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On the Application of Raised-Cosine Wavelets for Multicarrier Systems Design

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Abstract – *New orthogonal wavelet transforms can be designed by changing the wavelet basis functions or by constructing new low-pass filters (LPF). One family of wavelet may appeal, in use, to a particular application than another. In this study, the wavelet transform based on raised-cosine spectrum is used as an independent orthogonal wavelet to study multicarrier modulation behaviour over multipath channel environment. Then, the raised-cosine wavelet is compared with other well-known orthogonal wavelets that are used, also, to build multicarrier modulation systems. Traditional orthogonal wavelets do not have side-lobes, while the raised-cosine wavelets have lots of side-lobes; these characteristics influence the wavelet behaviour. It will be shown that the raised-cosine wavelet transform, as an orthogonal wavelet, does not support the design of multicarrier application well like the existing well-known orthogonal wavelets.*

***Keywords**: Raised-Cosine Filter, Orthogonal Wavelets, Wavelet Transforms, Multicarrier System, OFDM, Raised-Cosine Wavelet*

I. Introduction

Multicarrier systems require multiplexing many data bits over limited bandwidth. The selected wide-bandwidth is however divided into many narrow-bands such that the symbol time is as long as the reciprocal of the length of each of these narrow-bands, to increase symbol time. This is highly useful in the design of multicarrier systems such as the orthogonal frequency division multiplexing (OFDM) to combat intersymbol interference (ISI) due to multipath delay using the long symbol duration. OFDM is useful in almost all modern wireless communication systems such as the WiFi, WiMAX, LTE, UWB and also, in the LTE-Advanced and beyond. Traditionally, most well-known OFDM systems are designed using the discrete Fourier transform (DFT) with cyclic prefix (CP) to combat multipath distortions. Recently, different competitors have been introduced [1-3]. One of such competitors of the DFT is the wavelet transform (WT) and has the advantage of operating multicarrier systems without the CP. The transform operates with filter banks [1, 4-6]. Using these filter banks with CP would destroy the idea of perfect reconstruction of transmit signal in the receiver and so destroy the orthogonality among subcarrier of the transmitter and receiver after passing through dispersive channels [3]. This property further extends the advantage of multicarrier systems in exploiting the limited bandwidth.

In [7, 8], it was shown that wavelet-based multicarrier systems are better immune to multipath resulting intercarrier interference (ICI) and ISI effects than the FFT-based OFDM. This can be related to the fact that, in bandlimited signal processing, the frequency spectrum of every subcarrier signal in the FFT-OFDM is not a bandlimited sampling function, and so every subcarrier will produce side-lobes in the frequency domain resulting from the truncation of a rectangle or another type of window [7]. Low side-lobes imply reduced out-of-band power emission so that there are more power concentrations in the main-lobe. Meanwhile, the spectrum of an OFDM signal depends strongly on the windowing used [9].

With the breakthrough in the use of WT for multicarrier signalling in wireless communication systems [10] to exploit bandlimitedness for reasonable throughput, several ideas have been pursed for efficient multicarrier system designs. In the case of WT, the DFT bandpass filters are rather replaced using some WT filter bank of complementary lowpass and highpass filters [7, 8]. These filters are used to construct orthogonal wavelets. Sometimes, the wavelet filters may be independent of one another, and so does not provide necessary orthogonal requirements to construct multicarrier systems so that perfect reconstruction is not attained after passing the signal through multipath channel. These are mostly called the biorthogonal wavelets [11]. Thus, the multicarrier systems design using wavelets involves using different filters.

Wavelets are orthonormal functions. They may be designed either by changing the parent basis function or by constructing new filters [12]. Extensive account of some of these filters has been provided in [13-15]. In closed-form, [16] proposed the use of raised-cosine

function to design wavelets. Since then, no account has detailed its application, for instance in the design of multicarrier systems. Although traditional wavelets exist (see [11] and references therein), attempts like in [16-18] have pursued the construction of additional wavelets using different filter approximations.

