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Reconstruction of the complete characteristics of the hydro turbine based on inner energy loss

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Abstract: The power output characteristics of the hydro turbine is one of the core contents for transient calculation of the hydro turbine generating sets (HTGS). In particular, the hydro turbine operates far beyond the given parameters region during the load rejection transient. As such, obtaining the complete characteristics of the hydro turbine becomes one of the key issues in calculating the transient process. In this study, methods for calculating the energy losses are proposed by analyzing the general characteristics of the inner energy losses within the hydro turbine. Characteristic parameters in the hydro turbine power model are calculated from the synthetical characteristics of the model hydro turbine. The transient power model of the hydro turbine has been established and applied to calculate and reconstruct the complete characteristics of the hydro turbine. Furthermore, the relationship curve between the mechanical friction loss power and the rotation speed under different head can be established by combing the runaway curve with the proposed turbine power model. This relationship is applied to construct the complete characteristics of the mechanical friction loss. Combining the proposed two complete characteristics, the power model of the hydro turbine is suitable for simulation with a wide range of fluctuations as well as the load rejection transient. Details of the computational procedures are presented and demonstrated using a case study.

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***Keywords**: hydro turbine; inner energy loss; characteristic parameter; mechanical friction loss; complete characteristics.*

List of symbols

D	Equivalent damping coefficient of the generator
D_1	Diameter of turbine runner in m
$f(.)$	Stands for function
$F(.)$	Stands for function
ΔF	Correct function
f_p	Coefficient of the loss in the penstock
h_q	Transient head increment in relative value
h_Δ	Head convert coefficient
H_0	Static head of the hydro turbine in m
$H_{\max}$	Maximum head in m
$H_{\min}$	Minimum head in m
H_r	Rated head in m
H_t	Turbine head in m
k_h	Coefficient of the flow channel loss
k_v	Coefficient of the volume loss
n	Rotation speed of units in rpm
n_r.	Rated rotation speed of unit in rpm
n_1'	Unit speed in rpm
p_d	Resistance power in relative value
P_t	Turbine power output in KW
p_w	Water flow power in relative value
p_g	Power of the generator in relative value
P_d	Resistance power in KW
Q	Turbine flow discharge in m^3/s
Q_1	Turbine flow discharge in m^3/s
Q_2	Turbine flow discharge in m^3/s
Q_{nl}	No-load flow in m^3/s
Q_r	Rated flow in m^3/s
Q_z	Flow rate at the peak efficiency point m^3/s
Q_p	Flow rate at the runaway operation
Q_1'	Unit flow in m^3/s
Q_{zH}	Flow at the maximum efficiency point
ΔQ	Increment of turbine flow discharge in m^3/s
γ	Density of water in KN/m^3
ΔP_h	Turbine loss power in flow channel in KW
ΔP_i	Impact loss power in KW
ΔP_m	Mechanical friction loss power in KW
ΔP_γ	Volume loss power in KW
δP_v	Increment of volume loss power in KW
δP_h	Increment of turbine loss power in the flow channel in KW
δP_i	Increment of impact loss power in KW
T_j	Inertia time constant in s
ω	Angular speed of the unit in relative value

1 Introduction

Calculation of the power output of the hydro turbine is one of the most important issues in transient calculation of the hydro turbine generating sets (HTGS). Currently, there are two classes of computational methods. One focuses on the transient calculation of the hydro turbine head and establishes the power output model of the hydro turbine. Another approach is the calculus of the interpolation method based on the synthetical characteristics of the model turbine.

Among the first class methods, based on the hydraulic dynamic of the pipeline, the hydraulic dynamic of various hydraulic systems is converted as the water head at the inlet of the hydro turbine [1,2]. The power output of the hydro turbine can be calculated using the algebraic equation when the dynamic head and flow is obtained. Because the hydro turbine is approximately treated as a rigid component, the transient change of the hydro turbine power output is determined by the dynamic of the hydraulic system. In previous studies, there are two main model classes: one is the linearized model based on the transfer function of the hydraulic dynamic and another is the differential equation model with rigid water hammer. From the expression of the hydro turbine power output, the power model of the hydro turbine can be classified as linear model and nonlinear model [3,4,5,6]. Recently, several new methods and theory are developed to investigate the hydraulic dynamic and the turbine power output. Some novel description methods, such as, differential equation model with elastic water hammer [7], four-equation model by combining the fluid-structure interaction with water hamper model [8], the generalized Hamiltonian model [9,10,11], and the fractional-order model [12], have been proposed. These models are mainly applied to investigate the dynamic of the HGTS and control strategy.

In second class methods, the relationship of the unit speed，unit flow and unit torque are obtained from the synthetic characteristics of the model turbine, which is called the complete characteristics of the hydro turbine. This relationship can be used to interpolate the instantaneous turbine torque in transient according to the unit flow and the unit speed. During the load rejection transient, the amplitude of the head change is large [13,14], which can cause the hydro turbine operate beyond the region determined by the synthetical characteristics of the model turbine. On the other hand, there lacks of the turbine characteristics under small opening within the normal operation water head. For these reasons, the complete characteristics of the turbine power output are required to better deal with the small opening as well as high water head. The disadvantages of this method are: (i) the fitting algorithm of the flow and torque may result in large error when the characteristic curve of the hydro turbine is extended to small opening by means of the runaway characteristics; and (ii) the model synthetic characteristic curve of the hydro turbine is obtained for the steady operation state, therefore, the calculation method based on the model synthetic characteristic curve is essentially an approach to apply the steady-state characteristics to study transient problem.

