

The chemical effects of CO₂ addition to methane on aromatic chemistry

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Keywords:

Soot, PAH, Modeling, Chemical kinetics, Biogas, Bio-syngas

Abstract

This numerical study concerns the effect of CO₂ addition to CH₄ on aromatic chemistry. In the absence of any thermodynamical effects, *purely kinetic* factors cause increases in the amount of carbon dioxide to decrease benzene mole fraction. The reaction $\text{H} + \text{CO}_2 \rightleftharpoons \text{OH} + \text{CO}$ proved very important. Simulations under adiabatic conditions shows that raising the concentration of CO₂ leads to an *increase* in A₁ mole fractions. This is caused by CO₂ addition making the combustion less efficient, so that both the temperatures and the concentrations of the OH radical are lower than for pure methane. We also discovered that the CO₂ *kinetics* reduces the amount of benzene under isothermal conditions whereas it increases it under adiabatic conditions. Finally, we found that CO₂ addition to *benzene* has negligible kinetic effects on aromatic hydrocarbons, radicals and even CO. Overall, our study shows that the kinetic effects of carbon dioxide in biogas can be highly complex, non-linear and counter-intuitive.

1. Introduction

Owing to the dire repercussions that man-made global warming are likely to have on our environment, health, economies, and societies, most studies aimed at assessing the sustainability of potential renewable fuels have tended to focus on Greenhouse Gases (GHG) [1, 2]. Studies investigating the formation of other pollutants less directly relevant to climate change during the combustion of putative sustainable fuels tend to be rather rare in comparison.

Among potentially sustainable fuels, biogas and bio-syngas and related blends are attracting an increasing interest among researchers and practitioners alike [3–5]. Biogas is formed via the anaerobic degradation of organic material [6] and is essentially constituted of methane and carbon dioxide. Syngas (synthesis gas) is basically a blend of carbon

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1 monoxide and hydrogen which is employed for synthesising diverse products [7]. The gasification of coal is most often
2 utilised for its production. Bio-syngas is a form of synthesis gas which is formed through the thermal gasification of
3 biomass. It is made up of CO, CO₂, CH₄ and H₂ and is considered to be a cleaner fuel than biogas due to its higher
4 hydrogen content [8]. In three recent publications [9–11], we investigated the chemical kinetics of CH₄ - CO - H₂ -
5 CO₂ fuel mixtures by using ignition delay times measured inside shock tubes.

6 In the current work, we want to turn our attention to the effects of the use of these fuels on the formation of the
7 precursors of soot, the PAH. Along with CO, CO₂ and NO_x, soot is one of the most important and preoccupying
8 pollutants emitted in exhaust fumes [12, 13]. It stems from the incomplete combustion of fuel under rich conditions.
9 These prevail when, for instance, too high amounts of fuel have been introduced into a combustion chamber or the
10 mixing with the oxidant was insufficient [14]. In the industrial world, millions tonnes of soot are produced on a yearly
11 basis, mainly for the production of tires or coloured pigments such as black pigments used for printing [15]. Sooting
12 flames are characterised by a yellowish glow that can greatly contribute to heat transfer in industrial burners, thereby
13 raising their efficiency [16]. Nevertheless, soot production also naturally increases the number of airborne particles
14 hovering in the lower and upper atmosphere [17]. Upon inhalation, soot particles can penetrate into lungs and reach,
15 according to their size, different depths [18]. The farther they go, the harder they can be driven out of the lungs
16 or decomposed through natural purification processes [19]. Very small particles can go down to the alveoles and get
17 absorbed by cells where they can cause irritations and even lung cancers [20]. Aside from causing damages to human
18 health, soot has also a negative influence on the environment. Thus, soot particles relatively close to the ground play
19 a role in the emergence of smog [21]. Airborne soot particles reduces the reflection of the sun which is a further cause
20 of global warming [22]. They could also potentially contribute to the depletion of the ozone layer [23].

21 Consequently, when evaluating the local and global effects of allegedly renewable fuels, more studies need to
22 be devoted to the effects of their use in terms of soot production and balance and the related aromatic chemistry.
23 The purpose of our numerical investigation was to reach a better understanding of the **chemical kinetic** role of
24 carbon dioxide in soot formation by considering *homogeneous time-dependent* systems where chemical kinetics and
25 thermodynamics can be uncoupled from other phenomena (such as diffusion, turbulent mixing, convection and so on
26 and so forth) whose overlap can severely hamper the identification of empirical trends, the attribution of causes to
27 observations and the development of predictive models.

28 This article is organised as follows. Section 2 offers a review of the chemical kinetics of soot formation, especially
29 in relation to the combustion of CH₄-CO₂ blends. Section 3 lays out the methodology used to estimate the effects
30 of CO₂ addition on the formation of soot and its precursors. Section 4 compares the performance of five reaction

1 mechanisms aimed at simulating soot precursors with respect to the homogeneous combustion of biogas, bio-syngas
2 and related mixtures. Section 5 investigates numerically the **purely chemical** effects of CO₂ (25%, 50%, and 75%)
3 addition to methane under isothermal conditions. In section 6, the effects of carbon dioxide addition to CH₄ are
4 numerically studied under adiabatic conditions. In section 7, the direct addition of CO₂ to A₁ is investigated with
5 respect to aromatic chemistry. Finally, section 8 provides readers with the conclusion, the outlook and recommended
6 experimental studies to be carried out in the future.

7 **2. Existing literature related to soot formation and CO₂**

8 Historically, the formation of carbonaceous particulate matter during the pyrolysis and rich oxidation of fuels has
9 been attributed to either the formation and growth of poly-acetylene or polyynes ([24]) or to the appearance and growth
10 of polycyclic aromatic hydrocarbons ([14, 25]) which contain at least two benzene-type rings. Over the last years and
11 decades, numerous experimental and theoretical results ([26–29]) have revealed that soot is almost exclusively formed
12 through the aromatic pathway whereas the polyacetylene route could at most only play a minor role by leading to
13 the formation of specific PAH. Readers interested in the general aspects of soot formation are referred to the works of
14 McEnally and Pfefferle [30], Richter et al. [25], and Bockhorn [31]. Of greatest importance to the present study is the
15 *chemical* effect of the addition of CO₂ to methane (which may be used to represent various compositions of bio-syngas
16 and biogas [9], [10]) upon aromatic chemistry.

17 Several studies indicate that the presence of carbon dioxide in biogas leads to a more limited formation of soot as
18 compared to pure methane. Samanta et al. [32] performed a numerical analysis of methane-air laminar nonpremixed
19 flames with different levels of dilution in CO₂. They found out that increases in the original carbon dioxide mole
20 fraction go hand in hand with a decrease in soot and NO formation. Charest et al. [33] studied laminar diffusion
21 biogas flames with various amounts of CO₂ present in the fuel. They identified a soot suppression effect varying
22 linearly with the concentration of carbon dioxide. Whilst they do not rule out reactions between CO₂ and other
23 species, they explained this phenomenon in terms of the effects of *dilution* on chemical reaction rates. Apart from
24 these two studies, there does not seem to be, to the best of our knowledge, any further investigations into the effects
25 of carbon dioxide addition on the sooting tendency of flames having methane as a fuel.

