volatility pattern has a U-shape because traders tend to be very active at the opening and closing of every trading days. On the other hand, lunchtime is naturally associated with less trading activity. As trades are more frequent, their duration will be shorter; and vice versa.

In addition to the links that can be found from intraday patterns, duration can offer useful information for studying the intraday volatility. For example, Hautsch (2012) points out that, since a trade reflects demand for liquidity, a trade duration is associated with the intensity of liquidity demand. Liquidity and volatility tend to be positively correlated. Giot (1999) argues that a market featuring short durations is usually associated with possible informed trading. As mentioned by Admati and Pfleiderer (1988), a higher proportion of informed traders leads to higher adverse selection cost and higher volatility. So short duration indicates high volatility. Easley and O'HARA (1992) point out that short trade durations signify news arrival in the market that increases information-based trading. News and volatility tend to be positively correlated because the market maker needs to adjust his prices to reflect the increased uncertainty and risk of trading with informed traders. Engle (2000) supports the their idea based on empirical tests. He introduces an intraday GARCH model and incorporates the duration information in the model. He applies the model to IBM data and finds a statistically negative relation between duration and volatility. Zhang *et al.* (2001) introduce a TACD model and apply the model to IBM data. They find that fast trading regime is characterized by higher volatility, larger volume and wider spreads. Feng *et al.* (2004) examine the relationship between duration and volatility by regressing the realized volatility against the forecast of trade duration based on the SCD models. Using IBM, Boeing, and Coca Cola stocks data, they find a significantly negative relation between trade durations and volatility. Russell and Engle (2005) introduce an Autoregressive Conditional MultinomialAutoregressive Conditional Duration (ACM-ACD) model with three lags for the ACM model and two lags for the ACD model to investigate the relationship between duration and volatility, using Airgas stock traded on NYSE, they find that volatility per unit time is highest for short durations. That is consistent with the predictions of Easley and O'HARA (1992). Spierdijk (2004) extend the Dufour and Engle (2000) VAR model for five stocks using five NYSE stocks and find that volatility is higher when durations are short.

1.3 Data Description and Analysis

We use SPY and MSFT intraday data. The "Trade and Quote" (TAQ) database released by the NYSE is widely used. It consists of two parts: the quote database and the trade database. The trade database offers the transaction price, trading volumes, the exact time stamp used to calculate duration, and attribute information on the validity of the transaction. The quote database presents time stamped (best) bid and ask quotes, the volume for which the particular quote is valid (market depth), as well as additional information on the validity of the quotes.

As ACD models are based on tick data, researchers tend to use three months or less intraday data, such as Engle and Russell (1998), Bauwens and Giot (2000), Grammig and Maurer (2000) and Fernandes and Grammig (2006). Our intraday SV model with expected duration calculated from the ACD model is applied to SPY and MSFT from 4 Jan 2010 to 30 Apr 2010: a total of 17 weeks intraday data. Trades before 9:30 AM and after 4:00 PM are discarded. Estimation and comparison with the GARCH and GARCH duration models are based on different (rolling) forecasting horizons:

(1) in-sample: 1-13 weeks, out-of-sample: 14th week

(2) in-sample: 2-14 weeks, out-of-sample: 15th week

(3) in-sample: 3-15 weeks, out-of-sample: 16th week

(4) in-sample: 4-16 weeks, out-of-sample: 17th week

High frequency data tend to have recording errors and the same time stamp always has several records. Therefore, we follow the procedure for data clean offered by Barndorff-Nielsen *et al.* (2009). Sections 1.3.1 and 1.3.2 discusses the properties of the trade duration and intraday return series respectively.

1.3.1 Trade Duration

The trade duration, defined as the time between successive transactions, is incorporated as a component of our intraday SV model. Intraday financial data are irregularly time spaced as trades and quotes are recorded as soon as they are reported. As listed in the literature review, the intraday volatility models, either GARCH family or SV family, tend to be performed to an aggregated regularly spaced data, such as the 5-minute returns. If we focus only on the aggregated 5-minute return data and ignore the exact time stamp offered by the original TAQ data, information on trade duration is lost. Therefore, in order to incorporate the duration information that is most closely linked with intraday volatility, we use the tick data and consider the duration models before aggregating to the 5-minute data for volatility models. Following Hautsch (2012) we present statistical tables and histograms for trade duration. The properties of durations based on the table and figures are in line with Hautsch (2012)'s findings.

[Table 1.1 around here]

[Figure 1.1 around here]

Table 1.1 shows duration statistics of SPY and MSFT. We can find that the mean duration is relatively short. Compared with the SPY, MSFT trades occur less frequent with higher mean duration. It also can be reflected from the number of observations, as for the same length of sample time, SPY has more observations than MSFT. Both SPY and MSFT have high Kurtosis and positive skewness. Most of durations are around or less than the mean of them. Even though there exist much longer duration than the mean duration, they happen only very infrequently. Figure 1.1 shows the time plots of SPY and MSFT transaction durations.

[Figure 1.2 around here]

As the ACD model is proposed as a model for intertemporally correlated trading arrival times, we examine the dependence of duration by calculating its autocorrelations and partial autocorrelations. Figure 1.2 shows Autocorrelation functions of durations for the SPY and MSFT respectively. Both have significant positive autocorrelations revealing a strong persistence of the process. As mentioned by Engle and Russell (1998), intraday seasonality partly contributes to the autocorrelations, but the ACD model is used to fit the intraday seasonally-adjusted duration, so it is necessary to analyse the autocorrelations for the intraday seasonally-adjusted duration as well. Section 1.4.1 presents the calculations for the intraday seasonal pattern and the intraday seasonally-adjusted durations.

[Table 1.2 around here]

[Table 1.3 around here]

Tables 1.2 and 1.3 show the autocorrelations and partial autocorrelations of trade durations for the SPY and MSFT data respectively. As the four horizons have slightly different intraday seasonal pattern, we report the intraday pattern adjusted duration based on the first horizon sample data. The remaining horizons have the same conclusion for autocorrelations and partial autocorrelations of trade durations. The sample sizes for SPY and MSFT are 562608 and 125303 respectively. In the two tables, we can show the autocorrelations and partial autocorrelations are far from zero and all the signs are positive for both raw durations and intraday seasonally-adjusted durations (diurnally adjusted duration in tables). The columns "Q-Stat" and "P-value" are the Ljung-Box Q-statistics and their p-values. The Q-statistic at lag k is a test statistic for the null hypothesis that there is no autocorrelation up to order k. The null is very easily rejected with high values of Q-Stat and 0 p-values which further supports the existence of positive autocorrelations and partial autocorrelations. This suggests that the large Ljung-Box statistic observed for the raw durations is not a result of the intraday seasonality alone, which is in line with Engle and Russell (1998) and supports the existence of duration clustering. The ACD model is designed to capture the intertemporal autocorrelation.

1.3.2 Intraday Returns

The 5-minute and 10-minute returns are computed as $r_t = 100\,[\ln(P_t) - \ln(P_{t-1})]$, where P_t is the mean of stock prices during the t interval of 5 minutes and 10 minutes. Following Stoll and Whaley (1990) and Rahman *et al.* (2002), the first two 5-minute returns of the day are excluded from the analysis, because prices during these intervals may reflect the stale closing price of the previous day, and tend to be contaminated by the record errors.

[Table 1.4 around here]

Table 1.4 shows the intraday return statistics of SPY and MSFT. During the sample period, the mean of return for the SPY is higher than the MSFT, with lower standard deviation. Compared with the 5-minute return, the 10-minute return is more volatile and has a higher peak based on the larger kurtosis.

[Figure 1.3 around here]

Figure 1.3 show the density histograms of intraday returns for SPY and MSFT respectively. Data are sorted into a specified number of bins and the histogramplots the counts of observations falling in each bin. The histogram is normalized so that the area under the bars sums to one (essentially making it into a discrete probability density function). We also plot normal distributions for comparison. Compared with normal distributions, the intraday returns of both SPY and MSFT have higher peaks. Especially important is the fact that the histograms have fatter tails than the normal distribution. The fat tails should be carefully considered by the risk managers: it means that more extreme returns possibly to be happen than the normal distribution predicts.

1.4 Intraday SV Model with Duration

This section shows the details of the model structure. Section 1.4.1 shows the intraday patten adjustment for both duration and volatility and Section 1.4.2 shows the model structure of intraday SV.

1.4.1 Intraday Pattern Adjustment for Duration and Volatility

Duration is defined as the difference between successive transaction times $\Delta x_{i+1} = x_{i+1} - x_i$. Before applying the ACD model to duration, we should first deal with the intraday periodicity pattern in duration. We follow the detail for intraday adjustment for both duration and volatility from Giot (1999). His sequential method is based on the cubic spline, which is a widely used method for dealing with the intraday seasonal pattern (see, for example, Engle and Russell, 1998; Giot, 1999; Pacurar, 2008; Bauwens *et al.*, 2004).

Let $s_{d,i}$ be the intraday seasonal factor of duration at time i, and $\widetilde{\Delta x_i}$ be the the diurnally adjusted duration. Following Engle and Russell (1998), duration can be written as

$$\Delta x_i = \widetilde{\Delta x_i}\, s_{d,i} \tag{1.1}$$

For the return series, we consider sampling frequencies: 5-minute and 10-minute, denoted as r_t. We denote the intraday seasonal factor of variance at time t as $s_{v,t}$. Let $\widetilde{r_t}$ be the return after intraday pattern adjustment. Then the return can be written as

$$r_t = \widetilde{r}_t \sqrt{s_{v,t}} \tag{1.2}$$

We use $\sqrt{s_{v,t}}$ rather than $s_{v,t}$ in Equation 1.2 is that we use the squared returns to calculate the intraday volatility pattern rather than using return directly.

The intraday seasonal components for duration and volatility are calculated as follows (Giot, 1999):

1. We compute the average duration and average squared return for each 30 minutes interval, denoted respectively as $D_{m,n}$ and $V_{m,n}$ for the m interval on day n.

2. Take accounts of the day of the week. Let S_n be the set of daily time indexes for the same day of the week as time the index n. Let N_n be the number of time indexes to be found in S_n. Then we can calculate the crude pattern by

$$\overline{D}_m = \frac{1}{N_n}\sum_{S_n} D_{m,n} \qquad \overline{V}_m = \frac{1}{N_n}\sum_{S_n} V_{m,n} \tag{1.3}$$

3. The crude pattern is then smoothed by using cubic splines. The durations and squared returns are sampled with 10 minutes.

1.4.2 Model Structure

We use the information of expected duration calculated from the ACD model, as an input for the intraday SV model.

With Ψ_i defined as conditional expected duration, the ACD model is written as

$$\Psi_i = E[\Delta x_i|\Delta x_{i-1},\ldots,\Delta x_1] = E[\widetilde{\Delta x}_i|\widetilde{\Delta x}_{i-1},\ldots,\widetilde{\Delta x}_1]\; s_{d,i} = \widetilde{\Psi}_i\; s_{d,i} \tag{1.4}$$

$$\widetilde{\Psi}_{i+1} = \omega_d + \alpha_d(\widetilde{\Delta x_i}) + \beta_d\widetilde{\Psi}_i \tag{1.5}$$

The standardized residuals of the ACD model $\xi_i = \frac{\widetilde{\Delta x_i}}{\widetilde{\Psi}_i}$ are assumed to be i.i.d exponential distributed with $E(\xi_i) = 1$. The autoregressive structure on the conditional expectation of the durations implies the duration clustering. Based on Equation 1.5, we obtain the sequence of deseasonalized expected duration $\widetilde{\Psi}_i$. We then calculate the mean of corresponding 5-minute and 10-minute $\widetilde{\Psi}_i$, noted as $\overline{\Psi}_t$, as an input data for the intraday SV model for 5-minute and 10-minute returns respectively. We also need to calculate the mean of every 5-minute and 10-minute deseasonalized durations $\widetilde{\Delta x_i}$, denoted by $\overline{X}_t$. We transform the irregularly spaced duration information to regularly spaced duration information.

The mean of durations divide by the mean of expected durations $(\overline{X}_t/\overline{\Psi}_t)$ can be incorporated in the SV specification. We use this form following the structure of error terms in the ACD model. It measures the durations for each interval are generally longer or shorter than the expectation of the durations (the effects of surprises in durations). If the value of $(\overline{X}_t/\overline{\Psi}_t)$ is smaller than 1, then at time t the actual duration is smaller than the expected duration conditional on the past information, in other words, there is an unexpected decrease for the actual duration at time t. On the other hand, if the value of $(\overline{X}_t/\overline{\Psi}_t)$ is bigger than 1, then there is an unexpected increase for the actual duration at time t.

Following Tran (2006), we assume $E(r_t) = 0$ and choose to exclude the instantaneous expected rate of return. This practice is common when working with high frequency data (e.g. Bollerslev and Zhou, 2002; Engle and Sokalska, 2012). Merton (1980) firstly points out that when estimating the variance of asset return it is more accurate when leaving out the drift part. So the model is:

$$\widetilde{r}_t = e^{h_t/2}\varepsilon_t \tag{1.6}$$

$$h_t = \theta + \phi h_{t-1} + \kappa\left(\frac{\overline{X}_{t-1}}{\overline{\Psi}_{t-1}} - 1\right) + \sigma\eta_t \tag{1.7}$$

where $\varepsilon_t \sim i.i.d.N(0,1)$, $\eta_t \sim i.i.d.N(0,1)$. In order to illustrate the idea of the intraday SV model more clearly, we firstly use $(\overline{X}_{t-1}/\overline{\Psi}_{t-1} - 1)$ rather than $(\overline{X}_{t-1}/\overline{\Psi}_{t-1})$ in Equation (1.6). So when $(\overline{X}_{t-1}/\overline{\Psi}_{t-1} - 1) > 0$, there is an unexpected increase for actual duration; and when $(\overline{X}_{t-1}/\overline{\Psi}_{t-1} - 1) < 0$, there is an unexpected decrease for the actual duration. With negative κ, an unexpected increase for the lag duration tends to have a negative effect on the current volatility, because in that case the value of $(\overline{X}_{t-1}/\overline{\Psi}_{t-1} - 1)$ is positive which times the negative value of κ, the overall effect then is negative; whereas an unexpected decrease for the lag actual duration tends to positively impact the current volatility.

We can rewrite Equation (1.7) as

$$h_t = (\theta - \kappa) + \phi h_{t-1} + \kappa\frac{\overline{X}_{t-1}}{\overline{\Psi}_{t-1}} + \sigma\eta_t \tag{1.8}$$

We use γ to replace $(\theta - \kappa)$ which simplifies the model. So the volatility equation can be written as

$$h_t = \gamma + \phi h_{t-1} + \kappa\frac{\overline{X}_{t-1}}{\overline{\Psi}_{t-1}} + \sigma\eta_t \tag{1.9}$$

We report the empirical results using the Equation (1.9).

1.5 Intraday SV Model Estimation

We estimate the ACD model following the procedure described in Engle and Russell (1998). After getting the expected duration based on the estimated ACD model, we estimate the Intraday SV model following Harvey *et al.* (1994) QML method.

For the ACD model, the assumption that the error term follows the exponential distribution has an advantage of leading to a QML estimator for the parameters. The quasi log likelihood function is given by

$$\ln L(\boldsymbol{\Delta x};\boldsymbol{\theta_d}) = -\sum_{i=1}^{n}\left[\ln\Psi_i + \frac{\Delta x_i}{\Psi_i}\right] \tag{1.10}$$

where $\boldsymbol{\theta_d}$ is the set of ACD parameters and $\Psi_1 = \omega_d/(1-\beta_d)$. The QML estimator is based on the theorems introduced by Lee and Hansen (1994) and Lumsdaine (1996). As mentioned by Engle and Russell (1998), an important implication of the strong analogy between the Gaussian GARCH model and the Exponential ACD model is that the ACD model can be estimated with GARCH software by taking $\sqrt{\Delta x_i}$ as the dependent variable and setting the mean to zero.

For the extended SV model, the difficulty in using the maximum likelihood estimation (MLE) method is that the volatility terms of SV models are latent variables, the likelihood function is not available in a closed form (it is expressed as an analytically intractable T-dimensional integral, where T is the number of observations). In order to overcome this difficulty, other methods have been introduced to estimate SV models, see the summary of Broto and Ruiz (2004). Here we use the QML method to estimate the intraday SV model.

We first transform $\widetilde{r}_t = e^{h_t/2}\varepsilon_t$ to

$$\ln \widetilde{r}_t^2 = h_t + \ln \varepsilon_t^2 \tag{1.11}$$

where $\varepsilon_t \sim i.i.d.N(0,1)$, so $\ln \varepsilon_t^2 \sim i.i.d(-1.27, \pi/2)$. Then we rewrite equation (1.11) as

$$\ln \widetilde{r}_t^2 = -1.27 + h_t + \xi_t \tag{1.12}$$

where $\xi_t \sim i.i.d(0, \pi/2)$. We approximate $\xi_t \sim i.i.d.N(0, \pi/2)$, then the linearized SV model is approximated by a linear Gaussian State Space model. We estimate the State Space model based on the Kalman filter, see Harvey (1989), Hamilton (1994) and Koopman *et al.* (1999). We follow the standard notations of Koopman *et al.* (1999). The linear Gaussian State Space model is written as:

$$\alpha_{t+1} = d_t + T_t \alpha_t + H_t \varepsilon_t, \tag{1.13}$$

$$y_t = c_t + Z_t \alpha_t + G_t \varepsilon_t \tag{1.14}$$

where $\alpha_1 \sim N(a, P)$ and $\varepsilon_t \sim i.i.d.\ N(0, I)$, where α_t is a vector of unobserved state variables and y_t is an observation vector. c_t, d_t are exogenous variables. The deterministic matrices T_t, Z_t, H_t and G_t are referred to as system matrices and they are usually sparse selection matrices. For the intraday SV model, $\alpha_t = h_t$, $y_t = \ln \widetilde{r}_t^2$, $c_t = 1.27$, $d_t = \gamma + \kappa(\overline{X}_{t-1}/\overline{\Psi}_{t-1} - 1)$, $T_t = \phi$, $Z_t = 1$, $H_t = \sqrt{\pi/2}$ and $G_t = \sigma$.

For initial conditions, we can either use the usual diffuse prior or fix them at appropriate values to speed up convergence rate. The diffuse prior can be written as $\alpha \sim N(0, \kappa I)$, where κ is large, say $\kappa = 10^6$.

We define the mean and variance matrix of the conditional distribution respectively as

$$a_{t|s} = E_s(a_t) \tag{1.15}$$

$$P_{t|s} = E_s[(\alpha_t - a_{t|s})(\alpha_t - a_{t|s})'] \tag{1.16}$$

This allows us to obtain the one-step ahead mean $a_{t|t-1}$ and the one-step ahead variance $P_{t|t-1}$ by setting $s = t-1$. The one-step ahead prediction error is given by

$$\widetilde{\varepsilon}_t = \varepsilon_{t|t-1} = y_t - E_{t-1}(y_t) = y_t - E(y_t|a_{t|t-1}) = y_t - c_t - Z_t a_{t|t-1} \tag{1.17}$$

The prediction error variance is defined as

$$\widetilde{F}_t = F_{t|t-1} = var(\varepsilon_{t|t-1}) = Z_t P_{t|t-1} Z_t' + G_t G_t' \tag{1.18}$$

As shown in Koopman *et al.* (1999), the one-step ahead estimates of the state and the associated mean square error matrix can be written as

$$K_t = (T_t P_t Z_t' + H_t G_t') F_t^{-1} \tag{1.19}$$

$$a_{t+1|t} = d_t + T_t a_{t|t-1} + K_t \varepsilon_{t|t-1} \tag{1.20}$$

$$P_{t+1|t} = T_t P_{t|t-1} T_t' + H_t H_t' - K_t F_{t|t-1} K_t' \tag{1.21}$$

The process of using the sequence of data up to time period T to form expectations at any time period up to T is known as fixed-interval smoothing. We use $\hat{\alpha}_t = a_{t|T} = E_T(\alpha_t)$ to denote the smoothed estimates of the state and $V_t = var_T(\alpha_t)$ to denote the smoothed estimates of the state variances. We can use the smoothed values to form smoothed estimates of the signal variables,

$$\hat{y}_t = E(y_t|\hat{\alpha}_t) = c_t + Z_t\hat{\alpha}_t \tag{1.22}$$

and to compute the variance of the smoothed signal estimates:

$$S_t = var(\hat{y}_{t|T}) = Z_t V_t Z_t' \tag{1.23}$$

We can modify the expressions in (1.15)—(1.18) to get the n-step ahead state conditional mean and variance:

$$a_{t+n|t} = E_t(\alpha_{t+n}) \tag{1.24}$$

$$P_{t+n|t} = E_t[(\alpha_{t+n} - a_{t+n|t})(\alpha_{t+n} - a_{t+n|t})'] \tag{1.25}$$

the n-step ahead forecast,

$$y_{t+n|t} = E_t(y_{t+n}) = c_t + Z_t a_{t+n|t} \tag{1.26}$$

and the corresponding n-step ahead forecast MAE matrix

$$F_{t+n|t} = MSE(y_{t+n|t}) = Z_t P_{t+n|t} Z_t' + G_t G_t' \tag{1.27}$$

Based on one-step-ahead prediction error and the corresponding mean-squared error given in Equation (1.17) and Equation (1.18), the likelihood function is given by

$$\ln L(\theta) = -\frac{nT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(\ln|\widetilde{F}_t| + \widetilde{\varepsilon}_t'\widetilde{F}_t^{-1}\widetilde{\varepsilon}_t\right) \tag{1.28}$$

According to Ruiz (1994), the QML estimator is consistent and asymptotically normal. Jacquier *et al.* (1994) show that the QML procedure is inefficient as the method does not rely on the exact likelihood function, and support their argument by simulation results. However, Sandmann and Koopman (1998) and Breidt and Carriquiry (1996) state that although the QML is inefficient, it is not as bad as Jacquier *et al.* (1994) show. They suggest that the bad results of QML in Jacquier *et al.* (1994) may be due to an inefficient implementation of the procedure (such as poor starting values, different convergence criteria, etc.). Compared with GMM which is another popular nonlikelihood-based method for SV models, Andersen and Sørensen (1996) find that QML estimator is better for models with a high degree of persistence. Deo (2002) provides theoretical intuition for this finding. Despite the limitations, the QML estimator is very flexible and has been widely used for estimating SV models.

1.6 Empirical Illustration

This section outlines the empirical results. Section 1.6.1 shows the estimation results of the ACD and intraday SV models. Section 1.6.2 shows the GARCH and the GARCH-D models that are used to compare with the intraday SV model. Sections 1.6.3 and 1.6.4 respectively discusses the in-sample and out-of-sample forecasting results for different horizons and sampling frequencies.

1.6.1 Estimation Results

This subsection shows the in-sample results of the intraday SV model. We firstly see the results of the ACD model. After that, we list the results of the volatility part for the intraday SV model.

[Table 1.5 around here]

Table (1.5) shows the estimation results of the ACD models for SPY and MSFT respectively. The standard errors are given in parentheses. All the parameters of ACD models are highly significant. From the results of the four different horizons, the sums of α_d and β_d for SPY data are 0.9903, 0.9899, 0.9906 and 0.9908 respectively. For MSFT data, the sums of α_d and β_d are 0.9956, 0.9954, 0.9958 and 0.9957 respectively. The results indicate that the duration has a strong persistence. That is in line with Engle and Russell (1998). As summarized by Pacurar (2008), many studies found evidence of high persistence of trade durations, the sum of the autoregressive coefficients (i.e. $\alpha_d + \beta_d$) being close to one while still in the stationary region. It supports the existence of duration clustering that the long duration tends to be followed by long duration, and short duration tends to be followed by short duration. Based on the estimated ACD models, we can get the expected durations. After that, the effects of surprises in durations for 5-minute and 10-minute returns can be calculated as shown in Section 1.4.2.

[Tables 1.6 and 1.7 around here]

Tables 1.6 and 1.7 shows the estimation results of the intraday SV model for SPY and MSFT data respectively. Each table contains the estimation results for 5-minute returns and 10-minute returns. For 5-minute returns, the estimate values of parameter ϕ for both SPY and MSFT are around 0.98, indicating the existence of volatility clustering in the intradaily level and the volatility has strong persistence. For 10-minute returns, the values of ϕ are slightly lower compared with 5-minute returns. The lower effects of AR(1) can be partly contributed by the higher effects of surprises in durations.

The κ is the coefficient of the effects of surprises in durations $(\overline{X}_{t-1}/\overline{\Psi}_{t-1})$. The estimate values of κ are negative for SPY and MSFT. From Equation (1.7) we can find the relationship between the duration information and the unobserved log-volatilities h: if the duration in time $(t-1)$ is higher than the expectation, in other words, there is an unexpected increase in the duration, then the effect of duration information for volatility in time t is negative. On the other hand, if there is an unexpected decrease in the duration compared with the expectation, the effect of duration information is positive. Compared with the 5-minute returns, the absolute values of κ are bigger when we fit the model for 10-minute returns, indicating the higher effects of surprises in durations. The negative relationship between the unexpected duration and volatility is in line with Engle (2000). The unexpected decrease in duration is closely linked with unexpected news arrival in the market, which can increase the volatility. The unexpected decrease in duration indicates the possible informed trading. The higher informed trading leads to higher adverse selection cost and higher volatility. On the other hand, the unexpected increase in duration indicates the stable market. In this case, the confidence of investors and traders to the market increases so that they are more likely to hold their assets rather than trade.

1.6.2 Competing Models

This section shows the detail of the GARCH and the GARCH-D models that are used to compare with the intraday SV model. The GARCH model is introduced by Bollerslev (1986) based on ARCH (Engle, 1982). The GARCH-D model links the GARCH model and the expected duration calculated from the ACD model.

The GARCH(1,1) model is defined as follows:

$$\widetilde{r}_t = c + \alpha \widetilde{r}_{t-1} + e_t \tag{1.29}$$

Following Giot (1999), Rahman *et al.* (2002) and Worthington and Higgs (2009), we fit an AR(1) structure on the intraday returns. The variance equation is written as:

$$\sigma_t^2 = \lambda + \beta e_{t-1} + \gamma \sigma_{t-1}^2 \tag{1.30}$$

where $\widetilde{r}_t$ is the 5-minute intraday volatility pattern adjusted return at time t. The GARCH family models have been used for modeling intraday return volatility by many studies, such as Chan *et al.* (1991), Giot (1999), Martens *et al.* (2002), Darrat *et al.* (2003) and Darrat *et al.* (2007). For the method to deal with the intraday pattern of volatility, Chan *et al.* (1991) differs from the studies above. They use dummy variables to catch the intraday pattern of volatility. All others use the sequential method. Here we follow the general literature and use the sequential method.

We incorporate the duration information for the GARCH-D model which can be written as follows:

$$\widetilde{r}_t = c + \alpha \widetilde{r}_{t-1} + e_t \tag{1.31}$$

$$\sigma_t^2 = \lambda + \beta e_{t-1} + \gamma \sigma_{t-1}^2 + \theta \frac{\overline{X}_{t-1}}{\overline{\Psi}_{t-1}} \tag{1.32}$$

where $\overline{X}_t$ is the mean of every 5-minute deseasonalized durations and $\overline{\Psi}_t$ is the mean of every 5-minute deseasonalized expected durations. The expected duration can be calculated from the ACD model listed in Section 1.4.2. The in-sample estimation results of GARCH and GARCH-D are given in the Appendix.

1.6.3 The In-sample Fit

In this section, we perform diagnostic checks on the fit of the intraday SV, GARCH and GARCH-D models before discussing the forecasting performances of these models. Based on the in-sample fit test, we can assess the adequacy of the model and how well the fitted model accords with the observed data.

When the QML method is used to estimate SV models based on the State Space model and the Kalman filter, it is common to test the serial correlation of the standardized prediction residual (Krichene, 2003; Liesenfeld and Richard, 2003; Eratalay, 2012). The details of calculating the one-step ahead prediction error $\widetilde{\varepsilon}_t$ and the prediction error variance $\widetilde{F}_t$ are shown in Section 1.5. The standardized prediction residual is calculated as $e_t = \widetilde{\varepsilon}_t / \widetilde{F}_t$. When the model fits the data, the standardized prediction residuals are serially uncorrelated. We perform the Ljung-Box test to check whether or not there exists the autocorrelation up to order 10. For GARCH and GARCH-D models, the Ljung-Box test on the squared standardized residuals can be used to test for remaining

ARCH in the variance equation and to check the specification of the variance equation. If the model fit the data well, the squared standardized residuals should be uncorrelated.

[Tables 1.8 around here]

Table 1.8 shows the results of the Ljung-Box test on 10 lags for 5-minute sampling frequency and 10-minute sampling frequency respectively. We use 10 lags for the test following Bauwens and Giot (2000), Krichene (2003). As the observations for SV models based on QML method are log squared intraday pattern adjusted returns $\log(\widetilde{r}_t^{\,2})$, we also report the Ljung-Box test for $\log(\widetilde{r}_t^{\,2})$. The e_{isv} is the standardized prediction residual of Intraday SV model. $\widetilde{r}_t^{\,2}$ is the squared intraday pattern adjusted returns. SE_{garch} is the squared standardized residuals of GARCH model and SE_{garchd} is the squared standardized residuals of GARCH-D model. The p-value of the Ljung-Box Q-statistics are shown in the square brackets.

As the volatility clustering, we can find that the Q-statistics for $\log(\widetilde{r}_t^{\,2})$ and $\widetilde{r}_t^{\,2}$ are highly significant for both 5-minute returns and 10-minute returns. All the p-values of their Ljung-Box Q-statistics are negative. The Q-statistics are higher when we study 5-minute returns compared with 10-minute returns. For the intraday SV model, the Q-statistics of standardized prediction residuals e_t drop significantly compared with the Q-statistics fitted observations $\log(\widetilde{r}_t^{\,2})$. All their p-values are higher than 10%. So we cannot reject the null hypothesis of the Ljung-Box test that there is autocorrelation for the standardized prediction residuals. For the GARCH model, similarly, the Q-statistics of squared standardized residuals SE_1 are insignificant for all horizons and sampling frequency, indicating they are uncorrelated. We obtain the same conclusion for the GARCH-D model. The Q-statistics for squared standardized residuals support the GARCH and GARCH-D models fitted the data well.

However, based on the Ljung-Box test for the standardized prediction residual of intraday SV model and the squared standardized residuals of GARCH and GARCH-D models, we cannot directly compare the in-sample fit of the intraday SV model and GARCH, GARCH-D models, as the observations for the intraday SV model are $\log(\widetilde{r}_t^2)$ but for GARCH models are $\widetilde{r}_t$. In order to compare their in-sample fit, following Bhar and Lee (2009), Trolle and Schwartz (2009) Kosapattarapim *et al.* (2011), we compare the errors based on the volatilities calculated from fitted models and true values.

[Tables 1.9 around here]

Table 1.9 shows the comparison of in-sample fit for the intraday SV, GARCH and GARCH-D models based on the mean absolute error (MAE). With $\widetilde{v}_t$ be the variance of returns calculated from the estimated model at time t, and v_t be the actual in-sample values of variance. The MAE can be written as

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\widetilde{v}_t - v_t| \tag{1.33}$$

We transform the log squared intraday pattern adjusted returns calculated from intraday SV model to the intraday variance which also incorporates the intraday volatlity pattern as well. As the intraday volatility pattern is the same for both SV, GARCH, and GARCH-D models, so it does not influence the in-sample fit comparison. The lower value of MAE indicates the better in-sample fit. According to the table, the Intraday SV model fit the data better than the GARCH and GARCH-D models.

1.6.4 Forecasting Results

Although the good in-sample fits are important, when we choose a model for practical applications, the out-sample forecasting power should be considered as the ultimate test for comparing different models. So in this section, we compare the intraday SV model with the standard GARCH model and the GARCH duration (GARCH-D) model based on Engle (2000). In order to highlight the difference of forecasting power, we report the Diebold-Mariano test (DIEBOLD and MARIANO, 1995).

The MAE is a widely used measure to test the forecasting power of a model. Let $\widehat{v}_t$ be the one-period-ahead forecasts of return variance at time t, and v_t be the actual values of variance. The MAE for the out-of-sample forecasts can be written as

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |\widehat{v}_t - v_t| \tag{1.34}$$

Table 1.10 shows the forecasting performance of the intraday SV, GARCH, GARCH-D models based on MAE for 5-minute and 10-minute returns. The results of the four horizons are reported respectively. In addition, we also calculate the average values of MAE for the four horizons.

[Tables 1.10 around here]

For both the SPY and MSFT, the intraday SV model provides the lowest MAE's for all horizons compared with GARCH and GARCH-D, that supports the intraday SV model outperforms the GARCH and GARCH-D models. Compared with GARCH and GARCH-D, we can find that the GARCH-D's MAE's for the four horizons are slightly lower than the GARCH's MAE's, so the duration information is useful for forecasting. The conclusion is same for both the 5-minute and 10-minute returns. Compared with the two sampling frequencies, the 5-minute returns have the lower MAE than the 10-minute returns.

The Diebold-Mariano (DM) test is used to discover whether or not the forecasts of two models are equally good. Let v_t be the actual values of volatility; let $\widehat{v}_{1t}$ be the forecasts of the first model and $\widehat{v}_{2t}$ be the forecasts of the second model. The loss function $g(\widehat{v}_{it})$ of the DM test for model i ($i = 1,2$) is defined as

$$g(\widehat{v}_{it}) = |\widehat{v}_{it} - v_t| \tag{1.35}$$

The loss differential between the two forecasts is defined by

$$d_t = g(\widehat{v}_{1t}) - g(\widehat{v}_{2t}) \tag{1.36}$$

The two forecasts have equal accuracy if and only if the loss differential has zero expectation for all t. So the null hypothesis is

$$H_0 : E(d_t) = 0 \quad \forall t \tag{1.37}$$

against the alternative hypothesis

$$H_0 : E(d_t) \neq 0 \tag{1.38}$$

In other words, the null hypothesis is that the two forecasts have the same accuracy.

The alternative hypothesis is that the two forecasts have different levels of accuracy. Let $\overline{d}$ be the sample mean of loss differential and μ be the population mean of the loss differential; that is

$$\overline{d} = \sum_{t=1}^{T} d_t \tag{1.39}$$

and

$$\mu = E(d_t) \tag{1.40}$$

The spectral density of the loss differential at frequency 0 is

$$f_d(0) = \frac{1}{2\pi}\left(\sum_{k=-\infty}^{\infty} \gamma_d(k)\right) \tag{1.41}$$

where $\gamma_d(k)$ is the autocovariance of the loss differential at lag k. Assuming the loss differential series d_t is covariance stationary and short memory, we obtain

$$\sqrt{T}(\overline{d} - \mu) \to N(0, 2\pi f_d(0)) \tag{1.42}$$

Therefore, under H_0

$$\frac{\overline{d}}{[2\pi f_d(0)/T]^{1/2}} \to N(0,1) \tag{1.43}$$

For the one-step forecast, the DM test statistic is

$$DM = \frac{\overline{d}}{[2\pi \hat{f}_d(0)/T]^{1/2}} \tag{1.44}$$

where $\hat{f}_d(0)$ is a consistent estimate of $f_d(0)$ defined by

$$\hat{f}_d(0) = \frac{1}{2\pi}\hat{\gamma}_d(0) \tag{1.45}$$

where

$$\hat{\gamma}_d(0) = \frac{1}{T}\sum_{t=1}^{T}(d_t - \overline{d})^2 \tag{1.46}$$

Under the null hypothesis, the test statistics DM is asymptotically $N(0,1)$ distributed. We can calculate the p-value based on the computed DM statistic. Table 1.11 shows the results of DM test for the intraday SV model compared with GARCH and GARCH-D for 5-minute and 10-minute returns respectively.

[Tables1.11 around here]

We find that all the p-values are below 10% for both SPY and MSFT data, therefore we reject the null hypothesis that the two forecasts have the same accuracy. So the intraday SV model with lower MAE than GARCH and GARCH-D has better forecasting performance.

1.7 Conclusion

With the rapid development of algorithmic trading systems, the high-frequency trading increases the demand for the intraday volatility rather than only the daily volatility or the aggregated RV. Nowadays, thanks to the advanced trade recording technology, researchers can get access to the higher frequency data with fewer recorded errors, which boosts the recent high-frequency research. We extend the SV model for modeling and forecasting intraday return volatility. Unlike the traditional SV specification, we incorporate the duration information into the variance equation, because the duration is closely linked with volatility. As discussed by the literature, a trade duration is associate with the intensity of liquidity demand which is correlated to volatility. In addition, the short trade durations also signify news arrival in the market that increases information-based trading, which tends to increase the volatility.

The duration information includes both the lag duration and the lag expected duration. The expected duration is calculated from the ACD model. Although there are some literature supporting the negative relationship between duration and volatility, few studies use the expected duration as a component for the intraday volatility modeling. We consider the expected duration rather than only rely on the actual duration because the expected duration allows us to investigate the effects of surprises in durations on intraday return volatility. We transform the irregularly spaced duration information to regularly spaced duration information by using the mean of durations for a specific period divided by its corresponding mean of the expected durations. We use the QML method based on state space model and Kalman filter to estimate the Intraday SV model. The SPY and MSFT data are used for the empirical analysis.

We find that the parameter of the duration information is highly significant, and there is a negative relationship between the unexpected duration and volatility. It means that if there is an unexpected increase for the lag actual duration, the current volatility tends to decrease, and vice versa. For both the duration and volatility modeling, we adjust the intraday pattern before fit the model. The empirical results support that the Intraday SV model fits the data better than the GARCH and GARCH-D models. We also compare their out-of-sample forecasting performances and the Intraday SV model offers more accurate forecasts. The difference of forecasting power can be supported by the DM test. This chapter supports that when we investigate the intraday return volatility, the duration can offer useful information. The link between duration and volatility model might be interesting and useful for the future research.

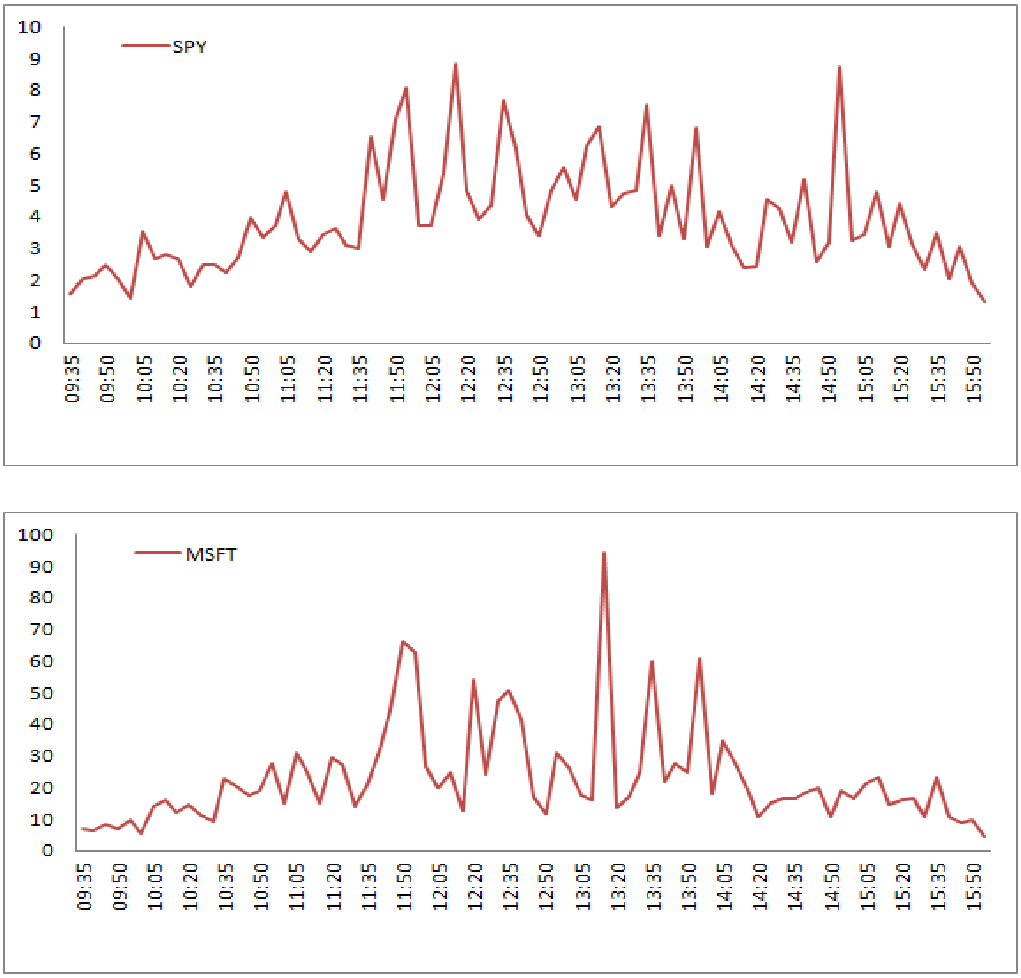

Figure 1.1: This figure show the time plots of SPY and MSFT transaction durations on 5th Jan 2010. As the duration is the irregularly spaced data, we use five minute average durations to show the changes for the whole trading day.

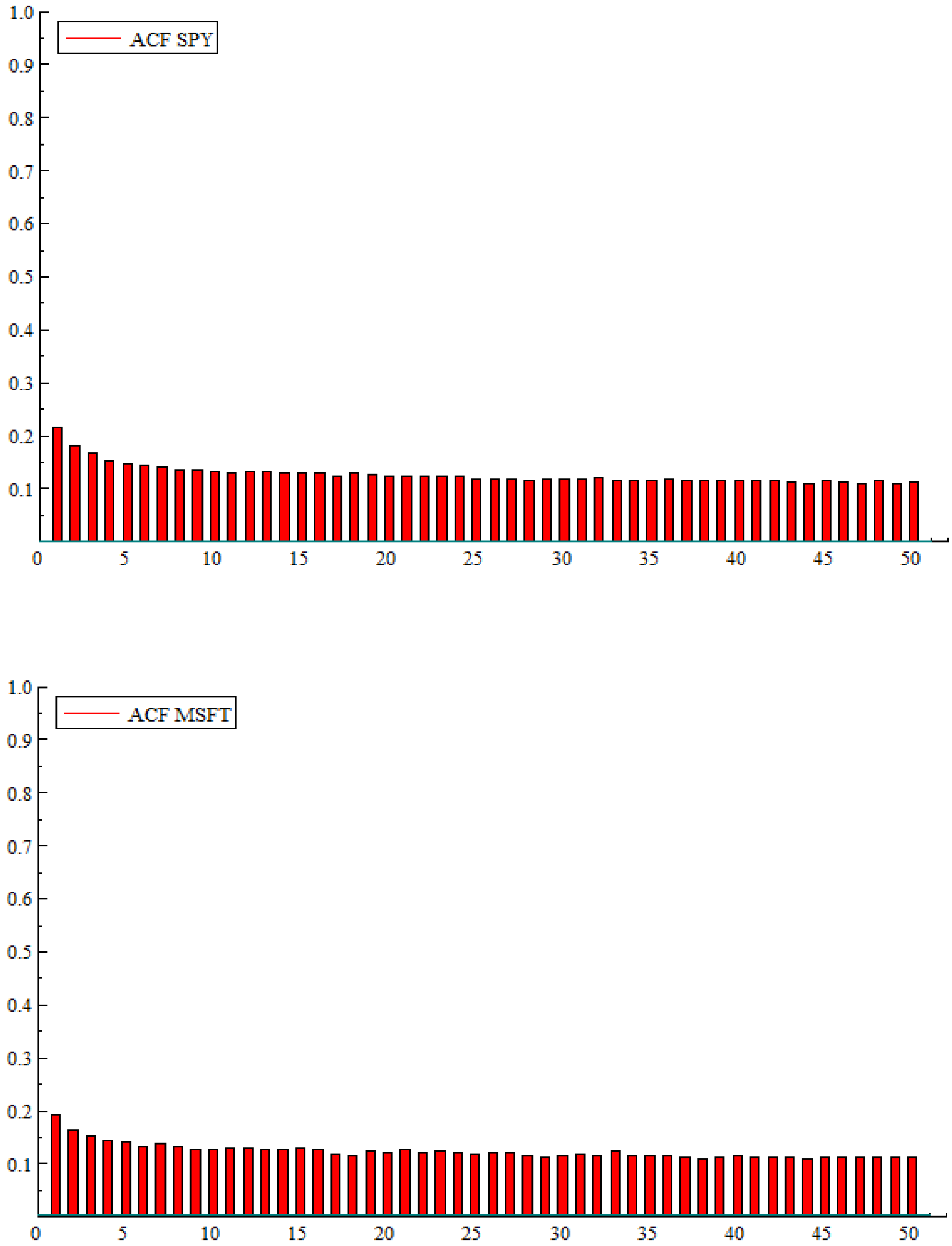

Figure 1.2: This figure shows the autocorrelation functions of trade durations for SPY and MSFT respectively

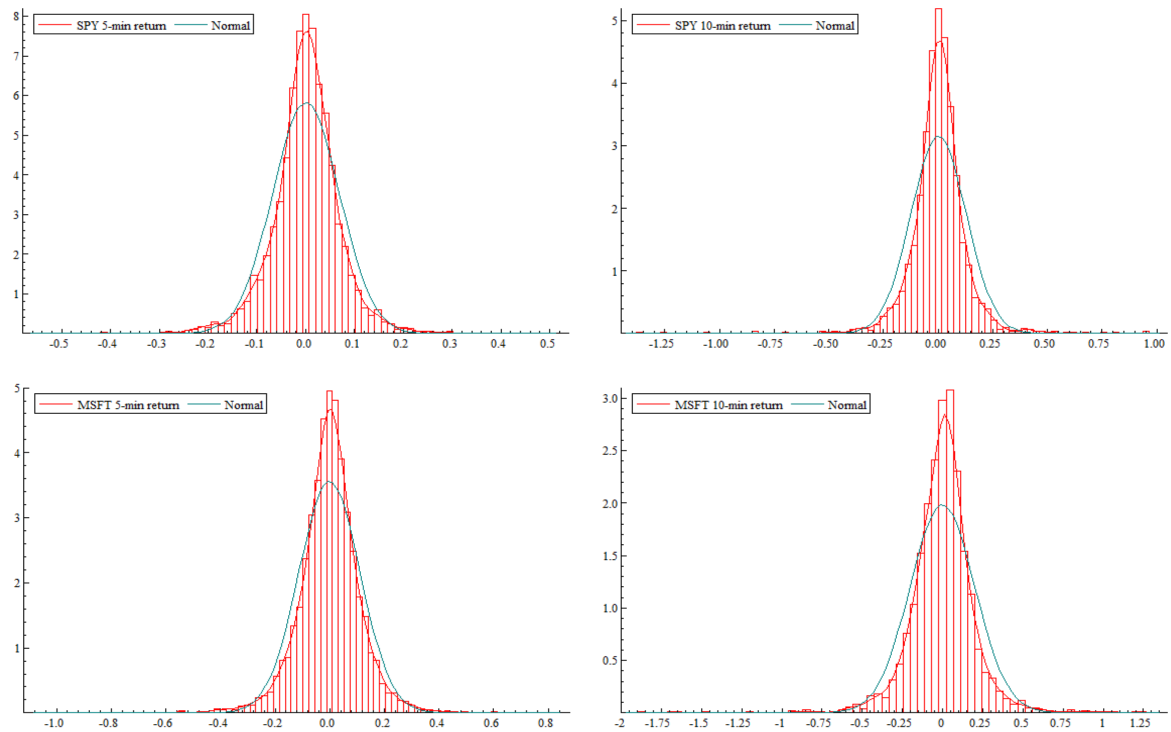

Figure 1.3: This figure shows the intraday return distribution for the SPY and MSFT. The left pannel shows the 5-minute returns and the right pannel shows the 10-minute returns.

Table 1.1: Trade duration: SPY and MSFT

	SPY	MSFT
observations	748069	167885
Mean	2.54	11.28
Standard dev.	3.13	18.60
Minimum	1	1
Maximum	99	427
Kurtosis	44.2499	40.7683
Skewness	4.8489	4.6627

Notes: The table reports the summary statistics for the duration data from 4 Jan 2010 to 30 April 2010.

Table 1.2: Autocorrelations and partial autocorrelations of trade durations (SPY)

SPY	Raw Duration				Diurnally Adjusted Duration			
	acf	pacf	Q-Stat	P-value	acf	pacf	Q-Stat	P-value
lag 1	0.215	0.215	26022	0.000	0.174	0.174	17012	0.000
lag 2	0.180	0.140	44153	0.000	0.138	0.111	27656	0.000
lag 3	0.166	0.110	59737	0.000	0.123	0.086	36162	0.000
lag 4	0.152	0.085	72678	0.000	0.109	0.066	42857	0.000
lag 5	0.147	0.075	84831	0.000	0.105	0.060	49031	0.000
lag 6	0.142	0.065	96118	0.000	0.100	0.052	54641	0.000
lag 7	0.139	0.059	106916	0.000	0.096	0.047	59859	0.000
lag 8	0.134	0.052	117051	0.000	0.093	0.042	64736	0.000
lag 9	0.132	0.049	126923	0.000	0.091	0.040	69420	0.000
lag 10	0.130	0.045	136402	0.000	0.088	0.035	73745	0.000
lag 11	0.128	0.042	145664	0.000	0.086	0.033	77911	0.000
lag 12	0.131	0.044	155286	0.000	0.088	0.035	82259	0.000
lag 13	0.132	0.043	165083	0.000	0.087	0.033	86541	0.000
lag 14	0.130	0.039	174578	0.000	0.087	0.032	90829	0.000
lag 15	0.129	0.036	183927	0.000	0.085	0.029	94927	0.000

Notes: The table reports autocorrelations and partial autocorrelations of original and diurnally adjusted trade durations for SPY.

Table 1.3: Autocorrelations and partial autocorrelations of trade durations (MSFT)

MSFT	Raw Duration				Diurnally Adjusted Duration			
	acf	pacf	Q-Stat	P-value	acf	pacf	Q-Stat	P-value
lag 1	0.194	0.194	4716.3	0.000	0.151	0.151	2852	0.000
lag 2	0.165	0.133	8141.8	0.000	0.127	0.107	4874	0.000
lag 3	0.154	0.106	11122	0.000	0.114	0.083	6488	0.000
lag 4	0.143	0.085	13683	0.000	0.102	0.065	7786	0.000
lag 5	0.145	0.083	16334	0.000	0.102	0.063	9098	0.000
lag 6	0.136	0.066	18641	0.000	0.095	0.052	10231	0.000
lag 7	0.141	0.069	21146	0.000	0.099	0.055	11459	0.000
lag 8	0.132	0.054	23323	0.000	0.093	0.046	12544	0.000
lag 9	0.128	0.048	25380	0.000	0.088	0.039	13510	0.000
lag 10	0.127	0.046	27414	0.000	0.085	0.035	14406	0.000
lag 11	0.131	0.048	29550	0.000	0.090	0.041	15426	0.000
lag 12	0.129	0.044	31623	0.000	0.089	0.038	16426	0.000
lag 13	0.128	0.042	33687	0.000	0.085	0.031	17320	0.000
lag 14	0.132	0.044	35861	0.000	0.089	0.035	18305	0.000
lag 15	0.129	0.039	37960	0.000	0.081	0.026	19133	0.000

Notes: The table reports autocorrelations and partial autocorrelations of original and diurnally adjusted trade durations for MSFT.

