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The Generalized Hamiltonian model for the shafting transient analysis of the hydro turbine generating sets

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Abstract: Traditional rotor dynamics mainly focuses on the steady state behavior of the rotor and shafting. However, for systems such as hydro turbine generating sets (HTGS) where the control and regulation is frequently applied, the shafting safety and stabilization in transient state is then a key factor. The shafting transient state inevitably involves multi-parameter domain, multi-field coupling and coupling dynamics. In this paper, the relative value form of the Lagrange function and its equations have been established by defining the base value system of the shafting. Taking the rotation angle and the angular speed of the shafting as a link, the shafting lateral vibration and generator equations are integrated into the framework of the generalized Hamiltonian system. The generalized Hamiltonian control model is thus established. To make the model be more general, additional forces of the shafting are taken as the input excitation in proposed model. The control system of the HTGS can be easily connected with the shafting model to form the whole simulation system of the HTGS. It is expected that this study will build a foundation for the coupling dynamics theory using the generalized Hamiltonian theory to investigate coupling dynamic mechanism between the shafting vibration, transient of hydro turbine generating sets and additional forces of the shafting.

Keywords: *hydro turbine generating sets; shafting; transient; the Lagrange relative value system; the generalized Hamiltonian model*

Nomenclature

c_1 damping coefficients of the generator rotor
c_2 damping coefficients of the turbine runner
D the damping coefficient
e_1 mass eccentricity of the generator rotor
e_2 mass eccentricity of the turbine runner
E_f output of excitation controller
E_q' internal transient voltage
F_{x1}, F_{y1} the x- and y-direction additional forces acting on the generator rotor
F_{x2}, F_{y2} the x-and y-direction additional forces acting on the hydro turbine runner
H the Hamiltonian function
J the rotary inertia of the HTGS
J_1 rotary inertia of the generator rotor
J_2 rotary inertia of the turbine runner
k_1 stiffness of the up guide bearing
k_2 stiffness of the lower guide bearing
k_3 stiffness of the hydro turbine bearing
L the Lagrange function
m_1 mass of the generator rotor
m_2 mass of the hydro turbine runner
M_gB the generator rated torque
M_g the generator magnetic torque.
M_t the hydro turbine torque
p_i the generalized momentums
Q_{x1}, Q_{y1} the external forces acting on the generator rotor
Q_{x2}, Q_{y2} the external forces acting on the hydro turbine runner
R_1 radius of the generator rotor
R_2 radius of the hydro turbine runner
r_1 radial displacement of the generator rotor
r_2 radial displacement of the turbine runner
r_3 radial displacement of the up guide bearing
r_4 radial displacement of lower guide bearing
r_5 radial displacement of turbine bearing
S_gB the generator rated power
T total kinetic energy of the HTGS
T_j inertia time constant of the generator
T_{j1} inertia time constant of the generator rotor
T_{j2} inertia time constant of the turbine runner
T_{d0}' the time constant
U elastic potential energy of the HTGS
U_s the infinite bus voltage
x_1, y_1 central coordinates of the generator rotor
x_{10}, y_{10} mass coordinates of the generator rotor
x_2, y_2 central coordinates of the turbine runner
x_{20}, y_{20} mass coordinates of the turbine runner
$X_{\mathrm{a}d}$ the d-axis armature reaction reactance
X_d the d-axis synchronous reactance
X_d' the d-axis transient reactance
X_f the excitation winding reactance
X_L the transmission line reactance
X_q the q-axis synchronous reactance
X_T reactance of transformer
δ rotor angle
φ rotation angle of the generator rotor
ω angular speed of the HTGS
ω_B basic value of electrical angular speed
ω_e electric angular speed
ω_mB basic value of mechanical angular speed

1 Introduction

The rotor dynamics mainly investigates the steady state behavior of the rotor and the shafting. However, for the system which frequently performs control and regulation, the shafting safety and stabilization in transient state is a key factor. Typical example is the hydro turbine generating sets (HTGS). The study for the shafting transient state inevitably involves multi-parameter domain, multi-field coupling and its coupling dynamics, which need to be integrated into a uniform framework.

With the development of the computational mechanics, methods based on finite element calculation (e.g. the shafting computational model [1,2]), the simulation computation [3,4], the faulty diagnosis [5,6], and the active control [7,8] have been developed to investigate the rotor dynamics. Though some achievements have been made using these approaches, it is still difficult to directly analyze the shafting transient state generated arising coming from control and regulation of the HTGS [9]. In the theories of the shafting vibration for the HTGS, the generator rotor, bearing and turbine runner are usually simplified as the equivalent elements to form the basic shafting model [10,11]. The central coordinates of the generator rotor and turbine runner are employed to build two group differential equations for motion, including the support action of the bearing [12,13]. Based on this, other factors arising from different purposes are transformed as additional forces and are added into the corresponding equations. For example, the magnetic pull is added into the motion equations of the generator rotor to consider the unbalance magnetic pull [14,15]; the sealing force is added into the motion equations of the hydro turbine to consider the sealing of hydro turbine [16]; the fluid inertia and angular momentum are added as additional force of the turbine runner [17]. The modeling of the multiple coupling vibration is similar to this approach [18,19]. As such, the shafting model is governed by more complex second order differential equations. If the magnetic transient of the generator is considered, the shafting model will be more complex [20]. In principal, these approaches transform the shafting system into autonomous system with no-input excitation. However, differential equations model cannot treat the effects and action mechanism between the basic shafting model and additional forces.

The development of nonlinear science, particularly the bifurcation and chaos theory, brings new approaches and ideas for studying the nonlinear dynamic characteristics of the rotor. The differential equations model of the system is established according to the structure characteristics of the rotor and shafting. Various factors, such as the interaction between the torsion and the lateral vibrations [21], the lateral-torsional coupling [22], nonlinear dynamics of rotor–bearing–seal system [23], the unbalanced rotor with nonlinear elastic restoring forces [24] and turbulent coupling stress fluid film journal bearings [25], are considered in the differential equation model. The nonlinear analysis method is then applied to analyze its dynamics behavior and the model is verified using the experiments. Although the dynamics characteristics of the system can be obtained from bifurcation and chaos method, its transient characteristics are not directly represented. Meanwhile, the differential equation model can not explicitly provide the inner coupling dynamics mechanism existed in multiple parameters domain and multi-fields.

The generalized Hamiltonian control system, an important branch of nonlinear science, has been developed in recent years. Its structure matrix provides the connection information for the system parameters; while its damping matrix provides the damping characteristics on port of system parameters. The effect of the external input is represented in its input matrix [26-28]. It has opened a new route for investigating the rotor dynamics. The new approach is to integrate the shafting of the HTGS and its relative subsystem into the framework of the generalized Hamiltonian to reveal the coupling dynamics mechanism between the shafting and its relative subsystems. This work includes three parts. In the first part, the transient control and regulation of the HTGS is introduced into the shafting model so that it can be applied to investigate the transient responses of the shafting vibration. In the second part, we define the Lagrange function and its equation in relative value form, which is further transformed into the commonly used form for the convenient application to the multiple domains and multiple coupling forces. In the third part, we establish the generalized Hamiltonian control model. The proposed model will provide a foundation for the coupling dynamics theory which applies the generalized Hamiltonian theory to investigate coupling dynamics mechanism between the shafting vibration, the transient state of the hydro turbine generating sets and additional forces of the shafting.

2 Shafting basic model

Fig.1 is the schematic diagram of the shafting structure of HTGS.

