Link to original published version: http://dx.doi.org/10.1680/macr.2008.00038

Citation: Ashour AF and Yang KH (2008) Application of Plasticity Theory to Reinforced Concrete Deep Beams: A Review. Magazine of Concrete Research, 60 (9): 657-664.

APPLICATION OF PLASTICITY THEORY TO REINFORCED CONCRETE DEEP BEAMS—A REVIEW

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Keywords: plasticity, deep beams, strut-and-tie, mechanism, capacity.

ABSTRACT

This paper reviews the application of the plasticity theory to reinforced concrete deep beams. Both the truss analogy and mechanism approach were employed to predict the capacity of reinforced concrete deep beams. In addition, most current codes of practice, for example Eurocode 1992 and ACI 318-05, recommend the strut-and-tie model for designing reinforced concrete deep beams.
Compared with methods based on empirical or semi-empirical equations, the strut-and-tie model and mechanism analyses are more rational, adequately accurate and sufficiently simple for estimating the load capacity of reinforced concrete deep beams. However, there is a problem of selecting the effectiveness factor of concrete as reflected in the wide range of values reported in the literature for deep beams.

1 INTRODUCTION

Reinforced concrete deep beams have many useful applications including transfer girders, pile caps, tanks, folded plates and foundation walls. The capacity of reinforced concrete deep beams is governed mainly by shear strength owing to their geometric proportions. The problem of shear in reinforced concrete was extensively studied for about a century [1] and it was suggested [1, 2] that the extension of plasticity theory would be one of the most important advances in understanding shear in reinforced concrete. The plastic theory was first applied to shear in reinforced concrete structures by research groups in Denmark [3] and Zurich [4]. Design philosophy for shear in reinforced concrete beams using plasticity theory comprises two branches: the truss analogy and the kinematic approach. The kinematic approach is far less developed than the static approach. This paper reviews the advances in the strut-and-tie and mechanism analyses for reinforced concrete deep beams.

2 EFFECTIVENESS FACTOR OF CONCRETE

The effective compressive strength f_c of concrete can be obtained by modifying the cylinder compressive strength $f_c^{'}$ with an effectiveness factor ν as given below:

$$f_c = \nu f_c^{'} \quad (1)$$

The reason for introducing the effectiveness factor ν is to account for the limited ductility of concrete and to absorb other shortcomings of applying the theory of plasticity to concrete. The value of the effectiveness factor of concrete normally depends on compressive strength, size, geometry, reinforcement and loading of the structure [3].

Different techniques were suggested for evaluating the effectiveness factor of concrete. Exner [5] proposed a computational method to calculate the effectiveness factor by comparing the area below the real uniaxial stress-strain curve for concrete in compression and an elastic-perfectly plastic stress-strain curve. The value of the effectiveness factor obtained is a function of the shape of the real stress-strain curve, compressive strength and the ultimate strain of concrete. Nielsen [3] proposed a formula for the effectiveness factor of concrete beams failing in shear as a function of cylinder compressive strength:

$$\nu = 0.8 - \frac{f_c^{'}}{200} \qquad (f_c^{'} \text{ in } N/mm^2) \tag{2}$$

The above formula indicates that increasing the concrete strength reduces the value of the effectiveness factor as the lower the concrete strength, the flatter the stress-strain curve, and the more observed ductility.

Although the plastic behaviour of reinforced concrete structures is mainly influenced by the amount of reinforcement, very few formulae proposed for the effectiveness factor consider the amount of reinforcement. Based on statistical analysis for the effectiveness factor of concrete in continuous deep beams, Ashour and Morley [6] proposed a formula for the effectiveness factor of concrete in continuous deep beams expressed in terms of concrete strength and amount of reinforcement:

$$\nu = 0.7 - \frac{f_c^{'}}{110} - \frac{\rho}{0.85} \qquad (f_c^{'} \text{ in } N/mm^2) \tag{3}$$

where ρ is a weighted-reinforcement ratio for the horizontal and vertical reinforcement based on their relative contribution to the load capacity of continuous deep beams.