With the development of the numerical computational method, the torque characteristics of the hydro turbine in the load rejection transient can be calculated [15,16]. However, it is still a challenging task to calculate the whole transient process with load rejection. Generally speaking, the complete characteristics of the turbine power output obtained from the synthetical characteristics of the model turbine is more accurate. Therefore, the complete characteristics of the turbine power output is of importance in the study of the hydro turbine. For the pump turbine, extensive studies on its complete characteristics have been conducted, partly due to its four-quadrant characteristics [17,18]. For traditional hydro turbine, a method was proposed in [19] to calculate the torque based on the turbine inner characteristics. The method employed the structure parameters of the hydro turbine to calculate the turbine torque. The inner energy composition within the hydro turbine is analyzed in [20] and the torque model of the hydro turbine based on the inner energy loss characteristics is proposed. The method can be applied to calculate the variation of the transient power output. Though these studies have demonstrated some features of the transient characteristics of the hydro turbine; studies and reports on the complete characteristics of the hydro turbine are still lack.

Based on the inner energy loss characteristics of the hydro turbine, this study proposes a method to calculate each loss power. An accurate model for calculating the hydro turbine power output is developed and applied to reconstruct the complete characteristics of the hydro turbine power. Detailed calculation procedures are presented and demonstrated using a case study of a hydropower station.

2 The turbine power model

2.1 General model

Refer to the turbine model with the inner energy loss characteristics in [20], the inner energy of the hydro turbine composition can be written as following:

$$P_{\mathrm{t}} = \gamma Q H_{\mathrm{t}} - \Delta P_{\mathrm{h}} - \Delta P_{v} - \Delta P_{\mathrm{m}} - \Delta P_{\mathrm{i}} \tag{1}$$

where H_t is the head of the hydro turbine, Q is turbine flow discharge, γ is the density of water, the volume loss power is defined as $\Delta P_\gamma=\gamma QH_{\mathrm{t}}k_{\mathrm{v}}$, the turbine loss power in the flow channel is defined as $\Delta P_{\mathrm{h}}=k_{\mathrm{h}}Q^3$, k_h is coefficient of floe channel loss,

The energy composition described by Eq. (1) and the calculation method of the characteristic parameters in the next section have been verified using an actual field measurement data for efficiency test of hydro turbine [20]. For details of the field measurements, readers are referred to [20].

Remark 1: At the optimum operation of the hydro turbine, there is no water flow impacting on the inlet of the turbine runner while water outflow moves along the normal direction at the outlet of the turbine runner. Under non-optimum operation, there exists the impact loss at both the inlet and outlet of the turbine runner. From the efficiency curve of the hydro turbine, the impact loss is almost the same for the same deviation of the flow rate away from the peak efficiency point. Therefore, it is possible to combine the impact loss at the inlet and the outlet, the effect of the draft tube and other losses except clear losses in Eq.(1) into the impact loss power ΔP_{i}. As such, ΔP_{i} is a function of flow and can be expressed as $\Delta P_{\mathrm{i}}=F[(Q\text{-}Q_{\mathrm{z}})]$.

The turbine power output expression can be written as following:

$$P_{\mathrm{t}} = \gamma(H_0 - \frac{H_{\mathrm{r}}}{Q_r^2} f_{\mathrm{p}} Q^2 - H_{\mathrm{r}} h_q) Q(1-k_{\mathrm{v}}) \\ - k_h Q^3 - F[(Q-Q_z)] \tag{2}$$

where $H_{\mathrm{t}}=H_0-f_pH_{\mathrm{r}}Q^2/Q_{\mathrm{r}}^2-H_{\mathrm{r}}h_qh_q$ is the transient head in relative value, the head convert coefficient is $h_\Delta=H_{\mathrm{r}}/H_{\mathrm{t}}$, the coefficient of the volume loss depends on the seal of the turbine.

In fact, Eq.(2) also provides the composition and calculation method of the hydro turbine head. The head of hydro turbine H_{t} equals to the difference of the static head of turbine H_0 and the sum of the friction loss head in pipeline $f_pH_{\mathrm{r}}Q^2/Q_{\mathrm{r}}^2$ and the transient head H_rh_q generated by flow change during unit regulation. Under steady state condition, the transient head H_rh_q 0, the head of hydro turbine is then equal to the static head of hydro turbine minus the friction loss head in pipeline.

Remark 2: Characteristic parameters in Eq.(2) are calculated under the rated head. Under arbitrary head, the turbine power can be approximately calculated by means of the head convert coefficient h_Δ. For example, the no-load flow under arbitrary head is expressed as $h_\Delta Q_{\mathrm{nl}}$, the flow at the peak efficiency point under arbitrary head is expressed as $Q_z/\sqrt{h_\Delta}$. On the other hand, the transient head $H_r h_q$ under steady state operation is zero ($H_r h_q$ =0). Under the transient state, using $H_r h_q$ as the transient head can reflect the transient change of the turbine power output. Thus Eq.(2) is suitable for both the large and small fluctuations under arbitrary head.

2.2 Computation of characteristic parameters

Applying equation (2) and using the data under steady state operation to calculate the characteristic parameters. Let h_q=0; the coefficient of volume loss k_v can be selected according to the seal characteristics of the turbine. Usually, k_v is taken as 0.002-0.004. The computational procedure of the characteristic parameters are presented as following.