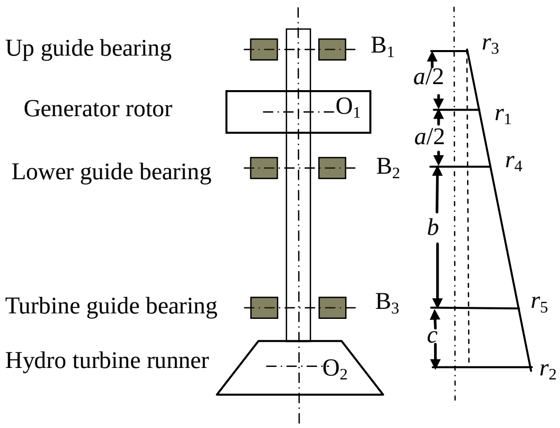


Fig.1 The shafting structure of the hydro turbine generating sets

In Fig.1, B_1, O_1, B_2, B_3 and O_2 are the geometric centers of the up guide bearing, the generator rotor, the lower guide bearing, the turbine bearing and the turbine runner, respectively.

The mass central coordinates of the generator rotor is (x_{10}, y_{10}), then one has $x_{10}=x_1+e_1\cos\varphi$, $y_{10}=y_1+e_1\sin\varphi$, $\varphi=\omega t$. As such, the mass central coordinate of the turbine runner is (x_{20}, y_{20}), then $x_{20}=x_2+e_2\cos\varphi$, $y_{20}=y_2+e_2\sin\varphi$.

Assumption 1: the rotary components are rigid element, effects of the thrust bearing and spindle mass are ignored, and the twist of axis is also ignored.

According to Assumption 1, the total kinetic energy of the HTGS, including the kinetic energy of the generator rotor and the hydro turbine runner, is:

$$T=\frac{1}{2}m_1(\dot{x}_{10}^2+\dot{y}_{10}^2)+\frac{1}{2}(J_1+m_1e_1^2)\omega^2 \\ +\frac{1}{2}m_2(\dot{x}_{20}^2+\dot{y}_{20}^2)+\frac{1}{2}(J_2+m_2e_2^2)\omega^2$$

$$
\begin{aligned}
&= \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + e_1^2 \omega^2 + 2e_1 \omega \dot{y}_1 \cos\varphi) \\
&+ \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2 + e_2^2 \omega^2 + 2e_2 \omega \dot{y}_2 \cos\varphi) \\
&- m_1 (e_1 \omega \dot{x}_1 \sin\varphi) - m_2 (e_2 \omega \dot{x}_2 \sin\varphi) \\
&+ \frac{1}{2} (J_1 + m_1 e_1^2) \omega^2 + \frac{1}{2} (J_2 + m_2 e_2^2) \omega^2
\end{aligned} \tag{1}
$$

where m_1 and m_2 are the mass of the generator rotor and the hydro turbine runner in kg, respectively; $J_1=m_1R_1^2/2$ and $J_2=m_2R_2^2/2$ are the rotary inertia of the generator rotor and the turbine runner in kg.m^2, respectively; R_1 and R_2 are the radius of the generator rotor and the turbine runner in m respectively.

In Fig.1, denote $|B_1O_1|=|O_1B_2|=a/2$, $|B_2B_3|=b$, $|B_3O_2|=c$, $r_1^2=(x_1^2+y_1^2)$, $r_2^2=(x_2^2+y_2^2)$. From geometrical relationship in Fig.1, we have $r_3 = \dfrac{2(a+b+c)r_1 - ar_2}{a+2b+2c}$; $r_4 = \dfrac{2(b+c)r_1 + ar_2}{a+2b+2c}$; $r_5 = \dfrac{2cr_1 + (a+2b)r_2}{a+2b+2c}$.

Assumption 2: The structure parameters of the shafting a, b and c, as well as the stiffness coefficient of the bearing k_1, k_2 and k_3 are constant. The change of gravitational potential energy is ignored in the HTGS operation. As such the potential energy of the shafting only includes elastic potential energy generated by the bearing.

According to Assumption 2, the elastic potential energy of the HTGS shafting can be expressed as:

$$
\begin{aligned}
U &= (x_1^2 + y_1^2) K_{11} + (x_2^2 + y_2^2) K_{22} \\
&\quad + \sqrt{x_1^2 + y_1^2} \sqrt{x_2^2 + y_2^2} K_{12}
\end{aligned} \tag{2}
$$

where

$$
K_{11} = \frac{[2k_1(a+b+c)^2 + 2k_2(b+c)^2 + 2k_3c^2]}{(a+2b+2c)^2},
$$

$$
K_{22} = \frac{[k_1a^2 + k_2a^2 + k_3(a+2b)^2]}{2(a+2b+2c)^2},
$$

$$
K_{12} = \frac{2[ak_2(b+c) + ck_3(a+2b) - ak_1(a+b+c)]}{(a+2b+2c)^2}
$$

Assumption 3: The various damping of the shafting can be converted to the damping of the generator rotor and the turbine runner, and can be simplified as linear damping. Other forces acting on the shaft can be converted to the force of the generator rotor and the turbine runner respectively.

Denote external forces acting on generator rotor be $Q_{x_1} = -c_1\dot{x}_1 + F_{x1}$, $Q_{y_1} = -c_1\dot{y}_1 + F_{y1}$, and external forces acting on turbine runner be $Q_{x_2} = -c_2\dot{x}_2 + F_{x2}$, $Q_{y_2} = -c_2\dot{y}_2 + F_{y2}$.

The additional forces acting on the generator include the unbalance magnetic pull. The additional forces acting on the hydro turbine runner include the sealing force and the unbalance force of the runner blade. These external forces keep their form and are taken as additional input excitation in the following derivation. As such, the proposed model is general and can be applied to analyze the multiple external forces.

The Lagrange function of the system is defined as the difference of the kinetic energy and the potential energy of the system:

$$
\begin{aligned}
L &= \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + e_1^2 \omega^2 + 2e_1 \omega \dot{y}_1 \cos\varphi) \\
&+ \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2 + e_2^2 \omega^2 + 2e_2 \omega \dot{y}_2 \cos\varphi) \\
&- m_1 (e_1 \omega \dot{x}_1 \sin\varphi) - m_2 (e_2 \omega \dot{x}_2 \sin\varphi) \\
&+ \frac{1}{2} (J_1 + m_1 e_1^2) \omega^2 + \frac{1}{2} (J_2 + m_2 e_2^2) \omega^2 \\
&- (x_1^2 + y_1^2) K_{11} - (x_2^2 + y_2^2) K_{22} \\
&- \sqrt{x_1^2 + y_1^2} \sqrt{x_2^2 + y_2^2} K_{12}
\end{aligned} \tag{3}
$$

The generalized coordinate is selected as $\boldsymbol{v}=\{x_1,y_1,x_2,y_2\}$. Denote external forces be $\boldsymbol{F}=\{Q_{x1}, Q_{y1}, Q_{x2}, Q_{y2}\}$. The Lagrange equation of the shafting is:

$$
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{v}_i}\right) - \frac{\partial L}{\partial v_i} = F_i \qquad i = 1,\dots,4 \tag{4}
$$

Expending Eq.(4) yields the differential equation model:

$$
\begin{aligned}
&m_1\ddot{x}_1 + c_1\dot{x}_1 - m_1e_1\dot{\omega}\sin\varphi + 2x_1K_{11} \\
&+ x_1\sqrt{\frac{x_2^2 + y_2^2}{x_1^2 + y_1^2}} K_{12} = m_1e_1\omega^2\cos\varphi + F_{x1}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&m_1\ddot{y}_1 + c_1\dot{y}_1 + m_1e_1\dot{\omega}\cos\varphi + 2y_1K_{11} \\
&+ y_1\sqrt{\frac{x_2^2 + y_2^2}{x_1^2 + y_1^2}} K_{12} = m_1e_1\omega^2\sin\varphi + F_{y1}
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&m_2\ddot{x}_2 + c_2\dot{x}_2 - m_2e_2\dot{\omega}\sin\varphi + 2x_2K_{22} \\
&+ x_2\sqrt{\frac{x_1^2 + y_1^2}{x_2^2 + y_2^2}} K_{12} = m_2e_2\omega^2\cos\varphi + F_{x2}
\end{aligned} \tag{7}
$$

$$m_2\ddot{y}_2 + c_2\dot{y}_2 + m_2 e_2 \dot{\omega}\cos\varphi + 2y_2 K_{22} + y_2\sqrt{\frac{x_1^2+y_1^2}{x_2^2+y_2^2}}K_{12} = m_2 e_2 \omega^2 \sin\varphi + F_{y2} \tag{8}$$

These equations are the basic forms of the HTGS shafting motion equations.

