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Modification Factor for Shear Capacity of Lightweight Concrete Beams

Keun-Hyeok Yang[a], and Ashraf F. Ashour[b]

[a] *Corresponding author, Department of Plant·Architectural Engineering, Kyonggi University, San 94-6, Iui-dong, Yeongtong-gu, Suwon, Kyonggi-do, South Korea 443-760. Tel: 82 (0)31 249 9703; E-mail: yangkh@kyonggi.ac.kr*

[b] *EDT 1, School of Engineering, Design and Technology, University of Bradford, Bradford, BD7 1DP, UK.*

BIOGRAPHY: **K. H. Yang** is a professor at Kyonggi University, South Korea. He received the Scientific Technician Medal for Most Meritorious Achievement in 2010 from the National Research Foundation of Korea. His recent research interests include the development of sustainable concrete materials and structural technology.

A. F. Ashour is a reader in structural engineering at the University of Bradford, UK. He obtained his BSc and MSc degrees from Mansoura University, Egypt and his PhD from Cambridge University, UK. His research interests include shear, plasticity and optimization of reinforced concrete structures, fibre reinforced composites and sustainable construction materials.

ABSTRACT

The validity of the modification factor specified in the ACI 318-11 shear provision for concrete members to account for the reduced frictional properties along crack interfaces is examined using a comprehensive database comprising of 1716 normal weight concrete (NWC), 73 all-lightweight concrete (ALWC) and 54 sand-lightweight concrete (SLWC) beam specimens without shear reinforcement. Comparisons of measured and predicted shear capacities of concrete beams in the database show that ACI 318-11 provisions for shear transfer capacity of concrete are more un-conservative for lightweight concrete (LWC) beams than in NWC beams. A rational approach based on the upper-bound theorem of concrete plasticity has been developed to assess the reduced aggregate interlock along the crack interfaces and predict the shear transfer capacity of concrete. A simplified model for the modification factor is then proposed as a function of the compressive strength and dry density of concrete and maximum aggregate size on the basis of analytical parametric studies on the ratios of shear transfer capacity of LWC to that of the companion NWC. The proposed modification factor decreases with the decrease in the dry density of concrete, gives closer predictions to experimental results than that in the ACI 318-11 shear provision and, overall, improves the safety of shear capacity of LWC beams.

Keywords: modification factor, lightweight concrete, shear capacity, beam, ACI 318-11.

INTRODUCTION

Aggregate interlock along inclined cracks in concrete beams without shear reinforcement is generally recognized to transfer a considerable amount of shear.[1-3] Taylor[1] concluded that up to 50% of the applied shear force can be transferred by aggregate interlock, whereas Sherwood et al.[3] showed that the increase of maximum aggregate size from 9.5 mm (0.37 in.) to 21 mm (0.82 in.) resulted in a 24% increase of shear strength of one-way reinforced concrete slabs. Shear transfer owing to aggregate interlock along inclined cracks in concrete beams is greatly affected by the strength and maximum size of aggregates, as roughness of crack planes is significantly dependent on whether cracks penetrate through coarse aggregates.[4] Yang et al.[5, 6] demonstrated that the failure plane of normal-strength normal-weight concrete beams is formed through the paste around aggregate particles, whereas that of lightweight concrete (LWC) beams mainly penetrates through coarse aggregate particles. However, the increase of lightweight aggregate size produces a slightly rougher failure surface.[6] Therefore, shear transfer by aggregate interlock is expected to be lower in LWC beams than normal-weight concrete (NWC) beams.

ACI 318-11[7] recommends a modification factor to account for the reduced shear transfer contribution of LWC owing to the softened aggregate interlock at inclined crack interfaces. This modification factor was introduced by Ivey and Buth[8] based on a regression analysis of limited 26 LWC beam specimens. However, the accuracy and reliability of this modification factor remain controversial, and their application can be problematic because of a lack of mathematical consensus on shear transfer mechanism along crack interfaces in LWC elements. Yang et al.[5, 6] showed that the modification factor specified in ACI 318-11 is unconservative for the LWC continuous beams tested and the lack of conservatism increases as the maximum aggregate size increases. Therefore, a more rational analytical model for the modification factor would be welcomed to reasonably explain the reduced friction properties along crack interfaces of LWC beams.

The present study evaluates the safety of the shear design provisions of ACI 318-11 against a comprehensive database comprising of 1716 normal weight concrete (NWC), 73 all-lightweight concrete (ALWC) and 54 sand-lightweight concrete (SLWC) beam specimens without shear reinforcement. The validity of the modification factor is examined through the comparisons of various statistical parameters according to the type of concrete. The statistical parameters include the average, standard deviation, coefficient of variation, and 5% and 95% fractiles of the ratios of measured and predicted shear capacities of concrete beams. A mathematical approach to explain the shear transfer contribution of aggregate interlock along crack interfaces in concrete members is derived based on the upper-bound theorem of concrete plasticity. A simple model for the modification factor is then formulated by analytical parametric study of the ratio of shear transfer capacities of LWC to the companion NWC.

SIGNIFICANCE OF RESEARCH

This investigation assesses the modification factor of ACI 318-11 shear provision for concrete members without shear reinforcement using a comprehensive database of NWC, ALWC, and SLWC. On the basis of the plasticity theorem of concrete, it was found that the shear transfer contribution of concrete owing to aggregate interlock along crack interfaces mainly depends on the density of concrete and maximum aggregate size. Overall, the proposed modification factor improves the safety of the shear provision of ACI 318-11 for LWC beams, but has little influence on the accuracy of the shear provision, especially for deep beams.

BEAM SHEAR DESIGN IN ACI 318-11

ACI 318-11 differentiates between deep and slender beams according to the load transfer mechanism. The applied load in deep beams having shear span-to-overall depth ratio a/h below 2.0 is commonly transferred to supports through diagonal concrete struts. On the other hand, the main load transfer mechanism in slender beams is the beam action characterised by inclined cracks owing to the combination of shear and flexural stresses. Consequently, ACI 318-11 formulates shear transfer capacity of concrete beams without shear reinforcement as a function of the capacity and inclination of struts for deep beams, whereas by a function of concrete compressive strength, longitudinal tensile reinforcement ratio and shear span-to-depth ratio for slender beams. ACI 318-11 also includes a modification factor λ to account for the reduced frictional properties along inclined crack interfaces of lightweight concrete, proposed by Ivey and Buth[8]. Therefore, ACI 318-11 specifies the shear transfer contribution V_c of concrete in beams as follows:

$$V_c = \lambda \nu_c f_c^{'} b_w w_s \sin\theta \qquad \text{for } a/h \le 2.0 \tag{1.a}$$

$$V_c = \left(0.16\lambda\sqrt{f_c^{'}} + 17\rho_s \frac{V_u d}{M_u}\right) b_w d \le 0.29\lambda\sqrt{f_c^{'}} b_w d \qquad \text{for } a/h > 2.0 \tag{1.b}$$

where ν_c = effectiveness factor of concrete, which is given as 0.6 for concrete struts having no shear reinforcement, $f_c^{'}$ = concrete compressive strength [in MPa (1MPa=145 psi)], b_w and d = beam section width and effective depth, respectively [in mm (1 mm=0.039 in.)], w_s and θ = concrete strut width [in mm (1 mm=0.039 in.)] and inclination, respectively, ρ_s = longitudinal reinforcement ratio, and V_u and M_u = factored shear force [in N (1 N=0.2248 lb)] and moment [in N·mm (1 N·mm= 0.7375 lb·ft)], respectively, at the beam section considered. The modification factor λ may be obtained from the splitting tensile strength f_{sp} [in MPa (1MPa=145 psi)] of LWC using the following formula: $\lambda = f_{sp} / \left(0.56\sqrt{f_c^{'}}\right) \leq 1.0$. Alternatively, for beams where splitting tensile strength of concrete is not measured, ACI 318-11 recommends that λ =0.75 for ALWC, 0.85 for SLWC and 1.0 for NWC. A linear interpolation between 0.85 and 1.0 can be followed to obtain a more representative value of λ according to the volumetric fractions of lightweight fine aggregates for concrete containing normal-weight and lightweight aggregates.

