Pelton Turbine: Identifying the Optimum Number of Buckets Using CFD

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Abstract

A numerical case study on identifying the optimum number of buckets for a Pelton turbine is presented. Three parameters: number of buckets, bucket radial position and bucket angular position are grouped since they are found to be interrelated. By identifying the best combination of the radial and angular position for each number of buckets it is shown that reduction in the number of buckets beyond the limit suggested by the available literature can improve the runner efficiency and be beneficial from the manufacturing complexity and cost point of view. The effect of this numerically suggested reduction in the amount of buckets was evaluated experimentally and confirmed that the efficiency was successfully increased.

1 Introduction

The design of Pelton turbines has been developed for more than a century [1] since its invention by Lester Pelton [2] in 1880. Available literature usually concentrates on distributor [3, 4], injector [5-12], bucket geometry [13-20] or turbine casing [21] analysis or design optimisation. However, not much work is published in terms of the optimum number of buckets.

Generally there is a tendency of fitting as many buckets on the runner as possible to ensure efficient transition of the jet from one bucket to another without wasting the energy of a water jet. However, there are energy losses associated with jet entering the bucket and providing some amount of counter-torque (Fig. 1.) as the outer side of the bucket hits the surface of the jet [22]. Therefore a minimum amount of buckets ensuring that no water particles are lost during the transition from one bucket to another should be identified [22-24].

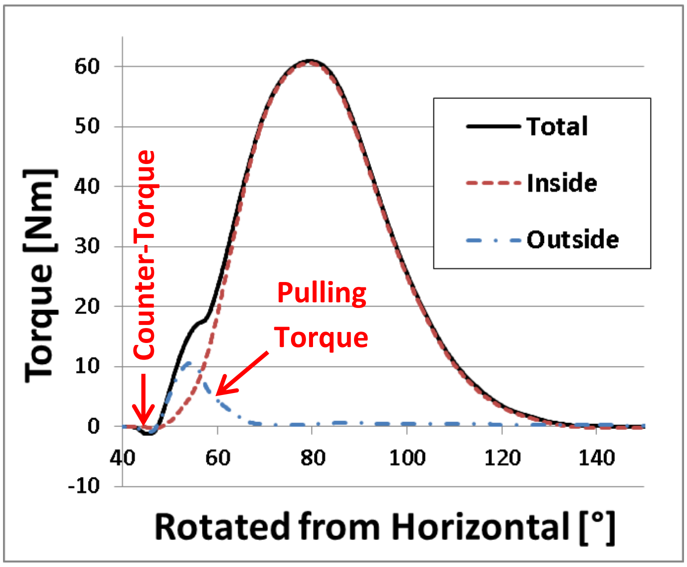


Fig. 1. Typical torque curves on a single Pelton bucket.

This paper concentrates on further developing a Pelton runner by identifying the optimum number of buckets and their mounting position after the geometry of the bucket has been modified. A Pelton runner was optimised at Lancaster University by parametrically modifying the shape of the bucket and then adjusting its mounting position and is presented as Case 2 in [25]. The study showed that after changes to the bucket geometry are made the runner efficiency might benefit from reassessing the number of buckets. The bucket shape of the modified runner described in Case 2 of [25] was further improved analytically by modifying the geometry of the cut-out, the splitter tip and the outer side of the bucket regions. To evaluate the effect of these modifications Computational Fluid Dynamics (CFD) was applied. As the final step it was decided to readjust the positioning and spacing of the bucket. The assumption is made that the three parameters: the number of buckets, the angular position and the radial position are highly interrelated and therefore should be analysed as a group in order to achieve the best result.

2 Background

There are theoretical suggestions on calculating the required amount of buckets that are derived by looking at the relative paths of the water particles. In 1957 M. Nechleba [24] suggested acceptable number of buckets (N_b) based on a ratio: jet diameter (d_0) over runner diameter (D) as shown in Table 1. This suggestion gives quite wide ranges of buckets per different d_0/D ratios therefore is not very exact. Since then the industry has developed more exact guidance to calculate the amount of buckets including additional parameters like bucket width to assist engineers. These methods correlate with suggested ranges by M. Nechleba however, they are not publically available.

Table 1 – Selecting the number of buckets.

Select Number of Buckets	
d_0/D	N_b
1/6	17 to 21
1/8	18 to 22
1/10	19 to 24
1/15	22 to 27
1/20	24 to 30
1/25	26 to 33

M. Eisenring [22] suggests Eq. (1) to calculate the optimum number of buckets by relating the length of the pitch circle to the optimum jet diameter.

$$N_b = \frac{\pi D_p}{2d_o} \tag{1}$$

Moreover, a statement is made that a minimum of at least 16 buckets should be installed. This statement does not agree with M. Nechleba [24] who suggests 17 buckets to be the minimum as presented in Table 1.

Work published by I. U. Atthanayake [26] suggests an empirical relationship given in Eq. (2) to select the number of buckets. However, no references are given to the work establishing this empirical relationship.

$$N_b = \frac{D_p}{d_j} + 15 \tag{2}$$

B. A. Nasir [27] has also published a paper that covers the number of buckets in which it is suggested to use Eq. (3) to calculate the optimum number of buckets.

$$N_b = \frac{D_p}{2d} + 15 \tag{3}$$

The fact that it is not clearly stated if *d* is the nozzle diameter or the jet diameter gives some uncertainty to this equation as the jet diameter might be different to the nozzle opening diameter. This difference is even more pronounced at the best efficiency point when the flow rate is not at its maximum and where the turbine is usually optimised. It will be therefore assumed that the nozzle diameter is to be used in Eq. (3) since it is a constant value.

The suggestions of all the authors reviewed in this section are taken into account and the suggested number of buckets is calculated according to each suggestion using the parameters of the Pelton turbine used in this case. The dimensions of the prototype runner are in accordance with the minimum required values for model size and test parameters (IEC 60193 [28]): pitch circle diameter = 320 mm, jet diameter at the best efficiency point = 30.1 mm and nozzle diameter = 46.9 mm. Table 2 provides the resultant number of buckets according to each suggestion.