In contrast to the current steady model, the angular acceleration $\dot{\omega}$ has been added into above equation. Since the angular speed ω is usually constant during the steady state operation, the angular acceleration $\dot{\omega}$ can then be approximated to zero when the state of the HTGS shafting is steady. Based on this approximation, the motion equations of the HTGS shafting are the differential equations with four variables x_1, y_1, x_2 and y_2, and the motion equation of the rotary angle is not included. In the transient state, however, the angular speed change is larger, thus the items containing $\dot{\omega}$ should be kept.

3 The relative value form of the Lagrange system

3.1 Defining relative value system

When the multiple subsystems are connected with the different argument regions, the parameter values can be several orders of magnitude or have different dimensional units. This may produce large calculation error or even unable to connect. In this case, the normalization method is a useful approach. The normalization method must keep the equivalence of their base value system. In this paper, the common motion between the shafting and the generator subsystems is the angular speed. Because the base value system of the generator subsystem has the whole definition, according to the equivalence principle, the generator rated power S_{gB} is then chosen as the base value of the shafting. The base value S_{gB} should be decomposed into the basic parameters of the generator rotor and the turbine runner to build the base value system for the shafting subsystem. As such, we have the following definitions.

Definition 1: The mass, the displacement, the mechanical angular speed, the speed and the power base values of the generator rotor are chosen as $m_{1B}=m_1$, $R_{1B}=R_1$, ω_{mB}, $R_1\omega_{mB}$ and S_{gB}, respectively. The inertia time constant of the generator rotor is then defined as:

$$T_{j1} = \frac{J_1\omega_{mB}^2}{S_{gB}} \tag{9}$$

Definition 2: The mass, the displacement and the speed base values of the turbine runner are selected as $m_{2B}=m_2$, $R_{2B}=R_2$, and $R_2\omega_{mB}$, respectively. The inertia time constant of the turbine runner is defined as:

$$T_{j2} = \frac{J_2\omega_{mB}^2}{S_{gB}} \tag{10}$$

Using above base value system, the relative value form of the Lagrange function can be derived from (3) by dividing S_{gB}.

$$\begin{aligned}\overline{L} &= T_{j1}(\dot{\overline{x}}_1^2 + \dot{\overline{y}}_1^2 + 2\overline{e}_1\overline{\omega}\dot{\overline{y}}_1\cos\varphi - 2\overline{e}_1\overline{\omega}\dot{\overline{x}}_1\sin\varphi) \\ &+ T_{j2}(\dot{\overline{x}}_2^2 + \dot{\overline{y}}_2^2 + 2\overline{e}_2\overline{\omega}\dot{\overline{y}}_2\cos\varphi - 2\overline{e}_2\overline{\omega}\dot{\overline{x}}_2\sin\varphi) \\ &+ \frac{1}{2}T_{j1}(1+4\overline{e}_1^2)\overline{\omega}^2 + \frac{1}{2}T_{j2}(1+4\overline{e}_2^2)\overline{\omega}^2 \\ &- 2T_{j1}(\overline{x}_1^2+\overline{y}_1^2)\overline{K}_{11} - 2T_{j2}(\overline{x}_2^2+\overline{y}_2^2)\overline{K}_{22} \\ &- 4T_{j1}T_{j2}\sqrt{\overline{x}_1^2+\overline{y}_1^2}\sqrt{\overline{x}_2^2+\overline{y}_2^2}\,\overline{K}_{12}\end{aligned} \tag{11}$$

where over bars denote the relative values of parameters, $\overline{e}_1 = e_1/R_1$ and $\overline{e}_2 = e_2/R_2$ are the mass eccentricity of the generator rotor and the hydro turbine runner respectively, $\overline{x}_1 = x_1/R_1$, $\overline{y}_1 = y_1/R_1$, $\overline{x}_2 = x_2/R_2$, $\overline{y}_2 = y_2/R_2$, $\overline{K}_{11} = K_{11}/(m_1\omega_{mB}^2)$, $\overline{K}_{22} = K_{22}/(m_2\omega_{mB}^2)$, $\overline{K}_{12} = S_{gB}K_{12}/(m_1 m_2 R_1 R_2 \omega_{mB}^4)$.

Multiplying the Eq. (4) by R_1/M_{gB} for i =1,2, and by R_2/M_{gB} for i=3,4 converts Eq. (4) into the relative value form of the Lagrange equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\overline{L}}{\partial\dot{\overline{v}}_i}\right) - \omega_{mB}\frac{\partial\overline{L}}{\partial\overline{v}_i} = \overline{F}_i \qquad i = 1,\ldots,4 \tag{12}$$

Accordingly, the external forces in relative values are converted to:

$$\overline{F} = \begin{cases} -\overline{c}_1\dot{\overline{x}}_1 + \overline{F}_{x1} \\ -\overline{c}_1\dot{\overline{y}}_1 + \overline{F}_{y1} \\ -\overline{c}_2\dot{\overline{x}}_2 + \overline{F}_{x2} \\ -\overline{c}_2\dot{\overline{y}}_2 + \overline{F}_{y_2} \end{cases} \tag{13}$$

where $\overline{c}_1 = c_1R_1^2\omega_{mB}/M_{gB}$, $\overline{c}_2 = c_2R_2^2\omega_{mB}/M_{gB}$, $\overline{F}_{x1} = F_{x1}R_1/M_{gB}$, $\overline{F}_{y1} = F_{y1}R_1/M_{gB}$, $\overline{F}_{x2} = F_{x2}R_2/M_{gB}$, $\overline{F}_{y2} = F_{y2}R_2/M_{gB}$, $M_{gB}=S_{gB}/\omega_{mB}$.

The dynamic system composed by the Lagrange function (3) and its equation (4) is equivalent to the dynamic system composed by (11) and (12) in relative values. This result can be verified by expending Eq.(11) and Eq.(12).

3.2 Angle and angular speed equation

The mechanical angle φ and angular speed ω are included in the energy function of the system (6). Thus the motion equation of the shafting should include the angle as a variable to reflect the effect of the energy on the shafting motion characteristics. Meanwhile, the angular speed ω is a key variable that relates to the transient state of the HTGS.

The external torque corresponding to the angle variable φ is M_t−M_g. Then the Lagrange equation of taking the angle as variable satisfies the following:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \omega}\right)-\frac{\partial L}{\partial \varphi}=M_{\mathrm{t}}-M_{\mathrm{g}} \tag{14}$$

Dividing equation (14) by the rated torque of the generator M_{gB} yields:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \overline{L}}{\partial \overline{\omega}}\right)-\frac{\partial \overline{L}}{\partial \overline{\varphi}}=\overline{m}_{\mathrm{t}}-\overline{m}_{\mathrm{g}} \tag{15}$$

where M_{gB}=S_{gB}/ω_{mB} is the base value of the generator rated torque, $M_{\mathrm{t}}=M_{\mathrm{gB}}\overline{m}_{\mathrm{t}}$, $M_{\mathrm{g}}=M_{\mathrm{gB}}\overline{m}_{\mathrm{g}}$, $\overline{\omega}=\omega/\omega_{\mathrm{mB}}$, $\overline{\varphi}=\varphi/\omega_{\mathrm{mB}}$.