26 The effects of CO₂ addition on the sooting tendencies of C₂H₄ have been extensively studied by a number of
27 authors who drew partially conflicting conclusions from their results. In an experimental study involving ethylene
28 spherical diffusion flames in micro-gravity, Berhan et al. [34] noticed that soot formation is inhibited by CO₂ addition
29 although they did not analyse the underlying causes. Kailasanathan et al. [35] investigated ethylene laminar diffusion

1 flames. They discovered that among four diluents (Argon, helium, nitrogen, and carbon dioxide), CO_2 is the one
2 leading to the highest suppression of soot. This effect was largely attributed to differences in the thermal diffusivities
3 of the four species, and it is not clear whether or not chemical reactions undergone by CO_2 also played a role in that
4 result. Liu et al. [36] also considered ethylene diffusion flames. They found that the chemical suppression of soot
5 formation can be traced back to the elementary steps $\text{CO}_2 + \text{H} \rightarrow \text{CO} + \text{OH}$ and $\text{CO}_2 + \text{CH} \rightarrow \text{HCO} + \text{CO}$. The higher
6 concentration of oxidising radicals, in turn, leads to lower concentrations of acetylene (thereby limiting the extent of
7 soot inception) and a stronger oxidation of the soot particles themselves. Mancarella et al. [37] studied CO_2 addition
8 to fuel-rich premixed ethylene flames. According to their numerical analysis, the reduction in the formation of soot is
9 mainly due to the oxidation of soot particles and their precursors PAH by hydroxyl radicals formed via the reaction
10 $\text{CO}_2 + \text{H} \rightarrow \text{CO} + \text{OH}$. Annunziata et al. [38] investigated ethylene laminar counter-flow diffusion flames where a
11 certain proportion of the inert carrier gas Argon had been replaced by varying amounts of carbon dioxide. They
12 attributed the decrease in soot concentration mainly to thermal effects stemming from the thermodynamic properties
13 of gaseous CO_2 when compared to those of Ar. Guo et al. [39] performed a numerical study involving ethylene
14 diffusion flames. They discovered that the reduced concentration of the hydrogen radical (via $\text{CO}_2 + \text{H} \rightarrow \text{CO} + \text{OH}$)
15 suppresses soot inception and lowers surface growth rates whereas the oxidation of the particles is only very weakly
16 influenced by the presence of carbon dioxide. Zhang et al. [40] considered a jet-stirred / plug-flow reactor. According
17 to their numerical analyses, the effect of the addition of carbon dioxide is to raise the OH / H ratio, diminish the overall
18 concentration of the O-H radical pool, and increase the availability of CO. This results in reduced concentrations of
19 C_2H_2 and PAHs.

20 **3. Modelling methodology**

21 Unlike their gaseous precursors PAH, soot particles can be extremely diverse in terms of their precise number
22 and the spatial arrangement of their atoms [15]. Consequently, the most detailed description of their growth usually
23 consists of polymerisation processes accounted for through various mathematical methods [13, 31, 41]. While these
24 approaches are suitable for simulating and numerically analysing soot formation and depletion in simplified systems
25 such as shock tubes [15] or laminar flames [14], it is really hard to apply them to the Computational Fluid Dynamics
26 (CFD) simulations of complex systems (such as car engines, incinerators or industrial burners) where the mere task of
27 modelling the underlying gas-phase chemistry can already be computationally prohibitive [13]. Furthermore, even if a
28 single CFD simulation of an industrial system based on a detailed or semi-detailed mechanism might be realisable with
29 the help of a supercomputer, such simulations could not be performed under a wide variety of conditions, as would be

1 necessary to reach a thorough understanding of the complex set of overlapping reactions and their interactions [33].
2 Consequently, in the frame of the present work, we decided to focus our attention on the chemical effects of CO₂ on
3 soot formation by considering the *homogeneous* combustion of CH₄-CO₂ blends of diverse compositions in systems
4 such as shock tubes or rapid compression machines where variables only depend on the reaction time. Proceeding
5 in this way, we were able to isolate the *chemistry* from thermodynamical aspects that might be entangled with it
6 otherwise [35, 38].

We further needed a way to qualify the sooting or aromatic tendency of a mixture at a given pressure and at a given temperature. We considered the *propensity to form soot* to be well represented by the mole fraction of benzene, the first aromatic ring.

$$Precursors(t) = X_{A_1}(t)$$

whereby A₁ stands for benzene. In this work, we first defined the propensity of a *given set of conditions* (encompassing initial mole fractions, pressure, temperature, and reaction time) to bring about aromatic compounds and soot as the *maximum* of benzene mole fraction over the entire time period, that is to say

$$Soot_p = \max\{X_{A_1}(t), t \in [0; t_{end}]\}$$

7 This choice was motivated by the fact that PAH formation and PAH concentrations determine soot formation [42–45]
8 to a great extent, that PAH peak concentrations are closely correlated to sooting limits and that the propensity of a
9 flame to form soot is characterised by its propensity to form PAH [46]. Several experimental and theoretical studies
10 have emphasised that the formation of soot and of PAH from aliphatic hydrocarbons (including, of course, CH₄ and
11 mixture thereof with CO and CO₂) is highly dependent on the formation of benzene A₁ [12, 29, 30, 43, 47–50]. We
12 tried to also consider aromatic species such as naphthalene (A₂), phenanthren (A₃), Acenaphthylen (A₂R₅) or Pyrene
13 (A₄) and their sum, but this did not bring in any additional information relevant to our qualitative comparison. What
14 is more, the very small values of some of these concentrations led to numerical problems and irregularities of the
15 profiles. Consequently, the computation of benzene A₁ appears to be ideal in terms of representing the formation of
16 aromatic compounds in CH₄ - CO₂ blends. Given the uncertainty of their coefficients [15, 31], describing the formation
17 and growth of soot particles would have considerably increased the duration of our computations without bringing in
18 any new reliable qualitative and quantitative information relevant to our concerns.

19 We considered four detailed reaction mechanisms describing the formation of PAH to perform our simulations:

- 20 • the model of Frenklach et al. [14, 51, 52] which was developed for describing soot formation during the com-

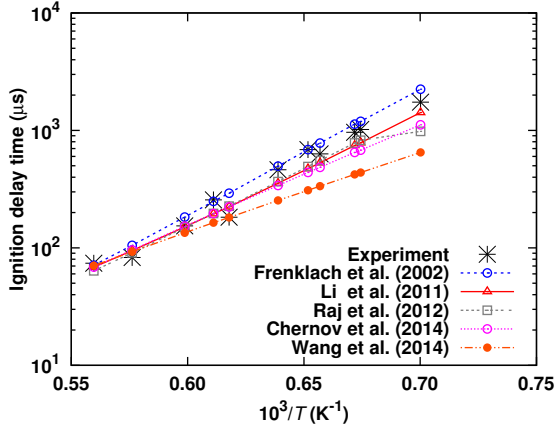
1 bustion of small hydrocarbons. It was validated through comparisons with experimental profiles of major and
2 minor chemical species, aromatic compounds, soot volume fractions, and soot particle diameters reported in the
3 literature for nine laminar premixed flames of ethane, ethylene, and acetylene.