(1) Determination of parameter k_h

Based on the definition of the impact loss power in Remark 1, assume that the impact loss be zero at the peak efficiency point for the same head and the impact loss be the same for the same flow deviation away from the peak efficiency point. Selecting the efficiency curve at some water head and taking a point at each side of the maximum efficiency point, the flow then satisfies the following

$$\begin{cases} Q_1 = Q_z + \Delta Q \\ Q_2 = Q_z - \Delta Q \end{cases} \tag{3}$$

The impact loss at these two points is equal to each other. From Eq. (2), one can obtain:

$$k_h = \frac{\gamma H_t (Q_1 - Q_2)(1 - k_v) - P_{t1} + P_{t2}}{Q_1^3 - Q_2^3} \tag{4}$$

where P_{t1}=9.81$Q_1 H\eta_1$, P_{t2}=9.81$Q_2 H\eta_2$, η_1 and η_2 is the efficiency corresponding to flow Q_1, and Q_2, respectively.

(2) Mechanical friction loss

Assume the impact loss ΔPi=0 at the maximum efficiency point, the loss power due to the mechanical friction at the maximum efficiency point can be calculated as:

$$\Delta P_m = \gamma H Q_z (1 - k_v) - k_h Q_z^3 - P_{tz} \tag{5}$$

where P_{tz} is the turbine power at the maximum efficiency point.

(3) No-load flow computation

The turbine power P-Q curve under the rated head can be obtained from the model synthetic characteristic curve. The no-load flow can then be obtained by extending P-Q curve to the point where P=0.

(4) Impact loss

The impact loss power can be written as:

$$\Delta P_i = \gamma Q H_t - \Delta P_m - \Delta P_h - \Delta P_v - P_t \tag{6}$$

Given flow Q, the impact loss power can be calculated by combining Eq.(6) with interpolation calculation of P-Q curve. Taking (Q-Q_z) as variable, the impact loss power is further fitted to impact loss function $F(Q-Q_z)$.

Remark 3: The calculation above for the characteristic parameters is carried out by means of the efficiency curve of the hydro turbine under the rated head. In order to obtain a uniform and suitable for arbitrary head model, the model of the impact loss power is obtained by extending and correcting the impact loss power under the rated head, which will be used in the case study. If the characteristic parameters are calculated using the measured efficiency curve of the hydro turbine, the power model should be transformed into the model under the rated head by using the head convert coefficient h_Δ if the turbine head is not equal to the rated head.

3 Mechanical friction loss

From equation (1), water flow power is the active power, while other loss powers are resistance power. Then one has:

$$P_w = \gamma Q H_t \tag{7}$$

$$P_d = \Delta P_h + \Delta P_v + \Delta P_m + \Delta P_i \tag{8}$$

Where the loss power ΔP_v, ΔP_h and ΔP_i are the mono-variable function of flow rate. The characteristic parameters are derived under the steady state operation, namely the rotation speed is the rated speed. In load rejection process, the sharp change of the rotation speed of HTGS deteriorates the flow regime in the runner, resulting in the change of the loss power. Assume the loss power change be $\Delta P_v+\delta P_v$, $\Delta P_h+\delta P_h$ and $\Delta P_i+\delta P_i$, the resistance power (8) can be expressed as:

$$\begin{aligned} P_d &= \Delta P_h + \delta P_h + \Delta P_v + \delta P_v + \Delta P_m + \Delta P_i + \delta P_i \\ &= \Delta P_h + \Delta P_v + \Delta P_i + (\Delta P_m + \delta P_i + \delta P_h + \delta P_v) \end{aligned} \tag{9}$$

The four items in bracket on the right hand side of (9) are related to the rotation speed of the unit. For simplification, these items are denoted as ΔP_m, and defined as the general mechanical friction loss power.

At the runaway operation, the active power is equal to the resistance power, namely P_w=P_d. The mechanical friction loss power can be expressed as:

$$\begin{aligned} \Delta P_m &= \gamma Q H_t - \Delta P_v - \Delta P_h - \Delta P_i \\ &= \lambda Q_p H_t - \gamma Q_p H_t k_v - k_h Q_p^3 - F[Q_p - Q_z] \end{aligned} \tag{10}$$

where Q_p is the flow rate at the runaway operation.

In Eq.(9), the volume loss, the flow channel loss and the impact loss are the mono-variable functions of the flow rate. The power balance relationship at the runaway operation reflects the internal relation between the mechanical friction loss and the rotation speed of the unit. Therefore, the relation between the mechanical friction loss and the rotation speed of unit can be calculated by means of the runaway characteristic curve of the hydro turbine.

Remark 4: The mechanical friction loss power ΔP_m defined in this paper only relates to the mechanical rotation speed. When HTGS operates under connecting with the power network, the change of the rotation speed of HTGS is small. Therefore, the mechanical friction loss power ΔP_m could be assumed as a constant. However, the rotation speed of HTGS in load rejection changes sharply. Meanwhile the change of the mechanical friction loss is also large. For this situation, the relation between the mechanical friction loss power and the rotation speed in transient state, $\Delta P_m=f(H, n)$, needs to be established.

Remark 5: After obtaining $\Delta P_m=f(H, n)$, the active power generated by water flow and the resistance power can be calculated by Eq.(7) and (9) respectively. Thus the motion equation of HTGS, which suits to various disturbances, can be obtained:

$$T_j \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{p_{\mathrm{w}}}{\omega} - \frac{p_{\mathrm{d}}}{\omega} - \frac{p_{\mathrm{g}}}{\omega} - D(\omega - 1) \qquad (11)$$

In load rejection transient, p_g=0, the damping item $D(\omega$-1) is combined into the resistance power $p_{d.}$ In this way, equation (11) can be used to simulate the transient process in load rejection.