Based on tests of concrete panels under shear, Vecchio and Collins [7] related the effectiveness factor to the tensile strain normal to the principal compressive strain as expressed below:

$$\nu = \frac{1}{1.0 + k_c k_f} \tag{4}$$

where $k_c = 0.35\left(-\frac{\varepsilon_1}{\varepsilon_3} - 0.28\right)^{0.8} \geq 1.0$, $k_f = 0.1825\sqrt{f_c^{'}} \geq 1.0$ and ε_1 and ε_3 = the principal tensile and compressive strains, respectively.

In many investigations, the value of the effectiveness factor of concrete is simply calculated by calibrating the failure loads obtained from the plasticity analysis against those from experiments. This technique shows significant variation of the effectiveness factor for different reinforced concrete structures. According to Oesterle et al. [8], the value of the effectiveness factor for concrete wall structures failing in shear varies between 0.16 and 0.49. Rogowsky and MacGregor [9] proposed values for the effectiveness factor varying from 0.25 to 0.85 depending on the concrete element in the plastic truss model used to predict the capacity of continuous deep beams. Ashour [10] concluded that the best mean value of the effectiveness factor of reinforced concrete deep beams with fixed ends is 0.5.

Foster and Malik [11] evaluated different effectiveness factor formulae used in strut-and-tie models of nonflexural members such as deep beams, corbels and nibs. They concluded that effectiveness factor models based primarily on concrete strength are found to have poor correlation with test results of 135 nonflexural structural elements. They recommended that effectiveness factor models, that account for the angle of the strut relative to the longitudinal axis of the structural member combined with models based on the modified compression field theory (Eq. 4 above), are found to give best correlation with the experimental results.

It is well documented that shear strength of reinforced concrete beams without web reinforcement appears to decrease as the beam depth increases [12, 13] and this size effect is less significant for beams with web reinforcement. Size effect could not be directly considered in the plasticity theory in which the nominal stress at failure must be independent on the size of structures. The only possibility to accommodate the effect of deep beam size would be to have the effectiveness factor depending on size. Yang et al. [14] modified the effectiveness factor proposed by Vecchio and Collins (Eq. 4 above) by a size effect factor which is a function of deep beam effective depth and maximum size of aggregate. They concluded that size effect is successfully represented in the modified effectiveness factor as the capacity of continuous deep beams with different section depths was accurately predicted. On the other hand, Tan and Cheng [15] suggested that size effect depends on factors such as the geometry of strut (width and length) and strut boundary conditions due to transverse web reinforcement (spacing and diameter). They proposed a modified strut and tie model that accurately predicts the size effect trends for deep beams, with a uniform safety margin for different member sizes considered.

3 LOWER BOUND ANALYSIS OF REINFORCED CONCRETE DEEP BEAMS

The lower-bound analysis of reinforced concrete deep beams is often developed from a hypothetical plastic truss model. In contrast to the situation for a truly plastic material, the validity of the chosen truss model for a reinforced concrete deep beam depends on whether the truss model represents the true situation reasonably close or not as reinforced concrete deep beams can undergo only a limited amount of redistribution of internal forces. Therefore, if the chosen truss requires excessive deformation to reach the fully plastic state assumed, the beam may fail prematurely at a load lower than that predicted by the truss. Extensive investigations have been carried out to improve the predictions from strut-and-tie models [9, 12, 16-18]. Strut-and-tie models developed in the literature for different reinforced concrete deep beam cases are presented and discussed below.

3.1 Strut-and-Tie Models of Deep Beams

Diagonal cracks in the web area of deep beams separate the concrete into a series of diagonal concrete struts which are assumed to resist compression. The diagonal concrete struts in deep beams commonly considered as bottle-shaped struts that are generally idealised as prismatic or uniformly tapered members within shear spans [9, 12, 17, 18, 19]. A tension tie represents one or several layers of steel reinforcement. Nodes are the joints where axial forces in struts and ties intersect. Marti [17] pointed out the importance of considering actual dimensions of compressive struts and tensile ties in formulating the truss models.

