

# A structural accounting framework for estimating the expected rate of return on equity

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## Abstract

This paper shows how the expected rate of return (ERR) on equity may be estimated using published accounting results, based on the information dynamics of reported earnings. As accounting-based valuation models conditional upon financial statement articulation lead to a rank deficient system of estimating equations, the paper introduces a nonlinear constraint on the articulation that allows the information system simultaneously to produce an estimate for the ERR by iteration, together with predictions for the key clean surplus forecasts of net earnings, net dividend and the book value of equity. Further decomposition produces estimates of expected capital gain, expected earnings and the expected change in equity book value, and by rearrangement, the expected change in unrecorded goodwill. The clean surplus relation is maintained in the forecast variables. Exploratory data methods are used to examine the nonlinear relationship between components of the accounting-based ERR and realized stock returns. Findings show that realized returns are higher (lower) than estimated ERR in expansionary (recessionary) periods, with evidence of a stronger returns impact in recessionary periods. For the large majority of firms, realized returns revert to the estimated ERR, and the time-varying accounting components are strongly related to future realized stock returns, consistent with time variation in the ERR around a long-run average. Predicted earnings and dividends provide useful additional information on short-run variations in the ERR.

**Keywords:** clean surplus, rank deficiency, expected rate of return, equity valuation.

# 1 Introduction

This paper is concerned with using only key accounting information to estimate the firm's long-run rate of return on its equity. Our analysis differs from previous research by focusing on how information dynamics can be adapted in order to estimate the expected return using published accounting data, in marked contrast to most of the extant accounting literature which reverse-engineers the expected return using earnings forecasts and stock prices.

We apply the Clubb (2013) development of the Ohlson (1995) linear information dynamics framework based on abnormal earnings (residual income) to extract an estimate of the firm's expected rate of return. In order to obtain estimates of the cost of equity under this approach, we convert the abnormal earnings information dynamics into a reported earnings information dynamics and then, following the methods proposed by Christodoulou and McLeay (2014), we introduce a constraint on accounting articulation into the resulting rank deficient equation system, which is then estimated on a firm-by-firm basis.

The linear information system simultaneously produces an estimate for the expected return together with predictions for future earnings, net dividend and closing book value of equity. We then apply the Easton, Harris, and Ohlson (1992) and Penman and Yehuda (2009) decomposition of stock returns to produce an estimate of expected change in price and employ exploratory data methods to examine the non-linear relationship between components of our accounting-based expected return estimates and realized stock returns.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on the use of accounting information to estimate expected returns. Section 3 develops the framework for estimating and analysing the long-run expected rate of return and explains the estimation issues related to implementation of this framework. Section 4 describes the data. Section 5 presents the results and Section 6 concludes.

## 2 Accounting information and expected returns

The relationship between the expected rate of return (ERR) on equity and information about the firm's fundamental economic performance and financial position is a considerably active area of theoretical and empirical research in finance and accounting. Some of this substantial literature is reviewed in papers by Callen (2016), Easton and Monahan (2016) and Penman (2016) in the current issue of this journal, so we limit ourselves here to reviewing only those aspects of the literature that motivate and locate the analysis in the current paper. Indeed, where possible, we exploit insights from these papers to highlight strengths and limitations of the approach to estimating expected equity returns adopted in our study. At the risk of over-simplification, we distinguish between 'finance' and 'accounting' perspectives in the literature on ERR in this section. We start by briefly outlining some important developments from the 'finance perspective' before focusing on issues raised by the 'accounting perspective' of particular relevance to our analysis.

The 'finance perspective' on ERR is focused on the development of asset pricing models to provide theoretical and empirical insights into the drivers of expected returns. In relation to the role of fundamental information, the seminal empirical studies by Fama and French (1992, 1993) highlighted the possibility of accounting information contributing to the identification of priced risk factors, in particular through the book-to-market factor. However, as indicated by Easton and Monahan (2016) in this issue and as discussed in previous work by Berk (1995) on testing asset pricing models, the interpretation of factors

such as the book-to-market and firm size as related to risk is somewhat controversial and lacks unambiguous theoretical backing. More recent asset pricing research, on the other hand, has provided important insight into why fundamentals-based variables, such as book-to-market, may be relevant for forecasting expected returns. Notably, [Campbell and Vuolteenaho \(2004\)](#) and [Campbell, Polk, and Vuolteenaho \(2009\)](#), building on previous theoretical work by [Campbell \(1991, 1993\)](#) on stock return variance decomposition and inter-temporal asset pricing, provide evidence to support the breakdown of the traditional CAPM beta into four components representing the covariance of firm-level returns (decomposed into cash flow news and discount rate news) with market returns (similarly decomposed into cash flow and discount rate news). Their analysis suggests that beta based on the covariance of a firm's stock returns with market cash flow news (referred to as 'bad beta') should be, and appears to be, more highly priced than beta based on the covariance of a firm's stock returns with market discount rate news (referred to as 'good beta'). Their results show that value stocks (with high book-to-market ratios) have had higher bad beta and lower good beta compared to growth stocks (with low book-to-market) since the 1960's, helping to explain the apparently anomalous higher average returns of value stocks during this period.

Research taking the 'finance perspective', such as that based on extensions of the CAPM alluded to above, not only indicates how asset pricing theory can explain the usefulness of accounting-based variables such as book-to-market for forecasting expected returns; perhaps more importantly, this research also makes use of accounting information in the measurement of risk. [Campbell and Vuolteenaho \(2004\)](#) and [Campbell et al. \(2009\)](#) make use of firm-level and market-level VAR models which combine stock return and accounting based data (specifically, book-to-market and accounting return on equity) to estimate cash flow news and discount rate news components of unexpected returns. In other words, income statement and balance sheet data play a major role in the decomposition of a firm's systematic risk in their analysis. Nevertheless, whilst making extensive use of accounting data to estimate beta risk components, the 'finance perspective' ultimately focuses on cash payoffs to investors and does not explicitly highlight conceptual issues in relation to the role of accounting information in predicting the amount, timing and riskiness of future firm cash flows.

The 'accounting perspective' on ERR emphasizes how accounting converts cash flow data into earnings and book values, which can then be used to assess the riskiness of a firm's operations and/or forecast future expected stock returns. An important ingredient in most accounting studies of expected returns (including the current paper) is the so-called 'clean surplus relation' (CSR) which links net dividends paid out to investors to accounting earnings and book value of equity. For example, [Penman \(2016\)](#) utilizes the CSR to analyze the possible role of earnings-to-price as an indicator of firm risk, while also arguing that the accounting 'structure must communicate risk that results in a discount to the denominating price (in the earnings-to-price ratio) to yield a higher expected return that reflects that risk'. [Penman \(2016\)](#) therefore argues for the need to go beyond the limited structure given to accounting by the CSR to gain an understanding of how accounting practices based on the risk-related deferral of income may generate information relevant to the assessment of firm risk. His analysis also highlights a growing empirical literature which investigates accounting information from this perspective and which broadly confirms the expectation that future realized stock returns are higher and riskier (both in terms of volatility and sensitivity to market movements) for firms with higher earnings growth related to earnings deferrals under conservative accounting.

As highlighted by [Easton and Monahan \(2016\)](#) and [Callen \(2016\)](#), there is also a substantial literature from the 'accounting perspective' which uses the residual income valuation model based on the CSR to reverse-engineer ERRs from stock prices or to analyze time variation in ERRs. Whilst the empirical research surveyed by [Callen \(2016\)](#) provides robust support for time-variation in ERRs consistent with

theoretical analysis of [Campbell \(1999\)](#) and [Vuolteenaho \(2002\)](#) (and with research based on the ‘finance perspective’ previously discussed), the large literature on estimation of implied cost of capital’ reviewed by [Easton and Monahan \(2016\)](#) indicates the continued importance of the simple constant discount rate model both for equity valuation and for management investment decision-making. The accounting perspective on expected stock returns in the current paper is consistent with the focus in the implied cost of capital literature on estimating an average or long-run expected rate of return. There are however also some important differences between our approach to ERR estimation and other approaches in the literature which we now briefly summarize before moving to a detailed outline of our research design.

The framework for estimating ERR adopted in the remainder of this paper aims to contribute to the ‘accounting perspective’ on ERR estimation and fundamental performance. More specifically, we use the linear information model based on abnormal earnings, net dividend, and book value of equity from [Clubb \(2013\)](#) to estimate the long-run expected return on equity on a firm-by-firm basis. This is achieved by replacing abnormal earnings with net earnings less lagged book value multiplied by ERR in the information dynamics; applying the CSR parameter constraints identified in [Clubb \(2013\)](#) to the adjusted dynamics; and applying estimation methods to deal with rank deficiency of the resulting equation system as in [Christodoulou and McLeay \(2014\)](#). This approach allows us to estimate a long-run ERR based purely on the firm’s accounting information dynamics and without reference to its stock price, thus avoiding the circularity problem of using reverse-engineered implied cost of capital estimates for equity valuation noted by [Penman \(2016\)](#) in this issue.

Furthermore, as discussed more fully in [Section 3](#), the [Clubb \(2013\)](#) dynamics is based on a generalization of the dynamics used in the seminal work of [Ohlson \(1995\)](#) and [Feltham and Ohlson \(1995\)](#), which facilitates understanding of implicit assumptions regarding dividend displacement and accounting conservatism in our analysis. There are of course some potential limitations associated with our approach. For example, we make simplified assumptions in relation to the information dynamics generating abnormal earnings (in particular, we define abnormal earnings in relation to a risk-adjusted discount rate of return, as opposed to the risk-free rate as advocated in research reviewed by [Callen \(2016\)](#)) and we do not provide an explicit risk-based explanation of our long-run ERR estimates. However, we believe that our focus on estimating the expected rate of return from linear information dynamics based on accounting fundamentals represents a novel approach which highlights the possibility of focusing on a firm’s performance in its product markets, as opposed to the capital market, to derive ERR estimates.

### 3 Structural estimation framework

This section begins with an outline of how accounting information models based on abnormal earnings are to extract estimates of the expected rate of return on equity. After providing key definitions for the clean surplus relation, we explain the rationale for our approach to ERR estimation by relating accounting information dynamics to the key valuation concepts of unbiased and conservative accounting. Second, we show how the assumed accounting information dynamics can be used not only to estimate the ERR but also to forecast the key components of the ERR, including forecast earnings. Third, the econometric issues related to the estimation of the accounting information dynamics and the implied ERR are discussed, and the estimation methods used in the study are explained.