The detailed analysis of the inner energy loss of the hydro turbine shows that the mechanical friction loss power varies with the rotation speed in the transient state. A complete characteristics of the hydro turbine is proposed in this study to describe and simulate this complex transient state, which is the main innovation of this study.

4 Reconstruction complete characteristics of turbine

The general description of the complete characteristics of the hydro turbine can be written as:

$$P_{\mathrm{t}} = f(Q, H) \qquad (12)$$

As the synthetical characteristics of the model turbine only provides the characteristics for a certain range of flow and water head, water head H and flow Q need to be extended for a wide range. Towards the small opening, the flow is usually extended to non-load flow Q_{nl}. Due to the limitation of the discharge capacity of the hydro turbine, the flow of the hydro turbine under the minimum water head will not exceed 150%Q_r (rated flow discharge). Therefore, the range of the flow rate for the complete characteristics of the hydro turbine can be taken from Q_{nl} to 1.5Q_r. For the extension of the water head, considering the large increase of the pressure in load rejection and the regulations for the pressure increase under various water head, the maximum water head should not exceed 2H_r (rated head). In order to avoid the occurrence of the negative pressure under the minimum water head, the range of the water head for the complete characteristics is selected as 0.9H_{min}~2H_r.

According to Remark 4, a novel complete characteristics of the hydro turbine can be written as:

$$\Delta P_m = f(H, n) \qquad (13)$$

In transient state, the rotation speed of the hydro turbine can reach two times of the rated rotation speed n_r. Therefore, the range of the rotation speed is selected as n_r～2n_r.

The unit speed and the unit flow corresponding to each opening can be obtained from the runaway curve. The flow rate, the rotation speed and the general mechanical loss power at the runaway operation point can be calculated based on the unit rotation speed ($n_1' = nD_1/\sqrt{H}$) and the unit flow ($Q_1' = Q/(D_1^2\sqrt{H})$) for the given water head H. The relationship (H，n)～ΔP_m can then be established by adding the no-load flow and the rated rotation speed. In the transient state, the mechanical friction loss power can be calculated by interpolating method.

To sum up, the procedures of reconstructing the complete characteristics of the hydro turbine is shown as following.

Step 1: Calculate the parameters Q_{nl}, Q_z, k_h and ΔP_m under the rated head using the synthetical characteristics of the model turbine.

Step 2: Obtain the impact loss power. Establish the impact loss function using the fitting and correcting method.

Step 3: Establish the accurate model of the hydro turbine power, which suits for a wide range of water head.

Step 4: Calculate the complete characteristics of the hydro turbine at the rated head using the proposed accurate model.

Step 5: Combining the proposed accurate model and the runaway curve, the complete characteristics of the mechanical friction loss $\Delta P_m=f(H, n)$ can be calculated.

5 Simulation example

To demonstrate the proposed model, a case study is carried out using the data from a hydropower station. The hydraulic system of the hydropower plant includes tunnel, penstock and two bifurcation penstocks with the Francis turbines. The total length is about 620m. The rated water head is H_r=69m, the maximum water head is H_{max}=79m, and the minimum water head is H_{min}=49.3m. The efficiency curve of the hydro turbine under the rated head has been calculated from the synthetic characteristics of the model turbine. These data are used to verify the validity and applicability of the descriptions and concepts proposed in this paper.

As the field test of the hydro turbine efficiency is conducted under a specific water head, it is difficult to obtain the whole complete characteristics of the hydro turbine. Furthermore, it is difficult to carry out the test and measurements for some special operation condition. As such, it is difficult to verify the model using the field data due to lack of field measurements. On the other hand, the synthetical characteristics of the model turbine has been provided by the hydro turbine manufacturing with full test operation conditions. The data can be regarded as laboratory test data and thus, can be used for verifying the proposed model. For this reason, in this study, the synthetical characteristics of the model turbine are used to verify the proposed model.

On the other hand, the efficiency of prototype hydro turbine is calculated from the synthetical characteristics of the model turbine and thus, needs correction.. Due to the difference of the installation and operation conditions of the prototype turbines, there exists error for modifying the efficiency using empirical approach. To avoid this, we propose a method, focusing on the efficiency of the hydro turbine to calculate and reconstruct the complete characteristics of the hydro turbine. The power model of the hydro turbine is calculated based on the field test efficiency, as such, there is no need of correction.

In next section, calculation and verification is based on the synthetic characteristics of the hydro turbine, which is, in principle, the characteristics of the model turbine.

5.1 Computation of the characteristic parameters

In the simulation, k_v is taken as 0. The determination of other parameters are as following.

(1) Determination of coefficient k_h

The unit speed under the rated head is n'_{10}=70.7869 r/min. The unit flow at the peak efficiency point is Q'_{10}=0.6606 m^3/s. From the synthetic characteristic curve of the hydro turbine and along the unit rotation line n'_{10}=70.7869 r/min, two efficiencies η_1 and η_2 are found, which correspond to two flow discharges $Q'_{10}+\Delta Q'_{10}$ and $Q'_{10}-\Delta Q'_{10}$, respectively. The power outputs P_{t1} and P_{t2} on the two sides of the maximum efficiency point can be calculated using $P_t=9.81QH\eta$. The coefficient k_h can then be obtained from Eq.(8). In order to improve the computational accuracy, ten unit flow increments $\pm\Delta Q'_{10}$ is selected from 0.01 to 0.10 m^3/s. The corresponding coefficient k_h is calculated using the ten increments. The mean value is then calculated and the result is k_h=0.5388.

