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1 Numerical simulation of gravity current descending a slope into a linearly stratified
2 environment

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4 Abstract: The accurate prediction of the dilution and motion of the produced denser
5 water (e.g. discharge of concentrated brine generated during solution mining and
6 desalination) is of importance for environmental protection. Boundary conditions and
7 ambient stratification can significantly affect the dilution and motion of gravity currents.
8 In this study, a multiphase model is applied to simulate the gravity current descending a
9 slope into a linearly stratified ambient. The $k-\omega$ turbulence model is used to better
10 simulate the near bed motion. The mathematical model, initial and boundary conditions
11 and the details of the numerical scheme are described. The time-dependent evolution of
12 the gravity current, the flow thickness and the velocity and density field are simulated
13 for a range of flow parameters. Simulations show that the Kelvin–Helmholtz billows
14 are generated at the top of trailing fluid by the interfacial velocity shear. The K-H type
15 instability becomes weaker with the slope distance from the source due to the decrease
16 of the interfacial velocity shear along slope. The ambient stratification restricts and
17 decreases the current head velocity as it descends slope, which differs from the situation
18 in homogenous ambient while the head velocity remains an approximately steady state.

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19 Motion of the descending flow into the stratified ambient has two stages: initial
20 acceleration and deceleration at later stage based on the balance of inertial, buoyancy
21 and friction forces. When the descending current approaches the initial neutral position
22 at later stage, it separates from the slope and spreads horizontally into environment. The
23 simulated results, such as vertical velocity and density profiles and front positions,
24 agree well with the measurements, indicating that the mathematical model can be
25 successfully applied to simulate the effect of the boundary condition and ambient
26 stratification on the dilution and propagation of gravity currents.

27

28 **Keywords:** Gravity current; numerical models; simulation; stratification

29

30 **Introduction**

31 Gravity currents are flows driven by density gradient and are frequently encountered in
32 both natural and man-made environments. Typical examples are saltwater intrusion in
33 estuaries; oil spillage in the oceans and brine discharges from desalination or solution
34 mining facilities. The saltwater wedge intrusion in estuaries occurs on non-uniform
35 slopes and often influences the overall water quality and environment of estuaries while
36 the discharge of denser water from desalination plants may greatly affect the
37 environment and ecology of the ambient receiving water body. Due to the practical
38 importance of gravity currents and their relevance and theoretical significance for a
39 variety of flow phenomena, many studies have been conducted over the last few
40 decades. Extensive studies have been conducted to investigate the simple gravity
41 current scenario, i.e. flow moving along a horizontal surface into a homogenous fluid

42 (e.g., Simpson 1982, 1997) or stratified ambient receiving fluid (e.g., Holyer and
43 Huppert 1980; Guo *et al.* 2000; Ungarish and Huppert 2002; Baines 2001, 2005;
44 Maxworthy *et al.* 2002; Birman *et al.* 2007; Munroe *et al.* 2009). For most real
45 situations (e.g., estuaries), however, the bottom solid boundary is not horizontal, and
46 the flow feature of gravity current descending a slope can be very different from that
47 over a horizontal surface. Such flow characteristics of the current descending a slope
48 have recently received increasing studies, primarily using laboratory experiments.

49

50 Ellison and Turner (1959) investigated the gravity currents descending a slope into a
51 tank using laboratory experiments. Based on the analysis of their experimental data,
52 they derived a dynamic model for investigating the bulk properties of the flow. They
53 found that the mean fluid velocity was only dependent on the local bulk Richardson
54 number, Ri and had no relation with the downslope distance. Britter and Linden (1980)
55 obtained slightly different results for small slope though their finding for larger slope
56 was similar to that of Ellison and Turner (1959). In their laboratory experiments, Britter
57 and Linden (1980) found that for the small slopes ($\theta < 0.5^\circ$), the head of the gravity
58 current decelerated with distance from the source while for larger slope, a steady head
59 velocity was generated as the buoyancy force was sufficiently large to overcome
60 frictional effects. Using internal hydraulic theory (Armi 1986), Lawrence (1993)
61 investigated the flow regimes of two layer flow over a fixed obstacle using laboratory
62 experiments. Such theory, however, cannot simulate the mixing at the interface of two
63 fluids (Zhu and Lawrence 1998; 2000). The internal hydraulic theory was extended by

64 Zhu and Lawrence (1998; 2000) to examine the effects of non-hydrostatic and friction
65 on exchange flow. They found that when the friction and non-hydrostatic effect were
66 considered, more accurate prediction of interfacial mixing in the exchange flow was
67 achieved. The method, however, requires information of the friction factor at the
68 interface which may be difficult to obtain. Similar method was applied by Cuthbertson
69 *et al.* (2004, 2006) who studied the buoyancy-driven exchange flow over a steadily
70 descending barrier using the laboratory experiments. Maxworthy and Nokes (2007) and
71 Maxworthy (2010) conducted laboratory experiments to investigate the propagation of
72 gravity currents descending a slope. The current was generated by releasing a fixed
73 volume of heavy fluid in a lock located at the top of the slope. They observed two flow
74 stages: initial acceleration stage and deceleration stage. Dai (2013) conducted similar
75 laboratory experiments and found that the flow patterns for gravity current descending
76 a slope qualitatively differed from those moving along a horizontal bottom. In above
77 studies, the ambient fluid was homogenous. Mitsudera and Baines (1992) firstly studied
78 the gravity current descending a slope into a continuously stratified environment using
79 laboratory experiments. This work was extended by Baines (2001; 2005) to investigate
80 in details the effect of slope and ambient stratification on the flow features. From the
81 experiments, Baines found that two flow regimes, gravity-current-like and plume-like
82 which depended on the balance of buoyancy and drag, were formed as the flow
83 descended the slope into a stratified ambient. A model was developed to calculate the
84 mixing of gravity current with ambient fluid. The effect of ambient two-layer
85 stratification on the motion of gravity currents was examined by Monaghan *et al.* (1999)

86 using laboratory experiments. They found that as the current gravity approached the
87 sharp density interface; it was split into two parts: one propagating along density
88 interface, another along the tank bottom.

89

90 With the development of computational science, mathematical models and numerical
91 methods, which have advantages of scaling, less expense, adaptability, noninvasion
92 and transportability (Falconer 1992; Guo *et al.* 2007), have provided an alternative
93 approach to simulate the motion of the gravity currents in past decades (Özgökmen *et*
94 *al.* 2006). Bournet *et al.* (1999) applied the $k-\varepsilon$ model to simulate the gravity
95 currents plunging into reservoirs. $k-\varepsilon$ model was also applied by Choi and Gracia
96 (2002) to investigate the two dimensional (2D) denser underflow descending a slope
97 into a homogenous environment. Zhang *et al.* (2008) applied the multiphase model to
98 simulate the flushing of trapped salt water from a bar-blocked estuary. Birman *et al.*
99 (2007) evaluated the effect of the slope on the front velocity by solving the
100 two-dimensional NS equations in a homogeneous ambient. They showed that
101 quasi-steady front velocity of the flow reached the maximum near the slope angle of 40
102 degree. Firoozabadi *et al.* (2009) simulated the 3D motion of denser underflows in a
103 straight channel by using the lower Reynolds number $k-\varepsilon$ model. Their simulation
104 was in good agreement with their experiments. Ooi *et al.* (2009) conducted 2D large
105 eddy simulation (LES) to model the motion of the gravity current generated by lock
106 exchange. They found that their 2D LES model can capture most important flow
107 features such as the front evolution and the formation of coherent billow structures at

108 the flow head. LES was also applied by Mahdinia *et al.* (2012) to investigate the lock
109 exchange flow in a curved channel. Dai *et al.* (2012) and Dai (2013) performed 3D
110 direct numerical simulation (DNS) for gravity currents generated from instantaneous
111 sources descending a slope into a homogeneous environment. They found that the flow
112 structure for lower slope angle was slightly different from that of steeper slope. Härtel
113 *et al.* (2000) performed 3D (for the lower Reynolds number up to 750) and 2D (for the
114 Reynolds number up to 30,000) DNS for lock exchange flow to investigate the
115 propagation of gravity current fronts. Their simulation showed that the 2D model was
116 able to capture essential flow features of the current front. More research work on the
117 motion of gravity currents and turbidity currents can be found in Simpson (1982, 1997)
118 and Meiburg and Kneller (2010).