Table 1.4: Statistical table of intraday returns

	5-minute		10-minute	
	SPY	MSFT	SPY	MSFT
Mean	0.0007	-0.0005	0.0018	-0.0007
Standard dev.	0.0676	0.1087	0.1284	0.1906
Minimum	-0.5314	-1.0327	-1.0726	-1.8665
Maximum	0.4849	0.7945	1.0932	1.2431
Kurtosis	7.8617	8.5561	21.1235	15.6330
Skewness	-0.4160	-0.3885	-0.7280	-0.7225

Notes: The table shows the 5-minute and 10-minute returns' summary statistics for SPY and MSFT respectively.

Table 1.5: The in-sample estimation results of the ACD model

	Horizons			
	(1)	(2)	(3)	(4)
SPY				
ω_d	0.0095***	0.0098***	0.0092***	0.0090***
	(0.0006)	(0.0006)	(0.0006)	(0.0006)
α_d	0.0607***	0.0640***	0.0639***	0.0629***
	(0.0013)	(0.0011)	(0.0011)	(0.0011)
β_d	0.9296***	0.9260***	0.9267***	0.9279***
	(0.0015)	(0.0015)	(0.0015)	(0.0015)
MSFT				
ω_d	0.00468***	0.0048***	0.0044***	0.0045***
	(0.0004)	(0.0004)	(0.0004)	(0.0004)
α_d	0.0557***	0.0560***	0.0546***	0.0539***
	(0.0014)	(0.0014)	(0.0014)	(0.0013)
β_d	0.9399***	0.9394***	0.9412***	0.9417***
	(0.0014)	(0.0015)	(0.0014)	(0.0014)

Notes: The table provides in-sample parameter estimates for the ACD model based on different horizons. The standard errors are in parentheses. *, **, *** denote significance of parameter at 10%, 5% and 1% levels, respectively.

Table 1.6: The in-sample estimation result of the Intraday SV model (SPY)

	Horizons			
	(1)	(2)	(3)	(4)
5-minute				
$\ln \sigma^2$	-5.7627***	-5.6629***	-5.4209***	-5.4421***
	(0.4198)	(0.4197)	(0.3671)	(0.3567)
γ	0.3289***	0.3837***	0.4063***	0.3301***
	(0.0913)	(0.0995)	(0.0988)	(0.0872)
ϕ	0.9803***	0.9778***	0.9772***	0.9805***
	(0.0053)	(0.0058)	(0.0055)	(0.0049)
κ	-0.3342***	-0.3904***	-0.4133***	-0.3362***
	(0.0928)	(0.1012)	(0.1005)	(0.0886)
10-minute				
$\ln \sigma^2$	-5.7411***	-6.9543*	-5.7693***	-5.8226***
	(1.1903)	(3.6209)	(1.2009)	(0.9069)
γ	0.8730***	0.9039***	0.9530***	0.6663***
	(0.2414)	(0.2421)	(0.2560)	(0.2128)
ϕ	0.9494***	0.9499***	0.9497***	0.9633***
	(0.0142)	(0.0141)	(0.0142)	(0.0121)
κ	-0.8365***	-0.8691***	-0.9181***	-0.6410***
	(0.2321)	(0.2330)	(0.2470)	(0.2048)

Notes: The table provides in-sample parameter estimates for the intraday SV model for SPY. The standard errors are in parentheses. *, **, *** denote significance of parameter at 10%, 5% and 1% levels, respectively.

Table 1.7: The in-sample estimation result of the Intraday SV model (MSFT)

	Horizons			
	(1)	(2)	(3)	(4)
5-minute				
$\ln\sigma^2$	-3.8879***	-4.1184***	-4.2916***	-4.3342***
	(0.3062)	(0.3138)	(0.3067)	(0.3089)
γ	0.08421***	0.0665***	0.0509**	0.0493**
	(0.0294)	(0.0253)	(0.0226)	(0.0223)
ϕ	0.9673***	0.9742***	0.9795***	0.9792***
	(0.0082)	(0.0069)	(0.0058)	(0.0058)
κ	-0.0839***	-0.0670***	-0.0512**	-0.0503**
	(0.0280)	(0.0240)	(0.0214)	(0.0212)
10-minute				
$\ln\sigma^2$	-2.9621***	-3.5012***	-3.7703***	-3.92477***
	(0.4130)	(0.4439)	(0.4402)	(0.4474)
γ	0.5053***	0.0499***	0.1523**	0.1801***
	(0.1078)	(0.0175)	(0.0664)	(0.0694)
ϕ	0.8818***	0.9480***	0.9625***	0.9583***
	(0.0256)	(0.0173)	(0.0135)	(0.0141)
κ	-0.3818***	-0.1447**	-0.1153**	-0.1389**
	(0.0864)	(0.0617)	(0.0542)	(0.0566)

Notes: The table provides in-sample parameter estimates for the Intraday SV model for MSFT. The standard errors are in parentheses. *, **, *** denote significance of parameter at 10%, 5% and 1% levels, respectively.

Table 1.8: The Ljung-Box test on 10 lags for returns and residuals

	Horizons			
	(1)	(2)	(3)	(4)
5-minute SPY				
$\log(\widetilde{r}_t^2)$	403.13 [0.000]	326.98 [0.000]	285.88 [0.000]	354.79 [0.000]
e_{isv}	13.955 [0.175]	13.425 [0.201]	11.978 [0.287]	9.3238 [0.502]
$\widetilde{r}_t^2$	737.05 [0.000]	718.37 [0.000]	577.94 [0.000]	774.88 [0.000]
SE_{garch}	7.617 [0.666]	11.216 [0.341]	15.773 [0.106]	9.6143 [0.475]
$SE_{garch\text{-}d}$	6.3257 [0.787]	8.3239 [0.597]	14.062 [0.170]	9.1306 [0.520]
5-minute MSFT				
$\log(\widetilde{r}_t^2)$	315.04 [0.000]	343.02 [0.000]	379.77 [0.000]	357.84 [0.000]
e_{isv}	15.876 [0.103]	11.648 [0.309]	15.810 [0.105]	15.667 [0.110]
$\widetilde{r}_t^2$	729.83 [0.000]	732.68 [0.000]	794.55 [0.000]	910.97 [0.000]
SE_{garch}	9.2915 [0.505]	10.638 [0.386]	8.3066 [0.599]	8.6939 [0.561]
$SE_{garch\text{-}d}$	8.0874 [0.620]	8.3207 [0.598]	6.2234 [0.796]	8.4404 [0.586]
10-minute SPY				
$\log(\widetilde{r}_t^2)$	119.23 [0.000]	100.27 [0.000]	120.76 [0.000]	110.31 [0.000]
e_{isv}	13.634 [0.190]	15.309 [0.121]	14.976 [0.133]	14.055 [0.170]
$\widetilde{r}_t^2$	46.840 [0.000]	41.273 [0.000]	44.219[0.000]	46.054 [0.000]
SE_{garch}	2.7088 [0.987]	2.9275 [0.983]	2.6775 [0.988]	3.2398 [0.975]
$SE_{garch\text{-}d}$	3.1428 [0.978]	2.5001 [0.991]	2.5808 [0.990]	3.8375 [0.954]
10-minute MSFT				
$\log(\widetilde{r}_t^2)$	116.49 [0.000]	133.55 [0.000]	142.12 [0.000]	131.37 [0.000]
e_{isv}	10.071 [0.434]	8.2531 [0.604]	11.334 [0.332]	13.064 [0.220]
$\widetilde{r}_t^2$	245.84 [0.000]	232.08 [0.000]	207.12 [0.000]	228.49 [0.000]
SE_{garch}	8.8087 [0.550]	8.3790 [0.592]	4.1598 [0.940]	4.1579 [0.940]
$SE_{garch\text{-}d}$	5.6449 [0.844]	5.7368 [0.837]	2.9719 [0.982]	2.9058 [0.984]

Notes: The table provides the Ljung-Box test on 10 lags for 5-minute and 10-minute sampling frequencies respectively. $\widetilde{r}_t$ is the intraday pattern adjusted return. The e_{isv} is the standardized prediction residual of Intraday SV model. SE_{garch} is the squared standardized residuals of GARCH model and $SE_{garch\text{-}d}$ is the squared standardized residuals of GARCH-D model. The p-value of the Ljung-Box Q-statistics are shown in the square brackets.

Table 1.9: The in-sample forecasting results based on the MAE

	Horizons				
	(1)	(2)	(3)	(4)	Average
5-minute SPY					
GARCH-D	0.0046	0.0048	0.0049	0.0045	0.0047
GARCH	0.0047	0.0048	0.0049	0.0045	0.0048
Intraday SV	**0.0042**	**0.0042**	**0.0043**	**0.0040**	**0.0042**
5-minute MSFT					
GARCH-D	0.0131	0.0131	0.0127	0.0121	0.0127
GARCH	0.0131	0.0191	0.0128	0.0121	0.0143
Intraday SV	**0.0117**	**0.0117**	**0.0112**	**0.0107**	**0.0113**
10-minute SPY					
GARCH-D	0.0176	0.0178	0.0179	0.0169	0.0175
GARCH	0.0181	0.0184	0.0186	0.0178	0.0182
Intraday SV	**0.0153**	**0.0155**	**0.0155**	**0.0148**	**0.0153**
10-minute MSFT					
GARCH-D	0.0444	0.0435	0.0434	0.0414	0.0432
GARCH	0.0443	0.0438	0.0440	0.0415	0.0434
Intraday SV	**0.0382**	**0.0380**	**0.0373**	**0.0355**	**0.0373**

Notes: The table shows the comparison of in-sample forecasts for the Intraday SV, GARCH and GARCH-D models based on the MAE.

Table 1.10: The out-of-sample forecasting results based on the MAE

	Horizons				
	(1)	(2)	(3)	(4)	Average
5-minute SPY					
GARCH-D	0.0024	0.0037	0.0039	0.0075	0.0044
GARCH	0.0026	0.0038	0.0041	0.0077	0.0045
Intraday SV	**0.0020**	**0.0032**	**0.0035**	**0.0065**	**0.0038**
5-minute MSFT					
GARCH-D	0.0079	0.0070	0.0105	0.0112	0.0091
GARCH	0.0079	0.0081	0.0106	0.0115	0.0095
Intraday SV	**0.0063**	**0.0050**	**0.0094**	**0.0081**	**0.0072**
10-minute SPY					
GARCH-D	0.0104	0.0113	0.0206	0.0273	0.0174
GARCH	0.0109	0.0120	0.0241	0.0303	0.0193
Intraday SV	**0.0078**	**0.0090**	**0.0180**	**0.0247**	**0.0149**
10-minute MSFT					
GARCH-D	0.0258	0.0343	0.0413	0.0343	0.0339
GARCH	0.0261	0.0363	0.0429	0.0352	0.0351
Intraday SV	**0.0196**	**0.0254**	**0.0367**	**0.0250**	**0.0267**

Notes: The table shows the comparison of out-of-sample forecasts for the Intraday SV, GARCH and GARCH-D models based on the MAE.

Table 1.11: The Diebold-Mariano Test

	Horizons			
	(1)	(2)	(3)	(4)
5-minute SPY				
SV vs GARCH-D	-4.6062	-3.0544	-2.2371	-3.9826
	(0.0000)	(0.0024)	(0.0259)	(0.0001)
SV vs GARCH	-5.4350	-3.3708	-3.2268	-4.2531
	(0.0000)	(0.0008)	(0.0014)	(0.0000)
5-minute MSFT				
SV vs GARCH-D	-5.1137	-7.55	-1.8505	-7.6955
	(0.0000)	(0.0000)	(0.0650)	(0.0000)
SV vs GARCH	-5.2948	-8.0764	-1.9044	-7.8363
	(0.0000)	(0.0000)	(0.0576)	(0.0000)
10-minute SPY				
SV vs GARCH-D	-5.6539	-4.6807	-5.0384	-4.5838
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
SV vs GARCH	-4.5166	-4.4038	-3.1358	-2.9324
	(0.0000)	(0.0000)	(0.0020)	(0.0038)
10-minute MSFT				
SV vs GARCH-D	-4.0401	-5.6711	-1.8056	-5.4942
	(0.0000)	(0.0000)	(0.0729)	(0.0000)
SV vs GARCH	-4.2556	-5.5048	-2.1597	-5.5569
	(0.0001)	(0.0000)	(0.0323)	(0.0000)

Notes: The table shows the results of the DM test for the intraday SV model compared with GARCH and GARCH-D for 5-minute and 10-minute returns respectively. The p-value of the test is shown in the bracket.

1.8 Appendix

Table 1.12: The in-sample estimation result of the GARCH model (SPY)

	Horizons			
	(1)	(2)	(3)	(4)
5-minute				
c	0.0257**	0.0262**	0.0298**	0.0353***
	(0.0122)	(0.0123)	(0.0121)	(0.0122)
α	0.2284***	0.2326***	0.2348***	0.2361***
	(0.0144)	(0.0144)	(0.0149)	(0.0149)
λ	0.0069***	0.0064***	0.0080***	0.0083***
	(0.0015)	(0.0015)	(0.0017)	(0.0018)
β	0.0455***	0.0434***	0.0555***	0.0526***
	(0.0044)	(0.0043)	(0.0047)	(0.0047)
γ	0.9471***	0.9497***	0.9363***	0.9385***
	(0.0052)	(0.0051)	(0.0054)	(0.0057)
10-minute				
c	0.0600*	0.0593*	0.0628**	0.0891***
	(0.0324)	(0.0325)	(0.0305)	(0.0313)
α	0.0435**	0.0450**	0.0480**	0.0485**
	(0.0212)	(0.0209)	(0.0213)	(0.0208)
λ	0.0149***	0.0202***	0.0343***	0.0152***
	(0.0021)	(0.0026)	(0.0027)	(0.0030)
β	0.0156***	0.0183***	0.0268***	0.0218***
	(0.0016)	(0.0019)	(0.0027)	(0.0022)
γ	0.9796***	0.9749***	0.9621***	0.9731***
	(0.0019)	(0.0022)	(0.0026)	(0.0022)

Notes: The table provides in-sample parameter estimates for the GARCH model for SPY. The standard errors are in parentheses. *, **, *** denote significance of parameter at 10%, 5% and 1% levels, respectively.

Table 1.13: The in-sample estimation result of the GARCH model (MSFT)

	Horizons			
	(1)	(2)	(3)	(4)
5-minute				
c	0.0041	0.0122	0.0111	0.0152
	(0.0044)	(0.0123)	(0.0122)	(0.0121)
α	0.2312***	0.2266***	0.2243***	0.2222***
	(0.0142)	(0.0140)	(0.0140)	(0.0140)
λ	0.0119***	0.0092***	0.0087***	0.0118***
	(0.0023)	(0.0020)	(0.0019)	(0.0023)
β	0.0526***	0.0460***	0.0464***	0.0510***
	(0.0053)	(0.0047)	(0.0047)	(0.0053)
γ	0.9353***	0.9442***	0.9444***	0.9358***
	(0.0063)	(0.0058)	(0.0057)	(0.0067)
10-minute				
c	0.0218	0.0455	0.0479	0.0606*
	(0.0309)	(0.0303)	(0.0310)	(0.0311)
α	0.0553***	0.0482**	0.0459**	0.0542***
	(0.0206)	(0.0203)	(0.0207)	(0.0207)
λ	0.0562***	0.0416***	0.0452***	0.0557***
	(0.0115)	(0.0095)	(0.0083)	(0.0098)
β	0.0502***	0.0469***	0.0444***	0.0431***
	(0.0068)	(0.0061)	(0.0059)	(0.0060)
γ	0.9303***	0.9379***	0.9392***	0.9355***
	(0.0088)	(0.0077)	(0.0075)	(0.0085)

Notes: The table provides in-sample parameter estimates for the GARCH model for MSFT. The standard errors are in parentheses. *, **, *** denote significance of parameter at 10%, 5% and 1% levels, respectively.

Table 1.14: The in-sample estimation result of the GARCH-D model (SPY)

	Horizons			
	(1)	(2)	(3)	(4)
5-minute				
c	0.0138	0.0129	0.0176	0.0248*
	(0.0142)	(0.0138)	(0.0131)	(0.0133)
α	0.2313***	0.2257***	0.2264***	0.2318***
	(0.0170)	(0.0171)	(0.0171)	(0.0160)
λ	0.9072***	1.0307***	0.9626***	0.8211***
	(0.0046)	(0.0425)	(0.0547)	(0.0189)
β	0.0719***	0.1089***	0.1335***	0.0591***
	(0.0117)	(0.01467)	(0.0156)	(0.0098)
γ	0.7931***	0.6951***	0.6725***	0.8220***
	(0.0167)	(0.0216)	(0.0214)	(0.0141)
θ	-0.7764***	-0.8460***	-0.7841***	-0.7166***
	(0.0112)	(0.0305)	(0.0440)	(0.0240)
10-minute				
c	0.0484	0.0511	0.0659*	0.0978***
	(0.0338)	(0.0348)	(0.0338)	(0.0323)
α	0.0544**	0.0508**	0.0528**	0.0562***
	(0.0220)	(0.0211)	(0.0213)	(0.0208)
λ	3.0116***	3.3340***	3.0064***	2.9935***
	(0.0107)	(0.0132)	(0.1383)	(0.0070)
β	0.0149***	0.0069	0.0091***	0.0150***
	(0.0047)	(0.0045)	(0.0041)	(0.0043)
γ	0.8842***	0.8770***	0.8967***	0.8873***
	(0.0070)	(0.0085)	(0.0081)	(0.0081)
θ	-2.7270***	-3.0063***	-2.7393***	-2.7294***
	(0.0006)	(0.0030)	(0.1219)	(0.0139)

Notes: The table provides in-sample parameter estimates for the GARCH-D model for SPY. The standard errors are in parentheses. *, **, *** denote significance of parameter at 10%, 5% and 1% levels, respectively.

Table 1.15: The in-sample estimation result of the GARCH-D model (MSFT)

	Horizons			
	(1)	(2)	(3)	(4)
5-minute				
c	0.0051	0.0099	0.0096	0.0149
	(0.0124)	(0.0123)	(0.0126)	(0.0122)
α	0.2270***	0.2218***	0.2172***	0.2187***
	(0.0144)	(0.0154)	(0.0155)	(0.0142)
λ	0.0550***	0.1155***	0.1527***	0.0407***
	(0.0091)	(0.0132)	(0.0116)	(0.0046)
β	0.0535***	0.1106***	0.1317***	0.0504***
	(0.0056)	(0.0122)	(0.0140)	(0.0055)
γ	0.9256***	0.7957***	0.7315***	0.9311***
	(0.0072)	(0.0179)	(0.0208)	(0.0069)
θ	-0.0323***	-0.0276***	-0.0275***	-0.0217***
	(0.0061)	(0.0063)	(0.0002)	(0.0025)
10-minute				
c	0.0188	0.0330	0.0300*	0.0473
	(0.0312)	(0.0303)	(0.0323)	(0.0316)
α	0.0584***	0.0511**	0.0473**	0.0557***
	(0.0218)	(0.0217)	(0.0224)	(0.0216)
λ	0.4999***	0.4515***	0.5025***	0.4490***
	(0.0545)	(0.0576)	(0.0494)	(0.0344)
β	0.0788***	0.0787	0.0788**	0.0519***
	(0.0103)	(0.0105)	(0.0102)	(0.0082)
γ	0.8545***	0.8609***	0.8483***	0.8926***
	(0.0154)	(0.0158)	(0.0164)	(0.0119)
θ	-0.2903***	-0.2638***	-0.2756***	-0.2790***
	(0.0300)	(0.0329)	(0.0273)	(0.0220)

Notes: The table provides in-sample parameter estimates for the GARCH-D model for MSFT. The standard errors are in parentheses. *, **, *** denote significance of parameter at 10%, 5% and 1% levels, respectively.

Chapter 2

Measuring financial contagion: A multivariate stochastic volatility approach

2.1 Introduction

The study of financial contagion has rapidly become one of the important research topics in financial economics. Compared with stable periods, the issues of risk management and asset allocation become more important to practitioners and academics during crises. Studying financial contagion is useful for these financial management tasks. The recent Global Financial Crisis in 2007 and the following Eurozone Sovereign Debt Crisis have once again highlighted the importance of this topic.

In this chapter, we investigate the contagion effects resulting from the US subprime crisis on a sample of EU countries (UK, Switzerland, Netherlands, Germany and France). Financial contagion is specified as a significant increase in cross-market correlations after a shock, so we can investigate the existence of contagion by testing the change of cross-market correlations[1]. Forbes and Rigobon (2002) find that the problem of heteroskedasticity can introduce general upwardly biased in the estimation of correlation coefficient for early studies, as some part of increased correlation that is due to an increase in volatility. They offer a method to remove the bias and calculate the adjusted correlation. Unlike the adjusted correlation approach, Engle (2002a) introduce the dynamic conditional correlation multivariate GARCH model (DCC-GARCH) which accounts for heteroskedasticity by estimating the dynamic correlation coefficients of the standardized residuals rather than the correlation coefficients of returns directly. The DCC-GARCH allows correlations time varying and is widely used by literature to study the financial contagion (e.g., Cappiello *et al.*, 2006; Chiang *et al.*, 2007; Dimitriou *et al.*,

[1]This definition of financial contagion is widely used and ignores fundamentals. We review more definitions in Section 2.2.1.

2013).

Stochastic volatility (SV) models offer powerful alternatives to GARCH type models in accounting for both the conditional and unconditional properties of volatility[2]. The advantages of SV type models compared with GARCH type models have been discussed by literature. As supported by Kim *et al.* (1998), the introduction of the additional error term makes SV models more flexible than the GARCH type models. In addition, Harvey *et al.* (1994) argue that SV models are the natural discrete-time versions of the continuous-time models upon which much of modern finance theory is based. Like the results of univariate SV and GARCH models, Danıelsson (1998) shows that the MSV models also outperform the MGARCH models. Based on the same idea of DCC-GARCH, dynamic correlation multivariate stochastic volatility models (DC-MSV) have been introduced by Yu and Meyer (2006) and Asai and McAleer (2009).

As mentioned by Yu and Meyer (2006), although the SV type models are considerable, compared to the multivariate GARCH (MGARCH) literature, the literature on multivariate SV (MSV) is rather limited. The main reason for this is that compared with MGARCH, the MSV models are harder to estimate as the likelihood function of the MSV has no closed form. To overcome the difficulty of the estimation, the Bayesian Markov chain Monte Carlo (MCMC) method has been introduced and it is generally regarded in the literature as the preferred estimation and inference technique (Jacquier *et al.*, 1994).

[2]For example, compared with GARCH models, the better performance of univariate SV models has been supported by Danielsson (1994) and Kim *et al.* (1998). In terms of multivariate SV models, Danıelsson (1998) find that the basic multivairate SV is superior to alternative multivairate GARCH models such as the vector GARCH, diagonal vector GARCH (Bollerslev *et al.*, 1988), Baba-Engle-Kraft-Kroner (BEKK) model (Engle and Kroner, 1995) and the constant conditional correlation (CCC) model (Bollerslev, 1990)

This chapter contributes to the existent literature in the following aspects. First, unlike most of the existing studies, we use the MSV rather than the MGARCH specification to obtain correlation estimates. We also directly compare the contagion effects detected by the DC-MSV and DCC-GARCH models. Although the better performance of the DC-MSV model compared with the DCC-GARCH model in terms of the in-sample fits and out-of-sample forecasts has been supported by Asai and McAleer (2009). However, due to the complicated estimation procedure of the SV specification, few studies consider the DC-MSV model to study financial contagion. To the best of our knowledge, only Gebka and Karoglou (2013) apply the DC-MSV model to study the changes of correlations during different market regimes for this topic. Existing studies have not directly compared the contagion effects estimated from MSV and MGARCH. So in this study we consider the DC-MSV model and find that the contagion effect is more significant based on the DC-MSV model compared with the DCC-GARCH model. We estimate the DC-MSV model based on Bayesian MCMC method as it is an efficient estimator compared with other methods for estimating SV models, such as the generalized method of moments or the quasi-maximum likelihood estimate (QMLE) for SV models.

Second, we extend the DC-MSV model by incorporating implied volatility information into the volatility equations. The implied volatility is calculated based on the corresponding stock option price, so it reflects market expectations regarding future price movements. It is also known as gauge to measure investors' fear of market crash. It has been supported that the implied volatilities are more informative than daily returns and provide better volatility forecasts, especially during turmoil periods (Fleming *et al.*, 1995; Fleming, 1998; Christensen and Prabhala, 1998; Blair *et al.*, 2001). Implied volatility information has been suggested for univariate stochastic volatility models by Koopman *et al.* (2005) and Jacquier and Paulson (2010). As summarized by Kolb (2011), the investors' negative expectation during financial crisis is an important channel for financial contagion, so it might be helpful for considering implied volatility information to study financial contagion. We compare the extended DC-MSV model (DC-MSV-IV) with the original DC-MSV model based on the Deviance Information Criterion (DIC) (Spiegelhalter *et al.*, 2002). The DIC support the better fit of the DC-MSV-IV model for every country pair, so we can get more accurate estimations for the dynamic correlations. Compared with the DC-MSV model, the contagion effect under the DC-MSV-IV model is more significant, so implied volatility information is useful for detecting financial contagion.

Third, we provide the evidence of contagion effects from USA to the investigated EU countries. We investigate the correlation changes during both the Global Financial Crisis (GFC) and the Eurozone Sovereign Debt Crisis (ESDC). We find the correlations are higher during the ESDC than GFC. For the five EU countries, the UK is most influenced by the contagion effect whereas Germany is least influenced. The empirical results also support that the strong contagion effect is not necessary as a result of high correlation. Although the correlation between Switzerland and USA is lowest among the sample countries, but it is highly influenced by the financial contagion. On the other hand, France is highest correlated to the USA but it is not highly influenced by the financial contagion. We further consider the correlations after financial crisis and find that the correlations tend to recover to a relatively lower level, but they are still higher than the pre-crisis correlations. We also investigate the relationship between financial contagion and crisis intensity, and find that the high contagion effect does not necessarily lead to the high crisis intensity.

The remainder of the chapter proceeds as follows. Section 2.2 review the relevant literature. Section 2.3 presents the data description and analysis. Section 2.4 provides a detailed description of the methodology, including the models, estimation method and procedure for testing the financial contagion. Section 2.5 outlines the empirical findings. Conclusion is provided in Section 2.6.

2.2 Literature Review

2.2.1 Financial Contagion

Generally, financial contagion refers to the spread of financial disturbances from one country to others. There are several formal definitions of financial contagion in the literature and the widely used one is based on the notion of "shift contagion", that is a statistically significant increase in cross-market correlations during the financial crisis period (Forbes and Rigobon, 2002): "...if two markets show a high degree of co-movement during periods of stability, even if the markets continue to be highly correlated after a shock to one market, this may not constitute contagion." According to this definition, the contagion exists if cross-market co-movement increases significantly after the shock. If the co-movement does not increase significantly, then any continued high level of market correlation suggests strong linkages between the two economies, but not contagion. This chapter uses the term interdependence to refer to this situation. We also use their definition of contagion in this chapter.

As mentioned by Kolb (2011), contagion is a fairly new concept in the economics literature-before 1990, it was scarcely mentioned. Earlier studies of this topic stemmed from international finance, so the financial contagion at the international level has always had a prominence in discussions of this topic. The recent global financial crisis of 2007-2009 also offer some evidence at the domestic level. There is no settled meaning for contagion in finance. In addition to Forbes and Rigobon (2002) definition, some studies fully embrace the disease metaphor, as mentioned by Allen and Gale (2000): "One theory is that small shocks which initially affect only a few institutions or a particular region of the economy, spread by contagion to the rest of the financial sector and then infect the larger economy." For others, contagion is merely the diffusion of financial stress, without connotations of disease. According to Caramazza *et al.* (2004), "the spread of financial difficulties from one economy to others in the same region and beyond in a process that has come to be referred to as 'contagion'."

Pericoli and Sbracia (2003) summarize five definitions of contagion that reflect the wide variety of meanings ascribed to this term: 1, Contagion is a significant increase in the probability of a crisis in one country, conditional on a crisis occurring in another country. 2, Contagion occurs when volatility of asset prices spills over from the crisis country to other countries. 3, Contagion occurs when cross-country co-movements of asset prices cannot be explained by fundamentals. 4, Contagion is a significant increase in co-movements of prices and quantities across markets, conditional on a crisis occurring in one market or group of markets. 5, Contagion occurs when the transmission channel intensifies or, more generally, changes after a shock in one market.

Following the shift contagion definition offered by Forbes and Rigobon (2002), many studies regard a change in the correlations among economic variables as a key for the financial contagion. This is reflected in the third and fourth definitions listed above by Pericoli and Sbracia (2003). This point also has been stressed by Kaminsky *et al.* (2003):“ Only if there is ‘excess comovement’ in financial and economic variables across countries in response to a common shock do we consider it contagion”. Forbes and Rigobon (2002) argue that the contagion is reflected by an increase in correlation among asset returns , after discounting any increased correlation that is due to an increase in volatility. Bekaert *et al.* (2005) follow and extend their idea. They assert that contagion is “excess correlation, that is, correlation over and above what one would expect from economic fundamentals”, and “Contagion is a level of correlation over what is expected”.

We use shift contagion in this chapter because it has three main advantages (Forbes and Rigobon, 2001): First, it is useful in evaluating the effectiveness of international diversification during a crisis. Second, the existence of financial contagion could justify multilateral intervention. Third, it provides a useful method of distinguishing between explanations of how shocks are transmitted across markets. As mentioned by Pericoli and Sbracia (2003), unlike other definition that set an unrealistically difficult test for the existence of contagion, this definition is empirically useful since it provides a straightforward method of testing.

2.2.2 Detecting Financial Contagion

According to the shift contagion, the correlation analysis is the most straightforward approach to test the existence of contagion. As mentioned by Forbes and Rigobon

(2002), some early literature directly measure the correlations in returns, and then test for a significant increase in this correlation coefficient after a shock. For example, King and Wadhwani (1990) investigate the stock markets of USA, UK and Japan, and find that cross-market correlations increased significantly after the USA market crash in 1987. Lee and Kim (1993) further extend this analysis to 12 major markets and also find the evidence of contagion: the weekly cross-market correlations increased from 0.23 before the 1987 USA crash to 0.39 afterward. Reinhart and Calvo (1996) study the correlation changes before and after the 1994 Mexican peso crisis. They find that cross-market correlations have increased in many emerging markets during the crisis. However, for the listed studies above and many other earlier studies, Forbes and Rigobon (2002) find that ignoring heteroskedasticity can introduce general upwardly biased in the estimation of correlation coefficient, because some part of the increased correlation is caused by an increase in volatility. They offer a adjusted correlation approach to deal with the heteroskedasticity problem.

When we consider the heightened correlation not due to an increase in overall volatility and/or not due to economic fundamentals, many of the early evidence supporting the existence of contagion is no longer valid. Forbes and Rigobon (2002) find that if taking the heteroskedasticity of returns into account, there was no contagion during the 1997 Asian crisis and earlier crisis. But according to Corsetti *et al.* (2005), the conclusion of ‘no contagion’ is a bit too strong. They generalize Forbes and Rigobon (2002) model to allow for a more general variance structure, nesting the Forbes and Rigobon (2002) model as a special case. They do find evidence of financial contagion during the 1997 Asian crisis, but their evidence is generally mixed.

Another method to deal with the bias introduced by the heteroskedasticity is based on the the dynamic correlation multivariate models. Unlike the adjusted correlation approach, it accounts for heteroskedasticity by estimating the dynamic correlation coefficients of the standardized residuals rather than the correlation coefficients of returns.

2.2.3 Dynamic Correlation-Multivariate Models

Engle (2002a) introduce the dynamic conditional correlation multivariate GARCH model (DCC-GARCH) which is a commonly used method in the topic of contagion. The DCC-GARCH model is estimated in two steps. In the first stage, a univariate GARCH model is fitted for each of the stock market returns. In the second stage, dynamic conditional correlation is estimated using the transformed stock-return residuals. Transformed stock return residuals are estimated by their standard deviations from the first stage. Based on the DCC-GARCH model, Chiang *et al.* (2007) find the evidence of contagion in Asian financial markets between 1996 and 2003.

Cappiello *et al.* (2006) extend the original DCC-GARCH model and use the asymmetric generalized dynamic conditional correlation (AG-DCC) specification to investigate asymmetries in conditional variances and correlation dynamics for three groups of countries (Europe, Australasia and North America). They find that while equity returns show strong evidence of asymmetries in conditional volatility, little is found for bond returns. Kenourgios *et al.* (2011) apply AG-DCC approach to study the contagion effect during the period 1995-2006, and find that the emerging BRIC (Brazil, Russia, India, China) markets are more prone to financial contagion compared with USA and UK.

As the DCC-GARCH model is based on the two stage estimation and has two parts, unlike the AG-DCC model, some studies try to make some extension for the GARCH part. Dimitriou *et al.* (2013) use Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) rather than GARCH model to calculate the stock-return residuals, that is used for the second stage to calculate the dynamic conditional correlation. They study the contagion of BRICS (BRIC plus South Africa) and USA for the recently global financial crisis in 2007.

Given the SV models is another main stream for modeling volatility, it is possible to study the dynamic correlations based on multivariate SV model (MSV). Danıelsson (1998) fit the standard MSV for foreign exchange rates (Deutschemark/Dollar, Yen/Dollar) and stock indices (S&P500 and Tokyo stock exchange), and find that the standard multivairate SV is superior to alternative GARCH models such as the vector GARCH, diagonal vector GARCH (Bollerslev *et al.*, 1988), Baba-Engle-Kraft-Kroner (BEKK) model (Engle and Kroner, 1995) and the constant conditional correlation (CCC) model (Bollerslev, 1990).

A weakness of the standard MSV model is that it has a conditional correlation matrix that is time-invariant, especially for the financial contagion topic. The additive factor MSV models accommodate both time-varying volatility and time-varying correlations. However, the correlation in the factor MSV models depends on the volatility of the factor, and the same set of parameters determines both the time-varying variance and time-varying correlation (Asai *et al.*, 2006). Yu and Meyer (2006) introduce a dynamic correlation MSV (DC-MSV) model. The generalization of this bivariate model to the higher dimensions is rather cumbersome as it is difficult to ensure the positive definiteness of the correlation matrix (Chib *et al.*, 2009). Gebka and Karoglou (2013) apply the DC-MSV model above to study the correlation dynamics during different time periods. They investigate the integration of the European peripheral financial markets GIPSI (Greece, Italy, Portugal, Spain and Ireland) with Germany, France and UK. They find that compared with other periods, the dynamic correlation is significantly higher during the 2007 global financial crisis period.

2.2.4 Crisis Period Identification

Before investigate financial contagion, we need to define the crisis period (the date of the outbreak of a crisis and the duration of a crisis). The identification of the crisis period is an important issue as tests of contagion are sensitive to the definition of the crisis period (Dungey *et al.*, 2005; Baur, 2012). In the literature, there are three ways to determine the crisis period length: a) determining the crisis length ad-hoc based on major economic and financial events (e.g., Engle, 2002a; Choudhry and Jayasekera, 2014); b) using the statistical approach (Markov Switching models) to identify the crisis period endogenously (e.g., Boyer *et al.*, 2006; Rodriguez, 2007); c) combining both the economic and the statistical approach (e.g., Baur, 2012; Dimitriou *et al.*, 2013; Kenourgios, 2014).

The economic approach relies on all major financial and economic news events representing the the Global Financial Crisis (GFC) and the Eurozone Sovereign Debt Crisis (ESDC). For GFC, we follow Baur (2012) that is based on official timelines provided by Federal Reserve Board of St. Louis (2009) and the Bank for International Settlements (BIS, 2009) among others. The BIS study separates the timeline in four phases from the third quarter in 2007 until the end of 2009. Phase 1 is described as "initial financial turmoil" and spans from Q3 in 2007 until the mid-September 2008. Phase 2 is described as "sharp financial market deterioration" and covers the period from mid-September 2008 until late 2008. Phase 3 is defined as "macroeconomic deterioration" (Q1 2009) and phase 4 is a phase of "stabilization and tentative signs of recovery" (from Q2 2009 onwards). Therefore, the GFC can be defined from August 2007 until March 2009 covering the first three phases. For ESDC, we follow Kenourgios (2014) that the ESDC timeline is constructed by merging two sources (European Central Bank and Reuters) as follows: Phase 1 spans a period from 5th November 2009 until 22nd April 2010, that begins with the announcement of the Greek budget deficit leading to a sharp increase of the regional sovereign risk. Phase 2 begins shortly before the Greek bailout in May 2010, when the Greek Prime Minister announced that the austerity packages are not enough and requested for a bailout plan from the Eurozone and the IMF (23rd April 2010 - 14th July 2011). Phase 3 (15 July 2011- 25 July 2012) starts when the European authorities published the banking stress-tests and Italy announced its first austerity package. Phase 4 is from 26 July 2012 onwards. On that day the European Central Bank (ECB) president Mario Draghi announced that the ECB was prepared within its mandate to do whatever it takes to preserve the euro. Shortly after, on September 6, 2012, the ECB formally announces the Outright Monetary Transactions (OMT) program although it has been shown that the markets had already anticipated this after Mr. Draghis announcement.

2.3 Data and Descriptive Statistics

The data employed consist of stock indices for six countries: USA, UK, Switzerland, Netherlands, Germany and France (S&P 500, FTSE 100, SMI, AEX, DAX and CAC 40), and their corresponding implied volatility indices: VIX, VFTSE, VSMI, VAEX, VDAX-NEW, and VCAC. The volatility indices have thirty days to maturity and reflect the volatility of the respective stock markets. All data are extracted from Bloomberg. The sample covers a period from 15th May 2003 to 25th November 2014 in order to study the recent global financial crisis and secure a sufficient number of observations. We use daily closing prices for our empirical analyses. The stock return is estimated as $r_t = 100[\log(p_t) - \log(p_{t-1})]$, where p_t is the price on date t. There is discrepancy between the closing times of the European exchanges and the US exchange, because of the different trading hours and different time zones. To take account the non-synchronicity issue, we follow Forbes and Rigobon (2002) and Kenourgios (2014), the stock returns and implied volatility indices are calculated as a two-day moving averages.

[Table 2.1 around here]

Table 2.1 presents the basic statistical features of the index return employed and their corresponding implied volatility indices. As well-documented in the literature, all the return series are not normal (higher peak and fatter tails). From the standard deviation of return series, the sequence from the least volatile to the most volatile is SMI (Switzerland), FTSE 100 (UK), S&P 500 (USA), AEX (Netherlands), CAC 40 (France), DAX (Germany). For the mean of implied volatility indices, the sequence from smallest to largest is VSMI, VFTSE, VIX, VAEX, VCAC, VDAX-NEW. We can find that the ranking of implied volatility indices are the same as the ranking of standard deviation of return series, because the implied volatility is the expectation of future return volatility which is closely related to the actual return volatility. According to the mean of return series, the German stock market has the highest returns compared with other markets, followed by USA and Switzerland. On the other hand, the Netherlands and France stock markets have lowest returns. We also report the full sample correlations between USA stock markets and EU countries, for return series and implied volatility series respectively. The correlations for the implied volatility tend to be higher than the returns.

[Figure 2.1 around here]

Figure 2.1 shows the implied volatilities for these countries. They share a similar pattern as the most volatile periods are around 2008-2009 and 2011-2012, corresponding to the Global Financial Crisis and the following Eurozone Sovereign Debt Crisis. The similar pattern can explain the high correlations for the implied volatilities.

[Figure 2.2 around here]

Figure 2.2 shows the autocorrelations of return series and implied volatilities for these countries respectively. The autocorrelations of return series are all close to 0. The absence of linear dependence indicates market efficiency. On the other hand, the implied volatility shows clearly positive autocorrelations.

2.4 Methodology

Section 2.4.1 outlines the models to be adopted. Section 2.4.2 describes the Bayesian methods for estimating and comparing the DC-MSV and DC-MSV-IV models. Section

2.4.3 shows the method for defining the beginning and ending days of crisis periods. Section 2.4.4 shows the test for financial contagion.

2.4.1 The Models

We firstly show the standard dynamic correlation multivariate models, the DCC-GARCH and DC-MSV respectively. Then we focus on our extended DC-MSV model by augmented the implied volatility information.

A. Dynamic Correlation Multivariate Models

The commonly used DCC-GARCH model proposed by Engle (2002a) can be written as

$$\boldsymbol{r}_t = \boldsymbol{C}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{v}_t \tag{2.1}$$

$$\boldsymbol{H}_t = \boldsymbol{D}_t^{1/2}\boldsymbol{R}_t\boldsymbol{D}_t^{1/2} \tag{2.2}$$

$$\boldsymbol{R}_r = diag(\boldsymbol{Q}_t)^{-1/2}\boldsymbol{Q}_t diag(\boldsymbol{Q}_t)^{-1/2} \tag{2.3}$$

$$\boldsymbol{Q}_t = (1-\lambda_1-\lambda_2)\boldsymbol{R} + \lambda_1\tilde{\boldsymbol{\varepsilon}}_{t-1}\tilde{\boldsymbol{\varepsilon}}'_{t-1} + \lambda_2\boldsymbol{Q}_{t-1} \tag{2.4}$$

where $\boldsymbol{r}_t$ is an $m\times 1$ vector of return series; $\boldsymbol{R}_r$ is an $m\times 1$ vector of dependent variables; $\boldsymbol{C}$ is an $m\times k$ matrix of parameters; $\boldsymbol{x}_t$ is a $k\times 1$ vector of independent variables, which may contain lags of $\boldsymbol{R}_r$; $\boldsymbol{H}_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\boldsymbol{H}_t$; $\boldsymbol{v}_t$ is an $m\times 1$ vector of normal, independent and identically distributed innovations; $\boldsymbol{D}_t$ is a diagonal matrix of conditional variances, $diag(\sigma^2_{1,t},\sigma^2_{2,t},...,\sigma^2_{m,t})$, in which each $\sigma^2_{i,t}$ is calculated from a univariate GARCH model; $\boldsymbol{R}_r$ is a matrix of conditional quasicorrelations; $\tilde{\boldsymbol{\varepsilon}}_t$ is an $m\times 1$ vector of standardized residuals, $\boldsymbol{D}_t^{-1/2}\boldsymbol{\varepsilon}_t$.

The original DC-MSV model proposed by Yu and Meyer (2006) can be written as follows

$$\boldsymbol{r}_t = \Omega_t \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t | \Omega_t \sim i.i.d.\ N(\boldsymbol{0}, \Sigma_{\varepsilon,t}) \quad \Sigma_{\varepsilon,t} = \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} \tag{2.5}$$

$$\boldsymbol{h}_{t+1} = \boldsymbol{\mu} + diag(\phi_{11}, \phi_{22})(\boldsymbol{h}_t - \boldsymbol{\mu}) + \boldsymbol{\eta}_t, \tag{2.6}$$

$$\boldsymbol{\eta}_t \sim i.i.d.\ N(\boldsymbol{0}, diag(\sigma_{\eta 1}^2, \sigma_{\eta 2}^2)) \tag{2.7}$$

$$q_{t+1} = \psi_0 + \psi(q_t - \psi_0) + \sigma_\rho v_t, \qquad v_t \sim i.i.d.\ N(0,1) \tag{2.8}$$

$$\rho_t = \frac{exp(q_t) - 1}{exp(q_t) + 1} \tag{2.9}$$

with $\boldsymbol{h_0} = \boldsymbol{\mu}$ and $q_0 = \psi_0$; where $\boldsymbol{r}_t = (r_{1t}, r_{2t})$ denotes the vector of return series; $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is the error term of returns; $\boldsymbol{h}_t = (h_{1t}, h_{2t})'$ is the vector of unobserved log-variance; $\boldsymbol{\mu} = (\mu_1, \mu_2)'$ is the mean term of log-variance; $\Omega_t = diag(\exp(\boldsymbol{h}_t/2))$ is a diagonal matrix of variances. From Equation 2.8, we can find that the current correlation is effected by previous correlation. From 2.9, the first derivative of ρ_t with respect to q_t is

$$\frac{d\rho_t}{dq_t} = \frac{(exp(q_t)-1)'(exp(q_t)+1) - (exp(q_t)-1)(exp(q_t)+1)'}{(exp(q_t)+1)^2} = \frac{2exp(q_t)}{(exp(q_t)+1)^2} \tag{2.10}$$

which is positive, so the correlation ρ_t is monotonically increasing with respect to q_t. Equations 2.8 and 2.9 allow the correlation coefficients to be time varying and ρ_t to be bounded by -1 and 1.

B. The DC-MSV-IV Model

To extend the DC-MSV model by incorporating implied volatility information, we firstly re-write Equation 2.6 as

$$\boldsymbol{h}_{t+1} = \boldsymbol{\alpha} + diag(\phi_{11}, \phi_{22})\boldsymbol{h}_t + \boldsymbol{\eta}_t, \tag{2.11}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)'$, so that $\alpha_1 = \mu_1 - \phi_{11}\mu_1$ and $\alpha_2 = \mu_2 - \phi_{22}\mu_2$. Let $\boldsymbol{x}_t = (\log s_{1t}^2, \log s_{2t}^2)'$ where s_{it}^2 is the implied volatility index of country i at time t. Then we incorporate implied volatility information $\boldsymbol{x}_t$ into the variance equation above to get

$$\boldsymbol{h}_{t+1} = \boldsymbol{\alpha} + diag(\phi_{11}, \phi_{22})\boldsymbol{h}_t + diag(\gamma_{11}, \gamma_{22})\boldsymbol{x}_t + \boldsymbol{\eta}_t, \tag{2.12}$$

Equation 2.12 shows that implied volatility information is used to describe the unobserved log-variance. As the h_t is log-variance and implied volatility index is the market expected variance, following Koopman *et al.* (2005) we take the logarithm of implied volatility indices. So the DC-MSV-IV model replace Equation 2.6 of DC-MSV model by Equation 2.12.

2.4.2 Bayesian Estimation

Although the better performance of SV type models compared with GARCH type models has been supported by literature (e.g. Danielsson, 1994; Kim *et al.*, 1998; Danıelsson, 1998) , the principal disadvantage of SV models is that they are difficult to estimate by maximum likelihood (Harvey *et al.*, 1994). The SV type models belong to the family of nonlinear non-Gaussian state space models. The maximum likelihood estimation method cannot be used directly, because volatility terms of SV models are latent variables, the likelihood function is not available in a closed form (it is expressed as an analytically intractable T-dimensional integral, where T is the number of observations). In order to overcome this difficulty, some other methods have been introduced to estimate SV models, such as Generalized Method of Moments (Melino and Turnbull, 1990), quasi-maximum likelihood method (Harvey *et al.*, 1994), and Bayesian MCMC method (Jacquier *et al.*, 1994 for the single-move Gibbs sampler; Kim *et al.*, 1998 for the multi-move Gibbs sampler).

Unlike the Bayesian MCMC method, both GMM and QML methods are found not to be efficient, and they suffer from some drawbacks as outlined by literature. For example, the quality of the (finite sample) GMM inference is quite sensitive to both the choice of the number of moments to include and the exact choice of moments among the natural candidates (Shephard and Andersen, 2009); for QML method, Harvey *et al.* (1994) and Ruiz (1994) point out that the adequacy of the approximation depends critically on the value of the stochastic process term for volatility, and approximating the $\log(\chi^2)$ density by a normal density could be rather inappropriate. Jacquier *et al.* (1994) compare the GMM, QML and Bayesian methods and find the Bayesian method to be superior. Therefore, we estimate the DC-MSV-IV model based on Bayesian MCMC method (the single move Gibbs sampler)[3]. In the literature of financial contagion, the commonly used two step quasi-likelihood estimators for DCC-GARCH model is also not efficient. In addition, it has been found that the two step quasi-likelihood estimators can introduce the downward bias (Engle and Sheppard, 2001). According to Aielli (2013), the second step of the DCC estimator can be inconsistent, and it is also shown

[3]Follow Yu and Meyer (2006), we make use of a freely available Bayesian Software, WinBUGS (Lunn *et al.*, 2000) , to estimate the model.

that the traditional interpretation of the dynamic correlation parameters can result in misleading conclusions.