(2) No-load flow Q_{nl}

From the model synthetic characteristics of the hydro turbine, the efficiency can be found for the corresponding unit flow rates. The turbine power P_t can then be calculated. As the model synthetic characteristics of the hydro turbine lacks of the data for the small opening, the flow rate and power need to be extended to include the situation of the small opening so that the relation (P_t, Q) suits low flow region. In order to reduce computational error, using polynomial of degree 1, degree 2 and degree 3 to fit the function $P_t=f(Q)$ respectively. The no-load flow Q_{nl} can be found when Q_{nl} satisfies $P_t=f(Q_{nl})=0$. The final no-load flow is obtained by averaging three no-load flow rates obtained by three fitting modes and calculations. The averaged no-load flow and P_t=0 are then added into the original data to compose new data group. The turbine power expression is obtained by using new data and polynomial of degree 3 as following:

$$N(Q) = -3.8913Q^3 + 19.1929Q^2 + 730.9058Q - 447.3741 \tag{14}$$

The final no-load flow obtained is Q_{nl}=0.6033m^3/s, which is equal to the relative opening 11.45%Y_r, where Y_r is the guide vane opening at the rated operation point. The variation of the turbine power with the flow rate is shown in Fig.1.

In Figure 1, the symbol "*" is the raw data which is obtained based on the synthetical characteristics of the model turbine, while the solid line is the fitted curve using Eq.(14). It is seen from Figure 1 that the turbine power output increases roughly linearly with the increase of the flow rate for various water heads.

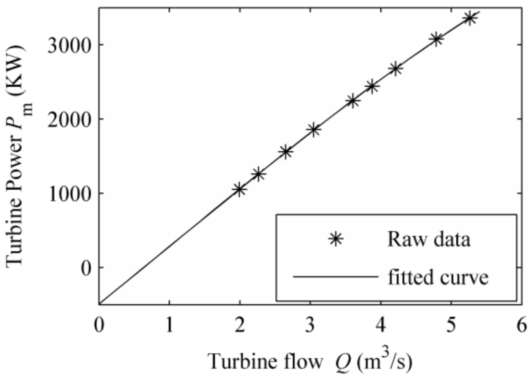

Fig.1 Variation of the turbine power with the flow rate

(3) Mechanical friction loss

The mechanical friction loss power is calculated using Eq.(7) at the maximum efficiency point. The calculated mechanical friction loss is ΔP_m=116.7398 KW. Using the rated power P_r as basic value, the mechanical friction loss power is then 3.48%P_r.

(4) Impact loss power

Combining Eq.(8) with Eq.(14), the impact loss power can be calculated for given flow rate Q. Taking (Q-Q_z) as variable, the impact loss power can be further fitted to the impact loss function $F(Q-Q_z)$.

$$F(Q-Q_z)=3.3524(Q-Q_z)^3+28.9476(Q-Q_z)^2 \\ -7.3239(Q-Q_z) \tag{15}$$

The variation of the impact loss power with the flow rate is shown in Fig.2. In Figure 2, the symbol "*" represents the raw data which is obtained based on the synthetical characteristics of the model turbine, while the solid line is the fitted curve by Eq.(15). It is seen that the impact loss power decreases with the increase of the flow rate until the flow rate reaches that of the peak efficiency point. After that, the impact power loss increases with the increase of flow rate.

Remark 6: The expression (14) of the impact loss power is obtained under the rated head. However, the flow rate Q_z at the maximum efficiency point is related to the turbine water head. It is not practical to calculate and fit the impact loss power for all water heads. On the other hand, it is known that from the characteristics of the hydro turbine the flow rate at the maximum efficiency point increases with the increase of the turbine water head. According to this fact and through the analysis of the simulation results, the flow rate Q_{zH} at the maximum efficiency point under the arbitrary water head can be expressed as $Q_{zH}=Q_z/\sqrt{h_\Delta}$. Simulation shows that the correction flow rate is approximately consistent with the actual system.

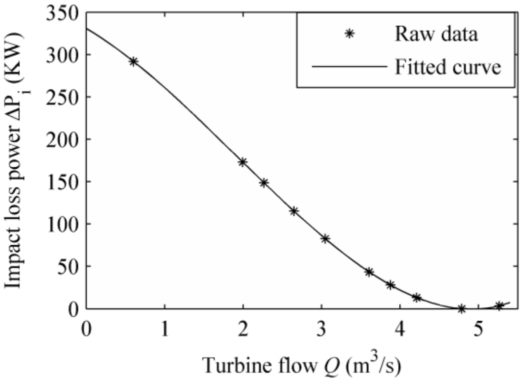

Fig.2 Variation of the impact loss power with the flow rate

For simplification, the impact loss power for arbitrary head can be estimated using the modified equation (15) for the rated head:

$$F(Q-Q_{zH})=3.3524(Q-Q_{zH})^3 \\ +28.9476(Q-Q_{zH})^2-7.3239(Q-Q_{zH}) \tag{16}$$

In order to verify rationality of this approximation, the impact loss power is calculated for both the minimum and the maximum heads from the model synthetic characteristic curve of the hydro turbine. The result is plotted in Fig.3.