119

120 Though these studies demonstrated some flow characteristics of gravity currents
121 moving in various boundary conditions, none of these numerical studies considered the
122 combined effect of ambient stratification and bottom slope on the movement of gravity
123 currents. Therefore, references to the numerical modelling studies for gravity currents
124 descending a slope into a stratified environment are still lacking. In fact, experiments of
125 Baines (2001, 2005) demonstrated that the stratification in receiving environment can
126 significantly influenced the motion of gravity current. Such effect of the combination of
127 ambient stratification and bed slope on the motion of gravity current was examined by
128 Özgökmen *et al.* (2006) who conducted the numerical simulation to investigate the
129 transport of large scale gravity currents in oceans. They found that when the gravity

130 currents separated from the slope bed, the transport of the flow only depended on the
131 strength of the ambient stratification. Their study only focused on the bulk properties of
132 the motion of the large scale gravity currents and didn't investigate the details of the
133 flow structure. Such information is important for predicting the dilution and motion of
134 the produced denser water (e.g. from desalination or mining solution) discharging into
135 the receiving water bodies, which is the major concern from the point of view of
136 environment protection. This is the motivation of this study in which a two-dimensional
137 multiphase model is employed to simulate the flow structures and density distribution
138 within the gravity current as it descends a slope into a linearly stratified environment.
139 The evolution of the gravity current and front motion, the flow thickness, the vertical
140 density distribution and velocity profiles within the flow are simulated for a range of
141 flow conditions. Simulated results are in good agreement with the laboratory
142 measurements of Mitsudera and Baines (1992) and Baines (2001; 2005).

143

144 **Multiphase model**

145 *Governing equations*

146 As different phases (inflow source water, surface salt water and the bottom salt water)
147 share the same velocity and pressure field, the governing equations are a single set of
148 momentum and continuity equations in conservative form (Ferziger and Perić 2002):

$$149 \quad \frac{\partial}{\partial t}(\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q) = 0 \quad (1)$$

$$150 \quad \frac{\partial}{\partial t}(\alpha_q \rho_q \vec{v}_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q \vec{v}_q) = -\alpha_q \nabla P + \nabla \cdot \vec{\tau}_q + \alpha_q \rho_q \vec{g} + \sum_{p=1}^3 K_{pq} (\vec{v}_p - \vec{v}_q) \quad (2)$$

$$151 \quad \bar{\tau}_q = \alpha_q \mu_q (\nabla \vec{v}_q + \nabla \vec{v}_q^T) + \alpha_q (\lambda_q - \frac{2}{3} \mu_q) \nabla \cdot \vec{v}_q \bar{I} \quad (3)$$

152 where p and q = the phase constituent; α_q = volume fraction of phase q (the volume
153 fractions for all phases sum to one); ρ_q =the density of phase q ; \vec{v} =the velocity vector;
154 $\bar{\tau}_q$ =the stress-strain tensor of phase q ; μ_q , λ_q = the coefficients of shear and bulk
155 viscosity of phase q , respectively; \bar{I} = unit tensor; P =the pressure shared by phases; \vec{g}
156 =the gravity acceleration; and K_{pq} = the momentum exchange coefficient between
157 phases.

158

159 ***Turbulence model***

160 In the simulation of gravity current, the near bed flow features have a significant effect
161 on the spreading and propagation of the gravity current. To accurately model this near
162 bed flow feature, a low Reynolds number k - ω model, which better models the near
163 wall flow, is applied. The governing equations are as following (Wilcox 2006; 2008):

$$164 \quad \frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}[(\mu + \frac{\mu_t}{\sigma_k}) \frac{\partial k}{\partial x_j}] + G_k - Y_k \quad (4)$$

$$165 \quad \frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_i}(\rho \omega u_i) = \frac{\partial}{\partial x_j}[(\mu + \frac{\mu_t}{\sigma_\omega}) \frac{\partial \omega}{\partial x_j}] + G_\omega - Y_\omega \quad (5)$$

$$166 \quad \rho = \sum_{p=1}^3 \alpha_p \rho_p \quad (6)$$

$$167 \quad u_i = \frac{\sum_{p=1}^3 \alpha_p \rho_p \vec{v}_p(i)}{\sum_{p=1}^3 \alpha_p \rho_p} \quad (7)$$

168 where ρ =the mixture density of all phases; k = the turbulent kinetic energy; μ =the
169 dynamic viscosity of water; t =the time; u_i =the component of velocity in the

170 x_i -direction; ω =the specific dissipation rate; μ_t =the turbulent (or eddy) viscosity; σ_k ,
 171 σ_ω =the turbulent Prandtl number for k and ω , respectively; G_k =the generation of k
 172 induced by the mean velocity gradients; G_ω =the generation of ω caused by the mean
 173 velocity gradients; Y_k =the dissipation of k due to turbulence; and Y_ω =the dissipation of
 174 ω due to turbulence.

175

176 The term of turbulent kinetic energy produced by the mean velocity gradients and
 177 turbulent viscosity can be determined by (Wilcox 2006; 2008):

$$178 \quad G_k = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \quad (8)$$

$$179 \quad \mu_t = \frac{0.144 \mu \omega + \rho k}{6 \mu \omega + \rho k} \frac{\rho k}{\omega} \quad (9)$$

$$180 \quad \omega = \frac{\varepsilon}{C_\mu k} \quad (10)$$

181 where ε = dissipation rate of k . The values of the constants are (Rodi 1993; Wilcox
 182 2006): $\sigma_k = 2.0$; $C_\mu = 0.09$; and $\sigma_\omega = 2.0$.

183

184 ***Numerical scheme***

185 The governing equations are solved by finite volume method (FVM). The discretized
 186 form of continuity equation can be expressed as (Versteeg and Malalasekera 1995):

$$187 \quad A_1^c \Omega^c = \sum_{nb} A_1^{nb} \Omega^{nb} \quad (11a)$$

$$188 \quad \Omega = (\alpha_1, \alpha_2, \alpha_3)^T \quad (11b)$$

189 A_1^c , A_1^{nb} = the coefficients matrices that contain the influence from transient and
 190 convection terms where superscript c refers to cell center and superscript nb refers to

191 cell neighbors, respectively; Ω = vector of phases. Applying Eq.(11) and $\sum_{p=1}^3 \alpha_p = 1$

192 yields the volume fractions of phases.

193

194 The transient, convection, pressure, diffusion, gravity and momentum exchange terms
195 in momentum equation can be discretized as (Cokljat *et al.* 2006)

$$196 \quad (\bar{A}^c - \bar{R}^c) \bar{U}_q^{c,*}(i) = \sum_{nb} \bar{A}^{nb} \bar{U}_q^{nb,*}(i) - \Omega^c \frac{\partial P^*}{\partial x_i} + \bar{B}_q^{c,n} \quad (12a)$$

$$197 \quad \bar{U}_q^*(i) = (\bar{v}_1^*(i), \bar{v}_2^*(i), \bar{v}_3^*(i))^T \quad (12b)$$

198 where \bar{A}^c , \bar{A}^{nb} = the coefficients matrices that contain the influence from transient,
199 convection and diffusion terms (superscript c and nb have the same meaning as in
200 Eq.(11); \bar{R}^c = the matrices representing the momentum exchange term; \bar{B}^c = gravity
201 term; \bar{U}_q = phase velocities vector. Superscript * represents the current iteration and n
202 refers to the previous iteration.