### 3.1 Clean surplus definitions

Clean surplus accounting prescribes the updating of the closing shareholders' equity position  $y_{t+1}$ , given the opening position  $y_t$  plus the intervening period net earnings  $x_{t+1}$  minus the net dividend distribution to shareholders  $d_{t+1}$ :

$$y_{it+1} \equiv x_{it+1} - d_{it+1} + y_{it} . \quad (1)$$

Specifically,  $x_{it}$  is defined as clean surplus comprehensive net earnings of firm  $i$  at time  $t$ ,  $d_{it}$  is dividend payout plus stock repurchases net of proceeds from new share issues, and  $y_{it}$  is book value of equity. The clean surplus relation can be re-expressed in terms of abnormal earnings, i.e. the difference between reported net earnings and the equity capital charge given the knowledge of the expected rate of return,  $x_{it+1}^a = x_{it+1} - r_i y_{it}$ :

$$y_{it+1} \equiv x_{it+1}^a - d_{it+1} + (r_i + 1)y_{it} , \quad (2)$$

where  $r_i$  is expected rate of return (the ERR) for firm  $i$ , which is assumed here to be inter-temporally constant over the firm-specific time period  $T_i$ .

### 3.2 Abnormal earnings valuation, accounting bias and the ERR

The seminal studies by [Ohlson \(1995\)](#) and [Feltham and Ohlson \(1995\)](#) demonstrate how linear information dynamics can be used to derive equity values which exhibit a long-run expected value either equal to book value (unbiased accounting) or in excess of book value (conservative accounting). In other words, these studies demonstrate how linear information dynamics can be used to model the joint impact of product market competition and accounting practices on the relationship between the long-run accounting rate of return (ARR) and the ERR, where unbiased accounting implies long-run convergence of the ARR to the ERR and conservative accounting implies long-run convergence of the ARR to a rate above the ERR.

In the case of unbiased accounting in [Ohlson \(1995\)](#), long-run reversion of the accounting rate of return to the ERR implies that abnormal earnings persistence,  $\omega$ , is less than 1 in the following autoregression of abnormal earnings:<sup>1</sup>

$$x_{it+1}^a = \omega x_{it}^a + \epsilon_{it+1}, \quad (3)$$

where  $\epsilon_{it+1}$  is a mean zero Normal disturbance term. Given that  $0 \leq \omega < 1$ , this model can be viewed as consistent with competitive product markets where the economic rate of return generated by the firm converges to the ERR in the long-run, and cost-based accounting valuation practices ensure that the long-run ARR closely approximates this long-run economic rate of return. Furthermore, equation 3 implies the following comprehensive earnings dynamic:

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<sup>1</sup>The original [Ohlson \(1995\)](#) analysis also allowed for 'other information' variables,  $v_t$ , which are assumed by [Ohlson \(1995\)](#) to converge to zero in the long-run.

$$x_{it+1} = \omega x_{it} + r_i y_{it} - \omega r_i y_{it-1} + \epsilon_{it+1}. \quad (4)$$

Hence it follows that the ERR,  $r_i$ , can be estimated as a composite coefficient on  $y_{it-1}$ , via the autoregression of net earnings, augmented by equity book value at  $t$ , and lagged book value at  $t - 1$ .

The information dynamics represented by equations 3 and 4 may not represent useful earnings forecast models for firms that operate in imperfectly competitive product markets and/or implement accounting valuation procedures that systematically value assets below cost. Such conditions where the long-run ARR is expected to be above the ERR are modelled by [Feltham and Ohlson \(1995\)](#), who assume the following abnormal earnings dynamic:

$$x_{it+1}^a = \omega x_{it}^a + \gamma y_{it} + \epsilon_{it+1}, \quad (5)$$

where  $0 \leq \omega < 1$  and  $\gamma > 0$ . Given that book value is expected to grow at a positive rate (i.e. it is not mean-reverting), then  $\gamma > 0$  implies non-convergence of abnormal earnings to zero in the long-run, which in turn implies ‘accounting conservatism’ where abnormal earnings are positive and expected ARR is greater than ERR. Extraction of the ERR from the [Feltham and Ohlson \(1995\)](#) model is based on:

$$x_{it+1} = \omega x_{it} + (r_i + \gamma) y_{it} - \omega r_i y_{it-1} + \epsilon_{it+1}, \quad (6)$$

where the ERR may be inferred from the regression coefficients for comprehensive earnings at  $t$  and book value of equity at date  $t - 1$  based on equation 6.

While [Feltham and Ohlson \(1995\)](#) provides one approach to modeling conservative accounting based on linear accounting information dynamics, a controversial feature of the implied equity valuation function is that a marginal dollar increase in net dividends reduces equity value by more than a dollar, i.e. that dividend displacement does not hold and net dividends turn out to be a negative indicator of value as a result of the assumption that  $\gamma > 0$ . An alternative approach to modeling conservative accounting highlighted by [Pope and Wang \(2005\)](#) and [Clubb \(2013\)](#), which is consistent with the [Miller and Modigliani \(1961\)](#) dividend displacement property, is to assume that abnormal earnings are generated by the following dynamic process:

$$x_{it+1}^a = \omega x_{it}^a + \phi d_{it} + \epsilon_{it+1}, \quad (7)$$

where the assumption that  $\phi > 0$  implies accounting conservatism, i.e. a long-run ARR above the ERR. The corresponding comprehensive earnings dynamic is:

$$x_{it+1} = \omega x_{it} + \phi d_{it} + r_i y_{it} - \omega r_i y_{it-1} + \epsilon_{it+1}, \quad (8)$$

where ERR is given by the regression coefficient  $r_i$  on book value. Given that equation 8 allows

estimation of long-run expected return on equity under the assumption that long-run ARR may exceed ERR, and avoids the implication of the [Feltham and Ohlson \(1995\)](#) model that net dividends reduce *cum div* equity value, we use equation 8 in our empirical analysis to estimate the ERR, as discussed below.

### 3.3 Estimation model

Our analysis employs the following linear information dynamics from [Clubb \(2013\)](#):<sup>2</sup>

$$x_{it+1}^a = \alpha_1 + \omega_{11}x_{it}^a + \omega_{12}d_{it} + \epsilon_{1it+1} \quad (9a)$$

$$d_{it+1} = \alpha_2 + \omega_{21}x_{it}^a + \omega_{22}d_{it} + \omega_{23}y_{it} + \epsilon_{2it+1} \quad (9b)$$

$$y_{it+1} = \alpha_3 + \omega_{31}x_{it}^a + \omega_{32}d_{it} + \omega_{33}y_{it} + \epsilon_{3it+1} , \quad (9c)$$

All error terms are assumed to be Normal mean zero disturbances. The estimation of the system of equations 9a - 9c requires knowledge of the rate of return on equity  $r_i$  in order to measure abnormal earnings,  $x_{it+1}^a = x_{it+1} - r_i y_{it}$ . However, rewriting the system in terms of reported earnings obviates the need to know  $r_i$ , which instead can be estimated as a free parameter using the restricted least squares approach from [Christodoulou and McLeay \(2014\)](#):

$$x_{it+1} = \alpha_1 + \omega_{11}x_{it} + \omega_{12}d_{it} + r_i y_{it} - \omega_{11}r_i y_{it-1} + \epsilon_{1it+1} \quad (10a)$$

$$d_{it+1} = \alpha_2 + \omega_{21}x_{it} + \omega_{22}d_{it} + \omega_{23}y_{it} - \omega_{21}r_i y_{it-1} + \epsilon_{2it+1} \quad (10b)$$

$$y_{it+1} = \alpha_3 + \omega_{31}x_{it} + \omega_{32}d_{it} + \omega_{33}y_{it} - \omega_{31}r_i y_{it-1} + \epsilon_{3it+1} , \quad (10c)$$

Note that equation 10a corresponds exactly to equation 8. Since earnings are measured on a clean surplus basis, as described in Section 3.1, [Clubb \(2013\)](#) notes that the linear information system of equations 10a - 10c implies that the following parameter restrictions must hold:

$$\alpha_3 = \alpha_1 - \alpha_2 \quad (11a)$$

$$\omega_{31} = \omega_{11} - \omega_{21} \quad (11b)$$

$$\omega_{32} = \omega_{12} - \omega_{22} \quad (11c)$$

$$\omega_{33} = (r_i + 1) - \omega_{23} . \quad (11d)$$

The linear information system described above is a rank deficient system of seemingly unrelated regressions. The system is characterized by a singular variance-covariance (VCE) matrix because it holds that  $0 + E(\epsilon_{1it+1}) - E(\epsilon_{2it+1}) = E(\epsilon_{3it+1})$ ; the inclusion of zero emphasizes that opening equity has zero residual because it is known and conforms with the clean surplus identity of  $y_{it} \equiv x_{it} - d_{it} + y_{it-1}$ . [Greene and Seaks \(1991\)](#) and [Greene \(2011\)](#) explain how such rank deficient systems can be estimated in one of two ways. We may impose the parameter restrictions of equations 11a - 11d and estimate the system of equations 10a - 10c via restricted least squares with the singular VCE. Alternatively, unrestricted least

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<sup>2</sup>The linear information dynamics may be extended to include other value relevant information given the valuation model.



squares can be applied to recover the estimates of two out of three equations, and then the estimates of the third equation are deduced from the known relationship between parameters. In this case, given the model, the predetermined opening equity  $y_{it}$  and the predicted  $\hat{x}_{it+1}$  and  $\hat{d}_{it+1}$ , then the predicted closing equity  $\hat{y}_{it+1}$  is simply deduced by means of the structural requirement of the clean surplus condition governing the accounting variables that are forecasted:<sup>3</sup>

$$y_{it} + \hat{x}_{it+1} - \hat{d}_{it+1} \equiv \hat{y}_{it+1} . \quad (12)$$

This articulation of predictions makes it clear that it is only necessary to estimate two out of three equations from the rank deficient system of linear information dynamics in equations 10a - 10c. Equation 12 can also be expressed in terms of abnormal earnings, without needing to predict abnormal returns as intended in the original linear information dynamics:

$$(1 + \hat{r}_i)y_{it} + \hat{x}_{it+1}^a - \hat{d}_{it+1} \equiv \hat{y}_{it+1} . \quad (13)$$

The individual predictions are analysed in detail in Section 5. The system of accounting information dynamics is not only rank deficient in its equations, but also contains rank deficient design matrices  $\mathbf{X}$ , and the common presence of  $r_i$  places additional cross-equation restrictions in non-linear estimation. These econometric implications are discussed further in Section 3.5.