In this study, the performance of the raised cosine wavelet in comparison to other traditional wavelets is presented. This presentation is strictly limited to using the raised-cosine functions proposed in [16] to construct orthogonal wavelets since orthogonal wavelets have been preferred against biorthogonal wavelets. The highpass and lowpass filters that construct orthogonal wavelets are complementary and satisfy the orthogonality requirement for multicarrier signaling, for instance, over multipath channels. These filters of the filter bank enable perfect signal reconstruction after the signals have traversed the hostile multipath environment [3].

A general linear system model used in this study is described in Section II for multicarrier design over multipath including additive white Gaussian noise (AWGN) channel. This linear model includes the wavelets discussed in Section III as the base signaling filter for the model. The system implementation is discussed in Section IV and then the performance bound is presented in Section V. The results discussed in Section VI with the conclusion following.

II. System Model

In a multipath-concerned design, a multicarrier system can be generally represented for a received signal of a single input single output (SISO) case as:

$$X = HS + Z \tag{1}$$

where $X \in C^{N\times 1}$ is the received signal, $diag\{H\} \in C^{N\times N}$ represents the channel, $S \in C^{N\times 1}$ is the transmitted signal and $Z \in C^{N\times 1}$ is the additive white Gaussian noise (AWGN). The AWGN is characterized with $Z \sim \mathcal{N}(0,\sigma^2 I_{N\times N})$ and satisfies $Z[i] \neq Z[j]$ for all $i, j = 0,1,\cdots,N-1$. Notice that $C^{N\times N}$ is a matrix that has complex elements of dimension $N\times N$. σ is the standard deviation and σ^2 is the variance of Z. Also, notice that $\sigma^2 I_{N\times N}$ is an identity matrix of dimension $N\times N$ with σ^2 as the diagonal elements.

From Equation 1, the time domain equivalent of the transmitted signal can further be expressed as:

$$s(t) = \sum_{m=0}^{N-1} a_{m,n}\,\varphi_{m,n}(t) \tag{2}$$

where $a_{m,n}(n \in Z, m = 0, 1, \cdots, N-1)$ represents the complex DFT processed symbol conveyed by m^{th} subcarrier during n^{th} symbol time and $\varphi_{m,n}(t)$ is the synthesis basis which is obtained by time-frequency translation of the prototype filter function $\varphi(t)$; this is the case of DFT-OFDM. In such conventional OFDM, the total symbol time $T_{tot} = T_s + T_{cp}$, and nT_{tot} sampling period. The prototype filter in this case can then be expressed as:

$$\varphi_{m,n}(t) = e^{j2\pi m f_{sc} t}\varphi(t - nT_{tot}) \tag{3}$$

where f_{sc} is the frequency spacing among the subcarriers. In this case, $\varphi(t)$ is rectangular function defined as [3]:

$$\varphi(t) = \begin{cases} \dfrac{1}{\sqrt{T_{tot}}}, & -T_{cp} \le t \le T \\ 0, & otherwise \end{cases} \tag{4}$$

Put Equation 4 into 3, and then put the result into Equation 2,

$$s(t) = \frac{1}{\sqrt{T_{tot}}}\sum_{m=0}^{N-1} a_{m,n} e^{j2\pi m f_{sc} t}\varphi(t - nT_{tot}) \tag{5}$$

Equation 5 is the analytical representation of the conventional CP-based OFDM. $\varphi(t)$ can assume other forms filter functions and this analytically characterizes different multicarrier system forms, for instance, the wavelet transform scaling function.

Since filter bank based multicarrier systems operate without a CP [3], we proceed to discussing signal transformation that precludes any CP. Hence, it is worth mentioning that wavelet multicarrier systems are filter bank based MCSs and so, can operate without the CP.

III. Wavelets for Multicarrier Systems

The basic theory of wavelet transform that can construct multicarrier systems of concern is described in this section. It includes the parent scaling function which is processed in multi-resolution analysis (MRA) to realize the transmission sub-channels; these scaling functions form the equivalent prototype filter functions that can be used in OFDM designs such as in Equation 5.