Table 2 – Resultant number of buckets as suggested by different sources.

Author	Suggested N_b
M. Nechleba	18 to 21
M. Eisenring	17
I. U. Atthanayake	26
B. A. Nasir	18

It is evident, that not only there is a strong disagreement between the suggested ways of identifying the optimum amount of buckets in the available literature sources but none of them provide any experimental or numerical research data to support their suggestions. Moreover, they do not take into account the fact that performance of runners with different amount of buckets should be compared when the bucket is mounted at its optimum radial and angular position which is different for each number of buckets because of different spacing. Fig. 2 provides a diagram showing the main dimensions of bucket positioning. The angular position describes at what angle is the bucket mounted on a runner and the splitter tip circle diameter describes the radial position of the bucket while keeping the pitch circle diameter fixed.

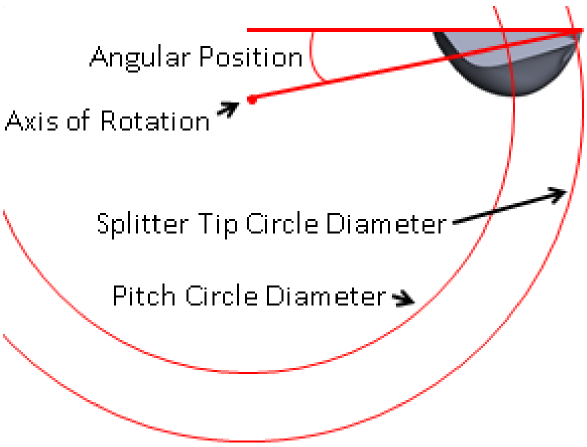


Fig. 2. Parameters used for bucket positioning.

It will be explained in section *3 Numerical Modelling* that in order to find the best angular and radial position a minimum of 9 data points per each number of buckets is required. Performing such study experimentally would be a costly and difficult process. That might explain the lack of experimentally established guidance on selecting the number of buckets and availability of theoretical suggestions only that are inevitably based on assumptions and are prone to have limitations. However, recent development in CFD methods and computational resources allows simulating the performance of Pelton runners that include complex phenomena like multiphase, free surface, highly turbulent flows for a relatively large number of design variations within a reasonable timescale.

3 Numerical Modelling

Computational Fluid Dynamics is used in this study to simulate the performance of the runner and calculate its efficiency at the best efficiency point for different combination of the three parameters of interest. According to the literature [1], the most widely used CFD code for numerical modelling of Pelton turbine and accurate prediction of its efficiency is ANSYS CFX [29]. The most recent publications on Pelton modelling with CFX use k-ω SST turbulence model and Homogeneous multiphase model. Since modelling of Pelton turbine performance requires very high computational resources many simplifications are introduced: such as assumption that gravity or surface tension is negligible or modelling of only few buckets of the runner and then constructing the torque of the whole runner. More detailed description of the CFD method used was described in the initial optimisation study [25] already mentioned in the introduction.

Design-Expert [30] software for Design of Experiments (DOE) approach is used to find the best combination of the radial and angular position per each number of buckets. The chosen method for this DOE analysis is the Central Composite Design (CCD) [31], which is designed to estimate the coefficients of a quadratic model and consists of three groups of design points:

a) two-level factorial or fractional factorial design points
b) axial points (sometimes called "star" points)
c) center points

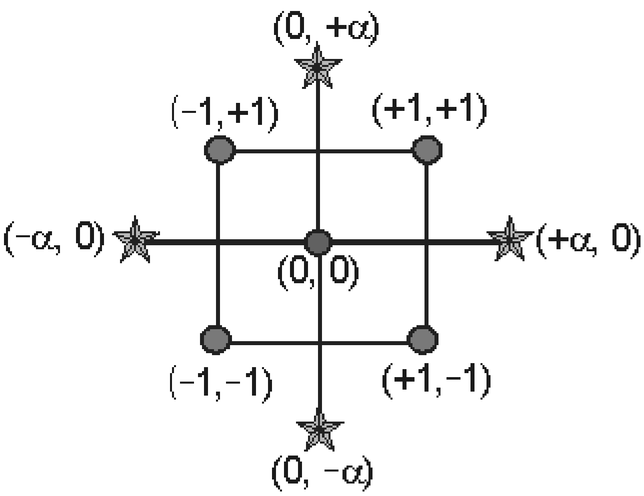
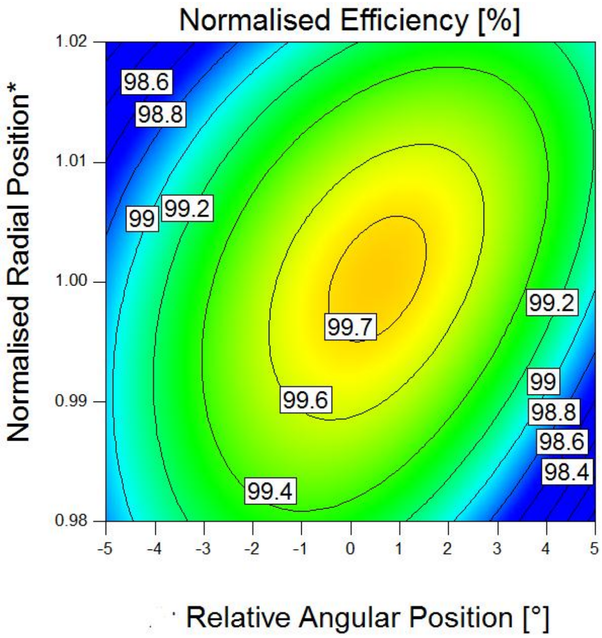


Fig. 3. Layout of the design points for the DOE study with two factors.