203

204 The turbulence equations can be discretized similarly to those used for continuity
205 equation. The pressure-velocity coupling is achieved with the use of the phased coupled
206 SIMPLE (PC-SIMPLE) (Vasquez and Ivanov 2000), an extension of the SIMPLE
207 algorithm (Spalding 1980) to multiphase flows. The QUICK scheme is applied for
208 spatial discretization of governing equations, while the second order implicit scheme is
209 used for temporal discretization (Ferziger and Perić 2002). The velocities are solved
210 and coupled by phases in a segregated fashion. Fluxes are reconstructed at the faces of
211 the control volume and then a pressure correction equation is built based on total

212 continuity. The coefficients of the pressure correction equations come from the coupled
 213 per phase momentum equations (Vasquez and Ivanov 2000). Body-fitted non-uniform
 214 meshes with arbitrarily spatially dependent size were used in order to accurately fit the
 215 slope bed in the computational domain. This allows for locally refining the concerned
 216 regions (e.g. near bed region) with small meshes and has the advantage of flexibly
 217 assigning meshes in the computational domain (Guo *et al.* 2008, 2012; Jing *et al.* 2009).
 218 Sensitivity analysis of mesh size was carried out by adapting and refining the meshes
 219 until no noticeable changes in the solution was achieved (Guo 2014). Several mesh
 220 sizes have been investigated and compared in terms of the simulation accuracy,
 221 convergence and computational time to determine the final meshes (see section: Results
 222 and discussion). The final meshes having 266000 elements were used in the simulation
 223 with the minimum and maximum grid size in x -direction being 0.0015m and 0.012m,
 224 and 0.00015m and 0.0006m in z -direction, respectively. The maximum residual for
 225 convergence was 10^{-5} with a constant time step being 10^{-4} s.

226

227 *Initial and boundary conditions*

228 The computational domain is shown in Figure 1. At the inlet boundary, velocity profile
 229 is specified using the experimental data. Turbulent kinetic energy k and specific
 230 dissipation rate ω are set as following (Ferziger and Perić 2002):

$$231 \quad u = u_0; w=0 \quad (13)$$

$$232 \quad k_m = 10^{-4} u_0^2 \quad (14)$$

$$233 \quad \omega_m = 10k_m^{0.5} / (c_\mu d_0) \quad (15)$$

234 where u_0 and w = the initial mean velocity in x - and z -direction at the inlet, respectively
235 (see Fig. 1); d_0 = the initial thickness of the inflow at the inlet. The pressure outlet
236 boundary condition is specified at the outlet in which a static pressure at the outlet
237 boundary is realized. At the free water surface, the atmospheric pressure is applied. The
238 non-slip boundary condition is applied on all solid walls. The standard wall function
239 law is used to estimate the velocity parallel to the slope bed at the first cell (Launder
240 and Spalding 1974).

241

242 In order to observe the evolution of the gravity current, the inflow source water, the
243 surface salt water and the bottom salt water in the tank are treated as three single
244 miscible phases. The densities of the surface and bottom salt water phases in the tank
245 are defined to generate the prescribed ambient stratification, which can be expressed as:

$$246 \quad \rho = \rho_2 - (\rho_2 - \rho_1)z / Z_0 \quad (16)$$

247 where ρ_1 and ρ_2 = the water density at the surface and the bottom of the tank,
248 respectively; Z_0 = water depth in the tank (see Figure 1).

249

250 *Procedure of solution*

251 The procedure of the solution for governing equations is:

- 252 1. Specify initial and boundary conditions
- 253 2. Solve the phase continuity equations
- 254 3. Construct the momentum equation matrix
- 255 4. Predict the pressure field.

- 256 5. Solve the momentum equation and obtain the velocity field
- 257 6. Correct the pressure and update the velocity field
- 258 7. Solve the transport equations for the turbulence quantities
- 259 8. Repeat steps 2 to 7 until the prescribed computational accuracy is achieved
- 260 9. Using the calculated variables from the current time step as initial conditions
- 261 and repeat steps 2 to 8 to calculate the variables of next time step until $t=t_{max}$.

262

263 **Experiments**

264 Laboratory experiments carried out by Mitsudera and Baines (1992) and Baines (2001;
265 2005) are used to validate the model. Though the details of the experiments can be
266 found in Mitsudera and Baines (1992) and Baines (2001; 2005); a brief description of
267 the experiments is presented for completeness and convenience. Figure 1 is a modified
268 sketch of the laboratory experiment under investigation. The experiments were carried
269 out in a rectangular tank of 38 cm wide, 299 cm long and 80 cm high. A thin vertical
270 partition was inserted to extend the effective working length. The tank was initially
271 filled with continuously/linearly stratified fluid using the two-tank technique (Davies *et*
272 *al.* 1995). The ambient stratification was measured by a conductivity probe and was
273 used for calculating the control parameters (see below). A horizontal platform of 40 cm
274 long (not shown in Fig. 1) was inserted from one end of tank and was connected with
275 the sloping bottom which extended into the main portion of tank. On the platform, a
276 water-tight removable sluice gate was installed at a distance of 31 cm from the tank end.
277 Denser water (dyed to facilitate the observations) was filled behind this removable gate.

278 The gravity current was generated and descended the slope into the initially quiescent
 279 stratified ambient when the gate was suddenly lifted. Constant denser flow rate was
 280 maintained and monitored by a flow meter in the inflow hose throughout the
 281 experiment. For more details of experiments, readers are referred to Baines (2001,
 282 2005).

283

284 To facilitate the description of the flow, the following parameters are defined (Baines,
 285 2001; 2005):

$$286 \quad N^2 = \frac{g'_0}{D} \quad (17)$$

$$287 \quad g'_0 = g \frac{\rho_{in} - \rho_{top}}{\bar{\rho}} \quad (18)$$

$$288 \quad B_0 = \frac{Q_0 N^3}{g'^2_0} \quad (19)$$

$$289 \quad Re = \frac{Q_0}{\nu} = \frac{u_0 d_0}{\nu} \quad (20)$$

290 where ρ_{top} = the density of ambient water in tank at the top of slope; ρ_{in} = the density of
 291 inflow which is equal to the water density in the tank at the vertical depth D from the
 292 top of the slope (see Figure 1); $\bar{\rho}$ = the mean density of ρ_{top} and ρ_{in} ; ν = the kinematic
 293 viscosity of water; N = the buoyancy frequency of the initially undisturbed density
 294 stratification in the tank; g'_0 = the reduced gravity acceleration; Q_0 = the initial
 295 volumetric flow rate per unit slot width; Re = the Reynolds number and B_0 the
 296 buoyancy number of the flow. From the definition, $B_0=0$ corresponds to a homogeneous
 297 environment and B_0 increases with the increase of the strength of the ambient
 298 stratification for the same initial volumetric flow rate.

299 The range of the experimental parameters was: volumetric flow rate
300 $Q_0=4\times 10^{-5}$ - 1.121×10^{-3} m²/s; inlet height $d_0=0.01$ m; inlet velocity $u_0=0.004$ - 0.1121 m/s;
301 the slope angle $\theta=6^\circ$; $D=0.10 - 0.206$ m; the reduced gravity acceleration of inlet salt
302 water $g'_0=5.75$ - 31.63 cm/s²; the water depth $Z_0=0.23$ m and the vertical distance
303 between the top of slope and the bottom of tank is 0.2 m. These values yield the
304 buoyancy number $B_0=0.0014$ - 0.0734 and the inflow Reynolds number $Re=40$ - 1121 .
305 The numerical simulation runs cover the range of these parameters.

306

307 **Results and discussion**

308 *Mesh sensitivity analysis*

309 To investigate the effect of mesh sizes on the computational accuracy and time as well
310 as the convergence, three meshes of coarse (106400), medium (266000) and fine
311 (500000) were used in the simulation. The corresponding minimum and maximum
312 mesh sizes in x - and z -directions are: 0.002m and 0.015m (x -direction), 0.000375m and
313 0.0008m (z -direction); 0.0015m and 0.012m (x -direction), 0.00015m and 0.0006m
314 (z -direction); and 0.0005 m and 0.008m (x -direction), and 0.000075m and 0.0008m
315 (z -direction); respectively. The simulations were performed on a PC workstation: HP
316 Z650 with 6 cores, CPU 2.30GHZ, 2 processors and 48GB memory. For all mesh sizes
317 simulated, a convergent solution was always obtained. The computational accuracy and
318 time, however, was different. Fig. 2 is the comparison of the simulated velocity profiles
319 at $x=0.7$ m using three meshes with the experimental measurements (Mitsudera and
320 Baines 1992) for the flow with initial $B_0=0.022$ and $Re=290$. It is seen that the

321 computational results using medium and fine meshes are similar and agree well with
322 the experimental results, while relatively large deviation exists between the simulated
323 results using coarse meshes and measurements. The computational times for 100s are
324 10.2 hours (coarse), 36.8 hours (medium) and 126.5 hours (fine) respectively.
325 Simulations performed for different flow parameters obtain the similar results.
326 Considering the computational accuracy and time, the final mesh used is 266000
327 (medium).