The variation of the normalized shear transfer capacity $V_c / b_w h f_c^{'}$ calculated using Eq. (1) against a/h for NWC, ALWC, and SLWC is shown in Fig. 1. The figure also presents the details of beams considered. The width and inclination of concrete struts required by Eq. 1(a) were determined using the schematic procedure proposed by Yang and Ashour[9]. The common features of shear provision specified in ACI 318-11 can be summarized as follows: 1) the shear capacity of LWC beams is lower by the modification factor than that of the companion NWC beams, regardless of a/h; 2) the shear capacity of concrete beams decreases with the increase of a/h up to 2.0, beyond which it remains constant; and 3) there is an unrealistic sudden discontinuity of shear capacity at a/h of 2.0. Therefore, the modification factor λ plays a significant role in determining the shear transfer capacity of LWC in accordance with ACI 318-11 shear provision.

Database of concrete beams without shear reinforcement

A total of 1716 NWC, 73 ALWC and 54 SLWC beams without shear reinforcement were compiled from different sources. In the database, there are 1310 and 406 NWC slender and deep beams, respectively, 38 and 35 ALWC slender and deep beams, respectively, and 43 and 11 SLWC slender and deep beams, respectively. The proportion of ALWC and SLWC beam specimens (6.9%) is very small compared with that of NWC beam specimens, indicating that further experimental investigations may be required to reasonably evaluate the shear transfer capacity of lightweight concrete in beams. Test results originally collected by Collins et al.[10], Yang and Ashour[11] and Yang et al.[12] were the main sources of NWC beams in the database. On the other hand, test results obtained from Ivey and Buth[8], Clarke[13], Hanson[14], Kim and Park[15], Kim et al.[16], Park et al.[17], Thorenfeldt and Stemland[18], and Yang et al.[5, 6] were the main sources for lightweight concrete beams. All test specimens in the database were reported to have failed in shear. The distributions of main parameters are summarized in Table 1 for deep and slender beams of different concrete types. The dry density ρ of concrete varies between 1236 kg/m^3 (76.6 lb/ft^3) and 1735 kg/m^3 (107.6 lb/ft^3) for ALWC, and between 1700 kg/m^3 (105.4 lb/ft^3) and 2024 kg/m^3 (125.5 lb/ft^3) for SLWC. The NWC beam specimens had a relatively low $f_c^{'}$ of 6 MPa (0.87 ksi) and very high $f_c^{'}$ of 130 MPa (18.85 ksi). On the other hand, the compressive strength of LWC beam specimens ranged from 20 MPa (2.90 ksi) to 40 MPa (5.80 ksi). The longitudinal tensile reinforcement ratio varied between 0.005 and 0.035 for NWC beams, and between 0.01 and 0.03 for LWC beams. Deep beams are primarily tested at a/h between 1.0 and 2.0, whereas a/h of slender beams mostly ranged between 2.5 and 5.0 for NWC and between 2.5 and 3.0 for LWC. All of the collected deep beams were reported to be failed owing to crushing and sliding of concrete struts joining load and reaction points.

Comparisons between ACI 318-11 predictions and test results

The ratio between measured $(V_c)_{Exp.}$ shear transfer capacities of concrete beams without shear reinforcement and predictions $(V_c)_{ACI318-11}$ obtained from ACI 318-11 shear provisions would produce the code safety factor γ_{cs} as defined below:

$$\gamma_{cs} = (V_c)_{Exp.} / (V_c)_{ACI318-11} \tag{2}$$

Values of γ_{cs} below 1.0 indicate unconservative predictions of shear transfer capacity of concrete in beams. A statistical evaluation of the distribution of γ_{cs} is conducted to grasp the safety and accuracy of the ACI 318-11 shear design guidelines. The statistical parameters evaluated include the average $\gamma_{cs,m}$, standard deviation $\gamma_{cs,s}$, coefficient of variation $\gamma_{cs,v}$, and 5% $\gamma_{cs,5\%}$ and 95% $\gamma_{cs,95\%}$ fractiles of γ_{cs} as test specimens collected from various sources have different geometrical dimensions, material properties and test setup. The 5% and 95% fractiles are calculated from statistics as follows[19]:

$$\gamma_{cs,5\%} = \gamma_{cs,m} - K_0 \gamma_{cs,s} \quad (3.\ a)$$

$$\gamma_{cs,95\%} = \gamma_{cs,m} + K_0 \gamma_{cs,s} \quad (3.\ b)$$

The coefficient K_0 depends on the number n of test data used to compute the average and standard deviation of the selected samples. According to the statistical theory[19], K_0 may be assumed as 1.645, 2.010 and 2.685 for $n \geq 120$, $n = 40$, and $n = 10$, respectively; K_0 values for n between the above stated values may be calculated from linear interpolation.

The distributions of γ_{cs} are plotted against f_c' and a/h in Figs. 2 and 3, respectively. Statistical parameters are also calculated for subsets of the test specimens and presented in the same figures. In addition, different statistical parameters for all test specimens in the database are listed in Table 2 for slender and deep beams and also different concrete mixes. For NWC beams, the 5% fractile decreases with the increase of f_c', regardless of the beam type, and the lowest 5% fractile is observed in beams having $1.0 \leq a/h \leq 2.0$. This indicates that the concrete shear capacity provision specified in ACI 318-11 is unconservative even for NWC beams, showing a higher un-safety in deep beams than in slender beams. For slender beams with f_c' between 20 (2.90) and 50 MPa (7.25 ksi), the lowest 5% fractile is exhibited by SLWC beams. The average, 95% fractile and 5% fractile are generally lower for LWC beams than NWC beams, especially for deep beams as indicated in

Table 2. Therefore, the ACI 318-11 provisions for shear transfer capacity of concrete becomes more un-conservative in LWC beams, in particular for deep beams.