Fig. 3 provides a typical layout of the design points for the DOE study containing two factors. Therefore, it requires 9 data points per each number of buckets to fit a surface representing the efficiency response to the angular and radial position. The chosen range for the number of buckets is 14 to 18 since 18 is the maximum number of buckets that can physically fit on this runner. Therefore a total of 45 data points is required for this study. A contour plot for a runner with 18 buckets is provided in Fig. 4 where efficiency is normalised to the overall maximum efficiency, the angular position is given as a relative measurement using the most optimum angular position for a runner with 18 buckets as a datum and the radial position is normalised using Eq. (4). The most optimum angular and radial position of the runner with 18 buckets was chosen to be used as a datum position because the original runner was containing this amount of buckets. Analogous contour plots are used to identify the maximum efficiency for each number of buckets.

$$R_N = \frac{D_t}{D_{t18}} \tag{4}$$



*it is a non-dimensional measure acquired using Eq. (4)

Fig. 4. Efficiency response contours to radial and angular position for a runner with 18 buckets.

By taking the maximum efficiencies of each number of buckets a graph given in Fig. 5 is created showing that the peak efficiency is achieved using 16 buckets instead of the initial number of 18. This result agrees with the initial assumption that 18 is no longer the optimum number of buckets since the bucket shape was modified and resulted in a larger size bucket. This reduction in the amount of buckets suggested by CFD results is in agreement with another similar case study by J. Veselý and M. Varner [32] where experimental results showed that less buckets were required after optimising the bucket shape.

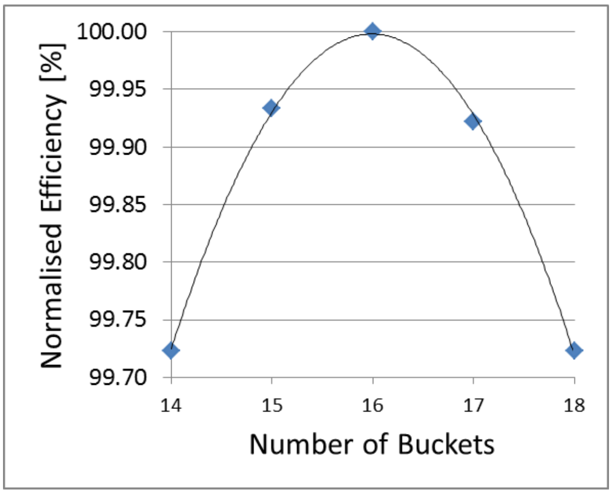


Fig. 5. Normalised efficiency vs. the number of buckets.

The results show that using the theoretical guidance provided by M. Nechleba [24] a number of buckets close to the optimum can be determined. However, the optimum value can only be achieved empirically because of the limitations in these theoretical methods caused by the assumptions made. When calculating the optimum number of buckets theoretically it is assumed that the water jet particle always remains in the plane parallel to the axis of the turbine. However, the trajectories of some of the water particles in the jet stream are slightly deflected because of the Coanda effect as water detaches from the outer side of the bucket when the jet is completely cut off. Also, only the torque from the positive pressure on the inside of the bucket is considered. However, CFD results show a noticeable amount of torque caused by the jet pulling the bucket on its outside when the bucket is cutting into the jet. The negative pressure region on the outside of the bucket is provided in Fig. 6. This phenomenon is also observed experimentally by [23, 33]. The typical torque curves (Fig. 1) acquired numerically on the inside and the outside of a single bucket give an indication on the amount of the torque caused by this negative pressure that is pulling the bucket.

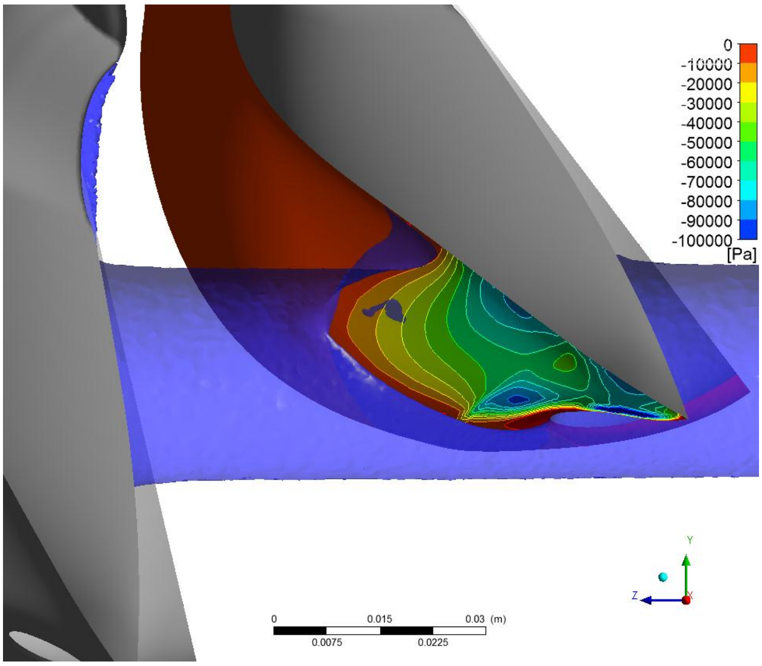


Fig. 6. Negative pressure on the outside of the bucket.

It can be seen from the efficiency response to the number of buckets presented in Fig. 5 that for this runner a number of 16 buckets would be the optimum when aiming at the most efficient turbine design at all costs. However, the variation in efficiency between 15, 16 and 17 buckets is less than 0.1%. This small variation could be treated as negligible. To be more exact according to these results if a number of 15 buckets was selected instead of 16, the resultant efficiency difference would be as small as 0.07 %. The difference is so small that a sensible decision from manufacturing and economic perspective could be to use a runner with 15 buckets which is the lowest number before a more noticeable drop in the efficiency occurs.