- 4 • the model of Li et al. [53] which aims at accounting for the combustion of toluene. It was tested against toluene-
5 O₂-Ar premixed laminar flames over a broad range of equivalence ratios ($\Phi = 0.75 - 1.75$) at low pressure. The
6 species profiles encompass H₂, H₂O, CO, CO₂, small aliphatic hydrocarbons and small aromatic compounds.
- 7 • the model of Raj et al. [45] which describes the combustion of gasoline surrogate fuels (n-heptane, iso-octane
8 and toluene). The mechanism has been validated for premixed laminar flames of n-heptane, iso-octane, benzene
9 and ethylene. The measured profiles include H₂, H₂O, CO, CO₂, C₂H₂ and aromatic compounds from benzene
10 (A₁) to coronene (A₇). Its prediction of PAH profiles follows the same trend as that of soot mole fractions in
11 the counterflow flames of n-heptane/toluene and iso-octane/toluene mixtures.
- 12 • the model of Chernov et al. [54] that was designed in order to simulate the formation of soot and its precursors
13 during the combustion of small aliphatic hydrocarbons (C₁ – C₂). It was validated with respect to the profiles
14 of small hydrocarbons (such as C₂H₂) and aromatic compounds in methane/oxygen, C₂H₆-CO₂-Ar premixed
15 flames and soot volume fractions in methane, ethane and ethylene co-flow flames.

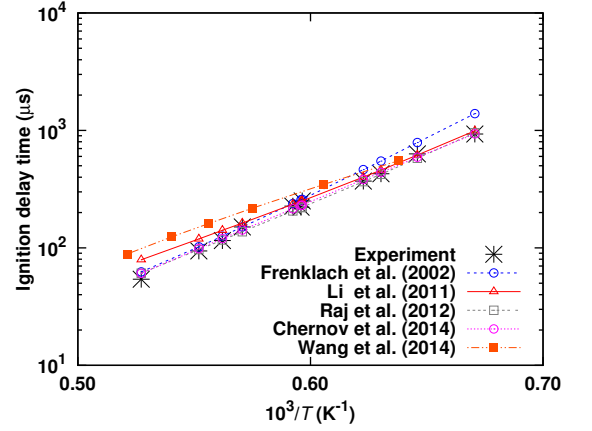
16 We also used a partially reduced mechanism developed by Wang et al. [55] to simulate PAH and soot formation
17 during the combustion of n-dodecane. It was validated through the ignition delay times of n-dodecane and n-decane
18 in shock tubes, species profiles measured during the combustion of n-dodecane in a jet stirred reactor and shock tube
19 (O₂, CO, CO₂, C₂H₂, C₂H₄, A₁) as well as soot volume fractions in n-dodecane spray flames.

As we want to predict the formation of aromatic species during the combustion of CH₄ – CO₂ blends, we first compared the performances of these five reaction mechanisms with respect to ignition delay times related to various mixtures involving methane, carbon monoxide and carbon dioxide. The simulations of the shock tube measurements have been carried out with the software Homrea [56–58] using homogeneous zero-dimensional assumptions involving a constant volume V and a constant internal energy U. For each experimental study we considered, we defined the modelled ignition delays in the same way the original authors had measured them (e.g. as corresponding to the peak concentration of OH or the greatest slope of $p(t)$). For a series of n measurements, we systematically calculated the *average logarithmic distance* between model and experiment $d(p)$

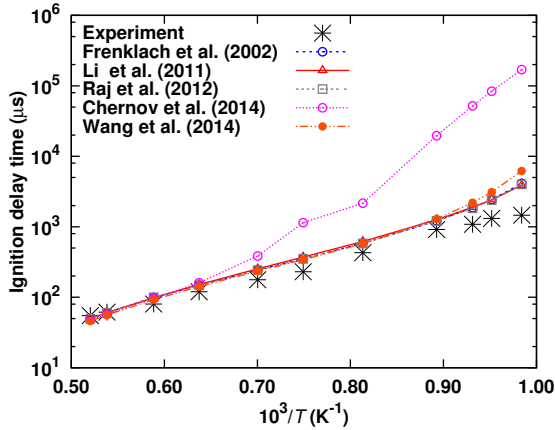
$$d(p) = \frac{\sum_{i=1}^n |\log_{10}(\tau_{m,i}) - \log_{10}(\tau_{e,i})|}{n}$$



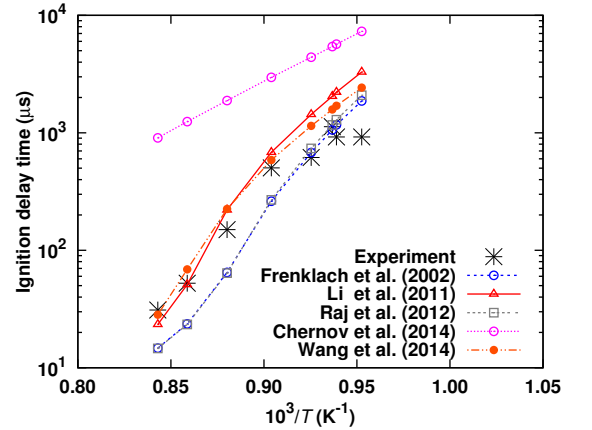
(a) Methane, first series



(b) Biogas, first series



(c) Bio-syngas, first series



(d) CH₄- CO- CO₂ -H₂, first series

Figure 1: Experiments and model predictions

1 whereby $\tau_{m,i}$ and $\tau_{e,i}$ are the model prediction and the experimental measurement, respectively.