328

329 *Evolution of the gravity current and the front motion*

330 To facilitate the analysis and compare with the experimental measurements, an along (s)
331 and normal to the slope (r) coordinate system s - r is used (see Figure 1. Note that the
332 simulation was performed in x - z coordinate system). In this coordinate system, u_s refers
333 to the downslope component of velocity. For a homogeneous environment, it is well
334 known that the typical motion of the gravity current descending a slope has a raised
335 head in the front, followed by a shallower steady current. This continues to flow to the
336 end of slope provided that the buoyancy is sufficiently large. However, for the cases of
337 the stratified environment, the situation is different. Figure 3 is a time series plot of the
338 simulated evolution of a gravity current descending a slope into a linearly stratified
339 environment in which the initial density of the current at the inlet is smaller than the
340 density of the ambient fluid near the tank bottom. Once the denser water intrudes the
341 ambient fluid, a front at the leading edge is quickly formed and flows down slope (see
342 Fig. 3a). A velocity shear layer is established at the interface between the flowing

343 current and initially quiescent ambient fluid. This shear velocity generates mixing at the
344 interface and entrains surrounding lighter fluid into the flow. As such, the flow,
345 particularly the front and leading part of the current, is diluted and grows as it moves
346 along the slope (see Fig. 3b, c, d). It is seen from Fig. 3c and d that the
347 Kelvin–Helmholtz billows (Baines 2001) are formed at the top of the trailing fluid – a
348 flow pattern also found in large scale simulation (Özgökmen *et al.* 2006). This means
349 that the local gradient Richardson number across the interface Ri_g
350 ($=\{g(\partial\rho/\partial z)/[\rho_r(\partial u/\partial z)^2]\}$, ρ_r =reference density, Moore and Long 1971) is sufficiently
351 low for the Kelvin-Helmholtz type billows to appear in the region of the trailing. As the
352 flow moves down slope, the velocity and density, thus the buoyancy and inertial, of the
353 leading front of flow decreases. This process continues until the inertia and buoyancy
354 of the flow front cannot overcome the bottom friction and ambient stratification. As a
355 result, the nose of the current thickens and separates from the bottom of slope (Figure
356 3c-d) and spreads horizontally into the environment before it reaches the end of the
357 slope. The ambient fluid below the position at which the flow separates from the slope
358 (separation point) is undisturbed. The position of the separation point partly depends on
359 the degree of the interfacial shear generated mixing and entrainment of the flow with
360 ambient fluid. This shear generated mixing and entrainment at the interface of flow and
361 ambient fluid is determined by the flow condition ($B_0=0.02$ and Re), the bed slope and
362 ambient stratification. Simulations have been performed for a range of parameters,
363 demonstrating that this position is usually not much lower than the neutral position
364 where the density of ambient fluid is equal to the initial density of the flow at the inlet

365 (e.g. the vertical extension of the flow from the top of the slope is usually not much
366 larger than the depth D). This means that no significant overshooting of the downflow
367 over its initial neutral level occurs for the range of parameters investigated here though
368 overshooting was usually observed in the experiments for high slope degree ($>30^\circ$) in
369 which higher inertia of the downflow was expected.

370

371 Britter and Linden (1980) found that the head velocity kept a nearly constant value
372 when slope angle $\theta \geq 0.5^\circ$ in homogenous environment. This means that a linear
373 relationship between the time and the slope distance that the head of the flow travels
374 exists. However, this is not the case when ambient fluid is stratified. Figure 4 plots the
375 dimensionless position of current head against the travelling time for various flow
376 conditions for the slope angle of 6 degree. Experimental results of Mitsudera and
377 Baines (1992) for $B_0=0.022$ and $Re=290$ are also plotted in Fig. 4 for comparison where
378 $s^*=s/D$. The slope distance corresponding to the buoyancy depth D (the initial neutral
379 depth) is $D/\sin(6^\circ)=9.567D$. It is seen from Fig. 4 that at early stage, the velocity of the
380 current head is roughly constant for all flow conditions simulated. As the buoyancy
381 number and the flow Reynolds number increases (e.g. larger buoyancy and initial and
382 more turbulent flow); the front of the current travels faster downslope, particularly at
383 larger times. As time goes, the current head decelerates. This flow deceleration is
384 caused by the decrease of the flow buoyancy and inertial along the slope due to (i) the
385 increase of the density of the ambient fluid along the slope and (ii) the decrease of the
386 flow velocity and density caused by the interfacial velocity shear generated mixing and

387 entrainment of flow fluid with ambient lighter fluid. The slowed current separates from
388 the slope and spreads into environment before it reaches the neutral level for $B_0=0.0072$
389 and $Re=267$. For larger buoyancy and more turbulent flow (e.g. $B_0=0.022$, $Re=290$; and
390 $B_0=0.0734$, $Re=839$), however, the current continues to flow down slope and slightly
391 overshoots the neutral level due to the flow inertia. The simulation demonstrates that
392 the distance of such overshooting increases with the increase of B_0 and Re (see Fig.4).
393 In general, the numerically simulated front position reasonably agrees with the
394 measured ones, particularly at the early stage. At larger times, the simulated distance
395 that the current head travels along the slope is slightly smaller than that of the
396 experimental measurements, indicating that the numerical model may slightly
397 overestimate the mixing which results in a slower motion of the flow.

398

399 *Vertical density profile*

400 The vertical density profiles within the gravity current are simulated for a range of flow
401 parameters, demonstrating similar interfacial shear generated vertical density
402 distribution within the current. Figure 5 is a typical example of the vertical density
403 profile (normal to the slope) at $s=0.7$ m for $B_0=0.022$ and $Re=290$ in which the depth
404 denotes the normal distance from the slope bottom. It is seen that a sharp density jump
405 takes place at about 0.01~0.015m from the slope bottom. This sharp density jump
406 interface divides flow into two parts: the upper turbulent mixing layer and the bottom
407 undisturbed/mixed or less disturbed/mixed current core whose density is almost the
408 same as that of the current at the inlet. This density interface almost coincides with the

409 velocity shear interface (see Fig. 8 below) where the mixing and entrainment of flow
410 fluid with ambient fluid takes place. Such mixing and entrainment, thereby, generates a
411 layer of weak density stratification/gradient or almost homogenous immediately above
412 the interface (see Fig.5). Above this well mixed layer, there is little motion and the
413 ambient stratification is almost not disturbed. The comparison shows that in general,
414 the simulated density distribution agrees well with the experimental measurements of
415 Mitsudera and Baines (1992).

416

417 ***Thickness of the current***

418 Figure 3c and d shows that when the current approaches the initially neutral level, the
419 front of current thickens and separates from the slope bed and then propagates
420 horizontally into environment. At this stage, the thickness of the gravity current along
421 the slope does not change significantly with time though some obvious spatial variation
422 exists along the slope. Relatively small thickness at the inlet is found while a thicker
423 gravity current takes place near the separation point (see Figure 3c and d and Figure 6a,
424 b). This may be ascribed to the fact that the ambient fluid entrained into the flow
425 increases the volumetric flux along the slope while the downslope flow velocity
426 decreases with the slope distance away from the source (see Figure 4 and 8), resulting
427 in the increase of the thickness of the flow along the slope. Figure 6 also demonstrates
428 that the thickness of the gravity current has a relation with the buoyancy number B_0 .
429 For relatively small B_0 (0.0072) and Re (267), the thickness of the current upstream of
430 the separation point is smaller; while the thickness is larger for relatively larger B_0

431 (0.0462) and Re (1121). To investigate the dependence of the current thickness on B_0 ,
432 Figure 7 plots the spatially averaged dimensionless thickness of the current (normalized
433 by the buoyancy depth D) versus B_0 in which the sharp density interface is used to
434 determine the boundary of the current (see Figure 5). The experimental results of
435 Baines' (2001) are included in Figure 7 for comparison. Though the data in Figure 7 is
436 somewhat scattered, it is seen that in general, both the simulated and measured
437 averaged thickness of the current increases with the increase of the flow buoyancy
438 number B_0 , which is consistent with Fig. 6. Figure 7 shows that the simulated spatially
439 averaged thickness of the current reasonably compares with the laboratory
440 measurements. For larger B_0 , however, the simulated thickness of the current is larger
441 than the measured ones, indicating that the numerical model overestimates the mixing,
442 which is consistent with the result of Figure 4.