III.1. Basic Wavelet Theory

Let $s(t)$ be the input signal that modulates the transforming function, $\varphi(t)$; this is the scaling function. Therefore, the basis function as in Equation 5 is the scaling function which is the prototype filter. If $\varphi(t)$ is spread over the wide-spectrum of interest, then there are narrowband functions $\psi(t)$ derivable from $\varphi(t)$; these are orthogonal wavelets used in our multicarrier system

design. Thus, if $\psi_{l,m}(t)$ belongs to a set of orthonormal functions, then [18];

$$\int \psi_{l,m}(t)\psi_{k,n}(t)dt = \delta(l-k)\delta(m-n) \tag{6}$$

where $\delta(.)$ is a Dirac delta. Equation 6 represents the orthogonality of two wavelets for a signal periodically band-limited with λ, $-2\pi \le \lambda \le 2\pi$. Since $\psi(t) \subseteq \varphi(t)$, where $\subseteq$ represents a proper subset of, the input signal with $\varphi(t)$ can be represented in discrete form as [19];

$$S_{DWT} = \sum_{m=0}^{N-1} a_{m,n}(t)\varphi_{m,n}(t) \tag{7}$$

where M is the length of the characteristic filter and $s[m]$ is the discrete equivalent of $s(t)$. The $\varphi(t)$ is weighted for each time-shift (m); the weight and $\varphi(t)$ are directly related as:

$$\varphi(t) = \sqrt{2}\sum_{m} b(m)\varphi(2t-m) \tag{8}$$

where $b(m)$ is the weight or LPF and $\varphi(t)$ is the scaling function. The LPFs construct the approximate coefficient part of the signal while the high-pass filters (HPFs) construct the detail coefficient part of the signal. The HPF can be formed from the LPF as [8]:

$$b(m) = (-1)^m g(M-1-m) \tag{9}$$

where $g(m)$ is a HPF and M is the length of the filter. Both the high-pass and low-pass filters constitute the filter bank [1].

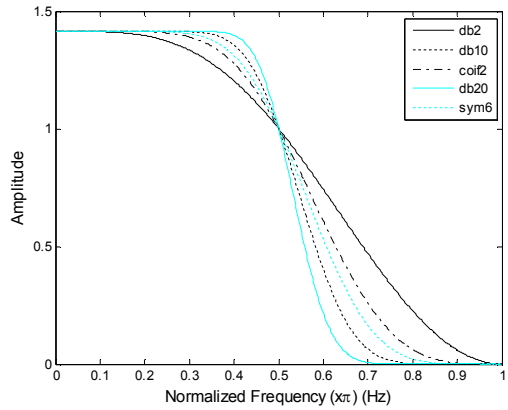

Figure 1: Examples of Orthogonal wavelet Filters

Examples of these orthogonal wavelet filters are shown in Figure 1. db2 is the Daubechies orthogonal wavelet with four filters. coif2 and sym6 are the coiflet and symlet wavelets with twelve and twelve filter coefficients respectively. In filter design for wavelets, the longer the filter length, the closer is the filter to the ideal filter. However, filters with shorter length will require shorter system runtime than the filters with longer lengths.

III.2. Raised-cosine function Wavelets

Both [16] and [20], constructed the scaling function to describe wavelets transform from root-raised cosine function. Later, [21] classified the resulting wavelet as a member of the Meyer functions provided β=1/3. The scaling function was defined as:

$$\varphi_{rc}(t) = \frac{\sin\pi(1-\beta)t + 4\beta t\cos\pi(1+\beta)t}{\pi t(1-(4\beta t)^2)} \tag{10}$$

where β is the roll-off factor of the raised cosine function. Using Equation 10 in Equation 2,

$$s(t) = \sum_{m=0}^{N-1} a_{m,n}(t)\varphi_{rc}(t) \tag{11}$$

For a sampling period of Δt, then $t = t_0 + m\Delta t$, $m = 0,1,\cdots,N-1$, where t_0 is the initial time. Thus,

$$s(m) = s(t_0 + m\Delta t), \qquad m = 0,1,\cdots,N-1 \tag{12}$$

If there are $n = 0,1,\cdots,N-1$ daughter wavelets, and there are also $m = 0,1,\cdots,N-1$ bits to be multiplexed, then each mth-discrete information will modulate n^{th}-daughter wavelet.