In Fig. 3, the thick solid lines are the result calculated using Eq.(16), while the dashed lines are the results obtained from the model synthetic characteristics of the hydro turbine for the maximum and minimum water heads. It is seen that when the impact loss power under different heads is calculated using equation (16), some errors appear in the low flow region with the maximum relative error being about 2%Pr. In load rejection transient, the turbine water head is significantly different from the rated head. In this situation, the error of the impact loss power under the low flow region may affect the computational result of the resistance power. Therefore, the error of the impact loss power due to the deviation from the rated head should be corrected, which is described as following.

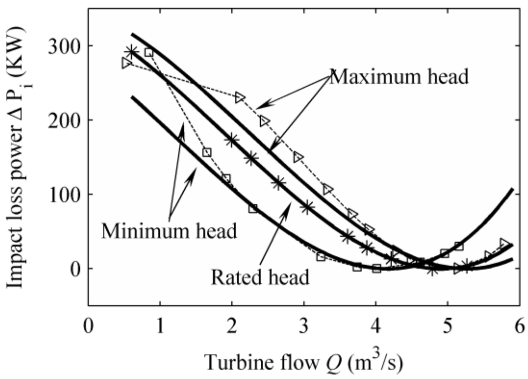

Fig.3 Variation of the impact loss power with the flow rate for different water heads

In this example, the impact loss power under the turbine water heads H_{min}=49.3m, H=75.0m and H_{max}=79.5m, are calculated from the model synthetic characteristics of the turbine respectively. According to the tendency of the impact loss power error shown in Fig.3, the correction function ΔF of the impact loss power under arbitrary head can be selected as:

$$\Delta F=sign(H-H_r)\sqrt{H-H_r}(Q-Q_{nl})(Q-Q_{zH})^2 \\ +[F(Q_{nl}h_\Delta-Q_z)-F(Q_{nl}h_\Delta-Q_{zH})]\frac{Q-Q_{zH}}{Q_{nl}h_\Delta-Q_{zH}} \tag{17}$$

where the function F of the impact loss power is Eq.(16), $sign(H\text{-}H_r)$ is a sign function, representing "+" for $H>H_r$ and"-" for $H<H_r$.

Combining Eq.(16) and Eq.(17), the corrected function of the impact loss power is:

$$F'(Q-Q_{zH})=F(Q-Q_{zH})+\Delta F \tag{18}$$

The impact loss power calculated using Eq.(18) is shown in Fig.4. It is seen from Fig.4 that the corrected algorithm demonstrated by Eq.(17) and Eq.(18) is appropriate.

In Fig.4, the solid line is calculated result using Eq.(18), while symbols are is the data obtained by interpolating calculation from the synthetical characteristic of model turbine.

Comparing Fig.4 with Fig.3 shows that the computational error of the impact loss generated by deviating rated head can be reduced by using the correction Eq.(17), indicating that the proposed correction method is appropriate.

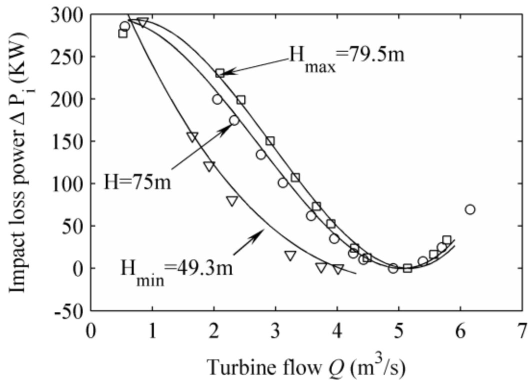

Fig.4 Correcting result of the impact loss power

5.2 The computational model of the turbine power output in transient state

In one-penstock-two-machines system of the hydropower plant, longer branch penstock with the Francis turbine is taken as the case study. The equivalent coefficient of the friction loss of the hydraulic system is fp=0.0924, Q_r=5.27m^3/s, $H_r f_p/Q_r^2$=0.2296, and Q_z=4.7867(m^3/s). Substituting these into Eq.(2) yields the turbine power:

$$P_t = 9.81(H_0 - 0.2296Q^2 - H_r h_q)Q - 0.5388Q^3 - F[Q - 4.7867/\sqrt{h_\Delta}] \quad (19)$$

Or:

$$P_t = 9.81H_t Q - k_h Q^3 - \Delta P_m - F(Q - Q_z/\sqrt{h_\Delta}) - \Delta F \quad (20)$$

Where the no-load flow under the rated head is Q_{nl}=0.6033m^3/s; the flow rate at the maximum efficiency point is Q_z=4.7867 m^3/s; and the coefficient of the hydraulic loss is k_h=0.5388. In this example, the rated flow of the hydro turbine is Q_r=5.27m^3/s. The error of the turbine water head, which is generated by the hydraulic loss item $0.2296Q^2$ in Eq.(19), between the head at the rated power operation and the head at no-load point is about 6.3m, namely the amplitude of water head change being about 9%H_r. On the other hand, the unit speed, which is used to calculate the turbine power from the model synthetic characteristic curve, is obtained at the same head. For convenient comparison, the hydraulic loss item is ignored when calculating the turbine power by Eq.(20); meanwhile H_0 is adjusted to maintain a constant head so that the computation is consistent.

The verification calculation is conducted at the steady state operation, h_q=0. The turbine power under different heads is shown in Fig.5 in which, the symbols are the raw data obtained from the synthetic characteristics of the model turbine and the solid line is the calculated results using Eq.(20). It is seen that the computed results from Eq. (20) agree well with those predicted by the synthetic characteristics of the model turbine.