2 4. Test and comparison of the reaction mechanisms

3 To test and compare the reaction mechanisms, we considered experiments involving methane [59], biogas [60],
 4 bio-syngas [61] and various mixtures of CH₄, CO, CO₂, and H₂ [62]. Due to space limitation, we only summarised our
 5 results here. The reader is referred to **Supplementary material 1 - Model Evaluation** to see the experimental
 6 conditions, the detailed results and their interpretation. Figure 1 illustrates the results we obtained. Table 1 contains
 7 the values of the average discrepancies. It can be seen that the model of Li et al. [53], closely followed by that of
 8 Raj et al. [45], leads to the best average performance. For methane mixtures, the results of Li et al. ($d = 0.084$) are
 9 slightly better than those of Raj et al. ($d = 0.095$). For biogas mixtures, the predictions of Raj et al. ($d = 0.057$) are
 10 marginally superior to the ones of Li et al. ($d = 0.063$). For bio-syngas mixtures, Li et al. provides us with better
 11 results ($d = 0.181$) than Raj et al. ($d = 0.137$). For CH₄ - CO₂ - CO - H₂ blends, the predictions of Li et al. ($d =$

Table 1: Average logarithmic discrepancies

Series	Frenklach et al.	Li et al.	Raj et al.	Chernov et al.	Wang et al.
Methane	0.129	0.084	0.095	0.098	0.216
Biogas	0.081	0.063	0.057	0.059	0.178
Bio-syngas	0.145	0.181	0.137	0.550	0.162
CH ₄ - CO- CO ₂ -H ₂	0.333	0.128	0.334	0.768	0.239
All	0.131	0.101	0.111	0.252	0.189

1 0.128) are largely superior to those of Raj et al. ($d = 0.334$).

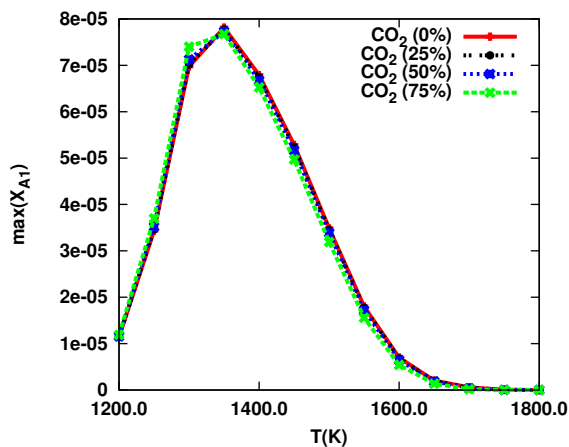
2 We decided to use the mechanism of Li et al. for all our computations. Besides its modest superiority to that of
3 Raj et al. in terms of predictions of the above measurements, it was specifically designed to simulate the pyrolysis
4 and oxidation of toluene. It thus naturally includes many elementary steps describing the growth and oxidation of
5 aromatic compounds. It also involves important reaction pathways responsible for the formation of the first aromatic
6 rings during the combustion of aliphatic fuels, such as those involving the radicals propargyl C₃H₃ [43, 63–66] and
7 cyclopentadienyl c – C₅H₅ [43, 67–69]. While the mechanism developed by Raj et al. presents many of the same
8 advantages, it contains considerably more reactions, most of which accounting for the combustion of n-heptane and
9 iso-octane. They would considerably raise the computational burden without playing any role in the burning of
10 CH₄ – CO₂ and CH₄ – CO mixtures.

11 5. Isothermal mixtures highly diluted in Argon

12 5.1. Numerical results

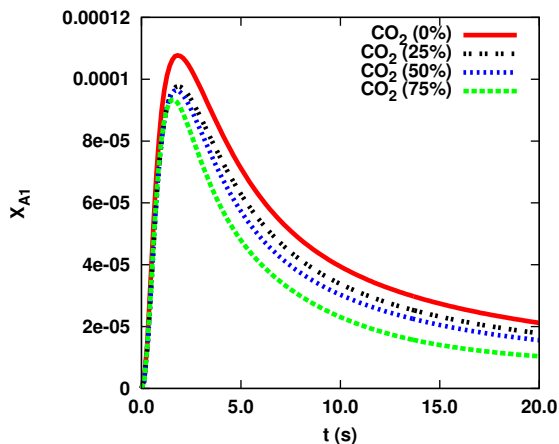
13 In order to decouple the chemical kinetics from the thermodynamics, we defined a series of **isothermal** trials
14 highly diluted in Argon at atmospheric pressure. Since for any given equivalence ratio, the initial concentration of
15 CH₄, O₂ and N₂ are the same, any differences in the temporal or temperature-dependent profiles of other species can
16 only stem from the *participation of carbon dioxide in chemical reactions* which changes with its initial amount. In
17 what follows, we considered three such equivalence ratios ($\phi = 1$, $\phi = 3$ and $\phi = 5$) and four fuels containing 0%, 25%,
18 50%, and 75% of CO₂. In order to have an overview of such effects, we first represented the temperature-dependent
19 profiles of the variable $\max\{X_{A1}(t), t \in [0; t_{end}]\}$ as illustrated in Figure 2 for a rich mixture ($\phi = 3$), whereby t_{end}
20 = 20 s so as to systematically capture the highest values of benzene mole fractions. The highest values of $\max(X_{A1})$
21 centres generally around a certain temperature T_{max} which equals **1350 K** for $\phi = 3$. Consequently, we decided to
22 perform our comparisons by considering the *temporal profiles* of benzene A₁ computed at the T_{max} corresponding to
23 each stoichiometry.

24 Different trials and errors we conducted showed us this is the optimal way to visualise the chemical effects of CO₂



(a) $\max(X_{A1}(t))$

Figure 2: A_1 formation for $\phi = 3$ under isothermal conditions



(a) X_{A1} at $T = 1450.0$ K

Figure 3: Time-dependent series for $\phi = 5$ at $p = 1$ atm and high dilution in Argon

1 addition on aromatic species, as they follow the same tendencies as benzene. It is important to always remember that
 2 *all computations included in our article* were performed at **atmospheric pressure** as this will not be repeated later
 3 on. Finally, it is noteworthy that the effect of CO_2 addition on the *maximum* mole fraction of benzene is scarcely
 4 noticeable for isothermal calculations, as can be clearly recognised in Figure 2. The temporal profile of benzene has been
 5 computed for $\phi = 5$ and is reported in Figure 3. Further results for $\phi = 1$ and $\phi = 3$ are described in **Supplementary**
 6 **material 2 - Additional results**. It is worth noting we obtained qualitatively similar results while performing
 7 simulations under lean conditions. We did not include them, however, because the very small concentrations of
 8 aromatic compounds make them prone to numerical errors, which results in unnatural discontinuities in the profiles.

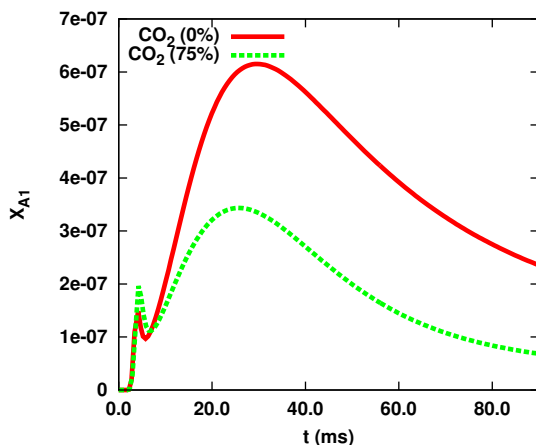


Figure 4: A_1 profiles for $\phi = 3$ and $T = 1700$ K and the same collision efficiencies for CO_2 and AR

1 5.2. Kinetics effects of CO_2 addition on PAH formation

2 As mentioned above, the isothermal conditions involving mixtures highly diluted in the inert gas Argon allow the
 3 chemical kinetics to completely govern the behaviour of the system. An analysis of the above results is following.