As such, the shafting motion differential equations, including the four displacement variables and the angle variable, have been integrated into the framework of the Lagrange system in relative values.

4. Energy function

4.1 Correction of the Lagrange function

The rotary kinetic energy $\overline{\omega}^2$ in the Lagrange function (11) can be directly substituted by $\omega_{\mathrm{B}}\overline{\omega}_1^{\,2}$, in which the ω_B=314rad/s is the basic value of the electrical angular speed. This modification is for connection to generator, the reason will be give in next section 5. Other items remain unchanged, and the Lagrange function (11) is then rewritten as:

$$\begin{aligned}
\overline{L}^{(1)} &= T_{j1}[\dot{\overline{x}}_1^2+\dot{\overline{y}}_1^2+2\overline{e}_1(\overline{\omega}_1+1)\dot{\overline{y}}_1\cos\varphi] \\
&-2T_{j1}\overline{e}_1(\overline{\omega}_1+1)\dot{\overline{x}}_1\sin\varphi \\
&+T_{j2}[\dot{\overline{x}}_2^2+\dot{\overline{y}}_2^2+2\overline{e}_2(\overline{\omega}_1+1)\dot{\overline{y}}_2\cos\varphi] \\
&-2T_{j2}\overline{e}_2(\overline{\omega}_1+1)\dot{\overline{x}}_2\sin\varphi \\
&+\frac{1}{2}T_{j1}\omega_{\mathrm{B}}(1+4\overline{e}_1^2)\overline{\omega}_1^{\,2}+\frac{1}{2}T_{j2}\omega_{\mathrm{B}}(1+4\overline{e}_2^2)\overline{\omega}_1^{\,2} \\
&-2T_{j1}(\overline{x}_1^2+\overline{y}_1^2)\overline{K}_{11}-2T_{j2}(\overline{x}_2^2+\overline{y}_2^2)\overline{K}_{22} \\
&-4T_{j1}T_{j2}\sqrt{\overline{x}_1^2+\overline{y}_1^2}\sqrt{\overline{x}_2^2+\overline{y}_2^2}\,\overline{K}_{12}
\end{aligned} \tag{16}$$

Above transformation should satisfy the basic hypothesis that the angular speed equation coincides with the different form of the Lagrange function. Therefore, the Lagrange equation of the angle variable is transformed into:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{\omega_{\mathrm{B}}}\frac{\partial \overline{L}^{(1)}}{\partial \overline{\omega}_1}\right)-\frac{\partial \overline{L}^{(1)}}{\partial \overline{\varphi}_1}=\overline{m}_{\mathrm{t}}-\overline{m}_{\mathrm{g}} \tag{17}$$

where $\overline{\varphi}_1=\overline{\omega}_1 t$, $\varphi=\omega_{\mathrm{mB}}(\overline{\omega}_1+1)t=\omega_{\mathrm{mB}}(\overline{\varphi}_1+t)$.

Substituting the Lagrange function (16) into (12), these equations can then be returned to the primary differential equations (5)~(8).

4.2 The Hamiltonian function

The generalized coordinates are selected as v={v_1, v_2, v_3, v_4, v_5}, v_1= $\overline{\varphi}_1$, v_2= $\overline{x}_1$, v_3= $\overline{y}_1$, v_4= $\overline{x}_2$, v_5= $\overline{y}_2$. The generalized momentums are defined as:

$$\begin{aligned}
p_1 &= \frac{\partial \overline{L}^{(1)}}{\partial \overline{\omega}_1} \\
&=2T_{j1}(\overline{e}_1\dot{\overline{y}}_1\cos\varphi-\overline{e}_1\dot{\overline{x}}_1\sin\varphi) \\
&+T_{j1}\omega_{\mathrm{B}}(1+4\overline{e}_1^2)\overline{\omega}_1 \\
&+2T_{j2}(\overline{e}_2\dot{\overline{y}}_2\cos\varphi-\overline{e}_2\dot{\overline{x}}_2\sin\varphi) \\
&+T_{j2}\omega_{\mathrm{B}}(1+4\overline{e}_2^2)\overline{\omega}_1
\end{aligned} \tag{18}$$

$$p_2=\frac{\partial \overline{L}^{(1)}}{\partial \dot{\overline{x}}_1}=2T_{j1}[\dot{\overline{x}}_1-\overline{e}_1(\overline{\omega}_1+1)\sin\varphi] \tag{19}$$

$$p_3=\frac{\partial \overline{L}}{\partial \dot{\overline{y}}_1}=2T_{j1}[\dot{\overline{y}}_1+\overline{e}_1(\overline{\omega}_1+1)\cos\varphi] \tag{20}$$

$$p_4=\frac{\partial \overline{L}}{\partial \dot{\overline{x}}_2}=2T_{j2}[\dot{\overline{x}}_2-\overline{e}_2(\overline{\omega}_1+1)\sin\varphi] \tag{21}$$

$$p_5 = \frac{\partial \bar{L}}{\partial \dot{\bar{y}}_2} = 2T_{j2}[\dot{\bar{y}}_2 + \bar{e}_2(\bar{\omega}_1 + 1)\cos\varphi] \tag{22}$$

In fact, p_2~p_5 are the momentum of the mass centre in relative values, indicating that the definition of the generalized momentum described with relative values is consistent with the definition in the traditional dynamics. Thus, the replacement of the angular speed variable in the Lagrange function doesn't change the energy characteristics of the shafting.

The Hamiltonian function is selected as:

$$H^{(1)} = \boldsymbol{p}^{\mathrm{T}}\dot{\boldsymbol{v}} - \bar{L}^{(1)} \tag{23}$$

Expanding above equation yields:

$$\begin{aligned} H^{(1)} &= T_{j1}[\dot{\bar{x}}_1^2 + \dot{\bar{y}}_1^2 + 2\bar{e}_1\bar{\omega}_1\dot{\bar{y}}_1\cos\varphi - 2\bar{e}_1\bar{\omega}_1\dot{\bar{x}}_1\sin\varphi] \\ &\quad - 2T_{j1}\bar{e}_1\bar{\omega}_1\dot{\bar{x}}_1\sin\varphi \\ &\quad + T_{j2}[\dot{\bar{x}}_2^2 + \dot{\bar{y}}_2^2 + 2\bar{e}_2\bar{\omega}_1\dot{\bar{y}}_2\cos\varphi - 2\bar{e}_2\bar{\omega}_1\dot{\bar{x}}_2\sin\varphi] \\ &\quad - 2T_{j2}\bar{e}_2\bar{\omega}_1\dot{\bar{x}}_2\sin\varphi \\ &\quad \frac{1}{2}T_{j1}\omega_{\mathrm{B}}(1+4\bar{e}_1^2)\bar{\omega}_1{}^2 + \frac{1}{2}T_{j2}\omega_{\mathrm{B}}(1+4\bar{e}_2^2)\bar{\omega}_1{}^2 \\ &\quad + 2T_{j1}(\bar{x}_1^2+\bar{y}_1^2)\bar{K}_{11} + 2T_{j2}(\bar{x}_2^2+\bar{y}_2^2)\bar{K}_{22} \\ &\quad + 4T_{j1}T_{j2}\sqrt{\bar{x}_1^2+\bar{y}_1^2}\sqrt{\bar{x}_2^2+\bar{y}_2^2}\bar{K}_{12} \end{aligned} \tag{24}$$

Differentiating (23) with p yields:

$$\frac{\partial H^{(1)}}{\partial \boldsymbol{p}} = \dot{\boldsymbol{v}} + \boldsymbol{p}^T\frac{\partial \dot{\boldsymbol{v}}}{\partial \boldsymbol{p}} - \frac{\partial \bar{L}^{(1)}}{\partial \boldsymbol{p}} = \dot{\boldsymbol{v}} \tag{25}$$

Obviously, ($\boldsymbol{v}$, $\boldsymbol{p}$) is still dual variable.