Figure 5 shows that the calculation is conducted for three characteristic water heads, covering the whole operation range of the hydro turbine. The good agreement between the simulated results of this method and the predictions using the synthetic characteristics of the model turbine for the whole operation range of the hydro turbine demonstrates that the proposed method has generic applicability and can be used to calculate the complete characteristics of the hydro turbine.

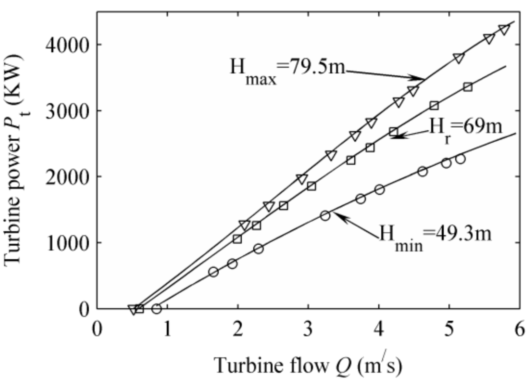

Fig.5 Turbine power under different heads

Traditional computational method of the turbine power from the model synthetic characteristic curve cannot produce the turbine characteristics for low load region. The method proposed in this study can generate the complete characteristics of the hydro turbine power. The turbine power given by Eq.(18) is a mono-variable function of the flow rate Q, which is convenient and ease to use with high computational accuracy.

5.3. Complete characteristics of the hydro turbine

The complete characteristics of the hydro turbine can be obtained using Eq.(20) for a wide range of the flow rate and water head. The computational results are plotted in Fig.6 in which the dark color is the interpolation data from the synthetical characteristics of the model turbine.

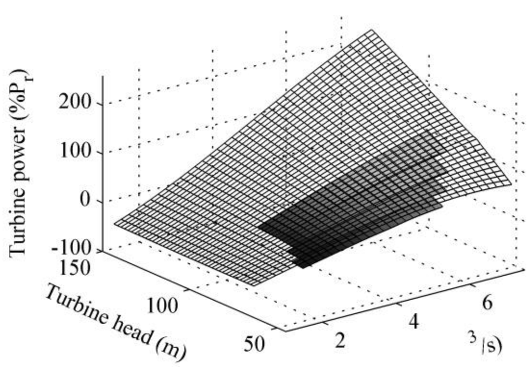

Fig.6 Complete characteristics of the hydro turbine

In order to verify the computational accuracy, the simulated results using Eq. (20) are compared with those obtained by using the traditional interpolation method for the range of the water head and flow rate limited by the synthetical characteristics of the model turbine. In traditional method, the complete characteristics of the hydro turbine is obtained by interpolating and smoothening around the power zero and runaway operation point. The comparison is shown in Fig.7 in which the relative error is calculated as the difference of the simulated power and the interpolated power divided by the result from the interpolation.

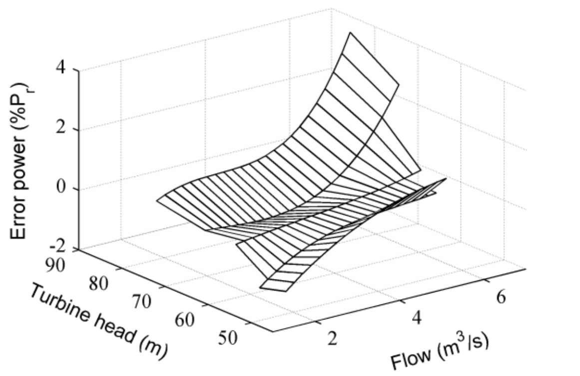

Fig.7 The variation of the computational relative error of the power with the turbine water head and flow rate

In Fig. 7, the rated head is H_r=69m, and the rated flow is Q_r=5.27m^3/s. It is seen that the computational error sharply increases when the flow rate and water head are significantly greater than the rated values of the flow and head with the maximum relative error being about 3.8%Pr. When the flow rate is smaller than the rated flow, the simulated power using Eq.(20) agrees well with that obtained by interpolation with the error being less than 1%Pr. The simulation demonstrates that the model of the hydro turbine proposed in this study has higher calculation accuracy than the traditional method.

5.4 Complete characteristics of the mechanical friction loss

At the runaway operation, the active power is equal to the resistance power, namely P_w=P_d. The mechanical friction loss power Eq.(10) can be rewritten as:

$$\Delta P_{\mathrm{m}} = 9.81 Q_p H_{\mathrm{t}} - k_{\mathrm{h}} Q_p{}^3 - F[Q_p - Q_z / \sqrt{h_\Delta}] \tag{21}$$

Equation (21) shows that the mechanical friction loss power relates to the water head and the flow rate at the runaway operation. As the impact loss power relates to the flow at the maximum efficiency point, it also relates to the head of the turbine. Consequently, the mechanical friction loss power relates to the head of the turbine.

The unit flow and the unit rotation speed corresponding to an opening can be obtained from the runaway characteristic curve; namely: unit flow $Q_1' = Q/(D_1^2\sqrt{H})$; unit speed $n_1' = nD_1\sqrt{H}$. With the known unit flow and head, the general mechanical friction loss ΔP_m at the runaway point can be calculated for giving head H. Adding the no-load flow Q_{nl} and the speed n_r into the data group, the relationship of (H, n)~ΔP_m can be obtained. In load rejection transient, ΔP_m is obtained by interpolating with (H, n). This can be demonstrated using an example. In this example, the minimum head is 49.3m, the rated head is 69m, and the rated speed is 600rpm. The region of (H, n) is extended to: H=49~130m and n=600~1200rpm. The computational result is shown in Fig. 8. The maximum rotation speed in low head region is less than 1200 rpm. Therefore, the delta region in the top left corner of Fig.8 reflects the limitation of the rotation speed.