Bayesian inference is based on the joint posterior distribution of all unobserved quantities in the model. So for the DC-MSV-IV model, the unobserved quantities include the unknown parameter $\boldsymbol{a} = (\alpha_1, \alpha_2, \phi_{11}, \phi_{22}, \gamma_{11}, \gamma_{22}, \psi_0, \psi, \sigma^2_{\eta 1}, \sigma^2_{\eta 2}, \sigma^2_{\rho})$, the vector of latent log-volatilities, $\boldsymbol{H} = (\boldsymbol{h_0}, \boldsymbol{h_1}, ..., \boldsymbol{h_T})$ and the dynamic correlations $\boldsymbol{\rho} = (\rho_0, \rho_1, ..., \rho_T)$. According to Bayes' rule, the joint posterior distribution can be calculated by multiplying prior and likelihood. By successive conditioning, the joint prior density $p(\boldsymbol{a}, \boldsymbol{H}, \boldsymbol{\rho})$ can be written as

$$\begin{aligned} p(\boldsymbol{a}, \boldsymbol{H}, \boldsymbol{\rho}) &= p(\boldsymbol{a}) p(\boldsymbol{h}_0) \textstyle\prod_{t=1}^T p(\boldsymbol{h}_t | \boldsymbol{h}_{t-1}, \boldsymbol{x}_{t-1}, \boldsymbol{a}) p(\rho_0) \prod_{t=1}^T p(\rho_t | q_{t-1}, \boldsymbol{a}) \\ &= p(\alpha_1) p(\alpha_2) p(\phi_{11}) p(\phi_{22}) p(\gamma_{11}) p(\gamma_{22}) p(\psi_0) p(\psi) p(\sigma^2_{\eta 1}) p(\sigma^2_{\eta 2}) \\ &\quad p(\sigma^2_{\rho}) p(\boldsymbol{h_0}) \textstyle\prod_{t=1}^T p(\boldsymbol{h}_t | \boldsymbol{h}_{t-1}, \boldsymbol{x}_{t-1}, \alpha_1, \alpha_2, \phi_{11}, \phi_{22}, \gamma_{11}, \gamma_{22}, \sigma^2_{\eta 1}, \sigma^2_{\eta 2}) \\ &\quad p(\rho_0) \textstyle\prod_{t=1}^T p(\rho_t | q_{t-1}, \psi_0, \psi, \sigma^2_{\rho}) \end{aligned} \tag{2.13}$$

For the prior distributions, we follow Yu and Meyer (2006) and set the new introduced parameters weakly informative. The prior distributions are listed in the Appendix. We denote return series $\boldsymbol{R} = (\boldsymbol{r}_0, \boldsymbol{r}_1, ..., \boldsymbol{r}_T)$. The likelihood function is

$$p(\boldsymbol{R} | \boldsymbol{H}, \boldsymbol{\rho}, \boldsymbol{a}) = \prod_{t=1}^{T} p(\boldsymbol{r}_t | \rho_t, \boldsymbol{h}_t, \boldsymbol{a}) \tag{2.14}$$

So the joint posterior distribution of the unobservables given the data is given by

$$\begin{aligned} p(\boldsymbol{H}, \boldsymbol{\rho}, \boldsymbol{a} | \boldsymbol{R}) &\propto p(\boldsymbol{a}, \boldsymbol{H}, \boldsymbol{\rho}) p(\boldsymbol{R} | \boldsymbol{H}, \boldsymbol{\rho}, \boldsymbol{a}) \\ &\propto p(\boldsymbol{a}) p(\boldsymbol{h}_0) \textstyle\prod_{t=1}^T p(\boldsymbol{h}_t | \boldsymbol{h}_{t-1}, \boldsymbol{x}_{t-1}, \boldsymbol{a}) p(\rho_0) \\ &\quad \textstyle\prod_{t=1}^T p(\rho_t | q_{t-1}, \boldsymbol{a}) \prod_{t=1}^T p(\boldsymbol{r}_t | \rho_t, \boldsymbol{h}_t, \boldsymbol{a}) \end{aligned} \tag{2.15}$$

If a Markov chain has a ergodic (irreducible and aperiodic) transition kernel P, it will have a unique stationary distribution π that can offer reliable estimations and satisfies $\pi = \pi P$. In this case, the Markov chain will converge to π irrespectively of our starting points. So if we can devise a Markov chain whose stationary distribution π is our desired posterior distribution $p(\boldsymbol{H}, \boldsymbol{\rho}, \boldsymbol{a}|\boldsymbol{R})$, then we can run this chain to get draws that are approximately from $p(\boldsymbol{H}, \boldsymbol{\rho}, \boldsymbol{a}|\boldsymbol{R})$ once the chain has converged. The single-move Gibbs sampler is used to generate the Markov chain and the detail of the method can be found in Appendix.

After a burn-in period of 10,000 iterations and a follow-up period of 100,000, we stored every 20th iteration. We use large number of iterations to allow for the well documented slow convergence of the single moving Gibbs sampler for SV models (Chib and Greenberg, 1996; Kim *et al.*, 1998). In case of the DC-MSV and DC-MSV-IV models, the reason for the slow convergence is driven by that the components of log variance and dynamic correlations are highly correlated. Then sampling each component from the full conditional distribution produces little movement in the draws, and hence slowly decaying autocorrelations.

2.4.3 Regime changes and Markov-Switching models

In this study, we consider both of the Global Financial Crisis (GFC) and the Eurozone Sovereign Debt Crisis (ESDC) for studying the contagion effects. We follow the c) method listed in Section 2.2.4 that is combining both an economic and a statistical approach. Step 1, we define a relatively long crisis period which includes all major financial and economic news events representing the GFC and ESDC[4]. Step 2, we

[4]The period timeline is also listed in Section 2.2.4.

identify the start date and end date of the GFC and ESDC via Markov Switching models (Hamilton, 1989), which takes into account endogenous structural breaks and thus allows the data to determine the beginning and ending of the crisis.

We consider the statistical approach (step 2) rather than only rely on economic approach (step 1) because the phases as identified by economic approach are treated as distinct and independent, so the lagged impact of the crisis on financial markets tends to be ignored by the economic approach. There are two ways in literature for applying Markov Switching models to identify the crisis regimes: the first one is applying Markov Switching models to the conditional volatility estimated from GARCH family models (e.g. Baur, 2012; Dimitriou *et al.*, 2013). In this case only intercept of the conditional volatilities are regime dependent as we focus on the higher volatility level.

$$h_t = \mu_{h(s_t)} + \sigma_h^2 \varepsilon_t \tag{2.16}$$

where $\varepsilon_t \sim N(0,1)$, h_t is the conditional volatility at time t. The Markov Switching model assumes the existence of two regimes ("stable" $i = 0$ and "volatile" $i = 1$) so that the intercept $\mu_{h(s_t)}$ depend on the state variable s_t which assume two values that representing the different regimes. It is used to check whether or not all the conditional volatility in regime $s_t = 1$ is allocated within the period based on the economic approach (Baur, 2012). The second way is applying Markov Switching models to return series (e.g. , Ahmad *et al.*, 2013). In this case, both the level of returns and its variance are considered regime dependent, so the model can be written as

$$r_t = \mu_{r(s_t)} + \sigma_{r(s_t)}^2 \varepsilon_t \tag{2.17}$$

where $\varepsilon_t \sim N(0,1)$, r_t is the return series at time t. It can be used to detect the beginning and ending of the relatively long crisis period based on the economic approach. In this study, we use this approach and identify the start date and end date of the GFC and ESDC based on the return series. According to the estimated results, the $\mu_{r(s_0)}$ is higher than $\mu_{r(s_1)}$, and $\sigma^2_{r(s_0)}$ is lower than $\sigma^2_{r(s_1)}$, which indicates the stable period has lower volatility and meanwhile has higher return. Figure 2.3 shows the smooth regime probabilities of returns.

[Figure 2.3 around here]

Following Kenourgios (2014), we ignore the regime with low persistence (below one week) and use USA data to identify the start date and end date. Based on the results of the Markov Switching model for return series (regime probabilities above 0.9), we incorporate the lagged impact of crisis and confirm the crisis period for GFC is from 26th July 2007 to 16th July 2009, and ESDC is from 5th November 2009 to 1st December 2011.

2.4.4 Testing for Contagion

Forbes and Rigobon (2002) use t-tests to evaluate if there is a significant increase in correlations during the turmoil period compared with the full period. They also mention that some other tests also can be considered, such as comparing the correlations during the turmoil period with that during the stable period (instead of the full period), which is also commonly used by the literature. It is straightforward to see that the first test is stronger for testing the contagion effect compared with the second one, as if the correlations during the turmoil period are significantly higher compared with full period, the difference of correlations will be even larger compared with stable period rather than the full period. Following Forbes and Rigobon (2002), we use the first t-test in this chapter to compare the difference of correlations.

As shown in Section 2.4.2, like the latent volatilities, the dynamic correlations are also treated as unknown parameters for Bayesian MCMC estimation. Unlike DCC-GARCH models that is based on the two steps estimation, we can get the posterior mean of dynamic correlations ρ_t from $t = 1$ to T after the estimation of DC-MSV-IV model, then we can construct the dynamic correlation time series by the posterior mean of ρ_t. As identified in Section 2.4.3, the GFC is from 26 July 2007 to 16 July 2009 and ESDC is from 5 November 2009 to 1 December 2011. The turmoil period should cover the two crisis so that we use the beginning date of GFC and the ending date of ESDC to define the turmoil period, and then the remaining sample is stable period [5]. Then we can calculate the mean and standard deviation of the dynamic correlation time series for each period.

We use ρ_f and ρ_h to denote the mean of correlations during the full period and turmoil period respectively, the test hypotheses are $H_0 : \rho_f > \rho_h$, $H_1 : \rho_f \leq \rho_h$. If the null hypothesis should be rejected according to the test results, then we can confirm the increase in correlations during the turmoil period which indicates the existence of financial contagion. We report the rest results in Section 2.5.3.

[5]We incorporate the time between the GFC and the ESDC into the turmoil period as we need consider the lagged impact of GFC.

2.5 Empirical Results

Section 2.5.1 firstly compares the dynamic correlations based on the standard dynamic correlation multivariate models, then discusses the improvement for the detecting contagion by incorporating the implied volatility information into the standard DC-MSV model. Section 2.5.2 shows the estimation results for the DC-MSV-IV model, and compare its in-sample fit with the standard DV-MSV model. Section 2.5.3 discusses the financial contagion detected by the DC-MSV-IV model in details.

2.5.1 Dynamic Correlations with Different Models

A. Dynamic Correlation Multivariate Models

The literature documents the superior performance of the DC-MSV compared with the DCC-GARCH (Asai and McAleer, 2009), but the two models have not been compared in terms of detecting financial contagion, that is the changes in correlations during different market regimes, so we investigate the difference in detecting financial contagion for the two models.

[Table 2.2 around here]

Table 2.2 shows the dynamic correlations estimated from the DCC-GARCH and DC-MSV models. We calculate the changes in correlations as $(\rho_{turmoil} - \rho_{full})/\rho_{full}$. As the financial contagion is measured by the significant increase in correlations after crisis, if the changes of correlations are larger, the contagion effect is stronger. The changes of correlations observed under the DC-MSV model are larger than the changes observed under the DCC-GARCH model. It indicates the contagion effect is more apparent when we estimate the time varying correlations based on the DC-MSV model.

B. DC-MSV vs DC-MSV-IV

We investigate whether or not incorporating the implied volatility can further improve the significance of financial contagion. We calculate the dynamic correlations based on the DC-MSV-IV model and compare with the standard DC-MSV model. The results are shown in Table 2.3

[Table 2.3 around here]

The changes of correlations from the DC-MSV-IV model are larger for all pairs. It indicates that the contagion effect is more apparent if we incorporate implied volatility information. The better performance of the DC-MSV-IV model compared with the standard DC-MSV model can be supported with the DIC, that we report in next section with estimation details.

2.5.2 DC-MSV augmented with Implied Volatility

[Table 2.4 around here]

Table 2.4 shows estimation results of the DC-MSV-IV model. The posterior mean of ψ for every pair are closed to 1. It indicates that the correlation processes are reasonably highly persistent which is in line with Yu and Meyer (2006) for DC-MSV model. We can find that the coefficients (γ_1 and γ_2) of implied volatility information for every pairs of countries are significant and positive, so today's return volatility can be influenced by previous market's expectation for future volatility. This is in line with expectation because the implied volatility is also known as "the investor fear gauge" and the investor sentiment is closely linked with the volatility of stock market in the future. Kolb (2011) argue that the investors' negative expectation during financial crisis is an important channel for financial contagion. The literature also support the implied volatilities are more informative than daily returns, especially during turmoil periods (Fleming *et al.*, 1995; Fleming, 1998; Christensen and Prabhala, 1998; Blair *et al.*, 2001).

[Table 2.5 around here]

In order to illustrate the gains obtained from augmenting the DC-MSV with implied volatility, we compare the DC-MSV-IV model with the original DC-MSV model based on Deviance Information Criterion (DIC). The DIC is introduced by Spiegelhalter *et al.* (2002) as a generalization of the Akaike information criterion (AIC; Akaike, 1973). The AIC deals with the trade-off between the goodness of fit of the model and the complexity of the model, measured by the number of free parameters. However, the AIC is not applicable for comparing SV models (more generally for all complex hierarchical Bayesian models) because the specification of the dimensionality of the parameter space is rather arbitrary: as for MSV models, the 2T latent volatilities are dependent, they cannot be counted as the 2T additional free parameters (Yu and Meyer, 2006). Therefore, DIC is introduced for comparing hierarchical models, and it also consider the measures goodness of fit and the penalty term for increasing model complexity.

As the idea of AIC, the DIC consists of two components,

$$DIC = \overline{D} + p_D \tag{2.18}$$

where $\overline{D}$ measures goodness of fit and p_D is a penalty term for increasing model complexity. Like AIC, the model with lower value of DIC is preferred. The performance of DIC relative to two posterior odd approaches (Newton and Raftery, 1994; Chib, 1995) has been investigated by Berg *et al.* (2004) in the context of univariate SV models. They found reasonably consistent performance of these three model comparison methods.

Table 2.5 shows the DIC together with $\overline{D}$ and p_D for DC-MSV-IV and DC-MSV models. The DC-MSV-IV model has higher p_D compared with DC-MSV as the incorporating of more parameters. After allowing for the the penalty of increasing model complexity, we find that the DC-MSV-IV model still achieves a lower DIC for every pair of countries compared with DC-MSV model, indicating the better performance. The results further highlight the added value of using implied volatility.

2.5.3 Dynamic Correlations and Contagion Analysis

After fitting the DC-MSV-IV model to each country pair, we obtain the dynamic correlations ρ_t. Figure 2.4 shows the dynamic correlations for each pair during the full sample period. According to the figure, we can find that the assumption of constant correlation is unreasonable.

[Figure 2.4 around here]

[Table 2.6 around here]

Table 2.6 shows the dynamic correlations and t-test based on the changes in correlations for financial contagion. We calculate the means of dynamic correlations denoted by ρ and its standard deviations for different sub-periods and full period. As shown in Section 2.4.4, the null hypothesis is that the cross-market correlations during the full period are significantly greater than during the turmoil (high volatility) period. It is clear that we should reject the null hypothesis for each pair, indicating the existence of financial contagion.

We discuss the strength of the contagion effect by investigating the extent of changes in correlations for each country pair. Compared with pre-crisis period, the correlations between USA and the five European countries increases during the GFC period, and are even higher during the ESDC period. According to the correlation changes for the turmoil period compared with pre-crisis period $(\rho_{turmoil} - \rho_{pre\ crisis})/\rho_{pre\ crisis}$, it can be seen that the UK is most influenced by the contagion effect compared with other countries. On the other hand, the least influenced country is Germany which has lowest increase in correlation compared with pre-crisis period.

A possible reason for this is that the UK has very high ratio of banking sector assets and liabilities to national income. Compared to other sectors, the banking system is highly globalized and tends to be highly influenced by the financial crisis. On the other hand, German has strong economic fundamentals than other countries which makes it less influenced by contagion.

We should note that the strong contagion effect is not necessary as a result of high correlation. For example, France is highest correlated to the USA but it is not highly influenced by the financial contagion. On the other hand, even though the correlation between Switzerland and USA during the turmoil period is lowest (0.755), but it highly influenced by the contagion effect. As outlined by Forbes and Rigobon (2002), "...if two markets show a high degree of co-movement during periods of stability, even if the markets continue to be highly correlated after a shock to one market, this may not constitute contagion." For this reason that we focus on the percentage changes of correlation to investigate the contagion effect.

It is natural to consider whether or not the high correlations will continue even after the turmoil period, so in order to check the further trends of dynamic correlations after the GFC and ESDC periods, we report the mean of correlations during the post-crisis period. So far, most studies in this topic consider the last day of their data sample as the end of the turmoil period, it is partly because the recently financial crises (including both GFC and ESDC) cover longer periods and in order to incorporate the stable period we need consider latest data sample. We compare the correlations of post-crisis with the correlations during turmoil period. From Table 2.6, we can find the correlations of all pairs decrease after the turmoil period. We calculate the decrease of correlations during the post crisis period compared with turmoil period as $(\rho_{post\ crisis} - \rho_{turmoil})/\rho_{turmoil}$. The largest decrease of correlations is UK with 12.4% followed by Switzerland with 12.3%, whereas the smallest decrease is Germany with 8.6%. The rank of the decreases in correlations is the same as the increases in correlations. We can confirm that after financial crisis (from December 2011 to November 2014) the correlations tend to revert back to a relatively lower level.

[Table 2.7 around here]

As we confirm the beginning of turmoil period (Crisis beginning date) based on Markov Switching model for return series (we consider both the return level and its volatility). However, according to BIS (2009), the official date of crisis start is 9 Aug 2007 (main event on that day: BNP Paribas, Frances largest bank, halts redemptions on three investment funds). So we further investigate the beginning date of turmoil period and also calculate the date for the five European countries. Then we compare them with the official timeline. From Table 2.7, we can find the crisis beginning dates are 25 or 26 July 2007, almost 2 weeks before the official date of crisis start. According to the timeline offered by Federal Reserve Board of St. Louis, the main event on July 24, 2007 is "Countrywide Financial Corporation warns of 'difficult conditions'." Then the transition probability increases and raises above 0.9 one or two days later. So the event on July 24, 2007 should be the real start of the turmoil period according to the data.

As the identified turmoil period covers a relatively long period, apparently not everyday during that period suffers low return and high volatility. The number of days in turmoil regime and the number of days in stable regime can be calculated based on the Markov Switching model. As shown in Pappas *et al.* (2015), then we can calculate the crisis intensity according to the identified regimes (days in turmoil regime divided by the days in turmoil period). From Table 2.7, we can find that USA, UK and Netherlands have relatively high values of crisis intensity which means they suffer relatively longer periods for low returns and high volatilities. On the other hand, Germany has lowest value of crisis intensity (26.94%). As identified before, Germany is also the country that is least influenced by the contagion effect. But we should note the relationship does not work for Switzerland. The crisis intensity is relatively low for Switzerland but the contagion effect has high impact on Switzerland.

2.6 Conclusion

This study examines financial contagion between stock markets of USA and five EU countries (UK, Switzerland, Netherlands, Germany and France). The sample covers a period from 2003 to 2014 in order to cover both the recent Global Financial Crisis

(GFC) and the European Sovereign Debt Crisis (ESDC). We contribute to the literature in a number of ways.

First, the existing literature have not directly compared the contagion effects estimating from multivariate GARCH and multivariate SV specifications. We compare the DCC-GARCH and DC-MSV models of estimating dynamic correlations and outline that the contagion effect is more significant based on the DC-MSV model.

Second, we extend the DC-MSV model by incorporating the implied volatility (DC-MSV-IV), and compare the contagion effect with the standard DC-MSV. The contagion effect is further more significant. Because DIC support the DC-MSV-IV model fits the data better than the DC-MSV model for every country pair, it offers more accurate estimations for the dynamic correlations. We confirm the implied volatility information is useful for detecting financial contagion.

Third, we offer the empirical evidence of the existence of contagion for the countries under investigation. We consider both GFC and the ESDC. Compared with the stable market regimes, the correlations are significantly higher during the crisis market regimes. The dynamic correlations are even higher during ESDC compared with GFC. For the five EU countries, the UK is most influenced by the financial contagion whereas Germany is least influenced. The dynamic correlations tend to recover to lower level after the turmoil period. We investigate the relationship between financial contagion and long-term correlation with USA, and support that the strong contagion effect is not necessary as a result of high correlation. In terms of the relationship between financial contagion and crisis intensity, the high contagion effect does not necessarily lead to the high crisis intensity.

The empirical results indicate that, if investors want to reduce their risk by portfolio diversification, they should carefully consider the contagion effect, because an investment strategy relies on the assumption of constant correlation between international markets might not work or lead to terrible performance during turmoil periods.

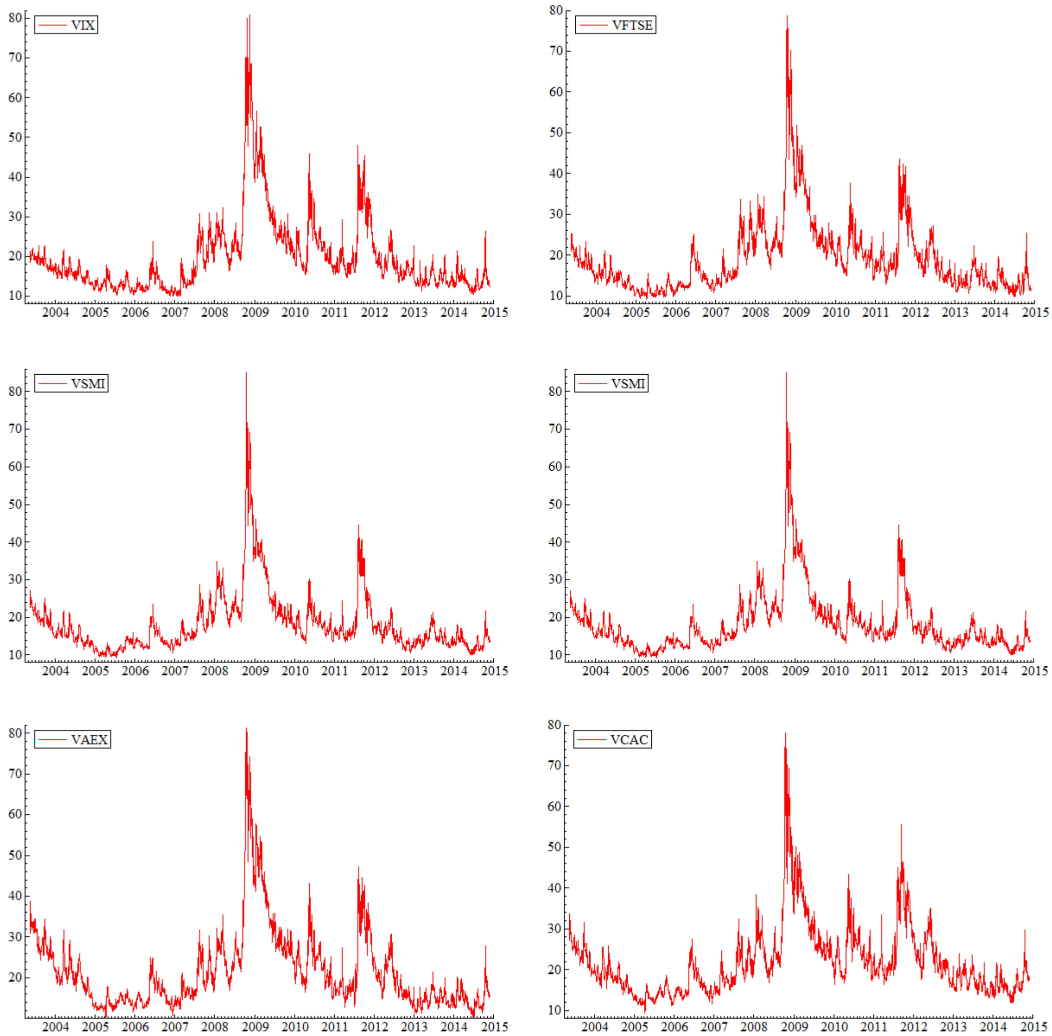

Figure 2.1: This figure show time series of impled volatilities for the sample countries: USA, UK, Switzerland, Netherlands, Germany and France (S&P 500, FTSE 100, SMI, AEX, DAX and CAC 40). The sample covers a period from 15th May 2003 to 25th November 2014.

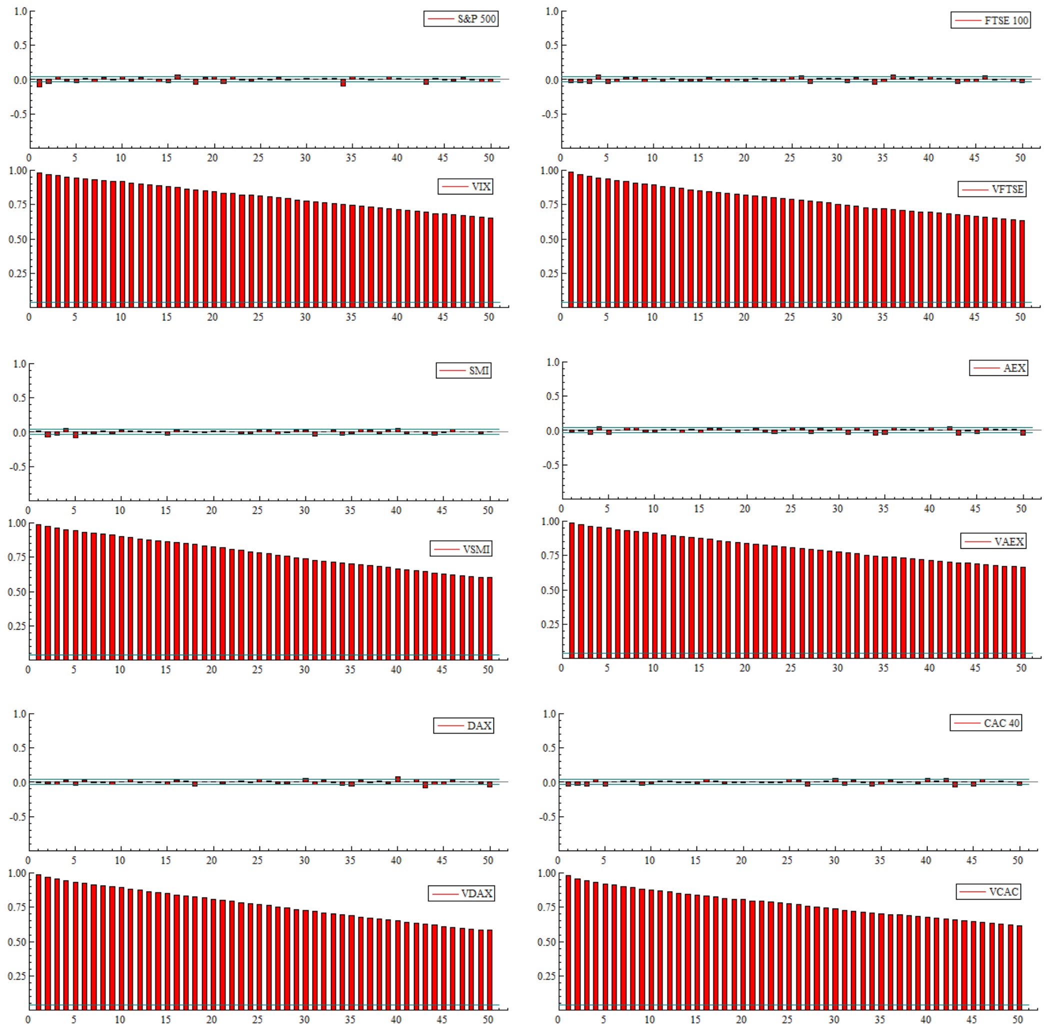

Figure 2.2: This figure shows autocorrelations of returns and implied volatilities for sample countries from 15th May 2003 to 25th November 2014. The top panel for each graph is the autocorrelation of return series, and the bottom panel is the autocorrelation of implied volatility series.

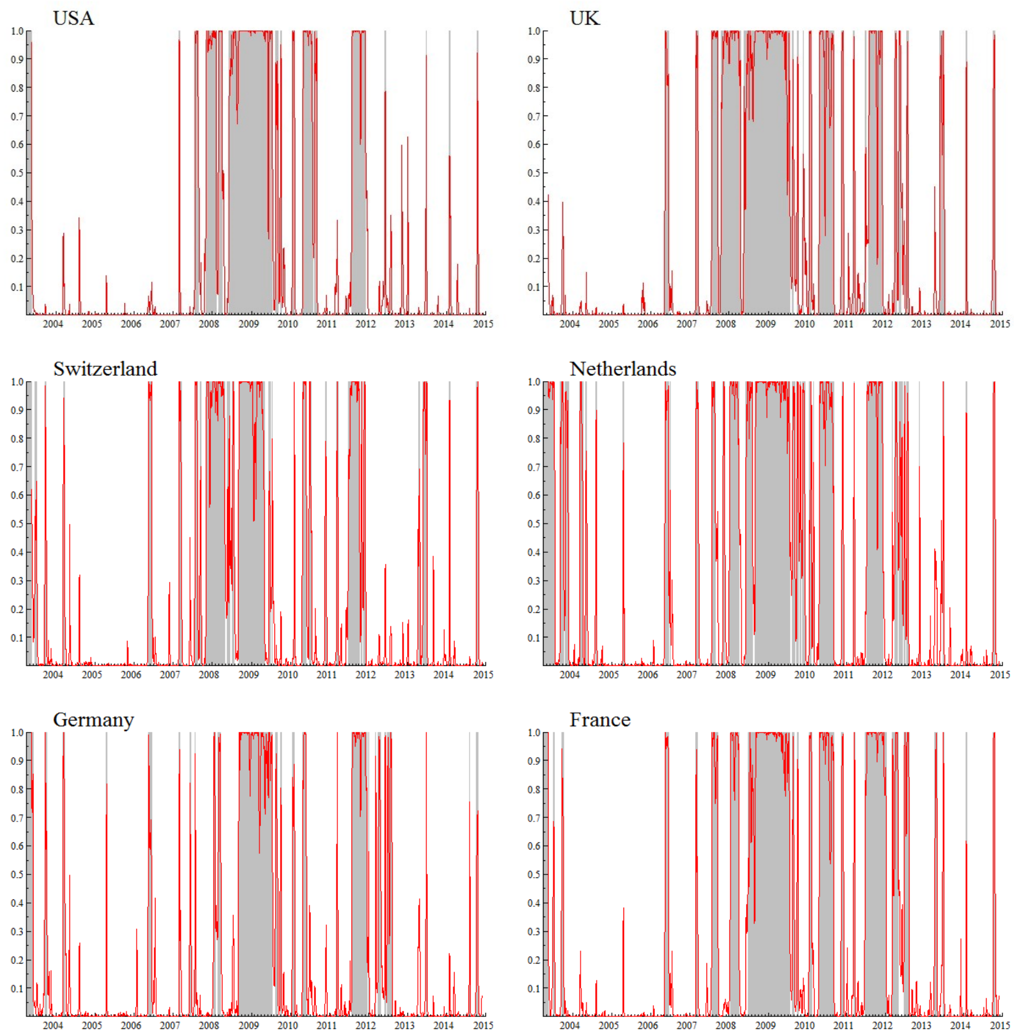

Figure 2.3: This figure shows Markov Switching regimes of return series for the sample countries from 15th May 2003 to 25th November 2014, the grey area is the regime with low returns and high volatilities.

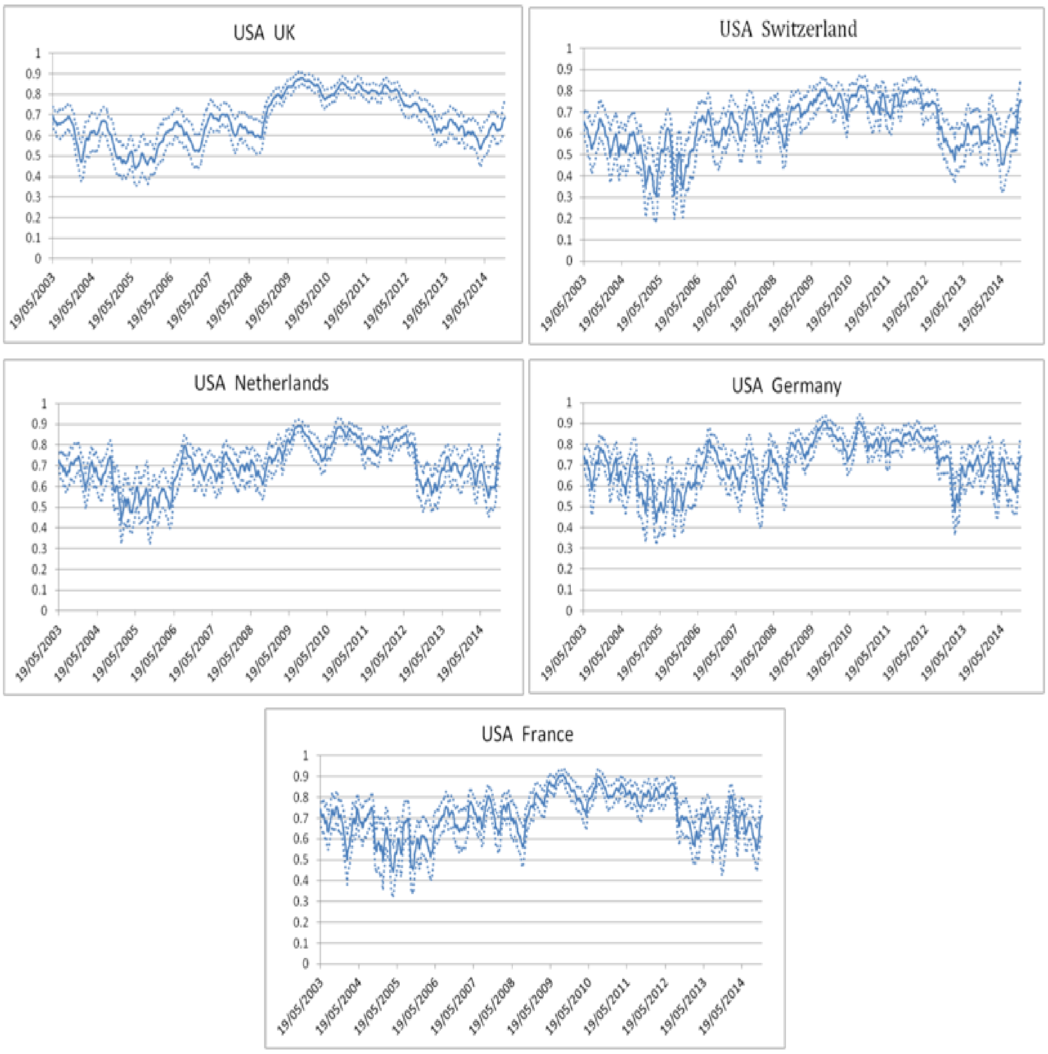

Figure 2.4: This figure shows dynamic correlations for each country pair from 15th May 2003 to 25th November 2014, as the correlations are estimated based on the Bayesian MCMC method, the dotted line is the 95% credible intervals of the posterior correlation estimation

Table 2.1: Summary statistics of return and implied volatility (2003-2014)

	USA	UK	Switzerland	Netherlands	Germany	France
	Return					
	S&P 500	FTSE 100	SMI	AEX	DAX	CAC 40
Mean	0.027	0.018	0.024	0.013	0.041	0.013
Std. Dev.	0.821	0.780	0.785	0.929	0.960	0.956
Skewness	-0.566	-0.354	-0.328	-0.650	-0.302	-0.230
Kurtosis	11.323	10.501	11.881	9.039	7.511	7.634
Jarque-Bera	8534	6874	9855	4697	2535	2668
Min	-6.629	-6.463	-6.368	-6.859	-5.875	-6.581
Median	0.066	0.056	0.065	0.063	0.104	0.068
Max	6.202	5.556	7.882	4.964	6.731	6.652
Correlation (USA)	1.000	0.735	0.687	0.736	0.748	0.745
	Implied Volatility					
	VIX	VFTSE	VSMI	VAEX	VDAX-NEW	VCAC
Mean	19.607	19.070	18.228	21.676	22.532	21.890
Std. Dev.	9.357	8.623	7.996	9.713	8.877	8.477
Skewness	2.486	2.360	2.793	2.143	2.247	1.986
Kurtosis	11.020	11.061	14.006	9.179	10.284	8.896
Jarque-Bera	10774	10569	18427	6962	8971	6221
Min	9.935	9.300	9.435	10.220	11.830	9.375
Median	16.862	16.692	15.925	18.760	20.045	19.953
Max	79.595	76.589	79.355	77.700	81.060	75.470
Correlation (USA)	1.000	0.972	0.955	0.953	0.936	0.953

Notes: The table shows the summary statistics of daily return and implied volatility of stock indices for six countries: USA, UK, Switzerland, Netherlands, Germany and France (S&P 500, FTSE 100, SMI, AEX, DAX and CAC 40). The sample covers a period from 15th May 2003 to 25th November 2014.

Table 2.2: Dynamic Correlations (DCC-GARCH vs DC-MSV)

DCC-GARCH	Dynamic Correlations			
USA-	Full	Stable	Turmoil	Changes(%)
UK	0.655	0.614	0.721	10.1
Switzerland	0.595	0.570	0.636	6.8
Netherlands	0.668	0.645	0.707	5.8
Germany	0.679	0.651	0.725	6.8
France	0.680	0.653	0.723	6.4
DC-MSV	Dynamic Correlations			
USA-	Full	Stable	Turmoil	Changes(%)
UK	0.668	0.612	0.759	13.5
Switzerland	0.626	0.593	0.681	8.7
Netherlands	0.694	0.663	0.745	7.3
Germany	0.709	0.68	0.757	6.8
France	0.709	0.672	0.769	8.5

Notes: "Full", "Stable" and "Turmoil" measures the mean of dynamic correlations for different regimes. The "Changes(%)" under "Dynamic Correlations" measures the increase of correlations during the turmoil period compared with the full period $((\rho_{Turmoil} - \rho_{Full})/\rho_{Full})$. The GFC is from 26 July 2007 to 16 July 2009 and ESDC is from 5 November 2009 to 1 December 2011. The turmoil period should cover the two crisis so that we use the beginning date of GFC and the ending date of ESDC to define the turmoil period, and then the remaining sample is stable period.

Table 2.3: Dynamic Correlations (DC-MSV vs DC-MSV-IV)

DC-MSV	Dynamic Correlations			
USA-	Full	Stable	Turmoil	Changes(%)
UK	0.668	0.612	0.759	13.5
Switzerland	0.626	0.593	0.681	8.7
Netherlands	0.694	0.663	0.745	7.3
Germany	0.709	0.68	0.757	6.8
France	0.709	0.672	0.769	8.5
DC-MSV-IV	Dynamic Correlations			
USA-	Full	Stable	Turmoil	Changes(%)
UK	0.678	0.621	0.771	13.7
Switzerland	0.636	0.584	0.722	13.4
Netherlands	0.702	0.658	0.775	10.4
Germany	0.711	0.671	0.777	9.3
France	0.715	0.671	0.786	10.0

Notes: "Full", "Stable" and "Turmoil" measures the mean of dynamic correlations for different regimes. The "Changes(%)" under "Dynamic Correlations" measures the increase of correlations during the turmoil period compared with the full period $((\rho_{Turmoil} - \rho_{Full})/\rho_{Full})$. The GFC is from 26 July 2007 to 16 July 2009 and ESDC is from 5 November 2009 to 1 December 2011. The turmoil period should cover the two crisis so that we use the beginning date of GFC and the ending date of ESDC to define the turmoil period, and then the remaining sample is stable period.

Table 2.4: The estimation result of the DC-MSV-IV model for all pairs of countries

		USA-UK	USA-Switzerland	USA-Netherlands	USA-Germany	USA-France
α_1	mean	-2.636	-2.454	-2.598	-2.644	-2.691
	SD	0.324	0.339	0.322	0.331	0.319
	95% CI	-3.295, -2.016	-3.131, -2.016	-3.251, -1.971	-3.310, -2.019	-3.310, -2.071
α_2	mean	-3.045	-2.500	-2.072	-1.736	-2.214
	SD	0.362	0.424	0.312	0.303	0.311
	95% CI	-3.777, -2.373	-3.343, -2.373	-2.707, -1.485	-2.368, -1.185	-2.870, -1.653
γ_1	mean	0.790	0.736	0.778	0.792	0.806
	SD	0.0973	0.102	0.097	0.010	0.096
	95% CI	0.603, 0.984	0.540, 0.984	0.588, 0.975	0.604, 0.993	0.621, 0.993
γ_2	mean	0.917	0.756	0.625	0.527	0.678
	SD	0.110	0.130	0.095	0.092	0.095
	95% CI	0.711, 1.139	0.507, 1.139	0.447, 0.818	0.360, 0.721	0.508, 0.880
ϕ_1	mean	0.660	0.684	0.661	0.655	0.650
	SD	0.041	0.044	0.042	0.043	0.042
	95% CI	0.576, 0.740	0.596, 0.740	0.572, 0.743	0.568, 0.737	0.568, 0.728
ϕ_2	mean	0.598	0.647	0.713	0.758	0.707
	SD	0.047	0.059	0.042	0.042	0.041
	95% CI	0.500, 0.686	0.531, 0.686	0.627, 0.792	0.670, 0.833	0.622, 0.781
ψ	mean	0.993	0.980	0.983	0.980	0.980
	SD	0.003	0.009	0.009	0.008	0.011
	95% CI	0.986, 0.998	0.959, 0.998	0.962, 0.994	0.960, 0.991	0.952, 0.995
ψ_0	mean	1.737	1.579	1.844	1.898	1.911
	SD	0.157	0.115	0.126	0.119	0.115
	95% CI	1.432, 2.043	1.353, 2.043	1.586, 2.091	1.669, 2.138	1.686, 2.152
σ_ρ	mean	0.063	0.096	0.095	0.113	0.108
	SD	0.008	0.021	0.022	0.021	0.032
	95% CI	0.051, 0.081	0.064, 0.145	0.066, 0.145	0.085, 0.166	0.061, 0.174
σ_{η_1}	mean	0.241	0.245	0.242	0.242	0.234
	SD	0.033	0.032	0.035	0.028	0.031
	95% CI	0.177, 0.306	0.186, 0.311	0.167, 0.309	0.191, 0.300	0.176, 0.295
σ_{η_2}	mean	0.299	0.352	0.308	0.270	0.273
	SD	0.034	0.033	0.030	0.030	0.031
	95% CI	0.229, 0.369	0.290, 0.422	0.248, 0.367	0.218, 0.335	0.212, 0.338

Notes: The table report means, standard errors, and 95% credible intervals of the posterior distributions for the DC-MSV-IV model among USA and the five European countries (UK, Switzerland, Netherlands, Germany and France). As we investigate the correlations between USA and the five European countries, for the DC-MSV-IV model r_{1t} is the USA return series and r_{2t} is the five European countries respectively. "mean" is the mean of the posterior distributions,"SD" is the posterior standard deviations, and "95% CI" is the 95% credible intervals of the posterior distributions.

Table 2.5: The comparison of the DC-MSV-IV and DC-MSV based on DIC

		USA-UK	USA-Switzerland	USA-Netherlands	USA-Germany	USA-France
DC-MSV-IV	DIC	8739.960	9128.390	9693.670	10003.100	9927.610
	$\overline{D}$	8323.450	8641.590	9209.160	9528.020	9474.750
	p_D	416.504	486.797	484.515	475.124	452.862
DC-MSV	DIC	8792.520	9138.500	9701.770	10003.700	9965.580
	$\overline{D}$	8462.070	8734.950	9279.310	9582.140	9570.770
	p_D	330.452	403.545	422.456	421.518	394.813

Notes: The DIC consists of two components: $\overline{D}$ measures goodness of fit and p_D is a penalty term for increasing model complexity. The model with lower value of DIC is preferred.

Table 2.6: The dynamic correlations during different periods and the test for contagion

	Pre-crisis		GFC		ESDC		Post-crisis	
USA-	ρ	Std. Dev.	ρ	Std. Dev.	ρ	Std. Dev.	ρ	Std. Dev.
UK	0.583	0.076	0.698	0.083	0.825	0.023	0.675	0.073
Switzerland	0.550	0.094	0.676	0.057	0.755	0.040	0.633	0.097
Netherlands	0.630	0.085	0.721	0.058	0.811	0.044	0.697	0.086
Germany	0.643	0.088	0.717	0.081	0.816	0.041	0.710	0.094
France	0.646	0.078	0.737	0.078	0.817	0.038	0.708	0.086
	Turmoil		Stable		Full			
USA-	ρ	Std. Dev.	ρ	Std. Dev.	ρ	Std. Dev.	T-test	Changes(%)
UK	0.771	0.089	0.621	0.087	0.678	0.114	-26.914	32.3, -12.4
Switzerland	0.722	0.064	0.584	0.104	0.636	0.113	-29.787	31.3, -12.3
Netherlands	0.775	0.072	0.658	0.091	0.702	0.102	-25.222	23.0, -10.1
Germany	0.777	0.084	0.671	0.096	0.711	0.105	-20.426	20.9, -8.6
France	0.786	0.077	0.671	0.087	0.715	0.100	-23.902	21.7, -9.9

Notes: ρ is the means of dynamic correlations and σ is its standard deviations. The test statistics are for one-sided t-tests examining if the cross-market correlations during the full period are significantly greater than during the turmoil (high volatility) period. “Changes(%)” measures the changes of ρ: the first number is the changes of ρ during the turmoil period compared with the pre-crisis period ($(\rho_{Turmoil} - \rho_{Pre\ Crisis})/\rho_{Pre\ Crisis}$), and the second number is the changes of ρ during the post-crisis compared with the turmoil period ($(\rho_{Post\ Crisis} - \rho_{Turmoil})/\rho_{Turmoil}$). The GFC is from 26 July 2007 to 16 July 2009 and ESDC is from 5 November 2009 to 1 December 2011. The turmoil period should cover the two crisis so that we use the beginning date of GFC and the ending date of ESDC to define the turmoil period, and then the remaining sample is stable period.

Table 2.7: Crisis Transition Dates, Duration and Intensity measures

	Crisis beginning date	Lead/Lag	Days in turmoil regime	Days in stable regime	Crisis intensity(%)
USA	26/07/2007	-10	495	604	45.0
UK	25/07/2007	-11	533	567	48.5
Switzerland	25/07/2007	-11	348	750	31.7
Netherlands	26/07/2007	-10	515	602	46.1
Germany	26/07/2007	-10	299	811	26.9
France	25/07/2007	-11	471	627	42.9

Notes: The Lead/Lag column reports the difference between the crisis beginning date (identified by Markov Switching model for return data) and the official date 9 Aug 2007 offered by BIS (2009). The negative number means the number of working days before the official date.

2.7 Appendix

The Prior Distribution

We follow Yu and Meyer (2006) and set the new introduced parameters weakly informative:

- $\alpha_1 \sim N(0,25); \quad \alpha_2 \sim N(0,25);$
- $\phi_{11}^* \sim beta(20,1.5), \text{where } \phi_{11}^* = (\phi_{11}+1)/2;$
- $\phi_{22}^* \sim beta(20,1.5), \text{where } \phi_{22}^* = (\phi_{22}+1)/2;$
- $\gamma_{11} \sim N(0,25); \quad \gamma_{22} \sim N(0,25)$
- $\psi_0 \sim N(0.7,10)$
- $\psi^* \sim beta(20,1.5), \text{where } \psi^* = (\psi+1)/2;$
- $\sigma_{\eta 1}^2 \sim \textit{Inverse-gamma}(2.5,0.025)$
- $\sigma_{\eta 2}^2 \sim \textit{Inverse-gamma}(2.5,0.025)$
- $\sigma_{\rho}^2 \sim \textit{Inverse-gamma}(2.5,0.025)$

Kim *et al.* (1998) discuss the choice of priors for autocorrelation parameters (here, ϕ_{11}, ϕ_{22} and ψ). They use the same prior as listed above, and mention that: the flat prior $\pi(\phi) \propto 1$ is attractive in that it leads to an analytically tractable full conditional density, but this prior can cause problems when the data are close to being non-stationary (Phillips, 1991; Schotman and Van Dijk, 1991). They argue that it is important from a data-analytic view to impose stationarity in the SV models.

The Gibbs Sampler

The Gibbs sampler is very useful when the joint posterior distribution does not take a convenient form, however, the full conditionals of the posterior for each parameter are relatively simple to draw from. The Gibbs sampler begins with the initialization of $(\boldsymbol{H}^{(0)}, \boldsymbol{\rho}^{(0)}, \boldsymbol{a}^{(0)})$[6], and then it draws from each of the following distributions:

For $s = 1, \dots, S$:

[6]We use superscripts to indicate draws, 0 draw means the initialization. We use a_1 ... a_k to represent the parameters of DC-MSV-IV model, and the data sample is $t = 1 \dots T$.

Take a random draw, $a_1^{(s)}$ from $p(a_1|a_2^{(s-1)}, a_3^{(s-1)}, ..., a_k^{(s-1)}, \boldsymbol{H}^{(s-1)}, \boldsymbol{\rho}^{(s-1)}, \boldsymbol{R})$.

Take a random draw, $a_2^{(s)}$ from $p(a_2|a_1^{(s-1)}, a_3^{(s-1)}, ..., a_k^{(s-1)}, \boldsymbol{H}^{(s-1)}, \boldsymbol{\rho}^{(s-1)}, \boldsymbol{R})$.

$\vdots$

Take a random draw, $a_k^{(s)}$ from $p(a_k|a_1^{(s-1)}, a_2^{(s-1)}, ..., a_{k-1}^{(s-1)}, \boldsymbol{H}^{(s-1)}, \boldsymbol{\rho}^{(s-1)}, \boldsymbol{R})$.

Take a random draw, $h_1^{(s)}$ from $p(h_1|\boldsymbol{a}^{(s-1)}, h_2^{(s-1)}, h_3^{(s-1)}, ..., h_{2T}^{(s-1)}, \boldsymbol{\rho}^{(s-1)}, \boldsymbol{R})$.

Take a random draw, $h_2^{(s)}$ from $p(h_2|\boldsymbol{a}^{(s-1)}, h_1^{(s-1)}, h_3^{(s-1)}, ..., h_{2T}^{(s-1)}, \boldsymbol{\rho}^{(s-1)}, \boldsymbol{R})$.

$\vdots$

Take a random draw, $h_{2T}^{(s)}$ from $p(h_2|\boldsymbol{a}^{(s-1)}, h_1^{(s-1)}, h_2^{(s-1)}, ..., h_{2T-1}^{(s-1)}, \boldsymbol{\rho}^{(s-1)}, \boldsymbol{R})$[7].

Take a random draw, $\rho_1^{(s)}$ from $p(\rho_1|\boldsymbol{a}^{(s-1)}, \boldsymbol{H}^{(s-1)}, \rho_2^{(s-1)}, \rho_3^{(s-1)}, ..., \rho_T^{(s-1)}, \boldsymbol{R})$

Take a random draw, $\rho_2^{(s)}$ from $p(\rho_2|\boldsymbol{a}^{(s-1)}, \boldsymbol{H}^{(s-1)}, \rho_1^{(s-1)}, \rho_3^{(s-1)}, ..., \rho_T^{(s-1)}, \boldsymbol{R})$

$\vdots$

Take a random draw, $\rho_T^{(s)}$ from $p(\rho_T|\boldsymbol{a}^{(s-1)}, \boldsymbol{H}^{(s-1)}, \rho_1^{(s-1)}, \rho_2^{(s-1)}, ..., \rho_{T-1}^{(s-1)}, \boldsymbol{R})$

Following these steps will yield a set of S draws. After dropping the first S_0 of a set of S draws to eliminate the effect of initialization $(\boldsymbol{H}^{(0)}, \boldsymbol{\rho}^{(0)}, \boldsymbol{a}^{(0)})$, the remaining S_1 draws can be averaged to create estimates of posterior features of interest. Dropping the first S_0 known as the burn-in. The reason to do this is that we need the draws from converged distribution and less dependent on the starting point. The posterior mean of $\boldsymbol{a}$ can be estimated by

$$\hat{\boldsymbol{a}} = \frac{1}{S_1} \sum_{s=S_0+1}^{S} \boldsymbol{a}^{(s)} \tag{2.19}$$

This is because under reasonably general conditions, the conditional density used in Gibbs sampling converges to the true marginal density as $S \to \infty$ (Casella and George, 1992).