PROPOSED MODIFICATION FACTOR FROM PLASTICITY ANALYSIS

Angle of concrete friction

According to Coulomb frictional hypothesis[20], the shear capacity of concrete beams governed by concrete web failure along inclined cracks significantly depends on cohesion and internal friction along crack interfaces. Furthermore, deep beams commonly fail along the inclined cracks joining the loading and supporting points within the concrete strut, which is accompanied by crushing of the concrete strut and sliding along concrete web cracks. The coefficient of friction of concrete can be obtained from the sliding resistance of concrete interfaces subjected to pure shear stress. If concrete is regarded as a perfectly plastic material obeying a modified Coulomb failure criteria, the condition for sliding failure under pure shear stresses can be expressed as follows (See Fig. 4):

$$\frac{1}{2}(\sigma_1 - \sigma_3) = c \cdot \cos\varphi \tag{4}$$

where σ_1 and σ_3 = principal stresses, which equal to $\pm v$ in case of pure shear stresses v, and c = cohesion of concrete. The cohesion of concrete with sliding failure is expressed as $f_c^* / \left[2\left(\mu + \sqrt{1+\mu^2}\right)\right]$, where f_c^* = effective compressive strength of concrete, $\mu(= \tan\varphi)$ = coefficient of friction, and φ = angle of friction[20]. The state of stress at the web of concrete beams may be reasonably assumed to be under pure shear as normal stresses applied at the web are very small enough to be negligible. Hence, the shear transfer stress v_c of concrete along the crack interface occurred at the beam web can be expressed as follows:

$$v_c = \frac{f_c^* \cos\varphi}{\left[2\left(\mu + \sqrt{1+\mu^2}\right)\right]} = \frac{f_c^* \cos\varphi}{\left[2\left(\tan\varphi + \sqrt{1+\tan^2\varphi}\right)\right]} \tag{5}$$

Based on the upper-bound theorem of concrete plasticity, Yang et al.[21] showed that the shear transfer stress v_c of concrete interface without shear reinforcement can be written in the following form:

$$v_c = \frac{1}{2} f_c^* \frac{1}{\cos\alpha}[l - m\sin\alpha] \tag{6}$$

where $l = 1 - 2\frac{f_t^*}{f_c^*}\frac{\sin\varphi}{1-\sin\varphi}$, $m = 1 - 2\frac{f_t^*}{f_c^*}\frac{1}{1-\sin\varphi}$, $\alpha = 2\tan^{-1}\left(\frac{x}{\sqrt{x^2+y^2}+y}\right)$, $x = m/l$, $y = \sqrt{l^2 - m^2}/l$, and f_t^* = effective tensile strength of concrete.

The effective strength of concrete can be determined from equating the area of the rigid-perfectly plastic stress-strain curve to that of the actual stress-strain curve. Yang et al.[21] derived a rational approach based on a numerical analysis to calculate the effectiveness factors in compression and tension of concrete using the stress-strain relationships. In this approach, the basic equations generalized by Yang et al.[22] and Hordijk[23] were, respectively, modified for compressive and tensile stress-strain relationships of concrete to account for the effect of dry density of concrete on the slopes of ascending and descending branches of the stress-strain curves. The primarily influencing parameters on the compressive and tensile effectiveness factors were found to be dependent on f_c' and ρ as well as maximum aggregate size d_a. Using the approach established by Yang et al.[21], a parametric study was carried out to generalize the effective strength ratio f_t^* / f_c^* required for the estimation of the two parameters l and m required by Eq. (6) in the ranges of f_c' between 20 (2.90) and 100 MPa (14.50 ksi), ρ between 1200 (74.4) and 2200 kg/m^3 (136.4 lb/ft^3), and d_a between 4 (0.16) and 40 mm (1.56 in.). To establish a simple model for f_t^* / f_c^*, each variable investigated (f_t', ρ , and d_a) was combined and tuned repeatedly by trial-and-error approach until a relatively acceptable correlation coefficient (R^2) was obtained. From a nonlinear multiple regression (NLMR) analysis of the mathematical results, f_t^* / f_c^* ratio can be obtained as below (see Fig. 5):

$$\frac{f_t^*}{f_c^*} = 0.03\left[\frac{f_c'(c_0 / d_a)}{f_{co} \times \eta_E{}^2}\right]^{-0.38} \tag{7}$$

$$\eta_E = (\rho / \rho_0)^2 \leq 1.0 \tag{8}$$

where f_{co} [= 10 MPa (1.45 ksi)]= reference concrete strength, ρ_0= dry density of normal weight concrete which can be generally assumed to be 2200 kg/m^3 (136.4 lb/ft^3)[24], and c_0 [= 25 mm (0.98 in.)] = reference size of aggregate.

Once f_t^* / f_c^* is calculated for given values of f_c', ρ, and d_a in a concrete beam, by equating Eq. (5) and Eq. (6) at sliding failure of concrete interface, the angle of friction φ can be obtained as below:

$$\frac{\cos\varphi}{\left(\tan\varphi + \sqrt{1+\tan^2\varphi}\right)} = \frac{l - m\sin\alpha}{\cos\alpha} \tag{9}$$

As mentioned in Eq. (6), the parameters l, m, and α are functions of f_t^* / f_c^* and φ. This indicates that φ in Eq. (9) is a function of only f_t^* / f_c^*; consequently, it can be numerically determined for each value of f_t^* / f_c^*. Linear regression analysis of the numerically determined φ against f_t^* / f_c^* produced the following simple formula (see Fig. 6):

$$\varphi = 22.9\left(f_t^* / f_c^*\right)^{-0.185} \tag{10}$$

Nielsen[20] assumed that concrete modelled as a modified Coulomb material has a constant value of φ as 37°, regardless of f_c', whereas Kahraman and Altindag[25] showed that φ commonly increases with the increase of the material brittleness. Equation (10) also indicates that φ varies according to the ratios of effective tensile and compressive strengths of concrete; i.e. φ slightly increases with the decrease in f_t^* / f_c^*.

Proposed equation for the modification factor

Equation (1) indicates that the modification factor λ is the ratio of shear transfer capacity of LWC to that of the companion NWC. Therefore, the modification factor λ to account for the reduced frictional properties along crack interfaces can be written in the following form:

$$\lambda = \frac{(v_c)_{LWC}}{(v_c)_{NWC}} \tag{11}$$

To determine λ using Eqs. (5) and (11), numerical parametric study was carried out for normal weight concrete having $f_c^{'}$ between 20 (2.90 ksi) and 80 MPa (11.60 ksi) and d_a between 4 (0.16) and 40 mm (1.56 in.), and lightweight concrete of the same condition but ρ between 1200 (74.4) and 2000 kg/m^3 (124.0 lb/ft^3). The results obtained from the parametric study were calibrated using a nonlinear multiple regression analysis, as plotted in Fig. 7, and thereby a new modification factor is finally proposed below:

$$\lambda = 0.82Ln\left[\left(\frac{\rho}{\rho_0}\right)^3 + \left(\frac{f_0}{f_c^{'}}\right)^{0.05}\left(\frac{d_a}{c_0}\right)^{0.05}\right] + 0.5 \le 1.0 \tag{12}$$

Figure 8 shows the comparison of the modification factors calculated by the ACI 318-11 provision and present study for the continuous lightweight concrete beams[5, 6]. The modification factor specified in ACI 318-11 provision is intermittently different from the test results. On the other hand, the proposed Eq. (12) gives more conservative values than ACI 318-11 provision. Furthermore, the general trend between ρ and λ is consistently reflected in Eq. (12), indicating that λ decreases with the decrease in ρ. Overall, the proposed modification factor is more rational than the empirical values specified in ACI 318-11 in explaining the reduced frictional properties along the inclined crack interfaces.