$$s(m) = \sum_{m=0}^{N-1} a_{m,n} \frac{\sin\pi(1-\beta)m + 4\beta m\cos\pi(1+\beta)m}{\pi m(1-(4\beta m)^2)} \tag{13}$$

The roll-off factor characterizes the filter properties used in the design of these wavelets. For instance, the side-lobe properties of the root-raised filter are controlled by the prevailing roll-off factor (see Figure 3).

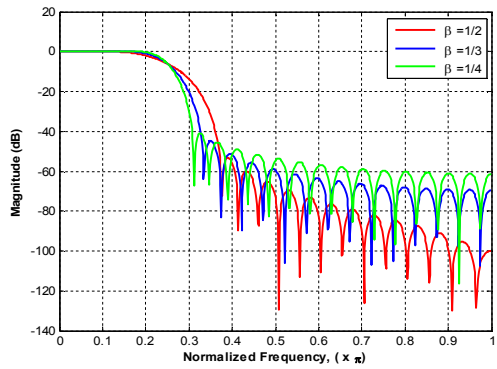

Figure 2: Side-lobes of root-raised cosine filter function

From Figure 2, the side-lobes of the roll-factor corresponding to ½ are most suppressed compared to that of 1/3 and ¼ but less spectrally efficient among all. In

that case, the 1/3 roll-off factor is more efficient than ½ but less efficient compared to ¼.

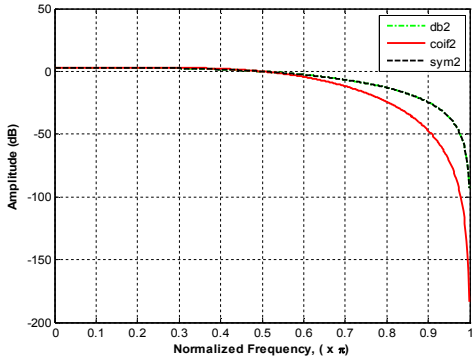

Figure 2b: Conventional wavelets showing no side-lobes

On the other hand, the side-lobes of the roll-factor corresponding to ½ are most well suppressed followed by those of 1/3. In all cases, 1/3 is most likely in trade-off in terms of efficiency and well-suppressed side-lobes compared ½ and ¼. In [16], the roll-off factor corresponding to 1/3 was used and will be adopted in this study. Based on Equation 10, the 1/3 roll-off factor is used to construct the mother wavelet that designs the multicarrier system applied in this study.

Thus, the roll-off factor influences the behaviour of any raised cosine-function filter and would influence the performance of the dependent wavelet, such as, the raised-cosine wavelet. For instance, it influences the spectrum of the raised-cosine filter function, and also the convergence of the filter tail (to zero); these are well explained in [22, 23] and [24]. In digital communications, the noise in a system is a function of the spectrum/bandwidth of transmission. Reducing the bandwidth will reduce the noise power in the system. However, reducing the bandwidth too much distorts the pulse-shape and introduces longer "ringing" of the sidelobes, and in turn introduces intersymbol interference. Sequel to the many sidelobes, there will be much out-of-band emissions that would influence energy in the adjacent band thus resulting in increased peak-to-average power ratio (PAPR) [23]. These limitations are absent in the case of the conventional wavelets as there are no sidelobes as shown in Figure 2b.

IV. System Implementation

In [18], the wavelet multicarrier system has been implemented over multicarrier environment. Figure 3 shows a schematic design of the multicarrier modulation transceiver using WT over a multipath channel. At first, the input signal are randomly generated and mapped using the BPSK mapping scheme.