Fig. 7 and Fig. 8 provide the optimum radial and angular position data taken from the contours for each number of buckets. The optimum positioning of the bucket is changing with the number of buckets and therefore must be taken into account when thoroughly looking for the optimum amount of buckets on the runner.

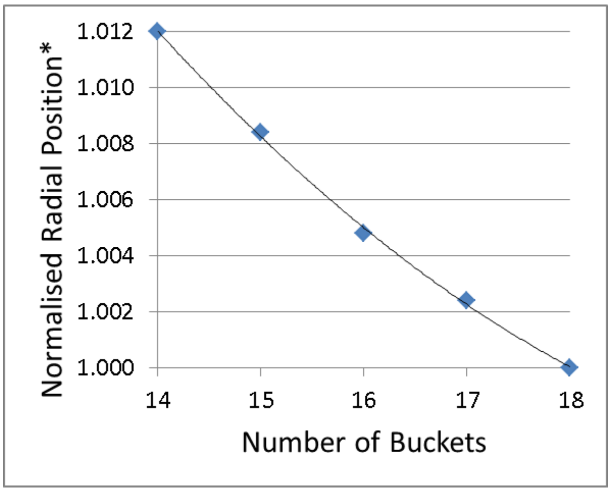


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Fig. 7. Optimum radial position (normalised with the initial position) for each number of buckets.

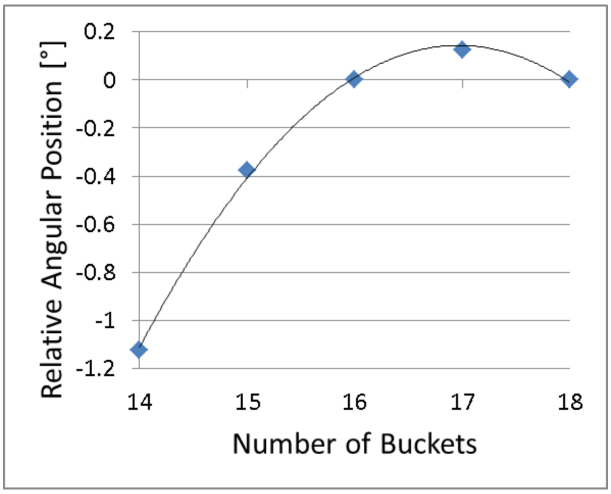

Fig. 8. Optimum angular position (relative to the initial position) for each number of buckets.

4 Experimental Testing

The CFD methods and solver settings used in this study have been previously validated and published by various authors [18, 23] showing that numerical modelling of similar problems has reached a state of reliable accuracy. However, the authors are aware of possible physical limitations therefore as a result experimental testing of two runners using 18 and 15 buckets was performed. It was expected that despite a possible systematic offset between numerical and experimental results, the increase in efficiency when going from 18 buckets design to 15 would be consistent between CFD and experiment.

The experimental testing was performed at the Laboratory of Hydraulic Turbo Machines of National Technical University of Athens. The test rig and the manufactured runners are provided in Fig. 9 and Fig. 10 respectively. The laboratory test rig and the measuring procedure complied with the international model test standards IEC 60193:1999 [34]. Table 3 provides the key characteristic dimensions of the testing facility and the model. All instruments were calibrated at the laboratory according to the IEC 60193:1999 standards. The calibration was performed before and after the experimental testing to confirm that the accuracy was within the range specified by the manufacturers. The total uncertainty of the measured efficiency was equal to ±1.0% while the random uncertainty in efficiency was investigated according to the IEC 60193:1999 standards and found equal to ±0.1% with the 95% confidence level. In this case, the random uncertainty was of primary importance as two designs were compared at identical conditions, hence the comparison was not influenced by the systematic uncertainty.

Table 3 – Characteristic parameters of the experimental test rig.

Test Head, H	60 m
Pitch Circle Diameter, D_p	320 mm
Bucket width, B	120 mm

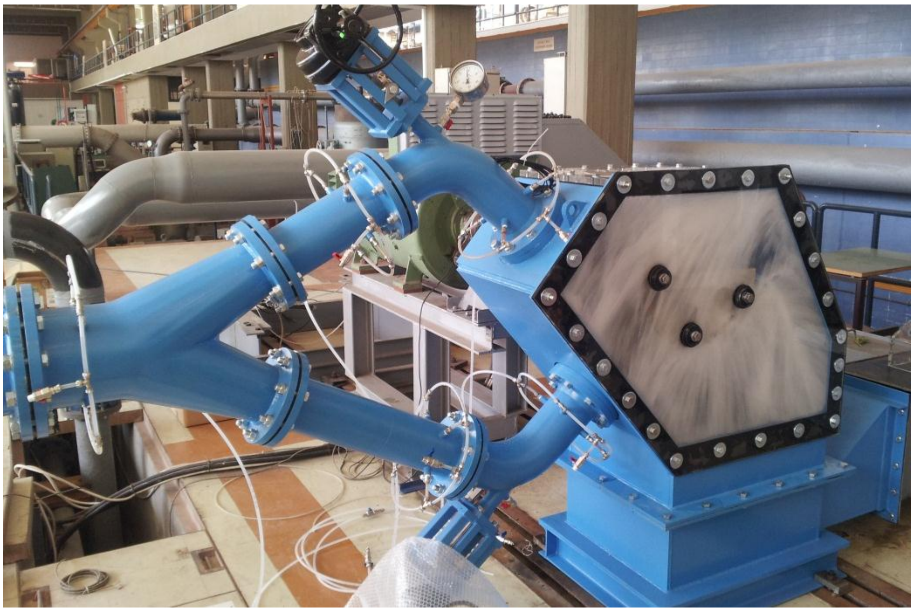

Fig. 9. Turbine testing facility at the Laboratory of Hydraulic Machines (NTUA).

N_b=18 N_b=15

Fig. 10. Prototype runners with 18 and 15 buckets.