4 The formation of benzene is very weak under the stoichiometric combustion of methane with or without added
 5 carbon dioxide. While the highest mole fraction of A_1 remains approximately the same, increasing the initial abundance
 6 of CO_2 delays its formation (see **Supplementary material 2 - Additional results**). For very rich mixtures (Figure
 7 3), the presence of additional carbon dioxide noticeably suppresses the formation of benzene. Thus, our numerical
 8 results strongly suggest that carbon dioxide addition has an inhibitive action on benzene which is purely chemical.
 9 This stands in contrast to studies mainly attributing this inhibition to thermal effects [33, 35, 38]. A possibility which
 10 must be considered is that while CO_2 does not take part in reactions as a reactant, it nonetheless influences the overall
 11 reactivity of the system because it does not have the same collision efficiencies in the definition of third bodies as
 12 Argon, nitrogen or other species. To evaluate the viability of this explanation, *we replaced the collision efficiencies of*
 13 *CO_2 by the values of AR* and compared the temporal profiles of benzene for $\phi = 3$ and $T = 1700$ K obtained with
 14 pure methane and a mixture of 25% of CH_4 and 75% of CO_2 . The results are reported in Figure 4. It can be clearly
 15 recognised that the differences between both profiles remain considerable despite argon and carbon dioxide having the
 16 same collision efficiencies. Comparisons performed under similar conditions deliver the same type of results. This,
 17 in turn, clearly demonstrates that the reaction mechanism predicts the chemical participation of CO_2 to reactions
 18 impacting benzene formation and depletion.

1 5.3. Sensitivity analyses related to A_1

2 To further evaluate the conclusions drawn above, sensitivity analyses were conducted at $T = 1350$ K for $t = 90.0$ s
3 where benzene is being consumed. The analyses were carried out with respect to pure methane and a blend containing
4 25% of methane and 75% of carbon dioxide as this yielded the greatest differences in terms of A_1 formation and
5 depletion. The results are displayed in Figure 5. In both cases, the chemistry related to the propargyl radical C_3H_3
6 proved of great importance to the chemistry of benzene. The peak concentration of A_1 was shown to be sensitive to the
7 reactions $CH_3 + C_2H_2 \rightleftharpoons H + PC_3H_4$, $CH_3 + PC_3H_4 \rightarrow CH_4 + C_3H_3$, $PC_3H_4 \rightarrow H + C_3H_3$, $H + PC_3H_4 \rightarrow H_2 + C_3H_3$,
8 $H + C_3H_3 \rightarrow H_2 + C_3H_2$, and $AC_3H_4 \rightleftharpoons PC_3H_4$. Such a trend has been identified in various other studies [43, 63–66].
9 Unlike these works, the self-recombination of propargyl $C_3H_3 + C_3H_3 \rightarrow A_1$ was found to be considerably less important
10 than $C_3H_3 + PC_3H_4 \rightarrow H + A_1$. The direct chemical action of carbon dioxide manifests itself through the reaction
11 $H + CO_2 \rightleftharpoons OH + CO$ which, in both directions, plays a remarkable role in the CH_4 (25%) - CO_2 (75%) mixture,
12 whereas its influence on benzene concentration is very small in the absence of CO_2 among the reactants. Furthermore,
13 our results confirm the findings of Liu et al. [70] and Mancarella et al. [37], according to which the suppression of soot
14 and PAH goes through an increase in the concentration of OH formed via $H + CO_2 \rightarrow OH + CO$, as the elementary steps
15 $OH + A_1 \rightleftharpoons H_2O + A_1-$, $H_2O + CH_3 \rightarrow OH + CH_4$, $OH + A_1 \rightleftharpoons H + A_1OH$, and $OH + CH_4 \rightleftharpoons H_2O + CH_3$ involving the
16 hydroxyl radical were all shown to significantly influence benzene concentration and have higher sensitivity coefficients
17 under the presence of carbon dioxide.

18 6. Adiabatic combustion

19 6.1. Temperature-dependent profiles for $\phi = 3$

20 Here, the effects of CO_2 addition on the adiabatic combustion of methane undiluted in argon have been investigated.
21 The initial mole fraction of methane always equals 15% whereas the concentration of carbon dioxide and oxygen are
22 varied according to the amount of CO_2 and the equivalence ratio, respectively. The rest of the reactive mixture is
23 constituted of N_2 in the absence of argon.

24 First, the temperature-dependent profile of $max(X_{A_1})$ has been represented at an equivalence ratio of $\phi = 3$. The
25 initial conditions of the adiabatic computations are given in Table 2. It has been contrasted with the isothermal results
26 of section 5 also obtained for $\phi = 3$. The results are shown in Figure 6.

27 A paradoxical effect emerges out of these data. For our isothermal and kinetically-controlled simulations, CO_2
28 addition has nearly no effect on the peak concentration of benzene as function of the temperature (and as we saw,
29 it leads to its kinetic suppression while considering temporal profiles). Yet, at the same time, CO_2 addition under