### 3.4 ERR structure

Given an estimate for  $\hat{r}_i$  from equations 10a - 10c, it is possible to use the relationship between observed stock returns and accounting earnings from Easton et al. (1992) and Penman and Yehuda (2009) to separate  $\hat{r}_i$  into forecast components for the expected earnings yield, the expected percentage change in market value of equity, and the expected change in book value to equity with respect to its current market value, where the decomposition follows from the clean surplus identity of equation 1:

$$\hat{r}_i = \frac{\hat{p}_{it+1} - p_{it}}{p_{it}} + \frac{\hat{d}_{it+1}}{p_{it}} = \frac{\hat{x}_{it+1}}{p_{it}} + \frac{\hat{p}_{it+1} - p_{it}}{p_{it}} - \frac{\hat{y}_{it+1} - y_{it}}{p_{it}} , \quad (14)$$

where  $\hat{\cdot}$  denotes predictions formed at  $t$  for  $t + 1$ ,  $\hat{r}_i$  is a parameter estimate from the linear information dynamics of equations 10a - 10c,  $\hat{x}_{it+1}/p_{it}$ ,  $\hat{d}_{it+1}/p_{it}$  and  $\hat{y}_{it+1}/p_{it}$  are the articulated predictions for reported earnings, net dividend and book value of equity, respectively, and  $p_{it}$  is market value of equity. The decomposition in equation 14 reflects the Easton et al. (1992) proposition that the expected return is driven by expected earnings adjusted for the change in expected unrecorded goodwill, as represented by the two last terms in equation 14. Note that the last term representing expected capital gain is estimated simply by rearranging the equation as follows:

<sup>3</sup>Greene (2011) discusses rank deficient systems of equations, explaining how the estimation of  $K - 1$  equations is sufficient for recovering the estimates of the  $K^{th}$  equation.

$$\frac{\hat{p}_{it+1} - p_{it}}{p_{it}} = \hat{r}_i - \frac{\hat{x}_{it+1}}{p_{it}} - \frac{\hat{y}_{it+1} - y_{it}}{p_{it}}. \quad (15)$$

### 3.5 Restricted non-linear least squares

The estimation model given by equations 10a - 10c is non-linear in its parameters and is estimated via iterative generalized non-linear least squares (IFGNLS), which is equivalent to maximum likelihood estimation (MLE) with multivariate normal disturbances across equations. It is important to iterate towards convergence to MLE, because the solution is invariant to choosing to estimate two out of three equations. As mentioned in Section 3.3, the third can be deduced using the parameter relations in equations 11a-11d, given the known opening equity and the singular system of linear information dynamics for clean surplus items.

The stacked system of equations represents a seemingly unrelated regression (SUR) estimator. In the absence of an integrated system, the SUR estimator makes the assumption of cross-equation error correlation for the  $it^{th}$  observation,  $E(\epsilon_{1it+1}, \epsilon_{2it+1}) \neq 0$ , and produces more efficient estimates than simple NLS when the equations are non-identical and non-nested (Zellner 1962; Zellner and Huang 1962; Zellner 1963).<sup>4</sup> The other key advantage to SUR is that it allows the imposition of cross-equation parameter constraints as required by the analytical framework. As with linear least squares, so too the consistency of NLS results requires proper specification so that the zero conditional mean assumption is satisfied,  $E(u|\mathbf{X}) = 0$ ; hence the inclusion of model intercepts,  $\alpha_k$ , not originally specified in the analytical work of Clubb (2013).

Estimation is performed at the firm level for  $T_i \geq 30$ . There is no need for robust correction of the VCE matrix because the estimated standard errors of the individual parameter estimates are not of interest under rank deficiency, as we cannot investigate individual statistical significance (Greene and Seaks 1991). We may only evaluate collective model significance, e.g. the portion of explained variability, which is reported in the table of estimates in the Appendix, for each firm-specific set of estimates.<sup>5</sup>

### 3.6 Parameter interpretation

As discussed above, given the clean surplus data-generating process, the design matrices  $\mathbf{X}$  of the regression equations 10a - 10c are rank deficient and, accordingly, estimation is only feasible via the imposition of parameter constraints. In this system, estimation is attainable through the imposition of the structural non-linear constraints of equations 11a - 11d, given the assumed linear information dynamics of equations 10a - 10c.

Christodoulou (2015) explains that the interpretation of individual estimated parameters is only meaningful if the imposed restriction identifies the simultaneous estimation of all slope parameters of the rank deficient  $\mathbf{X}$ , and the restriction is economically justified on the basis of the assumed valuation model.

However, regardless of the choice of parameter restriction towards achieving identification, the predicted values recovered from a rank deficient  $\mathbf{X}$  remain the same. Hence, the predictions  $\hat{x}_{it+1}$  and  $\hat{d}_{it+1}$ , and consequently of  $\hat{y}_{it+1}$  and  $\hat{x}_{it+1}^a$ , are still valid even if we were to question the underlying economic theory suggesting the above non-linear parameter relation and to dispute the interpretation of

<sup>4</sup>However, SUR assumes no correlation for  $E(\epsilon_{1it}, \epsilon_{2jt}) = 0$  for  $i \neq j$  and  $E(\epsilon_{1it}, \epsilon_{2is}) = 0$  for  $t \neq s$ .

<sup>5</sup>The predicted values suggested by equation 12 are valid even under singular VCE and the rank deficient design matrices  $\mathbf{X}$  (for discussion with examples see Christodoulou 2015).

the individual point estimates for  $\hat{r}_i$ .

The economic interpretation of the estimated parameters must account for the non-linear structure. Specifically, the parameter of the expected rate of return,  $r_i$ , is defined as follows:

$$\frac{\partial E(x_{it+1}|y_{it})}{\partial y_{it}} = r_i = -\frac{\partial E(x_{it+1}|y_{it-1})/\partial y_{it-1}}{\partial E(x_{it+1}|x_{it})/\partial x_{it}} = -\frac{\partial E(d_{it+1}|y_{it-1})/\partial y_{it-1}}{\partial E(d_{it+1}|x_{it})/\partial x_{it}}, \quad (16)$$

where  $r_i$  is equal to the marginal change in forward earnings given a change in current equity investment. At the same time,  $r_i$  is equal to the ratio of two partial derivatives, suggesting a constant marginal rate of substitution between the sensitivity of the next period's expected earnings  $x_{it+1}$  to a change in initial equity investment  $y_{it-1}$  and the sensitivity of  $x_{it+1}$  to a marginal sacrifice in current earnings  $x_{it}$ . The rate of substitution between this period's and the next period's net earnings in the denominator suggests a negative sign, hence the positive expected rate of return. The same marginal rate of substitution holds for  $d_{it+1}$ .

Finally, we suspect a non-linear relation between the accounting-based predictions of earnings, net dividend and book equity and the realized change in the market value of equity, as there is a known asymmetric *S*-shaped market response in interpreting earnings surprises (forecast deviations from earnings releases). Hence, a similar non-linear interpretation should be pertinent to the accounting fundamentals that give rise to earnings surprises. For this reason, rather than imposing an expected functional form on the relation between realized price changes and the components of equation 14, this paper will employ exploratory data methods to examine how the accounting-based estimates relate to market realisations.

## 4 Data

Annual financial statement and price data are obtained from Compustat for US non-financial and classified equities (i.e. exclude SIC codes 6000-6999 and 9000-9999), over the period 1964-2011. The clean surplus variables of equation 1 are defined as in Nissim and Penman (2001) and Penman and Yehuda (2009). The book value of equity  $y_{it}$  is defined as the common shareholders' residual claim on net operating assets, and is calculated as total common equity (item *ceq*) plus preferred treasury stock (item *tstkp*) minus preferred dividends in arrears (item *dvpa*). Comprehensive earnings  $x_{it}$  is defined as net income (item *ni*) minus preferred dividends (item *dvp*) plus the change in the marketable securities adjustment (item *msa*) minus the change in the cumulative translation adjustment in retained earnings (item *recta*). The net dividend  $d_{it}$  offsets dividend distributions with stock repurchases net of share issues and other transactions with shareholders as owners, and is deduced from the clean surplus identity.

All clean surplus variables are expressed per share by dividing by common shares outstanding (item *csho*) and also expressed in terms of price yield, i.e. deflated by opening price at financial year-end (item *prcc.f*). The initial sample of 34,309 observations comprises of 882 firm-specific time series with  $T_i \geq 30$ . This sample is screened for multiple outliers for the multivariate distribution  $f(x_{it}, d_{it}, y_{it})$  using the Hadi (1992, 1994) filter applied at the 5% level of significance. The filter detects 1,827 multivariate outliers, which are excluded from the analysis. The re-application of the sample selection criterion of  $T_i \geq 30$  further reduces the dataset to the estimation sample of 29,569 observations, comprising of 769 firms with  $30 \leq T_i \leq 47$ .

Table 1: Estimation sample summary statistics

$N = 29,569$	Min	$Q_1$	$Q_2$	Mean	$Q_3$	Max	St.dev.
<b>Clean surplus observables</b>							
$y_{it}/p_{it}$	-1.783	0.423	0.653	0.744	0.967	3.320	0.463
$x_{it+1}/p_{it}$	-0.358	0.048	0.076	0.082	0.114	0.512	0.079
$d_{it+1}/p_{it}$	-1.418	-0.033	0.073	0.085	0.198	1.699	0.284
$y_{it+1}/p_{it}$	-1.478	0.424	0.653	0.740	0.963	3.276	0.454
<b>Market observables</b>							
$(p_{it+1} - p_{it})/p_{it}$	-0.959	-0.167	0.033	0.066	0.240	26.923	0.426
$r_{it+1}^* = (p_{it+1} + d_{it+1} - p_{it})/p_{it}$	-1.908	-0.118	0.113	0.151	0.363	27.444	0.488
<b>Predictions</b>							
$\hat{y}_{it+1}/p_{it}$	-1.350	0.475	0.685	0.740	0.937	3.146	0.387
$\hat{x}_{it+1}/p_{it}$	-0.357	0.055	0.075	0.082	0.102	0.482	0.049
$\hat{d}_{it+1}/p_{it}$	-1.200	-0.005	0.058	0.085	0.150	1.711	0.180
$\hat{x}_{it+1}^a/p_{it}$	-0.330	0.010	0.033	0.038	0.059	0.697	0.058
$(\hat{p}_{it+1} - p_{it})/p_{it}$	-1.602	-0.112	-0.000	-0.028	0.080	1.284	0.190
$\Delta[(\hat{p}_{it+1} - \hat{y}_{it+1})/p_{it}]$	-2.634	-0.123	0.005	-0.000	0.129	3.398	0.322

Note: The statistics for observables and predictions are summarised for the total pooled sample of 29,569 observations. The clean surplus relation is reflected in the arithmetic means:  $0.744 + 0.082 - 0.085 = 0.740$ . The predictions are obtained as per equations 10. Table 3 reports descriptive statistics for the firm-specific parameter estimates, sample sizes and degree of explanatory power associate with these predictions.