443

444 ***Velocity profile***

445 The velocity field is simulated in the computational domain for a range of flow
446 parameters and compared with the available experimental data. Figure 8 is an example
447 to show the comparison of the simulated and measured (Mitsudera and Baines 1992)
448 vertical velocity profiles at four cross sections for $B_0=0.022$, $Re=290$ and the initial
449 current velocity of 0.029 m/s at the inlet. Figure 8 reveals that the similar velocity
450 profiles are found at the different distance from the inlet. Velocity profiles at various
451 positions demonstrate that the velocity increases sharply from zero on the slope bed
452 where the no-slip condition is applied and reaches the maximum value at about 0.01m.

453 The flow velocity then decreases sharply and reaches the minimal value at about the
454 height of 0.015m near the inlet ($s=0.1\text{m}$, Figure 8a) and of 0.02m away from the inlet
455 ($s=0.9\text{m}$, Figure 8e). Comparing velocity profiles at different distance from the source
456 shown in Fig. 8 reveals that the flow velocity decreases with the distance from the
457 source while the current thickness increases along slope (see also Fig. 6) due to the
458 mixing and entrainment of the flow with lighter ambient fluid. The velocity shear layer
459 in Figure 8 roughly coincides with the density interface shown in Figure 5. Figure 8
460 also shows that negative velocity appears near the free surface in both the simulation
461 and experiments. This may be caused by the confined geometry used in the experiments
462 and simulation where a reflection from the end wall takes place to respond the intrusion
463 of the current when it is released into the tank. This negative velocity may not exist in
464 the real situation in which the environment is sufficiently large to avoid any reflection.
465 However, the negative velocity is very small and has little effect on the motion of the
466 flow. In general, the simulated velocity profiles are in good agreement with the
467 experimental measurements, particularly at the distance close to the inlet. Some
468 discrepancy between the simulations and measurements exists at the position away
469 from the inlet, e.g. $s=0.7\text{m}$ and 0.9m . This discrepancy may be ascribed to the fact that
470 the present model is 2D which may not be able to capture the details of flow near the
471 neutral buoyancy level where stronger flow fluctuation takes place, indicating the 3D
472 flow features. To accurately simulate the details of the flow in this region, 3D
473 numerical model is required.

474

475 Numerical simulations are also performed for a range of flow parameters to evaluate
476 and investigate their effects on the maximum flow velocity along the slope. Figure 9
477 compares the simulated and measured (Mitsudera and Baines 1992) relative maximum
478 velocity (normalized by the initial flow velocity at the inlet) along the slope for various
479 inflow Re and B_0 . It is seen that when Re and B_0 are smaller ($Re=267$, $B_0=0.0072$; and
480 $Re=290$, $B_0=0.022$), the maximum velocity has a sharp increase near the inlet and
481 reaches the maximum value which is almost twice of the inlet flow velocity. This may
482 be ascribed to the decrease of the flow thickness in the region near the inlet (see also
483 Figure 6(a)) while the mass conservation makes the flow velocity increase. The
484 maximum velocity then gradually decreases with the distance from the source due to
485 the interfacial mixing and entrainment-induced decrease of buoyancy and the viscosity
486 loss. For larger Re and B_0 ($Re=839$, $B_0=0.0734$; and $Re=1121$, $B_0=0.0462$), however,
487 the situation is different. In these cases, the flow is more turbulent so that significant
488 mixing and entrainment between flow and ambient fluid takes place immediately as the
489 flow intrudes into the environment. As a result, the flow thickness increases with the
490 distance from the source (see Figure 6(b)), leading to the decrease of the flow velocity.
491 Figure 9 also demonstrates that the relative maximum velocity for smaller Re and B_0 is
492 larger than that for larger Re and B_0 flow as the latter generates stronger mixing and
493 entrainment along slope, thus slowing the current. In general, simulated relative
494 maximum velocity favorably compares with the experimental measurements.

495

496 **Conclusions**

497 Gravity currents are ubiquitous in both natural and man-made environments (e.g.
498 saltwater intrusion in estuaries; discharge of concentrated brine generated during
499 solution mining and desalination). The accurate prediction of the spreading and motion
500 is of importance from the point of view of protecting water quality in natural systems.
501 As such, details of velocity field and density distribution within the current are essential.
502 In this study, a multiphase model with $k-\omega$ turbulence model is applied to investigate
503 the gravity current descending a slope into linearly stratified environment. Velocity and
504 density fields are simulated for a wide range of flow parameters, including source
505 denser flow rate and density, initial buoyancy frequency of the ambient fluid. The
506 evolution of gravity current, head velocity, vertical velocity and density profiles are
507 simulated and compared with the available experimental measurements. The
508 simulations show that the flow characteristics can be described using a group of
509 dimensionless numbers, namely the flow Reynolds number and buoyancy number
510 defined by Baines (2001, 2005). Simulated results demonstrate that the ambient
511 stratification has significant effect on the gravity current: (1) the current head velocity
512 decreases along the slope in ambient stratification while in homogeneous environment,
513 the head velocity maintains roughly a constant value for slope angle $\theta \geq 0.5^\circ$ (Britter and
514 Linden 1980); (2) flow separation from the slope bed takes place when the current
515 approaches the initial neutral position. For smaller values of the flow Reynolds number
516 and buoyancy number, an initial acceleration of the flow near the source takes place,
517 which makes the maximum flow velocity being greater than the current velocity at the
518 inlet. The flow is then decelerated as the interfacial velocity shear generated mixing and

519 entrainment taking place along the slope. For larger flow Reynolds number and
520 buoyancy number, however, the flow is more turbulent and the shear-generated mixing
521 and entrainment occurs immediately as the flow intrudes into the environment. This
522 makes the maximum flow velocity at the region near the source being smaller than the
523 current velocity at the inlet. The shear-generated Kelvin–Helmholtz billows are seen to
524 appear at the top of the trailing fluid. Mixing and entrainment taking place at the
525 interface along the slope causes the increase of the current thickness with the distance
526 from the source. The simulations show that the spatially averaged thickness of the
527 current increases with the increase of the flow buoyancy number. Good agreement
528 between the numerical simulations and available laboratory measurements indicates
529 that the model can be applied to accurately simulate the spreading and motion of the
530 gravity current in complex environments. Some deviation between the simulated and
531 measured velocity takes place near the neutral buoyancy level where flow fluctuation is
532 strong. This discrepancy may suggest that the current 2D model may not be able to
533 capture the flow details near the neutral level and 3D numerical model will be required
534 for accurate simulation of the flow in this region. Comparison of the simulated and
535 measured velocity at large times indicates that the numerical model may overestimate
536 the mixing of flow with ambient fluid at that stage.

537

538 The flow simulated in this study has relatively low Reynolds number. For gravity
539 currents with the higher flow Reynolds number, the lobes and clefts may occur in the
540 front of the flow (Simpson 1997) and the current 2D model may not be able to capture

541 the details of such flow structures. In this case, 3D numerical models will be required to
542 run in order to capture these unstable events.