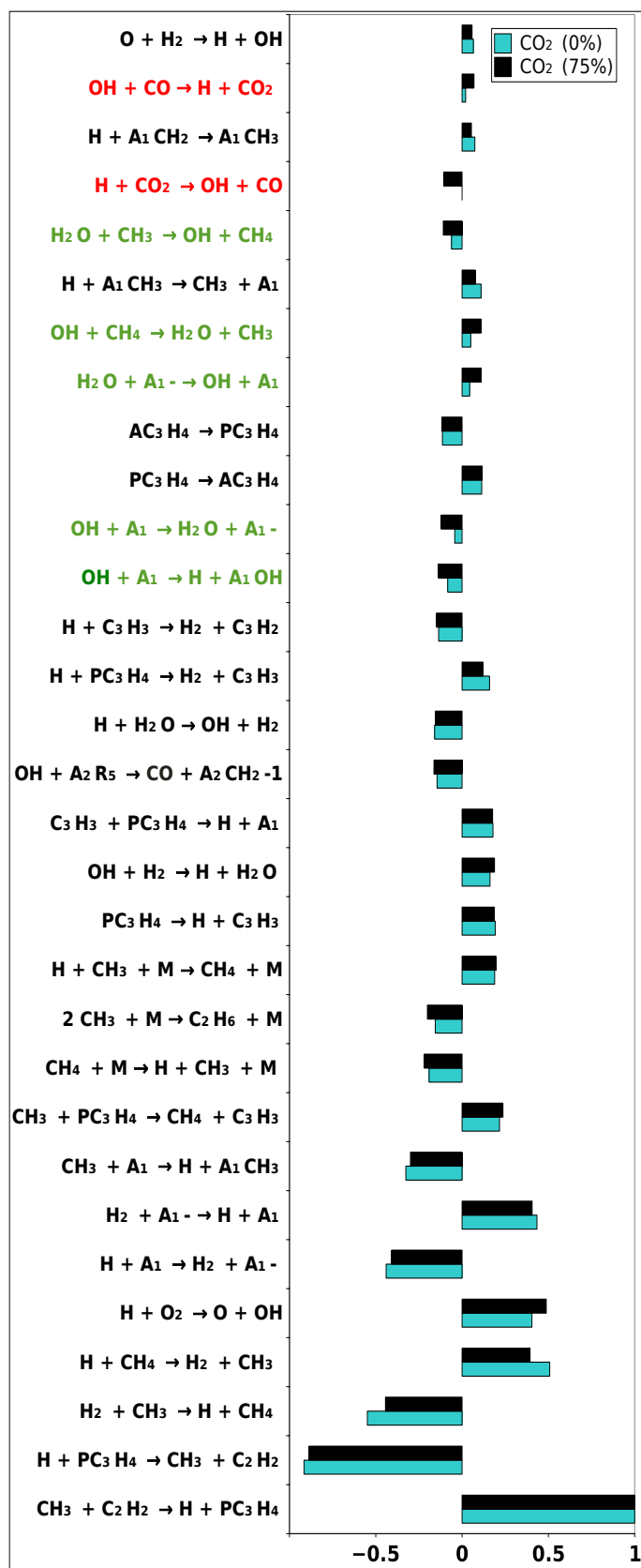


Figure 5: Reaction sensitivity analysis under isothermal conditions

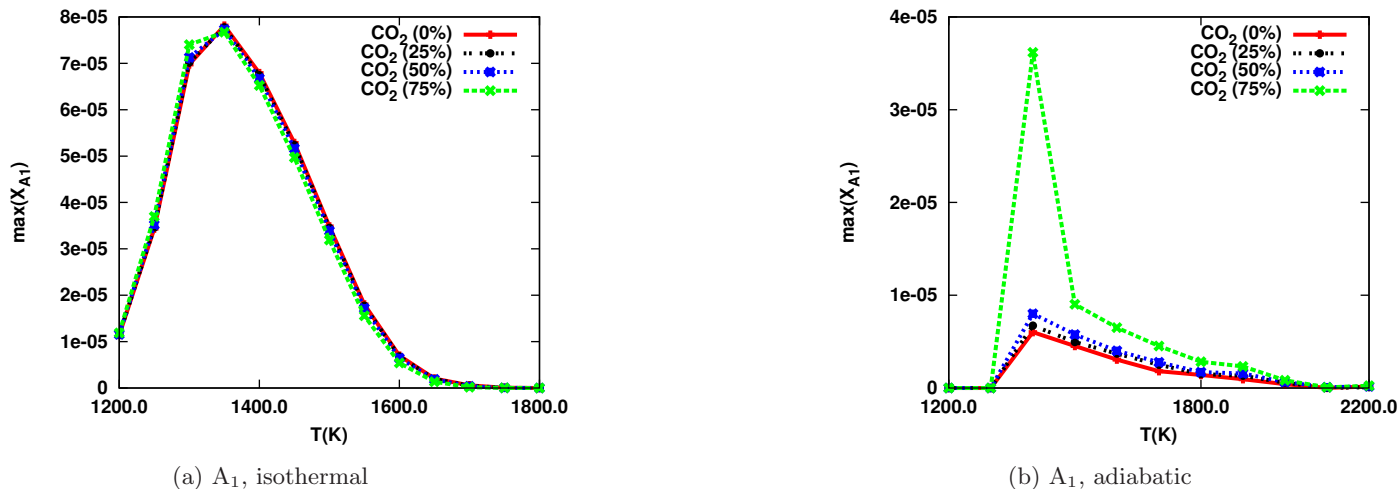


Figure 6: Comparison between isothermal and adiabatic simulations for $\phi = 3$

1 adiabatic conditions leads to an **increase** in the highest mole fraction of A_1 . The maximum concentration of benzene
 2 produced under the presence of 75% of carbon dioxide in the fuel even exceeds that produced in the absence of CO_2
 3 by a factor of 7. To understand the causes of this highly counter-intuitive result, the temporal adiabatic profiles of
 4 X_{A_1} , X_{OH} and $T(t)$ have been computed for $\phi = 3$ in section .

5 *6.2. Adiabatic time-dependent profiles for $\phi = 3$*

6 The temporal profiles of the mole fractions of benzene and the hydroxyl radical and of the temperature have
 7 been computed, based on the conditions of Table 2. They are reported in Figure 10. Considering simultaneously the
 8 evolution of these variables helps explain the peculiar trend discovered above. The presence of a large proportion of
 9 CO_2 among the reactants goes hand in hand with an overall diminution of the temperature, which does not increase
 10 as strongly as in the absence of carbon dioxide in the fuel. This, in turn, is correlated with a considerably smaller
 11 peak concentration of OH which consequently cannot potently oxidise A_1 and its precursors.

12 *6.3. Adiabatic combustion without the kinetics of CO_2*

13 In order to increase our understanding of these phenomena, we carried out the same simulations as in section 5
 14 but without any reactions involving carbon dioxide *as a reactant*. The modifications are reported in **Supplementary**

Table 2: Mole fractions of series 3 (Adiabatic)

CO_2 in Biogas	X_{CH_4}	X_{CO_2}	X_{O_2}	X_{N_2}
0.0%	0.15	0.00	0.10	0.75
25.0%	0.15	0.05	0.10	0.70
50.0%	0.15	0.15	0.10	0.60
75.0%	0.15	0.45	0.10	0.30