Table 1 gives a summary of key statistics over the entire estimation sample, for the observable clean surplus components, market capital gain and market rate of return, as well for their corresponding predictions. Table 2 reports the rank correlations for observables, between all reported clean surplus variables and the realized market rate of return. The table in the Appendix gives the firm-specific samples.

Table 2: Rank correlations for observables

$N = 29,569$		Spearman's $\rho$				
		$y_{it}$	$x_{it+1}$	$d_{it+1}$	$y_{it+1}$	$r_{it+1}^*$
<b>Kendall's <math>\tau</math></b>	$y_{it}$	1	0.4376	0.3869	0.8350	0.3217
	$x_{it+1}$	0.3130	1	0.1392	0.5207	0.3448
	$d_{it+1}$	0.2729	0.0996	1	-0.0838	0.4788
	$y_{it+1}$	0.6580	0.3815	-0.0541	1	0.1419
	$r_{it+1}^*$	0.2206	0.2418	0.3419	0.0985	1

Note: All variables are scaled by  $p_{it}$ . The rank correlations are reported for the pooled total estimation sample of 29,569 observations. The Spearman's  $\rho$  is reported above the diagonal, and Kendall's  $\tau$  is reported below the diagonal. All rank correlations are significant at the  $\alpha = 0.01$  level of significance.

## 5 Analysis

Table 3 provides a summarized report of key statistics for the estimated parameters from equations 10a - 10c, the coefficients of determination and firm-specific sample sizes. For the expected rate of return,  $\hat{r}_i$ , 85.44% of firms are recovered with quite reasonable estimates within the range of 0 to the maximum of 0.2134. The remaining 14.56% of firm-specific  $\hat{r}_i$  are recovered with a negative sign. The mean estimate is 0.0573 and the median is 0.0707. Past studies have chosen not to report on negative estimates of the expected rate of return, as it contradicts the intuition in economic theory that firms would not plan ahead to reduce their market value. As a result, the convention in published work to date is to use reduced datasets that are consistent with this intuition, particularly by excluding from the estimation sample any firm-year observation for which the forecast is negative, which would generate a negative estimated expected return. However, we do not place such restrictions on the explanatory variables, and report both positive and negative  $\hat{r}_i$ , which gives a more realistic reflection of realized returns, acknowledging at the same time the underlying complexity of the empirical study. Nevertheless, the advantage is that long-run estimates of the rate of return are retrieved from uninterrupted firm-year series, whereas much research in this area cannot draw generalised conclusions regarding the longer term given that specific firm-years are discarded from estimation when convenient, resulting in incomplete time series.

Table 3: Summary statistics for the 769 firm-specific estimates

	Min	Q <sub>1</sub>	Q <sub>2</sub>	Mean	Q <sub>3</sub>	Max	St.dev.
<b>Expected rate of return</b>							
$\hat{r}_i$	-0.345	0.029	0.071	0.057	0.098	0.213	0.068
<b>Parameter estimates</b>							
$\hat{\alpha}_1$	-0.123	0.006	0.020	0.022	0.037	0.156	0.031
$\hat{w}_{11}$	-0.697	0.049	0.218	0.228	0.415	0.964	0.270
$\hat{w}_{12}$	-0.254	-0.006	0.021	0.018	0.051	0.184	0.056
$\hat{\alpha}_2$	-0.946	-0.282	-0.171	-0.208	-0.093	0.111	0.160
$\hat{w}_{21}$	-2.553	0.230	0.809	0.980	1.545	6.925	1.146
$\hat{w}_{22}$	-1.850	-0.088	0.035	0.026	0.142	0.743	0.199
$\hat{w}_{23}$	-1.697	0.208	0.300	0.328	0.437	1.141	0.208
$\hat{\alpha}_3$	-0.022	0.115	0.195	0.229	0.304	1.094	0.161
$\hat{w}_{31}$	-7.085	-1.325	-0.586	-0.752	-0.013	3.367	1.204
$\hat{w}_{32}$	-0.774	-0.125	-0.010	-0.009	0.110	1.855	0.200
$\hat{w}_{33}$	-0.204	0.618	0.757	0.729	0.867	2.499	0.204
<b>Explanatory power and <math>T_i</math></b>							
$R_x^2$	0.002	0.218	0.339	0.364	0.512	0.854	0.196
$R_d^2$	0.015	0.275	0.361	0.369	0.447	0.808	0.132
$R_y^2$	0.062	0.436	0.593	0.570	0.725	0.948	0.192
$T_i$	30	35	39	39.14	43	47	5.108

Note: The statistics for parameter estimates and explanatory power reflect frequency-weighted summaries over firm-specific estimation samples. The realised rate of return  $r_{it+1}^*$  is from equation 17. The expected rate of return  $\hat{r}_i$  and the  $\hat{w}$  parameters are estimated as per equations 10.  $R_x^2$ ,  $R_d^2$  and  $R_y^2$  are the associated  $R$ -squares. It holds that  $\alpha_3 = \alpha_1 - \alpha_2$ ,  $w_{31} = w_{11} - w_{21}$ ,  $w_{32} = w_{12} - w_{22}$  and  $w_{33} = 1 + r - w_{23}$ .  $T_i$  is the firm-specific sample size. Table 1 reports the estimation sample statistics for the resulting predictions.

The Appendix reports a more detailed table of the 769 firm-specific estimates for the ERR,  $\hat{r}_i$ , with the respective sample sizes  $T_i$  and the  $R$ -squares. The explanatory power for the firm-specific regressions given  $T_i > 30$  is reasonably high.<sup>6</sup> The Appendix table also reports the firm-specific median value of the Hou, van Dijk, and Zhang (2012) composite measure of the implied cost of capital (ICC).<sup>7</sup> By means of comparison, the median of the ratio of our  $\hat{r}_i$  over the Hou, van Dijk, and Zhang (2012) median composite ICC measure is 0.9593 and the mean of this ratio is 1.0914, hence suggesting relatively close average estimates to the composite ICC, but not necessarily on a firm-by-firm basis.

### Evaluating $\hat{r}_i$ against its realisations

The estimated ERR,  $\hat{r}_i$ , with its predicted components from equation 14 are evaluated against its respective realisations:

$$r_{it+1}^* = \frac{d_{it+1}}{p_{it}} + \frac{p_{it+1} - p_{it}}{p_{it}} = \frac{x_{it+1}}{p_{it}} - \frac{y_{it+1} - y_{it}}{p_{it}} + \frac{p_{it+1} - p_{it}}{p_{it}}, \quad (17)$$

Figure 1 shows the year-specific difference between the realized rate or return  $r_{it+1}^*$  and the expected rate of return,  $\hat{r}_i$ , in terms of middle 90% (the 95<sup>th</sup> minus the 5<sup>th</sup> percentile), the interquartile range (IQR), the median and the arithmetic mean estimates. During recessionary periods,  $\hat{r}_i$  is much higher than  $r_{it+1}^*$  and during expansionary periods the opposite holds. This suggests a steeper impact on the market's realised rate of return when taking in bad news by comparison to good news, generally consistent with the Kahneman and Tversky (1979) Prospect Theory.<sup>8</sup>

To test the effectiveness of our accounting data-driven method to estimating a representative rate of expected return, we test whether the realized rate of returns,  $r_{it+1}^*$ , revert to their long-run average,  $\hat{r}_i$ , as estimated by the linear information system of equations 10a - 10c. If  $\hat{r}_i$  is a reasonable estimate of the long-run rate of the ERR, then the mean-reversion of the realized rate of return to  $\hat{r}_i$  should be strong. Returns are stationary over time and mean-reverting, hence a mean-reverting test can be stated in discrete time as a test for a stationary process (Cochrane 2001). In this respect, we may rewrite  $r_{it+1}^*$  as a weighted average of its past value and its expectation:

$$r_{it+1}^* = (1 - \phi_i) r_{it}^* + \phi_i \hat{r}_i + \epsilon_{it+1} \quad (18)$$

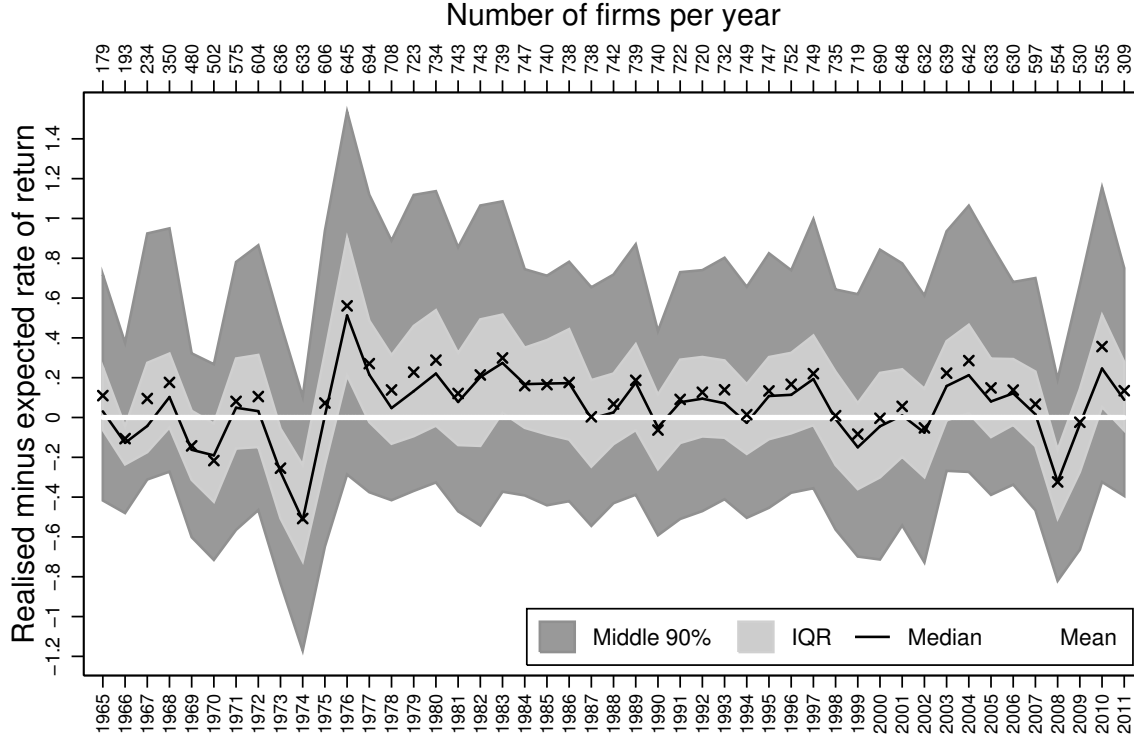
where  $\epsilon_{it+1}$  is a random Normal shock. The null hypothesis for mean-reversion is stated as  $H_0 : -\phi_i = -1$ . Given that  $\hat{r}_i$  is a firm-specific estimate, the regression test specified in equation 18 amounts to a regression on a slope coefficient with inverted sign. For the level of significance  $\alpha = 0.01$ , we fail to reject the null for 580 out of 769 firm-specific time series. Hence, the mean-reversion test suggests that for 75.42% of the firms with  $T_i > 30$  during 1964-2011, the future realized rate of return,  $r_{it+1}^*$ , reverts