One of the purposes defining the generalized momentum is that the substitution of the differential items in the Hamiltonian function and equation can reduce the order of the equation. In this paper, the motion equation of the variables $\bar{\varphi}_1$ and $\bar{\omega}_1$ will be substituted with the generator model while connecting the shafting model. Therefore, the speed item of the four axis variables will be substituted while the angular speed item will remain the same.

Expressions of $\dot{\bar{x}}_1$, $\dot{\bar{y}}_1$, $\dot{\bar{x}}_2$ and $\dot{\bar{y}}_2$ can be derived from the generalized momentum, and are used to replace the speed items in the Hamiltonian function. For the value of ω_{B}=314rad/s, $\frac{1}{2}T_{j1}\omega_{\mathrm{B}}(1+4\bar{e}_1^2)\bar{\omega}_1{}^2$ is much larger than $T_{j1}\bar{e}_1{}^2(\bar{\omega}_1{}^2-1)$. Therefore, $T_{j1}\bar{e}_1{}^2(\bar{\omega}_1{}^2-1)$ can be ignored. As such, the Hamiltonian function (24) can be written as the following:

$$\begin{aligned} H^{(1)} &= \frac{p_2^2}{4T_{j1}} + \frac{p_3^2}{4T_{j1}} + p_2\bar{e}_1\sin\varphi - p_3\bar{e}_1\cos\varphi \\ &\quad + \frac{p_4^2}{4T_{j2}} + \frac{p_5^2}{4T_{j2}} + p_4\bar{e}_2\sin\varphi - p_5\bar{e}_2\cos\varphi \\ &\quad + \frac{1}{2}[T_{j1}(1+4\bar{e}_1^2) + T_{j2}(1+4\bar{e}_2^2)]\omega_{\mathrm{B}}\bar{\omega}_1{}^2 \\ &\quad + 2T_{j1}(\bar{x}_1^2+\bar{y}_1^2)\bar{K}_{11} + 2T_{j2}(\bar{x}_2^2+\bar{y}_2^2)\bar{K}_{22} \\ &\quad + 4T_{j1}T_{j2}\sqrt{\bar{x}_1^2+\bar{y}_1^2}\sqrt{\bar{x}_2^2+\bar{y}_2^2}\bar{K}_{12} \end{aligned} \tag{26}$$

Meanwhile, the speed items of the four shafting variables included in the generalized momentum p_1 should be replaced in the same way. As such, Eq.(25) has been changed due to this substitution.

Combining the Hamiltonian function (24) with (18)~(22) yields the expressions of the generalized speed:

$$\bar{\omega}_1 = \frac{1}{T_j\omega_{\mathrm{B}}}\frac{\partial H^{(1)}}{\partial \bar{\omega}_1} \tag{27}$$

$$\dot{\bar{x}}_1 = \frac{\bar{e}_1\sin\varphi}{T_j\omega_{\mathrm{B}}}\frac{\partial H^{(1)}}{\partial \bar{\omega}_1} + \frac{\partial H^{(1)}}{\partial p_2} \tag{28}$$

$$\dot{\bar{y}}_1 = -\frac{\bar{e}_1\cos\varphi}{T_j\omega_{\mathrm{B}}}\frac{\partial H^{(1)}}{\partial \bar{\omega}_1} + \frac{\partial H^{(1)}}{\partial p_3} \tag{29}$$

$$\dot{\bar{x}}_2 = \frac{\bar{e}_2\sin\varphi}{T_j\omega_{\mathrm{B}}}\frac{\partial H^{(1)}}{\partial \bar{\omega}_1} + \frac{\partial H^{(1)}}{\partial p_4} \tag{30}$$

$$\dot{\bar{y}}_2 = -\frac{\bar{e}_2\cos\varphi}{T_j\omega_{\mathrm{B}}}\frac{\partial H^{(1)}}{\partial \bar{\omega}_1} + \frac{\partial H^{(1)}}{\partial p_5} \tag{31}$$

where $T_j = T_{j1}(1+4\bar{e}_1^2) + T_{j2}(1+4\bar{e}_2^2)$, is called the total inertia time constant.

Furthermore, some of transformation expressions can be obtained from (26): $\frac{\partial H^{(1)}}{\partial \bar{x}_1} = -\frac{\partial \bar{L}^{(1)}}{\partial \bar{x}_1}$, $\frac{\partial H^{(1)}}{\partial \bar{y}_1} = -\frac{\partial \bar{L}^{(1)}}{\partial \bar{y}_1}$, $\frac{\partial H^{(1)}}{\partial \bar{x}_2} = -\frac{\partial \bar{L}^{(1)}}{\partial \bar{x}_2}$, $\frac{\partial H^{(1)}}{\partial \bar{y}_2} = -\frac{\partial \bar{L}^{(1)}}{\partial \bar{y}_2}$.

Substituting the generalized momentum (18) into the Lagrange equation (17) yields:

$$\dot{p}_1 = \omega_{\mathrm{B}}\left(\frac{\partial \bar{L}^{(1)}}{\partial \bar{\varphi}_1} + \bar{m}_{\mathrm{t}} - \bar{m}_{\mathrm{g}}\right) \tag{32}$$

Substituting p_2, p_3, p_4 and p_5 into the Lagrange equations (12), and combining

(28)~(31) yields:

$$\dot{p}_2 = -\omega_{\mathrm{mB}} \frac{\partial H^{(1)}}{\partial \overline{x}_1} - \overline{c}_1 \frac{\partial H^{(1)}}{\partial p_2} - \overline{c}_1 \frac{\overline{e}_1 \sin\varphi}{T_j \omega_{\mathrm{B}}} \frac{\partial H^{(1)}}{\partial \overline{\omega}_1} + \overline{F}_{x1} \tag{33}$$

$$\dot{p}_3 = -\omega_{\mathrm{mB}} \frac{\partial H^{(1)}}{\partial \overline{y}_1} - \overline{c}_1 \frac{\partial H^{(1)}}{\partial p_3} + \overline{c}_1 \frac{\overline{e}_1 \cos\varphi}{T_j \omega_{\mathrm{B}}} \frac{\partial H^{(1)}}{\partial \overline{\omega}_1} + \overline{F}_{y1} \tag{34}$$

$$\dot{p}_4 = -\omega_{\mathrm{mB}} \frac{\partial H^{(1)}}{\partial \overline{x}_2} - \overline{c}_2 \frac{\partial H^{(1)}}{\partial p_4} - \overline{c}_2 \frac{\overline{e}_2 \sin\varphi}{T_j \omega_{\mathrm{B}}} \frac{\partial H^{(1)}}{\partial \overline{\omega}_1} + \overline{F}_{x2} \tag{35}$$

$$\dot{p}_5 = -\omega_{\mathrm{mB}} \frac{\partial H^{(1)}}{\partial \overline{y}_2} - \overline{c}_2 \frac{\partial H^{(1)}}{\partial p_5} + \overline{c}_2 \frac{\overline{e}_2 \cos\varphi}{T_j \omega_{\mathrm{B}}} \frac{\partial H^{(1)}}{\partial \overline{\omega}_1} + \overline{F}_{y2} \tag{36}$$

Equations (27)~(36) are expanded form of the Hamiltonian equation, and will be integrated into the generator model and rewritten as the standard form of the Hamiltonian model in next section.