Fig. 1 illustrates schematic strut-and-tie models for reinforced concrete deep beams subjected to two-point symmetrical top loads, as suggested by different researchers: Fig. 1(a) for simply supported deep beams [18, 16, 20, 21, 22] and Fig. 1(b) for continuous deep beams [9]. The contribution of web reinforcement to the beam capacity is ignored. The strut-and-tie model of a single span deep beam shown in Fig. 1(a) is composed of two diagonal concrete struts, a top flexural concrete strut and a horizontal bottom steel tie connected together at four nodal zones. Nodal zones at the applied load point would be classified as a CCC type, which is a hydrostatic node connecting two concrete struts and external applied load, whereas nodal zones at supports are CCT type anchoring the bottom horizontal tie to the diagonal concrete strut and support bearing area. In a CCC type nodal zone having equal stresses on all in-plane sides, the ratio of each face width of the hydrostatic node has to be the same as the ratio of forces meeting at the node to make the state of stresses in the whole node region constant [16, 17, 23]. Concrete stress levels in nodal zones must be controlled to allow for the safe transfer of forces, which depends on many factors, including the tensile straining from tension ties, confinement provided by reactions and concrete compression struts, and confinement provided by transverse reinforcement. A diverse range of limits on concrete stresses in different nodal zones was suggested in the literature [16, 12, 23]. Yun [23] conducted an extensive review on the approaches for evaluating nodal zone strength. He concluded that all approaches checked the concrete compressive stress in the nodal zone boundary and proposed that non linear finite element analysis of the nodal zone must be conducted to accurately predict the limiting stresses within the nodal zone.

The load transfer capacity of concrete struts depends on the strut area and effective concrete compressive strength and hence, by considering equilibrium of forces in concrete struts and steel ties, the load capacity of deep beams owing to crushing of concrete struts can be obtained.

The strut and tie model for continuous deep beams shown in Fig. 1(b) is statically indeterminate. Few techniques [21, 24, 25] were proposed to analyse statically indeterminate strut-and-tie models to determine the distribution of forces. One method is to assume the most heavily loaded ties yielded until the truss system becomes statically determinate [9]. Another technique is to decompose the statically indeterminate truss into several statically determinate trusses [21] and the third approach is to carry out stiffness analysis for the indeterminate truss [25].

Rogowsky and MacGregor [9] presented plastic truss models shown in Figs. 2 and 3 to predict the capacity of continuous deep beams with either vertical or horizontal web reinforcement. For beams with horizontal web reinforcement, there are two trusses to transfer the applied loads to supports as depicted in Fig. 2. In this case, it is expected that the bottom reinforcement would reach yield before the upper horizontal steel reinforcement. However, the additional deformations required for the upper steel reinforcement to yield so that the upper truss can reach its full capacity would generally be large enough to cause beam collapse. Therefore, it was suggested that horizontal web reinforcement would be neglected in the truss model [9]. For deep beams with vertical reinforcement, the applied load is transferred to supports by direct diagonal struts and two compression fans radiating from the applied point load and support reaction as shown in Fig. 3.

Other strut-and-tie models for reinforced concrete deep beams [20, 21, 26] were also proposed for estimating the load transfer mechanism of both horizontal and vertical web reinforcement and in case of deep beams with web openings. However, the detailing of such models considering the actual sizes of struts and ties proved to be complicated, therefore they were presented in a load path form.

Some research investigations [27, 28] have focused on automatic generation of optimal strut-and tie models using topology optimisation algorithms, especially for deep beams with web openings interrupting the follow of diagonal struts between applied loads and supports. The optimal strut-and-tie model is produced by gradually removing concrete regions that are ineffective in carrying loads based on overall stiffness performance criteria. Such techniques heavily utilise the finite element method as a modelling and analytical tool.

3.2 Code Modelling of Deep Beams

Most current codes of practice [19, 29-31] recommend the use of strut-and-tie models for designing reinforced concrete deep beams. Nonetheless, they do not provide specific guidance on suitable strut-and-tie models for different deep beam cases. For example, no specific guidelines on the truss action identifying the load transfer mechanism of horizontal and vertical shear reinforcement and in case of web openings are provided.