Experimental testing results are provided in form of efficiency hill charts in terms of the unit speed n_{11} (eq. 5) and the unit flow rate specified to the bucket width and one jet Q_{11k} (eq. 7). Equations used to define these parameters characterising the operating conditions of a turbine are based on the Affinity laws [24, 35] and most of them can be found in the IEC 60193:1999 standards.

Unit speed:

$$n_{11} = \frac{n \times D}{\sqrt{H}} \tag{5}$$

Unit flow rate:

$$Q_{11} = \frac{Q}{D^2 \times \sqrt{H}} \tag{6}$$

In the IEC 60193:1999 standards the minimum model size of Pelton turbine is specified in terms of bucket width (as oppose to the reference diameter which is a characteristic minimum dimension for other turbines). This suggests that bucket width represents the Pelton turbine better than the reference diameter; hence following the same logic additional unit flow rate definition was made to allow more generic comparison of Pelton runners. This equation of unit flow rate specified to the bucket width and one jet allows comparison between different specific speed Pelton runners.

Unit flow rate specified to the bucket width and one jet:

$$Q_{11k} = \frac{Q/N_j}{B^2 \times \sqrt{H}} \tag{7}$$

Each runner was tested at single jet (lower jet) and two jet operation. A test plan consisted of 61 data point (6 different rotational speed values for each 10 flow rate values plus the original best efficiency point that was used for CFD study). Testing sequence and the data points are provided in Fig. 11.

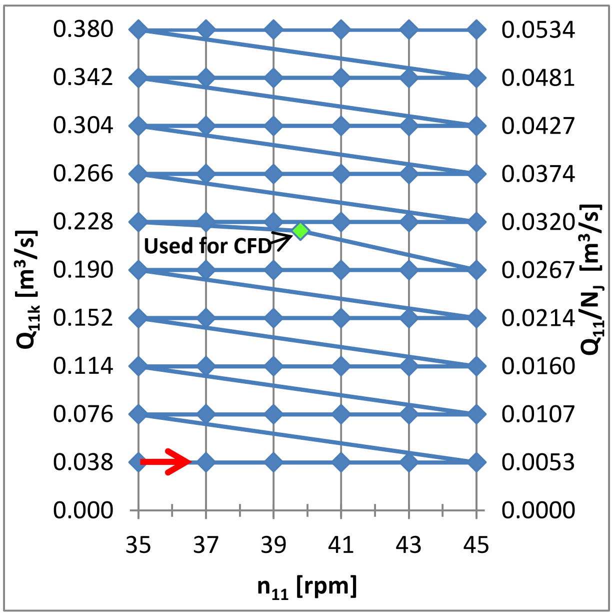

Fig. 11. Test plan and testing sequence.

5 Results and Discussion

Normalised experimental results of runners with 18 and 15 buckets are provided in Fig. 12 (two jet operation) and Fig. 13 (single jet operation). The datum for normalising of all experimental results was the measured best efficiency point of the runner with 18 buckets under the two jet operation. The efficiency increase at the best efficiency point was 0.4% under the two jet operation and 0.8% under the single jet operation, showing that the peak efficiency has increased as the number of buckets was drastically reduced.

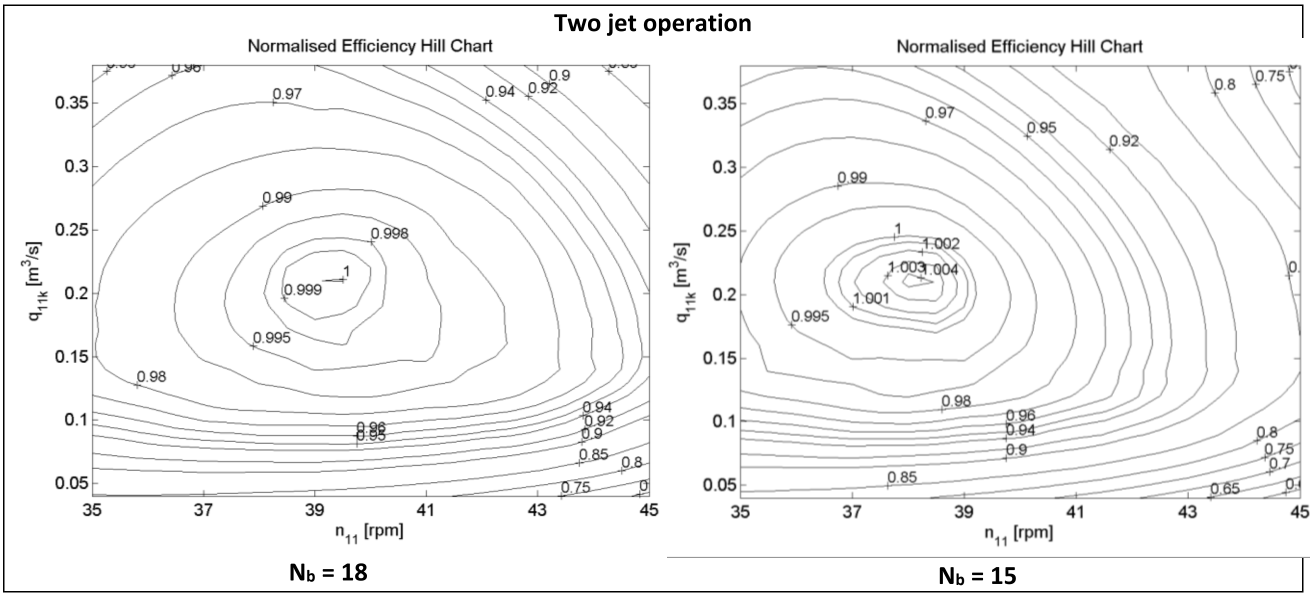

Fig. 12. Normalised efficiency hill charts of runners with 18 and 15 buckets under two jet operation.