Shear capacity prediction of LWC beams according to proposed modification factor

To examine the accuracy of the proposed modification factor, the shear capacity of LWC beams calculated using Eq. (1) combined with Eq. (12) is compared with that of beams in the database.

The distribution of the ratio between $(V_c)_{Exp}/(V_c)_{ACI\,318\text{-}11}$ for LWC beams with the new proposed modification factor is presented against the variation of a/h in Fig. 9. The variation of statistical parameters for the proposed modification factor is also shown in Fig. 10 for ALWC and SLWC beams. When the proposed modification factor for LWC slender beams is used, most of statistical parameters are practically improved, resulting in approximately 15% increase in the 5% fractile compared with that using the original modification factor specified in ACI 318-11 (compare Figs. 3 and 9). This increasing rate of 5% fractile is more notable for ALWC beams with a/h between 3.0 and 4.5 and for SLWC beams with a/h between 2.0 and 3.0, as shown in Fig. 9. For deep beams using the proposed modification factor, the average and 95% fractile values increase from 0.75 to 1.0 and from 1.35 to 1.84, respectively, for ALWC, and 0.66 to 0.83 and 1.31 to 1.56, respectively, for SLWC, whereas the 5% fractile is practically similar, compared with these using the modification factor of ACI 318-11. Overall, it can be concluded that the proposed modification factor improves the safety of shear provision for LWC beams, with the 15% increase in the 5% fractile for slender beams and 19 to 37% increase in the average and 95% fractile for deep beams.

CONCLUSIONS

The safety of shear provisions specified in ACI 318-11 is examined using a comprehensive database comprising 1716 normal weight concrete (NWC), 73 all-lightweight concrete (ALWC), and 54 sand-lightweight concrete (SLWC) beam specimens. Based on the upper-bound theorem of concrete plasticity, a simple equation for the modification factor is proposed as a function of the compressive strength and dry density of concrete and maximum aggregate size. Different statistical parameters for the safety factor of ACI 318-11 shear provision are compared according to the proposed and original modification factors. Based on the analytical solution and comparisons with the database, the following conclusions may be drawn:

1. For NWC beams, the 5% fractile decreases with the increase of concrete compressive strength, regardless of the beam type, and the lowest 5% fractile is observed in beams with shear span-to-depth ratios between 1.0 and 2.0.
2. The average, 95% fractile and 5% fractile for all the lightweight concrete (LWC) beam specimens are lower than those for NWC beams, in particular for deep beams. Overall, ACI 318-11 provisions for shear transfer capacity of concrete become more un-conservative in LWC beams than in NWC beams due to the overestimation of the modification factor.
3. The proposed modification factor improves the safety of shear provision for LWC beams, for example, an increase of 15% in the 5% fractile for slender beams and 19 to 37% increase in the average and 95% fractile for deep beams.

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NOTATION

a = shear span of beam

b_w = width of beam section

c = cohesion of concrete

c_0 = reference aggregate size [= 25 mm (0.98 in.)]

d = effective depth of beam section

d_a = maximum size of aggregate

f_c' = concrete compressive strength

f_0 = reference concrete compressive strength [= 10 MPa (1.45 ksi)]

f_c^* = effective compressive strength of concrete

f_t^* = effective tensile strength of concrete

f_{sp} = splitting tensile strength of concrete

h = ovrall depth of beam section

M_u = factored moment

n = number of test data

V_c = shear capacity of beam without shear reinforcement

V_u = factored shear force

v_c = shear transfer stress of concrete

w_s = width of concrete strut of deep beam

γ_{cs} = ratio of measured shear capacity of beam and prediction obtained from ACI 318-11

$\gamma_{cs,m}$ = average of γ_{cs}

$\gamma_{cs,s}$ = standard deviation of γ_{cs}

$\gamma_{cs,v}$ = coefficient of variation of γ_{cs}

$\gamma_{cs,5\%}$ = 5% fractile of γ_{cs}

$\gamma_{cs,95\%}$ = 95% fractile of γ_{cs}

θ = inclination of concrete strut of deep beam

λ = modification factor

ν_c = effectiveness factor of concrete

ρ = dry density of concrete

ρ_0 = reference dry density of concrete [= 2200 kg/m^3 (136.4 lb/ft^3)]

ρ_s = longitudinal reinforcement ratio of beams

φ = friction angle of concrete

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Table 1-Distribution of different parameters in the 1391 slender and 452 deep beams without shear reinforcement. (1 mm =0.039 in.; 1 MPa=145 psi)

b_w mm	Range		≤ 50	50-75	75-100	100-125	125-150	150-200	200-250	250-300	≥ 300		Total
	Slender beams	NWC	45	17	6	129	95	553	143	49	273		1310
		ALWC	-	-	-	1	2	34	1	-	-		38
		SLWC	-	-	-	21	11	11	-	-	-		43
	Deep beams	NWC	-	1	24	34	33	220	36	22	36		406
		ALWC	-	-	-	-	4	31	-	-	-		35
		SLWC	-	-	-	-	5	6	-	-	-		11
h mm	Range		≤ 100	100-200	200-300	300-400	400-550	550-700	700-900	900-1100	≥ 1100		
	Slender beams	NWC	24	183	358	411	187	32	14	72	29		1310
		ALWC	-	-	3	34	1	-	-	-	-		38
		SLWC	-	-	37	6	-	-	-	-	-		43
	Deep beams	NWC	-	14	48	146	99	59	15	25	-		406
		ALWC	-	-	4	31	-	-	-	-	-		35
		SLWC	-	-	9	2	-	-	-	-	-		11
$f_c^{'}$ MPa	Range		≤ 20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	≥ 100	
	Slender beams	NWC	117	397	319	117	108	82	58	48	49	15	1310
		ALWC	2	16	18	1	-	-	-	-	1	-	38
		SLWC	-	7	12	8	14	2	-	-	-	-	43
	Deep beams	NWC	63	166	86	27	14	24	18	5	1	2	406
		ALWC	-	13	14	3	2	-	1	2	-	-	35
		SLWC	-	-	6	1	4	-	-	-	-	-	11
ρ_s (%)	Range		≤ 0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-3.5	3.5-4.0	4.0-5.0	≥ 5.0	
	Slender beams	NWC	25	185	164	398	139	161	149	27	34	28	1310
		ALWC	-	3	23	-	4	8	-	-	-	-	38
		SLWC	-	5	4	24	4	6	-	-	-	-	43
	Deep beams	NWC	19	79	51	90	38	43	43	17	16	10	406
		ALWC	-	3	8	-	3	16	-	-	-	5	35
		SLWC	-	3	1	4	1	-	2	-	-	-	11
a/h	Range		0.2-0.5	0.5-1.0	1.0-1.5	1.5-2.0	2.0-2.5	2.5-3.0	3.0-4.0	4.0-5.0	5.0-6.0	≥ 6.0	
	NWC		34	92	166	114	387	460	324	89	34	16	1716
	ALWC		-	2	-	33	-	15	-	22	-	-	73
	SLWC		-	-	6	5	5	29	9	-	-	-	54

Table 2-Statistical parameters for code safety factor for NWC, ALWC and SLWC beams.