<sup>6</sup>A more complete report, including firm-specific estimates for the  $\omega$  parameters from equations 10a - 10c, is available upon request.

<sup>7</sup>This composite measure is the average ICC as estimated by Easton (2004), Gordon and Gordon (2002), Claus and Thomas (2001), Gebhardt, Lee, and Swaminathan (2001), and Ohlson and Juettner-Nauroth (2005). With thanks to Kewei Hou for providing the firm-specific estimates from Hou, van Dijk, and Zhang (2012).

<sup>8</sup>Using the National Bureau of Economic Research definition of business cycle, the list of US recessions during the time period under investigation include Dec 1969-Nov 1970, Nov 1973-Mar 1975, Jan-July 1980, July 1981-Nov 1982, July 1990-Mar 1991, March 2001-Nov 2001, and Dec 2007-June 2009; see <http://www.nber.org/cycles/recessions.html>.

Figure 1: Difference between realized and expected rate of return per year



The  $y$ -axis gives the year-specific differences between the realized rate of return  $r_{it+1}^*$  from equation 17 and the expected rate of return  $\hat{r}_i$ . The differences are shown for the middle 90% (the 95<sup>th</sup> minus 5<sup>th</sup> percentile), the interquartile range (IQR), the median and the arithmetic mean estimates. The superimposed white line indicates  $r_{it+1}^* - \hat{r}_i = 0$ . The bottom  $x$ -axis shows the timeline where the top  $x$ -axis reports the year-specific sample size (number of firms per year).

to  $\hat{r}_i$  on average, and that past changes in price do not help predict future returns in the market. The Appendix reports the firm-specific estimates for  $\hat{\phi}_i$ .<sup>9</sup>

### Evaluating the components of $\hat{r}_i$

To evaluate the relation between the components of the predicted ERR,  $\hat{r}_i$  (equation 14), against the respective realisations as well against the realized rate of return,  $r_{it+1}^*$  (equation 17), we apply the exploratory method of *portfolio smoothing*. The total sample of  $N = 29,569$  observations is grouped into 296 portfolios each containing 99 or 100 observations summing up to the total. For every  $y = f(x)$  relation that is graphed, we first calculate the median expectation for the  $x$ -axis variable per portfolio, and on the basis of this median we calculate the three quartiles of the  $y$ -axis variable.<sup>10</sup> The resulting graph is a smoothed summary of central tendency for consecutive localities of the bivariate distribution. In addition, we plot the non-parametric estimates for cubic splines with five knots in order to assess the

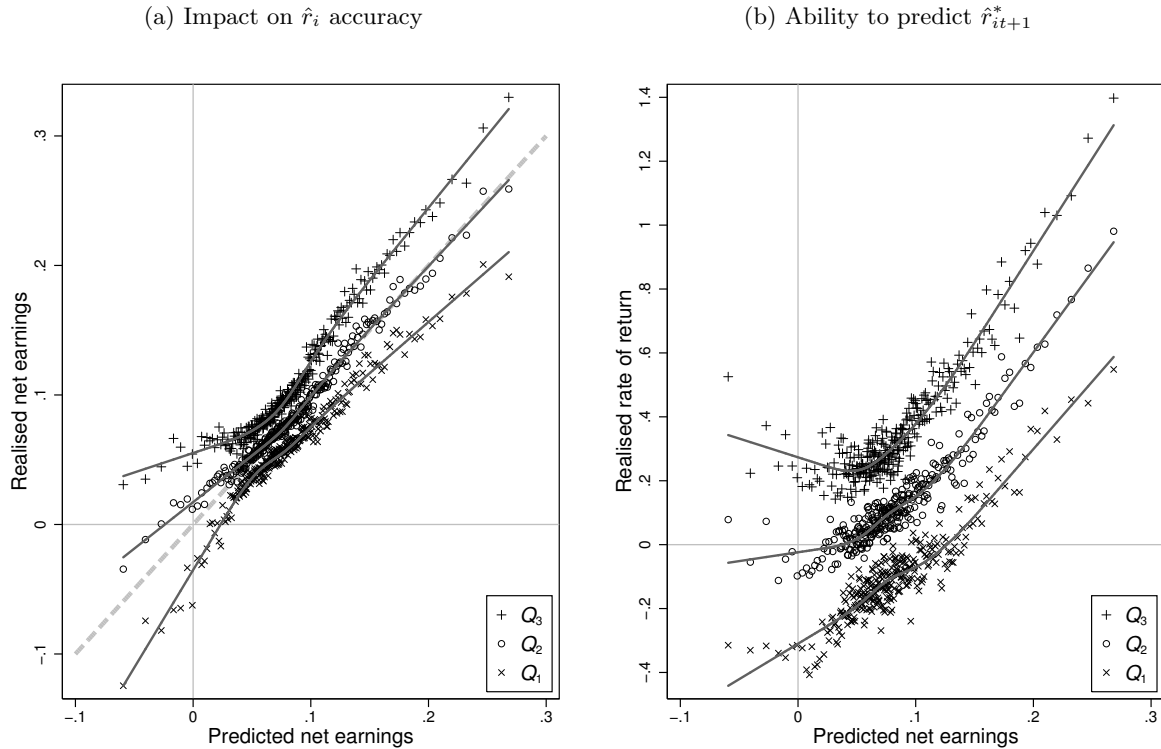
<sup>9</sup>Standard error estimates are corrected via the Eicker-Huber-White ‘sandwich’ robust estimator and are available upon request.

<sup>10</sup>Portfolio smoothing summarizes consecutive localities of the bivariate distribution and therefore eliminates noise hence revealing hidden patterns in the data. For the origins of this method, sometimes also referred to as *quantile smoothing*, see Tukey (1977) and for examples using financial data see Christodoulou (2016).

nature of relationship without having to impose a rigid functional form.<sup>11</sup> This exploratory approach is applied in a consistent manner for Figures 4 to 7 for the predicted components of  $\hat{r}_i$ , that is for the predicted capital gain, net earnings, net dividend, change in book value of equity, abnormal earnings and change in unrecorded goodwill, respectively.

Figures 2-5 evaluate the various components of  $\hat{r}_i$  in terms of (i) their impact on the accuracy of  $\hat{r}_i$  as a predictor of one-year ahead realized stock returns (left-hand side plots), and (ii) their own ability to predict future realized stock returns  $\hat{r}_{it+1}^*$  (right-hand side plots).

Figure 2: Predicted net earnings



The  $y$ -axis of the left-hand side graph gives the quartiles of realized net earnings  $x_{it+1}/p_{it}$ . The  $y$ -axis of the right-hand side graph gives the quartiles of the realized rate of return  $r_{it+1}^*$ . The quartiles of both  $y$ -axes variables are calculated on the basis of the portfolio-specific median of the  $x$ -axis variable of the predicted net earnings,  $\hat{x}_{it+1}/p_{it}$ . There are 296 portfolios each with 99 or 100 observations summing to the total of  $N = 29,569$ . To assist visual representation, the graph suppresses the display of the two most extreme portfolios, those with the minimum and maximum  $\hat{x}_{it+1}/p_{it}$ . The overlaid lines reflect estimation of cubic splines with five knots. The diagonal dashed line references the  $45^\circ$  degree line where  $y = x$ .

Focusing first on the impact of each component of  $\hat{r}_i$  given by equation 14 on the accuracy of  $\hat{r}_i$  as a predictor of one-year ahead realized stock returns, Figures 2a and 3a confirm a strong positive association between predicted earnings and predicted change in book value and their corresponding component of realized stock returns. Figure 4a, on the other hand, indicates that, while there is broad agreement between the predicted and the realized capital gain for the typical portfolio i.e. high bivariate density concentration around zero, there is a negative relation between predicted and realized capital gains for the median realized capital gain observed in each portfolio. In particular, for portfolios with significantly negative predicted capital gains, the median realized capital gain turns out to be significantly positive. These results suggest that the substantial deviations of  $\hat{r}_{it+1}^*$  from  $\hat{r}_i$  documented in Figure 1 may be principally explained by the capital gain component of  $\hat{r}_i$  as opposed to the accounting components.