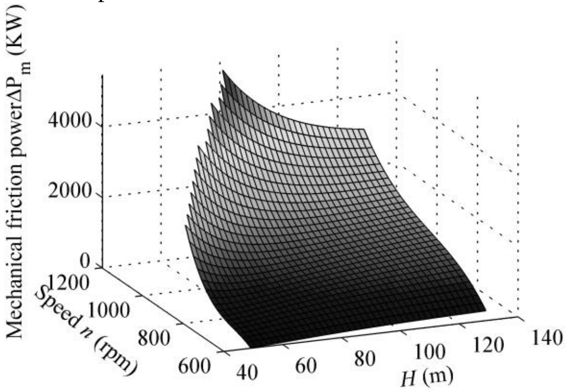

Fig. 8 Variation of the mechanical friction loss power with the rotation speed and flow rate

5.5 Computation of the transient speed in load rejection

Each loss in the power model of the hydro turbine can be calculated by using the proposed method. In load rejection, the rotation speed of HTGS is calculated by Eq.(11). The active power of the turbine in relative value is p_w=γQH/Pr, where H is the approximate pressure at the end of the pipeline. The relative resistance power p_d is P_d/P_r. Fig. 9 is the computational result in which the vertical dashed line indicates the total power=0, corresponding to the maximum rotation speed.

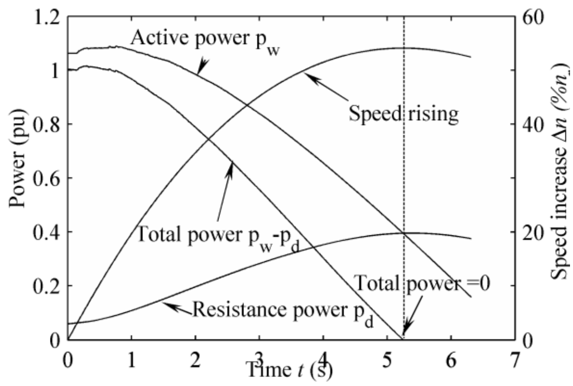

Fig. 9 Power and speed change in load rejection transient

The load rejection is a specific operation condition, where the flow Q and head H in transient can be calculated by using the classic pipeline method of characteristics. The head H and flow Q are then taken as transient data and used in calculation.

Under the lower no-load, the mechanical friction loss and the impact loss is complex and difficult to calculate. Meanwhile, it is unnecessary to accurately calculate the characteristics of the loss below the no-load operation. Therefore, the computation of the rotation speed increase in transient beginning will stop when the closing guide vane reaches the no-load opening.

It is difficult to measure in-situ the rotation speed increase of HTGS during the load rejection. Therefore, in this study the maximum rotation speed increase is calculated using the traditional empirical formula and is used as the reference to verify the result obtained by this method. The results show that the calculated maximum rotation speed increase agrees well with that predicted using the empirical formula.

Remark 7: As aforementioned, the impact loss power should be corrected when the water head deviates from the rated head. It is interesting to investigate how the correction affects the simulation accuracy. To this end, the impact loss power are calculated with and without the correction. Results show that the relative error of the maximum rotation speed increase calculated with and without correct is about 2%. On the other hand, it is seen from Eq. (8) and Eq. (10) that the algebraic relationship for the impact loss power items is subtraction. This means that the effect of the calculation error of the impact loss power on the resistance power is reduced. Therefore, whether to modify the impact loss power depends on the purpose of calculation.

Remark 8: Under the condition that HTGS is connected with the large power network and/or the isolated power grid, the frequency fluctuation of the power network/grid is required to be within the prescribed range. The mechanical friction power is approximately invariant due to the small fluctuation of the unit speed. , Therefore, the mechanical friction loss power ΔP_m could be considered as constant. The transient power of the hydro turbine under both the large and small fluctuations can be directly calculated using Eq.(20). In load rejection, the transient power of the hydro turbine is calculated by combining Eq.(20) with the complete characteristics of the mechanical friction loss $\Delta P_m=f(H, n)$. Therefore, from the point of the view of the transient calculation, it is not necessary to reconstruct the complete characteristics $P_t=f(H, Q)$. However, the reconstruction of the complete characteristics of the mechanical friction loss power $\Delta P_m=f(H, n)$ is necessary while the power model of the hydro turbine Eq.(20) is applied to load rejection case.

6. Conclusions

Based on the calculation of the hydro turbine power, the reconstruction of the complete characteristics of the hydro turbine is proposed in this paper. Although the proposed approach needs to calculate various power losses and its computational process is slightly complex; it solves two key issues: (i) the steady state characteristics are used to calculate the power in the transient state. The proposed method is suitable for calculating the transient turbine power under arbitrary water head with high calculation accuracy; (ii) the fitting error of the unit flow and unit torque produced during the generation of the complete characteristics of the hydro turbine could be controlled to some extent. Meanwhile, the complete characteristics of the mechanical friction loss reveals the law between the mechanical friction loss and the rotation speed in load rejection transient. The proposed method produces good results.

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