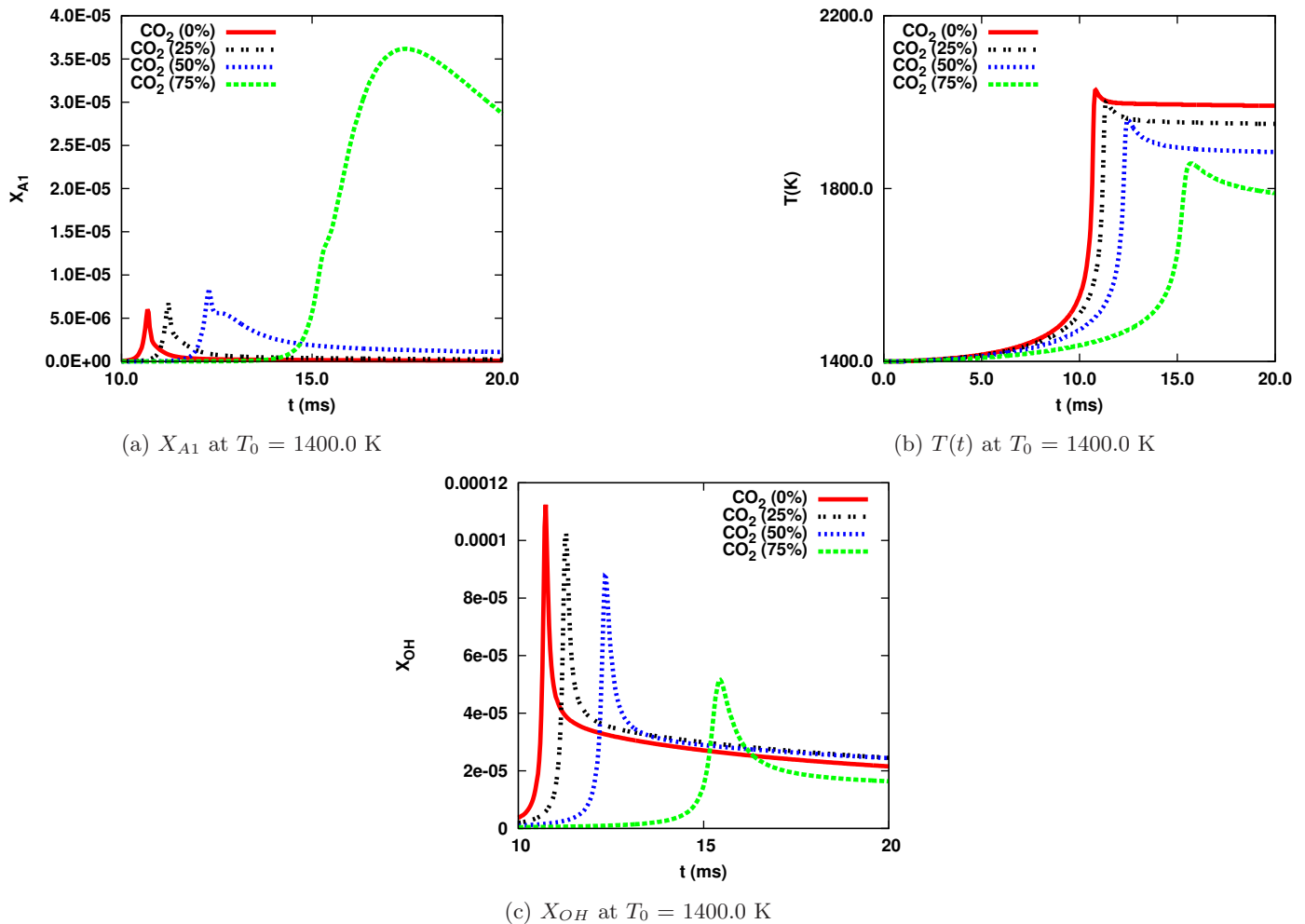
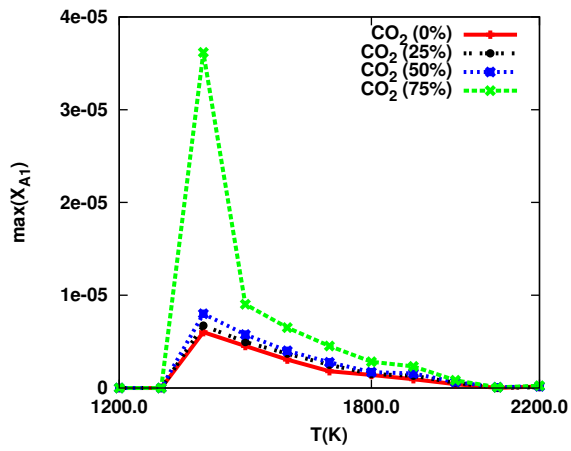
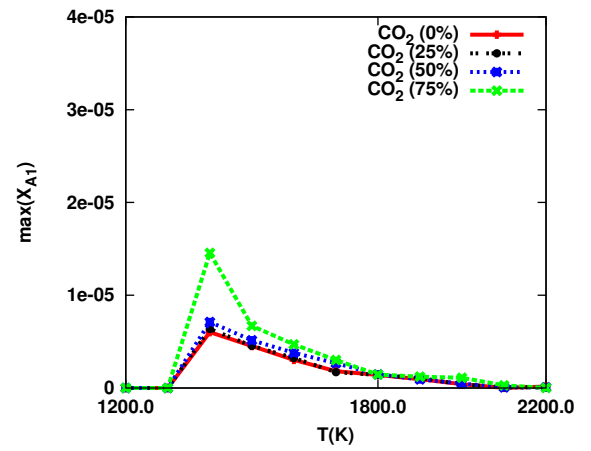


Figure 7: Time-dependent series for $\phi = 3$ at $p = 1$ atm under adiabatic conditions

1 **material 3 - Reactions with CO_2 .** Using the conditions of Table 2, we computed the temperature-dependent
 2 evolution of $\max(X_{A1})$ without CO_2 -kinetics for $\phi = 3$ and $p = 1$ atm under adiabatic conditions. The results are
 3 shown in Figure 8, where they are contrasted with those obtained in Figure 6. Moreover, the temporal profile of
 4 the mole fractions of benzene has been computed in the absence of reactions involving CO_2 as a reactant, based on
 5 the conditions of Table 2, that is $\phi = 3$, $p = 1$ atm and $T_0 = 1400$ K under adiabatic conditions. It is displayed in
 6 Figure 9. Whilst the deactivation does not affect the profile of benzene during the combustion of pure methane and
 7 $CH_4(75\%) - CO_2(25\%)$ blend, it results in a very strong increase of A_1 mole fraction if the fuel contains 75% of carbon
 8 dioxide. This suggests that, under adiabatic conditions, these chemical reactions enhance benzene formation instead
 9 of inhibiting it as they do under isothermal conditions.