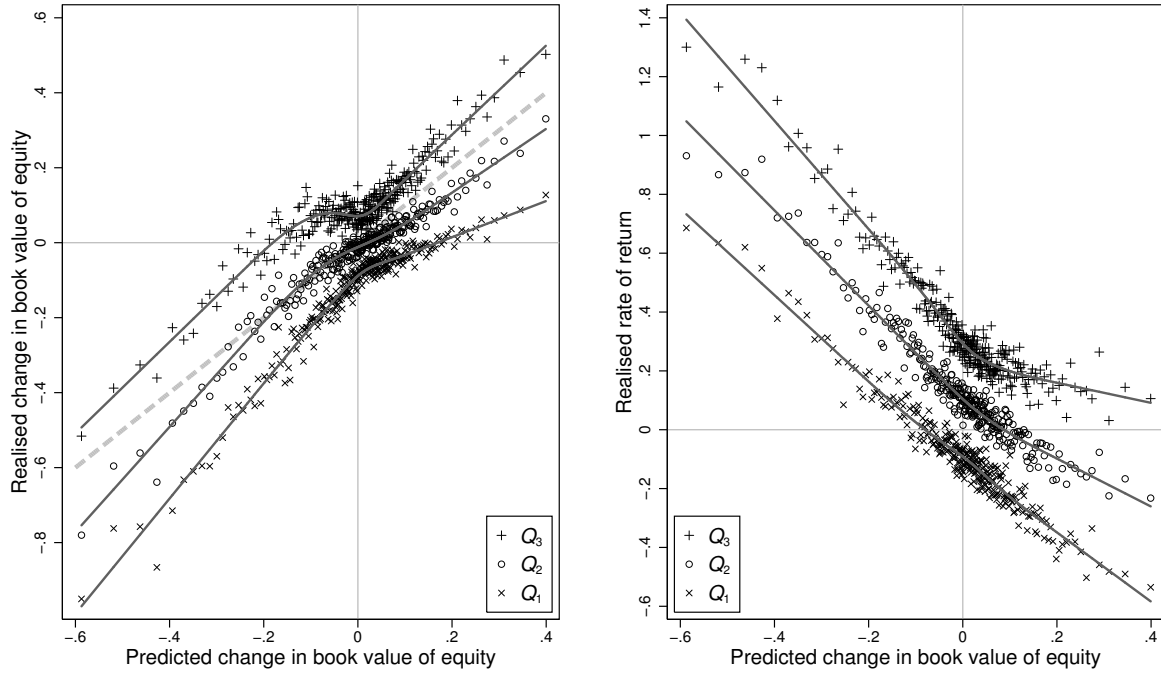
<sup>11</sup>For a practical coverage on cubic splines see de Boor (2001).



Figure 3: Predicted change in book value of equity

(a) Impact on  $\hat{r}_i$  accuracy

(b) Ability to predict  $\hat{r}_{it+1}^*$



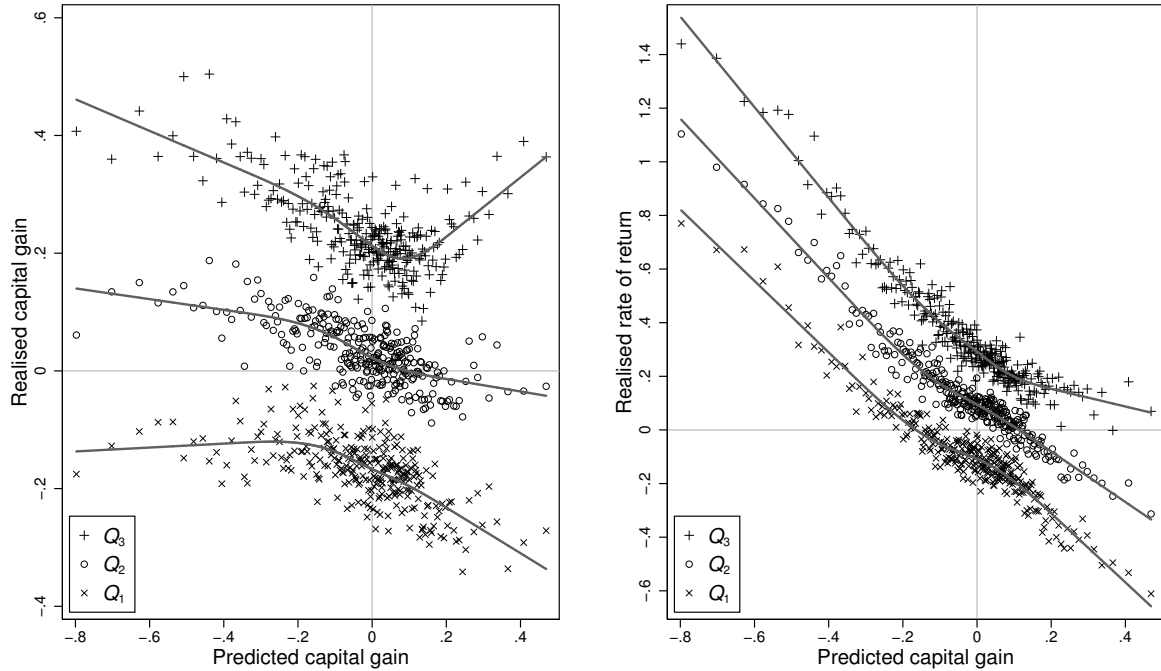
The  $y$ -axis of the left-hand side graph gives the quartiles of realized change in book value of equity  $(y_{it+1} - y_{it})/p_{it}$ . The  $y$ -axis of the right-hand side graph gives the quartiles of the realized rate of return  $r_{it+1}^*$ . The quartiles of both  $y$ -axes variables are calculated on the basis of the portfolio-specific median of the  $x$ -axis variable of the predicted change in book value of equity,  $(\hat{y}_{it+1} - y_{it})/p_{it}$ . There are 296 portfolios each with 99 or 100 observations summing to the total of  $N = 29,569$ . To assist visual representation, the graph suppresses the display of the two most extreme portfolios, those with the minimum and maximum  $(\hat{y}_{it+1} - y_{it})/p_{it}$ . The overlaid solid lines reflect estimation of cubic splines with five knots. The diagonal dashed line references the  $45^\circ$  degree line where  $y = x$ .

Given that  $\hat{r}_i$  is an average long-run measure of the ERR, it is not surprising that it is a noisy predictor of short-run realized stock returns,  $\hat{r}_{it+1}^*$ . Specifically, consistent with perspectives of Penman (2016) and Callen (2016) in this issue, if there are short-run variations in the ERR and these are positively (negatively) associated with the net earnings (change in book value of equity) components of  $\hat{r}_i$ , this will both weaken the association of  $\hat{r}_i$  with realized stock returns and potentially create a negative association between the capital gain component of  $\hat{r}_i$  and realized stock returns. Evidence in Figures 2b and 3b strongly suggest that net earnings (change in book value of equity) scaled by opening market value of equity are positively (negatively) related to future stock returns and this in turn results in the apparently perverse result in Figure 4b that the predicted capital gain component of  $\hat{r}_i$  strongly negatively related to future stock returns. The latter is simply due to the indirect estimation of the predicted capital gain component as  $\hat{r}_i$  minus the predicted net earnings component plus the predicted change in book value of equity component i.e. if the net earnings component is positively related to the short-run ERR (and to one-year ahead realized stock returns) and if the book value of equity component is negatively related to the short-run ERR (and the one year ahead realized stock returns), then it follows that the predicted capital gain component will be negatively related to short-run ERR (and one year ahead stock returns) as shown in Figure 4b.

Figure 4: Predicted capital gain

(a) Impact on  $\hat{r}_i$  accuracy

(b) Ability to predict  $\hat{r}_{it+1}^*$



The  $y$ -axis of the left-hand side graph gives the quartiles of the realized capital gain  $(p_{it+1} - p_{it})/p_{it}$ . The  $y$ -axis of the right-hand side graph gives the quartiles of the realized rate of return  $r_{it+1}^*$ . The quartiles of both  $y$ -axes variables are calculated on the basis of the portfolio-specific median of the  $x$ -axis variable of the predicted capital gain,  $(\hat{p}_{it+1} - p_{it})/p_{it}$ . There are 296 portfolios each with 99 or 100 observations summing to the total of  $N = 29,569$ . To assist visual representation, the graph suppresses the display of the two most extreme portfolios, those with the minimum and maximum  $(\hat{p}_{it+1} - p_{it})/p_{it}$ . The overlaid lines reflect estimation of cubic splines with five knots.

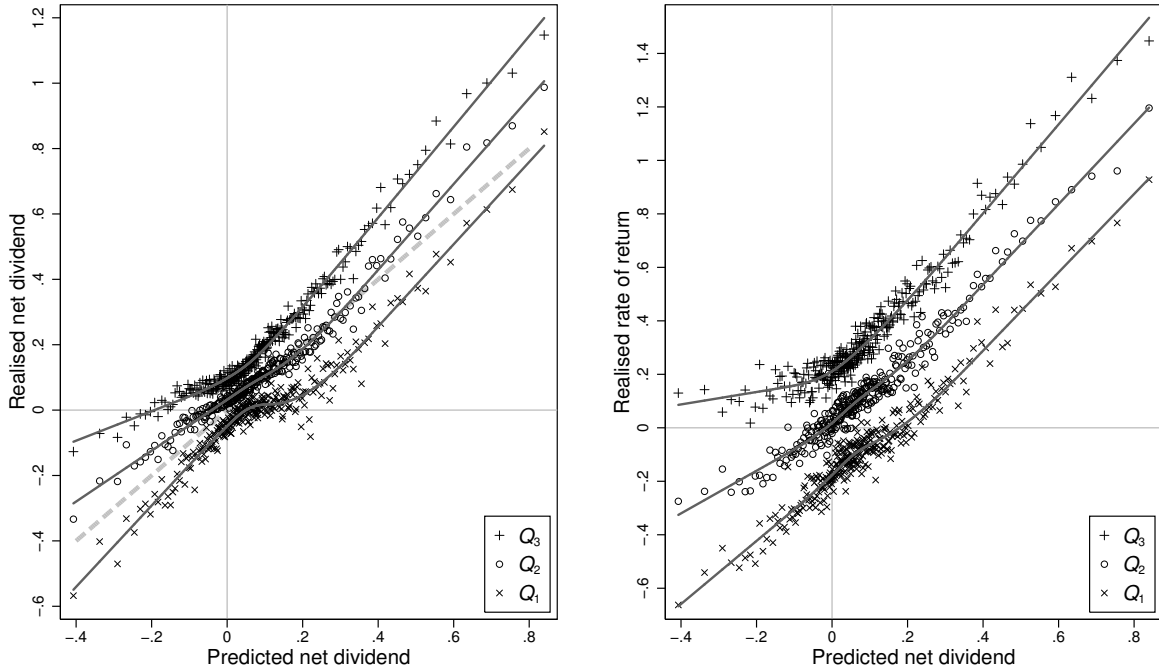
Further results reported in Figures 5-7 support the view that the time-varying accounting components of  $\hat{r}_i$  capture the short-run variation of ERR around its long-run expected rate. First, Figure 5 indicates that when predicted earnings and predicted change in book value of equity are combined to form predicted net dividends, this composite accounting component is strongly associated with future realized net dividends and, more importantly, strongly related to the realized stock returns. Second, Figure 6 provides further evidence on the role of predicted abnormal earnings as a predictor of future stock returns which is broadly consistent with, but somewhat weaker than, the results reported for predicted net earnings.