543

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549

550 **Notation**

551 *The following symbols are used in this paper:*

552 B_0 =buoyancy number of the inflow

553 D =vertical distance from the top of slope to the position where the density of ambient
554 fluid equals to the density of inflow

555 d =the thickness of gravity current along slope

556 d_0 =initial thickness of the inflow at the inlet

557 \vec{g} = vector of gravity acceleration

558 g'_0 =the reduced gravity acceleration

559 K_{pq} = the momentum exchange coefficient between phases

560 k = turbulent kinetic energy

561 k_{in} =turbulent kinetic energy at the inlet

562 N = buoyancy frequency of ambient fluid

- 563 P =pressure shared by phases
- 564 Q_0 =initial volumetric flux per unit slot width
- 565 Re =the Reynolds number
- 566 Ri_g = the local gradient Richardson number across the interface
- 567 s = slope distance from the top of slope
- 568 s^* =dimensionless slope distance from the top of slope
- 569 t =time
- 570 u_0 =initial flow velocity along the x direction at the inlet
- 571 u_s =velocity component along the slope
- 572 u_{sm} =the maximum velocity along the slope
- 573 \vec{v} =velocity vector
- 574 w =velocity in z - direction
- 575 x =horizontal coordinate
- 576 Z_0 =water depth in the tank
- 577 z =vertical coordinate
- 578 α_q =volume fraction of phase q
- 579 θ =slope angle
- 580 μ = dynamic viscosity of water
- 581 μ_t = turbulent (or eddy) viscosity
- 582 ν =kinetic viscosity of water
- 583 ρ_1 = water density at the free surface
- 584 ρ_2 =water density at the bottom of the tank

585 ρ_{in} = the initial density of inflow fluid
586 ρ_q = density of phase q
587 ρ_{top} = density of ambient fluid at the top of slope
588 $\bar{\rho}$ = mean density of the ρ_{top} and ρ_{in}
589 σ_k = turbulent Prandtl number for k
590 σ_ω = turbulent Prandtl number for ω
591 $\bar{\tau}_q$ = stress-strain tensor of phase q
592 ε = dissipation rate of the turbulent kinetic energy
593 ω = the specific dissipation rate
594 ω_{in} = specific dissipation rate of at the inlet

595

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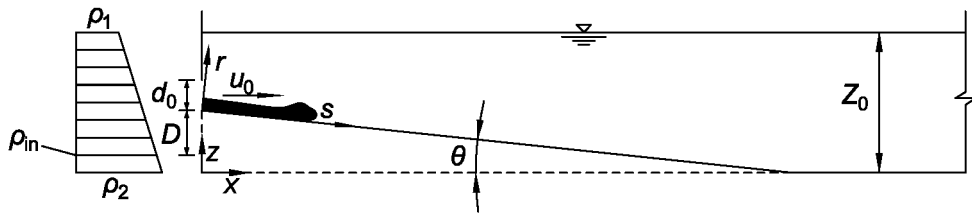


Figure 1. Sketch of the physical system under investigation

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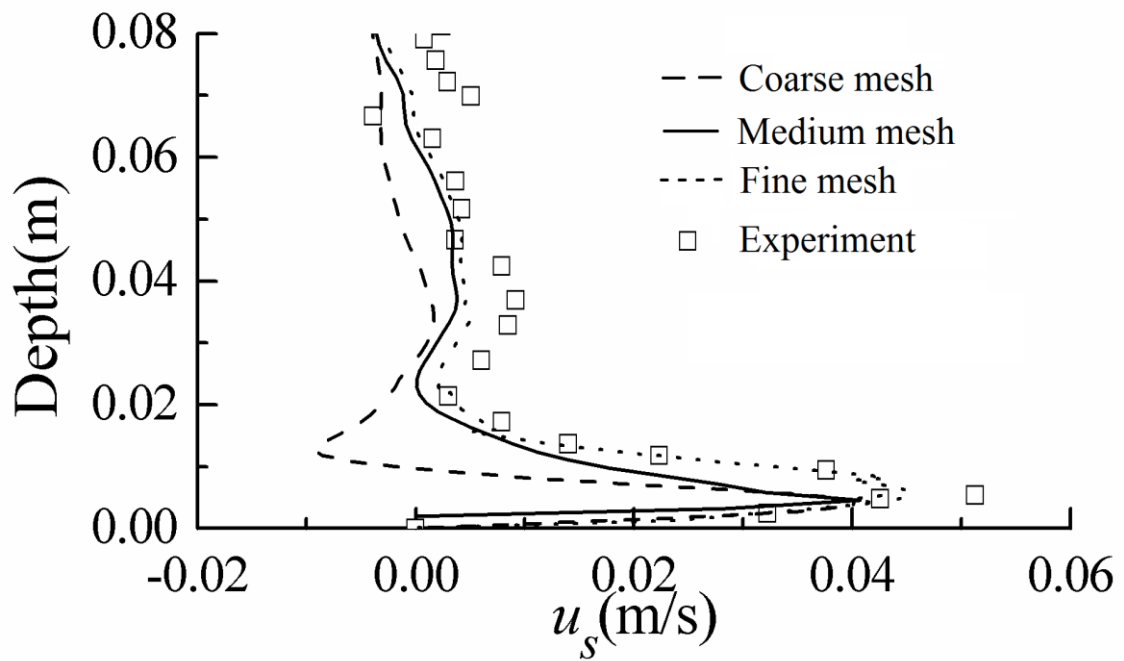


Figure 2. Mesh sensitivity study: effect of mesh size on the computational accuracy for gravity current descending a slope into a linearly stratified ambient, $B_0=0.022$ and $Re=290$

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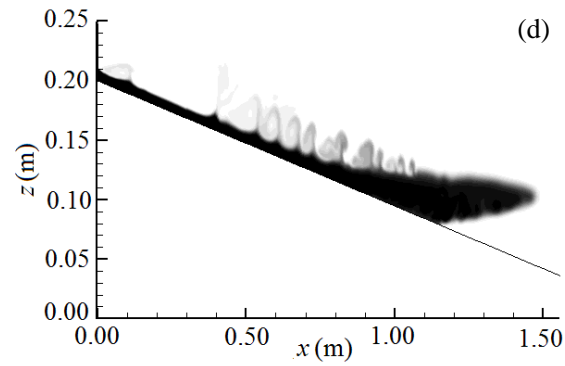
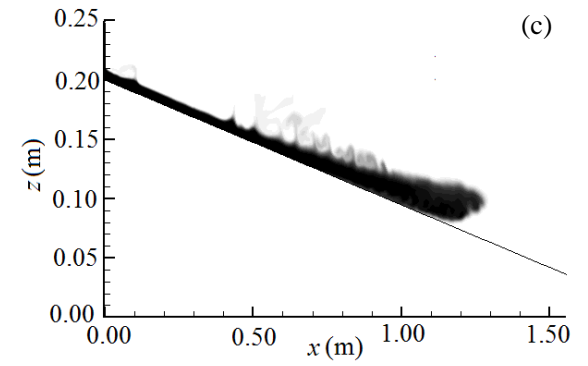
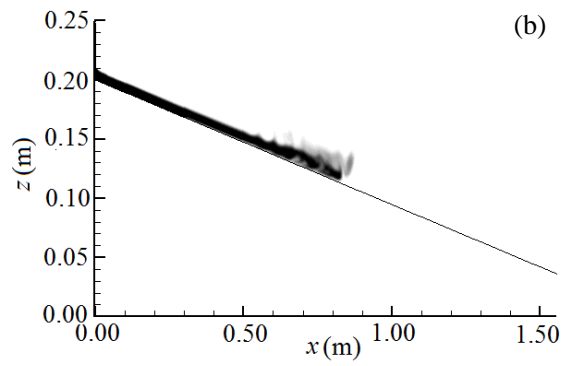
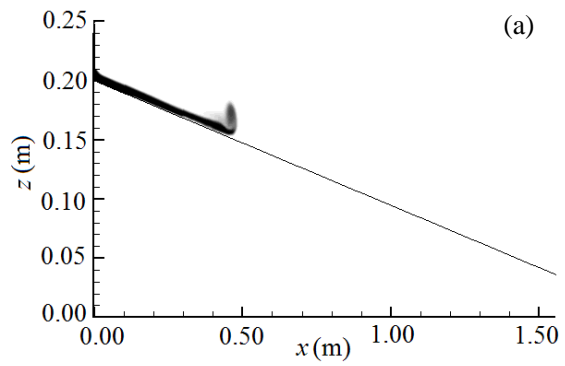


Figure 3. Evolution of the gravity current at various times for $B_0=0.022$ and $Re=290$. Note that for the sake of the clarity, the ambient density stratification is omitted. (a) $t= 12.5s$; (b) $t=27.5s$; (c) $t=70s$ and (d) $t=100s$.