[7] As the $\boldsymbol{h}_t$ includes h_{1t} and h_{2t}, so we have $2T$ for the latent volatility term.

Chapter 3

A time varying HAR model for realized volatility forecasting

3.1 Introduction

As a measure of the risk of financial assets, volatility plays an important role in many practical financial management decisions. For example, volatility is a key parameter for pricing financial derivatives. Volatility is also pivotal for asset allocation and risk management. Therefore, accurately measuring and forecasting financial volatility is of crucial importance for financial market participants.

Volatility measures and models are classified under parametric approaches (e.g., GARCH or stochastic volatility) and nonparametric approaches (e.g., Realized volatility). As discussed by Andersen *et al.* (2001), Barndorff-Nielsen and Shephard (2001, 2002a), by the theory of quadratic variation and under suitable conditions, realized volatility (RV) is an unbiased and highly efficient estimator of return volatility. RV is defined as the sum of squared intra-day returns. Andersen *et al.* (2003) and Andersen *et al.* (2004) find that simple models of RV outperform the popular GARCH and related stochastic volatility models in out-of-sample forecasting. The availability of high quality intra-day data has raised the popularity of RV, which is now widely investigated. In this chapter, we focus on modelling and forecasting RV.

Long-range dependence is a well documented stylized fact of RV. Fractionally integrated ARFIMA models are shown by Andersen *et al.* (2003) to characterize this strong dependency. Early studies have employed ARFIMA models for modelling and forecasting RV (e.g. Koopman *et al.*, 2005; Martens and Zein, 2004; Pong *et al.*, 2004). Despite the success, the ARFIMA models are difficult to extend to multivariate processes. In addition, they lack a clear economic interpretation. Fractionally integrated models are also difficult to estimate and forecast. Recent studies treat the simple and easy-to-estimate approximate long-memory Heterogeneous AR (HAR) model of Corsi (2009) as the preferred specification for RV based forecasting (e.g. Chen and Ghysels, 2011; Duong and Swanson, 2015; Liu *et al.*, 2015). Andersen *et al.* (2007) and Corsi *et al.* (2010) extend the HAR model by considering jumps. Busch *et al.* (2011) introduce a Vector-HAR model and consider the implied volatility information for forecasting RV. Patton and Sheppard (2015) introduce a Semivariance-HAR model. Bollerslev *et al.* (2016) propose a HARQ model which uses the estimated degree of measurement error to adjust the weight of daily lags.

In this chapter, we extend the HAR model to allow the autocorrelation parameter of daily lags to be time varying (TV-HAR). We observe a regular pattern of RV which is captured by the TV-HAR model: if there is an increase in the lag daily RV compared with its longer-term average level (monthly RV), the current RV tends to decrease rapidly to its long term level; conversely, if there is a decrease in the lag daily RV compared with its longer-term average level (monthly RV), that reversion takes longer. The pattern can be supported by the data summary statistics. The observations of $RV_{d,t} > RV_{m,t}$ are significantly fewer than the $RV_{d,t} > RV_{m,t}$, where $RV_{d,t}$ is the daily RV and $RV_{m,t}$ is its long-term moving average monthly RV. Our model can capture this pattern: the magnitude of changes for the daily lags is based on the absolute difference between the long-term (monthly) RV and the short-term (daily) RV. The weight of daily lags is highest when the RV is equal to its longer-term level. The lower weight can make the forecasts quickly mean reverting when daily RV is bigger than its monthly RV, and slowly mean reverting when daily RV is smaller than its monthly RV.

To highlight the significance of forecasting improvements, we use simulation and empirical data to investigate the performance of the TV-HAR model. The simulation uses the two-factor stochastic volatility model to generate the intraday log price. The empirical analysis relies on high-frequency data of the S&P 500, SPY indices and ten individual stocks from 2000 to 2010. We consider different sampling frequencies of RV: 150, 300, 450 and 900 seconds. We also investigate the model performance over different sub-sample periods: the pre-crisis period from 2000 to 2006, and the crisis period from 2007 to 2010. We compare the TV-HAR model with the standard HAR and HARQ models. We find that for the in-sample fits, the TV-HAR and HARQ have similar gains compared with the HAR model. For the out-of-sample forecasts, the TV-HAR model generally outperforms the HAR and HARQ models for both the full sample and sub-sample periods based on the S&P 500, SPY and ten stocks.

The rest of chapter is organized as follows. Section 2 provides the theoretical framework of RV. Section 3 discuss the HAR, HARQ and TV-HAR models. Section 4 shows a simulation study for the performance of the TV-HAR model. Section 5 describes the data set employed in the empirical study. Section 6 reports the empirical results and Section 7 concludes.

3.2 Literature Review

Volatility has been one of the most active research areas in both theoretical and empirical finance during the past decade. Approaches to the measurement and estimation of volatility can be classified into parametric and non-parametric approaches. The parametric approach relates to the estimation of parametric models, where there are two main model streams: one is ARCH (GARCH) models (Engle, 1982; Bollerslev, 1986); the other is Stochastic Volatility models (Taylor, 1986; Hull and White, 1987). Volatility is considered as an unobserved variable under the parametric approach. On the other hand, the non-parametric approach treats volatility as an observable variable, which allows us to directly analyze, model and forecast volatility itself. In this study, we focus on the RV which belongs to the non-parametric approach.

Some very early studies based on the non-parametric approach use price change or absolute price change to measure volatility (e.g. Ying, 1966; Clark, 1973). Later studies use historical, ex-post sample variances computed from higher frequency return data as lower frequency volatility measures. For example, Poterba and Summers (1984), French *et al.* (1987), Pagan and Schwert (1990), and Schwert (1989) calculate monthly sample variances from daily returns. Thanks to the technological process in trade recording, the available of high-frequency data allow the calculation of daily variance from intraday returns (e.g., Schwert, 1990; Hsieh, 1991; Taylor and Xu, 1997). Andersen and Bollerslev (1998), Andersen *et al.* (2001) Barndorff-Nielsen and Shephard (2002a, 2002b) and Comte and Renault (1998) discuss the theoretical properties of RV. They show that under general conditions, as the number of intraday returns increases, the sum of intraday squared returns converges to the relevant notion of volatility of the interval. Therefore, RV provides us, in principle, with a consistent nonparametric measure of volatility.

Since the RV is calculated from the high-frequency data, there is a trade-off between accuracy and microstructure bias. On one hand, efficiency considerations suggest the use of a very high number of intraday return observations to reduce the stochastic error of volatility estimation. On the other hand, a bias introduced by market microstructure grows as the sampling frequency increases. Aït-Sahalia *et al.* (2005) and Bandi and Russell (2006, 2008) introduce techniques for determining the optimal sampling frequencies. They find that the 5-minute RV is an empirically satisfactory frequency. Liu *et al.* (2015) compare different sampling frequencies and realized measures. They use 5-minute RV as their benchmark and find little evidence that it is outperformed by any other measures.

The long-range dependence is an important stylized fact of RV. It displays significant autocorrelations even at very long lags. This stylized fact can be captured by long memory models. Andersen *et al.* (2003) introduce the ARFIMA model to model and forecast the RV of Deutschemark/Dollar and Yen/Dollar exchange rates. There are many applications and extensions of ARFIMA models to RV and other realized measures. For example, Li (2002) show that the forecasting performance of the ARFIMA model is better than that of implied volatility from options on currencies. Martens and Zein (2004) find that the ARFIMA model outperforms the daily GARCH model for three separate asset classes, equity, foreign exchange, and commodities. Koopman *et al.* (2005) compare the forecasting power of RV based on the ARFIMA model, daily volatility based on GARCH and SV models, and implied volatility calculated from option price. They find that the ARFIMA model performs best. Based on three exchange rates, Pong *et al.* (2004) compare the forecasts from a short memory ARMA model, a long memory ARFIMA model, a GARCH model and option implied volatilities. They find intraday rates provide the most accurate forecasts for the one-day and one-week forecast horizons while implied volatilities are at least as accurate as the historical forecasts for the one-month and three-month horizons.

Corsi *et al.* (2012) argue that the long-range dependence stylized fact might be due to a genuine long-memory data generating process or, alternatively, that it can be explained as a combination of different short-memory processes. Although Granger (1980) shows that a true long-memory process requires the aggregation of an infinite number of short-memory process, LeBaron *et al.* (2001) find that an approximated long-memory process can be obtained by aggregating only a few heterogeneous timescales, that offers the econometric foundation of the HAR model. On the other hand, the theoretical foundation of the HAR model is motivated by the heterogenous market hypothesis of Müller *et al.* (1993). The need for multiple components in the volatility process has also been advocated by Engle and Lee (1993), Müller *et al.* (1997), Bollerslev and Wright (2001), Calvet and Fisher (2004).

Most recent literature tends to use the HAR model and its extensions for forecasting RV, treating them as the benchmark model. For example, Chen and Ghysels (2011) treat the HAR family models as the benchmark model to investigate whether or not news can differently influence future volatility. They find that moderately good news reduces volatility, while both very good news and bad news increase volatility, with the latter having a more severe impact. The asymmetries disappear over longer horizons. Andersen *et al.* (2011) augment the HAR-J model with a GARCH(1,1)-t error structure to describe the dynamic dependencies in the daily continuous sample path variability, while they model the overnight returns by an augmented GARCH type structure. Duong and Swanson (2015) use the HAR model to investigate the impact of different jumps on volatility. They find that past large jump power variations help less in the prediction of future realized volatility, than past small jump power variations. In addition, incorporation of downside and upside jump power variations can improve volatility forecasting. Liu *et al.* (2015) study the accuracy of a variety of estimators of asset price variation constructed from high-frequency data, and compare them with the simple RV measure. For volatility forecasting, they also rely on the HAR model.

3.3 Theoretical Framework

Let us start by considering an asset for which the return process s_t is determined by the stochastic differential equation:

$$ds_t = \mu_t d_t + \sigma_t dW_t + k_t dq_t \tag{3.1}$$

where μ_t and σ_t represent the drift and the instantaneous volatility processes respectively, W_t is a standard Brown motion which is assumed to be independent of σ_t, $q(t)$ is a pure jump process with time varying intensity and k_t is the jump size.

To simplify the notation, we normalize the unit time interval to a day, the quadratic return variation process (QV) can be written as the sum of the diffusive intergrated variance (IV) and the cumulative squared jumps:

$$QV_t = \int_{t-1}^{t} \sigma_s^2 d_s + \sum_{t-1 \leq s \leq t} J_s^2 = IV_{t,k} + \sum_{t-1 \leq s \leq t} J_s^2 \tag{3.2}$$

where $J_t = k_t dq_t$ is non-zero only if there is a jump at time t. The RV_t provides a consistent estimator of the QV_t as the number of intraday observation increases, according to a series of seminal papers by Andersen and Bollerslev (1998), Andersen *et al.* (2001) Barndorff-Nielsen and Shephard (2002a, 2002b) and Comte and Renault

(1998). The RV_t is defined by the summation of high-frequency returns,

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2 \tag{3.3}$$

where $M = 1/\Delta$, and the Δ-period intraday return is defined by $r_{t,i} = \log(P_{t-1+i\Delta}) - \log(P_{t-1+(i-1)\Delta})$, so for $\Delta \to 0$, $RV_t \to QV_t$. It means that RV_t approximates QV_t arbitrarily well as the sampling frequency increases. However, as summarized by Andersen and Teräsvirta (2009), there are two issues complicating the application of this result. First, even for the most liquid assets a continuous price record is unavailable. We should recognize the presence of a measurement error because this limitation introduces an inevitable discretization error in the RV measures. Second, spurious autocorrelations in the ultra-high frequency return series can be induced by a wide array of microstructure effects. Such "spurious" autocorrelations can inflate the RV measures and then generate a traditional type of bias-variance trade off. Therefore, in this chapter, we consider different sampling frequencies to investigate the model performances.

3.4 Models

3.4.1 The HAR

The standard HAR model introduced by Corsi (2009) has arguably emerged as the most popular model for daily realized volatility based forecasting. The HAR model uses the autoregressive processes of realized volatility on three volatility components (the daily RV, weekly RV and monthly RV) to model the long-memory behavior of volatility. The weekly RV and monthly RV are moving averages of daily RV. Thus, the weekly realized volatility (RV_w) at time t is

$$RV_{w,\,t} = \frac{1}{5}(RV_t + RV_{t-1} + \ldots + RV_{t-4}) \tag{3.4}$$

and the monthly realized volatility (RV_m) at time t is can be written as

$$RV_{m,\,t} = \frac{1}{22}(RV_t + RV_{t-1} + \ldots + RV_{t-21}) \tag{3.5}$$

Based on the weekly and monthly RV as calculated above, the HAR model is defined as

$$RV_{d,\,t} = \beta + \beta_d RV_{d,\,t-1} + \beta_w RV_{w,\,t-1} + \beta_m RV_{w,\,t-1} + u_t \tag{3.6}$$

where $RV_{d,\,t}$ is the daily RV at day t, so that $RV_{d,\,t} = RV_t$. The HAR model can be easily estimated by simple OLS.

The choice of daily, weekly and monthly lags can conveniently capture the approximate long-memory dynamic dependencies observed in most RV series. The simple additive model defined as the sum of only three different AR(1) processes displays a decaying memory pattern, as discussed by LeBaron *et al.* (2001). Based on the HAR model, the appearance of long-memory of RV might be due to a combination of different short-memory processes rather than a genuine long-memory data-generating process.

The motivating idea of the HAR model stems from the heterogenous market hypothesis (Müller *et al.*, 1993). Participants in financial markets have different trading frequencies that may influence volatility differently, so the HAR model uses different autoregressive parameters for daily, weekly and monthly RVs respectively.

The autocorrelation parameters are assumed to be constant over time in the standard HAR model. However, this assumption is suboptimal from a forecasting perspective. For example, when lag daily RV significantly increases, we should put less weight on it, otherwise the forecast for the current RV tends to exceed the true value.

3.4.2 The HARQ

As the constant weight of daily lags is suboptimal, Bollerslev *et al.* (2016) introduce the HARQ model which allows the weight to be time varying. The model is based on the idea that RV is equal to the sum of two components: the true latent IV and a time varying measurement error. It allows the autocorrelation parameter to vary with the estimated degree of measurement error. The model is defined as

$$RV_{d,\,t} = \beta + \underbrace{(\gamma + \beta_q RQ_{t-1}^{1/2})}_{\beta_{d,t}} RV_{d,\,t-1} + \beta_w RV_{w,\,t-1} + \beta_m RV_{m,\,t-1} + u_t \tag{3.7}$$

where the Realized Quarticity (RQ) is used to consistently estimate the Integrated Quarticity (IQ). The RQ can be calculated as

$$RQ = \frac{M}{3}\sum_{i=1}^{M} r_{t,i}^4 \tag{3.8}$$

Unlike the HAR model with constant β_d and which assumes that the variance of the measurement error is constant, the HARQ model considers heteroskedasticity in the error, by allowing for the $\beta_{d,t}$ is high when the variance of the measurement error is low, and adjusted downward on days when the variance of measurement error is high. The β_q is assumed to be negative. According to Equation 3.7, the measurement error is measured by $RQ_{t-1}^{1/2}$. Therefore, the HARQ model places more weight on the daily lags when the RQ is low, and less weight on the daily lags when the RQ is high. Bollerslev *et al.* (2016) argue that theoretically when IQ is low, daily lag RV provides a strong signal about the true IV, and when IQ is high, the signal is weak.

According to Equation 3.8, the level of RQ is positively related to the level of RV. Therefore, the HARQ model tends to place more weight on daily lags RV when RQ is lower, and less weight on daily lags RV when RQ is higher. In this case, the theoretical maximum $\beta_{d,t}$ happens when the RQ is equal to 0 (if so RV is also equal to 0), and then $\beta_{d,t} = \gamma$.

As the monthly RV is the long-term moving average of daily RV, theoretically, the number of observations that daily RV ($RV_{d,\, t}$) is higher than its moving average monthly RV ($RV_{m,\, t}$) should approximately equal to the number of observations that daily RV is lower than its moving average monthly RV. However, according to the data summary statistics shown in Section 3.6, the the number of observations for $RV_{d,\, t} < RV_{m,\, t}$ is significantly higher than the number of observations for $RV_{d,\, t} > RV_{m,\, t}$. This regular pattern holds for S&P 500, SPY and all ten individual stocks. It means that if there is an increase in lag RV compared with its longer-term average level monthly RV, current RV tends to quickly revert back, in this case, the autoregressive parameter for $RV_{d,\, t-1}$ should be low and the model quickly mean reverting. On the other hand, if there is a decrease in lag RV compared with lag monthly RV, current RV tends to take longer to revert back. However, according to the HARQ model structure, when the daily lags is low, it tends to put more weight on daily lags and might not be able to catch the regular pattern very well.

3.4.3 The TV-HAR

Motivated by the regular pattern of RV discussed above, which indicates that (i) the HAR model should put less weight when the daily lag is greater than the monthly lag and (ii) the HARQ model should put less weight when the daily lag is lower than the montly lag. We introduce a TV-HAR model which allow the weight of daily lag to be time varying. The HAR model is popular because it is simpler to estimate than fractionally integrated processes (Corsi, 2009; Liu *et al.*, 2015). Our extension retains this advantage and we can use the OLS method to estimate the model. Compared with the HARQ model, the weight changes of daily lags depends on the absolute difference between daily RV and monthly RV rather than the level of RQ. The TV-HAR model structure can be written as

$$RV_{d,\,t} = \beta + \underbrace{(\gamma + \alpha|RV_{d,\,t-1} - RV_{m,\,t-1}|)}_{\beta_{d,t}} RV_{d,\,t-1} + \beta_w RV_{w,\,t-1} + \beta_m RV_{m,\,t-1} + u_t \tag{3.9}$$

As the monthly RV is the moving average of daily RV, the monthly RV is the long-term trend and smoothed version of daily RV. So the absolute difference between daily RV and monthly RV is the short-term changes in daily RV. With negative α, the theoretical maximum $\beta_{d,t}$ occurs when the $RV_{d,\,t-1} = RV_{m,\,t-1}$, and then $\beta_{d,t} = \gamma$. The lower weight on daily lags can produce lower forecasts for the current RV. Therefore, when $RV_{d,\,t-1} > RV_{m,\,t-1}$, the lower weight on the higher $RV_{d,\,t-1}$ can lead to the current RV reverting rapidly to its long-term average level. In this scenario, the TV-HAR model is similar to the HARQ model because the high RV tends to come with high RQ which also decreases the weight of daily lags. However, when $RV_{d,\,t-1} < RV_{m,\,t-1}$, the TV-HAR model works differently because the lower weight on the smaller $RV_{d,\,t-1}$ can further cause the current RV reverting more slowly to its long-term average level.

To better illustrate the inner workings of the TV-HAR model, Figures 3.1 and 3.2 for S&P 500 and SPY respectively show the time varying autoregressive parameter of daily RV in the TV-HAR model ($\beta_{d,t}$), along with the daily and monthly realized volatilities. We also compare the resulting one-day-ahead RV forecasts for the TV-HAR model with the HAR and the HARQ models.

[Figure 3.1 around here]

Figure 1 shows the two model estimates for the S&P 500 for ten successive trading days . We first compare the TV-HAR model with the HAR model based on the left panel (a). The top (a1) shows daily and monthly RVs. The middle (a2) shows the autocorrelation parameter for the daily lags. Unlike the constant autoregressive parameter of daily RV in the standard HAR model β_d, the value of $\beta_{d,t}$ in the TV-HAR model is time varying, as shown in the figure. It is apparently from middle panel that on day 6, the value of $\beta_{d,t}$ drops to a relatively low level, below 0.15, from around 0.45 on the previous trading day. According to the model structure, this is caused by the wider gap between the lagged daily RV and the lagged monthly RV, which can be supported by the daily and monthly RV from the top panel (a1) on day 5. If there is a big increase in one day's RV, the RV for the next day tends to quickly revert back to the longer-term average level. This is not inconsistent with volatility clustering theory. For example, during a high volatility period, the longer-term average level of RV (monthly RV) is higher, although the short-term fluctuations of RV still exist. The RV for day 5 is far above its monthly average level, the RV for the next trading day 6 is more likely to be lower. Therefore, for the HAR model, less weight should be placed on the daily RV on day 5 when we forecast the daily RV on day 6. The TV-HAR model can decrease the weight of daily RV on day 6 with the negative estimated value of α. We can compare the one-day-ahead forecasting results of the standard HAR model and the TV-HAR model based on the bottom panel (a3). Due to the constant β_d of the standard HAR model, the high RV on day 5 leads to an increase in the fitted value for the next day. On the other hand, the decreased $\beta_{d,t}$ of the TV-HAR model puts less weight on the high RV on day 5, then the forecast RV on day 6 is lower and closer to the true value which is shown in the top panel. It is also the case for the forecasts of RV on day 10.

The right panel (b) of Figure 3.1 compares the inner workings of the TV-HAR model with those of the HARQ model. Unlike panel (a), for panel (b), the $RV_d < RV_m$. As shown in the top (b1), although from day 1 to day 2, the daily RV decrease significantly, it reverts back slowly and on day 3 it remains almost same level. In order to capture the slowly meaning reverting in this scenario, we should still put less weight on the daily lags which can lead to lower forecasts. The TV-HAR can capture the pattern as the theoretical maximum $\beta_{d,t}$ happens when the $RV_{d,\ t-1} = RV_{m,\ t-1}$, the absolute difference between $RV_{d,\ t-1}$ and $RV_{m,\ t-1}$ still decreases the weight of daily lags. On the other hand, the HARQ model tends to put more weight in this scenario as the RQ also tends to be lower when the RV is lower. In order to investigate the weight of daily lags for the two models, we compare the weight changes based on the percentage of their theoretical maximum $\beta_{d,t} = \gamma$. The (b2) shows the weight changes of daily lags for the TV-HAR and HARQ models. We find that the weight of daily lags for the HARQ model is higher than for the TV-HAR model. On day 3 and day 4, the $\beta_{d,t}$ of the HARQ model is very close to its theoretical maximum level, because the RV one day 2 and day 3 is close to 0, so the level of RQ on that two days also tends to be very low. In this case, the HARQ model puts more weight on the daily lags which leads to a higher forecast level. For the TV-HAR model, the weight of daily lags is much lower than its theoretical maximum level as the different model inner working. Due to the difference in daily lag weights, the fitted value of RV is also different. As shown in (b3), the TV-HAR model forecasts a lower value than the HARQ model, and the fitted value of TV-HAR is closer to the actual RV.

[Figure 3.2 around here]

The inner workings of the TV-HAR model compared with the HAR and HARQ models for the SPY are shown in Figure 3.2. It shares the same analysis process and conclusion as we discussed for the S&P 500 above. As a summary, when $RV_{d,\,t-1} > RV_{m,\,t-1}$ the TV-HAR model can let the forecasts quickly mean reverting that the HAR model cannot, and when $RV_{d,\,t-1} < RV_{m,\,t-1}$ the TV-HAR model can let forecasts slowly mean reverting that the HARQ cannot.

3.5 Simulation study

3.5.1 Design and settings

This section presents a simulation study to further investigate the performance of the TV-HAR model. Our simulations are based on a two-factor stochastic volatility model that is commonly used in the literature to generate intraday returns (e.g. Huang and Tauchen, 2005; Barndorff-Nielsen *et al.*, 2008; Gonçalves and Meddahi, 2009; Patton, 2011; Bollerslev *et al.*, 2016). Following Bollerslev *et al.* (2016), we also consider the intraday pattern and noise for the two-factor stochastic volatility model. The model can be written as

$$d\log P_t = \mu dt + \sigma_{ut} v_t (\rho_1 dW_{1t} + \rho_2 dW_{2t} + \sqrt{1-\rho_1^2-\rho_2^2} dW_{3t}) \tag{3.10}$$

$$v_t^2 = s\text{-}\exp(\beta_0 + \beta_1 v_{1t} + \beta_2 v_{2t}) \tag{3.11}$$

$$dv_{1t} = \alpha_1 v_{1t} dt + dW_{1t} \tag{3.12}$$

$$dv_{2t} = \alpha_2 v_{2t} dt + (1 + \phi v_{2t}) dW_{2t} \tag{3.13}$$

$$\sigma_{ut} = C + A\exp(-at^*) + B\exp(-b(1-t^*)) \tag{3.14}$$

where W_{1t}, W_{2t}, and W_{3t} are standard Brownian motions, s-exp denotes the exponential function with a polynomial splined at high values to avoid explosive behavior. The function is defined as

$$s\text{-}\exp(x) = \begin{cases} 1.5\sqrt{1 - log(1.5) + x^2/\log(1.5)}, & \text{if } x > 1.5. \\ \exp(x), & \text{otherwise.} \end{cases} \tag{3.15}$$

The process v_{1t} is the persistent factor and the process v_{2t} is the strongly mean-reverting factor. Following Huang and Tauchen (2005), we set $\mu = 0.03$, $\beta_0 = -1.2$, $\beta_1 = 0.04$, $\beta_2 = 1.5$, $\alpha_1 = -0.00137$, $\alpha_2 = -1.386$, $\phi = 0.25$, and $\rho_1 = \rho_2 = -0.3$, where the parameters are expressed in daily units. The σ_{ut} is the diurnal U-shape function and it can model the intraday volatility pattern. Following Andersen *et al.* (2012), we set $A = 0.75$, $B = 0.25$, $C = 0.88929198$ and $a = b = 10$, respectively. The simulations are generated based on the Euler scheme. Therefore, Equation 3.10 can be transform to

$$\log P_t = \log P_{t-1} + \mu\Delta t + \sigma_{ut} v_t(\rho_1 Z_1\sqrt{\Delta t} + \rho_2 Z_2\sqrt{\Delta t} + \sqrt{1-\rho_1^2-\rho_2^2} Z_3\sqrt{\Delta t}) \quad (3.16)$$

And Equations 3.12 and 3.13 can be transformed to

$$v_{1,\,t} = v_{1,\,t-1} + \alpha_1 v_{1,\,t-1}\Delta t + Z_1\sqrt{\Delta t} \quad (3.17)$$

$$v_{2,\,t} = v_{2,\,t-1} + \alpha_2 v_{2,\,t-1}\Delta t + (1+\phi v_{2,\,t-1}) Z_2\sqrt{\Delta t} \quad (3.18)$$

We simulate data for the unit interval [0, 1] and normalize 1 second to be 1/23400 ($\Delta t = 1/23400$), so that [0, 1] is to span 6.5 hours. The t^* in the intraday pattern equation means that, at the start of each interval, we initialize the $\Delta t = 1/23400$, so that the intraday pattern is the same for every day. The empirical evidence of the noise term has been discussed by Bandi and Russell (2006) and Hansen and Lunde (2006). We mirror the design of Barndorff-Nielsen *et al.* (2008) and generate an i.i.d. noise term $u_{t,i} \sim N(0, \omega_t^2)$ with $\omega_t^2 = \xi^2 \int_{t-1}^{t} v_s^2\, ds$. The $\xi^2 = 0.01$ following Bandi and Russell (2006). According to the noise equations, the variance of noise is constant throughout the day, but changes from day to day. The noise is then added to the log price process to obtain the series of actual high-frequency simulated prices.

We initialized the persistent factor v_1 by drawing $v_{1,0} \sim N(0, \frac{-1}{2\alpha_1})$ from its unconditional distribution. The strongly mean-reverting factor v_2 is initialized at 0. We consider the $T = 2000$ days for the simulation study. Then the two-factor stochastic volatility generates 23400×2000 second log price observations. Then we aggregate these prices to sparsely sampled 390, 156, 78, 52, 26 and 13 observations per day, corresponding to 60 seconds, 150 seconds, 300 seconds, 450 seconds, 900 seconds and 1800 seconds returns respectively. The simulation is based on 100 replications for every sampling frequency.

3.5.2 Monte Carlo Results

We use the simulated RV to compare the forecasting performances of the HAR, HARQ an TV-HAR models. The comparison is based on the MSE and different sampling frequencies. We consider one-day-ahead rolling forecasting and the rolling window is the previous 1000 days. Thus the forecasting period is from 1001 to 2000 days.

[Table 3.1 around here]

Table 3.1 shows the simulation results for different sampling frequencies and the average value across these frequencies. The numbers in bold represent the model with best forecasting performance. The relative MSE means the ratio of the losses for the different models relative to the losses of the HAR model. We also calculate the gains of the forecasting accuracy based on the loss function. For example, the MSE gains of the HARQ model compared with the HAR model is reported as ”HAR, HARQ” and calculated as $(MSE_{HAR} - MSE_{HARQ})/MSE_{HAR}$. Therefore, the positive number means the HARQ model offers better forecasting performance than the HAR model by a certain percent. According to the simulated data, for all frequencies, the TV-HAR model offers better forecasts than the standard HAR model and the HARQ model. The HARQ model outperforms the HAR model. Compared with the HAR model, the gains for the HARQ model are lower at the very low frequencies and very high frequencies, which is in line with the results offered by Bollerslev *et al.* (2016). Similarly the TV-HAR model also offers lower improvement compared with the HAR model for the very low and very high sampling frequencies.

3.6 Data Description

The empirical analysis relies on the S&P 500 equity index, SPY and ten stocks data from different sectors. The sample period is from January 3, 2000 to December 31, 2010, giving 2767 observations of daily RV. In this chapter, we use four different sampling frequencies of RV, (150, 300, 450 and 900 seconds) to investigate the model performance. We also extract the two subsample periods from the full sample to further investigate the model performance: the pre-crisis is from 2000 to 2006 and the crisis period is from 2007 to 2010. The S&P 500 equity index and ten stocks data come from Tick Data which provides data on a commercial basis for futures, indices and equity markets. The Tick database is sourced from the NYSE's TAQ (Trade and Quote) database. Tick adjusts TAQ for ticker mapping, code filtering, price splits and dividend payments. We did not consider any adjustments beyond that provided by the database. The SPY data come from the TAQ data. The trading day is from 9:30am to 16:00pm, which amounts to 23000 seconds.

[Tables 2.1 to 2.4 around here]

Tables 2.1 to 2.4 show the summary statistics of RV for the four sampling frequencies. We report the number of observations, mean, standard deviation, minimum and maximum values for daily, weekly and monthly RVs. As we use the absolute difference of daily and month lags to adjust the weight of daily lags, we also report the summary statistics for the absolute difference. In order to show the regular pattern of RV discussed above, we also compare the summary statistics of $(RV_d - RV_m)$ when $(RV_d - RV_m) > 0$ represented by $(RV_d - RV_m)^+$, and $(RV_d - RV_m)$ when $(RV_d - RV_m) < 0$ represented by $(RV_d - RV_m)^-$.

[Figure 3.3 around here]

The weekly and monthly RVs are the moving average of daily RV. Although the daily, weekly and monthly RVs have similar averages for every sampling frequency, the standard deviations are significantly different. The standard deviation of daily RV is higher than the standard deviations of weekly RV and monthly RV. The weekly RV and monthly RV are less volatile because the short-term fluctuations are removed by smoothing out the daily RV. We use the 300 seconds RV for S&P 500 as an example: the means are 0.8660 (daily), 0.8669 (weekly) and 0.8695 (monthly) which are similar. The standard deviations decrease from 1.9010 (daily) to 1.6228 (weekly) and 1.4620 (monthly). Compared with the four sampling frequencies, we find that, with the increase of the sampling frequencies, the standard deviations for daily, weekly and monthly RVs generally tend to decrease. Figure 3.3 shows the autocorrelations for daily, weekly and monthly RVs. For the weekly and monthly RVs, as the short term fluctuations have been removed to some extent, they have longer memory than the daily RV.

Although the monthly RV is the moving average of daily RV, we find that the observations of $(RV_d - RV_m)^+$ are significantly lower than the observations of $(RV_d - RV_m)^-$ for all the S&P 500, SPY indices and the ten stocks. The observations of $(RV_d - RV_m)^+$ are around 1000 whereas the observations of $(RV_d - RV_m)^+$ are around 1700 for all sampling frequencies and indices or stocks. The absolute mean of $(RV_d - RV_m)^+$ is also higher compared with $(RV_d - RV_m)^-$ for all data. We use the

300 seconds RV for S&P 500 as an example: the observations of $(RV_d - RV_m)^+$ are 1036 and the absolute mean is 0.5962, whereas the observations of $(RV_d - RV_m)^-$ are 1710 and the absolute mean is 0.3668. This indicates the different pattern for the daily RV reverting back to its long term average level.

3.7 Empirical Results

3.7.1 In-sample estimation results

In this section, we show the estimation results and also investigate the model performance for in-sample fits. Like the standard HAR model, the TV-HAR model can be easily estimated by the standard OLS method.

[Tables 3.3 to 3.5 around here]

Tables 3.3 and 3.4 show the parameter estimates, with standard errors in parentheses, of the standard HAR, the HARQ and the TV-HAR models for the S&P 500 and SPY indices. The adjusted R-squared values and Akaike information criterion (AIC) are followed by the estimated parameters in the table to compare the in-sample fit. The in-sample estimation results for the ten stocks are shown in the Appendix. Table 3.5 summaries and compares the in-sample fits of different models for the ten stocks. We discuss the in-sample estimation results for the HAR, HARQ and TV-HAR models as follows.

A. The HAR

We begin with the in-sample estimates for the HAR model. For the S&P 500, the

HAR model places more weight on the weekly lags according to the full sample period estimates. Compared with other sampling frequencies, the HAR model fits the data better with the 300 seconds RV. In terms of different market regimes, according to the estimations for the HAR model, because the pre-crisis period is less volatile, the daily lags seems more informative than them in the crisis period, the HAR model tends to put more weight on the daily lags for the pre-crisis period than the crisis period. It seems that the market microstructure noise is greater for the crisis period as the HAR model fits the data best for the 150 seconds RV during pre-crisis period but for the crisis period the best fits sampling frequency is 300 seconds RV. For SPY, the estimation results share similar pattern compared with the S&P 500. The difference is that the HAR model does not place largest weight on the daily lags for the pre-crisis period.

B. The HARQ

Next we discuss the in-sample estimation results of the HARQ model. Compared with other sampling frequencies, the HARQ model fits the data better with the 150 seconds RV for both indices. In line with Bollerslev *et al.* (2016), the values of β_q are all negative and significant. This indicates that the uninformative days with large measurement errors have a smaller impact on the forecasts than days where RV is estimated precisely. The value of γ is the theoretical maximum weight of daily lags, which are higher for the crisis period than the pre-crisis period, because the crisis period has some days which the daily lags tend to be very uninformative and requires larger adjustment for their weights. Compared with the HAR model, the HARQ model tends to allocate lower weight on the weekly and monthly lags, but a greater average weight on the daily lags, so the HARQ model generally allows for a more rapid response, except when the signal is poor.

C. The TV-HAR

In this section we focus on the in-sample estimation results of the TV-HAR model. Similar to the HARQ model, the TV-HAR model fits the data better with the 150 seconds RV for both indices. As expected, the value of α of the TV-HAR model is negative and strongly significant for all sampling frequencies and subsample periods. Therefore, the absolute difference of daily and monthly lags is negatively related to the weight of daily lags, which is consistent with the inner working of the TV-HAR model. Like the HARQ model, the theoretical maximum weight of daily lags measured by γ is larger for the crisis period than the pre-crisis period. Compared with the HAR model, the TV-HAR model allocates lower weight to the weekly and monthly lags, but a greater average weight to the daily lags. One possible explanation for this is that the TV-HAR model allows the weight of daily lags to change according to the different scenarios, so the daily lags can offer more accurate information for forecasting future RV. The theoretical maximum weight of daily lags is generally larger for the TV-HAR model compared with the HARQ model.

D. Comparison

Finally, we compare the in-sample fits of the HAR, HARQ and TV-HAR models. For the S&P 500, the TV-HAR model generally performs better than the HAR and HARQ models. It fits the data best for the 150, 450 and 900 seconds RV. The HARQ model performs best for the 300 seconds RV. Both the HARQ and TV-HAR model outperform the HAR model. In order to better compare these models, we calculate the average

R-square and AIC across different sampling frequencies. Based on the average values, the TV-HAR model performs best for the S&P 500. In terms of SPY, the HARQ model generally performs better than the HAR and TV-HAR models according to the average level. The TV-HAR model performs best for the 900 seconds RV. Then we compare the average in-sample fits for the 10 stocks shown in Table 3.5. There are five stocks support the TV-HAR model and five stocks support the HARQ model. The HARQ model generally fits the data better for the pre-crisis period, and the TV-HAR model fits the data better for the crisis period. Therefore, in terms of the in-sample fits, the TV-HAR and HARQ models obtain similar gains compared with the HAR model. As the financial market participants care more about the out-of-sample forecasting, in order to further compare the TV-HAR and HARQ models, in the next section we investigate the forecasting performances.

3.7.2 Out-of-sample forecasting results

This section shows the out-of-sample forecasting results for the HAR, HARQ and TV-HAR models. The models are re-estimated daily on a moving window of 1000 observations. We then perform the one-day-ahead forecasts.

[Tables 3.6 to 3.8 around here]

Tables 3.6 and 3.8 show the forecasting results for the S&P 500 and SPY respectively. Table 3.8 shows the forecasting results based on the loss functions averaged the ten stocks. We calculate the commonly used loss functions, the MAE and MSE, as the performance measures. To make the comparison of different models more clearly, we calculate the Relative MAE and MSE, and Gains(%). The details of these measures are the same as shown in Section 3.5.2. As the MAE and MSE have different level for the ten stocks, in order to put the same weight on the different stocks, in Table 3.8 we use the average values of relative losses rather than the original losses for comparing the model performances. The detailed forecasting results for every stock are given in the Appendix.

[Figure 3.4 around here]

For both indices, the losses are much lower for the pre-crisis period compared with the crisis period, especially for the MSE. We compare the forecasting performances of the HAR, HARQ and TV-HAR models for different sample frequencies and sub-sample periods. We calculate the average losses of the different sampling frequencies in order to compare the overall forecasting performances. As shown in Tables 3.6 and 3.7, for both S&P 500 and SPY, the MAE and MSE show that the TV-HAR model outperforms the HAR and the HARQ models for the full sample periods. For the sub-sample period performance, only the MSE for S&P 500 favours the HAR for the pre-crisis period, all other losses of the S&P 500 and SPY sub-sample periods support the better performance of the TV-HAR model. Compared with the HARQ and HAR models, for S&P 500, both MAE and MSE show that the HARQ model gives more accurate forecasts for full sample and sub-sample periods except the MSE for the pre-crisis period. In terms of SPY, the HARQ model has lower MAE compared with the HAR model for full sample and sub-sample periods. The original MSE indicates that the HARQ model only outperform the HAR model for the pre-crisis period, but according to the relative MSE, the HARQ model still outperform the HAR model for the full sample and sub-samples. The difference is due to that the averages of the relative measures allocate same weight for every sampling frequencies compared with the original losses. In order to investigate the model performances more clearly, Figure 3.4 shows the forecasting accuracy gains of the HARQ and TV-HAR models compared with the standard HAR model. The gains for the TV-HAR model is larger compared with the HARQ model and almost all the gains are positive for the TV-HAR model. For the average of ten stocks, the TV-HAR model also outperform the HAR and HARQ models for the full period and crisis period, and the HARQ model performs better for the pre-crisis period.

3.8 Conclusions

In this chapter, we focus on modeling and forecasting the RV which is an unbiased and highly efficient estimator of return volatility. The HAR model and its variants are commonly used by most recently studies to forecast the RV. The standard HAR model assumes that the autocorrelation parameters are constant over time. However, this assumption is suboptimal from a forecasting perspective because if there is an increase in lag RV compared with its longer-term average level monthly RV, current RV tends to quickly revert back. Another scenario is that, if there is a decrease in lag RV compared with its longer-term average level monthly RV, the current RV tends to slowly revert back. The recently introduced HARQ model does not capture this scenario very well because it tends to allocate greater weight to the daily lags when the RV is low. In this chapter, we introduce the TV-HAR model which allows the autocorrelation parameter of daily RV time varying according to the absolute difference between the lagged daily and lagged monthly RVs. The TV-HAR model can successfully capture the two scenarios. When RVd is above RVm, the RVd tends to be more fluctuated. Large increases in RVd tend to be followed by large decreases. The increases of RVd lead to higher difference between RVd and RVm, so that the TV-HAR assigns a lower weight on the daily lags which generates lower forecast, which makes the model quickly mean reversion. On the other hand, when RVd is below RVm, the RVd takes a long duration to recover back to its long term average level, RVm. As the difference between RVd and RVm is above 0 which leads to a lower weight on daily lags, therefore the TV-HAR can still generate a lower forecast, which makes the model slower mean reversion. The economic intuition for the pattern is that the extreme values or jumps of volatility increase the long term average level of realized volatility, so the volatility during stable periods tends to below the long term average level.

We use simulations to investigate the model performances. The simulation is based on a two factor stochastic volatility model which is used to generate the intraday log prices. The simulation results show that the TV-HAR model performs better than the HAR and HARQ models. This also holds true for the empirical data. Our empirical analysis is based on the S&P 500, SPY and ten stocks data. We consider different sampling frequencies and sub-sample periods. We find that both the TV-HAR and HARQ models fit the data better than the standard HAR model. The TV-HAR model fits the data best for the S&P 500; and the HARQ model fits the data best for the SPY. For the out-of-sample forecasts,the TV-HAR model generally outperforms the HAR and HARQ models. According to the average losses across different sampling frequencies, for the S&P 500 and SPY, the TV-HAR model offers most accurate forecasts for both the full sample period and sub-sample periods. In terms of the ten stocks, the TV-HAR model performs best for the full sample period and crisis period, and the HARQ model performs best for the pre-crisis period. The TV-HAR model is very easy to implement, and can be useful for forecasting the RV.

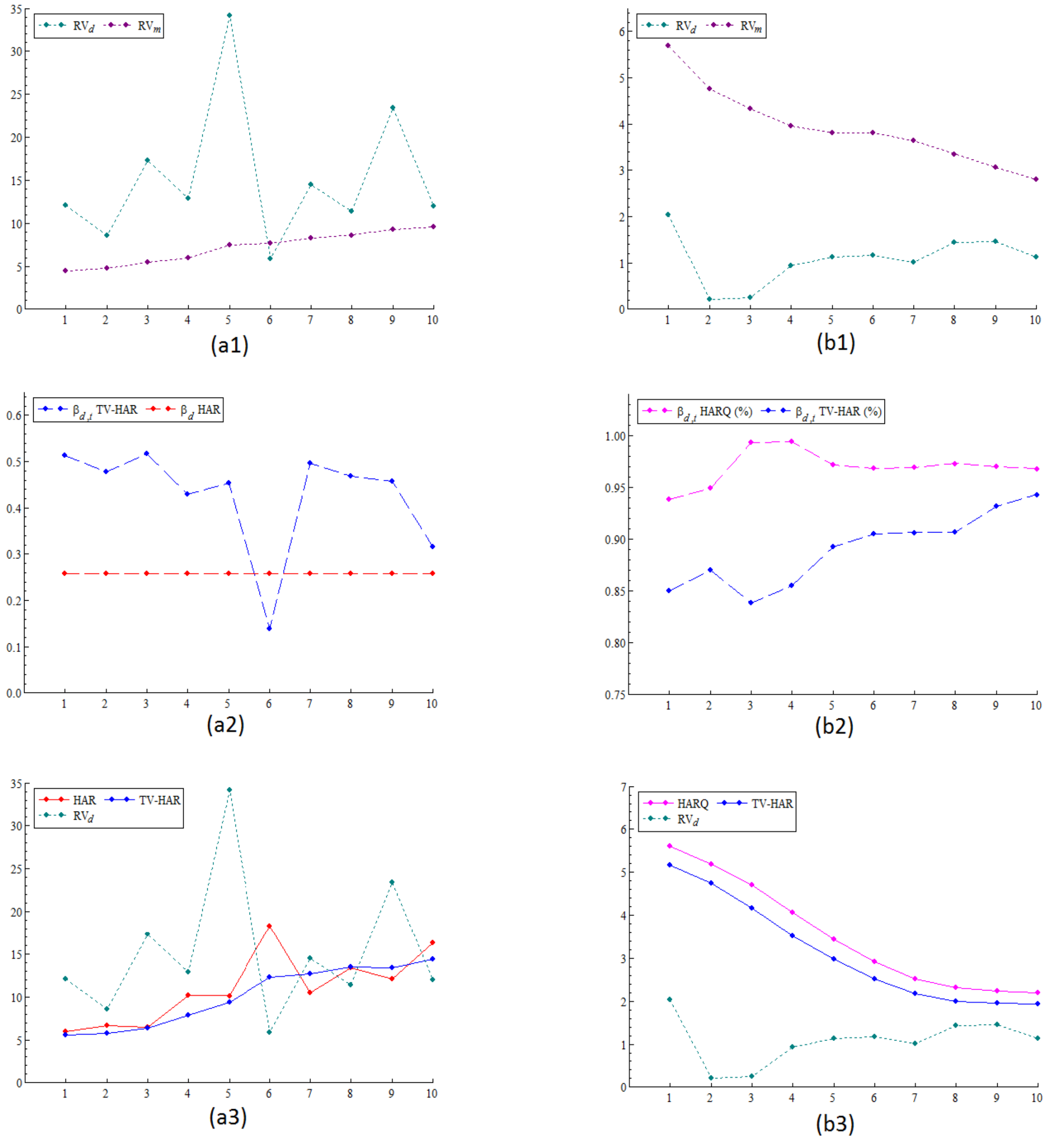

Figure 3.1: The left panel compares the TV-HAR model and the HAR model for ten successive trading days begin from 6 October 2008. The right panel compares the TV-HAR model and the HARQ model for ten successive trading days begin from 23 December 2008. The top shows daily and monthly realized volatilities. The middle left shows the time varying AR of daily RV ($\beta_{d,t}$) estimates from the TV-HAR model and the AR of daily RV (β_d) estimates from the HAR model. The middle right shows the time varying AR of daily RV estimates from the TV-HAR model and the HARQ model in percentage ($\beta_{d,t}/\gamma$, where the γ is the theoretical maximum value of time varying $\beta_{d,t}$, so we compare the percentage weights of daily lags of the two models). The bottom left compares the resulting one-day-ahead RV forecasts from the TV-HAR and the HAR models. The bottom right compares the resulting one-day-ahead RV forecasts from the TV-HAR and the HAR models.

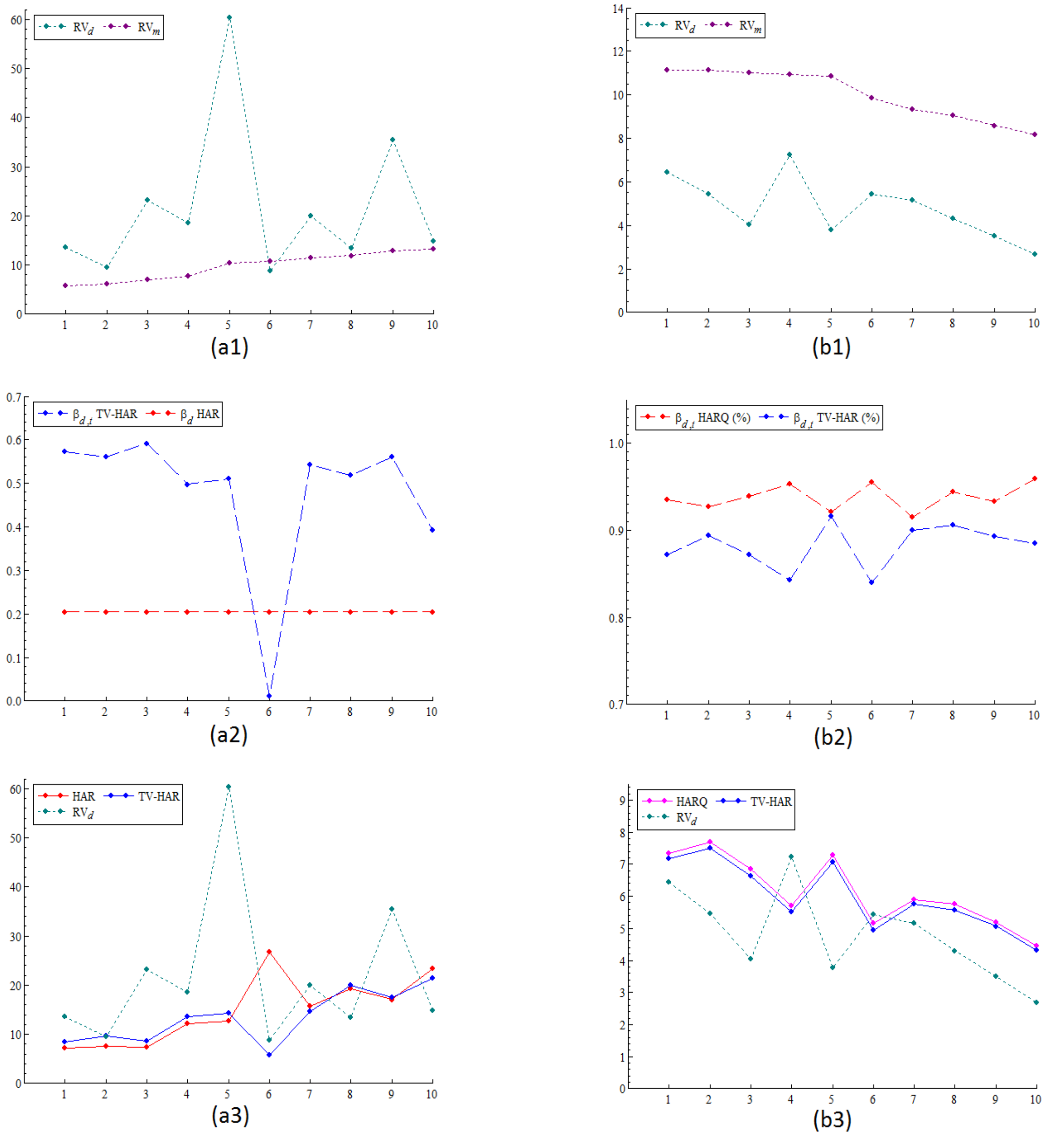

Figure 3.2: The left panel compares the TV-HAR model and the HAR model for ten successive trading days begin from 6 October 2008. The right panel compares the TV-HAR model and the HARQ model for ten successive trading days begin from 9 December 2008. The top shows daily and monthly realized volatilities. The middle left shows the time varying AR of daily RV ($\beta_{d,t}$) estimates from the TV-HAR model and the AR of daily RV (β_d) estimates from the HAR model. The middle right shows the time varying AR of daily RV estimates from the TV-HAR model and the HARQ model in percentage ($\beta_{d,t}/\gamma$, where the γ is the theoretical maximum value of time varying $\beta_{d,t}$, so we compare the percentage weights of daily lags of the two models). The bottom left compares the resulting one-day-ahead RV forecasts from the TV-HAR and the HAR models. The bottom right compares the resulting one-day-ahead RV forecasts from the TV-HAR and the HAR models.