Finally, Figure 7 shows that when the last two components of  $\hat{r}_i$  in equation 14 are combined to form predicted change in unrecorded goodwill, this composite variable broadly shares the negative association with its own future realisation as for predicted capital gains (however there are some evidence of a positive association for the majority of portfolios with small positive predicted changes in goodwill as shown in Figure 7a), but is less strongly negatively associated with future stock returns than was the case for predicted capital gains as shown in Figure 7b. Interestingly, the latter implies that if our long-run ERR estimate is simply dichotomised into a predicted earnings component and an unrecorded goodwill component, then comparison of Figures 2b and 7b provide strong evidence that the earnings component has a stronger and more consistently positive association with short-run variation in the ERR reflected in realized one year ahead stock returns than the unrecorded goodwill component.

Figure 5: Predicted net dividend

(a) Impact on  $\hat{r}_i$  accuracy

(b) Ability to predict  $\hat{r}_{it+1}^*$



The  $y$ -axis of the left-hand side graph gives the quartiles of realized net dividend  $d_{it+1}/p_{it}$ . The  $y$ -axis of the right-hand side graph gives the quartiles of the realized rate of return  $r_{it+1}^*$ . The quartiles of both  $y$ -axes variables are calculated on the basis of the portfolio-specific median of the  $x$ -axis variable of the predicted net dividend,  $\hat{d}_{it+1}/p_{it}$ . There are 296 portfolios each with 99 or 100 observations summing to the total of  $N = 29,569$ . To assist visual representation, the graph suppresses the display of the two most extreme portfolios, those with the minimum and maximum  $\hat{d}_{it+1}/p_{it}$ . The overlaid solid lines reflect estimation of cubic splines with five knots. The diagonal dashed line references the  $45^\circ$  degree line where  $y = x$ .

In summary, our empirical analysis has provided firm-specific estimates of the long-run expected return on equity, based on the constrained estimation of a system of accounting-based forecast models, which are related to firm-specific implied cost of capital estimates generated in prior research. Our analysis indicates that realized stock returns on average revert to the estimated ERR but that, consistent with prior research by [Easton and Monahan \(2005\)](#) on the implied cost of capital, there is no evidence that our estimate of ERR predicts one-year ahead realized stock returns. Further analysis, however, indicates that the time-varying accounting components of our firm-specific ERR estimates are strongly related at a portfolio level to future realized stock returns. This is consistent with time variation in ERRs around a long-run average where predicted net earnings and predicted net dividends scaled by equity value provide useful additional information on short-run variations in the ERR.

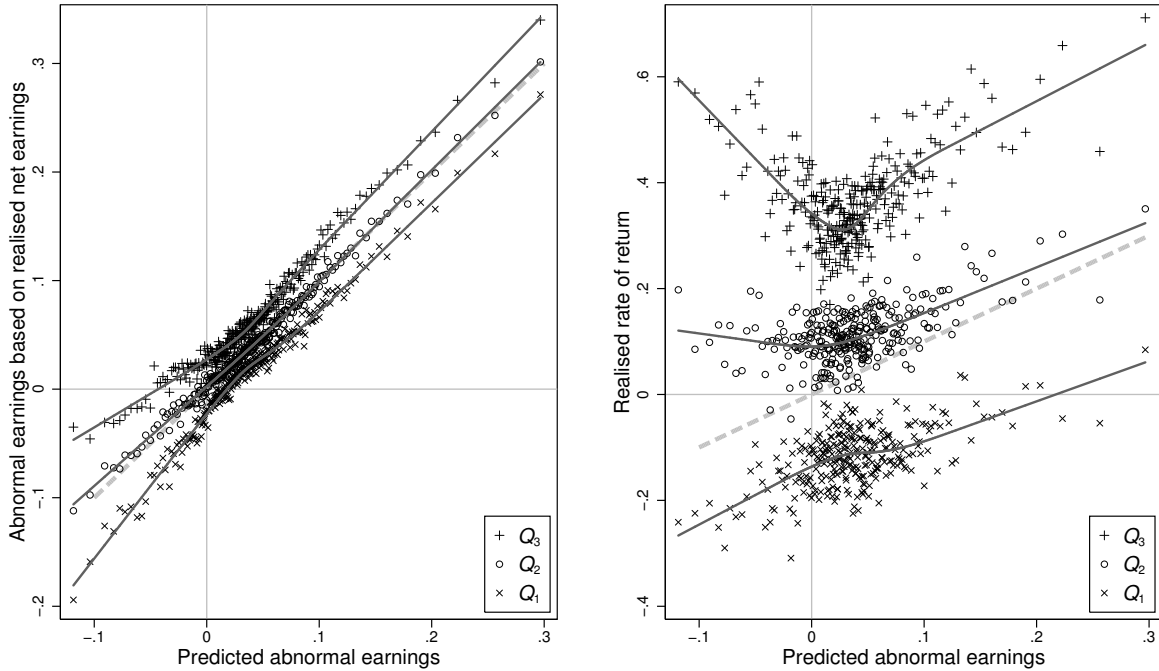
## 6 Conclusions

The method for estimating the expected rate of return that is described in this paper adapts the linear information model that is based on abnormal earnings to a reported earnings basis, and then imposes the clean surplus parameter constraints identified in [Clubb \(2013\)](#). The resulting equation system is rank deficient and, following the methods described by [Christodoulou and McLeay \(2014\)](#), we identify its parameters by requiring the predicted accounting variables to articulate in accordance with the underlying

Figure 6: Predicted abnormal earnings

(a) Impact on  $\hat{r}_i$  accuracy

(b) Ability to predict  $\hat{r}_{it+1}^*$



The  $y$ -axis of the left-hand side graph gives the quartiles of abnormal earnings based on realized net earnings  $x_{it+1}^a/p_{it} = x_{it+1}/p_{it} - r_i \times y_{it}/p_{it}$ . The  $y$ -axis of the right-hand side graph gives the quartiles of the realized rate of return  $r_{it+1}^*$ . The quartiles of both  $y$ -axes variables are calculated on the basis of the portfolio-specific median of the  $x$ -axis variable of the predicted abnormal earnings,  $\hat{x}_{it+1}^a/p_{it}$ , from equation 13. There are 296 portfolios each with 99 or 100 observations summing to the total of  $N = 29,569$ . To assist visual representation, the graph suppresses the display of the two most extreme portfolios, those with the minimum and maximum  $\hat{x}_{it+1}^a/p_{it}$ . The overlaid lines reflect estimation of cubic splines with five knots. The diagonal dashed line references the  $45^\circ$  degree line where  $y = x$ .

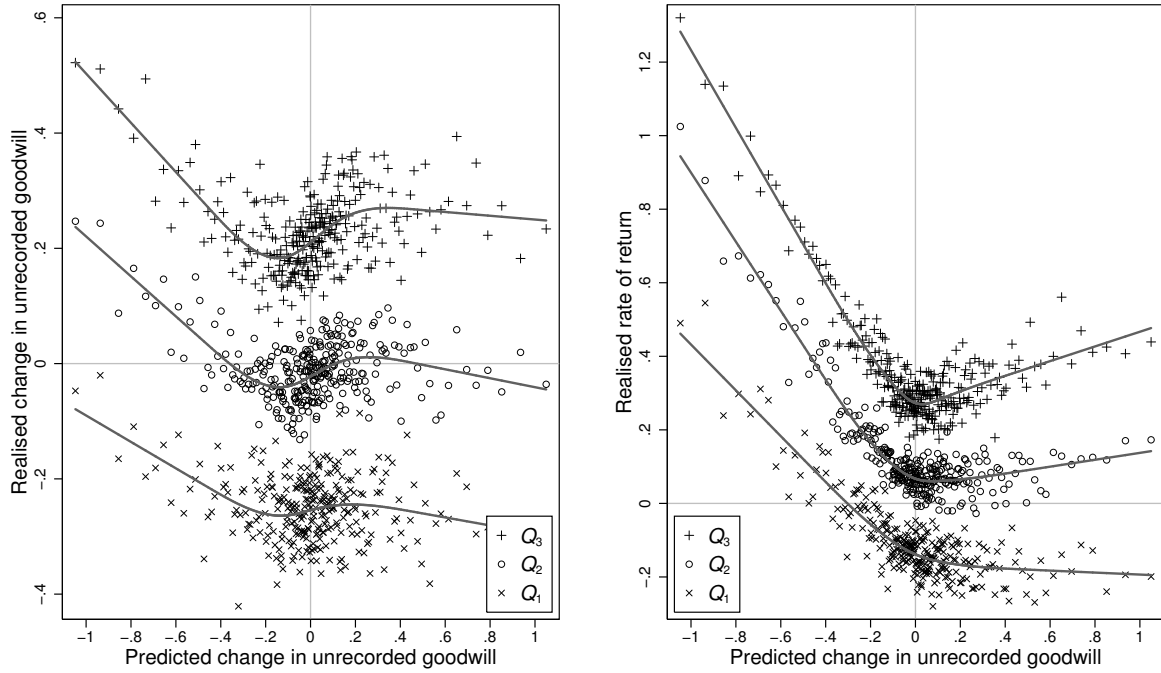
accounting identity. In contrast to previous research on the ‘implied cost of capital’, the long-run ERR estimates reported in this paper are based purely on the firm’s accounting information dynamics and without reference to its stock price or market analysts’ forecasts. In other words, these estimates are based on evidence regarding the persistence of each firm’s accounting performance in its product markets, as opposed to assessments by the capital market of the firm’s future performance. Interestingly, the average of our firm-specific ERRs is similar to the average based on prior research on the implied cost of capital, although -not surprisingly- there are differences at the firm level.