Quantity	NWC		ALWC		SLWC	
	Slender beams	Deep beams	Slender beams	Deep beams	Slender beams	Deep beams
Average ($\gamma_{cs,m}$)	1.33	0.97	1.11	0.74	1.22	0.66
Standard deviation ($\gamma_{cs,s}$)	0.42	0.46	0.23	0.28	0.32	0.25
Coefficient of variation ($\gamma_{cs,v}$)	0.31	0.47	0.21	0.37	0.26	0.38
95% fractile ($\gamma_{cs,95\%}$)	2.01	1.72	1.57	1.32	1.85	1.31
5% fractile ($\gamma_{cs,5\%}$)	0.64	0.21	0.64	0.16	0.58	0.01

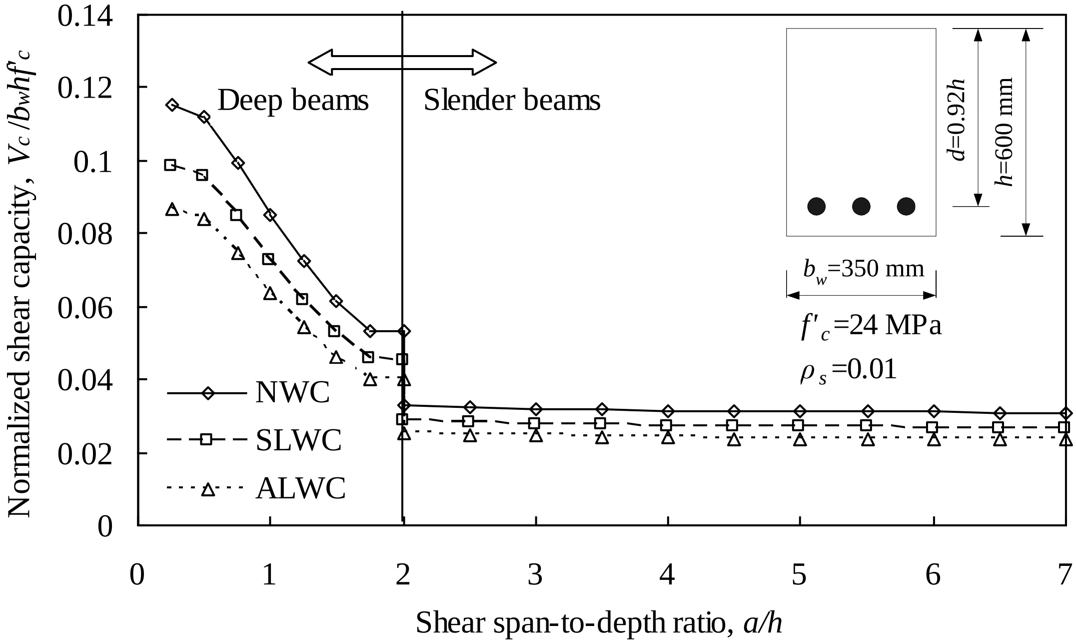
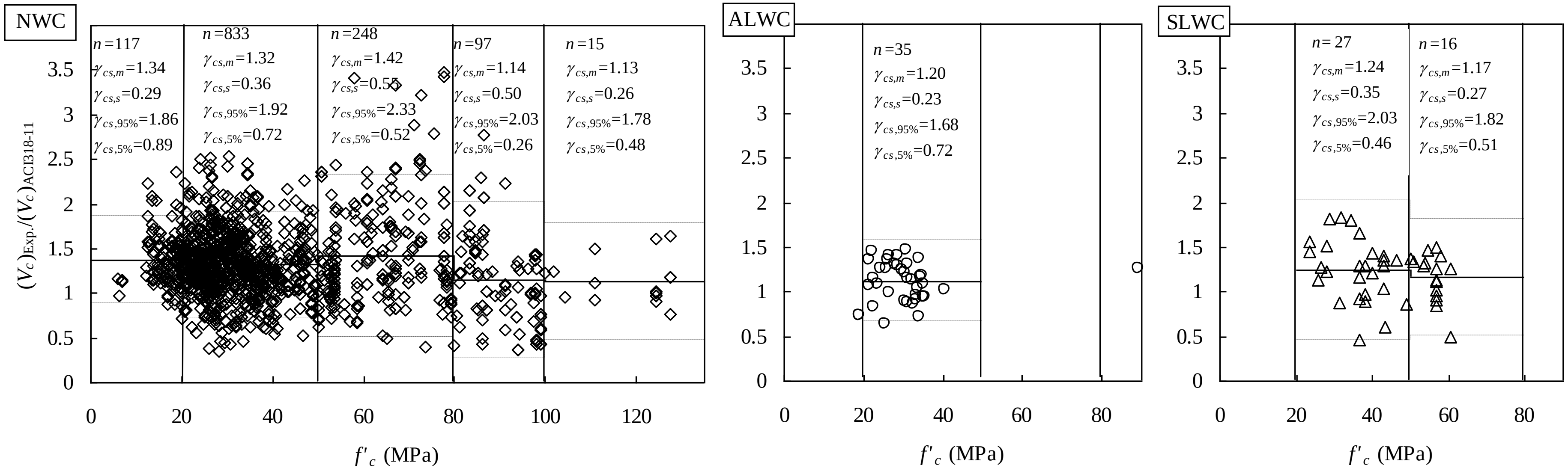
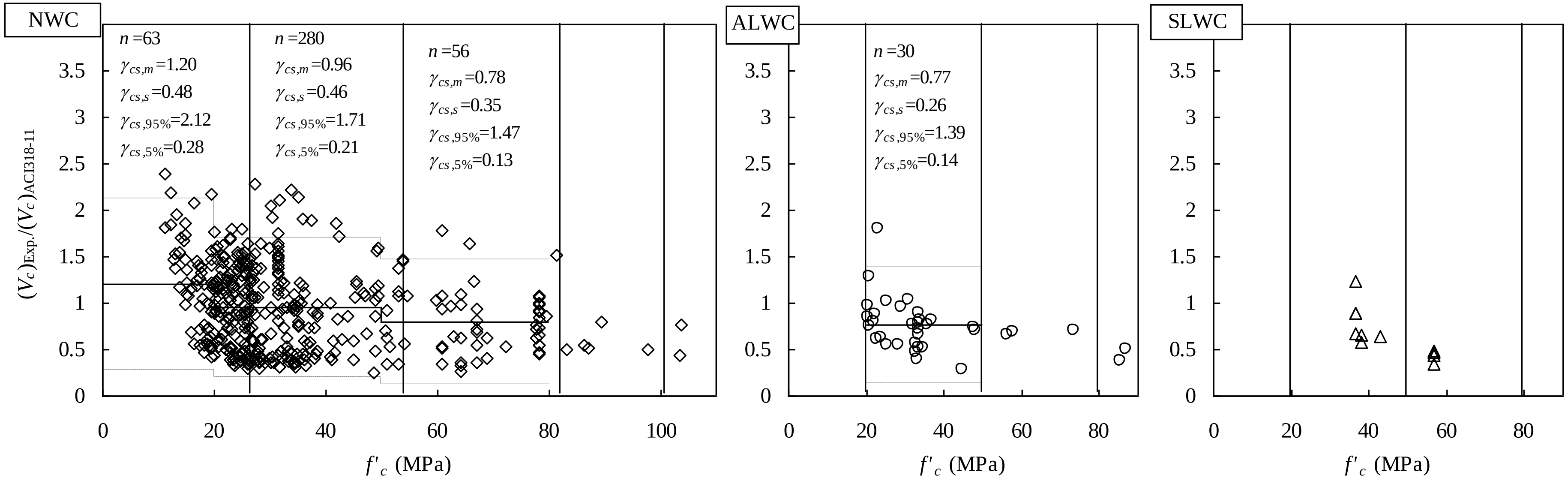


Fig. 1-Shear capacity of concrete beams calculated using ACI 318-11 equations.