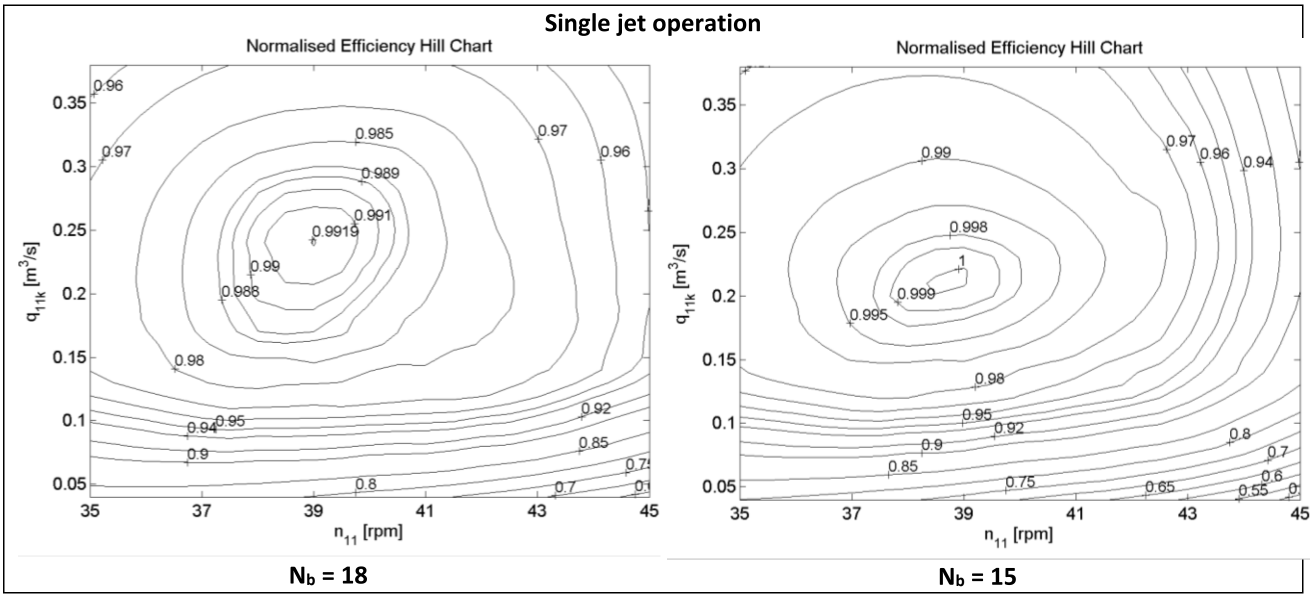

Fig. 13. Normalised efficiency hill charts of runners with 18 and 15 buckets under single jet operation.

To represent the performance increase under complete range of flow rates and the possible change of the best efficient unit speed, efficiency vs. Q_{11k} graphs at constant n_{11} values were produced (Fig. 14 and Fig. 15). In both figures three curves are presented:

black – performance with original number of buckets N_b = 18 at the best efficient n_{11} value,

blue – performance with reduced number of buckets N_b = 15 at the original best efficient n_{11} value,

red – performance with reduced number of buckets N_b = 15 at the best efficient n_{11} value for the reduced number of buckets design.

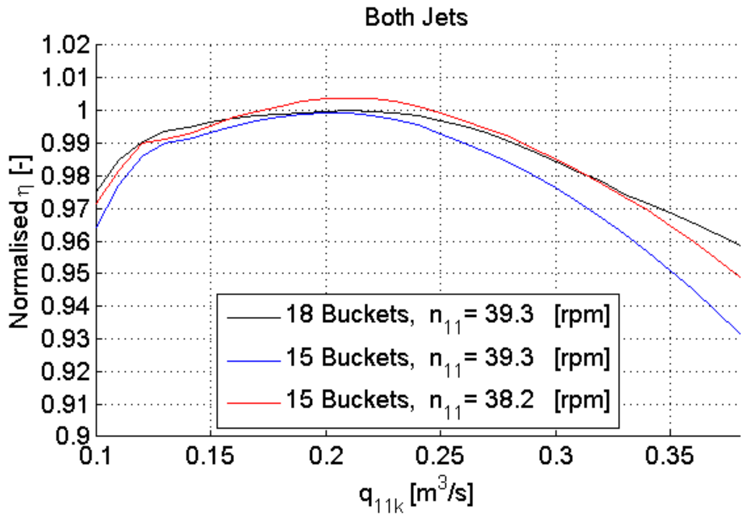

Fig. 14. Comparison of runner performance at the best efficient n_{11} using 18 and 15 buckets – twin jet in operation.

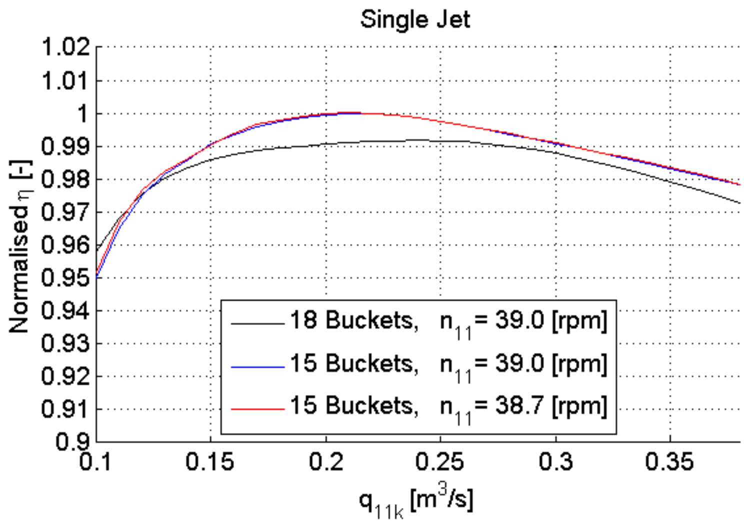

Fig. 15. Comparison of runner performance at the best efficient n_{11} using 18 and 15 buckets – single jet operation.