5 The Hamiltonian model for the generator

As the Hamiltonian function and equation is not sole, selected different Hamiltonian function will yield different Hamiltonian equation. The Hamiltonian control model of the third order generator derived from basic energy relationship is as following [29]:

$$\begin{bmatrix} \dot{\delta} \\ \dot{\overline{\omega}}_1 \\ \dot{E}_q^{/} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{1}{T_j} & 0 \\ -\dfrac{1}{T_j} & -\dfrac{D}{T_j^2 \omega_{\mathrm{B}}} & 0 \\ 0 & 0 & -\dfrac{\omega_{\mathrm{B}} X_{\mathrm{ad}}^2}{T_{\mathrm{d0}}' X_f} \end{bmatrix} \begin{bmatrix} \dfrac{\partial H^{(2)}}{\partial \delta} \\ \dfrac{\partial H^{(2)}}{\partial \overline{\omega}_1} \\ \dfrac{\partial H^{(2)}}{\partial E_q^{/}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{1}{T_j} & 0 \\ 0 & \dfrac{\omega_{\mathrm{B}}}{T_{\mathrm{d0}}'} \end{bmatrix} \begin{bmatrix} \overline{m}_{\mathrm{t}} \\ \overline{E}_f \end{bmatrix} \tag{37}$$

The Hamiltonian function of the system is

$$H^{(2)} = \frac{1}{2} T_j \omega_{\mathrm{B}} \overline{\omega}_{1e}^2 + \frac{1}{2} U_{\mathrm{s}}^2 \frac{X_{q\Sigma} - X_{d\Sigma}}{X_{d\Sigma} X_{q\Sigma}} \cos^2\delta + \frac{U_{\mathrm{s}}^2}{2X_{q\Sigma}} + \frac{1}{2} \frac{\left(X_{\mathrm{ad}}^2 U_{\mathrm{s}} \cos z_1 - X_{d\Sigma} X_f z_3\right)^2}{X_{d\Sigma} X_{d\Sigma}' X_f X_{\mathrm{ad}}} \tag{38}$$

where $\overline{\omega}_{1e} = \overline{\omega}_e - 1$, $\overline{\omega}_e = \omega_e/\omega_B$, $T_j=J*\omega_{mB}^2/S_{gB}$, $X_{d\Sigma}=X_d+X_T+X_L$, $X_{d\Sigma}'=X_d'+X_T+X_L$, $X_{q\Sigma}=X_q+X_T+X_L$, $X_{d\Sigma}'=X_d'+X_T+X_L$.

The link between the generator model and the shafting model is the angular speed motion equation. In order to explicit the connection, the Eq. (37) is restored back to the differential equation form:

$$\begin{aligned} &\dot{\delta} = \omega_{\mathrm{B}} \overline{\omega}_{1e} \\ &\dot{\overline{\omega}}_{1e} = \frac{1}{T_j} (\overline{m}_{\mathrm{t}} - \overline{m}_{\mathrm{g}} - D\overline{\omega}_{1e}) \\ &\dot{E}_q' = -\frac{\omega_{\mathrm{B}}}{T_{d0}'} \frac{X_{d\Sigma}}{X_{d\Sigma}'} E_q' + \frac{\omega_{\mathrm{B}}}{T_{d0}'} \frac{X_{d\Sigma} - X_{d\Sigma}'}{X_{d\Sigma}'} U_{\mathrm{s}} \cos\delta + \frac{\omega_{\mathrm{B}}}{T_{d0}'} \overline{E}_f \end{aligned} \tag{39}$$

where $D\omega_1$ is the additional modified item.

The first item in the Hamiltonian function (38) should be the rotary kinetic energy of the HGTS. The Hamiltonian model for the generator is mainly applied to study its transient action. The rotary kinetic energy is much larger than other energy items in the Hamiltonian function, indicating that the impact of other energy item on the system transient is likely to be masked. On the other hands, the angular speed increment in relative value reflects the variations of the rotary kinetic energy in transient. Thus, the rotary kinetic energy can be replaced by the angular speed increment in relative value. Meanwhile, the angular speed change is very small when the HTGS is connected with the power system. The angular speed increment is multiplied by ω_B to reflect the influence of the rotary kinetic energy. The readers are referred to [30] for the details of the explanation of the rationality of this description.

The angular speed in Eq.(39) is the electric angular speed, and is denoted with subscript 'e'. The relationship between the mechanical angle φ_m and the electric angle φ_e is $\varphi_m=\varphi_e/p_p$, where the p_p is the pole numbers of the generator. The relationship between the mechanical angular speed ω_m and the electric angular speed ω_e is $\omega_m=\omega_e/p_p$; while the relationship between the base value of the mechanical angular speed ω_{mB} and the base value of the electric angular speed ω_{eB} is $\omega_{mB}=\omega_{eB}/p_p$. Thus, the mechanical angular speed is equal to the electric angular speed in relative value, $\overline{\omega}_e = \overline{\omega}_m$. In the shafting model, the subscript m of the mechanical angular speed is omitted.

6. Uniform generalized Hamiltonian model

From the generator model (39), the equation of the rotor angle δ is similar to the equation of the shafting angle $\overline{\varphi}_1$. So the order of the system can be reduced by directly calculating $\overline{\varphi}_1$ from δ. The relationship between the variable δ and the mechanical angle φ is:

$$\dot{\delta} = \omega_{\mathrm{B}}(\overline{\omega} - 1) = \omega_{\mathrm{B}}\overline{\omega}_1 = \omega_{\mathrm{B}}\dot{\overline{\varphi}}_1 = p_p\dot{\varphi} - \omega_{\mathrm{B}} \quad (40)$$

Integrating (40) yields:

$$\varphi(t_i) = \varphi(t_{i-1}) + \omega_{\mathrm{mB}}\Delta t + [\delta(t_i) - \delta(t_{i-1})]/\,p_p \quad (41)$$

Therefore, the generator subsystem can be combined with the shafting subsystem to form the uniform generalized Hamiltonian control model.

As the increment in relative value of the electric angular speed is equal to one of the mechanical angular speed, $\overline{\omega}_{1e} = \overline{\omega}_1$, new variables are then selected as $z_1=\delta$, $z_2=\overline{\omega}_1$, $z_3=E_q'$, $z_4=\overline{x}_1$, $z_5=\overline{y}_1$, $z_6=\overline{x}_2$, $z_7=\overline{y}_2$, $z_8=p_2$, $z_9=p_3$, $z_{10}=p_4$, $z_{11}=p_5$.

Because the rotary kinetic energy of the generator model and the shafting model is equal, summing their Hamiltonian functions yields the uniform Hamiltonian function:

$$\begin{aligned} H = {} & \frac{1}{2}T_j\omega_{\mathrm{B}}z_2^2 + \frac{1}{2}U_{\mathrm{s}}^2\frac{X_{q\Sigma} - X_{d\Sigma}}{X_{d\Sigma}X_{q\Sigma}}\cos^2 z_1 + \frac{1}{2}\frac{1}{X_{q\Sigma}}U_{\mathrm{s}}^2 + \frac{1}{2}\frac{\left(X_{\mathrm{ad}}^2U_{\mathrm{s}}\cos z_1 - X_{d\Sigma}X_f z_3\right)^2}{X_{d\Sigma}X_{d\Sigma}'X_f X_{\mathrm{ad}}} \\ & + \frac{z_8^2}{4T_{j1}} + \frac{z_9^2}{4T_{j1}} + z_8\overline{e}_1\sin\varphi - z_9\overline{e}_1\cos\varphi + \frac{z_{10}^2}{4T_{j2}} + \frac{z_{11}^2}{4T_{j2}} + z_{10}\overline{e}_2\sin\varphi - z_{11}\overline{e}_2\cos\varphi 2T_{j1}(z_4^2 + z_5^2)\overline{K}_{11} \\ & + 2T_{j2}(z_6^2 + z_7^2)\overline{K}_{22} + 4T_{j1}T_{j2}\sqrt{z_4^2 + z_5^2}\sqrt{z_6^2 + z_7^2}\,\overline{K}_{12} \end{aligned} \quad (42)$$