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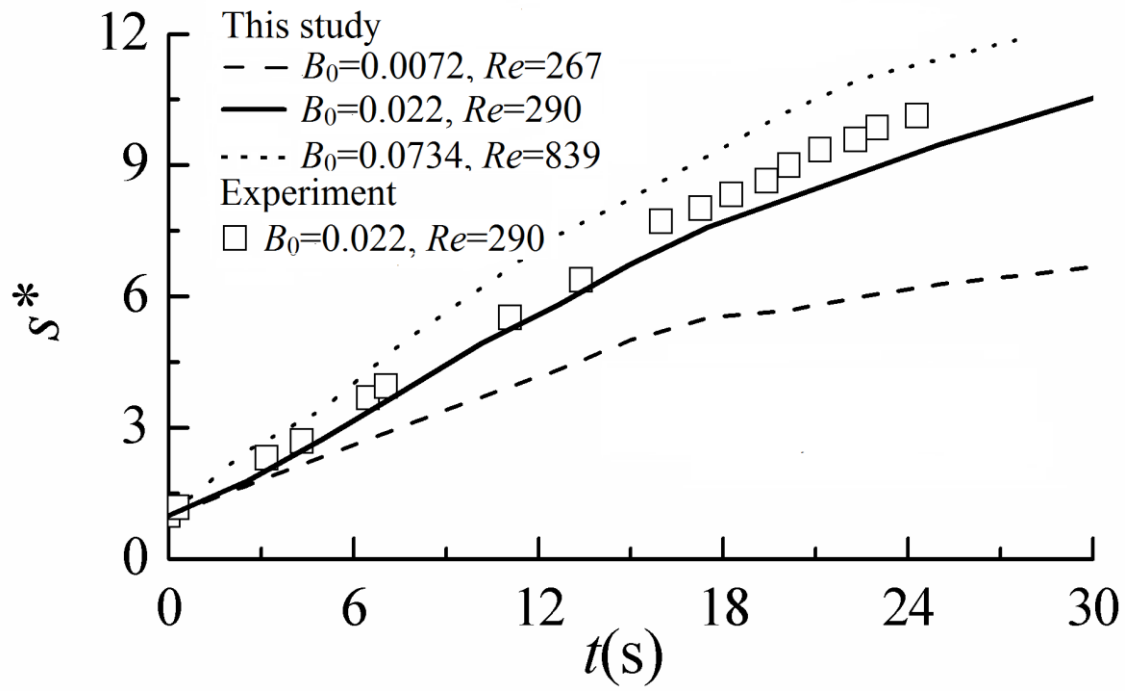


Figure 4. Dimensionless position of the current head versus time. Note that the time starts when the current reaches one D distance along the slope.

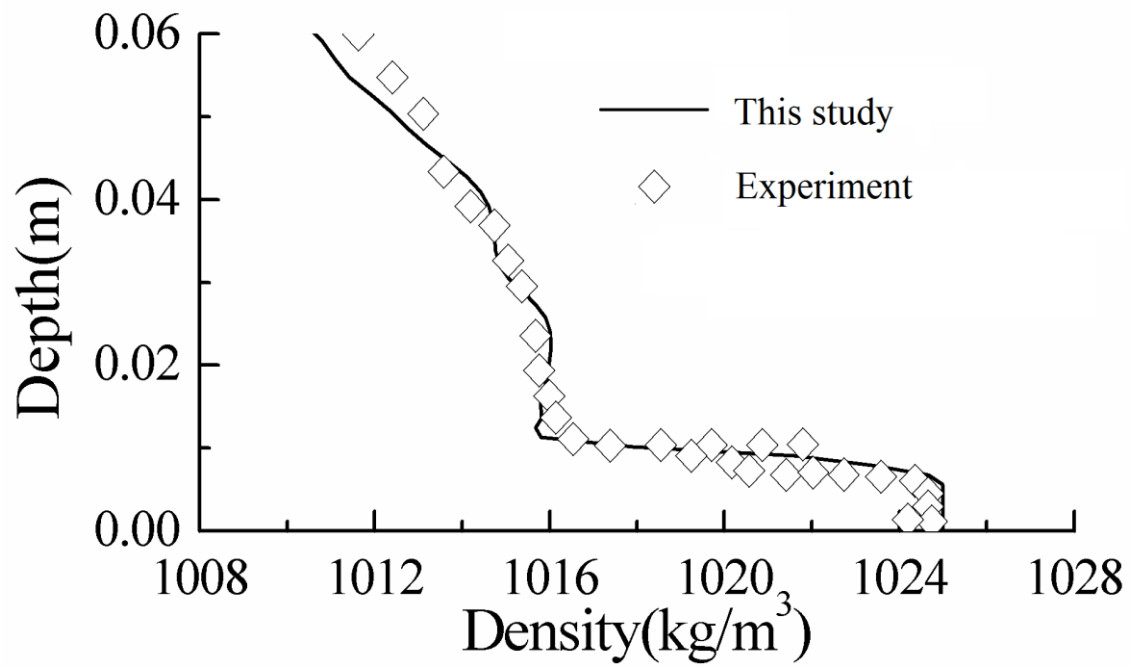


Figure 5. Vertical density profile at $s=0.7$ m for $B_0=0.022$ and $Re=290$.

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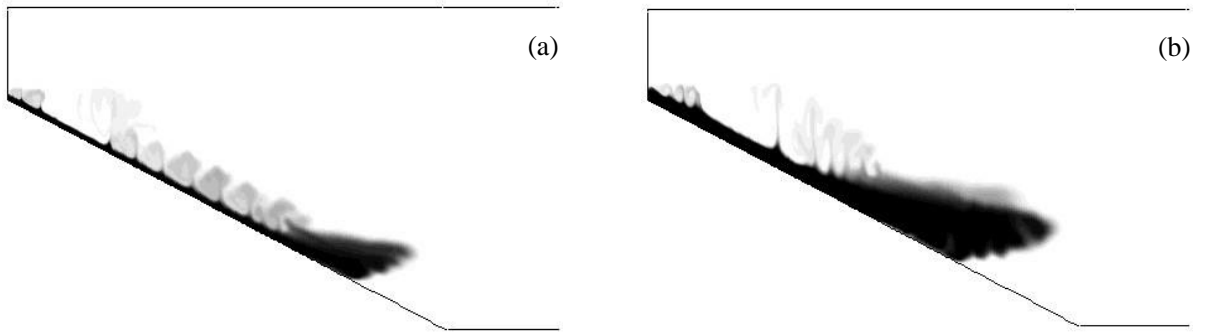


Figure 6. Typical shapes of gravity current near the inlet. Note that the vertical scale is five times of the horizontal scale. (a) $B_0=0.0072$, $Re=267$; (b) $B_0=0.0462$, $Re=1121$