In addition to providing accounting-based estimates of firm-specific ERRs, we investigate the relationship between our ERR estimates and future realised one-year ahead stock returns in line with prior research on the implied cost of capital, and make use of our accounting-based approach to analyse the usefulness of ERR components with respect to predictions of earnings, dividends and book value. Consistent with previous research on the market reaction to released accounting information, there is no simple linear relation between our ERR estimates and realised stock returns, but we do find evidence that average realised stock returns are related to average ERR over the full sample period 1961-2011. Using a portfolio smoothing method, evidence suggests that the articulated components of the estimated ERR (i.e. articulated such that predicted net earnings, less predicted change in book value of equity is equal to predicted net dividends, and where each is scaled by opening market value of equity) are strongly positively related to one year ahead realised stock returns, but that predicted capital gains and predicted change in unrecorded goodwill (similarly scaled) are negatively or ambiguously related to one year ahead

Figure 7: Predicted change in unrecorded goodwill

(a) Impact on  $\hat{r}_i$  accuracy

(b) Ability to predict  $\hat{r}_{it+1}^*$



The  $y$ -axis of the left-hand side graph gives the quartiles of realized change in unrecorded goodwill  $\Delta[(p_{it+1} - y_{it+1})/p_{it}]$ . The  $y$ -axis of the right-hand side graph gives the quartiles of the realized rate of return  $r_{it+1}^*$ . The quartiles of both  $y$ -axes variables are calculated on the basis of the portfolio-specific median of the  $x$ -axis variable of the predicted change in unrecorded goodwill,  $\Delta[(\hat{p}_{it+1} - \hat{y}_{it+1})/p_{it}]$ . There are 296 portfolios each with 99 or 100 observations summing to the total of  $N = 29,569$ . To assist visual representation, the graph suppresses the display of the two most extreme portfolios, those with the minimum and maximum  $\Delta[(\hat{p}_{it+1} - \hat{y}_{it+1})/p_{it}]$ . The overlaid solid lines reflect estimation of cubic splines with five knots. The diagonal dashed line references the  $45^\circ$  degree line where  $y = x$ .

realised stock returns. We interpret this as consistent with short-term variation from the long-run ERR (presumably related to changes in risk), and also as evidence of the usefulness of accounting predictions in forecasting short-run stock returns.

We conclude that this study provides a promising new approach to the estimation of a long-run expected rate of return, which highlights the important role of accounting fundamentals in the assessment of firm risk. While our approach avoids the use of stock prices to reverse-engineer expected rates of return, the underlying role of abnormal earnings dynamics clearly emphasises the importance of the capital market's return expectations in influencing competition between firms and hence firm performance. We hope that future research might further explore the relationship between, and relative performance of, capital market and accounting-based estimates of expected stock returns.











... table continued from previous page.

Company name	$T_i$	$\hat{r}_i$	$ICC$	$\phi_i$	$R_x^2$	$R_d^2$	$R_y^2$	Company name	$T_i$	$\hat{r}_i$	$ICC$	$\phi_i$	$R_x^2$	$R_d^2$	$R_y^2$
Todd Shipyards	35	-0.045	0.109	-0.48	0.30	0.42	0.36	Tootsie Roll	44	0.075	0.097	-0.59	0.69	0.21	0.76
Trane	32	0.041	0.083	-0.92*	0.30	0.57	0.86	Trinity Industries	42	0.051	0.078	-0.82*	0.35	0.38	0.17
True North Comm.	32	0.188	0.108	-0.74*	0.34	0.43	0.28	Twin Disc	41	0.079	0.131	-0.97*	0.10	0.43	0.23
Tyler Technologies	41	0.057	0.089	-0.86*	0.27	0.31	0.37	Tyson Foods	38	0.073	0.089	-0.84*	0.13	0.57	0.63
U S Lime & Minerals	35	0.048	0.150	-0.52	0.16	0.30	0.28	UGI	44	0.043	0.100	-0.74*	0.12	0.17	0.72
URS	36	0.038	0.095	-1.01*	0.52	0.52	0.63	USG	38	0.101	0.086	-0.87*	0.46	0.22	0.68
UST	43	0.063	0.073	-1.05*	0.26	0.23	0.86	Uil Hldg	46	0.097	0.125	-1.01*	0.31	0.52	0.57
Unicom	35	0.102	0.097	-0.58	0.43	0.25	0.74	Unilever	46	0.065	0.807	-0.96*	0.58	0.25	0.80
Unilever Nv	43	0.075	0.564	-1.02*	0.65	0.24	0.90	Union Camp	31	-0.219	0.071	-0.36	0.40	0.36	0.39
Union Pacific	47	0.013	0.064	-0.84*	0.06	0.18	0.72	Unisource Energy	43	0.057	0.098	-0.87*	0.33	0.33	0.65
Unisys	37	0.085	0.056	-1.03*	0.20	0.10	0.68	United Continental	33	-0.026	0.093	-0.90*	0.19	0.17	0.50
United Industrial	38	0.068	0.125	-0.61	0.35	0.40	0.60	United Technologies	47	0.079	0.084	-0.97*	0.31	0.26	0.71
United-Guardian	30	0.134	0.117	-0.98*	0.61	0.42	0.53	Universal Corp/Va	43	0.068	0.091	-0.91*	0.31	0.38	0.54
Unocal	38	0.091	0.082	-0.84*	0.62	0.23	0.75	Upper Penins. Energy	30	0.100	0.214	-0.59	0.39	0.42	0.76
VF	43	0.152	0.081	-0.90*	0.51	0.43	0.46	Valmont Industries	41	0.108	0.098	-0.86*	0.23	0.55	0.58
Valpey-Fisher	42	0.064	0.143	-0.73*	0.13	0.33	0.52	Valspar	41	0.077	0.099	-0.49	0.60	0.46	0.89
Varian Medical Systems	42	0.024	0.062	-0.99*	0.13	0.48	0.86	Vectren	44	0.121	0.106	-0.96*	0.68	0.30	0.55
Veeco Instruments	32	0.213	0.077	-0.93*	0.68	0.51	0.37	Village Super Market	34	0.043	0.257	-0.88*	0.39	0.26	0.61
Virco Mfg.	36	0.103	0.153	-1.00*	0.46	0.44	0.66	Vishay Intertechnology	34	0.047	0.067	-0.85*	0.29	0.59	0.11
Vulcan Materials	47	0.083	0.076	-0.75*	0.77	0.36	0.73	Wackenhut -Ser A	33	0.020	0.136	-0.74*	0.44	0.33	0.60
Wal-Mart Stores	39	0.195	0.037	-0.93*	0.62	0.43	0.65	Walgreen	43	0.103	0.065	-0.88*	0.81	0.55	0.88
Wallace Computer Svcs	34	-0.198	0.080	-0.45	0.54	0.48	0.23	Warner-Lambert	35	0.003		-0.87*	0.04	0.14	0.85
Washington Post	37	0.143	0.074	-0.61	0.66	0.42	0.62	Watkins-Johnson	30	0.029	0.077	-0.79*	0.08	0.52	0.59
Watsco	43	0.079	0.274	-0.64	0.26	0.36	0.42	Wausau Paper	30	0.078	0.069	-0.88*	0.32	0.63	0.57
Wd-40	36	-0.196	0.070	-0.44	0.44	0.60	0.60	Weis Markets	42	-0.071	0.071	-0.30	0.73	0.35	0.55
Wells-Gardner Electr.	30	0.085	0.109	-0.90*	0.25	0.34	0.44	Wesco Financial	37	0.015	0.051	-0.36	0.10	0.43	0.71
West Pharmaceutical	38	0.135	0.075	-0.83*	0.40	0.29	0.53	Westar Energy	42	0.089	0.100	-0.70	0.25	0.47	0.47
Westmoreland Coal	31	0.038	0.070	-0.83*	0.11	0.07	0.74	Weyco Group	42	0.090	0.154	-0.87*	0.60	0.35	0.67
Weyerhaeuser	44	0.000	0.058	-1.03*	0.21	0.34	0.46	Wgl Hldg	45	0.072	0.097	-0.75*	0.65	0.23	0.66
Whirlpool	47	0.060	0.070	-0.84*	0.10	0.37	0.40	Wiley (John) & Sons	40	0.108		-0.68*	0.75	0.36	0.75
Williams Cos	39	0.043	0.077	-0.96*	0.11	0.34	0.41	Winn-Dixie Stores	40	-0.267		-0.49	0.33	0.67	0.89
Winnebago Industries	38	0.059	0.061	-0.75*	0.15	0.39	0.42	Wisconsin Energy	47	0.131	0.084	-0.76*	0.85	0.30	0.77
Witco	34	0.134	0.088	-0.90*	0.30	0.30	0.54	Wolverine World Wide	40	0.003	0.093	-0.52	0.39	0.51	0.82
Woodhead Industries	31	-0.098	0.110	-0.78*	0.20	0.56	0.15	Worldwide Restaurant	32	0.137	0.084	-0.50	0.36	0.36	0.35
Worthington Industries	37	0.093		-0.95*	0.58	0.57	0.55	Wrigley (Wm) Jr	41	0.097	0.062	-1.17*	0.70	0.36	0.89
Wsi Industries	33	0.037	0.210	-0.78*	0.29	0.34	0.19	Wyeth	44	0.041	0.051	-1.20*	0.58	0.46	0.68
XTRA	32	0.132	0.103	-0.65*	0.48	0.32	0.60	Xcel Energy	46	0.057	0.093	-1.09*	0.21	0.51	0.22
Xerox	45	0.015	0.063	-0.92*	0.03	0.24	0.51	Yrc Worldwide	43	-0.075	0.073	-0.90*	0.04	0.44	0.53
ZALE	33	0.067	0.056	-0.76*	0.25	0.34	0.71	Zareba Systems	32	0.083	0.451	-0.66*	0.17	0.59	0.28
Zemex Cda	31	0.033	0.186	-0.61	0.18	0.58	0.33								

Note: Some company names are abbreviated.  $T_i$  is the firm-specific sample size.  $\hat{r}_i$  is estimated expected return.  $ICC$  is the firm-specific median estimate of the composite measure of implied cost of capital from Hou et al. (2012).  $\phi_i$  is the estimated coefficient from equation 18; an asterisk indicates significance at  $\alpha = 0.01$  via a  $t$ -statistic with  $N - 1$  degrees of freedom.  $R_x^2$ ,  $R_d^2$  and  $s_d^2$  are the associated  $R$ -squares. It holds that  $\alpha_3 = \alpha_1 - \alpha_2$ ,  $w_{31} = w_{11} - w_{21}$ ,  $w_{32} = w_{12} - w_{22}$  and  $w_{33} = 1 + r - w_{23}$ .

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