Integrating equations (28)~(31), (33)~(36) and (39) yields:

$$\dot{\boldsymbol{z}} = \boldsymbol{T}(\boldsymbol{z})\frac{\partial H}{\partial \boldsymbol{z}} + \boldsymbol{G}(\boldsymbol{z})\boldsymbol{u}(\boldsymbol{z}) \quad (43)$$

where：

$$\boldsymbol{T}(\boldsymbol{z})=\begin{bmatrix} 0 & \frac{1}{T_j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{T_j} & -\frac{D}{T_j^2\omega_{\mathrm{B}}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\omega_{\mathrm{B}}X_{\mathrm{ad}}^2}{T_{\mathrm{d0}}'X_f} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\bar{e}_1\sin\varphi}{T_j\omega_{\mathrm{B}}} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{\bar{e}_1\cos\varphi}{T_j\omega_{\mathrm{B}}} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{\bar{e}_2\sin\varphi}{T_j\omega_{\mathrm{B}}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{\bar{e}_2\cos\varphi}{T_j\omega_{\mathrm{B}}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -\bar{c}_1\frac{\bar{e}_1\sin\varphi}{T_j\omega_{\mathrm{B}}} & 0 & -\omega_{\mathrm{mB}} & 0 & 0 & 0 & -\bar{c}_1 & 0 & 0 & 0 \\ 0 & \bar{c}_1\frac{\bar{e}_1\cos\varphi}{T_j\omega_{\mathrm{B}}} & 0 & 0 & -\omega_{\mathrm{mB}} & 0 & 0 & 0 & -\bar{c}_1 & 0 & 0 \\ 0 & -\bar{c}_2\frac{\bar{e}_2\sin\varphi}{T_j\omega_{\mathrm{B}}} & 0 & 0 & 0 & -\omega_{\mathrm{mB}} & 0 & 0 & 0 & -\bar{c}_2 & 0 \\ 0 & \bar{c}_2\frac{\bar{e}_2\cos\varphi}{T_j\omega_{\mathrm{B}}} & 0 & 0 & 0 & 0 & -\omega_{\mathrm{mB}} & 0 & 0 & 0 & -\bar{c}_2 \end{bmatrix}$$

$$\boldsymbol{G}(\boldsymbol{z})=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{T_j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\omega_{\mathrm{B}}}{T_{\mathrm{d0}}'} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T, \boldsymbol{u}(\boldsymbol{z})=\begin{bmatrix} \overline{m}_{\mathrm{t}} \\ \overline{E}_f \\ \overline{F}_{x1} \\ \overline{F}_{y1} \\ \overline{F}_{x2} \\ \overline{F}_{y2} \end{bmatrix}$$

Using the basic transformation of the following:

$$\begin{cases} \boldsymbol{J}(\boldsymbol{z})=\frac{1}{2}[\boldsymbol{T}(\boldsymbol{z})-\boldsymbol{T}^T(\boldsymbol{z})] \\ \boldsymbol{R}(\boldsymbol{z})=-\frac{1}{2}[\boldsymbol{T}(\boldsymbol{z})+\boldsymbol{T}^T(\boldsymbol{z})] \end{cases} \tag{44}$$

The model (43) can then be transformed into the standard form of generalized Hamiltonian control model:

$$\dot{\boldsymbol{z}}=[\boldsymbol{J}(\boldsymbol{z})-\boldsymbol{R}(\boldsymbol{z})]\frac{\partial H}{\partial \boldsymbol{z}}+\boldsymbol{G}(\boldsymbol{z})\boldsymbol{u}(\boldsymbol{z}) \tag{45}$$

where $\boldsymbol{J}(\boldsymbol{z})$ is the antisymmetric matrix, $\boldsymbol{R}(\boldsymbol{z})$ is the symmetric matrix.

Equation (45) can be proved by expanding it to restore back to the primary differential equation.

Remark 1: The input control includes the hydro turbine torque $\overline{m}_{\mathrm{t}}$ and the generator excitation control $\overline{E}_f$, which means that the transient regulation and control of the HTGS is introduced into the shafting model. Furthermore, the hydro turbine and its hydraulic system and the governor can be introduced into the shafting model by means of $\overline{m}_{\mathrm{t}}$ while the excitation control system and the power system can be introduced into the shafting model by means of $\overline{E}_f$. Thus, the proposed model provides a foundation for investigating the effects of the HTGS transient regulation, the HTGS objects and the HTGS controller on the shafting motion.

Remark 2: The external forces $\overline{F}_{x1}$, $\overline{F}_{y1}$, $\overline{F}_{x2}$ and $\overline{F}_{y2}$ are taken as additional input control to improve the generality and the feasibility of the model. On one hand, if the external force acting on the shafting is considered, the equation (45) can be applied to simulate the effect of the external force on the shafting motion. On the other hand, the external force acting on the shafting can be decomposed and merged into the structure and damping matrix of the Hamiltonian system. As such, the effects and action mechanism between the shafting inner parameters and the external force can then be investigated by employing the structure analysis theory of the generalized Hamiltonian system.

Remark 3: If the multiple fields coupling need to be considered in the rotor shafting modeling, the action forces of the multiple fields coupling can be transformed into the relative value and introduced into the shafting model through additional input control. Thus, equation (45) provides an approach for modelling the rotor shafting under the multiple fields coupling.

7 Simulation

In order to simulate the operation characteristics of the HTGS under the control and regulation, the whole HTGS system is used in the simulation, shown as in Fig.2. The governor uses classical parallel PID controller, and the excitation is a PI controller of reactive power. The hydro turbine and its hydraulic system is differential equation model with elastic water column. The generator model is classical one machine and infinite bus system with the third order.

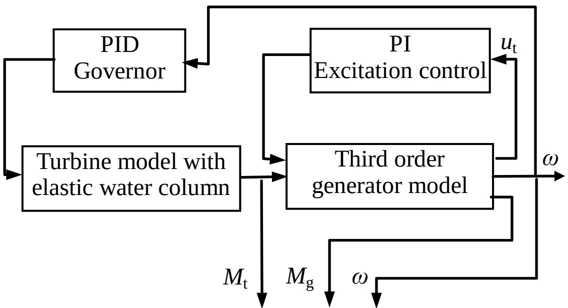


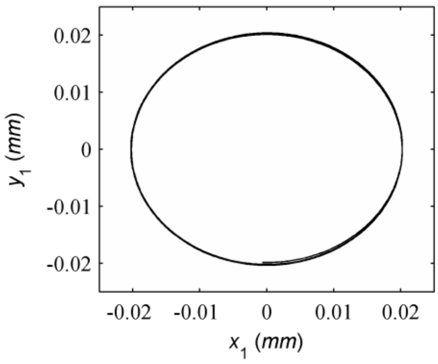
Fig.2 Sketch of the simulation system

The actual hydro turbine generator set is taken as an example. Main parameters are: S_{gB}=150MW, n_r=125r/min, m_1=7.32×10^{5}kg, m_2=2.4×10^{5}kg, R_1=4.646m, R_2=1.708m, J_1=7.9×10^{6}N·m^2, J_2=3.5×10^{5}N·m^2, p_p=24, ω_{mB}=13.09rad/s, k_1=0.2×10^{9}N/m, k_2=0.2×10^{9}N/m, k_3=0.35×10^{9}N/m, c_1=0.35×10^{7}N·s/m, c_2=0.25×10^{7}N·s/m, a=4m, b=3m, c=1.2m, e_1=1.0mm, e_2=0.5mm.

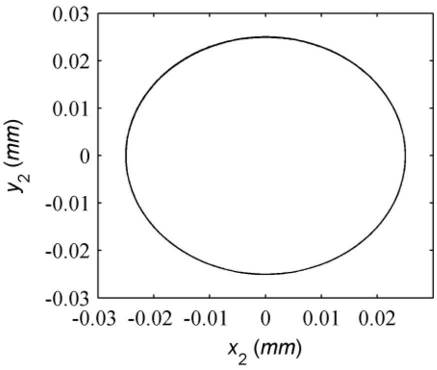
Three cases are simulated to verify the model. Case 1 considers the steady state for testing the model. Case 2 simulates the transient control to verify whether the model can reflect the change of the transient vibration of the shafting. Case 3 is to examine whether the model can reflect the effect of the external force.