For single jet operation there is almost no difference in the best efficient n_{11} of the runners with 18 and 15 buckets. However, when both jets are operating, the best efficient n_{11} for 18 and 15 buckets is quite different. I.e. there is a reduction in the best efficient n_{11} for the runner with 15 buckets. Moreover, in two jet operation the efficiency increase is local and drops to negative for very high flow rates. Higher improvement in the efficiency for the single jet operation and the reduction of the best efficient n_{11} for two jet operation indicate that the runner with 15 buckets is experiencing some problems in the two jet operation.

The problem was identified to be the jet interference during the operation with both jets. As the number of buckets is reduced the angle between the buckets is increased. This means that the first jet is entering the bucket for slightly longer time before it gets cut off by the following bucket. Consequently the water from the first jet stays longer in the bucket as presented in Fig. 16. Therefore there is a possibility that the second jet starts entering the bucket before the water from the first jet has cleared. The problem of jet interference in the two jet operation was investigated by Wei, Yang et al. [36] and indeed showed reduction in the torque produced by the second jet in the case where the angle between the jets is too small (Fig. 17). This interference can only be expected to increase as the flow rate is increased and more water is entering the bucket.

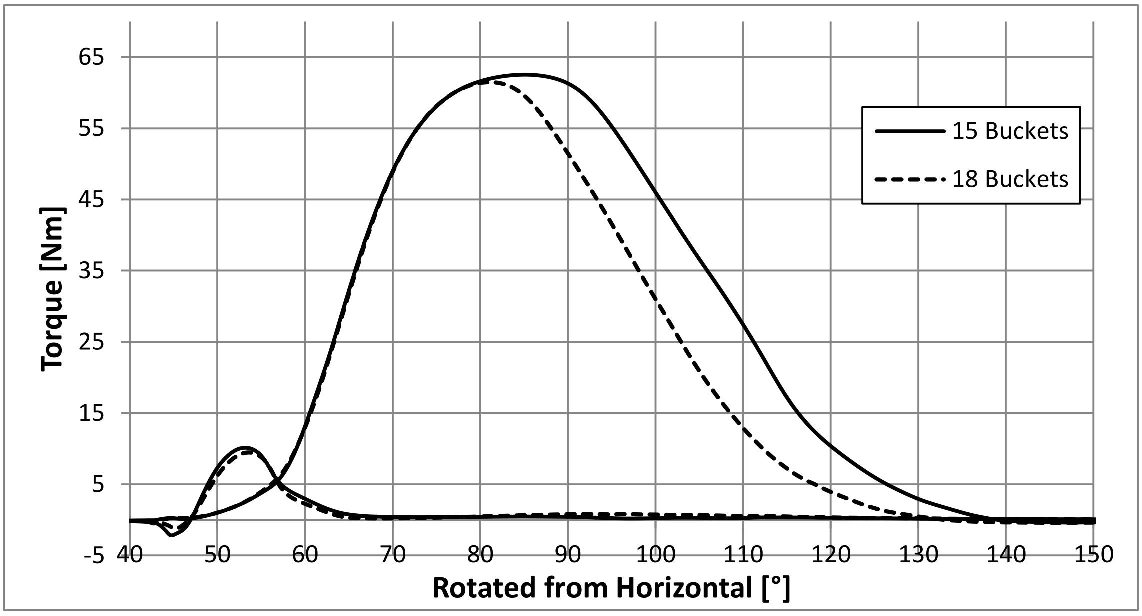

Fig. 16. Torque curves on a single bucket in a runner made of different amount of buckets.

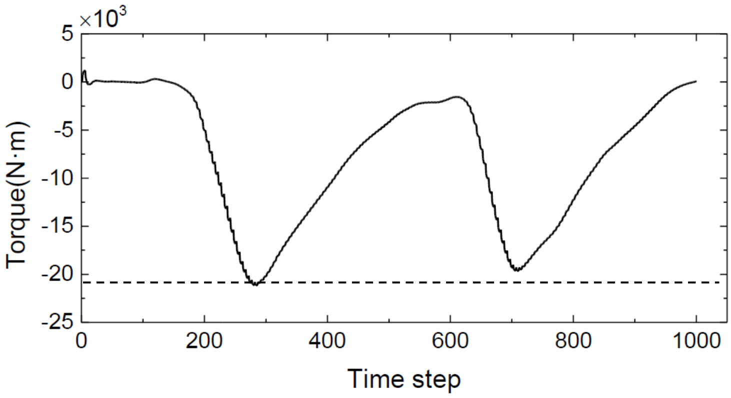

Fig. 17. Two torque peaks (inverted) generated by two jets on a single bucket [36].

To check if the interference between the jets is the case in the current runner with 15 buckets, the torque curve from a single jet operation was copied and shifted by 80° which was the angle between the jets in the test rig. Fig. 18 presents the two torque curves on the runner with the original number of buckets N_b=18. As expected the transition from the first jet entering the bucket to the second was swift. I.e. the water from the first jet has left the bucket just before the second jet was entering showing that the angle between the jets was appropriate for the original runner. However the torque curves provided in Fig. 19 indicate that there is a potential for interference between the first and second jets. This suggests that if the angle between the jets was increased to eliminate the interference between them, the efficiency increase provided by the runner with 15 buckets under the operation with both jets can be expected to be higher than 0.4 % at the BEP and consistent over the whole range of flow rates as in the single jet operation.

Overall, the experimental results show that the runner was successfully optimised by reducing its number of buckets from the original 18 to 15 which is beyond any suggestion found in Pelton design guidelines available in the public domain.