(1 mm =0.039 in.; 1 MPa=145 psi)



(a) Slender beams



(b) Deep beams

Fig. 2-Variation of the code safety factor according to concrete compressive strength.

(1 MPa=145 psi)

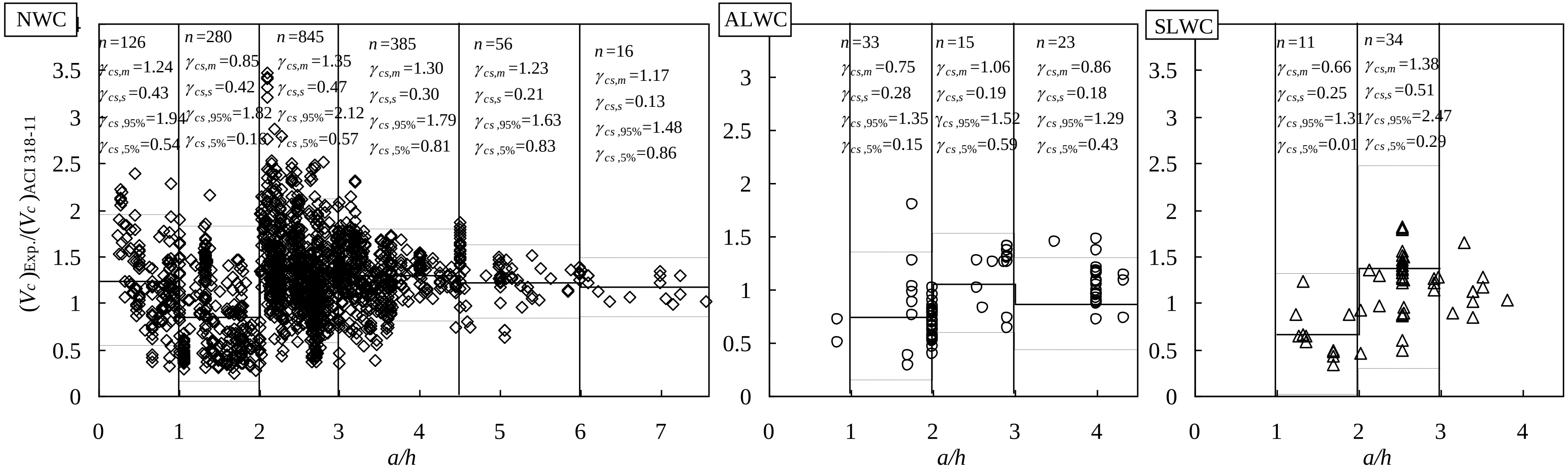


Fig. 3-Code safety factor according to the shear span-to-overall depth ratio of beam.

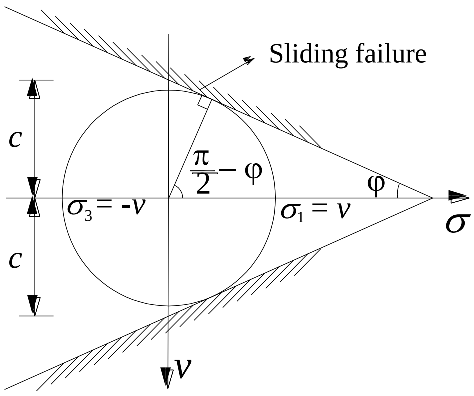


Fig. 4-Mohr's circle for sliding failure under pure shear.

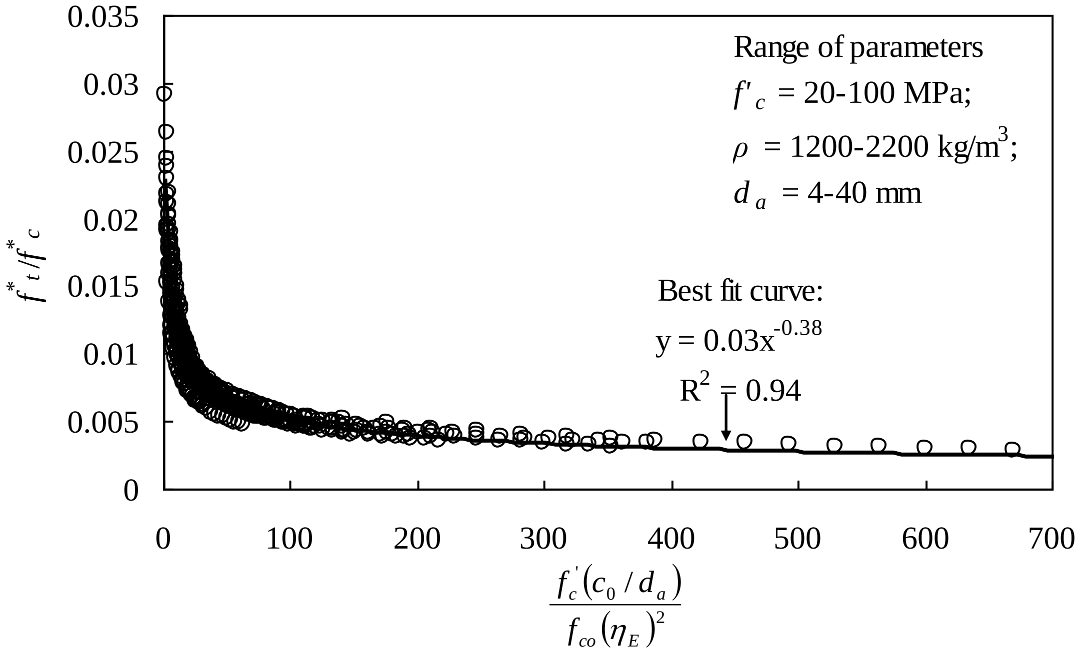


Fig. 5-Regression analysis for f_t^* / f_c^*.