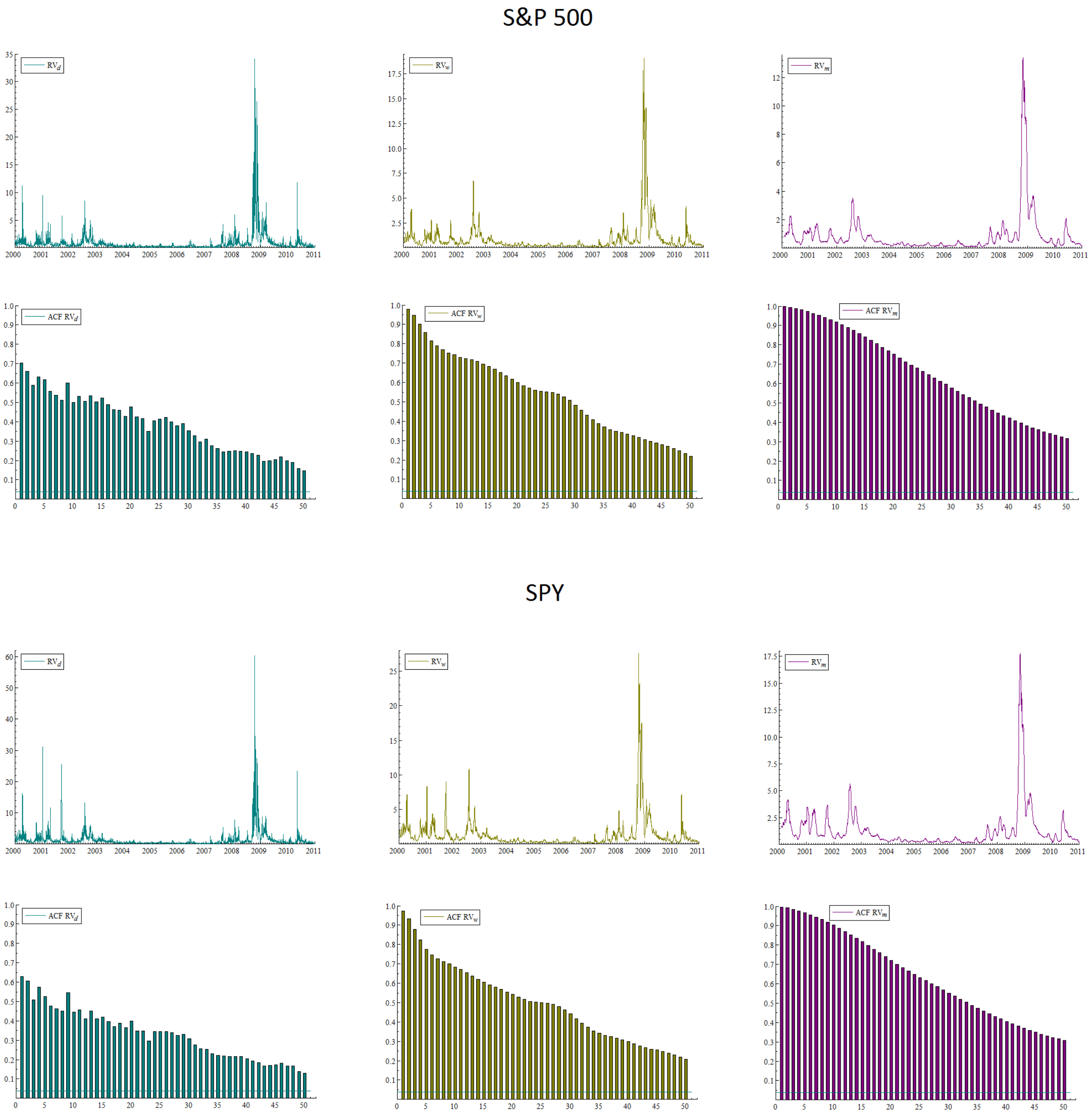

Figure 3.3: This figure shows the daily, weekly and monthly RVs and their autocorrelations for S&P 500 and SPY respectively. The left shows the daily RV and its autocorrelations. The middle shows the weekly RV and its autocorrelations. The right shows the monthly RV and its autocorrelations.

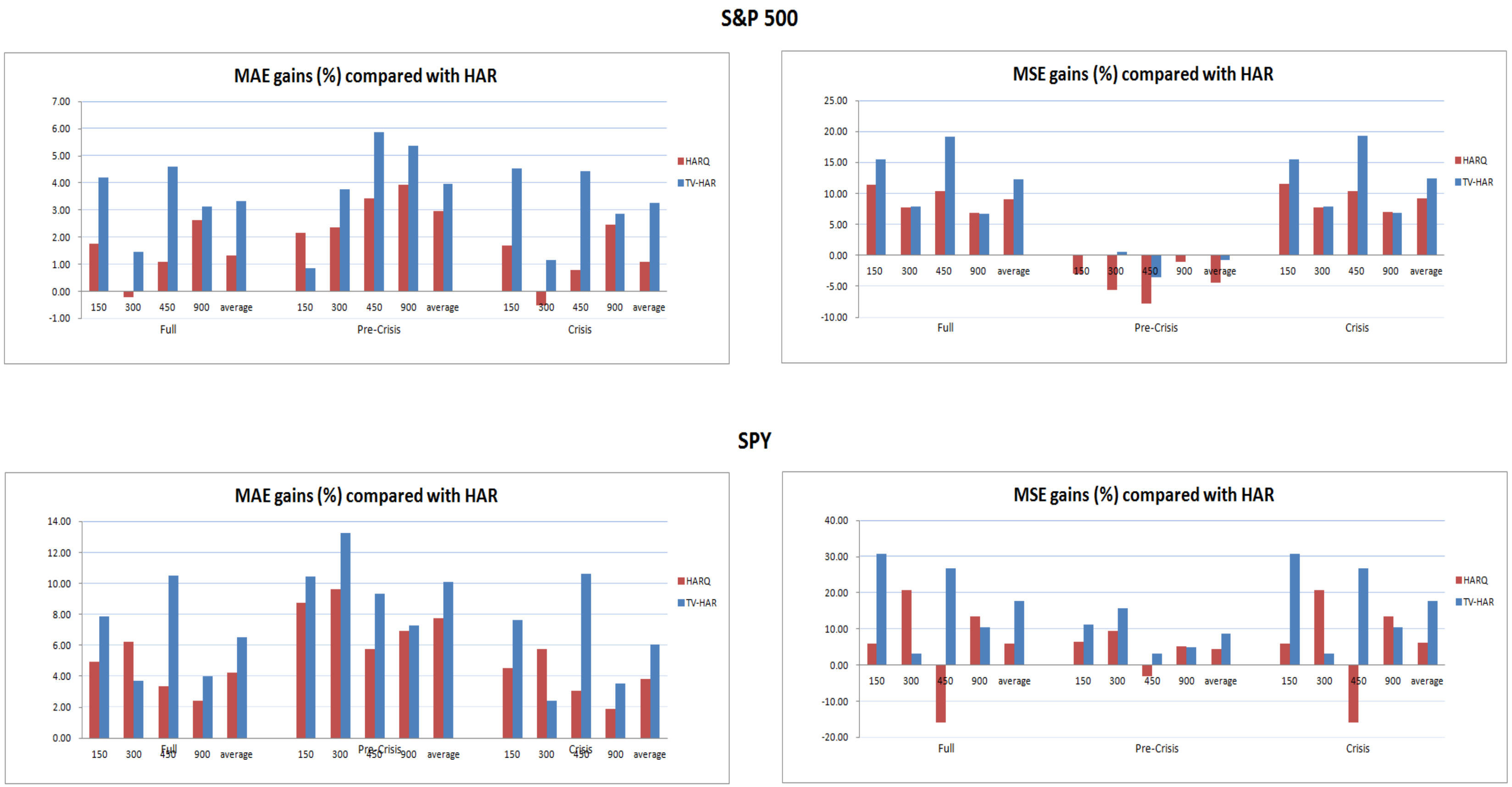

Figure 3.4: This figure shows that compared with the HAR model, the gains of forecasting accuracy for the HARQ model and TV-HAR model respectively. For example, the gains of the HARQ model compared with the HAR model based on the MAE is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$. The top panel shows the gains for MAE and MSE for S&P 500 and the bottom shows the gains for MAE and MSE for SPY

Table 3.1: Simulation results

	Sampling frequencies (seconds)						
MSE	60	150	300	450	900	1800	average
HAR	25.1468	6.3087	2.6586	1.6990	1.0161	0.7911	6.2701
HARQ	25.1437	6.3053	2.6486	1.6914	1.0105	0.7886	6.2647
TVHAR	**25.1252**	**6.2932**	**2.6483**	**1.6906**	**1.0103**	**0.7885**	**6.2594**
	Sampling frequencies (seconds)						
Relative MSE	60	150	300	450	900	1800	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.9999	0.9995	0.9963	0.9955	0.9944	0.9969	0.9991
TVHAR	**0.9991**	**0.9975**	**0.9961**	**0.9951**	**0.9943**	**0.9967**	**0.9983**
	Sampling frequencies (seconds)						
MSE Gains(%)	60	150	300	450	900	1800	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	0.0123	0.0542	0.3746	0.4481	0.5588	0.3127	0.0857
TV-HAR	**0.0857**	**0.2460**	**0.3867**	**0.4933**	**0.5714**	**0.3333**	**0.1706**

Notes: The table reports the simulation results for the HAR, HARQ and TV-HAR models. The relative MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The numbers in bold represent the model with best forecasting performance. The MSE Gains(%) measure the difference of losses, for example, the MSE gains of HAR, HARQ is calculated as follows: $(MSE_{HAR} - MSE_{HARQ})/MSE_{HAR}$.

Table 2.1: Summary statistics (150 seconds)

Company	Symbol	RV_d					RV_w					RV_m				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.8298	1.8864	0.0192	44.5257	2746	0.8305	1.6018	0.0337	19.9711	2746	0.8327	1.4344	0.1055	13.0667
SPY		2746	1.3559	2.8131	0.0398	80.6724	2746	1.3578	2.3060	0.0643	31.2816	2746	1.3629	2.0182	2.0182	18.6772
3M Company	MMM	2746	2.5019	5.3743	0.1294	197.6791	2746	2.5047	3.6327	0.3255	43.2008	2746	2.5160	2.9100	0.5957	25.6036
Amazon.com Inc	AMZN	2746	11.6188	16.9863	0.3606	167.7904	2746	11.6389	15.1712	0.8045	104.4095	2746	11.7096	14.1227	1.4835	73.9042
Merck	MRK	2746	3.2313	6.3881	0.1691	181.5483	2746	3.2351	4.4129	0.3757	61.7866	2746	3.2462	3.4567	0.6619	30.4799
Boeing	BA	2746	3.7285	4.5454	0.1414	70.2961	2746	3.7317	3.9830	0.4075	37.8564	2746	3.7444	3.5741	0.7007	27.9544
Microsoft	MSFT	2746	3.4308	4.3918	0.0855	79.7645	2746	3.4355	3.8481	0.4273	38.3609	2746	3.4575	3.4708	0.6045	25.4920
Coca-Cola	KO	2746	2.1226	4.0702	0.0941	138.6231	2746	2.1271	2.7682	0.1348	32.5139	2746	2.1410	2.2683	0.3276	16.5752
ExxonMobil	XOM	2746	2.5736	5.8451	0.1653	233.7847	2746	2.5773	4.4088	0.2234	78.8877	2746	2.5863	3.6723	0.4280	39.3795
DuPont	DD	2746	3.6263	5.0896	0.1405	126.5660	2746	3.6319	4.2535	0.4023	52.3977	2746	3.6502	3.7527	0.6675	31.5986
Verizon	VZ	2746	3.2280	4.9357	0.2481	136.0271	2746	3.2320	4.0685	0.3662	50.1012	2746	3.2414	3.6030	0.5159	29.6507
Pfizer	PFE	2746	3.0701	4.0909	0.3235	113.5654	2746	3.0762	3.1847	0.4662	39.3571	2746	3.1023	2.7950	0.6454	21.3392
		$\|RV_d - RV_m\|$					$(RV_d - RV_m)^+$					$(RV_d - RV_m)^-$				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.4215	1.2599	0.0001	36.5218	1073	0.5356	1.7543	0.0002	36.5218	1673	-0.3483	0.7867	-11.0856	-0.0001
SPY		2746	0.6570	1.9995	0.0000	69.1197	1074	0.8310	2.8664	0.0000	69.1197	1672	-0.5453	1.1225	-15.4049	-0.0001
3M Company	MMM	2746	1.1929	4.5098	0.0004	187.5184	1082	1.4958	6.8508	0.0004	187.5184	1664	-0.9959	1.7216	-20.5558	-0.0037
Amazon.com Inc	AMZN	2746	4.7069	9.1394	0.0016	128.7462	1090	5.8146	12.3386	0.0016	128.7462	1656	-3.9777	6.0849	-42.8295	-0.0043
Merck	MRK	2746	1.6850	5.3006	0.0002	159.4901	970	2.3640	8.4435	0.0002	159.4901	1776	-1.3141	2.0349	-24.8230	-0.0009
Boeing	BA	2746	1.4785	2.7213	0.0001	53.1779	1098	1.8289	3.7321	0.0024	53.1779	1648	-1.2450	1.7114	-19.1706	-0.0001
Microsoft	MSFT	2746	1.3762	2.6690	0.0002	63.6499	1130	1.6397	3.5415	0.0024	63.6499	1616	-1.1920	1.8048	-20.1835	-0.0002
Coca-Cola	KO	2746	0.9252	3.3400	0.0001	131.1389	1021	1.2194	5.2235	0.0035	131.1389	1725	-0.7510	1.2400	-13.3346	-0.0001
ExxonMobil	XOM	2746	1.1805	4.6954	0.0001	207.1188	1094	1.4656	6.9244	0.0002	207.1188	1652	-0.9917	2.1963	-31.8499	-0.0001
DuPont	DD	2746	1.4873	3.3876	0.0002	107.3862	1041	1.9301	4.9258	0.0003	107.3862	1705	-1.2169	1.8668	-23.5736	-0.0002
Verizon	VZ	2746	1.3705	3.3221	0.0002	117.4352	1062	1.7545	4.7826	0.0005	117.4352	1684	-1.1283	1.8519	-22.5424	-0.0002
Pfizer	PFE	2746	1.2565	2.8938	0.0001	97.7100	1016	1.6545	4.3885	0.0001	97.7100	1730	-1.0228	1.3569	-16.7589	-0.0001

Notes: The table provides summary statistics for the 150 seconds sampling frequency RV, including the number of observations, mean, standard deviation, minimum and maximum values. RV_d represents the daily realized volatility, RV_w represents the weekly realized volatility, and RV_m represents the monthly realized volatility. $(RV_d - RV_m)^+$ stands for the $(RV_d - RV_m)$ when $RV_d > RV_m$. $(RV_d - RV_m)^-$ stands for the $(RV_d - RV_m)$ when $RV_d < RV_m$.

Table 2.2: Summary statistics (300 seconds)

		RV_d					RV_w					RV_m				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.8660	1.9010	0.0164	34.1518	2746	0.8669	1.6228	0.0307	19.0655	2746	0.8695	1.4620	0.1050	13.3710
SPY		2746	1.3072	2.6876	0.0425	60.3263	2746	1.3089	2.2070	0.0637	27.5883	2746	1.3136	2.2070	0.0637	27.5883
3M Company	MMM	2746	2.3778	3.8086	0.0824	91.9551	2746	2.3811	3.1315	0.3433	41.0810	2746	2.3923	2.7347	0.5929	25.0027
Amazon.com Inc	AMZN	2746	11.3270	17.5638	0.2919	229.2436	2746	11.3437	15.1668	0.7527	118.0673	2746	11.4164	13.8925	1.4008	73.1124
Merck	MRK	2746	3.1508	6.3411	0.1370	223.2551	2746	3.1549	4.2504	0.2908	56.3544	2746	3.1659	3.2979	0.6123	27.0872
Boeing	BA	2746	3.5787	3.5787	0.1666	55.5697	2746	3.5812	3.8709	0.3986	40.5371	2746	3.5931	3.4316	0.6588	27.2926
Microsoft	MSFT	2746	3.3363	4.4776	0.0829	62.3858	2746	3.3416	3.8549	0.3717	35.4059	2746	3.3644	3.4701	0.5104	23.9948
Coca-Cola	KO	2746	1.9882	2.9940	0.0456	58.8085	2746	1.9928	2.4589	0.1388	29.7524	2746	2.0056	2.1695	0.3345	16.2009
ExxonMobil	XOM	2746	2.4521	4.7536	0.1548	141.1297	2746	2.4564	3.9508	0.2067	65.3502	2746	2.4665	3.4004	0.3866	36.3412
DuPont	DD	2746	3.4952	4.7785	0.1003	83.4874	2746	3.5010	4.0800	0.3443	47.6385	2746	3.5188	3.6145	0.6402	29.7545
Verizon	VZ	2746	3.1083	4.5549	0.1580	102.2209	2746	3.1127	3.8113	0.3230	39.6343	2746	3.1220	3.4037	0.4827	26.5760
Pfizer	PFE	2746	2.8742	3.7083	0.2247	62.6970	2746	2.8797	2.9314	0.3945	28.6037	2746	2.9028	2.5597	0.5987	18.3445
		$\lvert RV_d - RV_m \rvert$					$(RV_d - RV_m)^+$					$(RV_d - RV_m)^-$				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.4534	1.2324	0.0001	26.6613	1036	0.5962	1.7116	0.0001	26.6613	1710	-0.3668	0.8034	-11.7608	-0.0008
SPY		2746	0.6714	1.8814	0.0001	49.9368	1058	0.8629	2.6903	0.0001	49.9368	1688	-0.5514	1.0897	-15.0502	-0.0002
3M Company	MMM	2746	1.1011	2.6544	0.0002	79.1980	1066	1.3996	3.8222	0.0011	79.1980	1680	-0.9117	1.4698	-20.4248	-0.0002
Amazon.com Inc	AMZN	2746	5.0587	10.3262	0.0008	190.8051	1059	6.4428	14.3813	0.0036	190.8051	1687	-4.1898	6.4702	-43.6638	-0.0008
Merck	MRK	2746	1.7140	5.3111	0.0001	208.5760	978	2.3852	8.4320	0.0001	208.5760	1768	-1.3428	2.0304	-21.1981	-0.0008
Boeing	BA	2746	1.5633	2.7799	0.0001	41.1240	1089	1.9528	3.8149	0.0016	41.1240	1657	-1.3073	1.7557	-19.6321	-0.0001
Microsoft	MSFT	2746	1.4488	2.7747	0.0001	48.7349	1111	1.7558	3.7125	0.0001	48.7349	1635	-1.2403	1.8609	-19.4535	-0.0003
Coca-Cola	KO	2746	0.8556	2.0219	0.0000	50.9593	1035	1.1119	2.9604	0.0000	50.9593	1711	-0.7005	1.0952	-12.8000	-0.0006
ExxonMobil	XOM	2746	1.1713	3.4696	0.0004	119.3460	1071	1.4831	4.9281	0.0038	119.3460	1675	-0.9719	2.0286	-29.0863	-0.0004
DuPont	DD	2746	1.5101	3.0533	0.0002	66.7032	1054	1.9364	4.3229	0.0004	66.7032	1692	-1.2445	1.8202	-22.8639	-0.0002
Verizon	VZ	2746	1.3859	2.9105	0.0006	86.9591	1065	1.7690	4.1098	0.0025	86.9591	1681	-1.1432	1.7295	-20.4593	-0.0006
Pfizer	PFE	2746	1.3024	2.5298	0.0001	56.3627	985	1.7755	3.8080	0.0014	56.3627	1761	-1.0378	1.2959	-13.5763	-0.0001

Notes: The table provides summary statistics for the 300 seconds sampling frequency RV, including the number of observations, mean, standard deviation, minimum and maximum values. RV_d represents the daily realized volatility, RV_w represents the weekly realized volatility, and RV_m represents the monthly realized volatility. $(RV_d - RV_m)^+$ stands for the $(RV_d - RV_m)$ when $RV_d > RV_m$. $(RV_d - RV_m)^-$ stands for the $(RV_d - RV_m)$ when $RV_d < RV_m$.

Table 2.3: Summary statistics (450 seconds)

		RV_d					RV_w					RV_m				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.8659	1.9218	0.0175	43.4451	2746	0.8669	1.6301	0.0319	20.0766	2746	0.8695	1.4523	0.1085	13.4844
SPY		2746	1.2907	3.1320	0.0405	108.7012	2746	1.2924	2.3798	0.0577	36.7878	2746	1.2967	2.0213	0.1659	19.5537
3M Company	MMM	2746	2.2930	3.9736	0.1277	117.9914	2746	2.2965	3.0737	0.3033	43.8363	2746	2.3079	2.6165	0.5571	23.6597
Amazon.com Inc	AMZN	2746	11.2594	18.5054	0.2991	249.3781	2746	11.2772	15.5552	0.7695	125.6467	2746	11.3467	14.1081	1.3815	74.7395
Merck	MRK	2746	3.0077	3.0077	0.1127	179.9697	2746	3.0114	3.9643	0.6148	29.6327	2746	3.0222	3.1861	0.6148	29.6327
Boeing	BA	2746	3.4954	4.5448	0.1699	63.6756	2746	3.4980	3.8274	0.4091	38.7852	2746	3.5109	3.3394	0.6585	27.3585
Microsoft	MSFT	2746	3.2855	4.7170	0.1010	89.4953	2746	3.2901	3.9709	0.3263	40.7815	2746	3.3131	3.5151	0.4618	24.8051
Coca-Cola	KO	2746	1.9177	3.0113	0.0430	76.2785	2746	1.9226	2.4736	0.1513	35.0772	2746	1.9356	2.1265	0.3159	16.5425
ExxonMobil	XOM	2746	2.3771	5.5754	0.1079	220.1092	2746	2.3817	4.2628	0.1874	80.2940	2746	2.3921	3.4951	0.3701	38.3663
DuPont	DD	2746	3.4200	5.0460	0.0805	125.7009	2746	3.4257	4.1305	0.3043	53.8371	2746	3.4446	3.6049	0.5655	30.1586
Verizon	VZ	2746	2.9596	4.9298	0.1418	155.1925	2746	2.9636	3.8385	0.2901	49.5433	2746	2.9727	3.3356	0.4092	27.8103
Pfizer	PFE	2746	2.7686	3.9975	0.1495	106.2600	2746	2.7754	2.9934	0.3918	38.0877	2746	2.7972	2.5419	0.5918	19.9804
		$\lvert RV_d - RV_m\rvert$					$(RV_d - RV_m)^+$					$(RV_d - RV_m)^-$				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.4669	1.2847	0.0000	35.7030	1045	0.6087	1.7838	0.0006	35.7030	1701	-0.3797	0.8313	-12.3374	0.0000
SPY		2746	0.6871	2.4286	0.0000	96.7746	1058	0.8839	3.5849	0.0004	96.7746	1688	-0.5637	1.2270	-17.4268	0.0000
3M Company	MMM	2746	1.1226	2.9782	0.0000	105.8665	1068	1.4241	4.3778	0.0000	105.8665	1678	-0.9307	1.4932	-20.0271	-0.0001
Amazon.com Inc	AMZN	2746	5.3549	11.5260	0.0007	210.7692	1072	6.7466	16.1111	0.0007	210.7692	1674	-4.4637	7.0548	-49.8341	-0.0061
Merck	MRK	2746	1.6388	4.4651	0.0001	161.6738	965	2.3109	7.0666	0.0002	161.6738	1781	-1.2746	1.8232	-23.8343	-0.0001
Boeing	BA	2746	1.6168	2.9715	0.0002	50.4956	1064	2.0664	4.1606	0.0002	50.4956	1682	-1.3324	1.8065	-21.5007	-0.0010
Microsoft	MSFT	2746	1.5069	3.1033	0.0001	75.0796	1089	1.8651	4.2615	0.0009	75.0796	1657	-1.2715	1.9729	-21.2653	-0.0001
Coca-Cola	KO	2746	0.8810	2.1310	0.0001	67.0539	1023	1.1583	3.1233	0.0001	67.0539	1723	-0.7163	1.1735	-13.3672	-0.0002
ExxonMobil	XOM	2746	1.1822	4.5013	0.0003	196.2076	1051	1.5248	6.7155	0.0003	196.2076	1695	-0.9699	2.1820	-30.7417	-0.0016
DuPont	DD	2746	1.5452	3.4594	0.0007	108.8356	1064	1.9622	5.0096	0.0007	108.8356	1682	-1.2814	1.8689	-22.1796	-0.0022
Verizon	VZ	2746	1.3834	3.5524	0.0001	138.6766	1059	1.7765	5.2161	0.0001	138.6766	1687	-1.1366	1.8204	-22.4782	-0.0003
Pfizer	PFE	2746	1.3524	2.9382	0.0001	92.2066	986	1.8434	4.4892	0.0002	92.2066	1760	-1.0773	1.4059	-16.4218	-0.0001

Notes: The table provides summary statistics for the 450 seconds sampling frequency RV, including the number of observations, mean, standard deviation, minimum and maximum values. RV_d represents the daily realized volatility, RV_w represents the weekly realized volatility, and RV_m represents the monthly realized volatility. $(RV_d - RV_m)^+$ stands for the $(RV_d - RV_m)$ when $RV_d > RV_m$. $(RV_d - RV_m)^-$ stands for the $(RV_d - RV_m)$ when $RV_d < RV_m$.

Table 2.4: Summary statistics (900 seconds)

		RV_d					RV_w					RV_m				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.8781	2.0887	0.0140	41.8510	2746	0.8791	1.7426	0.0345	20.5337	2746	0.8818	1.5344	0.1059	14.4925
SPY		2746	1.2255	2.6870	0.0367	59.2785	2746	1.2269	1.2269	0.0618	28.6059	2746	1.2310	1.9302	0.1489	18.1126
3M Company	MMM	2746	2.2072	3.6356	0.0611	88.6850	2746	2.2107	2.8471	0.3218	35.4077	2746	2.2229	2.4357	0.5146	21.0156
Amazon.com Inc	AMZN	2746	11.0905	19.9610	0.2073	291.4926	2746	11.1077	16.2089	0.6757	148.2630	2746	11.1847	14.3540	1.3587	79.8037
Merck	MRK	2746	2.9190	4.7473	0.1151	85.6970	2746	2.9230	3.6077	0.3264	42.7294	2746	2.9339	3.0060	0.5844	26.2185
Boeing	BA	2746	3.3975	4.6908	0.1864	67.9505	2746	3.4003	3.7923	0.3584	39.2561	2746	3.4123	3.2980	0.5610	27.9624
Microsoft	MSFT	2746	3.1391	4.4819	0.0790	59.6027	2746	3.1436	3.7531	0.2769	36.2599	2746	3.1654	3.3578	0.3824	22.7149
Coca-Cola	KO	2746	1.8520	2.8907	0.0284	46.0981	2746	1.8560	2.3556	2.3556	26.3453	2746	1.8673	2.0434	0.3026	15.1583
ExxonMobil	XOM	2746	2.2504	4.4821	0.0788	120.9934	2746	2.2554	3.7853	0.1838	63.8851	2746	2.2650	3.1951	0.3742	34.5925
DuPont	DD	2746	3.2896	4.8409	0.0250	99.3736	2746	3.2955	3.9380	0.2908	49.1450	2746	3.3132	3.4209	0.5594	27.7393
Verizon	VZ	2746	2.8471	4.5800	0.0984	127.9788	2746	2.8515	3.5960	0.2508	43.0768	2746	2.8610	3.1164	0.4026	24.6481
Pfizer	PFE	2746	2.6249	3.6972	0.1197	57.8745	2746	2.6295	2.7396	0.3726	24.4129	2746	2.6484	2.3208	0.5722	16.7887

		$\lvert RV_d - RV_m \rvert$					$(RV_d - RV_m)^+$					$(RV_d - RV_m)^-$				
Company	Symbol	obs	mean	std	min	max	obs	mean	std	min	max	obs	mean	std	min	max
S&P 500		2746	0.5090	1.4397	0.0002	33.8892	1039	0.6678	1.9936	0.0002	33.8892	1707	-0.4124	-0.4124	-13.3537	-0.0002
SPY		2746	0.6691	1.9058	0.0001	49.0296	1066	0.8546	2.6723	0.0001	49.0296	1680	-0.5513	1.1716	-16.1721	-0.0001
3M Company	MMM	2746	1.1678	2.6348	0.0001	78.1059	1027	1.5403	3.8740	0.0001	78.1059	1719	-0.9452	1.4131	-18.6131	-0.0004
Amazon.com Inc	AMZN	2746	5.9822	13.3558	0.0016	247.8109	1026	7.8793	19.1979	0.0016	247.8109	1720	-4.8506	7.8516	-54.5427	-0.0027
Merck	MRK	2746	1.6819	3.4669	0.0006	70.8920	976	2.3452	5.2537	0.0025	70.8920	1770	-1.3162	1.7497	-21.3200	-0.0006
Boeing	BA	2746	1.7493	3.1314	0.0008	62.8438	1009	2.3603	4.4978	0.0011	62.8438	1737	-1.3945	1.8481	-21.7729	-0.0008
Microsoft	MSFT	2746	1.5425	2.8532	0.0005	42.3295	1088	1.9134	3.8250	0.0005	42.3295	1658	-1.2991	1.9336	-19.9975	-0.0008
Coca-Cola	KO	2746	0.9234	1.9836	0.0000	37.8275	1040	1.1990	2.8410	0.0005	37.8275	1706	-0.7554	1.1583	-13.0521	0.0000
ExxonMobil	XOM	2746	1.1877	3.2817	0.0005	101.5293	1059	1.5210	4.6463	0.0005	101.5293	1687	-0.9785	1.9682	-27.7955	-0.0031
DuPont	DD	2746	1.5963	3.2868	0.0004	84.6653	1030	2.0964	4.7819	0.0004	84.6653	1716	-1.2961	1.8252	-22.2100	-0.0007
Verizon	VZ	2746	1.4642	3.2123	0.0010	114.4350	1034	1.9258	4.7082	0.0018	114.4350	1712	-1.1855	1.7218	-18.4093	-0.0010
Pfizer	PFE	2746	1.4008	2.6595	0.0017	52.9311	991	1.9082	4.0186	0.0018	52.9311	1755	-1.1143	1.3139	-14.8587	-0.0017

Notes: The table provides summary statistics for the 900 seconds sampling frequency RV, including the number of observations, mean, standard deviation, minimum and maximum values. RV_d represents the daily realized volatility, RV_w represents the weekly realized volatility, and RV_m represents the monthly realized volatility. $(RV_d - RV_m)^+$ stands for the $(RV_d - RV_m)$ when $RV_d > RV_m$. $(RV_d - RV_m)^-$ stands for the $(RV_d - RV_m)$ when $RV_d < RV_m$..

Table 3.3: In-sample estimation (S&P 500)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.0668	0.2407		0.4909	0.1864	0.5701	3.2649	0.0453	0.3662		0.2959	0.2480	*0.5513*	*1.1159*	0.1203	0.2295		0.5075	0.1740	0.5500	4.1932
	(0.0273)	(0.0228)		(0.0382)	(0.0334)			(0.0153)	(0.0280)		(0.0463)	(0.0417)			(0.0728)	(0.0377)		(0.0633)	(0.0557)		
300 sec	0.0645	0.2585		0.4703	0.1951	*0.5865*	*3.2415*	0.0624	0.3258		0.2651	0.2987	0.4608	1.7613	0.1055	0.2454		0.5044	0.1719	*0.5849*	*4.0902*
	(0.0272)	(0.0227)		(0.0383)	(0.0334)			(0.0206)	(0.0279)		(0.0483)	(0.0460)			(0.0688)	(0.0375)		(0.0632)	(0.0549)		
450 sec	0.0720	0.2390		0.4989	0.1773	0.5660	3.3115	0.0772	0.2758		0.2659	0.3248	0.3849	2.1107	0.1128	0.2270		0.5455	0.1423	0.5790	4.1026
	(0.0282)	(0.0230)		(0.0383)	(0.0332)			(0.0242)	(0.0280)		(0.0501)	(0.0497)			(0.0691)	(0.0382)		(0.0628)	(0.0535)		
900 sec	0.0810	0.2309		0.4501	0.2249	0.5213	3.5761	0.0954	0.1988		0.2666	0.3725	0.2930	2.5061	0.1239	0.2383		0.4809	0.1885	0.5467	4.3331
	(0.0318)	(0.0230)		(0.0389)	(0.0348)			(0.0291)	(0.0281)		(0.0533)	(0.0554)			(0.0767)	(0.0382)		(0.0631)	(0.0552)		
average	0.0711	0.2423		0.4775	0.1959	0.5610	3.3485	0.0701	0.2916		0.2734	0.3110	0.4225	1.8735	0.1156	0.2351		0.5096	0.1692	0.5652	4.1798
	(0.0286)	(0.0229)		(0.0384)	(0.0337)			(0.0223)	(0.0280)		(0.0495)	(0.0482)			(0.0719)	(0.0379)		(0.0631)	(0.0548)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	−0.0608	0.7854	−0.0143	0.3591	0.0231	*0.6126*	*3.1611*	0.0162	0.5836	−0.0267	0.2354	0.1926	*0.5659*	*1.0833*	−0.0787	0.9110	−0.0168	0.3420	−0.0272	0.6041	4.0661
	(0.0270)	(0.0381)	(0.0008)	(0.0371)	(0.0331)			(0.0155)	(0.0394)	(0.0035)	(0.0462)	(0.0416)			(0.0704)	(0.0679)	(0.0014)	(0.0610)	(0.0550)		
300 sec	−0.0703	0.6628	−0.0141	0.3934	0.1110	0.6106	3.1817	0.0307	0.5620	−0.0197	0.1887	0.2363	0.4853	1.7154	−0.1287	0.8168	−0.0181	0.3919	0.0533	0.6184	4.0072
	(0.0283)	(0.0379)	(0.0011)	(0.0376)	(0.0331)			(0.0204)	(0.0376)	(0.0022)	(0.0479)	(0.0454)			(0.0705)	(0.0705)	(0.0019)	(0.0617)	(0.0541)		
450 sec	−0.0692	0.6660	−0.0131	0.4518	0.0457	0.6070	3.2127	0.0404	0.5364	−0.0183	0.1914	0.2451	0.4128	2.0649	−0.1317	0.8362	−0.0165	0.4612	−0.0403	*0.6382*	*3.9520*
	(0.0281)	(0.0334)	(0.0008)	(0.0365)	(0.0325)			(0.0240)	(0.0396)	(0.0020)	(0.0496)	(0.0493)			(0.0669)	(0.0592)	(0.0013)	(0.0586)	(0.0516)		
900 sec	−0.0633	0.6403	−0.0125	0.4059	0.1145	0.5564	3.5004	0.0646	0.4494	−0.0116	0.1871	0.2867	0.3225	2.4639	−0.1231	0.8066	−0.0158	0.4126	0.0386	0.5963	4.2181
	(0.0322)	(0.0355)	(0.0008)	(0.0376)	(0.0344)			(0.0287)	(0.0397)	(0.0013)	(0.0530)	(0.0551)			(0.0757)	(0.0624)	(0.0014)	(0.0599)	(0.0538)		
average	−0.0659	0.6886	−0.0135	0.4025	0.0735	0.5966	3.2640	0.0380	0.5328	−0.0190	0.2007	0.2402	0.4466	1.8319	−0.1156	0.8427	−0.0168	0.4019	0.0061	**0.6142**	**4.0609**
	(0.0289)	(0.0362)	(0.0009)	(0.0372)	(0.0333)			(0.0222)	(0.0391)	(0.0022)	(0.0492)	(0.0479)			(0.0708)	(0.0650)	(0.0015)	(0.0603)	(0.0536)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	−0.0131	0.6421	−0.0188	0.3801	0.0458	*0.6137*	*3.1584*	0.0252	0.5153	−0.0402	0.2629	0.1953	*0.5569*	*1.1040*	0.0025	0.6915	−0.0203	0.3789	0.0149	0.6016	4.0724
	(0.0263)	(0.0314)	(0.0011)	(0.0368)	(0.0327)			(0.0158)	(0.0418)	(0.0084)	(0.0466)	(0.0429)			(0.0693)	(0.0537)	(0.0018)	(0.0606)	(0.0542)		
300 sec	−0.0236	0.5955	−0.0211	0.3578	0.1304	0.6092	3.1852	0.0187	0.6195	−0.0515	0.1910	0.2014	0.4818	1.7221	−0.0276	0.6712	−0.0240	0.3543	0.0933	0.6136	4.0197
	(0.0273)	(0.0345)	(0.0017)	(0.0382)	(0.0329)			(0.0208)	(0.0443)	(0.0061)	(0.0481)	(0.0465)			(0.0682)	(0.0609)	(0.0028)	(0.0633)	(0.0537)		
450 sec	−0.0155	0.6171	−0.0199	0.4186	0.0374	0.6134	3.1961	0.0188	0.6445	−0.0491	0.1766	0.1999	0.4194	2.0535	−0.0199	0.7121	−0.0222	0.4240	−0.0292	*0.6408*	*3.9447*
	(0.0270)	(0.0299)	(0.0011)	(0.0364)	(0.0323)			(0.0242)	(0.0453)	(0.0048)	(0.0495)	(0.0498)			(0.0646)	(0.0510)	(0.0017)	(0.0587)	(0.0511)		
900 sec	−0.0222	0.6230	−0.0214	0.3554	0.1167	0.5628	3.4857	0.0403	0.5975	−0.0362	0.1531	0.2326	0.3352	2.4450	−0.0265	0.7191	−0.0235	0.3494	0.0631	0.5991	4.2111
	(0.0311)	(0.0327)	(0.0013)	(0.0376)	(0.0340)			(0.0287)	(0.0466)	(0.0034)	(0.0528)	(0.0553)			(0.0733)	(0.0551)	(0.0020)	(0.0604)	(0.0531)		
average	−0.0186	0.6194	−0.0203	0.3780	0.0826	**0.5998**	**3.2564**	0.0257	0.5942	−0.0442	0.1959	0.2073	**0.4483**	**1.8312**	−0.0179	0.6985	−0.0225	0.3767	0.0356	0.6138	4.0620
	(0.0279)	(0.0321)	(0.0013)	(0.0373)	(0.0330)			(0.0224)	(0.0445)	(0.0057)	(0.0492)	(0.0486)			(0.0688)	(0.0552)	(0.0021)	(0.0608)	(0.0530)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.4: In-sample estimation (SPY)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	−0.1357	0.1623		0.5257	0.2096	0.4896	4.2356	0.1073	0.2044		0.3894	0.2986	*0.4438*	*2.9826*	0.2015	0.1521		0.5537	0.1874	*0.4858*	*5.0418*
	(0.0463)	(0.0233)		(0.0399)	(0.0359)			(0.0375)	(0.0288)		(0.0514)	(0.0480)			(0.1120)	(0.0386)		(0.0656)	(0.0588)		
300 sec	0.1247	0.2045		0.4791	0.2189	*0.5002*	*4.1233*	0.1392	0.1979		0.3116	0.3482	0.3372	3.3602	0.1681	0.2055		0.5306	0.1711	*0.5363*	*4.7754*
	(0.0438)	(0.0230)		(0.0395)	(0.0356)			(0.0447)	(0.0285)		(0.0528)	(0.0528)			(0.0982)	(0.0381)		(0.0638)	(0.0561)		
450 sec	0.1710	0.0853		0.5157	0.2644	0.3665	4.6666	0.1173	0.2588		0.2806	0.3361	0.3968	3.0667	0.2620	0.0572		0.5550	0.2445	0.3505	5.5324
	(0.0566)	(0.0232)		(0.0428)	(0.0411)			(0.0387)	(0.0282)		(0.0511)	(0.0498)			(0.1415)	(0.0385)		(0.0712)	(0.0686)		
900 sec	0.1257	0.1729		0.5167	0.2060	0.4909	4.1413	0.1219	0.2458		0.2802	0.3359	0.3619	3.0501	0.1858	0.1491		0.5748	0.1709	0.5045	4.9048
	(0.0434)	(0.0233)		(0.0394)	(0.0352)			(0.0384)	(0.0281)		(0.0515)	(0.0513)			(0.1037)	(0.0389)		(0.0645)	(0.0566)		
average	0.1393	0.1563		0.5093	0.2247	0.4618	4.2917	0.1214	0.2267		0.3155	0.3297	0.3849	3.1149	0.2043	0.1410		0.5535	0.1935	0.4693	5.0636
	(0.0475)	(0.0232)		(0.0404)	(0.0369)			(0.0398)	(0.0284)		(0.0517)	(0.0505)			(0.1139)	(0.0385)		(0.0663)	(0.0600)		
	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	−0.0330	0.7033	−0.0064	0.3808	0.0041	*0.5579*	*4.0925*	0.0544	0.5673	−0.0078	0.1982	0.2196	*0.4852*	*2.9057*	−0.0523	0.8244	−0.0072	0.3810	−0.0705	0.5725	4.8581
	(0.0439)	(0.0341)	(0.0003)	(0.0378)	(0.0348)			(0.0364)	(0.0413)	(0.0007)	(0.0520)	(0.0467)			(0.1037)	(0.0587)	(0.0005)	(0.0610)	(0.0565)		
300 sec	−0.0219	0.6601	−0.0076	0.3334	0.0916	0.5452	4.0294	0.0918	0.4952	−0.0051	0.1655	0.2747	0.3753	3.3015	−0.0793	0.8125	−0.0100	0.3652	−0.0099	*0.5942*	*4.6430*
	(0.0428)	(0.0353)	(0.0005)	(0.0387)	(0.0349)			(0.0437)	(0.0399)	(0.0005)	(0.0531)	(0.0517)			(0.0941)	(0.0618)	(0.0008)	(0.0613)	(0.0546)		
450 sec	0.0375	0.5448	−0.0032	0.3752	0.0902	0.4189	4.5806	0.0659	0.5193	−0.0105	0.1770	0.2742	0.4248	3.0198	0.0458	0.6745	−0.0038	0.3628	0.0166	0.4165	5.4262
	(0.0548)	(0.0367)	(0.0002)	(0.0420)	(0.0409)			(0.0382)	(0.0394)	(0.0011)	(0.0512)	(0.0491)			(0.1357)	(0.0682)	(0.0004)	(0.0698)	(0.0684)		
900 sec	−0.0640	0.6086	−0.0108	0.4560	0.0737	0.5254	4.0716	0.0345	0.6306	−0.0217	0.1881	0.2136	0.3950	2.9974	−0.1414	0.8274	−0.0145	0.4543	−0.0245	0.5568	4.7941
	(0.0440)	(0.0381)	(0.0008)	(0.0382)	(0.0353)			(0.0385)	(0.0479)	(0.0022)	(0.0510)	(0.0515)			(0.1026)	(0.0721)	(0.0013)	(0.0619)	(0.0564)		
average	−0.0203	0.6292	−0.0070	0.3864	0.0649	**0.5118**	**4.1935**	0.0616	0.5531	−0.0113	0.1822	0.2455	**0.4201**	**3.0561**	−0.0568	0.7847	−0.0089	0.3908	−0.0220	**0.5350**	**4.9303**
	(0.0464)	(0.0360)	(0.0004)	(0.0392)	(0.0365)			(0.0392)	(0.0421)	(0.0011)	(0.0518)	(0.0497)			(0.1090)	(0.0652)	(0.0008)	(0.0635)	(0.0590)		
	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.0111	0.5947	−0.0099	0.3987	0.0384	*0.5469*	*4.1169*	0.0349	0.6500	−0.0332	0.2094	0.1585	*0.4830*	*2.9101*	0.0051	0.7382	−0.0115	0.3731	−0.0376	0.5651	4.8753
	(0.0442)	(0.0319)	(0.0005)	(0.0382)	(0.0350)			(0.0367)	(0.0477)	(0.0029)	(0.0520)	(0.0479)			(0.1040)	(0.0559)	(0.0009)	(0.0618)	(0.0565)		
300 sec	0.0067	0.5759	−0.0130	0.3621	0.1070	0.5370	4.0473	0.0516	0.6782	−0.0255	0.1208	0.2027	0.3887	3.2799	0.0046	0.6578	−0.0138	0.3806	0.0375	*0.5815*	*4.6739*
	(0.0429)	(0.0335)	(0.0009)	(0.0388)	(0.0351)			(0.0436)	(0.0482)	(0.0021)	(0.0530)	(0.0521)			(0.0946)	(0.0564)	(0.0013)	(0.0623)	(0.0548)		
450 sec	0.0500	0.5260	−0.0062	0.3770	0.0934	0.4170	4.5839	0.0428	0.5961	−0.0273	0.1712	0.2353	0.4238	3.0216	0.0698	0.6388	−0.0072	0.3674	0.0251	0.4134	5.4315
	(0.0548)	(0.0362)	(0.0004)	(0.0421)	(0.0409)			(0.0387)	(0.0464)	(0.0030)	(0.0514)	(0.0500)			(0.1358)	(0.0667)	(0.0007)	(0.0700)	(0.0685)		
900 sec	0.0147	0.5224	−0.0128	0.4338	0.0835	0.5272	4.0677	0.0615	0.5220	−0.0224	0.2072	0.2408	0.3851	3.0137	0.0272	0.6035	−0.0144	0.4489	0.0207	0.5512	4.8068
	(0.0426)	(0.0329)	(0.0009)	(0.0383)	(0.0350)			(0.0384)	(0.0437)	(0.0028)	(0.0513)	(0.0517)			(0.0999)	(0.0577)	(0.0014)	(0.0626)	(0.0558)		
average	0.0206	0.5547	−0.0105	0.3929	0.0806	0.5070	4.2039	0.0477	0.6116	−0.0271	0.1772	0.2093	0.4201	3.0563	0.0267	0.6596	−0.0117	0.3925	0.0114	0.5278	4.9469
	(0.0461)	(0.0336)	(0.0007)	(0.0394)	(0.0365)			(0.0394)	(0.0465)	(0.0027)	(0.0519)	(0.0504)			(0.1086)	(0.0592)	(0.0011)	(0.0642)	(0.0589)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.5: In-sample fits

	Full			Pre-Crisis			Crisis		
R^2	HAQ	HARQ	TV-HAR	HAQ	HARQ	TV-HAR	HAQ	HARQ	TV-HAR
MMM	0.3792	0.4038	**0.4061**	0.4718	**0.4886**	0.4760	0.3639	0.3934	**0.3954**
AMZN	0.5879	**0.5950**	0.5890	0.5849	**0.5915**	0.5851	0.5235	0.5544	**0.5661**
MRK	0.2679	0.3060	**0.3099**	0.2503	**0.2649**	0.2615	0.2714	0.3266	**0.3368**
BA	0.5611	**0.5710**	0.5685	0.5307	**0.5436**	0.5326	0.5821	0.5914	**0.5959**
MSFT	0.5885	0.6045	**0.6080**	0.5991	0.6125	**0.6139**	0.5852	0.6059	**0.6091**
KO	0.6025	**0.6108**	0.6103	0.5784	**0.5887**	0.5821	0.6288	0.6426	**0.6453**
XOM	0.6961	0.6970	**0.6979**	0.6099	0.6101	**0.6102**	0.7119	0.7139	**0.7149**
DD	0.5082	**0.5324**	0.5281	0.5353	**0.5510**	0.5490	0.4824	**0.5196**	0.5115
VZ	0.6199	0.6206	**0.6213**	0.5738	**0.5758**	0.5744	0.6602	0.6643	**0.6674**
PFE	0.4050	**0.4402**	0.4356	0.3456	**0.3707**	0.3655	0.4754	**0.5356**	0.5246
Average	0.5216	**0.5381**	0.5375	0.5080	**0.5197**	0.5150	0.5285	0.5548	**0.5567**
S&P500	0.5610	0.5966	**0.5998**	0.4225	0.4466	**0.4483**	0.5652	**0.6142**	0.6138
SPY	0.4618	**0.5118**	0.5070	0.3849	**0.4201**	0.4201	0.4693	**0.5350**	0.5278

	Full			Pre-Crisis			Crisis		
AIC	HAQ	HARQ	TV-HAR	HAQ	HARQ	TV-HAR	HAQ	HARQ	TV-HAR
MMM	5.1951	5.1540	**5.1515**	4.0171	**3.9846**	4.0102	5.9305	5.8806	**5.8798**
AMZN	7.7431	**7.7250**	7.7404	8.0170	**8.0011**	8.0172	6.9606	6.8902	**6.8628**
MRK	6.0127	5.9611	**5.9554**	4.9495	**4.9300**	4.9346	6.7281	6.6523	**6.6374**
BA	5.0457	**5.0216**	5.0277	4.7019	**4.6722**	4.6981	5.4504	5.4276	**5.4159**
MSFT	4.9335	4.8925	**4.8840**	4.7815	4.7470	**4.7436**	5.1174	5.0667	**5.0588**
KO	3.7936	**3.7747**	3.7755	3.5837	**3.5621**	3.5776	4.0620	4.0275	**4.0190**
XOM	4.1938	4.1910	**4.1879**	3.1905	3.1905	**3.1902**	4.9174	4.9119	**4.9087**
DD	5.3200	**5.2689**	5.2785	4.4125	**4.3772**	4.3829	6.0256	**5.7928**	5.9685
VZ	4.5845	4.5828	**4.5811**	4.2844	**4.2803**	4.2839	4.9322	4.9209	**4.9115**
PFE	4.9572	**4.8964**	4.9045	4.7919	**4.7532**	4.7614	5.1522	**5.0326**	5.0563
Average	5.1779	**5.1468**	5.1486	4.6730	**4.6498**	4.6600	5.5277	**5.4603**	5.4719
S&P500	3.3485	3.2640	**3.2564**	1.8735	1.8319	**1.8312**	4.1798	**4.0609**	4.0620
SPY	4.2917	**4.1935**	4.2039	3.1149	**3.0561**	3.0563	5.0636	**4.9303**	4.9469

Notes: The table provides the in-sample fit averaged across different sampling frequencies for the standard HAR, the HARQ and the TV-HAR models. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The sub-sample of the pre-crisis period is from 2000 to 2006. The sub-sample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The Average means the mean of R^2 and AIC for the ten stocks.