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757
758

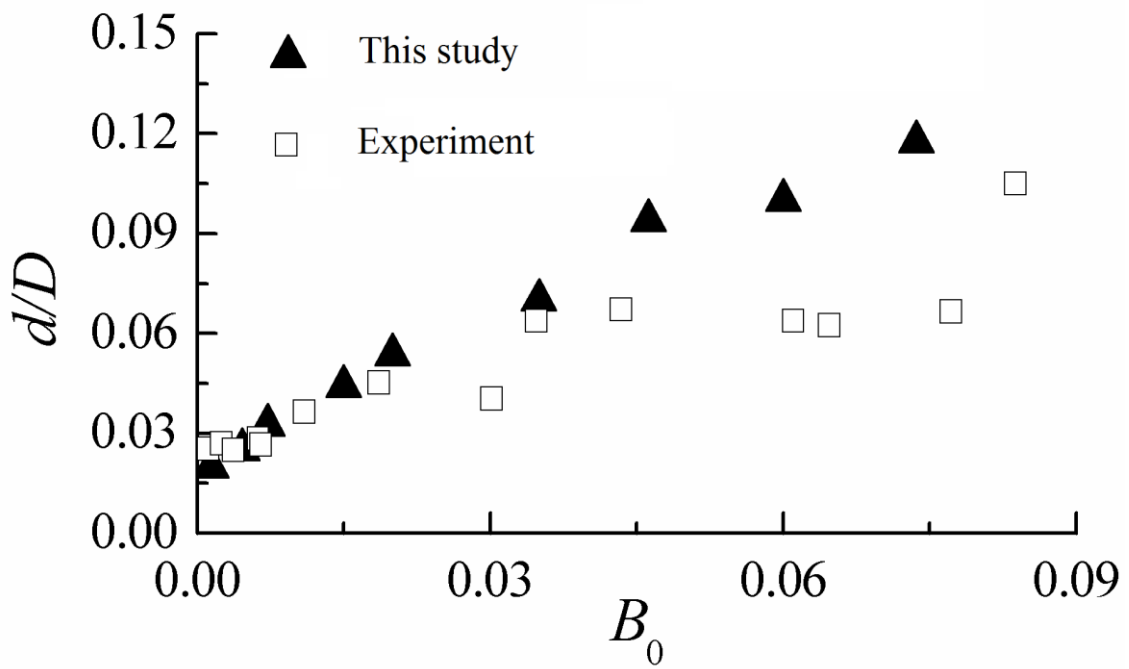


Figure 7. Comparison of the simulated and measured spatially averaged thickness of the gravity current versus B_0 .

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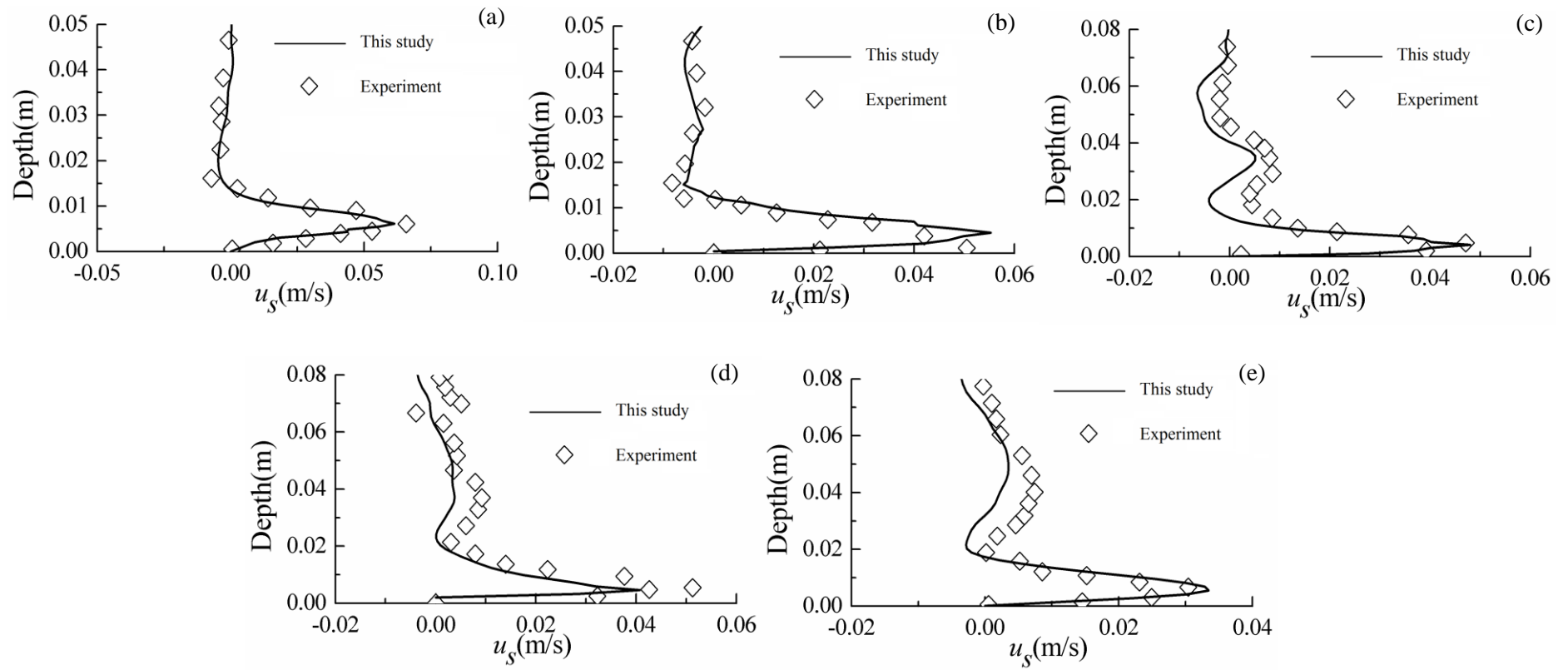


Figure 8. Comparisons of the simulated and measured (Mitsudera and Baines 1992) velocity profiles at different positions with $B_0=0.022$, $Re=290$. (a) $s=0.1\text{m}$; (b) $s=0.3\text{m}$; (c) $s=0.5\text{m}$; (d) $s=0.7\text{m}$; (e) $s=0.9\text{m}$.

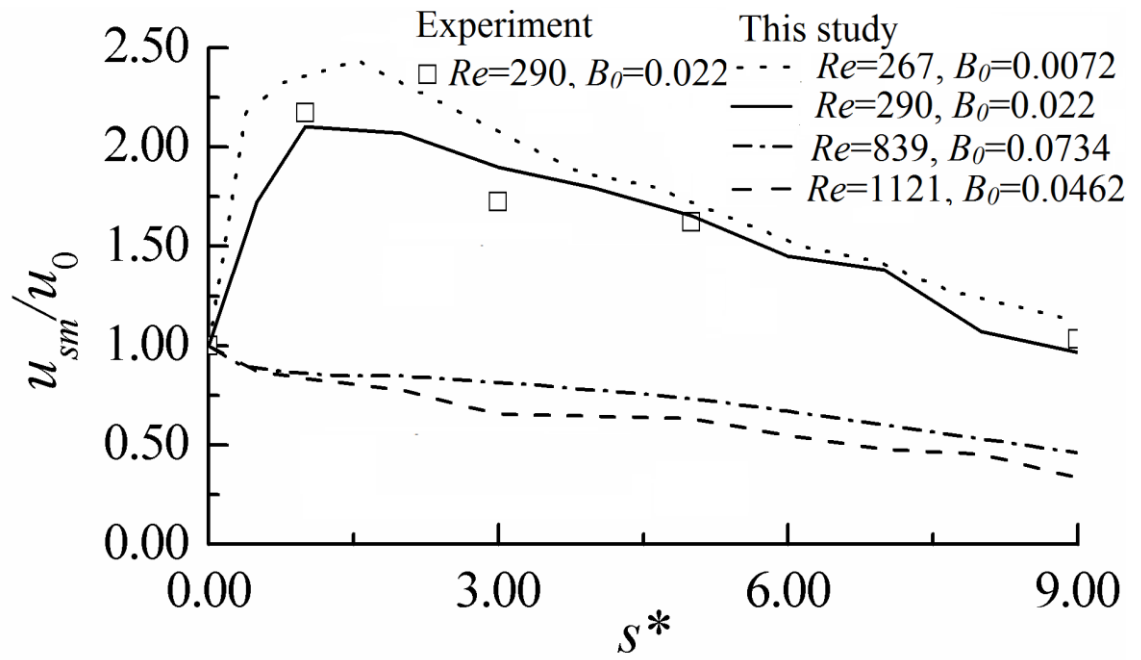


Figure 9. Comparison of the simulated and measured relative maximum velocity along the distance from the top of slope.

765