(1 MPa=145 psi; 1 mm =0.039 in.; 1 kg/m^3=0.062 lb/ft^3)

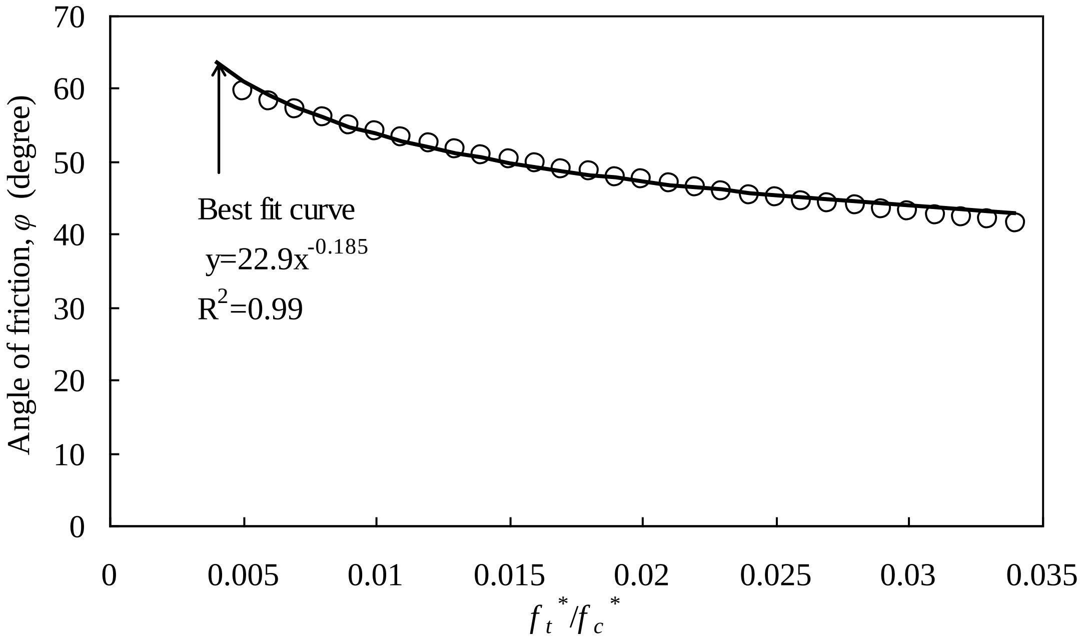


Fig. 6-Relationship of f_t^* / f_c^* and φ.

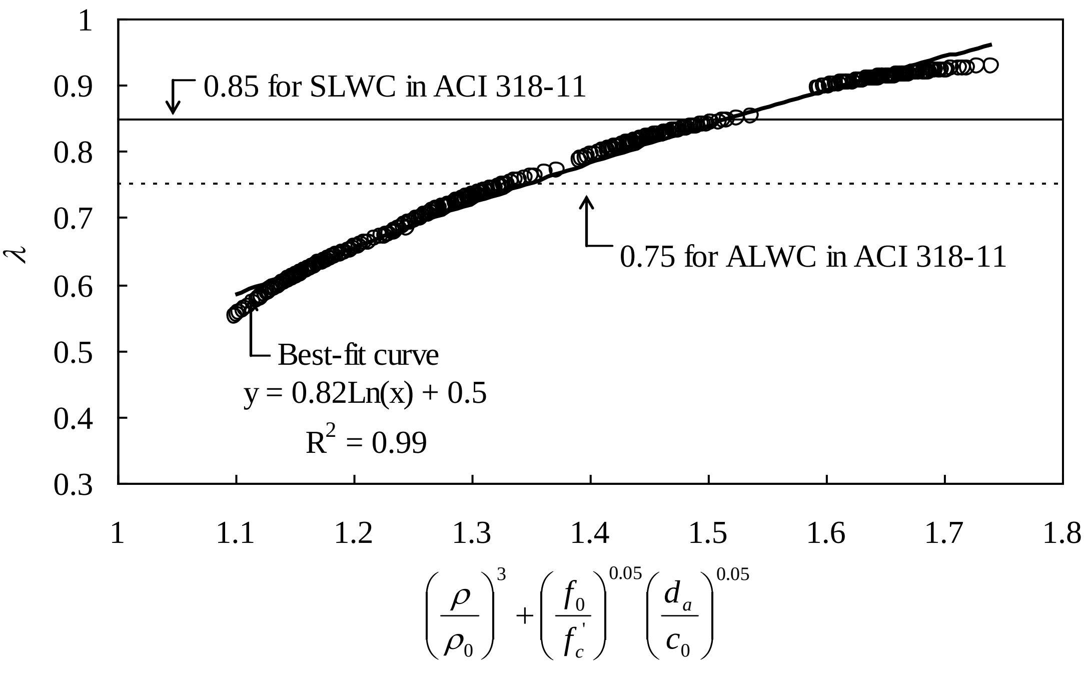


Fig. 7-Proposed modification factor for lightweight concrete.

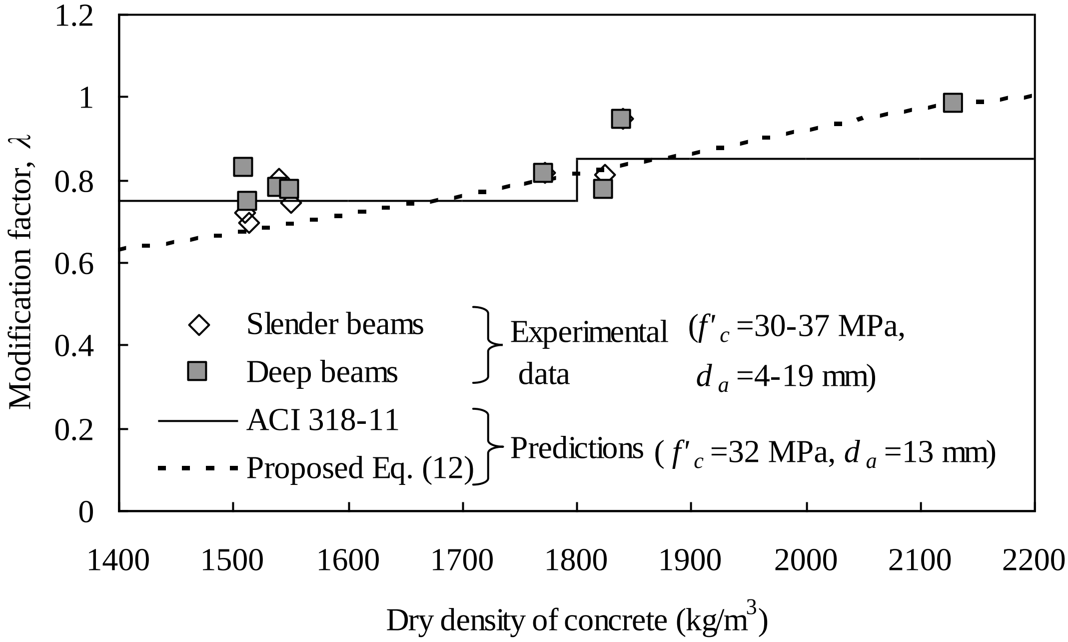


Fig. 8-Comparison of the measured modification factor and predictions from the current investigation and ACI 318-11.