Table 3.6: Out-of-sample forecasts (S&P 500)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.4356	0.4328	0.4442	0.4980	0.4526	0.0938	0.1067	0.1192	0.1268	0.1116	0.6930	0.6783	0.6890	0.7775	0.7094
HARQ	0.4280	0.4337	0.4394	0.4850	0.4465	**0.0918**	0.1042	0.1151	0.1218	0.1082	0.6812	0.6818	0.6837	0.7585	0.7013
TV-HAR	**0.4173**	**0.4265**	**0.4238**	**0.4825**	**0.4375**	0.0930	**0.1027**	**0.1122**	**0.1200**	**0.1070**	**0.6615**	**0.6704**	**0.6585**	**0.7554**	**0.6864**

	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	2.8848	2.3424	2.6749	3.1065	2.7522	0.0160	0.0207	**0.0256**	0.0281	**0.0226**	5.0450	4.0906	4.6698	5.4245	4.8075
HARQ	2.5549	2.1625	2.3989	**2.8925**	2.5022	0.0165	0.0219	0.0276	0.0284	0.0236	4.4663	3.7743	4.1844	**5.0492**	4.3685
TV-HAR	**2.4401**	**2.1579**	**2.1629**	2.8967	**2.4144**	**0.0160**	**0.0206**	0.0265	**0.0281**	0.0228	**4.2654**	**3.7673**	**3.7715**	5.0567	**4.2152**

	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.9826	1.0021	0.9892	0.9739	0.9870	**0.9786**	0.9764	0.9656	0.9607	0.9703	0.9830	1.0051	0.9923	0.9755	0.9890
TV-HAR	**0.9579**	**0.9856**	**0.9542**	**0.9688**	**0.9666**	0.9915	**0.9623**	**0.9414**	**0.9463**	**0.9604**	**0.9545**	**0.9884**	**0.9558**	**0.9715**	**0.9675**

	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.8856	0.9232	0.8968	**0.9311**	0.9092	1.0300	1.0564	1.0779	1.0110	1.0438	0.8853	0.9227	0.8961	**0.9308**	0.9087
TV-HAR	**0.8458**	**0.9212**	**0.8086**	0.9325	**0.8770**	**0.9991**	**0.9947**	1.0362	**0.9991**	1.0073	**0.8455**	**0.9210**	**0.8076**	0.9322	**0.8766**

	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	1.7401	-0.2103	1.0781	2.6104	1.3046	**2.1446**	2.3609	3.4399	3.9268	2.9681	1.6984	-0.5145	0.7705	2.4501	1.1011
TV-HAR	**4.2057**	**1.4396**	**4.5846**	**3.1224**	**3.3381**	0.8489	**3.7662**	**5.8564**	**5.3698**	**3.9603**	**4.5484**	**1.1647**	**4.4189**	**2.8499**	**3.2455**

	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	11.4358	7.6802	10.3191	**6.8888**	9.0810	-2.9988	-5.6357	-7.7929	-1.0997	-4.3818	11.4708	7.7324	10.3938	**6.9186**	9.1289
TV-HAR	**15.4153**	**7.8765**	**19.1432**	6.7536	**12.2971**	**0.0939**	**0.5259**	-3.6181	**0.0890**	-0.7273	**15.4529**	**7.9035**	**19.2371**	6.7791	**12.3432**

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The Relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.7: Out-of-sample forecasts (SPY)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.6288	0.5883	0.7077	0.6266	0.6379	0.1505	0.1658	0.1591	0.1616	0.1593	0.9889	0.9064	1.1209	0.9767	0.9982
HARQ	0.5977	**0.5518**	0.6841	0.6113	0.6113	0.1374	0.1499	0.1499	0.1504	0.1469	0.9444	**0.8545**	1.0864	0.9584	0.9609
TV-HAR	**0.5792**	0.5664	**0.6333**	**0.6017**	**0.5951**	**0.1348**	**0.1438**	**0.1442**	**0.1499**	**0.1432**	**0.9137**	0.8845	**1.0016**	**0.9420**	**0.9354**
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	8.5777	5.1910	15.1145	5.5846	8.6170	0.0539	0.0528	0.0444	0.0698	0.0552	14.9972	9.0616	26.4620	9.7395	15.0651
HARQ	8.0552	**4.1260**	17.5343	**4.8303**	8.6365	0.0415	0.0628	0.0458	0.0632	0.0533	14.0901	**7.1872**	30.7027	**8.4214**	15.1004
TV-HAR	**5.9376**	5.0233	**11.0824**	4.9967	**6.7600**	**0.0341**	**0.0374**	**0.0430**	**0.0488**	**0.0408**	**10.3830**	8.7781	**19.3948**	8.7231	**11.8197**
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.9506	**0.9380**	0.9667	0.9756	0.9577	0.9128	0.9041	0.9423	0.9308	0.9225	0.9550	**0.9427**	0.9693	0.9812	0.9620
TV-HAR	**0.9211**	0.9627	**0.8949**	**0.9603**	**0.9347**	**0.8958**	**0.8674**	**0.9067**	**0.9273**	**0.8993**	**0.9240**	0.9758	**0.8936**	**0.9644**	**0.9395**
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.9397	**0.7936**	1.1601	**0.8650**	0.9396	0.9359	0.9059	1.0314	0.9486	0.9555	0.9397	**0.7932**	1.1603	**0.8647**	0.9395
TV-HAR	**0.6927**	0.9683	**0.7332**	0.8959	**0.8225**	**0.8892**	**0.8436**	**0.9672**	**0.9518**	**0.9129**	**0.6924**	0.9688	**0.7329**	0.8957	**0.8224**
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	4.9365	**6.1976**	3.3337	2.4370	4.2262	8.7159	9.5947	5.7711	6.9183	7.7500	4.5030	**5.7303**	3.0728	1.8807	3.7967
TV-HAR	**7.8931**	3.7295	**10.5125**	**3.9706**	**6.5264**	**10.4232**	**13.2614**	**9.3258**	**7.2710**	**10.0703**	**7.6035**	2.4183	**10.6389**	**3.5588**	**6.0549**
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	6.0317	**20.6414**	-16.0096	**13.5010**	6.0411	6.4070	9.4103	-3.1409	5.1386	4.4538	6.0312	**20.6831**	-16.0258	**13.5297**	6.0546
TV-HAR	**30.7266**	3.1685	**26.6774**	10.4101	**17.7457**	**11.0845**	**15.6373**	**3.2793**	**4.8213**	**8.7056**	**30.7637**	3.1231	**26.7070**	10.4307	**17.7561**

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.8: Out-of-sample forecasts (average stock)

	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	1.0000	1.0000
HARQ	**0.9689**	0.9950	0.9945	0.9931	0.9878	**0.9616**	**0.9700**	**0.9614**	**0.9731**	**0.9665**	0.9719	1.0021	1.0052	0.9992	0.9946
TV-HAR	0.9706	**0.9935**	**0.9934**	**0.9923**	**0.9874**	0.9738	0.9777	0.9734	0.9816	0.9766	**0.9705**	**0.9979**	1.0001	**0.9956**	**0.9910**
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.9481	0.9693	1.0551	0.9940	0.9916	**0.9782**	**0.9829**	**0.9729**	**0.9822**	**0.9790**	0.9459	0.9693	1.0589	0.9920	0.9915
TV-HAR	**0.9073**	**0.9418**	**0.9846**	**0.9788**	**0.9531**	0.9853	0.9836	0.9805	0.9848	0.9836	**0.9029**	**0.9393**	**0.9865**	**0.9769**	**0.9514**
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0000	0.0000
HARQ	**3.1132**	0.5039	0.5496	0.6934	1.2150	**3.8384**	**3.0005**	**3.8624**	**2.6902**	**3.3479**	2.8137	-0.2125	-0.5191	0.0800	0.5405
TV-HAR	2.9399	**0.6471**	**0.6638**	**0.7714**	**1.2556**	2.6228	2.2258	2.6625	1.8355	2.3367	**2.9471**	**0.2146**	-0.0090	**0.4379**	**0.8977**
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	5.1853	3.0715	-5.5065	0.5971	0.8369	**2.1839**	**1.7130**	**2.7086**	**1.7848**	**2.0976**	5.4118	3.0674	-5.8876	0.7962	0.8469
TV-HAR	**9.2722**	**5.8231**	**1.5368**	**2.1216**	**4.6884**	1.4714	1.6390	1.9508	1.5152	1.6441	**9.7085**	**6.0654**	**1.3488**	**2.3135**	**4.8591**

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE averaged across the individual stocks. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

3.9 Appendix

Individual stocks in-sample estimation results

Table 3.9: In-sample estimation (MMM)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.4628	0.0817		0.3442	0.3862	0.2211	5.9531	0.1989	0.3947		0.2628	0.2461	*0.5312*	*3.8071*	0.6770	0.0555		0.3340	0.3934	0.1768	6.8735
	(0.1198)	(0.0226)		(0.0455)	(0.0490)			(0.0628)	(0.0277)		(0.0457)	(0.0418)			(0.2954)	(0.0372)		(0.0769)	(0.0856)		
300 sec	0.2432	0.1743		0.5505	0.1707	*0.4990*	*4.8230*	0.2024	0.3309		0.3004	0.2692	0.4979	3.9655	0.3063	0.1081		0.6542	0.1289	*0.4988*	*5.5064*
	(0.0685)	(0.0232)		(0.0389)	(0.0346)			(0.0663)	(0.0279)		(0.0477)	(0.0442)			(0.1484)	(0.0387)		(0.0646)	(0.0567)		
450 sec	0.2924	0.1027		0.5397	0.2275	0.3927	5.1003	0.2101	0.3042		0.3137	0.2756	0.4729	4.0097	0.3803	0.0428		0.6117	0.2042	0.3674	5.8576
	(0.0789)	(0.0232)		(0.0414)	(0.0391)			(0.0674)	(0.0282)		(0.0483)	(0.0451)			(0.1772)	(0.0385)		(0.0696)	(0.0662)		
900 sec	0.2716	0.1090		0.5192	0.2456	0.4038	4.9040	0.2352	0.2205		0.3120	0.3440	0.3854	4.2861	0.3336	0.0432		0.6335	0.1927	0.4128	5.4845
	(0.0726)	(0.0232)		(0.0410)	(0.0386)			(0.0764)	(0.0282)		(0.0516)	(0.0506)			(0.1496)	(0.0388)		(0.0676)	(0.0624)		
average	0.3175	0.1170		0.4884	0.2575	0.3792	5.1951	0.2116	0.3126		0.2972	0.2837	0.4718	4.0171	0.4243	0.0624		0.5584	0.2298	0.3639	5.9305
	(0.0849)	(0.0230)		(0.0417)	(0.0403)			(0.0682)	(0.0280)		(0.0483)	(0.0454)			(0.1926)	(0.0383)		(0.0697)	(0.0677)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.3369	0.4279	−0.0006	0.2170	0.2398	0.2508	5.9145	0.0484	0.6478	−0.0080	0.2231	0.1645	*0.5496*	*3.7676*	0.5400	0.4262	−0.0006	0.2117	0.2295	0.2029	6.8423
	(0.1181)	(0.0398)	(0.0001)	(0.0462)	(0.0500)			(0.0641)	(0.0403)	(0.0009)	(0.0451)	(0.0421)			(0.2916)	(0.0735)	(0.0001)	(0.0785)	(0.0888)		
300 sec	0.0200	0.5468	−0.0044	0.4548	0.0417	*0.5284*	*4.7630*	0.0155	0.6382	−0.0108	0.2498	0.1800	0.5178	3.9256	−0.0545	0.7421	−0.0058	0.4631	−0.0772	*0.5412*	*5.4191*
	(0.0686)	(0.0363)	(0.0003)	(0.0385)	(0.0350)			(0.0686)	(0.0453)	(0.0013)	(0.0471)	(0.0446)			(0.1468)	(0.0752)	(0.0006)	(0.0649)	(0.0583)		
450 sec	0.1688	0.3894	−0.0018	0.4301	0.1299	0.4148	5.0636	0.0002	0.6394	−0.0148	0.2545	0.2010	0.4887	3.9798	0.1965	0.4704	−0.0021	0.4443	0.0624	0.3923	5.8185
	(0.0784)	(0.0362)	(0.0002)	(0.0420)	(0.0396)			(0.0722)	(0.0532)	(0.0020)	(0.0482)	(0.0456)			(0.1760)	(0.0759)	(0.0003)	(0.0730)	(0.0684)		
900 sec	0.1244	0.3604	−0.0041	0.4671	0.1551	0.4211	4.8749	0.0717	0.5003	−0.0114	0.2627	0.2718	0.3984	4.2652	0.0953	0.4723	−0.0053	0.5185	0.0495	0.4375	5.4425
	(0.0734)	(0.0358)	(0.0005)	(0.0408)	(0.0393)			(0.0800)	(0.0530)	(0.0018)	(0.0517)	(0.0514)			(0.1507)	(0.0744)	(0.0008)	(0.0684)	(0.0647)		
average	0.1625	0.4311	−0.0027	0.3923	0.1416	0.4038	5.1540	0.0340	0.6064	−0.0113	0.2475	0.2043	**0.4886**	**3.9846**	0.1943	0.5278	−0.0035	0.4094	0.0660	0.3934	5.8806
	(0.0846)	(0.0370)	(0.0003)	(0.0419)	(0.0410)			(0.0712)	(0.0480)	(0.0015)	(0.0480)	(0.0459)			(0.1913)	(0.0748)	(0.0005)	(0.0712)	(0.0701)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.2681	0.5707	−0.0032	0.1766	0.1762	0.2632	5.8979	0.1574	0.4792	−0.0067	0.2432	0.2162	*0.5329*	*3.8040*	0.4365	0.6404	−0.0036	0.1559	0.1322	0.2188	6.8221
	(0.1175)	(0.0447)	(0.0003)	(0.0462)	(0.0505)			(0.0645)	(0.0415)	(0.0025)	(0.0462)	(0.0431)			(0.2896)	(0.0868)	(0.0005)	(0.0786)	(0.0906)		
300 sec	0.0975	0.4654	−0.0063	0.4474	0.0741	*0.5248*	*4.7705*	0.1404	0.4539	−0.0099	0.2761	0.2250	0.5017	3.9585	0.0870	0.5620	−0.0076	0.4717	−0.0078	*0.5341*	*5.4343*
	(0.0677)	(0.0328)	(0.0005)	(0.0388)	(0.0346)			(0.0681)	(0.0430)	(0.0026)	(0.0480)	(0.0456)			(0.1452)	(0.0638)	(0.0009)	(0.0657)	(0.0569)		
450 sec	0.1805	0.3796	−0.0039	0.4295	0.1318	0.4142	5.0646	0.1713	0.3794	−0.0066	0.3021	0.2478	0.4743	4.0076	0.2190	0.4400	−0.0045	0.4488	0.0723	0.3914	5.8199
	(0.0783)	(0.0357)	(0.0004)	(0.0421)	(0.0396)			(0.0693)	(0.0424)	(0.0028)	(0.0484)	(0.0466)			(0.1756)	(0.0729)	(0.0007)	(0.0729)	(0.0681)		
900 sec	0.1735	0.3219	−0.0052	0.4592	0.1629	0.4222	4.8730	0.1382	0.4156	−0.0135	0.2624	0.2885	0.3952	4.2706	0.1891	0.3736	−0.0060	0.5205	0.0750	0.4373	5.4428
	(0.0723)	(0.0322)	(0.0006)	(0.0409)	(0.0390)			(0.0779)	(0.0458)	(0.0025)	(0.0520)	(0.0512)			(0.1480)	(0.0623)	(0.0009)	(0.0683)	(0.0635)		
average	0.1799	0.4344	−0.0047	0.3782	0.1362	**0.4061**	**5.1515**	0.1518	0.4320	−0.0092	0.2709	0.2444	0.4760	4.0102	0.2329	0.5040	−0.0054	0.3992	0.0679	**0.3954**	**5.8798**
	(0.0840)	(0.0363)	(0.0004)	(0.0420)	(0.0409)			(0.0700)	(0.0432)	(0.0026)	(0.0487)	(0.0466)			(0.1896)	(0.0714)	(0.0007)	(0.0714)	(0.0698)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.10: In-sample estimation (AMZN)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.5340	0.2601		0.4573	0.2333	*0.6848*	*7.3498*	0.5825	0.2618		0.4350	0.2573	*0.6870*	*7.5915*	0.6257	0.2509		0.5391	0.1258	*0.6030*	*6.7187*
	(0.2366)	(0.0231)		(0.0381)	(0.0315)			(0.3451)	(0.0290)		(0.0482)	(0.0401)			(0.2902)	(0.0379)		(0.0610)	(0.0511)		
300 sec	0.6268	0.2849		0.3522	0.3042	0.6136	7.6205	0.7213	0.2975		0.3238	0.3216	0.6108	7.8899	0.6270	0.2220		0.4899	0.2016	0.5497	6.8648
	(0.2699)	(0.0226)		(0.0387)	(0.0339)			(0.3989)	(0.0284)		(0.0486)	(0.0428)			(0.3112)	(0.0375)		(0.0641)	(0.0567)		
450 sec	0.7299	0.2666		0.3324	0.3325	0.5601	7.8547	0.8586	0.2668		0.3229	0.3426	0.5543	8.1417	0.6844	0.2643		0.3801	0.2598	0.5047	7.0146
	(0.3008)	(0.0225)		(0.0398)	(0.0360)			(0.4479)	(0.0283)		(0.0502)	(0.0456)			(0.3345)	(0.0370)		(0.0647)	(0.0593)		
900 sec	0.8789	0.2724		0.2868	0.3575	0.4932	8.1475	1.0600	0.2863		0.2694	0.3603	0.4877	8.4451	0.7505	0.1889		0.3889	0.3144	0.4366	7.2444
	(0.3441)	(0.0223)		(0.0398)	(0.0374)			(0.5145)	(0.0280)		(0.0500)	(0.0472)			(0.3708)	(0.0373)		(0.0666)	(0.0631)		
average	0.6924	0.2710		0.3572	0.3069	0.5879	7.7431	0.8056	0.2781		0.3378	0.3205	0.5849	8.0170	0.6719	0.2315		0.4495	0.2254	0.5235	6.9606
	(0.2879)	(0.0226)		(0.0391)	(0.0347)			(0.4266)	(0.0284)		(0.0493)	(0.0439)			(0.3267)	(0.0374)		(0.0641)	(0.0575)		
	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	−0.0201	0.4610	−0.0011	0.4388	0.1584	*0.6927*	*7.3249*	0.0548	0.4448	−0.0010	0.4192	0.1847	*0.6934*	*7.5715*	−0.8383	0.7990	−0.0043	0.4911	−0.0377	*0.6441*	*6.6102*
	(0.2427)	(0.0329)	(0.0001)	(0.0377)	(0.0323)			(0.3524)	(0.0416)	(0.0002)	(0.0478)	(0.0414)			(0.3062)	(0.0621)	(0.0004)	(0.0579)	(0.0507)		
300 sec	0.2339	0.4429	−0.0007	0.3294	0.2447	0.6208	7.6020	0.3080	0.4538	−0.0007	0.3022	0.2597	0.6177	7.8726	−0.4157	0.6637	−0.0032	0.4073	0.0907	0.5821	6.7911
	(0.2727)	(0.0311)	(0.0001)	(0.0385)	(0.0346)			(0.4020)	(0.0393)	(0.0001)	(0.0483)	(0.0438)			(0.3220)	(0.0614)	(0.0004)	(0.0624)	(0.0560)		
450 sec	0.1952	0.4800	−0.0009	0.2932	0.2597	0.5697	7.8328	0.3197	0.4690	−0.0008	0.2887	0.2685	0.5626	8.1235	−0.3941	0.7054	−0.0027	0.2544	0.1889	0.5423	6.9367
	(0.3051)	(0.0350)	(0.0001)	(0.0397)	(0.0368)			(0.4533)	(0.0446)	(0.0001)	(0.0501)	(0.0469)			(0.3426)	(0.0600)	(0.0003)	(0.0637)	(0.0575)		
900 sec	0.4918	0.4172	−0.0006	0.2652	0.3059	0.4970	8.1404	0.5772	0.4535	−0.0007	0.2460	0.2974	0.4922	8.4368	−0.0210	0.4850	−0.0021	0.3163	0.2730	0.4492	7.2228
	(0.3528)	(0.0383)	(0.0001)	(0.0400)	(0.0389)			(0.5259)	(0.0498)	(0.0002)	(0.0501)	(0.0495)			(0.3991)	(0.0709)	(0.0004)	(0.0675)	(0.0629)		
average	0.2252	0.4503	−0.0009	0.3316	0.2422	**0.5950**	**7.7250**	0.3149	0.4553	−0.0008	0.3140	0.2526	**0.5915**	**8.0011**	−0.4173	0.6633	−0.0031	0.3673	0.1287	0.5544	6.8902
	(0.2933)	(0.0343)	(0.0001)	(0.0389)	(0.0357)			(0.4334)	(0.0439)	(0.0001)	(0.0491)	(0.0454)			(0.3425)	(0.0636)	(0.0004)	(0.0629)	(0.0568)		
	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.3389	0.3663	−0.0015	0.4430	0.1810	*0.6872*	*7.3426*	0.4881	0.3130	−0.0007	0.4285	0.2305	*0.6874*	*7.5908*	−0.4680	0.7024	−0.0081	0.4759	−0.0197	*0.6538*	*6.5827*
	(0.2394)	(0.0322)	(0.0003)	(0.0380)	(0.0333)			(0.3490)	(0.0410)	(0.0004)	(0.0483)	(0.0428)			(0.2855)	(0.0512)	(0.0007)	(0.0572)	(0.0492)		
300 sec	0.5283	0.3391	−0.0006	0.3423	0.2793	0.6142	7.6193	0.6662	0.3272	−0.0003	0.3185	0.3072	0.6108	7.8905	−0.6699	0.7644	−0.0096	0.3691	0.0792	0.5970	6.7547
	(0.2731)	(0.0327)	(0.0003)	(0.0389)	(0.0356)			(0.4029)	(0.0418)	(0.0003)	(0.0489)	(0.0454)			(0.3175)	(0.0611)	(0.0009)	(0.0616)	(0.0548)		
450 sec	0.5979	0.3372	−0.0006	0.3196	0.3000	0.5612	7.8524	0.7798	0.3081	−0.0003	0.3161	0.3221	0.5544	8.1419	−0.5535	0.7650	−0.0074	0.2256	0.1875	0.5564	6.9054
	(0.3039)	(0.0333)	(0.0002)	(0.0400)	(0.0377)			(0.4519)	(0.0427)	(0.0003)	(0.0505)	(0.0483)			(0.3365)	(0.0579)	(0.0007)	(0.0628)	(0.0565)		
900 sec	0.7914	0.3124	−0.0003	0.2805	0.3399	0.4935	8.1473	0.9764	0.3220	−0.0003	0.2642	0.3438	0.4878	8.4456	−0.0634	0.5232	−0.0046	0.3005	0.2582	0.4572	7.2082
	(0.3486)	(0.0341)	(0.0002)	(0.0400)	(0.0391)			(0.5205)	(0.0438)	(0.0002)	(0.0502)	(0.0497)			(0.3866)	(0.0649)	(0.0007)	(0.0669)	(0.0626)		
average	0.5641	0.3387	−0.0007	0.3463	0.2750	0.5890	7.7404	0.7276	0.3176	−0.0004	0.3318	0.3009	0.5851	8.0172	−0.4387	0.6887	−0.0074	0.3428	0.1263	**0.5661**	**6.8628**
	(0.2912)	(0.0331)	(0.0002)	(0.0392)	(0.0364)			(0.4311)	(0.0423)	(0.0003)	(0.0495)	(0.0466)			(0.3315)	(0.0588)	(0.0007)	(0.0621)	(0.0558)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.11: In-sample estimation (MRK)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.6199	0.1581		0.2872	0.3603	0.2328	6.2835	0.4175	0.2167		0.2429	0.3764	*0.2873*	*4.7709*	0.9213	0.1478		0.2948	0.3422	0.2080	7.1388
	(0.1466)	(0.0226)		(0.0436)	(0.0474)			(0.1112)	(0.0279)		(0.0535)	(0.0563)			(0.3560)	(0.0374)		(0.0723)	(0.0802)		
300 sec	0.6367	0.1495		0.2729	0.3729	0.2091	6.2991	0.4594	0.2195		0.2364	0.3609	0.2599	4.9271	0.9327	0.1355		0.2796	0.3598	0.1842	7.1294
	(0.1493)	(0.0224)		(0.0445)	(0.0494)			(0.1176)	(0.0279)		(0.0533)	(0.0576)			(0.3588)	(0.0370)		(0.0741)	(0.0841)		
450 sec	0.5194	0.1550		0.3443	0.3254	0.2717	5.9570	0.4588	0.2204		0.1973	0.3960	0.2467	4.9852	0.6961	0.1318		0.3924	0.2938	0.2675	6.6877
	(0.1252)	(0.0227)		(0.0430)	(0.0452)			(0.1195)	(0.0278)		(0.0539)	(0.0589)			(0.2823)	(0.0377)		(0.0712)	(0.0744)		
900 sec	0.3880	0.2024		0.3338	0.3281	*0.3582*	*5.5111*	0.4992	0.1725		0.2301	0.3885	0.2071	5.1150	0.4444	0.2243		0.3781	0.2779	*0.4259*	*5.9566*
	(0.1015)	(0.0227)		(0.0412)	(0.0411)			(0.1262)	(0.0279)		(0.0555)	(0.0620)			(0.1995)	(0.0381)		(0.0654)	(0.0621)		
average	0.5410	0.1662		0.3096	0.3467	0.2679	6.0127	0.4587	0.2073		0.2267	0.3805	0.2503	4.9495	0.7486	0.1599		0.3362	0.3184	0.2714	6.7281
	(0.1307)	(0.0226)		(0.0431)	(0.0458)			(0.1186)	(0.0279)		(0.0540)	(0.0587)			(0.2991)	(0.0376)		(0.0708)	(0.0752)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.3274	0.5696	−0.0011	0.2073	0.1628	0.2886	6.2084	0.3625	0.4301	−0.0018	0.1575	0.2975	*0.3088*	*4.7409*	0.4950	0.6789	−0.0013	0.1953	0.0910	0.2801	7.0444
	(0.1426)	(0.0354)	(0.0001)	(0.0423)	(0.0476)			(0.1098)	(0.0399)	(0.0002)	(0.0539)	(0.0565)			(0.3420)	(0.0636)	(0.0001)	(0.0696)	(0.0805)		
300 sec	0.4099	0.5235	−0.0007	0.1815	0.1920	0.2642	6.2274	0.3914	0.4145	−0.0017	0.1630	0.2903	0.2805	4.8994	0.5770	0.6485	−0.0009	0.1597	0.1107	0.2585	7.0350
	(0.1449)	(0.0338)	(0.0001)	(0.0434)	(0.0493)			(0.1164)	(0.0388)	(0.0002)	(0.0536)	(0.0577)			(0.3439)	(0.0620)	(0.0001)	(0.0717)	(0.0839)		
450 sec	0.2818	0.4729	−0.0017	0.2771	0.1919	0.2988	5.9194	0.3808	0.4015	−0.0028	0.1477	0.3306	0.2582	4.9704	0.2424	0.6803	−0.0025	0.2645	0.0715	0.3123	6.6256
	(0.1249)	(0.0379)	(0.0002)	(0.0427)	(0.0462)			(0.1195)	(0.0440)	(0.0005)	(0.0543)	(0.0598)			(0.2791)	(0.0766)	(0.0003)	(0.0707)	(0.0771)		
900 sec	0.2086	0.4332	−0.0030	0.2903	0.2500	*0.3723*	*5.4892*	0.4453	0.2818	−0.0016	0.1990	0.3515	0.2120	5.1093	0.0859	0.6318	−0.0049	0.3075	0.1365	*0.4557*	*5.9043*
	(0.1029)	(0.0368)	(0.0004)	(0.0411)	(0.0418)			(0.1268)	(0.0422)	(0.0005)	(0.0561)	(0.0627)			(0.2001)	(0.0659)	(0.0007)	(0.0644)	(0.0633)		
average	0.3069	0.4998	−0.0016	0.2390	0.1992	0.3060	5.9611	0.3950	0.3820	−0.0020	0.1668	0.3175	**0.2649**	**4.9300**	0.3501	0.6599	−0.0024	0.2318	0.1024	0.3266	6.6523
	(0.1288)	(0.0360)	(0.0002)	(0.0424)	(0.0462)			(0.1181)	(0.0412)	(0.0004)	(0.0545)	(0.0592)			(0.2913)	(0.0670)	(0.0003)	(0.0691)	(0.0762)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.2079	0.6653	−0.0043	0.1914	0.1256	0.3007	6.1912	0.3070	0.4691	−0.0110	0.1620	0.2873	*0.3036*	*4.7484*	0.1961	0.9255	−0.0058	0.1480	−0.0079	0.3121	6.9988
	(0.1423)	(0.0378)	(0.0003)	(0.0420)	(0.0475)			(0.1113)	(0.0479)	(0.0017)	(0.0543)	(0.0574)			(0.3369)	(0.0719)	(0.0005)	(0.0684)	(0.0800)		
300 sec	0.3689	0.5638	−0.0030	0.1783	0.1731	0.2689	6.2210	0.3399	0.4507	−0.0081	0.1619	0.2849	0.2768	4.9045	0.4925	0.7370	−0.0037	0.1478	0.0681	0.2700	7.0193
	(0.1447)	(0.0350)	(0.0002)	(0.0432)	(0.0494)			(0.1178)	(0.0452)	(0.0013)	(0.0540)	(0.0582)			(0.3418)	(0.0653)	(0.0003)	(0.0712)	(0.0840)		
450 sec	0.3549	0.3948	−0.0024	0.2824	0.2240	0.2935	5.9270	0.3401	0.4211	−0.0090	0.1478	0.3315	0.2555	4.9740	0.3898	0.5430	−0.0032	0.2735	0.1276	0.3030	6.6391
	(0.1246)	(0.0342)	(0.0003)	(0.0429)	(0.0458)			(0.1215)	(0.0514)	(0.0019)	(0.0546)	(0.0602)			(0.2786)	(0.0678)	(0.0004)	(0.0714)	(0.0762)		
900 sec	0.2238	0.4358	−0.0066	0.2820	0.2457	*0.3767*	*5.4823*	0.4377	0.2766	−0.0038	0.2030	0.3554	0.2102	5.1116	0.1846	0.5859	−0.0086	0.2845	0.1514	*0.4622*	*5.8923*
	(0.1017)	(0.0341)	(0.0007)	(0.0410)	(0.0415)			(0.1279)	(0.0465)	(0.0014)	(0.0562)	(0.0630)			(0.1956)	(0.0571)	(0.0010)	(0.0643)	(0.0620)		
average	0.2888	0.5149	−0.0041	0.2335	0.1921	**0.3099**	**5.9554**	0.3562	0.4044	−0.0080	0.1687	0.3148	0.2615	4.9346	0.3157	0.6978	−0.0053	0.2134	0.0848	**0.3368**	**6.6374**
	(0.1283)	(0.0353)	(0.0004)	(0.0423)	(0.0461)			(0.1196)	(0.0478)	(0.0016)	(0.0548)	(0.0597)			(0.2882)	(0.0655)	(0.0006)	(0.0688)	(0.0755)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.12: In-sample estimation (BA)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.2715	0.3489		0.3952	0.1815	*0.6284*	*4.8778*	0.2917	0.3535		0.4536	0.1090	*0.6265*	*4.4490*	0.2735	0.3450		0.3342	0.2524	*0.6289*	*5.3507*
	(0.0767)	(0.0228)		(0.0361)	(0.0303)			(0.0874)	(0.0287)		(0.0431)	(0.0353)			(0.1467)	(0.0376)		(0.0625)	(0.0535)		
300 sec	0.2911	0.3552		0.3776	0.1844	0.5954	4.9399	0.3405	0.3646		0.4140	0.1200	0.5714	4.6040	0.2734	0.3466		0.3308	0.2513	0.6122	5.3429
	(0.0791)	(0.0226)		(0.0361)	(0.0309)			(0.0946)	(0.0285)		(0.0433)	(0.0370)			(0.1459)	(0.0372)		(0.0626)	(0.0543)		
450 sec	0.3374	0.3129		0.4255	0.1637	0.5478	5.0740	0.4080	0.2947		0.4651	0.1160	0.4954	4.8592	0.3060	0.3312		0.3789	0.2088	0.5902	5.3660
	(0.0848)	(0.0227)		(0.0367)	(0.0321)			(0.1067)	(0.0286)		(0.0450)	(0.0405)			(0.1487)	(0.0375)		(0.0624)	(0.0538)		
900 sec	0.3622	0.2552		0.4012	0.2352	0.4726	5.2910	0.4161	0.2502		0.4511	0.1648	0.4296	4.8955	0.3652	0.2588		0.3555	0.2894	0.4970	5.7418
	(0.0937)	(0.0227)		(0.0394)	(0.0364)			(0.1110)	(0.0285)		(0.0477)	(0.0451)			(0.1776)	(0.0375)		(0.0672)	(0.0614)		
average	0.3155	0.3180		0.3999	0.1912	0.5611	5.0457	0.3641	0.3157		0.4459	0.1275	0.5307	4.7019	0.3045	0.3204		0.3499	0.2505	0.5821	5.4504
	(0.0836)	(0.0227)		(0.0371)	(0.0324)			(0.0999)	(0.0286)		(0.0448)	(0.0395)			(0.1547)	(0.0375)		(0.0637)	(0.0558)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.0984	0.5327	−0.0025	0.3739	0.1009	*0.6426*	*4.8392*	0.1767	0.4918	−0.0021	0.4581	0.0238	*0.6448*	*4.3994*	−0.0005	0.6397	−0.0036	0.2727	0.1535	*0.6427*	*5.3140*
	(0.0770)	(0.0284)	(0.0002)	(0.0355)	(0.0307)			(0.0861)	(0.0315)	(0.0002)	(0.0420)	(0.0356)			(0.1504)	(0.0597)	(0.0006)	(0.0621)	(0.0548)		
300 sec	0.0844	0.5326	−0.0032	0.3671	0.1171	0.6058	4.9141	0.1966	0.5050	−0.0030	0.4328	0.0360	0.5869	4.5677	−0.0320	0.6141	−0.0043	0.2818	0.1821	0.6201	5.3233
	(0.0817)	(0.0304)	(0.0004)	(0.0356)	(0.0315)			(0.0945)	(0.0329)	(0.0004)	(0.0426)	(0.0378)			(0.1584)	(0.0680)	(0.0009)	(0.0628)	(0.0557)		
450 sec	0.1445	0.4870	−0.0030	0.4209	0.0880	0.5589	5.0495	0.2604	0.4359	−0.0027	0.4791	0.0357	0.5086	4.8334	0.0311	0.5846	−0.0040	0.3433	0.1240	0.6011	5.3399
	(0.0869)	(0.0306)	(0.0004)	(0.0363)	(0.0330)			(0.1075)	(0.0349)	(0.0004)	(0.0445)	(0.0416)			(0.1555)	(0.0602)	(0.0007)	(0.0620)	(0.0554)		
900 sec	0.2336	0.3697	−0.0021	0.3896	0.1961	0.4766	5.2837	0.3078	0.3556	−0.0030	0.4572	0.1165	0.4340	4.8885	0.2005	0.4186	−0.0023	0.3337	0.2327	0.5019	5.7331
	(0.0973)	(0.0332)	(0.0004)	(0.0393)	(0.0372)			(0.1142)	(0.0398)	(0.0008)	(0.0476)	(0.0467)			(0.1837)	(0.0613)	(0.0007)	(0.0672)	(0.0635)		
average	0.1402	0.4805	−0.0027	0.3879	0.1255	**0.5710**	**5.0216**	0.2354	0.4471	−0.0027	0.4568	0.0530	**0.5436**	**4.6722**	0.0498	0.5642	−0.0036	0.3079	0.1731	0.5914	5.4276
	(0.0857)	(0.0307)	(0.0004)	(0.0367)	(0.0331)			(0.1006)	(0.0348)	(0.0004)	(0.0442)	(0.0404)			(0.1620)	(0.0623)	(0.0007)	(0.0635)	(0.0574)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.0838	0.5533	−0.0081	0.3550	0.1006	*0.6413*	*4.8429*	0.2042	0.4644	−0.0064	0.4518	0.0446	*0.6310*	*4.4377*	0.0162	0.6362	−0.0094	0.2662	0.1454	*0.6492*	*5.2956*
	(0.0777)	(0.0304)	(0.0008)	(0.0357)	(0.0308)			(0.0888)	(0.0371)	(0.0014)	(0.0429)	(0.0377)			(0.1465)	(0.0527)	(0.0012)	(0.0614)	(0.0539)		
300 sec	0.1392	0.4944	−0.0062	0.3553	0.1371	0.6004	4.9277	0.3026	0.4052	−0.0024	0.4149	0.0979	0.5717	4.6037	0.0183	0.5878	−0.0091	0.2725	0.1863	0.6234	5.3147
	(0.0826)	(0.0324)	(0.0010)	(0.0361)	(0.0318)			(0.0977)	(0.0388)	(0.0015)	(0.0433)	(0.0397)			(0.1509)	(0.0569)	(0.0016)	(0.0625)	(0.0548)		
450 sec	0.1708	0.4757	−0.0064	0.4116	0.0931	0.5563	5.0553	0.3424	0.3619	−0.0032	0.4646	0.0819	0.4968	4.8570	0.0471	0.5969	−0.0090	0.3443	0.0979	0.6087	5.3207
	(0.0870)	(0.0317)	(0.0009)	(0.0364)	(0.0332)			(0.1100)	(0.0401)	(0.0013)	(0.0449)	(0.0429)			(0.1500)	(0.0529)	(0.0013)	(0.061)	(0.0549)		
900 sec	0.2708	0.3519	−0.0035	0.3880	0.1969	0.4760	5.2849	0.3619	0.3127	−0.0034	0.4473	0.1370	0.4308	4.8940	0.2442	0.3989	−0.0040	0.3363	0.2288	0.5021	5.7326
	(0.0958)	(0.0317)	(0.0008)	(0.0394)	(0.0373)			(0.1137)	(0.0407)	(0.0016)	(0.0477)	(0.0469)			(0.1803)	(0.0559)	(0.0012)	(0.0671)	(0.0637)		
average	0.1662	0.4688	−0.0061	0.3775	0.1319	0.5685	5.0277	0.3028	0.3861	−0.0038	0.4447	0.0904	0.5326	4.6981	0.0815	0.5549	−0.0079	0.3048	0.1646	**0.5959**	**5.4159**
	(0.0858)	(0.0315)	(0.0009)	(0.0369)	(0.0333)			(0.1026)	(0.0392)	(0.0015)	(0.0447)	(0.0418)			(0.1569)	(0.0546)	(0.0013)	(0.0631)	(0.0568)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.13: In-sample estimation (MSFT)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.2561	0.3340		0.4322	0.1565	*0.6295*	*4.8059*	0.2042	0.4831		0.2422	0.2110	*0.6833*	*4.4116*	0.3154	0.1997		0.6108	0.0973	0.5861	5.2314
	(0.0721)	(0.0228)		(0.0360)	(0.0300)			(0.0782)	(0.0272)		(0.0427)	(0.0358)			(0.1383)	(0.0386)		(0.0616)	(0.0511)		
300 sec	0.2604	0.3191		0.4186	0.1815	0.5970	4.9288	0.2416	0.3907		0.2831	0.2498	0.6028	4.8012	0.2845	0.2133		0.6031	0.0960	0.5952	5.1063
	(0.0757)	(0.0225)		(0.0367)	(0.0315)			(0.0924)	(0.0276)		(0.0458)	(0.0404)			(0.1296)	(0.0382)		(0.0609)	(0.0506)		
450 sec	0.2614	0.3596		0.3713	0.1861	0.5980	4.9139	0.2463	0.3822		0.2870	0.2518	0.5919	4.8716	0.2798	0.3179		0.4899	0.1029	*0.6085*	*4.9830*
	(0.0748)	(0.0224)		(0.0360)	(0.0310)			(0.0945)	(0.0277)		(0.0460)	(0.0407)			(0.1224)	(0.0378)		(0.0586)	(0.0483)		
900 sec	0.2825	0.1911		0.5051	0.2105	0.5296	5.0853	0.2597	0.2354		0.3755	0.3015	0.5185	5.0417	0.3134	0.1061		0.6923	0.0991	0.5508	5.1490
	(0.0807)	(0.0231)		(0.0391)	(0.0344)			(0.1019)	(0.0283)		(0.0503)	(0.0459)			(0.1321)	(0.0396)		(0.0633)	(0.0524)		
average	0.2651	0.3010		0.4318	0.1836	0.5885	4.9335	0.2380	0.3728		0.2969	0.2535	0.5991	4.7815	0.2983	0.2093		0.5990	0.0988	0.5852	5.1174
	(0.0758)	(0.0227)		(0.0370)	(0.0317)			(0.0917)	(0.0277)		(0.0462)	(0.0407)			(0.1306)	(0.0386)		(0.0611)	(0.0506)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	−0.0044	0.6427	−0.0051	0.3603	0.0558	*0.6559*	*4.7322*	0.0367	0.7156	−0.0061	0.1914	0.1404	*0.6943*	*4.3767*	−0.0817	0.6221	−0.0053	0.5017	−0.0246	*0.6256*	*5.1322*
	(0.0718)	(0.0305)	(0.0004)	(0.0350)	(0.0297)			(0.0796)	(0.0396)	(0.0008)	(0.0425)	(0.0363)			(0.1376)	(0.0559)	(0.0005)	(0.0597)	(0.0502)		
300 sec	0.1263	0.5187	−0.0030	0.3537	0.1239	0.6126	4.8898	0.1564	0.5865	−0.0028	0.2123	0.1831	0.6214	4.7540	−0.0077	0.5003	−0.0048	0.5312	0.0333	0.6112	5.0669
	(0.0753)	(0.0291)	(0.0003)	(0.0365)	(0.0314)			(0.0907)	(0.0342)	(0.0003)	(0.0454)	(0.0401)			(0.1347)	(0.0579)	(0.0007)	(0.0607)	(0.0505)		
450 sec	−0.0513	0.6618	−0.0081	0.3469	0.0930	0.6108	4.8820	−0.0591	0.7572	−0.0107	0.2504	0.1185	0.6087	4.8302	−0.0916	0.6098	−0.0070	0.4636	0.0392	0.6194	4.9556
	(0.0805)	(0.0386)	(0.0008)	(0.0356)	(0.0321)			(0.0990)	(0.0510)	(0.0012)	(0.0452)	(0.0427)			(0.1384)	(0.0651)	(0.0013)	(0.0580)	(0.0490)		
900 sec	0.1109	0.3901	−0.0047	0.4706	0.1524	0.5388	5.0658	0.1559	0.3980	−0.0038	0.3379	0.2496	0.5257	5.0272	−0.0784	0.4392	−0.0080	0.6616	0.0186	0.5675	5.1122
	(0.0832)	(0.0351)	(0.0006)	(0.0390)	(0.0349)			(0.1031)	(0.0419)	(0.0007)	(0.0504)	(0.0467)			(0.1438)	(0.0656)	(0.0013)	(0.0623)	(0.0530)		
average	0.0453	0.5533	−0.0052	0.3829	0.1063	0.6045	4.8925	0.0725	0.6143	−0.0058	0.2480	0.1729	0.6125	4.7470	−0.0649	0.5428	−0.0063	0.5395	0.0166	0.6059	5.0667
	(0.0777)	(0.0333)	(0.0005)	(0.0365)	(0.0320)			(0.0931)	(0.0417)	(0.0008)	(0.0459)	(0.0415)			(0.1386)	(0.0611)	(0.0010)	(0.0602)	(0.0507)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.0475	0.6082	−0.0094	0.3478	0.0662	*0.6568*	*4.7298*	0.0598	0.6943	−0.0135	0.1851	0.1426	*0.6940*	*4.3776*	−0.0069	0.5913	−0.0096	0.4760	−0.0125	*0.6256*	*5.1322*
	(0.0708)	(0.0287)	(0.0006)	(0.0351)	(0.0295)			(0.0790)	(0.0379)	(0.0017)	(0.0426)	(0.0363)			(0.1351)	(0.0528)	(0.0009)	(0.0600)	(0.0498)		
300 sec	0.0555	0.6074	−0.0110	0.3184	0.1032	0.6189	4.8733	0.0790	0.6937	−0.0131	0.1795	0.1510	0.6274	4.7380	0.0402	0.4996	−0.0093	0.4942	0.0419	0.6134	5.0613
	(0.0754)	(0.0317)	(0.0009)	(0.0365)	(0.0313)			(0.0908)	(0.0388)	(0.0012)	(0.0454)	(0.0402)			(0.1314)	(0.0556)	(0.0013)	(0.0615)	(0.0501)		
450 sec	0.0754	0.5871	−0.0105	0.3222	0.1118	0.6121	4.8786	0.0779	0.6336	−0.0129	0.2249	0.1672	0.6064	4.8360	0.0484	0.5584	−0.0096	0.4371	0.0368	0.6241	4.9432
	(0.0757)	(0.0316)	(0.0010)	(0.0357)	(0.0314)			(0.0951)	(0.0414)	(0.0016)	(0.0458)	(0.0414)			(0.1250)	(0.0522)	(0.0015)	(0.0580)	(0.0484)		
900 sec	0.1150	0.4134	−0.0095	0.4457	0.1461	0.5441	5.0542	0.1422	0.4277	−0.0090	0.3147	0.2474	0.5278	5.0228	0.0663	0.3824	−0.0103	0.6191	0.0302	0.5733	5.0986
	(0.0815)	(0.0328)	(0.0010)	(0.0390)	(0.0346)			(0.1028)	(0.0429)	(0.0015)	(0.0508)	(0.0464)			(0.1331)	(0.0539)	(0.0014)	(0.0625)	(0.0519)		
average	0.0734	0.5540	−0.0101	0.3585	0.1069	**0.6080**	**4.8840**	0.0897	0.6123	−0.0121	0.2260	0.1771	**0.6139**	**4.7436**	0.0370	0.5079	−0.0097	0.5066	0.0241	**0.6091**	**5.0588**
	(0.0759)	(0.0312)	(0.0009)	(0.0366)	(0.0317)			(0.0919)	(0.0403)	(0.0015)	(0.0462)	(0.0411)			(0.1312)	(0.0536)	(0.0013)	(0.0605)	(0.0500)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.14: In-sample estimation (KO)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.1224	0.3456		0.3923	0.1981	*0.6570*	*3.6179*	0.1087	0.3188		0.3768	0.2466	*0.6671*	*3.2911*	0.1328	0.3676		0.3984	0.1640	0.6471	4.0145
	(0.0401)	(0.0226)		(0.0358)	(0.0300)			(0.0468)	(0.0285)		(0.0468)	(0.0395)			(0.0725)	(0.0372)		(0.0575)	(0.0480)		
300 sec	0.1192	0.3260		0.4053	0.2036	0.6450	3.6274	0.1133	0.3308		0.3741	0.2326	0.6474	3.2987	0.1239	0.3218		0.4299	0.1810	0.6418	4.0272
	(0.0397)	(0.0226)		(0.0363)	(0.0307)			(0.0468)	(0.0284)		(0.0461)	(0.0392)			(0.0720)	(0.0374)		(0.0594)	(0.0501)		
450 sec	0.1269	0.3537		0.3978	0.1772	0.6384	3.6002	0.1370	0.2994		0.3942	0.2299	0.5922	3.5088	0.1141	0.4249		0.3790	0.1306	*0.6882*	*3.7371*
	(0.0392)	(0.0226)		(0.0354)	(0.0297)			(0.0513)	(0.0286)		(0.0468)	(0.0407)			(0.0625)	(0.0369)		(0.0552)	(0.0443)		
900 sec	0.1863	0.1980		0.4352	0.2624	0.4694	4.3288	0.1983	0.1962		0.3318	0.3614	0.4070	4.2359	0.1722	0.1962		0.5341	0.1715	0.5381	4.4693
	(0.0545)	(0.0231)		(0.0393)	(0.0361)			(0.0717)	(0.0285)		(0.0516)	(0.0501)			(0.0875)	(0.0391)		(0.0623)	(0.0534)		
average	0.1387	0.3058		0.4077	0.2103	0.6025	3.7936	0.1393	0.2863		0.3692	0.2676	0.5784	3.5837	0.1357	0.3276		0.4354	0.1618	0.6288	4.0620
	(0.0434)	(0.0227)		(0.0367)	(0.0316)			(0.0542)	(0.0285)		(0.0478)	(0.0424)			(0.0736)	(0.0376)		(0.0586)	(0.0490)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.0156	0.5458	−0.0067	0.3688	0.1259	*0.6622*	*3.6030*	0.0042	0.5612	−0.0113	0.3504	0.1515	*0.6737*	*3.2717*	−0.0608	0.7346	−0.0100	0.3425	0.0573	0.6575	3.9856
	(0.0429)	(0.0378)	(0.0010)	(0.0357)	(0.0317)			(0.0495)	(0.0492)	(0.0019)	(0.0465)	(0.0422)			(0.0793)	(0.0750)	(0.0018)	(0.0575)	(0.0509)		
300 sec	−0.0070	0.5366	−0.0078	0.3864	0.1333	0.6519	3.6081	0.0348	0.4974	−0.0089	0.3630	0.1681	0.6507	3.2899	−0.1253	0.7536	−0.0128	0.3790	0.0592	0.6587	3.9799
	(0.0428)	(0.0360)	(0.0011)	(0.0360)	(0.0318)			(0.0502)	(0.0489)	(0.0021)	(0.0459)	(0.0420)			(0.0785)	(0.0708)	(0.0018)	(0.0585)	(0.0518)		
450 sec	0.0692	0.4435	−0.0043	0.3966	0.1471	0.6394	3.5976	0.0061	0.5656	−0.0158	0.3799	0.1328	0.5999	3.4902	0.0326	0.5547	−0.0050	0.3720	0.0948	*0.6894*	*3.7344*
	(0.0435)	(0.0374)	(0.0014)	(0.0354)	(0.0313)			(0.0555)	(0.0534)	(0.0027)	(0.0464)	(0.0435)			(0.0727)	(0.0700)	(0.0023)	(0.0552)	(0.0472)		
900 sec	0.0224	0.5022	−0.0082	0.3885	0.1591	0.4898	4.2900	0.0727	0.5248	−0.0101	0.2562	0.2424	0.4303	4.1965	−0.0496	0.5993	−0.0093	0.4720	0.0527	0.5650	4.4102
	(0.0557)	(0.0368)	(0.0008)	(0.0388)	(0.0368)			(0.0719)	(0.0478)	(0.0012)	(0.0514)	(0.0511)			(0.0894)	(0.0633)	(0.0012)	(0.0610)	(0.0540)		
average	0.0250	0.5071	−0.0068	0.3851	0.1413	**0.6108**	**3.7747**	0.0295	0.5372	−0.0115	0.3374	0.1737	**0.5887**	**3.5621**	−0.0508	0.6605	−0.0093	0.3914	0.0660	0.6426	4.0275
	(0.0462)	(0.0370)	(0.0011)	(0.0365)	(0.0329)			(0.0568)	(0.0498)	(0.0020)	(0.0476)	(0.0447)			(0.0800)	(0.0698)	(0.0018)	0.0580)	(0.0510)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.0421	0.5196	−0.0155	0.3485	0.1416	*0.6631*	*3.6004*	0.0810	0.3953	−0.0108	0.3711	0.2066	*0.6682*	*3.2885*	−0.0361	0.7458	−0.0259	0.2685	0.0878	0.6646	3.9645
	(0.0413)	(0.0332)	(0.0022)	(0.0360)	(0.0308)			(0.0480)	(0.0411)	(0.0042)	(0.0467)	(0.0423)			(0.0744)	(0.0631)	(0.0035)	(0.0588)	(0.0479)		
300 sec	0.0285	0.4969	−0.0167	0.3687	0.1537	0.6516	3.6089	0.1024	0.3568	−0.0039	0.3720	0.2208	0.6474	3.2994	−0.0558	0.6840	−0.0273	0.3273	0.1076	0.6611	3.9730
	(0.0413)	(0.0324)	(0.0023)	(0.0363)	(0.0312)			(0.0485)	(0.0418)	(0.0046)	(0.0461)	(0.0416)			(0.0739)	(0.0599)	(0.0036)	(0.0594)	(0.0497)		
450 sec	0.0811	0.4364	−0.0089	0.3836	0.1537	0.6397	3.5968	0.1140	0.3546	−0.0071	0.3869	0.2076	0.5927	3.5083	0.0111	0.6124	−0.0167	0.3329	0.0984	*0.6933*	*3.7218*
	(0.0414)	(0.0333)	(0.0026)	(0.0356)	(0.0304)			(0.0530)	(0.0428)	(0.0041)	(0.0470)	(0.0426)			(0.0667)	(0.0579)	(0.0040)	(0.0558)	(0.0446)		
900 sec	0.0794	0.4507	−0.0113	0.3552	0.1874	0.4867	4.2960	0.1258	0.3984	−0.0098	0.2723	0.2868	0.4201	4.2141	0.0295	0.5424	−0.0140	0.4075	0.0945	0.5622	4.4166
	(0.0547)	(0.0347)	(0.0012)	(0.0395)	(0.0364)			(0.0719)	(0.0426)	(0.0015)	(0.0519)	(0.0509)			(0.0873)	(0.0598)	(0.0019)	(0.0630)	(0.0530)		
average	0.0578	0.4759	−0.0131	0.3640	0.1591	0.6103	3.7755	0.1058	0.3763	−0.0079	0.3506	0.2304	0.5821	3.5776	−0.0128	0.6462	−0.0209	0.3341	0.0971	**0.6453**	**4.0190**
	(0.0447)	(0.0334)	(0.0021)	(0.0369)	(0.0322)			(0.0553)	(0.0421)	(0.0036)	(0.0479)	(0.0444)			(0.0756)	(0.0602)	(0.0032)	(0.0592)	(0.0488)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.15: In-sample estimation (XOM)