(a) X_{A1} with CO_2 -kinetics



(b) X_{A1} without CO_2 -kinetics

Figure 8: Adiabatic combustion with and without CO_2 -kinetics

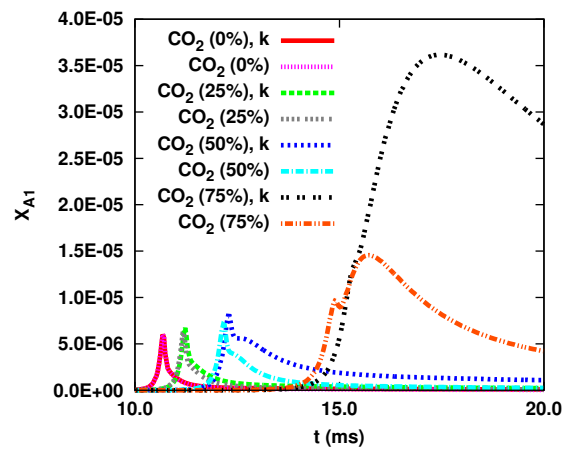


Figure 9: Temporal profile of X_{A1} with and without CO_2 -kinetics

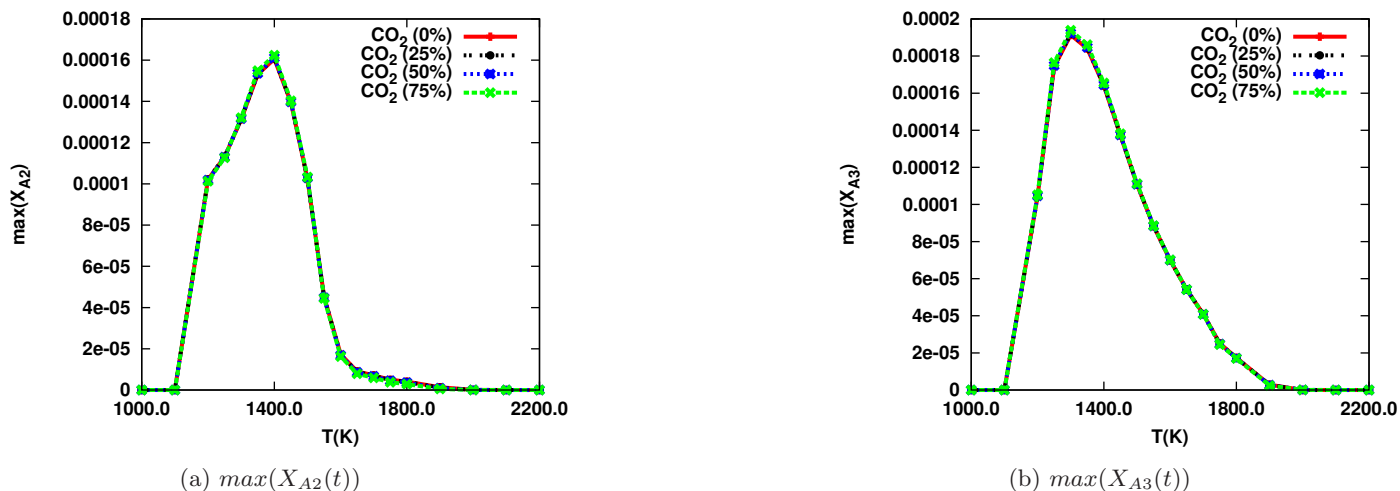


Figure 10: Combustion of A_1 - CO_2 blends for $\phi = 3$

1 7. Chemical effects of CO_2 addition to A_1

2 In spite of all our efforts, we could not find any studies considering the effects of CO_2 -addition to benzene during
3 the combustion of the latter. Rosso et al. [71] investigated the reaction of carbon dioxide radical anion with substituted
4 benzenes in aqueous solutions. There does not appear to be any direct connections between their results and our main
5 concerns in this article. Consequently, we considered it worthwhile to perform a chemical-kinetic isotherm studies of
6 the effects of CO_2 -addition to benzene while using conditions closely similar to those of section 5. The highest mole
7 fractions of naphthalene A_2 and phenanthrene A_3 have been represented as function of the temperature, for $\Phi = 3$,
8 in Figure 10. The addition of carbon dioxide to benzene does not cause any noticeable differences to the chemistry
9 of growing aromatic compounds ultimately leading to soot. The same observation was made while producing similar
10 curves for $\phi = 1$ and $\phi = 5$. The temporal profiles of A_2 , A_3 , A_4 , OH, H and CO have been computed for $\phi = 3$ at $T =$
11 1400 K where the concentrations of the aromatic species reach high values. The results (shown in **Supplementary**
12 **material 2 - Additional results**) confirm that the changes in all these profiles are barely noticeable.

13 The main result of this section is that the addition of carbon dioxide to benzene (and probably also to higher
14 aromatic compounds) has either a very weak or a non-existent influence on the combustion chemistry *if thermodynamic*
15 *effects do not occur*, i.e. the temperature remains constant. As a consequence, during the combustion of biogas, the
16 direct chemical action of carbon dioxide involves reactions preceding the formation of the first aromatic ring, at the
17 level of small aliphatic hydrocarbons and their radicals.

1 8. Conclusion and outlook

2 The goal of the present article was to increase our understanding of the formation of the first aromatic species
3 (represented by benzene) through chemical reactions CO_2 takes part in. In Section 5, we investigated the *purely*
4 *chemical* effects of carbon dioxide addition to methane by performing isothermal simulations involving various CO_2
5 amounts and equivalence ratios. We found that increasing the concentration of carbon dioxide reduces the concentra-
6 tion of benzene. This effect seems to be due to a growth in the concentration of the oxidising radical OH via the step
7 $\text{CO}_2 + \text{H} \rightarrow \text{CO} + \text{OH}$. In section 6, we investigated CO_2 addition under adiabatic conditions. We identified a very
8 peculiar phenomenon: while higher initial concentrations of carbon dioxide under isothermal conditions chemically
9 inhibit the formation of benzene, they *greatly increase* it under adiabatic conditions. We could show this is caused by
10 CO_2 thermally inhibiting the whole combustion, thereby leading to lower levels of OH radicals which cannot oxidise
11 benzene and its precursors as efficiently as when pure methane is burnt. Even more unexpected was our discovery
12 of the two natures of the role plaid by reactions involving CO_2 as reactants (such as $\text{CO}_2 + \text{OH} \rightarrow \text{CO} + \text{HO}_2$ or
13 $\text{CO}_2 + \text{H} \rightarrow \text{CO} + \text{OH}$). While they reduce the concentration of benzene under isothermal conditions, they increase
14 it under adiabatic conditions. Eventually, in Section 7, we found that the direct addition of CO_2 to benzene does not
15 cause any significant differences in the profiles of aromatic species and small radicals. Thus, during the combustion
16 of biogas, CO_2 can only chemically participate in reactions with smaller aliphatic hydrocarbons and radicals which
17 precede steps involving aromatic compounds.

18 While we mainly presented results related to benzene, our computations showed us that the same trends can be
19 observed for higher aromatic species. These, in turn, determine the sooting behaviour of any flames [26–29]. Whilst
20 we had interesting and unexpected results, our study was numerical and qualitative. As a consequence, we believe
21 future kinetic experiments under conditions close to those we considered could prove extremely valuable. Besides con-
22 firming or mitigating the conclusions we drew, they would also provide experimental information for the estimation
23 and optimisation of kinetic parameters. For example, isothermal experiments entirely controlled by chemical kinetics
24 (such as simulated in Section 5) could be performed in flow reactors where the effects of mass diffusion and wall
25 surface reactions can be neglected or accurately assessed [72, 73]. Undiluted adiabatic experiments could, for instance,
26 be conducted in shock tubes where the assumptions of constant volume and constant energy remain approximately
27 valid during the measurements of the profiles [74–76]. We believe that such experiments could prove very valuable
28 to applied scientists and practitioners alike who are interested in assessing the complex consequences of using biogas,
29 both in specific situations and on a global scale.

30

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