(1 MPa=145 psi; 1 mm =0.039 in.; 1 kg/m^3=0.062 lb/ft^3)

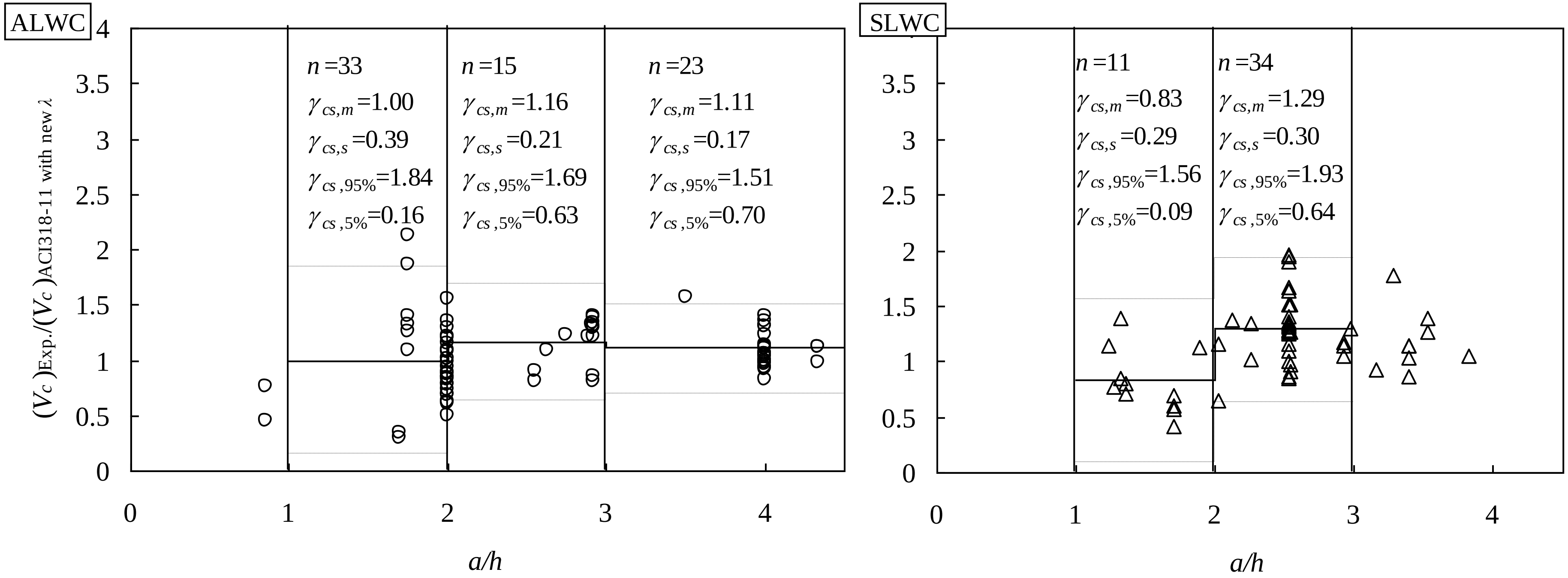
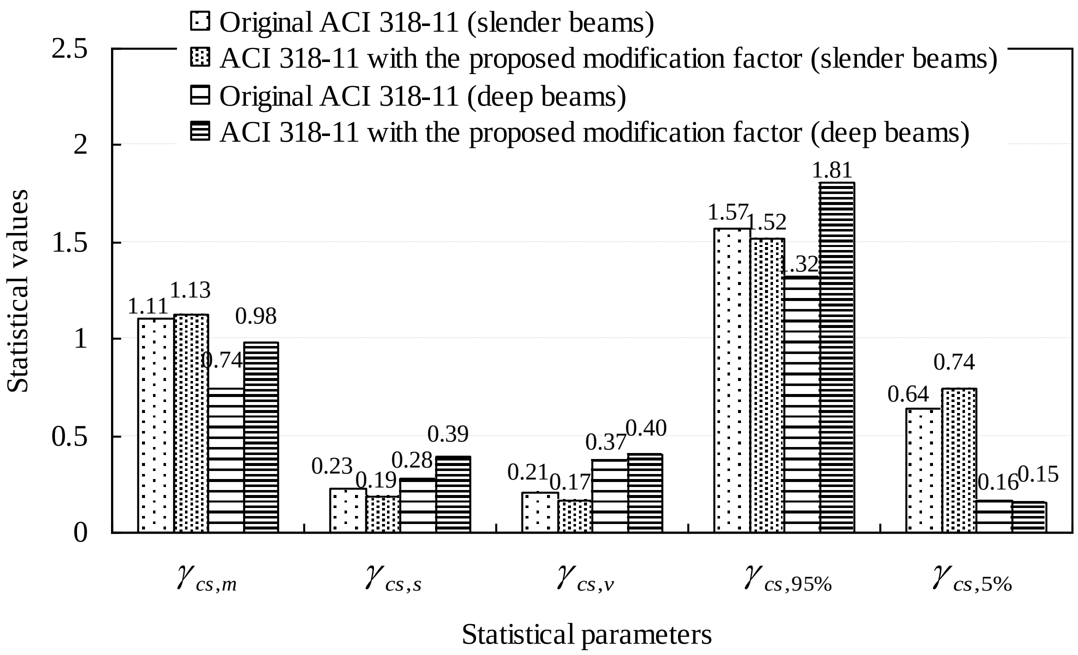
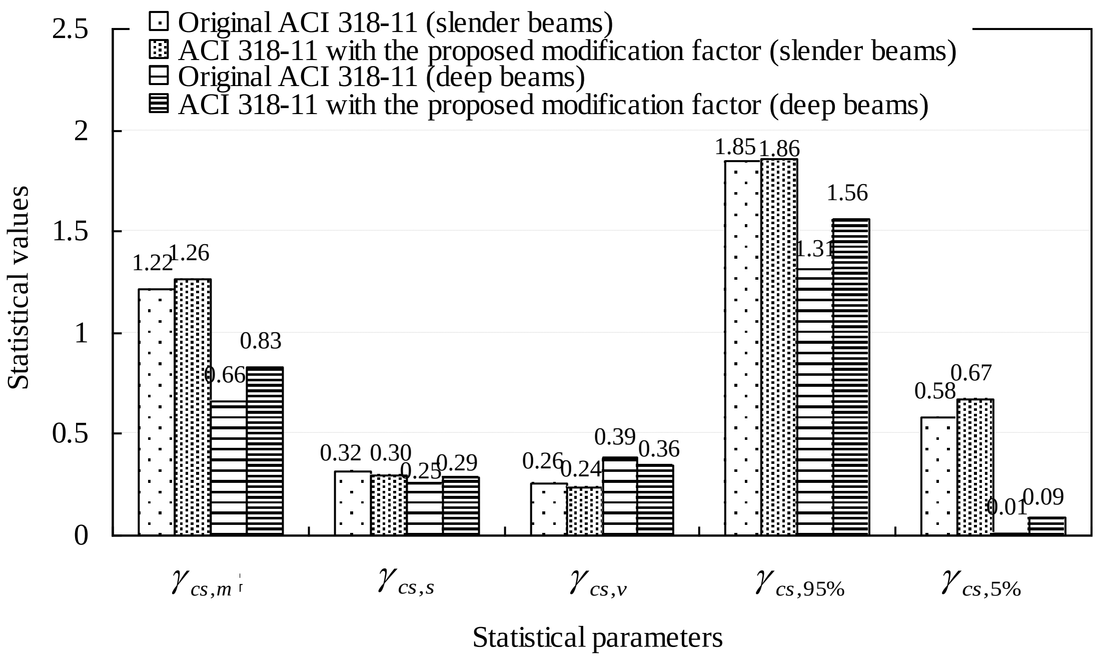


Fig. 9-Variation of code safety factor of LWC beams when the proposed modification factor is used.



(a) ALWC



(b) SLWC

Fig. 10-Comparisons of statistical parameters for LWC beams according to the modification factors.

Tables and Figures

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