HAR	Full							Pre-Crisis							Crisis						
	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.1696	0.3878		0.4791	0.0631	0.7012	4.2062	0.1314	0.3219		0.5309	0.0830	*0.7123*	*3.0103*	0.2243	0.4016		0.4685	0.0560	0.6926	4.9997
	(0.0496)	(0.0223)		(0.0337)	(0.0264)			(0.0425)	(0.0293)		(0.0437)	(0.0333)			(0.1142)	(0.0365)		(0.0552)	(0.0438)		
300 sec	0.1708	0.4533		0.3804	0.0929	0.6904	4.2535	0.1503	0.2957		0.5195	0.1073	0.6527	3.1009	0.2229	0.4875		0.3504	0.0872	0.6934	5.0322
	(0.0501)	(0.0221)		(0.0329)	(0.0262)			(0.0453)	(0.0293)		(0.0445)	(0.0354)			(0.1150)	(0.0359)		(0.0534)	(0.0427)		
450 sec	0.1558	0.5002		0.3634	0.0665	*0.7228*	*4.0309*	0.1748	0.2562		0.5188	0.1314	0.5797	3.2730	0.1863	0.5857		0.2976	0.0512	*0.7584*	*4.6602*
	(0.0449)	(0.0217)		(0.0314)	(0.0243)			(0.0501)	(0.0291)		(0.0454)	(0.0382)			(0.0954)	(0.0345)		(0.0489)	(0.0370)		
900 sec	0.1689	0.4641		0.3577	0.0990	0.6698	4.2847	0.1961	0.2377		0.4655	0.1845	0.4947	3.3780	0.2098	0.5323		0.3103	0.0822	0.7034	4.9777
	(0.0501)	(0.0220)		(0.0324)	(0.0262)			(0.0539)	(0.0290)		(0.0475)	(0.0425)			(0.1105)	(0.0355)		(0.0510)	(0.0403)		
average	0.1663	0.4513		0.3952	0.0803	0.6961	4.1938	0.1631	0.2779		0.5087	0.1266	0.6099	3.1905	0.2108	0.5018		0.3567	0.0692	0.7119	4.9174
	(0.0487)	(0.0220)		(0.0326)	(0.0258)			(0.0480)	(0.0292)		(0.0453)	(0.0374)			(0.1088)	(0.0356)		(0.0521)	(0.0409)		

HARQ	Full							Pre-Crisis							Crisis						
	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.0192	0.5368	−0.0030	0.4629	0.0222	0.7045	4.1956	0.0947	0.3681	−0.0027	0.5364	0.0637	*0.7125*	*3.0101*	−0.0810	0.6954	−0.0052	0.4349	−0.0210	0.7001	4.9758
	(0.0562)	(0.0348)	(0.0005)	(0.0336)	(0.0273)			(0.0487)	(0.0420)	(0.0018)	(0.0438)	(0.0356)			(0.1275)	(0.0677)	(0.0010)	(0.0549)	(0.0458)		
300 sec	0.1575	0.4652	−0.0003	0.3797	0.0898	0.6903	4.2542	0.1560	0.2885	0.0005	0.5190	0.1098	0.6525	3.1020	0.1312	0.5681	−0.0015	0.3443	0.0673	0.6939	5.0318
	(0.0570)	(0.0328)	(0.0005)	(0.0330)	(0.0270)			(0.0539)	(0.0468)	(0.0026)	(0.0446)	(0.0376)			(0.1291)	(0.0628)	(0.0010)	(0.0535)	(0.0445)		
450 sec	0.2027	0.4635	0.0010	0.3602	0.0767	*0.7230*	*4.0304*	0.1106	0.3392	−0.0062	0.5233	0.1051	0.5805	3.2718	0.1277	0.6324	−0.0011	0.2987	0.0387	*0.7584*	*4.6610*
	(0.0518)	(0.0297)	(0.0006)	(0.0314)	(0.0249)			(0.0594)	(0.0506)	(0.0031)	(0.0454)	(0.0404)			(0.1093)	(0.0548)	(0.0010)	(0.0489)	(0.0387)		
900 sec	0.2215	0.4199	0.0010	0.3522	0.1144	0.6702	4.2838	0.1508	0.2998	−0.0049	0.4646	0.1693	0.4950	3.3780	0.1645	0.5715	−0.0007	0.3124	0.0691	0.7034	4.9789
	(0.0561)	(0.0306)	(0.0005)	(0.0325)	(0.0272)			(0.0626)	(0.0524)	(0.0035)	(0.0475)	(0.0438)			(0.1225)	(0.0579)	(0.0009)	(0.0511)	(0.0432)		
average	0.1502	0.4714	−0.0003	0.3888	0.0758	0.6970	4.1910	0.1280	0.3239	−0.0033	0.5108	0.1120	0.6101	3.1905	0.0856	0.6169	−0.0021	0.3476	0.0386	0.7139	4.9119
	(0.0553)	(0.0320)	(0.0005)	(0.0326)	(0.0266)			(0.0562)	(0.0479)	(0.0027)	(0.0454)	(0.0393)			(0.1221)	(0.0608)	(0.0010)	(0.0521)	(0.0430)		

TV-HAR	Full							Pre-Crisis							Crisis						
	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.0571	0.5296	−0.0063	0.4320	0.0368	0.7048	4.1947	0.1599	0.2791	0.0074	0.5278	0.1041	*0.7126*	*3.0096*	0.0227	0.6645	−0.0101	0.3783	0.0111	0.7005	4.9745
	(0.0529)	(0.0330)	(0.0011)	(0.0344)	(0.0267)			(0.0454)	(0.0379)	(0.0042)	(0.0437)	(0.0354)			(0.1190)	(0.0616)	(0.0019)	(0.0571)	(0.0440)		
300 sec	0.1095	0.5222	−0.0032	0.3621	0.0803	0.6913	4.2510	0.1929	0.2319	0.0109	0.5193	0.1331	0.6537	3.0986	0.0803	0.6539	−0.0066	0.3033	0.0585	0.6970	5.0213
	(0.0541)	(0.0319)	(0.0011)	(0.0334)	(0.0265)			(0.0485)	(0.0392)	(0.0045)	(0.0445)	(0.0369)			(0.1210)	(0.0583)	(0.0018)	(0.0546)	(0.0432)		
450 sec	0.2486	0.4133	0.0052	0.3687	0.0886	*0.7249*	*4.0237*	0.1697	0.2638	−0.0012	0.5190	0.1283	0.5795	3.2741	0.2471	0.5256	0.0030	0.3050	0.0653	*0.7588*	*4.6594*
	(0.0490)	(0.0285)	(0.0011)	(0.0313)	(0.0246)			(0.0536)	(0.0403)	(0.0043)	(0.0454)	(0.0399)			(0.1021)	(0.0499)	(0.0018)	(0.0491)	(0.0379)		
900 sec	0.2278	0.4031	0.0030	0.3626	0.1155	0.6707	4.2823	0.2151	0.2066	0.0050	0.4661	0.1960	0.4947	3.3786	0.2108	0.5312	0.0000	0.3105	0.0825	0.7031	4.9797
	(0.0539)	(0.0304)	(0.0010)	(0.0324)	(0.0268)			(0.0570)	(0.0418)	(0.0048)	(0.0475)	(0.0439)			(0.1169)	(0.0550)	(0.0017)	(0.0514)	(0.0417)		
average	0.1608	0.4671	−0.0003	0.3814	0.0803	**0.6979**	**4.1879**	0.1844	0.2453	0.0055	0.5081	0.1404	**0.6102**	**3.1902**	0.1402	0.5938	−0.0034	0.3243	0.0543	**0.7149**	**4.9087**
	(0.0525)	(0.0309)	(0.0011)	(0.0329)	(0.0262)			(0.0511)	(0.0398)	(0.0045)	(0.0453)	(0.0390)			(0.1148)	(0.0562)	(0.0018)	(0.0531)	(0.0417)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.16: In-sample estimation (DD)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.3209	0.1952		0.4832	0.2304	0.5220	5.3559	0.1837	0.2460		0.3984	0.2892	*0.6000*	*4.2014*	0.5037	0.1813		0.5040	0.2068	0.4806	6.1389
	(0.0938)	(0.0231)		(0.0389)	(0.0347)			(0.0754)	(0.0287)		(0.0494)	(0.0433)			(0.2223)	(0.0384)		(0.0641)	(0.0581)		
300 sec	0.2869	0.2581		0.4584	0.1989	*0.5666*	*5.1318*	0.2009	0.2433		0.3925	0.2899	0.5679	4.3400	0.4267	0.2633		0.4765	0.1650	*0.5504*	*5.7993*
	(0.0839)	(0.0230)		(0.0373)	(0.0324)			(0.0790)	(0.0286)		(0.0496)	(0.0441)			(0.1887)	(0.0381)		(0.0604)	(0.0524)		
450 sec	0.3415	0.1625		0.5004	0.2343	0.4808	5.4215	0.2230	0.1429		0.4826	0.2903	0.5206	4.5118	0.5202	0.1687		0.5037	0.2090	0.4488	6.1333
	(0.0961)	(0.0234)		(0.0398)	(0.0359)			(0.0847)	(0.0291)		(0.0519)	(0.0466)			(0.2214)	(0.0387)		(0.0650)	(0.0594)		
900 sec	0.3295	0.1722		0.4456	0.2789	0.4636	5.3710	0.2352	0.1466		0.3808	0.3791	0.4525	4.5969	0.4929	0.1826		0.4602	0.2417	0.4498	6.0310
	(0.0943)	(0.0231)		(0.0398)	(0.0368)			(0.0898)	(0.0286)		(0.0543)	(0.0512)			(0.2122)	(0.0385)		(0.0641)	(0.0592)		
average	0.3197	0.1970		0.4719	0.2356	0.5082	5.3200	0.2107	0.1947		0.4136	0.3121	0.5353	4.4125	0.4859	0.1990		0.4861	0.2056	0.4824	6.0256
	(0.0920)	(0.0232)		(0.0389)	(0.0349)			(0.0822)	(0.0287)		(0.0513)	(0.0463)			(0.2111)	(0.0384)		(0.0634)	(0.0573)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.0027	0.6142	−0.0034	0.3342	0.1066	0.5655	5.2609	0.1461	0.4560	−0.0017	0.2836	0.2257	*0.6210*	*4.1479*	−0.2117	0.8319	−0.0052	0.3029	0.0353	0.5501	5.9962
	(0.0915)	(0.0335)	(0.0002)	(0.0382)	(0.0339)			(0.0735)	(0.0351)	(0.0002)	(0.0495)	(0.0426)			(0.2146)	(0.0631)	(0.0004)	(0.0618)	(0.0558)		
300 sec	0.0481	0.5304	−0.0036	0.3894	0.1157	*0.5845*	*5.0901*	0.1502	0.4254	−0.0023	0.3079	0.2333	0.5850	4.3003	0.1082	0.4603	−0.0060	0.4098	0.1263	*0.5792*	*5.1026*
	(0.0850)	(0.0336)	(0.0003)	(0.0370)	(0.0326)			(0.0776)	(0.0353)	(0.0003)	(0.0496)	(0.0437)			(0.0850)	(0.0317)	(0.0007)	(0.0371)	(0.0329)		
450 sec	0.1312	0.4573	−0.0024	0.4167	0.1232	0.5049	5.3742	0.1719	0.3090	−0.0023	0.4095	0.2387	0.5336	4.4850	0.1328	0.5774	−0.0029	0.3904	0.0663	0.4842	6.0680
	(0.0956)	(0.0341)	(0.0002)	(0.0395)	(0.0364)			(0.0839)	(0.0372)	(0.0003)	(0.0523)	(0.0466)			(0.2192)	(0.0616)	(0.0004)	(0.0644)	(0.0600)		
900 sec	0.1390	0.3978	−0.0033	0.4143	0.1870	0.4747	5.3504	0.1692	0.3178	−0.0034	0.3227	0.3204	0.4643	4.5757	0.1508	0.5014	−0.0042	0.4195	0.1208	0.4648	6.0043
	(0.0966)	(0.0372)	(0.0004)	(0.0396)	(0.0383)			(0.0894)	(0.0394)	(0.0005)	(0.0545)	(0.0515)			(0.2186)	(0.0702)	(0.0008)	(0.0636)	(0.0625)		
average	0.0802	0.4999	−0.0032	0.3887	0.1331	**0.5324**	**5.2689**	0.1594	0.3771	−0.0024	0.3309	0.2545	**0.5510**	**4.3772**	0.0450	0.5927	−0.0046	0.3807	0.0872	**0.5196**	**5.7928**
	(0.0922)	(0.0346)	(0.0003)	(0.0386)	(0.0353)			(0.0811)	(0.0368)	(0.0003)	(0.0515)	(0.0461)			(0.1843)	(0.0566)	(0.0006)	(0.0567)	(0.0528)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.0868	0.5093	−0.0053	0.3978	0.0958	0.5584	5.2771	0.1193	0.4781	−0.0111	0.3121	0.1927	*0.6151*	*4.1633*	0.1105	0.5903	−0.0060	0.3906	0.0471	0.5314	6.0370
	(0.0915)	(0.0305)	(0.0004)	(0.0378)	(0.0345)			(0.0743)	(0.0396)	(0.0013)	(0.0495)	(0.0440)			(0.2144)	(0.0534)	(0.0006)	(0.0618)	(0.0572)		
300 sec	0.1082	0.4603	−0.0060	0.4098	0.1263	*0.5792*	*5.1026*	0.1334	0.4250	−0.0095	0.3365	0.2137	0.5779	4.3173	0.1237	0.5362	−0.0069	0.4050	0.0818	*0.5688*	*5.7585*
	(0.0850)	(0.0317)	(0.0007)	(0.0371)	(0.0329)			(0.0788)	(0.0399)	(0.0015)	(0.0497)	(0.0451)			(0.1904)	(0.0556)	(0.0010)	(0.0601)	(0.0528)		
450 sec	0.1884	0.3941	−0.0039	0.4288	0.1422	0.5006	5.3829	0.1467	0.3616	−0.0101	0.4119	0.2009	0.5343	4.4833	0.2338	0.5082	−0.0047	0.3912	0.0884	0.4806	6.0750
	(0.0954)	(0.0318)	(0.0004)	(0.0396)	(0.0363)			(0.0842)	(0.0417)	(0.0014)	(0.0521)	(0.0476)			(0.2180)	(0.0571)	(0.0006)	(0.0647)	(0.0597)		
900 sec	0.2160	0.3385	−0.0041	0.4068	0.2091	0.4742	5.3513	0.1563	0.3688	−0.0112	0.3173	0.2858	0.4686	4.5675	0.3053	0.4125	−0.0047	0.3971	0.1567	0.4652	6.0036
	(0.0946)	(0.0319)	(0.0005)	(0.0397)	(0.0376)			(0.0891)	(0.0414)	(0.0015)	(0.0542)	(0.0520)			(0.2120)	(0.0566)	(0.0009)	(0.0642)	(0.0604)		
average	0.1499	0.4255	−0.0048	0.4108	0.1434	0.5281	5.2785	0.1389	0.4084	−0.0105	0.3444	0.2233	0.5490	4.3829	0.1933	0.5118	−0.0055	0.3960	0.0935	0.5115	5.9685
	(0.0916)	(0.0315)	(0.0005)	(0.0386)	(0.0353)			(0.0816)	(0.0407)	(0.0014)	(0.0514)	(0.0472)			(0.2087)	(0.0557)	(0.0008)	(0.0627)	(0.0575)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.17: In-sample estimation (VZ)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.1865	0.3436		0.3803	0.2155	*0.6561*	*4.5980*	0.1554	0.2208		0.4543	0.2715	*0.6524*	*4.1027*	0.2123	0.4154		0.3314	0.1883	0.6612	5.1070
	(0.0638)	(0.0224)		(0.0363)	(0.0309)			(0.0702)	(0.0289)		(0.0508)	(0.0436)			(0.1236)	(0.0363)		(0.0566)	(0.0481)		
300 sec	0.1826	0.3393		0.3585	0.2406	0.6403	4.5746	0.1686	0.2335		0.3798	0.3270	0.6005	4.2803	0.1941	0.4297		0.3155	0.1930	0.6741	4.9285
	(0.0633)	(0.0223)		(0.0370)	(0.0321)			(0.0761)	(0.0285)		(0.0518)	(0.0461)			(0.1139)	(0.0362)		(0.0564)	(0.0479)		
450 sec	0.1775	0.3019		0.3961	0.2384	0.6261	4.4940	0.1698	0.2089		0.3762	0.3509	0.5672	4.2410	0.1829	0.3925		0.3638	0.1819	*0.6743*	*4.8087*
	(0.0610)	(0.0226)		(0.0374)	(0.0324)			(0.0755)	(0.0284)		(0.0529)	(0.0481)			(0.1073)	(0.0372)		(0.0570)	(0.0472)		
900 sec	0.2031	0.2610		0.3807	0.2833	0.5570	4.6715	0.2095	0.1947		0.3083	0.4156	0.4751	4.5137	0.1987	0.3406		0.3892	0.2016	0.6310	4.8846
	(0.0670)	(0.0227)		(0.0388)	(0.0350)			(0.0864)	(0.0282)		(0.0540)	(0.0514)			(0.1126)	(0.0376)		(0.0590)	(0.0500)		
average	0.1874	0.3114		0.3789	0.2445	0.6199	4.5845	0.1758	0.2145		0.3797	0.3413	0.5738	4.2844	0.1970	0.3946		0.3500	0.1912	0.6602	4.9322
	(0.0638)	(0.0225)		(0.0374)	(0.0326)			(0.0770)	(0.0285)		(0.0524)	(0.0473)			(0.1144)	(0.0368)		(0.0572)	(0.0483)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.0831	0.4475	−0.0021	0.3670	0.1843	*0.6576*	*4.5938*	0.0377	0.3422	−0.0042	0.4493	0.2347	*0.6548*	*4.0964*	−0.0633	0.7216	−0.0048	0.2739	0.1104	0.6687	5.0858
	(0.0697)	(0.0362)	(0.0006)	(0.0364)	(0.0320)			(0.0773)	(0.0444)	(0.0012)	(0.0507)	(0.0446)			(0.1348)	(0.0726)	(0.0010)	(0.0572)	(0.0502)		
300 sec	0.1630	0.3570	−0.0005	0.3576	0.2355	0.6402	4.5752	0.0895	0.3188	−0.0031	0.3730	0.3038	0.6015	4.2784	0.0930	0.5268	−0.0020	0.3044	0.1690	0.6749	4.9272
	(0.0698)	(0.0347)	(0.0007)	(0.0370)	(0.0330)			(0.0835)	(0.0468)	(0.0013)	(0.0518)	(0.0472)			(0.1264)	(0.0641)	(0.0011)	(0.0567)	(0.0496)		
450 sec	0.0753	0.3919	−0.0028	0.3992	0.2095	0.6273	4.4911	0.0825	0.3086	−0.0040	0.3698	0.3225	0.5689	4.2377	−0.1021	0.6514	−0.0066	0.3591	0.1096	*0.6802*	*4.7914*
	(0.0690)	(0.0364)	(0.0009)	(0.0374)	(0.0336)			(0.0817)	(0.0458)	(0.0014)	(0.0528)	(0.0491)			(0.1244)	(0.0692)	(0.0015)	(0.0565)	(0.0496)		
900 sec	0.1376	0.3163	−0.0022	0.3871	0.2653	0.5574	4.6711	0.0809	0.3408	−0.0071	0.3100	0.3706	0.4780	4.5086	0.0084	0.5063	−0.0053	0.3990	0.1536	0.6334	4.8791
	(0.0762)	(0.0381)	(0.0012)	(0.0390)	(0.0363)			(0.0946)	(0.0527)	(0.0022)	(0.0538)	(0.0530)			(0.1320)	(0.0712)	(0.0019)	(0.0589)	(0.0528)		
average	0.1148	0.3782	−0.0019	0.3777	0.2237	0.6206	4.5828	0.0727	0.3276	−0.0046	0.3755	0.3079	**0.5758**	**4.2803**	−0.0160	0.6015	−0.0047	0.3341	0.1357	0.6643	4.9209
	(0.0712)	(0.0363)	(0.0008)	(0.0375)	(0.0338)			(0.0843)	(0.0474)	(0.0015)	(0.0523)	(0.0485)			(0.1294)	(0.0693)	(0.0014)	(0.0573)	(0.0505)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.1205	0.4167	−0.0042	0.3652	0.1934	*0.6573*	*4.5946*	0.1308	0.2555	−0.0042	0.4510	0.2579	*0.6526*	*4.1029*	0.0231	0.6443	−0.0100	0.2698	0.1347	0.6686	5.0860
	(0.0667)	(0.0313)	(0.0013)	(0.0365)	(0.0316)			(0.0727)	(0.0393)	(0.0033)	(0.0509)	(0.0448)			(0.1284)	(0.0594)	(0.0021)	(0.0574)	(0.0489)		
300 sec	0.1258	0.4003	−0.0042	0.3502	0.2214	0.6412	4.5724	0.1387	0.2819	−0.0052	0.3728	0.3090	0.6009	4.2799	0.0312	0.6130	−0.0097	0.2780	0.1482	0.6797	4.9122
	(0.0663)	(0.0310)	(0.0015)	(0.0370)	(0.0327)			(0.0782)	(0.0410)	(0.0032)	(0.0520)	(0.0474)			(0.1191)	(0.0556)	(0.0023)	(0.0566)	(0.0486)		
450 sec	0.0999	0.3791	−0.0060	0.3885	0.2162	0.6277	4.4902	0.1585	0.2280	−0.0022	0.3737	0.3439	0.5670	4.2419	−0.0499	0.6315	−0.0142	0.3191	0.1342	*0.6838*	*4.7800*
	(0.0648)	(0.0314)	(0.0017)	(0.0374)	(0.0330)			(0.0779)	(0.0428)	(0.0036)	(0.0531)	(0.0495)			(0.1137)	(0.0563)	(0.0025)	(0.0567)	(0.0473)		
900 sec	0.1179	0.3536	−0.0075	0.3737	0.2560	0.5591	4.6671	0.1564	0.2847	−0.0089	0.3001	0.3805	0.4769	4.5107	0.0220	0.5322	−0.0125	0.3602	0.1604	0.6375	4.8678
	(0.0706)	(0.0333)	(0.0020)	(0.0388)	(0.0356)			(0.0885)	(0.0440)	(0.0034)	(0.0540)	(0.0530)			(0.1188)	(0.0576)	(0.0029)	(0.0589)	(0.0504)		
average	0.1160	0.3874	−0.0055	0.3694	0.2218	**0.6213**	**4.5811**	0.1461	0.2625	−0.0051	0.3744	0.3228	0.5744	4.2839	0.0066	0.6052	−0.0116	0.3068	0.1444	**0.6674**	**4.9115**
	(0.0671)	(0.0318)	(0.0016)	(0.0375)	(0.0332)			(0.0793)	(0.0418)	(0.0034)	(0.0525)	(0.0487)			(0.1200)	(0.0572)	(0.0024)	(0.0574)	(0.0488)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Table 3.18: In-sample estimation (PEF)

	Full							Pre-Crisis							Crisis						
HAR	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC	β	β_d		β_w	β_m	R^2	AIC
150 sec	0.3228	0.1686		0.3547	0.3662	0.4091	5.1306	0.2903	0.2542		0.2400	0.4014	*0.4374*	*4.6459*	0.3613	0.1091		0.4405	0.3348	0.3918	5.6384
	(0.0897)	(0.0228)		(0.0427)	(0.0407)			(0.0989)	(0.0280)		(0.0511)	(0.0488)			(0.1719)	(0.0381)		(0.0727)	(0.0695)		
300 sec	0.2903	0.2455		0.2939	0.3551	0.4350	4.8899	0.3566	0.2417		0.2360	0.3895	0.3551	4.8172	0.2469	0.2491		0.3646	0.3018	0.5206	5.0068
	(0.0805)	(0.0225)		(0.0407)	(0.0389)			(0.1083)	(0.0279)		(0.0518)	(0.0517)			(0.1271)	(0.0376)		(0.0662)	(0.0604)		
450 sec	0.2737	0.2681		0.2761	0.3511	*0.4413*	*4.7612*	0.3293	0.2159		0.2735	0.3814	0.3618	4.6681	0.2364	0.3370		0.2687	0.3090	*0.5253*	*4.9015*
	(0.0759)	(0.0223)		(0.0403)	(0.0386)			(0.1014)	(0.0281)		(0.0516)	(0.0511)			(0.1209)	(0.0365)		(0.0648)	(0.0601)		
900 sec	0.3364	0.1836		0.2725	0.4112	0.3347	5.0473	0.4561	0.1625		0.2008	0.4536	0.2279	5.0364	0.2587	0.2182		0.3553	0.3295	0.4640	5.0621
	(0.0874)	(0.0225)		(0.0431)	(0.0434)			(0.1242)	(0.0280)		(0.0559)	(0.0603)			(0.1295)	(0.0375)		(0.0680)	(0.0638)		
average	0.3058	0.2165		0.2993	0.3709	0.4050	4.9572	0.3581	0.2186		0.2376	0.4065	0.3456	4.7919	0.2758	0.2283		0.3573	0.3188	0.4754	5.1522
	(0.0834)	(0.0225)		(0.0417)	(0.0404)			(0.1082)	(0.0280)		(0.0526)	(0.0530)			(0.1373)	(0.0374)		(0.0679)	(0.0635)		

	Full							Pre-Crisis							Crisis						
HARQ	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC	β	γ	β_q	β_w	β_m	R^2	AIC
150 sec	0.0632	0.5976	−0.0032	0.2506	0.1753	0.4598	5.0413	0.1683	0.5253	−0.0038	0.1572	0.2999	*0.4573*	*4.6104*	−0.1190	0.8711	−0.0043	0.2632	−0.0092	0.4819	5.4790
	(0.0873)	(0.0344)	(0.0002)	(0.0413)	(0.0407)			(0.0983)	(0.0435)	(0.0005)	(0.0513)	(0.0495)			(0.1627)	(0.0674)	(0.0003)	(0.0685)	(0.0692)		
300 sec	0.1071	0.5713	−0.0042	0.1942	0.2465	*0.4676*	*4.8308*	0.2448	0.5135	−0.0033	0.1434	0.2906	0.3841	4.7718	−0.0866	0.7279	−0.0066	0.2447	0.1482	0.5659	4.9084
	(0.0794)	(0.0332)	(0.0003)	(0.0402)	(0.0387)			(0.1066)	(0.0405)	(0.0004)	(0.0516)	(0.0517)			(0.1252)	(0.0587)	(0.0006)	(0.0641)	(0.0594)		
450 sec	0.0970	0.5098	−0.0041	0.2366	0.2614	0.4660	4.7164	0.1715	0.5207	−0.0075	0.1858	0.2959	0.3859	4.6302	−0.0755	0.7923	−0.0054	0.2083	0.1071	*0.5827*	*4.7736*
	(0.0759)	(0.0305)	(0.0004)	(0.0395)	(0.0386)			(0.1012)	(0.0459)	(0.0009)	(0.0517)	(0.0512)			(0.1164)	(0.0516)	(0.0005)	(0.0610)	(0.0589)		
900 sec	0.1221	0.5313	−0.0065	0.1983	0.2907	0.3675	4.9972	0.3321	0.4486	−0.0054	0.1288	0.3440	0.2556	5.0005	−0.1013	0.7243	−0.0091	0.2668	0.1567	0.5119	4.9694
	(0.0871)	(0.0364)	(0.0005)	(0.0425)	(0.0435)			(0.1229)	(0.0448)	(0.0007)	(0.0556)	(0.0608)			(0.1287)	(0.0621)	(0.0009)	(0.0655)	(0.0633)		
average	0.0974	0.5525	−0.0045	0.2199	0.2435	**0.4402**	**4.8964**	0.2292	0.5020	−0.0050	0.1538	0.3076	**0.3707**	**4.7532**	−0.0956	0.7789	−0.0063	0.2458	0.1007	**0.5356**	**5.0326**
	(0.0824)	(0.0337)	(0.0004)	(0.0409)	(0.0404)			(0.1073)	(0.0437)	(0.0006)	(0.0525)	(0.0533)			(0.1332)	(0.0599)	(0.0006)	(0.0648)	(0.0627)		

	Full							Pre-Crisis							Crisis						
TV-HAR	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC	β	γ	α	β_w	β_m	R^2	AIC
150 sec	0.1220	0.5199	−0.0062	0.2603	0.2042	*0.4557*	*5.0488*	0.2116	0.4745	−0.0086	0.1586	0.3172	0.4545	4.6156	−0.0142	0.7287	−0.0079	0.2829	0.0467	0.4733	5.4954
	(0.0871)	(0.0316)	(0.0004)	(0.0414)	(0.0405)			(0.0979)	(0.0405)	(0.0012)	(0.0515)	(0.0493)			(0.1627)	(0.0609)	(0.0006)	(0.0689)	(0.0687)		
300 sec	0.1084	0.5629	−0.0110	0.1961	0.2442	*0.4681*	*4.8299*	0.2523	0.4892	−0.0090	0.1473	0.3020	0.3794	4.7794	−0.0430	0.6954	−0.0145	0.2459	0.1397	*0.5700*	*4.8990*
	(0.0794)	(0.0326)	(0.0008)	(0.0402)	(0.0387)			(0.1070)	(0.0405)	(0.0011)	(0.0519)	(0.0518)			(0.1234)	(0.0546)	(0.0013)	(0.0637)	(0.0591)		
450 sec	0.0967	0.5251	−0.0133	0.2062	0.2794	0.4580	4.7311	0.1762	0.4833	−0.0174	0.1945	0.3092	0.3788	4.6418	−0.0363	0.7478	−0.0168	0.1532	0.1858	0.5563	4.8349
	(0.0772)	(0.0354)	(0.0014)	(0.0404)	(0.0388)			(0.1024)	(0.0474)	(0.0025)	(0.0522)	(0.0515)			(0.1212)	(0.0601)	(0.0020)	(0.0641)	(0.0599)		
900 sec	0.1934	0.4595	−0.0104	0.2005	0.3099	0.3606	5.0079	0.3597	0.3983	−0.0091	0.1351	0.3616	0.2493	5.0089	0.0553	0.5710	−0.0128	0.2673	0.2025	0.4987	4.9961
	(0.0868)	(0.0341)	(0.0010)	(0.0428)	(0.0436)			(0.1233)	(0.0432)	(0.0013)	(0.0559)	(0.0609)			(0.1275)	(0.0555)	(0.0015)	(0.0666)	(0.0636)		
average	0.1301	0.5169	−0.0102	0.2158	0.2594	0.4356	4.9045	0.2500	0.4613	−0.0110	0.1589	0.3225	0.3655	4.7614	−0.0095	0.6857	−0.0130	0.2373	0.1437	0.5246	5.0563
	(0.0826)	(0.0334)	(0.0009)	(0.0412)	(0.0404)			(0.1077)	(0.0429)	(0.0015)	(0.0529)	(0.0534)			(0.1337)	(0.0578)	(0.0014)	(0.0658)	(0.0628)		

Notes: The table provides in-sample parameter estimates and measure of fit for the standard HAR, the HARQ and the TV-HAR models. The standard errors are in parentheses. R^2 stands for the adjusted R-squared. AIC denotes the Akaike information criterion. The subsample of the pre-crisis period is from 2000 to 2006. The subsample of the crisis period is from 2007 to 2010. The numbers in bold represent that this model fits the data better than the other two models for the average of different frequencies. The numbers in italic represent that for one specific model, which sampling frequency can offer the better fit than other sampling frequencies.

Individual stocks out-of-sample forecast results

Table 3.19: Out-of-sample forecasts (MMM)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.1365	0.9294	0.9678	1.0105	1.0110	0.4023	0.4449	0.4554	0.5141	0.4542	1.6893	1.2942	1.3536	1.3843	1.4303
HARQ	**1.0181**	0.9240	0.9848	1.0165	0.9859	**0.3936**	**0.4365**	**0.4410**	**0.5035**	**0.4436**	1.4884	1.2911	1.3944	1.4028	1.3942
TV-HAR	1.0208	**0.9188**	**0.9418**	**1.0003**	**0.9704**	0.4002	0.4417	0.4528	0.5103	0.4513	**1.4881**	**1.2781**	**1.3101**	**1.3692**	**1.3614**
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	37.2711	10.8604	15.8844	10.5021	18.6295	**0.3933**	**0.5158**	**0.5543**	0.7763	**0.5599**	65.0393	18.6497	27.4276	17.8253	32.2355
HARQ	**32.3932**	10.4042	22.3326	10.8672	18.9993	0.3990	0.5318	0.5583	**0.7743**	0.5658	**56.4841**	17.8379	38.7281	18.4669	32.8793
TV-HAR	32.4450	**9.5722**	**14.7722**	**9.7000**	**16.6224**	0.3955	0.5195	0.5560	0.7814	0.5631	56.5776	**16.3887**	**25.4766**	**16.4155**	**28.7146**
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	**0.8959**	0.9942	1.0176	1.0060	0.9784	**0.9783**	**0.9811**	**0.9683**	**0.9792**	**0.9767**	0.8811	0.9976	1.0301	1.0134	0.9806
TV-HAR	0.8982	**0.9886**	**0.9732**	**0.9899**	**0.9625**	0.9948	0.9928	0.9943	0.9925	0.9936	**0.8809**	**0.9875**	**0.9678**	**0.9892**	**0.9564**
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	**1.0000**	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	**0.8691**	0.9580	1.4059	1.0348	1.0670	1.0143	1.0311	1.0071	**0.9973**	1.0124	**0.8685**	0.9565	1.4120	1.0360	1.0682
TV-HAR	0.8705	**0.8814**	**0.9300**	**0.9236**	**0.9014**	1.0055	1.0073	1.0030	1.0065	1.0056	0.8699	**0.8788**	**0.9289**	**0.9209**	**0.8996**
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	**10.4119**	0.5801	-1.7612	-0.5958	2.1587	**2.1727**	**1.8884**	**3.1687**	**2.0776**	**2.3269**	11.8893	0.2414	-3.0101	-1.3434	1.9443
TV-HAR	10.1771	**1.1392**	**2.6804**	**1.0112**	**3.7520**	0.5158	0.7192	0.5694	0.7516	0.6390	**11.9095**	**1.2479**	**3.2152**	**1.0838**	**4.3641**
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	**0.0000**	**0.0000**	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	**13.0878**	4.2005	-40.5942	-3.4766	-6.6956	-1.4262	-3.1100	-0.7097	**0.2680**	-1.2445	**13.1539**	4.3528	-41.2011	-3.5994	-6.8235
TV-HAR	12.9486	**11.8615**	**7.0020**	**7.6370**	**9.8623**	-0.5542	-0.7300	-0.3016	-0.6491	-0.5587	13.0101	**12.1237**	**7.1132**	**7.9088**	**10.0390**

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.20: Out-of-sample forecasts (AMZN)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	2.3162	2.5051	2.6659	3.0013	2.6221	1.3762	1.4652	1.6102	1.8473	1.5747	3.0240	3.2881	3.4608	3.8703	3.4108
HARQ	2.2981	**2.4718**	**2.5797**	**2.9521**	**2.5754**	**1.3112**	**1.4088**	**1.5186**	**1.7526**	**1.4978**	3.0411	**3.2721**	**3.3786**	3.8554	**3.3868**
TV-HAR	**2.2938**	2.5102	2.6196	2.9711	2.5987	1.3449	1.4406	1.5761	1.8069	1.5421	**3.0082**	3.3156	3.4053	**3.8478**	3.3942
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	32.5901	36.5768	43.4159	53.4816	41.5161	3.5172	3.9143	4.6656	6.3905	4.6219	54.4813	61.1709	72.5940	88.9401	69.2966
HARQ	32.8004	**36.0394**	40.4686	52.8421	40.5376	**3.3854**	**3.7767**	**4.4283**	**6.2293**	**4.4549**	54.9492	**60.3324**	67.6061	87.9405	67.7071
TV-HAR	**30.2987**	37.4240	**39.3789**	**52.6757**	**39.9444**	3.4666	3.8681	4.5926	6.3338	4.5653	**50.5027**	62.6908	**65.5722**	**87.5701**	**66.5840**
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.9922	**0.9867**	**0.9676**	**0.9836**	**0.9825**	**0.9527**	**0.9615**	**0.9431**	**0.9487**	**0.9515**	1.0057	**0.9951**	**0.9762**	0.9962	**0.9933**
TV-HAR	**0.9903**	1.0021	0.9826	0.9900	0.9912	0.9772	0.9832	0.9788	0.9781	0.9793	**0.9948**	1.0084	0.9839	**0.9942**	0.9953
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	1.0065	**0.9853**	0.9321	0.9880	0.9780	**0.9625**	**0.9648**	**0.9491**	**0.9748**	**0.9628**	1.0086	**0.9863**	0.9313	0.9888	0.9787
TV-HAR	**0.9297**	1.0232	**0.9070**	**0.9849**	**0.9612**	0.9856	0.9882	0.9843	0.9911	0.9873	**0.9270**	1.0248	**0.9033**	**0.9846**	**0.9599**
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	0.7845	**1.3299**	**3.2359**	**1.6384**	**1.7472**	**4.7259**	**3.8473**	**5.6918**	**5.1262**	**4.8478**	-0.5661	**0.4852**	**2.3756**	0.3849	**0.6699**
TV-HAR	**0.9702**	-0.2055	1.7379	1.0048	0.8769	2.2796	1.6797	2.1173	2.1857	2.0656	**0.5215**	-0.8381	1.6051	**0.5805**	0.4672
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	-0.6452	**1.4694**	6.7885	1.1957	2.2021	**3.7469**	**3.5163**	**5.0862**	**2.5229**	**3.7181**	-0.8587	**1.3707**	6.8709	1.1239	2.1267
TV-HAR	**7.0308**	-2.3161	**9.2984**	**1.5069**	**3.8800**	1.4370	1.1811	1.5657	0.8869	1.2677	**7.3027**	-2.4846	**9.6726**	**1.5404**	**4.0078**

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.21: Out-of-sample forecasts (MRK)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.8267	1.8758	1.6712	1.6364	1.7525	0.9847	1.0170	1.1030	1.0871	1.0479	2.4606	2.5225	2.0990	2.0501	2.2830
HARQ	**1.7635**	1.8015	1.6192	1.5912	1.6938	0.9545	1.0070	1.0727	1.0806	1.0287	**2.3727**	**2.3997**	2.0307	1.9756	**2.1947**
TV-HAR	1.7655	**1.7971**	**1.6155**	**1.5848**	**1.6907**	**0.9450**	**0.9921**	**1.0654**	**1.0761**	**1.0197**	2.3832	2.4033	**2.0296**	**1.9678**	2.1960
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	55.5580	71.9409	33.1289	17.0851	44.4282	6.5013	6.9400	8.3676	7.2428	7.2629	92.4966	120.8851	51.7736	24.4961	72.4128
HARQ	48.5127	47.5582	30.9971	16.4167	35.8712	6.3279	7.0083	8.3117	7.2633	7.2278	80.2768	78.0913	48.0787	23.3090	57.4389
TV-HAR	**44.7759**	**46.6737**	**30.8540**	**16.2452**	**34.6372**	**6.3137**	**6.8971**	**8.2362**	**7.2322**	**7.1698**	**73.7370**	**76.6246**	**47.8846**	**23.0318**	**55.3195**
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	**0.9654**	0.9604	0.9689	0.9723	0.9668	0.9693	0.9901	0.9726	0.9941	0.9815	**0.9643**	**0.9513**	0.9675	0.9637	**0.9617**
TV-HAR	0.9665	**0.9580**	**0.9667**	**0.9685**	**0.9649**	**0.9597**	**0.9755**	**0.9660**	**0.9900**	**0.9728**	0.9685	0.9527	**0.9669**	**0.9599**	0.9620
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	0.8732	0.6611	0.9357	0.9609	0.8577	0.9733	1.0098	0.9933	1.0028	0.9948	0.8679	0.6460	0.9286	0.9515	0.8485
TV-HAR	**0.8059**	**0.6488**	**0.9313**	**0.9508**	**0.8342**	**0.9711**	**0.9938**	**0.9843**	**0.9985**	**0.9869**	**0.7972**	**0.6339**	**0.9249**	**0.9402**	**0.8240**
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	**3.4577**	3.9638	3.1091	2.7662	3.3242	3.0689	0.9877	2.7439	0.5937	1.8485	**3.5749**	**4.8673**	3.2536	3.6336	**3.8324**
TV-HAR	3.3504	**4.1967**	**3.3338**	**3.1533**	**3.5086**	**4.0316**	**2.4498**	**3.4035**	**1.0043**	**2.7223**	3.1452	4.7270	**3.3063**	**4.0113**	3.7974
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	12.6811	33.8927	6.4349	3.9121	14.2302	2.6665	-0.9841	0.6680	-0.2824	0.5170	13.2111	35.4004	7.1367	4.8460	15.1486
TV-HAR	19.4070	35.1222	6.8670	4.9160	16.5780	2.8853	0.6182	1.5706	0.1470	1.3053	20.2814	36.6137	7.5115	5.9777	17.5961

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.22: Out-of-sample forecasts (BA)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.1250	**1.1709**	**1.2212**	1.4407	**1.2395**	0.5604	0.6203	0.6781	**0.8011**	0.6650	**1.5502**	**1.5855**	**1.6302**	1.9223	**1.6721**
HARQ	1.1309	1.1926	1.2487	**1.4325**	1.2512	**0.5385**	**0.6002**	**0.6505**	0.7887	**0.6444**	1.5770	1.6387	1.6992	**1.9172**	1.7080
TV-HAR	**1.1175**	1.1876	1.2407	1.4471	1.2482	0.5426	0.6176	0.6678	0.8037	0.6579	1.5504	1.6167	1.6720	1.9316	1.6927
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	8.0374	7.8581	**8.2595**	11.7079	**8.9657**	0.5771	0.7361	0.8235	1.1778	0.8286	13.6549	13.2209	**13.8587**	19.6368	**15.0928**
HARQ	7.8226	7.8598	10.4614	**11.6793**	9.4558	0.5710	0.7339	**0.8082**	**1.1739**	**0.8217**	13.2830	13.2254	17.7300	**19.5896**	15.9570
TV-HAR	**7.6069**	**7.7509**	8.8926	11.7697	9.0050	**0.5699**	**0.7337**	0.8169	1.1794	0.8250	**12.9056**	**13.0347**	14.9734	19.7439	15.1644
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	**1.0000**	**1.0000**	1.0000	**1.0000**	1.0000	1.0000	1.0000	**1.0000**	1.0000	**1.0000**	**1.0000**	**1.0000**	1.0000	**1.0000**
HARQ	1.0052	1.0185	1.0225	**0.9943**	1.0101	**0.9610**	**0.9676**	**0.9593**	0.9844	**0.9681**	1.0173	1.0336	1.0423	**0.9974**	1.0226
TV-HAR	**0.9933**	1.0142	1.0159	1.0044	1.0070	0.9684	0.9956	0.9849	1.0032	0.9880	1.0001	1.0197	1.0256	1.0048	1.0126
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	**1.0000**	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	1.0000	**1.0000**
HARQ	0.9733	1.0002	1.2666	**0.9976**	1.0594	0.9894	0.9970	**0.9814**	**0.9967**	**0.9911**	0.9728	1.0003	1.2793	**0.9976**	1.0625
TV-HAR	**0.9464**	**0.9864**	1.0766	1.0053	1.0037	**0.9877**	**0.9968**	0.9920	1.0014	0.9945	**0.9451**	**0.9859**	1.0804	1.0055	1.0042
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	**0.0000**	**0.0000**	0.0000	**0.0000**	0.0000	0.0000	0.0000	**0.0000**	0.0000	**0.0000**	**0.0000**	**0.0000**	0.0000	**0.0000**
HARQ	-0.5237	-1.8549	-2.2506	**0.5721**	-1.0143	**3.9038**	**3.2439**	**4.0710**	1.5550	**3.1934**	-1.7287	-3.3570	-4.2304	**0.2636**	-2.2631
TV-HAR	**0.6674**	-1.4230	-1.5913	-0.4442	-0.6978	3.1624	0.4367	1.5149	-0.3232	1.1977	-0.0117	-1.9709	-2.5641	-0.4822	-1.2572
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	**0.0000**	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0000	**0.0000**
HARQ	2.6723	-0.0207	-26.6584	**0.2442**	-5.9407	1.0552	0.2992	**1.8632**	**0.3284**	**0.8865**	2.7237	-0.0341	-27.9345	**0.2404**	-6.2511
TV-HAR	**5.3558**	**1.3642**	-7.6649	-0.5281	-0.3683	**1.2315**	**0.3206**	0.7955	-0.1374	0.5526	**5.4870**	**1.4079**	-8.0435	-0.5457	-0.4236

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.23: Out-of-sample forecasts (MSFT)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.9398	0.9413	**0.9622**	1.0360	0.9698	0.3760	0.4095	0.4237	0.4412	0.4126	1.3643	**1.3417**	**1.3677**	**1.4838**	**1.3894**
HARQ	0.9352	0.9568	0.9739	1.0371	0.9757	**0.3487**	0.3879	0.3977	**0.4145**	0.3872	1.3769	1.3851	1.4077	1.5059	1.4189
TV-HAR	**0.9269**	**0.9407**	0.9697	**1.0298**	**0.9668**	0.3539	**0.3822**	**0.3911**	0.4198	**0.3868**	**1.3584**	1.3612	1.4053	1.4891	1.4035

	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	7.8400	6.4018	**5.7300**	6.6837	6.6639	0.2429	0.2944	0.3027	0.3297	0.2924	13.5605	11.0005	9.8165	11.4681	11.4614
HARQ	9.1598	7.5335	6.2997	6.6170	7.4025	0.2323	0.2844	0.2814	**0.3056**	0.2759	15.8820	12.9919	10.8314	11.3693	12.7687
TV-HAR	**7.2368**	**6.2084**	6.0562	**6.5961**	**6.5244**	**0.2321**	**0.2776**	**0.2776**	0.3077	**0.2737**	**12.5113**	**10.6741**	**10.4073**	**11.3311**	**11.2309**

	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**
HARQ	0.9952	1.0165	1.0122	1.0010	1.0062	**0.9275**	0.9474	0.9388	**0.9393**	0.9383	1.0092	1.0323	1.0293	1.0148	1.0214
TV-HAR	**0.9863**	**0.9993**	1.0078	**0.9941**	**0.9969**	0.9411	**0.9335**	**0.9232**	0.9515	**0.9373**	**0.9957**	1.0145	1.0275	1.0036	1.0103

	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	1.0000	1.0000
HARQ	1.1683	1.1768	1.0994	0.9900	1.1086	0.9565	0.9661	0.9296	**0.9270**	0.9448	1.1712	1.1810	1.1034	0.9914	1.1117
TV-HAR	**0.9231**	**0.9698**	1.0569	**0.9869**	**0.9842**	**0.9555**	**0.9429**	**0.9169**	0.9332	**0.9371**	**0.9226**	**0.9703**	1.0602	**0.9881**	**0.9853**

	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	**0.0000**	**0.0000**	**0.0000**
HARQ	0.4825	-1.6457	-1.2182	-0.1031	-0.6211	**7.2496**	5.2578	6.1181	**6.0678**	6.1733	-0.9218	-3.2320	-2.9294	-1.4848	-2.1420
TV-HAR	**1.3699**	**0.0661**	-0.7808	**0.5941**	**0.3124**	5.8887	**6.6544**	**7.6793**	4.8466	**6.2673**	**0.4322**	-1.4477	-2.7541	-0.3580	-1.0319