Case 1:

The HTGS operates at the steady state with the active power being p_e=0.8. All additional forces are not considered, namely F_{x1}= F_{y1}=F_{x2}=F_{y2}=0.



(a) Central trajectory of the generator rotor



(b) Central trajectory of the turbine runner

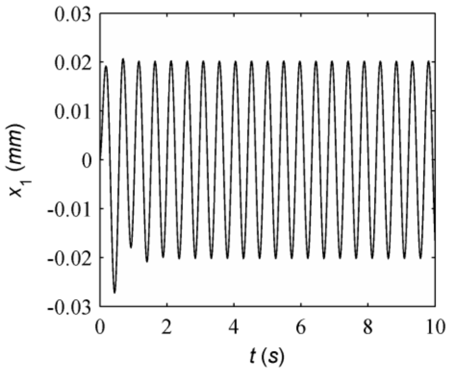
Fig.3 Center trajectories at the steady operation

The central trajectory of the generator rotor and the hydro turbine runner is shown in Fig.3(a) and (b), respectively.

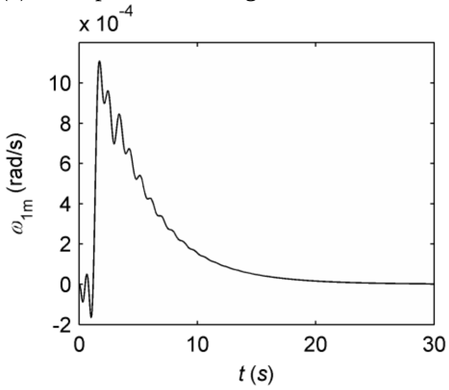
The central trajectory of both the generator rotor and the turbine runner is closely related with the damping coefficients c_1, c_2 and the stiffness k_1, k_2, k_3. When feature parameters are invariant and without the shafting additional forces, the vibration amplitude of the central trajectory is stable. The calculation result shown in Fig.3 is consistent with the actual situation.

Case 2:

The active power regulates from p_e=0.8 to p_e=1.0. All additional forces are not considered, namely F_{x1}=F_{y1}=F_{x2}=F_{y2}=0.



(a) Amplitude of the generator rotor



(b) Increment variation of the mechanical angular speed

Fig.4 The variation of the amplitude and the mechanical angular speed under regulation active power versus time

The vibration amplitude in x direct of the generator rotor at first 10 second is shown in Fig. 4(a). The variation of the mechanical angular speed increment of the HTGS ω_{1m} is shown in Fig.4(b).

Under the governor PID controller, the variation of the angular speed in the transient state is very small. The vibration amplitude of the generator rotor and the turbine runner is small and similar to that under the steady state operation. Fig.4 shows that the shafting transient model can reflect the transient change of the shafting vibration in regulation process.

In large disturbance, such as throw load, faulty at power grid side and low frequency oscillation, the variation of the angular speed is large, and so is the shafting vibration. The transient model proposed in this paper can be better applied to analysis large disturbance transient.

Case 3:

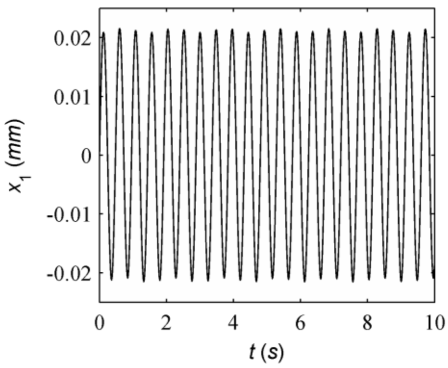
The HTGS operates at the steady state with the active power being p_e=0.5. Additional forces acting on the generator rotor are not considered, namely $F_{x1}=F_{y1}=0$; while additional forces acting on the hydro turbine runner is considered. Here, the pressure impulse of the draft tube is also considered.

According to the pressure impulse characteristics of the draft tube, the equivalent action force of the pressure impulse is assumed as:

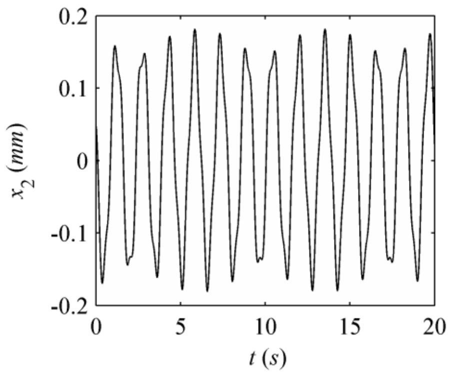
$$\overline{F}_{2x} = 0.01\cos(2\pi \times 0.65t)$$

$$\overline{F}_{2y} = 0.01\sin(2\pi \times 0.65t),$$

The above assumption indicates that the frequency of the pressure impulse of the draft tube is 0.65Hz. The central vibration in the x direction of the generator rotor and the hydro turbine runner is shown in Fig.5(a) and Fig.5(b) respectively.



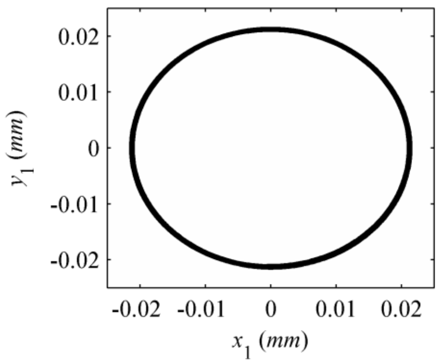
(a) Amplitude of the generator rotor



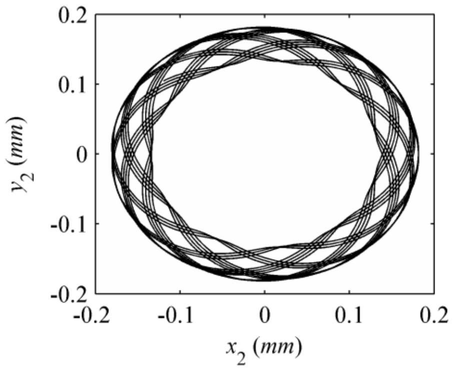
(b) Amplitude of the hydro turbine runner

Fig.5 The shafting vibration u under the pressure impulse of the draft tube

In Fig.(5), the amplitude of the generator rotor is approximately invariant. The amplitude period of the turbine runner vibration is consistent with the period of the pressure impulse in the draft tube. In order to be clear, central trajectory of the generator rotor and the hydro turbine runner under the pressure impulse of the draft tube are plotted in Fig.6 (a) and (b) respectively.



(a) Central trajectory of the generator rotor



(b) Central trajectory of the turbine runner

Fig.6 The central trajectory under the pressure impulse of the draft tube

In Fig.6, the vibration of the turbine runner is obvious, and the period feature is clear. Comparing the Fig.3 (a) and Fig.6 (a), the central trajectory line is thicker, indicating that the trajectory circle is slight swing due to the pressure impulse of the draft tube. This kind of vibration difference relates to the shafting geometry structure and acted position of the external force. The transient model better reflects the vibrations case.

These simulations show that the proposed model is flexible and can be applied to investigate the various shafting issues.

8. Conclusions

The following conclusions can be drawn from this study:

(1). Different variable domains can be transformed into relative value form by using the equivalent base value transformation system. As such, the connection of the multiple subsystems is realized. In proper base value system, the form of the Lagrange function and equation keep their basic forms.

(2). The proposed generalized Hamiltonian control model includes additional input item, which opens a new research approach and modeling method for simulating the shafting under the multiple domains and factors.

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