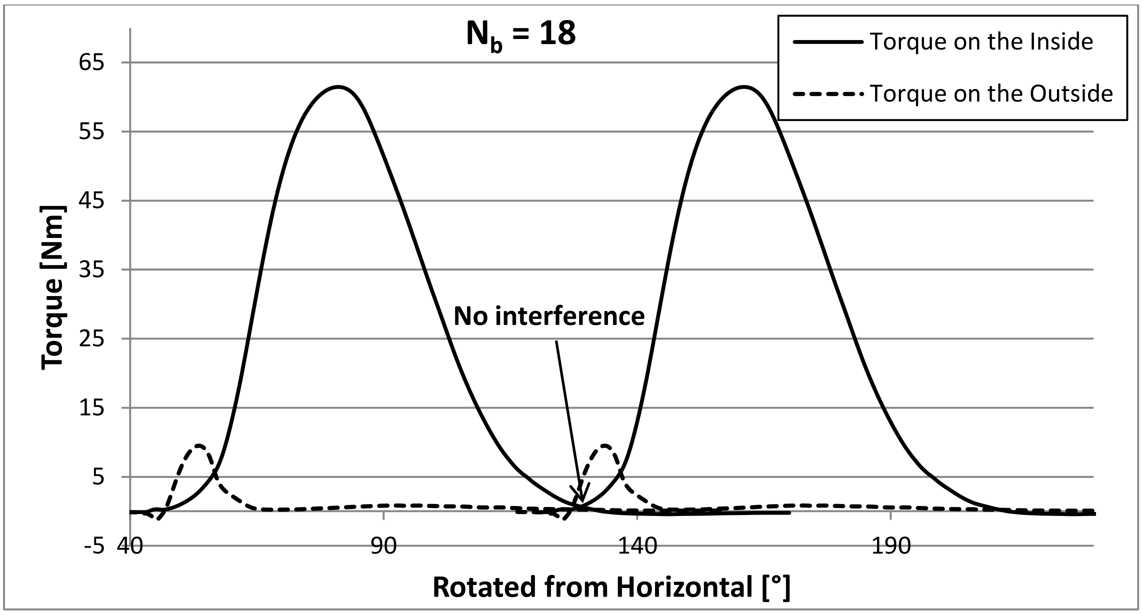

Fig. 18. Two torque peaks taken from the single jet simulation and manually shifted by the angle between the jets (80°). Runner with 18 buckets.

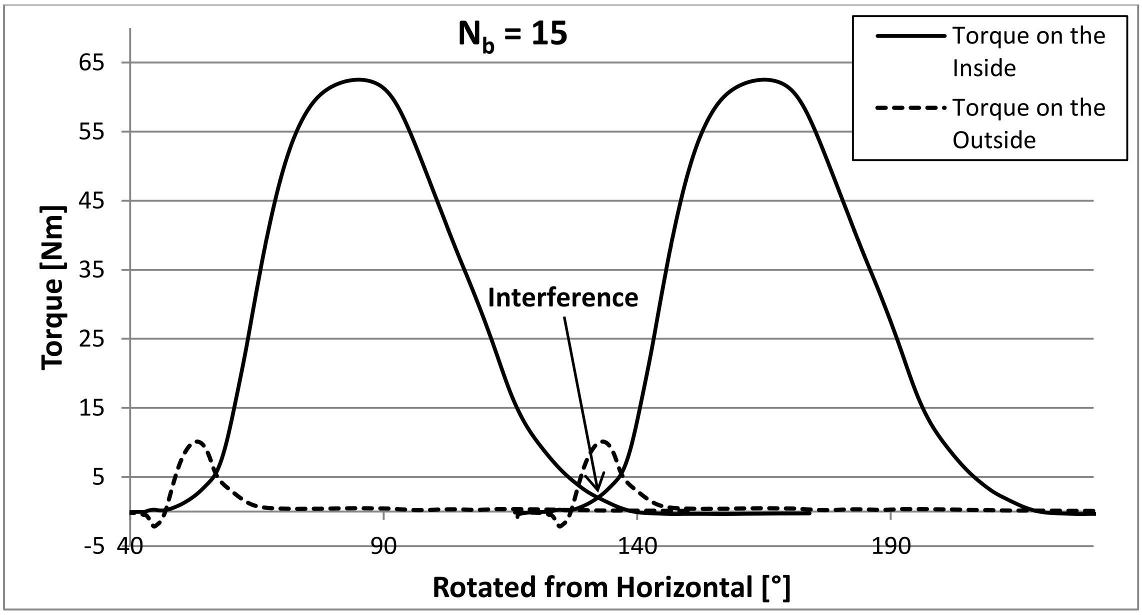

Fig. 19. Two torque peaks taken from the single jet simulation and manually shifted by the angle between the jets (80°). Runner with 15 buckets.

6 Conclusions

The number of buckets is an important parameter when optimising a Pelton turbine runner. However, no consistent guidance based on experimental or numerical research data is available in the public domain. The case study described in this paper draws attention to the inconsistency in the available guidance and provides an example of how the optimum number of buckets and their mounting position could be identified numerically after the bucket geometry is modified.

Experimental results show that readjusting these parameters has additionally increased runner efficiency by 0.8% under the single jet operation and 0.4% under the two jet operation. It is explained that the efficiency increase under the operation with both jets could be higher and most probably similar to the single jet operation if the angle between the jets was increased to eliminate the interference between the jets. In addition to the efficiency increase which is a highly desirable achievement itself, the reduction of the number of buckets from 18 to 15 reduces the complexity and the cost of runner manufacturing.

Acknowledgements

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Nomenclature

Notation	Units	Description
N_b	-	Number of buckets
D_p	m	Pitch circle diameter
d	m	Jet or nozzle diameter
d_j	m	Jet diameter
d_o	m	Optimum jet diameter
R_N	-	Normalized radial position
D_t	m	*S*plitter tip circle diameter
D_{t18}	m	Optimum splitter tip circle diameter for a runner with 18 buckets.
H	m	Pressure head
B	m	Bucket width
n	rpm	Rotational speed
n_{11}	rpm	Unit speed
Q	m^3/s	Flow rate
Q_{11}	m^3/s	Unit flow rate
Q_{11k}	m^3/s	Unit flow rate specified to the bucket width and one jet
N_J	-	Number of jets

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