	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0000	0.0000
HARQ	-16.8340	-17.6784	-9.9433	0.9979	-10.8645	4.3479	3.3916	7.0450	**7.3045**	5.5222	-17.1196	-18.1030	-10.3378	0.8614	-11.1748
TV-HAR	**7.6937**	**3.0214**	-5.6932	**1.3109**	**1.5832**	**4.4488**	**5.7078**	**8.3082**	6.6797	**6.2861**	**7.7375**	**2.9673**	-6.0183	**1.1947**	**1.4703**

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.24: Out-of-sample forecasts (KO)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.5974	0.6036	0.5704	0.6292	0.6002	0.3623	0.3467	0.3556	0.3869	0.3629	0.7745	0.7970	**0.7322**	**0.8117**	0.7789
HARQ	**0.5810**	0.6083	**0.5670**	**0.6290**	**0.5963**	**0.3376**	**0.3329**	**0.3358**	**0.3786**	**0.3462**	0.7643	0.8156	0.7410	0.8175	0.7846
TV-HAR	0.5858	**0.5992**	0.5736	0.6316	0.5975	0.3529	0.3437	0.3508	0.3839	0.3578	**0.7612**	**0.7916**	0.7413	0.8181	**0.7781**
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	2.3054	2.2310	**1.7567**	2.0232	2.0791	0.7204	0.4757	0.5448	0.5755	0.5791	3.4988	3.5526	**2.6692**	**3.1133**	3.2085
HARQ	2.2349	2.1587	1.8021	2.0227	2.0546	**0.6787**	**0.4532**	**0.5151**	**0.5625**	**0.5524**	3.4067	3.4429	2.7712	3.1222	3.1858
TV-HAR	**2.2169**	**2.1376**	1.8032	**2.0220**	**2.0449**	0.7138	0.4727	0.5390	0.5634	0.5722	**3.3487**	**3.3911**	2.7552	3.1202	**3.1538**
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	1.0000
HARQ	**0.9725**	1.0078	**0.9939**	**0.9996**	**0.9934**	**0.9317**	**0.9603**	**0.9443**	**0.9786**	**0.9537**	0.9868	1.0233	1.0120	1.0072	1.0073
TV-HAR	0.9806	**0.9928**	1.0054	1.0037	0.9956	0.9740	0.9915	0.9864	0.9923	0.9861	**0.9829**	**0.9932**	1.0124	1.0078	**0.9991**
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	1.0000
HARQ	0.9694	0.9676	1.0259	0.9997	0.9907	**0.9420**	**0.9526**	**0.9454**	**0.9775**	**0.9544**	0.9737	0.9691	1.0382	1.0028	0.9960
TV-HAR	**0.9616**	**0.9581**	1.0265	**0.9994**	**0.9864**	0.9908	0.9937	0.9893	0.9790	0.9882	**0.9571**	**0.9545**	1.0322	1.0022	**0.9865**
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	**0.0000**	0.0000
HARQ	**2.7508**	-0.7766	**0.6122**	**0.0364**	**0.6557**	**6.8251**	**3.9729**	**5.5707**	**2.1357**	**4.6261**	1.3155	-2.3320	-1.2010	-0.7169	-0.7336
TV-HAR	1.9404	**0.7200**	-0.5446	-0.3719	0.4360	2.5968	0.8542	1.3570	0.7665	1.3936	**1.7092**	**0.6760**	-1.2399	-0.7805	**0.0912**
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	**0.0000**	0.0000
HARQ	3.0569	3.2390	-2.5864	0.0259	0.9338	**5.7994**	**4.7371**	**5.4591**	**2.2510**	**4.5616**	2.6317	3.0879	-3.8230	-0.2838	0.4032
TV-HAR	**3.8384**	**4.1868**	-2.6507	**0.0611**	**1.3589**	0.9236	0.6294	1.0708	2.0953	1.1798	**4.2904**	**4.5455**	-3.2227	-0.2221	**1.3478**

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.25: Out-of-sample forecasts (XOM)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.8387	**0.8741**	**0.8293**	0.9000	0.8605	**0.4114**	**0.4508**	0.4792	0.5196	**0.4653**	1.1604	**1.1929**	**1.0929**	**1.1864**	**1.1581**
HARQ	0.8365	0.8780	0.8296	0.9225	**0.8451**	0.4128	0.4520	0.4796	0.5207	0.4663	1.1555	1.1988	1.0931	1.2250	1.1681
TV-HAR	**0.8337**	0.8867	0.8400	0.9112	0.8485	0.4135	0.4530	**0.4789**	**0.5163**	0.4654	**1.1501**	1.2133	1.1119	1.2086	1.1710
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	**5.4459**	**5.6783**	**4.0226**	**5.4953**	**5.1605**	**0.3734**	**0.4365**	**0.5085**	0.5688	0.4718	**9.2653**	**9.6253**	**6.6686**	**9.2049**	**8.6910**
HARQ	5.5249	5.8939	4.1268	6.0216	5.3918	0.3776	0.4385	0.5156	0.5714	0.4758	9.4008	10.0017	6.8460	10.1255	9.0935
TV-HAR	5.4655	6.0175	4.2157	5.7227	5.3553	0.3736	0.4365	0.5091	**0.5665**	**0.4714**	9.2995	10.2198	7.0066	9.6052	9.0328
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	1.0000	1.0000	**1.0000**	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**
HARQ	0.9974	1.0044	1.0003	1.0249	1.0068	1.0033	1.0026	1.0008	1.0021	1.0022	0.9958	1.0049	1.0002	1.0325	1.0084
TV-HAR	**0.9941**	1.0144	1.0129	1.0125	1.0085	1.0050	1.0049	**0.9993**	**0.9936**	1.0007	**0.9912**	1.0172	1.0174	1.0187	1.0111
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	1.0000	1.0000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
HARQ	1.0145	1.0380	1.0259	1.0958	1.0435	1.0113	1.0046	1.0139	1.0046	1.0086	1.0146	1.0391	1.0266	1.1000	1.0451
TV-HAR	1.0036	1.0597	1.0480	1.0414	1.0382	1.0007	1.0001	1.0012	**0.9959**	**0.9995**	1.0037	1.0618	1.0507	1.0435	1.0399
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.0000	0.0000	**0.0000**	0.0000	**0.0000**	**0.0000**	**0.0000**	**0.0000**
HARQ	0.2610	-0.4427	-0.0335	-2.4949	-0.6775	-0.3340	-0.2610	-0.0796	-0.2097	-0.2211	0.4198	-0.4944	-0.0182	-3.2485	-0.8353
TV-HAR	**0.5916**	-1.4443	-1.2915	-1.2483	-0.8481	-0.5003	-0.4886	**0.0733**	**0.6360**	-0.0699	**0.8832**	-1.7163	-1.7422	-1.8697	-1.1112
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.0000	0.0000	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
HARQ	-1.4521	-3.7965	-2.5911	-9.5768	-4.3541	-1.1285	-0.4564	-1.3893	-0.4605	-0.8587	-1.4619	-3.9105	-2.6601	-10.0009	4.5084
TV-HAR	-0.3602	-5.9728	-4.8001	-4.1373	-3.8176	-0.0651	-0.0053	-0.1229	**0.4084**	**0.0538**	-0.3691	-6.1766	-5.0687	-4.3488	-3.9908

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.26: Out-of-sample forecasts (DD)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.3914	**1.3108**	1.3994	1.4607	1.3906	0.4739	0.5069	0.5388	0.6169	0.5341	2.0822	**1.9161**	2.0475	2.0961	2.0355
HARQ	**1.3161**	1.3304	**1.3635**	**1.4399**	**1.3625**	0.4659	0.4984	0.5261	0.6056	0.5240	**1.9563**	1.9570	**1.9940**	**2.0680**	**1.9938**
TV-HAR	1.3381	1.3333	1.3700	1.4555	1.3742	**0.4618**	**0.4945**	**0.5214**	**0.6041**	**0.5205**	1.9979	1.9649	2.0090	2.0965	2.0171
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	19.8313	12.3804	18.4184	15.4229	16.5133	**0.4715**	**0.5252**	0.5462	0.7470	**0.5725**	34.4087	21.3072	31.8758	26.4735	28.5163
HARQ	**15.1980**	**11.8152**	**16.3627**	**14.8184**	**14.5486**	0.4867	0.5441	**0.5404**	**0.7451**	0.5791	**26.2753**	**20.3020**	**28.2764**	**25.4152**	**25.0672**
TV-HAR	16.0452	12.3708	16.4521	15.1435	15.0029	0.4944	0.5446	0.5489	0.7512	0.5848	27.7545	21.2757	28.4269	25.9805	25.8594
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	1.0000	1.0000	1.0000
HARQ	**0.9459**	1.0150	**0.9743**	**0.9857**	**0.9802**	0.9833	0.9832	0.9765	0.9818	0.9812	**0.9395**	1.0213	**0.9739**	**0.9866**	**0.9803**
TV-HAR	0.9617	1.0172	0.9790	0.9964	0.9886	**0.9746**	**0.9756**	**0.9677**	**0.9793**	**0.9743**	0.9595	1.0255	0.9812	1.0002	0.9916
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	1.0000	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	**0.7664**	**0.9543**	**0.8884**	**0.9608**	**0.8925**	1.0322	1.0360	**0.9895**	**0.9975**	1.0138	**0.7636**	**0.9528**	**0.8871**	**0.9600**	**0.8909**
TV-HAR	0.8091	0.9992	0.8932	0.9819	0.9209	1.0485	1.0370	1.0050	1.0057	1.0240	0.8066	0.9985	0.8918	0.9814	0.9196
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000
HARQ	**5.4099**	-1.4978	**2.5702**	**1.4270**	**1.9773**	1.6747	1.6833	2.3493	1.8209	1.8821	**6.0500**	-2.1315	**2.6140**	**1.3397**	**1.9680**
TV-HAR	3.8309	-1.7180	2.1024	0.3596	1.1437	**2.5385**	**2.4432**	**3.2268**	**2.0679**	**2.5691**	4.0523	-2.5469	1.8796	-0.0190	0.8415
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	**0.0000**	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	**23.3634**	**4.5659**	**11.1615**	**3.9196**	**10.7526**	-3.2218	-3.6020	**1.0538**	**0.2544**	-1.3789	**23.6378**	**4.7175**	**11.2919**	**3.9975**	**10.9112**
TV-HAR	19.0917	0.0776	10.6758	1.8118	7.9142	-4.8482	-3.7002	-0.4978	-0.5705	-2.4042	19.3387	0.1477	10.8199	1.8624	8.0422

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.27: Out-of-sample forecasts (VZ)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.9796	0.9636	**0.9468**	1.0335	0.9809	0.4696	0.4950	0.5163	0.5656	0.5116	1.3636	1.3164	**1.2709**	**1.3857**	1.3342
HARQ	**0.9634**	**0.9568**	0.9473	1.0348	**0.9756**	**0.4506**	**0.4807**	**0.5017**	**0.5540**	**0.4968**	**1.3496**	1.3153	1.2827	1.3968	1.3361
TV-HAR	0.9720	0.9596	0.9501	**1.0325**	0.9785	0.4666	0.4909	0.5179	0.5626	0.5095	1.3526	**1.3125**	1.2755	1.3862	**1.3317**
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	**6.1433**	**5.1869**	4.6750	5.0415	**5.2617**	0.5203	0.5617	0.7093	0.8963	0.6719	**10.3773**	**8.6696**	7.6610	8.1627	**8.7177**
HARQ	6.2779	5.4946	4.7059	5.0885	5.3917	**0.5115**	**0.5540**	**0.6995**	0.9004	**0.6663**	10.6199	9.2148	7.7226	8.2421	8.9498
TV-HAR	6.1595	5.2982	**4.6597**	**5.0143**	5.2829	0.5167	0.5582	0.7079	**0.8959**	0.6697	10.4084	8.8673	**7.6353**	**8.1153**	8.7566
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	1.0000
HARQ	**0.9835**	**0.9930**	1.0005	1.0013	**0.9946**	**0.9596**	**0.9711**	**0.9718**	**0.9794**	**0.9705**	**0.9897**	0.9992	1.0093	1.0080	1.0015
TV-HAR	0.9922	0.9958	1.0035	**0.9990**	0.9976	0.9936	0.9917	1.0031	0.9947	0.9958	0.9919	**0.9970**	1.0036	1.0004	**0.9982**
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	**1.0000**	**1.0000**	1.0000	1.0000	**1.0000**	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	1.0000	1.0000	**1.0000**
HARQ	1.0219	1.0593	1.0066	1.0093	1.0243	**0.9832**	**0.9862**	**0.9861**	1.0046	**0.9900**	1.0234	1.0629	1.0080	1.0097	1.0260
TV-HAR	1.0026	1.0214	**0.9967**	**0.9946**	1.0039	0.9932	0.9937	0.9980	**0.9996**	0.9961	1.0030	1.0228	**0.9966**	**0.9942**	1.0042
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	**0.0000**	0.0000
HARQ	**1.6499**	**0.7018**	-0.0516	-0.1290	**0.5428**	**4.0359**	**2.8886**	**2.8213**	**2.0632**	**2.9522**	**1.0312**	0.0826	-0.9304	-0.8028	-0.1548
TV-HAR	0.7778	0.4177	-0.3473	**0.0959**	0.2360	0.6415	0.8336	-0.3126	0.5294	0.4230	0.8132	**0.2999**	-0.3579	-0.0374	**0.1795**
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	**0.0000**	**0.0000**	0.0000	0.0000	**0.0000**	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	**0.0000**	0.0000	0.0000	**0.0000**
HARQ	-2.1910	-5.9319	-0.6615	-0.9327	-2.4293	**1.6794**	**1.3813**	**1.3878**	-0.4589	**0.9974**	-2.3371	-6.2887	-0.8044	-0.9719	-2.6005
TV-HAR	-0.2633	-2.1450	**0.3266**	**0.5393**	-0.3856	0.6785	0.6286	0.1955	**0.0409**	0.3859	-0.2989	-2.2803	**0.3358**	**0.5805**	-0.4157

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Table 3.28: Out-of-sample forecasts (PEF)

	Full					Pre-Crisis					Crisis				
MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0967	1.0781	1.0460	1.1804	1.1003	0.7937	0.8535	0.8018	0.9884	0.8594	1.3249	1.2472	**1.2299**	1.3249	1.2817
HARQ	**1.0260**	1.0277	1.0326	**1.1353**	**1.0554**	0.7536	0.7981	0.7524	0.9323	0.8091	**1.2312**	**1.2005**	1.2436	**1.2882**	**1.2409**
TV-HAR	1.0340	**1.0272**	**1.0320**	1.1384	1.0579	**0.7535**	**0.7965**	**0.7457**	**0.9302**	**0.8065**	1.2452	1.2009	1.2476	1.2951	1.2472
	Full					Pre-Crisis					Crisis				
MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	14.4748	8.8819	6.6204	9.1039	9.7703	6.6833	7.6515	3.8176	7.8585	6.5027	20.3417	9.8084	8.7309	10.0417	12.2307
HARQ	**11.8529**	7.9249	**6.3831**	**8.2243**	**8.5963**	6.1272	**6.7366**	3.5648	7.3776	5.9516	**16.1641**	8.8197	**8.5052**	**8.8619**	**10.5877**
TV-HAR	11.8724	**7.7245**	6.4875	8.3667	8.6128	**6.1101**	6.7532	**3.5533**	**7.3673**	**5.9460**	16.2112	**8.4559**	8.6970	9.1192	10.6208
	Full					Pre-Crisis					Crisis				
Relative MAE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	1.0000	1.0000
HARQ	**0.9355**	0.9532	0.9872	**0.9618**	**0.9594**	0.9494	0.9350	0.9383	0.9433	0.9415	**0.9293**	**0.9625**	1.0111	**0.9723**	**0.9688**
TV-HAR	0.9428	**0.9528**	**0.9866**	0.9644	0.9616	**0.9493**	**0.9332**	**0.9300**	**0.9411**	**0.9384**	0.9398	0.9628	1.0144	0.9775	0.9736
	Full					Pre-Crisis					Crisis				
Relative MSE	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HARQ	**0.8189**	0.8923	**0.9641**	**0.9034**	**0.8947**	0.9168	**0.8804**	0.9338	0.9388	0.9175	**0.7946**	0.8992	**0.9741**	**0.8825**	**0.8876**
TV-HAR	0.8202	**0.8697**	0.9799	0.9190	0.8972	**0.9142**	0.8826	**0.9308**	**0.9375**	**0.9163**	0.7969	**0.8621**	0.9961	0.9081	0.8908
	Full					Pre-Crisis					Crisis				
MAE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**	0.0000	0.0000
HARQ	**6.4478**	4.6808	1.2835	**3.8166**	**4.0572**	5.0620	6.4958	6.1685	5.6713	5.8494	**7.0730**	**3.7455**	-1.1146	**2.7748**	**3.1197**
TV-HAR	5.7233	**4.7221**	**1.3390**	3.5594	3.8360	**5.0740**	**6.6756**	**6.9966**	**5.8898**	**6.1590**	6.0163	3.7155	-1.4383	2.2504	2.6360
	Full					Pre-Crisis					Crisis				
MSE Gains(%)	150	300	450	900	average	150	300	450	900	average	150	300	450	900	average
HAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HARQ	**18.1140**	10.7748	**3.5851**	**9.6617**	**10.5339**	8.3201	**11.9567**	6.6220	6.1201	8.2547	**20.5369**	10.0805	**2.5853**	**11.7486**	**11.2378**
TV-HAR	17.9792	**13.0312**	2.0074	8.0980	10.2790	**8.5768**	11.7401	**6.9245**	**6.2505**	**8.3730**	20.3053	**13.7896**	0.3885	9.1867	10.9175

Notes: The table reports the out-of-sample performances of the standard HAR, the HARQ and the TV-HAR models for different subsample periods, based on MAE and MSE. The numbers in bold represent the model with best forecasting performance. The subsample of the pre-crisis period is from 2004 to 2006. The subsample of the crisis period is from 2007 to 2010. The relative MAE and MSE means the ratio of the losses for the different models relative to the losses of the HAR model. The MAE and MSE Gains(%) measure the gains of forecasting accuracy compared with the HAR model, for example, the MAE gains of HARQ is calculated as follows: $(MAE_{HAR} - MAE_{HARQ})/MAE_{HAR}$.

Concluding Remarks

This thesis investigates financial volatility and cross-market correlation by using different volatility measures and models. We consider both parametric (Chapters 1 and 2) and non-parametric (Chapter 3) volatility measurement; high-frequency (Chapters 1 and 3) and daily (Chapter 2) data; univariate (Chapters 1 and 3) and multivariate (Chapter 2) models; conventional (Chapters 1 and 3) and Bayesian (Chapter 2) methods.

In Chapter 1, we introduce the Intraday SV specification which incorporates the duration information to model and forecast intraday return volatility. The duration information includes both the lag duration and the lag expected duration calculated from the ACD model. We consider the expected duration rather than only rely on the actual duration because the expected duration allows us to investigate the effects of surprises in durations on intraday return volatility. We find there is a negative relationship between the unexpected duration and volatility. This chapter supports that when we investigate the intraday return volatility, the duration can offer useful information.

In Chapter 2, we examines financial contagion between stock markets of USA and five EU countries. We consider both the recent Global Financial Crisis (GFC) and the European Sovereign Debt Crisis (ESDC). We compare the contagion effects estimating from the DCC-GARCH and DC-MSV models and outline that financial contagion is more significant based on the DC-MSV model. We extend the DC-MSV model by incorporating the implied volatility (DC-MSV-IV), and compare the contagion effect with the standard DC-MSV. The contagion effect is further more significant under the DC-MSV-IV model. We confirm the implied volatility information is useful for detecting financial contagion. We offer the empirical evidence of the existence of contagion for the countries under investigation. Compared with the stable market regimes, the correlations are significantly higher during the crisis market regimes. The dynamic correlations are even higher during ESDC compared with GFC. For the five EU countries, the UK is most influenced by the financial contagion whereas Germany is least influenced.

In Chapter 3, we introduce a TV-HAR model to forecast RV. We observe a regular pattern of RV that can be captured by the TV-HAR model: if there is an increase in the lag daily RV compared with its longer-term average level (monthly RV), the current RV tends to decrease rapidly to its long term level; conversely, if there is a decrease in the lag daily RV compared with its longer-term average level (monthly RV), that reversion takes longer. The TV-HAR model allows the weight of daily lags to vary according to the absolute difference between the long-term (monthly) RV and the short-term (daily) RV. The weight of daily lags is highest when the RV is equal to its longer-term level. The lower weight can make the forecasts quickly mean reverting when daily RV is bigger than its monthly RV, and slowly mean reverting when daily RV is smaller than its monthly RV. We compare the TV-HAR model with the standard HAR and recently introduced HARQ models. The better performance of the TV-HAR model can be supported by both the simulated and empirical data.

References

Admati, A. R. and Pfleiderer, P., 1988. A theory of intraday patterns: Volume and price variability. Review of Financial studies. 1, 3–40.

Ahmad, W., Sehgal, S. and Bhanumurthy, N., 2013. Eurozone crisis and briicks stock markets: Contagion or market interdependence?. Economic Modelling. 33, 209–225.

Aielli, G. P., 2013. Dynamic conditional correlation: on properties and estimation. Journal of Business & Economic Statistics. 31, 282–299.

Aït-Sahalia, Y., Mykland, P. A. and Zhang, L., 2005. How often to sample a continuous-time process in the presence of market microstructure noise. Review of Financial studies. 18, 351–416.

Akaike, H., 1973. Information theory and an extension of the maximum likelihood principle. in *Second International Symposium on Information Theory*. Akademinai Kiado. pp. 267–281.

Alizadeh, S., Brandt, M. W. and Diebold, F. X., 2002. Range-based estimation of stochastic volatility models. The Journal of Finance. 57, 1047–1091.

Allen, D., Chan, F., McAleer, M. and Peiris, S., 2008. Finite sample properties of the qmle for the log-acd model: application to australian stocks. Journal of Econometrics. 147, 163–185.

Allen, F. and Gale, D., 2000. Financial contagion. Journal of political economy. 108, 1–33.

Andersen, T. G. and Bollerslev, T., 1997. Intraday periodicity and volatility persistence in financial markets. Journal of empirical finance. 4, 115–158.

Andersen, T. G. and Bollerslev, T., 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. International economic review. pp. 885–905.

Andersen, T. G., Bollerslev, T. and Diebold, F. X., 2007. Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. The Review of Economics and Statistics. 89, 701–720.

Andersen, T. G., Bollerslev, T., Diebold, F. X. and Labys, P., 2001. The distribution of realized exchange rate volatility. Journal of the American statistical association. 96, 42–55.

Andersen, T. G., Bollerslev, T., Diebold, F. X. and Labys, P., 2003. Modeling and forecasting realized volatility. Econometrica. 71, 579–625.

Andersen, T. G., Bollerslev, T. and Huang, X., 2011. A reduced form framework for modeling volatility of speculative prices based on realized variation measures. Journal of Econometrics. 160, 176–189.

Andersen, T. G., Bollerslev, T. and Lange, S., 1999. Forecasting financial market volatility: Sample frequency vis-a-vis forecast horizon. Journal of Empirical Finance. 6, 457–477.

Andersen, T. G., Bollerslev, T. and Meddahi, N., 2004. Analytical evaluation of volatility forecasts*. International Economic Review. 45, 1079–1110.

Andersen, T. G., Dobrev, D. and Schaumburg, E., 2012. Jump-robust volatility estimation using nearest neighbor truncation. Journal of Econometrics. 169, 75–93.

Andersen, T. G. and Sørensen, B. E., 1996. Gmm estimation of a stochastic volatility model: a monte carlo study. Journal of Business & Economic Statistics. 14, 328–352.

Andersen, T. G. and Teräsvirta, T., 2009. Realized volatility. in *Handbook of Financial Time Series*. Springer. pp. 555–575.

Andreou, E. and Ghysels, E., 2002. Rolling-sample volatility estimators. Journal of Business & Economic Statistics. 20.

Areal, N. M. and Taylor, S. J., 2002. The realized volatility of ftse-100 futures prices. Journal of Futures Markets. 22, 627–648.

Asai, M. and McAleer, M., 2009. The structure of dynamic correlations in multivariate stochastic volatility models. Journal of Econometrics. 150, 182–192.

Asai, M., McAleer, M. and Yu, J., 2006. Multivariate stochastic volatility: a review. Econometric Reviews. 25, 145–175.

Baillie, R. T., Bollerslev, T. and Mikkelsen, H. O., 1996. Fractionally integrated generalized autoregressive conditional heteroskedasticity. Journal of econometrics. 74, 3–30.

Bandi, F. M. and Russell, J. R., 2006. Separating microstructure noise from volatility. Journal of Financial Economics. 79, 655–692.

Bandi, F. M. and Russell, J. R., 2008. Microstructure noise, realized variance, and optimal sampling. The Review of Economic Studies. 75, 339–369.

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A. and Shephard, N., 2008. Designing realized kernels to measure the ex post variation of equity prices in the presence of noise. Econometrica. pp. 1481–1536.

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A. and Shephard, N., 2009. Realized kernels in practice: Trades and quotes. The Econometrics Journal. 12, C1–C32.

Barndorff-Nielsen, O. E. and Shephard, N., 2001. Non-gaussian ornstein–uhlenbeck-based models and some of their uses in financial economics. Journal of the Royal Statistical Society: Series B (Statistical Methodology). 63, 167–241.

Barndorff-Nielsen, O. E. and Shephard, N., 2002a. Econometric analysis of realized volatility and its use in estimating stochastic volatility models. Journal of the Royal Statistical Society: Series B (Statistical Methodology). 64, 253–280.

Barndorff-Nielsen, O. E. and Shephard, N., 2002b. Estimating quadratic variation using realized variance. Journal of Applied econometrics. 17, 457–477.

Baur, D. G., 2012. Financial contagion and the real economy. Journal of Banking & Finance. 36, 2680–2692.

Bauwens, L. and Giot, P., 2000. The logarithmic acd model: an application to the bid-ask quote process of three nyse stocks. Annales d'Economie et de Statistique. pp. 117–149.

Bauwens, L. and Giot, P., 2001. *Econometric modelling of stock market intraday activity*. vol. 38. Springer.

Bauwens, L. and Giot, P., 2003. Asymmetric acd models: introducing price information in acd models. Empirical Economics. 28, 709–731.

Bauwens, L., Giot, P., Grammig, J. and Veredas, D., 2004. A comparison of financial duration models via density forecasts. International Journal of Forecasting. 20, 589–609.

Bauwens, L. and Veredas, D., 2004. The stochastic conditional duration model: a latent variable model for the analysis of financial durations. Journal of econometrics. 119, 381–412.

Bekaert, G., Harvey, C. R. and Ng, A., 2005. Market integration and contagion. Journal of Business. 78.

Beltratti, A. and Morana, C., 1999. Computing value at risk with high frequency data. Journal of empirical finance. 6, 431–455.

Berg, A., Meyer, R. and Yu, J., 2004. Deviance information criterion for comparing stochastic volatility models. Journal of Business & Economic Statistics. 22, 107–120.

Bhar, R. and Lee, D., 2009. Comparing estimation procedures for stochastic volatility models of short-term interest rates. Working Paper.

BIS, 2009. The international financial crisis: timeline, impact and policy responses in asia and the pacific. Bank for International Settlements.

Black, F., 1976. Studies of stock price volatility changes, proceedings of the 1976 meetings of the business and economics statistics section. American Statistical Association. pp. 177–181.

Blair, B. J., Poon, S.-H. and Taylor, S. J., 2001. Forecasting s&p 100 volatility: the incremental information content of implied volatilities and high-frequency index returns. Journal of Econometrics. 105, 5–26.

Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of econometrics. 31, 307–327.

Bollerslev, T., 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model. The Review of Economics and Statistics. pp. 498–505.

Bollerslev, T., Engle, R. F. and Wooldridge, J. M., 1988. A capital asset pricing model with time-varying covariances. The Journal of Political Economy. pp. 116–131.

Bollerslev, T., Patton, A. J. and Quaedvlieg, R., 2016. Exploiting the errors: A simple approach for improved volatility forecasting. Journal of Econometrics. 192, 1–18.

Bollerslev, T. and Wright, J., 2001. Volatility forecasting, high-frequency data, and frequency domain inference. Review of Economics and Statistics. 83, 4.

Bollerslev, T. and Zhou, H., 2002. Estimating stochastic volatility diffusion using conditional moments of integrated volatility. Journal of Econometrics. 109, 33–65.

Boyer, B. H., Kumagai, T. and Yuan, K., 2006. How do crises spread? evidence from accessible and inaccessible stock indices. The Journal of Finance. 61, 957–1003.

Breidt, F. J. and Carriquiry, A. L., 1996. Improved quasi-maximum likelihood estimation for stochastic volatility models. in *Modelling and Prediction Honoring Seymour Geisser*. Springer. pp. 228–247.

Broto, C. and Ruiz, E., 2004. Estimation methods for stochastic volatility models: a survey. Journal of Economic Surveys. 18, 613–649.

Busch, T., Christensen, B. J. and Nielsen, M. Ø., 2011. The role of implied volatility in forecasting future realized volatility and jumps in foreign exchange, stock, and bond markets. Journal of Econometrics. 160, 48–57.

Calvet, L. E. and Fisher, A. J., 2004. How to forecast long-run volatility: regime switching and the estimation of multifractal processes. Journal of Financial Econometrics. 2, 49–83.

Cappiello, L., Engle, R. F. and Sheppard, K., 2006. Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial econometrics. 4, 537–572.

Caramazza, F., Ricci, L. and Salgado, R., 2004. International financial contagion in currency crises. Journal of International Money and Finance. 23, 51–70.

Casella, G. and George, E. I., 1992. Explaining the gibbs sampler. The American Statistician. 46, 167–174.

Chan, K., Chan, K. C. and Karolyi, G. A., 1991. Intraday volatility in the stock index and stock index futures markets. Review of Financial Studies. 4, 657–684.

Chen, X. and Ghysels, E., 2011. News–good or bad–and its impact on volatility predictions over multiple horizons. Review of Financial Studies. 24, 46–81.

Chiang, T. C., Jeon, B. N. and Li, H., 2007. Dynamic correlation analysis of financial contagion: Evidence from asian markets. Journal of International Money and Finance. 26, 1206–1228.

Chib, S., 1995. Marginal likelihood from the gibbs output. Journal of the American Statistical Association. 90, 1313–1321.

Chib, S. and Greenberg, E., 1996. Markov chain monte carlo simulation methods in econometrics. Econometric theory. 12, 409–431.

Chib, S., Nardari, F. and Shephard, N., 2002. Markov chain monte carlo methods for stochastic volatility models. Journal of Econometrics. 108, 281–316.

Chib, S., Omori, Y. and Asai, M., 2009. Multivariate stochastic volatility. in *Handbook of Financial Time Series*. Springer. pp. 365–400.

Choudhry, T. and Jayasekera, R., 2014. Returns and volatility spillover in the european banking industry during global financial crisis: Flight to perceived quality or contagion?. International Review of Financial Analysis.

Christensen, B. J. and Prabhala, N. R., 1998. The relation between implied and realized volatility. Journal of Financial Economics. 50, 125–150.

Clark, P. K., 1973. A subordinated stochastic process model with finite variance for speculative prices. Econometrica: journal of the Econometric Society. pp. 135–155.

Comte, F. and Renault, E., 1998. Long memory in continuous-time stochastic volatility models. Mathematical Finance. 8, 291–323.

Corsetti, G., Pericoli, M. and Sbracia, M., 2005. some contagion, some interdependence: More pitfalls in tests of financial contagion. Journal of International Money and Finance. 24, 1177–1199.

Corsi, F., 2009. A simple approximate long-memory model of realized volatility. Journal of Financial Econometrics. p. nbp001.

Corsi, F., Audrino, F. and Renò, R., 2012. Har modeling for realized volatility forecasting. Handbook of Volatility Models and Their Applications. pp. 363–382.

Corsi, F., Pirino, D. and Reno, R., 2010. Threshold bipower variation and the impact of jumps on volatility forecasting. Journal of Econometrics. 159, 276–288.

Danielsson, J., 1994. Stochastic volatility in asset prices estimation with simulated maximum likelihood. Journal of Econometrics. 64, 375–400.

Danıelsson, J., 1998. Multivariate stochastic volatility models: estimation and a comparison with vgarch models. Journal of Empirical Finance. 5, 155–173.

Darrat, A. F., Rahman, S. and Zhong, M., 2003. Intraday trading volume and return volatility of the djia stocks: A note. Journal of Banking & Finance. 27, 2035–2043.

Darrat, A. F., Zhong, M. and Cheng, L. T., 2007. Intraday volume and volatility relations with and without public news. Journal of Banking & Finance. 31, 2711–2729.

Deo, R. S., 2002. On gmm and qml estimation for the long memory stochastic volatility model. Working Paper.

DIEBOLD, F. X. and MARIANO, R. S., 1995. Comparing predictive accuracy. Journal of Business & Economic Statistics. 13.

Dimitriou, D., Kenourgios, D. and Simos, T., 2013. Global financial crisis and emerging stock market contagion: A multivariate fiaparch–dcc approach. International Review of Financial Analysis. 30, 46–56.

Ding, Z. and Granger, C. W., 1996. Modeling volatility persistence of speculative returns: a new approach. Journal of econometrics. 73, 185–215.

Drost, F. C. and Werker, B. J. M., 2004. Semiparametric duration models. Journal of Business & Economic Statistics. 22, 40–50.

Dufour, A. and Engle, R. F., 2000. Time and the price impact of a trade. The Journal of Finance. 55, 2467–2498.

Dungey, M., Fry, R., González-Hermosillo, B. and Martin, V. L., 2005. Empirical modelling of contagion: a review of methodologies. Quantitative Finance. 5, 9–24.

Duong, D. and Swanson, N. R., 2015. Empirical evidence on the importance of aggregation, asymmetry, and jumps for volatility prediction. Journal of Econometrics. 187, 606–621.

Easley, D. and O'HARA, M., 1992. Time and the process of security price adjustment. The Journal of finance. 47, 577–605.

Ederington, L. and Lee, J. H., 2001. Intraday volatility in interest-rate and foreign-exchange markets: Arch, announcement, and seasonality effects. Journal of Futures Markets. 21, 517–552.

Engle, R., 2002a. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. Journal of Business & Economic Statistics. 20, 339–350.

Engle, R., 2002b. New frontiers for arch models. Journal of Applied Econometrics. 17, 425–446.

Engle, R. F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. Econometrica: Journal of the Econometric Society. pp. 987–1007.

Engle, R. F., 2000. The econometrics of ultra-high-frequency data. Econometrica. 68, 1–22.

Engle, R. F. and Kroner, K. F., 1995. Multivariate simultaneous generalized arch. Econometric theory. 11, 122–150.

Engle, R. F. and Lee, G. G., 1993. A permanent and transitory component model of stock return volatility. University of California at San Diego, Economics Working Paper Series.

Engle, R. F. and Russell, J. R., 1998. Autoregressive conditional duration: a new model for irregularly spaced transaction data. Econometrica. pp. 1127–1162.

Engle, R. F. and Sheppard, K., 2001. Theoretical and empirical properties of dynamic conditional correlation multivariate garch. Tech. rep.. National Bureau of Economic Research.

Engle, R. F. and Sokalska, M. E., 2012. Forecasting intraday volatility in the us equity market. multiplicative component garch. Journal of Financial Econometrics. 10, 54–83.

Eraker, B., Johannes, M. and Polson, N., 2003. The impact of jumps in volatility and returns. The Journal of Finance. 58, 1269–1300.

Eratalay, M. H., 2012. Estimation of multivariate stochastic volatility models: A comparative monte carlo study. Eratalay MH, Estimation of Multivariate Stochastic Volatility Models: A Comparative Monte Carlo Study. p. 50.

Feng, D., Jiang, G. J. and Song, P. X.-K., 2004. Stochastic conditional duration models with leverage effect for financial transaction data. Journal of financial econometrics. 2, 390–421.

Fernandes, M. and Grammig, J., 2006. A family of autoregressive conditional duration models. Journal of Econometrics. 130, 1–23.

Fleming, J., 1998. The quality of market volatility forecasts implied by s&p 100 index option prices. Journal of Empirical Finance. 5, 317–345.

Fleming, J., Ostdiek, B. and Whaley, R. E., 1995. Predicting stock market volatility: a new measure. Journal of Futures Markets. 15, 265–302.

Forbes, K. and Rigobon, R., 2001. Measuring contagion: conceptual and empirical issues. in *International financial contagion*. Springer. pp. 43–66.

Forbes, K. J. and Rigobon, R., 2002. No contagion, only interdependence: measuring stock market comovements. The journal of finance. 57, 2223–2261.

French, K. R., Schwert, G. W. and Stambaugh, R. F., 1987. Expected stock returns and volatility. Journal of financial Economics. 19, 3–29.

Gebka, B. and Karoglou, M., 2013. Have the gipsi settled down? breaks and multivariate stochastic volatility models for, and not against, the european financial integration. Journal of Banking & Finance. 37, 3639–3653.

Gerhard, F. and Pohlmeier, W., 2002. On the simultaneity of absolute price changes and transaction durations. Discussion paper.

Giot, P., 1999. *Time transformations, intraday data and volatility models*. Citeseer.

Giot, P., 2005. Market risk models for intraday data. The European Journal of Finance. 11, 309–324.

Gonçalves, S. and Meddahi, N., 2009. Bootstrapping realized volatility. Econometrica. 77, 283–306.

Grammig, J. and Maurer, K.-O., 2000. Non-monotonic hazard functions and the autoregressive conditional duration model. The Econometrics Journal. 3, 16–38.

Granger, C. W., 1980. Long memory relationships and the aggregation of dynamic models. Journal of econometrics. 14, 227–238.

Hamilton, J. D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica: Journal of the Econometric Society. pp. 357–384.

Hamilton, J. D., 1994. *Time series analysis*. vol. 2. Princeton university press Princeton.

Hansen, P. R. and Lunde, A., 2006. Realized variance and market microstructure noise. Journal of Business & Economic Statistics. 24, 127–161.

Harvey, A., Ruiz, E. and Shephard, N., 1994. Multivariate stochastic variance models. The Review of Economic Studies. 61, 247–264.

Harvey, A. C., 1989. *Forecasting, structural time series models and the Kalman filter*. Cambridge university press.

Harvey, A. C., 2002. Long memory in stochastic volatility. Forecasting Volatility in the Financial Markets. p. 351.

Harvey, A. C. and Shephard, N., 1996. Estimation of an asymmetric stochastic volatility model for asset returns. Journal of Business & Economic Statistics. 14, 429–434.

Hautsch, N., 2012. *Econometrics of financial high-frequency data*. Springer.

Henker, T. and Wang, J.-X., 2006. On the importance of timing specifications in market microstructure research. Journal of Financial Markets. 9, 162–179.

Hsieh, D. A., 1991. Chaos and nonlinear dynamics: application to financial markets. The journal of finance. 46, 1839–1877.

Huang, X. and Tauchen, G., 2005. The relative contribution of jumps to total price variance. Journal of financial econometrics. 3, 456–499.

Hull, J. and White, A., 1987. The pricing of options on assets with stochastic volatilities. The journal of finance. 42, 281–300.

Jacquier, E. and Miller, S., 2010. The information content of realized volatility. working paper.

Jacquier, E. and Paulson, N., 2010. Bayesian econometrics in finance.

Jacquier, E., Polson, N. G. and Rossi, P. E., 1994. Bayesian analysis of stochastic volatility models. Journal of Business & Economic Statistics. pp. 371–389.

Jacquier, E., Polson, N. G. and Rossi, P. E., 2004. Bayesian analysis of stochastic volatility models with fat-tails and correlated errors. Journal of Econometrics. 122, 185–212.

Jasiak, J., 1999. Persistence in intertrade durations. in *Finance*. Citeseer.

Kaminsky, G. L., Reinhart, C. M. and Vegh, C. A., 2003. The unholy trinity of financial contagion. Journal of Economic Perspectives. 17, 51–74.

Karanasos, M., 2004. The statistical properties of long-memory and exponential acd models. WSEAS Trans Bus Econ.

Kenourgios, D., 2014. On financial contagion and implied market volatility. International Review of Financial Analysis. 34, 21–30.

Kenourgios, D., Samitas, A. and Paltalidis, N., 2011. Financial crises and stock market contagion in a multivariate time-varying asymmetric framework. Journal of International Financial Markets, Institutions and Money. 21, 92–106.

Kim, S., Shephard, N. and Chib, S., 1998. Stochastic volatility: likelihood inference and comparison with arch models. The Review of Economic Studies. 65, 361–393.

King, M. A. and Wadhwani, S., 1990. Transmission of volatility between stock markets. Review of Financial studies. 3, 5–33.

Kolb, R. W., 2011. What is financial contagion?. Robert W. Kolb, Financial Contagion-the Viral Threat to the Wealth of the Nations, John Wiley & Sons, Inc.

Koopman, S. J., Jungbacker, B. and Hol, E., 2005. Forecasting daily variability of the s&p 100 stock index using historical, realised and implied volatility measurements. Journal of Empirical Finance. 12, 445–475.

Koopman, S. J., Shephard, N. and Doornik, J. A., 1999. Statistical algorithms for models in state space using ssfpack 2.2. The Econometrics Journal. 2, 107–160.

Kosapattarapim, C., Lin, Y.-X. and McCrae, M., 2011. Evaluating the volatility forecasting performance of best fitting garch models in emerging asian stock markets.

Krichene, N., 2003. Modeling stochastic volatility with application to stock returns. International Monetary Fund working paper.

LeBaron, B. *et al.*, 2001. Stochastic volatility as a simple generator of apparent financial power laws and long memory. Quantitative Finance. 1, 621–631.

Lee, C. and Ready, M. J., 1991. Inferring trade direction from intraday data. The Journal of Finance. 46, 733–746.

Lee, S. B. and Kim, K. J., 1993. Does the october 1987 crash strengthen the co-movements among national stock markets. Review of Financial Economics. 3, 89–102.

Lee, S.-W. and Hansen, B. E., 1994. Asymptotic theory for the garch (1, 1) quasi-maximum likelihood estimator. Econometric theory. 10, 29–29.

Li, K., 2002. Long-memory versus option-implied volatility predictions. The Journal of Derivatives. 9, 9–25.

Liesenfeld, R. and Richard, J.-F., 2003. Univariate and multivariate stochastic volatility models: estimation and diagnostics. Journal of empirical finance. 10, 505–531.

Liu, L. Y., Patton, A. J. and Sheppard, K., 2015. Does anything beat 5-minute rv? a comparison of realized measures across multiple asset classes. Journal of Econometrics. 187, 293–311.

Louis, F. R. B. o. S., 2009. The financial crisis. a timeline of events and policy actions. St. Louis: FRBS.

Lumsdaine, R. L., 1996. Consistency and asymptotic normality of the quasi-maximum likelihood estimator in igarch (1, 1) and covariance stationary garch (1, 1) models. Econometrica: Journal of the Econometric Society. pp. 575–596.

Lunde, A., 1999. A generalized gamma autoregressive conditional duration model. Department of Economics, Politics and Public Administration, Aalbourg University, Denmark.

Lunn, D. J., Thomas, A., Best, N. and Spiegelhalter, D., 2000. Winbugs-a bayesian modelling framework: concepts, structure, and extensibility. Statistics and computing. 10, 325–337.

Manganelli, S., 2005. Duration, volume and volatility impact of trades. Journal of Financial markets. 8, 377–399.

Martens, M., Chang, Y.-C. and Taylor, S. J., 2002. A comparison of seasonal adjustment methods when forecasting intraday volatility. Journal of Financial Research. 25, 283–299.

Martens, M. and Zein, J., 2004. Predicting financial volatility: High-frequency time-series forecasts vis-à-vis implied volatility. Journal of Futures Markets. 24, 1005–1028.

McAleer, M. and Medeiros, M. C., 2008. Realized volatility: A review. Econometric Reviews. 27, 10–45.

McMillan, D. G. and Garcia, R. Q., 2009. Intra-day volatility forecasts. Applied Financial Economics. 19, 611–623.

Meitz, M. and Teräsvirta, T., 2006. Evaluating models of autoregressive conditional duration. Journal of Business & Economic Statistics. 24.

Melino, A. and Turnbull, S. M., 1990. Pricing foreign currency options with stochastic volatility. Journal of Econometrics. 45, 239–265.

Merton, R. C., 1980. On estimating the expected return on the market: An exploratory investigation. Journal of Financial Economics. 8, 323–361.

Müller, U., Dacorogna, M., Davé, R., Pictet, O., Olsen, R. and Ward, J., 1993. Fractals and intrinsic time: A challenge to econometricians. in *Discussion Paper Presented at the 1993 International Conference of the Applied Econometrics Association*. Citeseer.

Müller, U. A., Dacorogna, M. M., Davé, R. D., Olsen, R. B., Pictet, O. V. and von Weizsäcker, J. E., 1997. Volatilities of different time resolutionsanalyzing the dynamics of market components. Journal of Empirical Finance. 4, 213–239.

Newton, M. A. and Raftery, A. E., 1994. Approximate bayesian inference with the weighted likelihood bootstrap. Journal of the Royal Statistical Society. Series B (Methodological). pp. 3–48.

Omori, Y., Chib, S., Shephard, N. and Nakajima, J., 2007. Stochastic volatility with leverage: Fast and efficient likelihood inference. Journal of Econometrics. 140, 425–449.

Oomen, R. C., 2004. Modelling realized variance when returns are serially correlated. Tech. rep.. Social Science Research Center Berlin (WZB).

Pacurar, M., 2008. Autoregressive conditional duration models in finance: a survey of the theoretical and empirical literature. Journal of Economic Surveys. 22, 711–751.

Pagan, A. R. and Schwert, G. W., 1990. Alternative models for conditional stock volatility. Journal of econometrics. 45, 267–290.

Pappas, V., Ingham, H., Izzeldin, M. and Steele, G., 2015. Will the crisis tear us apart? evidence from the eu. International Review of Financial Analysis.

Patton, A. J., 2011. Data-based ranking of realised volatility estimators. Journal of Econometrics. 161, 284–303.

Patton, A. J. and Sheppard, K., 2015. Good volatility, bad volatility: Signed jumps and the persistence of volatility. Review of Economics and Statistics. 97, 683–697.

Pericoli, M. and Sbracia, M., 2003. A primer on financial contagion. Journal of Economic Surveys. 17, 571–608.

Phillips, P. C., 1991. To criticize the critics: An objective bayesian analysis of stochastic trends. Journal of Applied Econometrics. 6, 333–364.

Pong, S., Shackleton, M. B., Taylor, S. J. and Xu, X., 2004. Forecasting currency volatility: A comparison of implied volatilities and ar (fi) ma models. Journal of Banking & Finance. 28, 2541–2563.

Poterba, J. M. and Summers, L. H., 1984. The persistence of volatility and stock market fluctuations.

Rahman, S., Lee, C.-f. and Ang, K. P., 2002. Intraday return volatility process: Evidence from nasdaq stocks. Review of Quantitative Finance and Accounting. 19, 155–180.

Reinhart, C. and Calvo, S., 1996. Capital flows to latin america: Is there evidence of contagion effects?.

Renault, E., 1997. Econometric models of option pricing errors. Econometric Society Monographs. 28, 223–278.

Renault, E. and Werker, B. J., 2011. Causality effects in return volatility measures with random times. Journal of Econometrics. 160, 272–279.

Robinson, P. M. and Henry, M., 1999. Long and short memory conditional heteroskedasticity in estimating the memory parameter of levels. Econometric theory. 15, 299–336.

Rodriguez, J. C., 2007. Measuring financial contagion: A copula approach. Journal of Empirical Finance. 14, 401–423.

Ruiz, E., 1994. Quasi-maximum likelihood estimation of stochastic volatility models. Journal of econometrics. 63, 289–306.

Russell, J. R. and Engle, R. F., 2005. A discrete-state continuous-time model of financial transactions prices and times. Journal of Business & Economic Statistics. 23.

Sandmann, G. and Koopman, S. J., 1998. Estimation of stochastic volatility models via monte carlo maximum likelihood. Journal of Econometrics. 87, 271–301.

Schotman, P. and Van Dijk, H. K., 1991. A bayesian analysis of the unit root in real exchange rates. Journal of Econometrics. 49, 195–238.

Schwert, G. W., 1989. Why does stock market volatility change over time?. The journal of finance. 44, 1115–1153.

Schwert, G. W., 1990. Stock volatility and the crash of87. Review of financial Studies. 3, 77–102.

Shephard, N. and Andersen, T. G., 2009. *Stochastic volatility: origins and overview*. Springer.

Spiegelhalter, D. J., Best, N. G., Carlin, B. P. and Van Der Linde, A., 2002. Bayesian measures of model complexity and fit. Journal of the Royal Statistical Society: Series B (Statistical Methodology). 64, 583–639.

Spierdijk, L., 2004. An empirical analysis of the role of the trading intensity in information dissemination on the nyse. Journal of Empirical Finance. 11, 163–184.

Stoll, H. R. and Whaley, R. E., 1990. The dynamics of stock index and stock index futures returns. Journal of Financial and Quantitative Analysis. 25, 441–468.

Stroud, J. and Johannes, M., 2014. Bayesian modeling and forecasting of 24-hour high-frequency volatility: A case study of the financial crisis. Working paper, George Washington University and Columbia Business School.

Taylor, S., 2005. Asset price dynamics, volatility and prediction.

Taylor, S. J., 1982. Financial returns modelled by the product of two stochastic processes a study of daily sugar prices 1961-79. Anderson, O. D. (Ed.): Time Series Analysis: Theory and Practice 1. pp. 203–226.

Taylor, S. J., 1986. Modelling financial time series.

Taylor, S. J. and Xu, X., 1997. The incremental volatility information in one million foreign exchange quotations. Journal of Empirical Finance. 4, 317–340.

Tran, D. T., 2006. Stochastic volatility, intraday seasonality, and market microstructure noise. Working paper, Duke University.

Trolle, A. B. and Schwartz, E. S., 2009. A general stochastic volatility model for the pricing of interest rate derivatives. Review of Financial Studies. 22, 2007–2057.

Wood, R. A., McInish, T. H. and Ord, J. K., 1985. An investigation of transactions data for nyse stocks. The Journal of Finance. 40, 723–739.

Worthington, A. C. and Higgs, H., 2009. Modeling the intraday return volatility process in the australian equity market: An examination of the role of information arrival in s&p/asx 50 stocks. International Business Research. 1, P87.

Ying, C. C., 1966. Stock market prices and volumes of sales. Econometrica: Journal of the Econometric Society. pp. 676–685.

Yu, J. and Meyer, R., 2006. Multivariate stochastic volatility models: Bayesian estimation and model comparison. Econometric Reviews. 25, 361–384.

Zhang, M. Y., Russell, J. R. and Tsay, R. S., 2001. A nonlinear autoregressive conditional duration model with applications to financial transaction data. Journal of Econometrics. 104, 179–207.