Different effectiveness factors for concrete are proposed in codes of practice. AASHTO LRFD Specification [30] and CSA A23.3-04 [31] consider the effectiveness factor as a function of the amount of transverse tensile strain, whereas Eurocode 1992 [29] gives it as a function of concrete strength. On the other hand, ACI 318-05 [19] allows the use of effectiveness factor of 0.75 for concrete struts having a minimum amount of shear reinforcement, regardless of concrete strength and the amount of transverse tensile strain. The value of the effectiveness factor drops to 0.6 if the minimum shear reinforcement specified in ACI 318-05 is not provided. This implies that the arrangement of shear reinforcement recommended by ACI 318-05 allows the load capacity of deep beams predicted by the strut-and-tie model to be increased by 25%.

ACI 318-05 stipulates that the concrete effective stresses in the nodal zones should not exceed a certain value: $0.85f_c^{'}$ in CCC type nodes bounded by compression struts and bearing areas as shown in the top node of Fig. 1(a), and $0.68f_c^{'}$ in CCT type nodes anchoring only one tension tie as shown in the bottom nodes of Fig. 1. However, Eurocode 1992 proposed that the concrete effective stresses in CCC and CCT type nodes are limited to $vf_c^{'}$ and $0.85vf_c^{'}$, respectively, where v is the effectiveness factor of concrete.

Most codes of practices limit the angle between concrete struts and steel ties joined at the same nodal zone. ACI 318-05 limits this angle to values greater than 25 degrees, whereas Eurocode 1992 recommends limiting values for the angle of the inclined struts to be between 22.5 and 45 degrees.

4 UPPER BOUND ANALYSIS OF REINFORCED CONCRETE DEEP BEAMS

The first upper bound analysis for shear strength of reinforced concrete beams was developed by Nielsen and his associates in Denmark [3]. Kemp and Al-Safi [32] derived an upper bound solution for reinforced concrete beams based on rotation and translation of rigid blocks. The first attempt to generalise the upper bound analysis for plane stress problem was made by Zainai and Morley [33], when they derived upper bound solutions for deep beams with web openings considering more than one yield line.

4.1 Failure Mechanisms of Deep Beams

The failure mechanism of reinforced concrete deep beams is idealised as an assemblage of rigid blocks moving in the beam plane, separated by yield lines. The yield line is a theoretical representation of the narrow discontinuity zone with many criss-crossing cracks and crushing zones which occurs in reality [3, 34]. It was proved [35, 36] that the optimum shape of the yield line is a hyperbola as the energy dissipated along it is less than that dissipated in a straight yield line. This hyperbolic yield line turns into two straight segments when the instantaneous centre (I.C.) of relative rotation of rigid blocks lies inside or on the circle whose diameter is the straight line between the end terminals of the yield line. As a special case, when the instantaneous centre (I.C.) approaches infinity, the hyperbolic yield line reduces to a straight yield line between the edges of point load and support plates. In this case, there is a pure translation of rigid blocks relative to each other as considered by Nielsen et al. [3, 34].

Figs. 4 to 8 show idealised mechanisms of failure of reinforced concrete deep beams with different end conditions and with or without web openings studied in the literature [3, 10, 36-39]. These mechanisms were experimentally observed at failure.

4.2 Upper Bound on Load Capacity of Deep Beams

In general, each rigid block formed at failure in the mechanisms shown in Figs. 4 to 8 has two transitional and one rotational displacement components. Considering boundary conditions at supports and/or beam symmetry, the independent displacement components for each block can be reduced.

The load capacity obtained from the mechanisms presented above is obtained by equating the total internal energy dissipated in concrete and steel reinforcement along yield lines to the external work done by applied loads. All steel reinforcing bars crossing yield lines are assumed to be yielded. Although, the energy dissipated depends on the rigid block displacements and the location of the instantaneous centre (I.C.) of relative rotation of blocks, the load capacity obtained for each mechanism is generally expressed as a function of the concrete and steel properties, and position of the instantaneous centre, (X_{ic}, Y_{ic}). Table 1 presents the normalised load capacity, $\lambda = P/(bhf_c^{'})$, of deep beams with different end conditions, and with or without web openings obtained from the mechanism approach.