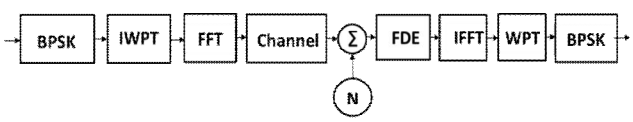

Figure 3: Multicarrier modulation using wavelet transform over multipath channel

The inverse-WPT (IWPT) is applied and the resulting signal transformed into frequency domain. The channel transfer function of the Rayleigh multipath channel model is then convolved with the resulting frequency-domain contents of the signal. Some AWGN noise is added and the signal received in the receiver. A frequency domain equalization is performed before transforming the signals back into the wavelet-domain for onward WPT demodulation. Using the BPSK mapping scheme, the received signals are demapped and error calculation performed. This process is repeated for QPSK.

We follow the FDE equalization method described in [18]. If the wavelet-transformed signal is $s(t)$, then an FFT will be required to obtain the frequency domain equivalent; $S(f) \overset{FT}{\Leftrightarrow} s(t)$, with FT as the Fourier transform. The wavelet does not operate with a cyclic prefix (CP), so the convolution with the channel transfer function becomes;

$$X(f) = H(f) \otimes S(f) + Z(f) \tag{14}$$

Equation 14 is the received signal in the frequency domain with $Z(f)$ as the AWGN. In the receiver, the equalization follows as:

$$\hat{S} = \frac{H^H \times X}{\|H\|^2} = \frac{H^H \times H \otimes S}{\|H\|^2} + \frac{H^H \times Z}{\|H\|^2} \tag{15}$$

where $(\cdot)^H$ is a Hermitian operator, $\otimes$ and (.)* are convolution and conjugation operators respectively, and $\|\cdot\|$ is the absolute operator. If the channel response could negligible, then $|H| \to 0$, then Equation 15 can be modified to include some error correction parameter, ε, such that:

$$\hat{S} = \frac{H^H \times X}{\|H\|^2 + \varepsilon} = \frac{H^H \times H \otimes S}{\|H\|^2 + \varepsilon} + \frac{H^H \times Z}{\|H\|^2 + \varepsilon}, \forall 0 \le \varepsilon \le 1 \tag{16}$$

Equation 16 represents the FDE equalization. $\hat{X}$ is the frequency domain content of the received signal after equalization. The received signals were transformed into the wavelet domain by the inverse FFT (IFFT) before demodulating by the forward WPT and demapping using BPSK. In a second case, these stages are repeated for a QPSK mapping scheme.

V. Performance Bound of the Model

The performance bound of wavelet-based multicarrier system has been studied in [18] using the Cramer Rao

Lower Bound (CRLB) of estimation theory. Let the transmit symbol be estimated from Equation (14) as:

$$\begin{aligned}\hat{S} &= \left(H^H H\right)^{-1} H^H X \\ &= \left(H^H H\right)^{-1} H^H H\, S + \left(H^H H\right)^{-1} H^H Z\end{aligned} \tag{17}$$

The CRLB is defined as:

$$CRLB = R_{\hat{S}-S} \geq J^{-1} \tag{18}$$

where J is the Fisher information matrix which describes the autocorrelation ($R_{\hat{S}-S}$) of the estimation error $(\hat{S} - S)$. This is defined as:

$$J = E\left[\left(\frac{\partial \log_e f(X \mid S)}{\partial S}\right)^H \left(\frac{\partial \log_e f(X \mid S)}{\partial S}\right)\right]$$

where $E[.]$ is the expectation value, $(.)^H$ is a Hermitian operator and $f(X|S)$ is the pdf of X. The Fisher information matrix requires that [18]:

$$J^{-1} = \left(H^H C^{-1} H\right)^{-1} \tag{19}$$

where C is the AWGN covariance. Since Equation 19 is independent of the transmit symbols, the detection method attains minimum variance unbiased estimation (MVUE) criteria needed in CRLB theory. The mean squared error (MSE) is discussed in terms of the CRLB and can be expressed in terms of the energy in the received symbol and the noise power as shown in Figure 4.

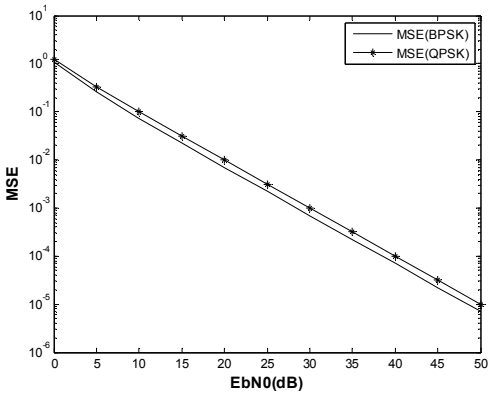

Fig 4: CRLB for wavelet-based OFDM over BPSK and QPSK mapping schemes

Figure 4 depicts the performance bound expected of the wavelet multicarrier scheme and will be used to assess the results of our design. In [25], we compared the performance of the new wavelet proposed in [18] and the raised cosine wavelet for multicarrier applications. Although, it was shown in [18] that new wavelet only outperformed the traditionally well-known wavelets slightly, the new wavelet is better than the raised-cosine wavelet. The performance of these well-known wavelets shwon in [25] corroborate the performance reported in [18] although the well-known wavelets outperformed the raised-cosine wavelet as shown in Figure 4 [25];

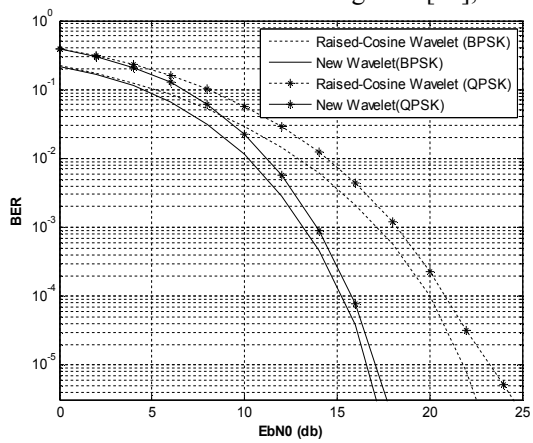

Figure 4: Comparison of ideal-filter wavelet with raised Cosine Wavelet (β=1/3) over multipath channel with AWGN for φ_{rc} [25].

Since the performance of the new wavelet (as in Figure 4) above the well-known wavelets was only slightly, then we follow [25] to compared the raised-cosine wavelet with the traditional wavelets.

VI. Simulation Results and Discussion

At first, the system is modelled for a BPSK mapping scheme with 128 symbol length averaged over 35000 symbols. Subsequently, the design is extended to a QSPK mapping scheme with similar design parameters.

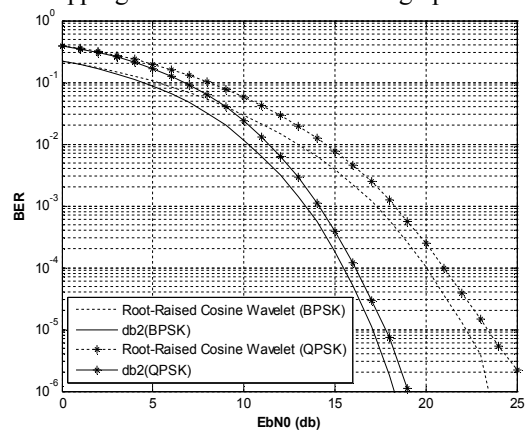

Figure 5a: Comparison of db2 with raised-cosine wavelet over multipath channel with AWGN

In Figures 5a – 5c, the results of our simulations comparing the raised-cosine wavelet with other well-known wavelets over multipath channel with AWGN are shown in terms of BER statistics. Since the raised-raised wavelets are compactly supported, some lower order filter lengths have been preferred to reduce the simulation run-time. Obviously, the concept would

equally reduce the effective real-life system resource consumption, such as the battery life.