According to the upper-bound theorem of the plasticity theory, the collapse occurs at the least strength [3]. The minimum value of the load capacity is obtained by varying the position (X_{ic}, Y_{ic}) of the instantaneous centre in the vertical plane of the deep beam [6, 10, 24, 36, 37]. This is normally achieved by numerical optimisation techniques. Alternatively, if main longitudinal steel bars are sufficiently strong not to yield, the instantaneous centre of rotation can be located at their level (i.e., steel does not yield). In deep beam cases where more than one mechanism of failure may occur, the governing (guide) mechanism would be the one predicting the lowest capacity.

5 PREDICTIONS OF DEEP BEAM CAPACITY USING PLASTICITY THEORY

The accuracy of predictions of deep beam capacity obtained from both strut-and-tie models and mechanism analysis was studied in many investigations [20-22, 24, 37,38, 40-42]. Table 2 presents summary of statistical parameters of comparisons between the predicted capacity and experiments. Most validations were carried out for simply supported deep beams without web openings. The statistical parameters show the efficiency of both the strut and tie models and mechanism analysis in predicting the deep beam capacity. In addition, the plasticity-based analyses of deep beams accurately predicted the trend of deep beam capacity against different influencing parameters, for example, the decrease of deep beam capacity with the increase of shear-span-to-depth ratio [3, 12, 6, 37] as experimentally observed in many investigations.

6 CORRELATION AND CONCLUSIONS

The following correlation and conclusions are deduced from the survey on the application of plasticity theory to reinforced concrete deep beams presented in this paper:

- Upper and lower bound analyses are powerful tools for predicting load capacity of reinforced concrete deep beams. Strut-and-tie models give a better understanding of the distribution of internal forces within deep beams, whereas upper bound analysis would encourage designers to examine different failure mechanisms of deep beams. Compared with methods based on empirical or semi-empirical equations, upper and lower bound analyses are more rational, adequately accurate and sufficiently simple for reinforced concrete deep beams.
- Strut-and-tie models received more attention in recent years than the mechanism analysis. This may be attributed to the fact that plasticity-based strut-and-tie models theoretically produce safe, lower bound designs. However, mechanism analysis would also produce a conservative prediction when the effectiveness factor is carefully selected.
- Although more than one admissible strut and tie model for deep beams can be developed, designers should be careful that the chosen model would represent the true situation

reasonably close as reinforced concrete deep beams have a limited ductility. On the other hand, different mechanisms of failure have to be examined before identifying the deep beam capacity.

- There is a problem of selecting the effectiveness factor of concrete as reflected in the wide range of values reported for deep beams in the literature.
- The plasticity models for deep beams assume that horizontal and vertical steel bars crossing failure zones have yielded. An assumption has not been experimentally fulfilled in case of shear failure of reinforced concrete deep beams, and significantly contributes to inconsistent prediction in few cases.
- The detailing of the strut-and-tie model is strongly influenced by the size of the loading plates and the location of main longitudinal reinforcement. In addition, the main longitudinal reinforcing bars in deep beams are not carried through to anchor plates as the stress fields pretend.
- Strut-and-tie models are generally more difficult to develop for deep beams with orthogonal web reinforcement or web openings than the mechanism approach.

REFERENCES

[1] Regan, P. E., "Research on Shear: a Benefit to Humanity or a Waste of Time?", *The Structural Engineer*, 71, 19, 1993, pp. 337-346.

[2] MacGregor, J. G., "Challenge and Changes in the Design of Concrete Structures", *Concrete International*, 1984, 6, 2, pp 48-52.

[3] Nielsen M. P., *Limit Analysis and Concrete Plasticity*, Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1984, 420pp.

[4] Grob, J., and Thürlimann, B., ''Ultimate Strength and Design of Reinforced Concrete Beams under Bending and Shear'', *IABSE Periodica*, Zurich, 36-II, 1976, 105–120.