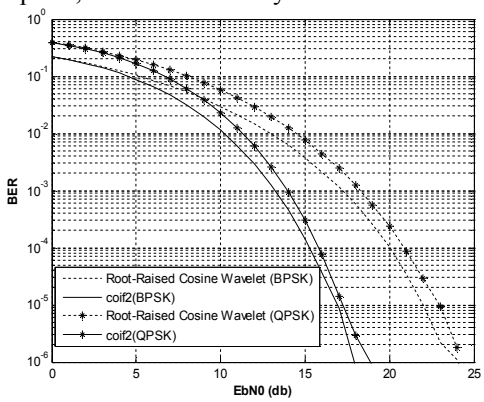

Figure 5b: Comparison of coif2 with raised -cosine wavelet over multipath channel with AWGN.

Over a multipath channel with AWGN, the results of using the orthogonal wavelet built from raised-cosine function to design multicarrier system are compared with other orthogonal wavelet (e.g. db2, sym2, and coif2) in Figure 5a – 5c. It is found that the raised-cosine wavelet perform less than the rest orthogonal wavelets by about 1db. The roll-off factor determines the behaviour of a raised cosine filter in terms of the bandwidth, sidelobe and also influences the length of "ringing" of the sidelobes of the filter. Except for selected bandwidth and windowing method, using the roll-off factor is a significant way to bandlimit subcarriers.

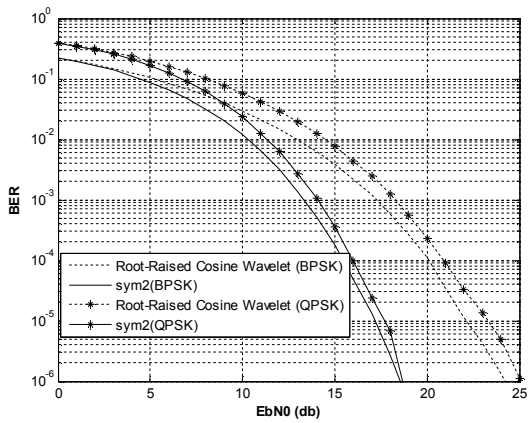

Figure 5c: Comparison of sym2 with raised -cosine wavelet over multipath channel with AWGN

This is because in band-limited signal processing, the frequency spectrum of every subcarrier signal in OFDM is not a band-limited sampling function, and so every subcarrier will produce side-lobes in the frequency domain resulting from the truncation of a rectangular or another type of window [7].

It is well-know that reducing the bandwidth will reduce the noise power in the system. However, reducing the bandwidth too much distorts the pulse-shape and introduces longer "ringing" of the sidelobes, and in turn introduces ISI. This would likely to affect the PAPR due to out-of-band emissions from the *ringings* that may increase the energy in the adjacent subcarrier mainlobes. Although the 1/3 roll-off factor provides a trade-off in the raised-cosine filter design, the BER statistic reflects that the raised-cosine wavelet is less in performance than the rest orthogonal wavelets. It follows therefore that the raised-cosine function cannot be more suitable than the rest orthogonal wavelets, and thus limits its suitability in multicarrier transmissions. The raised-cosine wavelets may find excellent performance in other design niches for application, but has not shown recommendable competence from the foregoing discussions.

Though the wavelets from raised cosine spectrum may well appeal to other applications in science or engineering, this study shows that the wavelet transform based on raised cosine spectrum is not suitable for multicarrier modulations when compared to other well-known (already used) wavelet families, such as the orthogonal wavelets used in this study as shown in Figs. 5a – 5c.

VII. Conclusion

Multicarrier system design using the wavelet transform has been presented and evaluated. The designs involved the use of wavelet transform constructed from the raised-cosine function in comparison with other well-known wavelet transforms. Both families of wavelets were discussed as orthogonal wavelets. Results showed that wavelet transform constructed from the raised-cosine function is less suitable for multicarrier wireless communications design by circa 1db compared to the earlier well-known orthogonal wavelet transforms. It is possible that the raised-cosine function finds suitable application(s) in other fields, meanwhile, it is demonstrated in this study that the raised-cosine wavelets are less suitable for multicarrier design application in comparison to other well-known orthogonal wavelets.

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