[5] Exner, H., "On the Effectiveness Factor in Plastic Analysis of Concrete", *Plasticity in reinforced concrete,* IABSE, Zurich, 1979, pp. 35-42.

[6] Ashour, A. F. and Morley, C. T., "Effectiveness Factor of Concrete in Continuous Deep Beams", *Journal of Structural Engineering, ASCE*, 122, 2, 1996, pp 169-178.

[7] Vecchio F. J. and Collins M. P., "Compression Response of Cracked Reinforced Concrete", *Journal of Structural Engineering, ASCE*, 119, 12, 1993, pp 3590-3610.

[8] Oesterle, R. G., Aristizabal-Ochoa, J. D., Shiu, K. N. and Corley, W. G., "Web Crushing of Reinforced Concrete Structural Walls", *ACI Structural Journal*, 81, 3, 1984, pp 231-241.

[9] Rogowsky, D. M. and MacGregor, J. G., "Design of Reinforced Concrete Deep Beams", *Concrete International: Design and Construction*, 8, 8, 1986, pp 49-58.

[10] Ashour, A. F., "Upper Bound Analysis for Reinforced Concrete Deep Beams with Fixed End Supports", *ACI Structural Journal*, 96, 2, 1999, pp 167-173.

[11] Foster, S. J. and Malik, A. R., "Evaluation of Efficiency Factor Models used in Strut-and-Tie Modeling of Nonflexural Members", *Journal of Structural Engineering*, ASCE, 128, 5, 2002, pp 569-577.

[12] ASCE–ACI Committee 445 on Shear and Torsion, ''Recent Approaches to Shear Design of Structural Concrete'', *Journal of Structural Engineering, ASCE,* 124, 12, 1998, pp 1375–1417.

[13] Bažant Z. P., and Kim J. K., "Size Effect in Shear Failure of Longitudinally Reinforced Beams", ACI Journal, 81, 5, 1984, pp 456-468.

[14] Yang, K. H., Chung, H. S. and Ashour, A. F., "Influence of Section Depth on the Structural Behavior of Reinforced Concrete Continuous Deep Beams", Magazine of Concrete Research, 2007, Vol. 59, No. 8, pp. 575–586..

[15] Tan, K.H. and Cheng, G.H., "Size effect on shear strength of deep beams: Investigating with strut-and-tie model", *Journal of Structural Engineering*, ASCE, 132, 5, 2006, pp. 673-685.

[16] ACI Committee 445, "Examples for the Design of Structural Concrete with Strut-and-Tie Models", ACI Special publication, SP-208, ed.: Reineck, K., 2002, 242pp.

[17] Marti, P., "Basic Tools of Reinforced Concrete Beam Design", *ACI Journal,* 82, 1, 1985, pp. 45-56.

[18] Schlaich, J., Schäfer, K., and Jennewein, M., "Toward a Consistent Design of Structural Concrete", *Journal of the Prestressed Concrete Institute*, 32, 3, 1987, pp. 74-150.

[19] ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05)" American Concrete Institute, 2005.

[20] Hwang, S-J, Lu, W-Y and Lee, H-J, "Shear Strength Prediction for Deep Beams", *ACI Structural Journal*, 97, 3, 2000, pp 367-376.

[21] Matamoros, A. B. and Wong, K. H., “Design of Simply Supported Deep Beams Using Strut-and-Tie Models”, *ACI Structural Journal*, 100, 6, 2003, pp 704-712.
[22] Tan, K. H., Weng, L. W. and Teng, S., “A Strut-and-Tie Model for Deep Beams Subjected to Combined Top-and-Bottom Loading”, *The Structural engineer*, 75, 13, 1997, pp. 215-225.
[23] Yun, Y. M., “Strength of Two-Dimensional Nodal Zones in Strut-Tie Models”, *Journal of Structural Engineering*, 132, 11, 2006, pp 1764-1783.
[24] Yang, K. H. and Ashour, A. F., “Load Capacity of Reinforced Concrete Continuous Deep Beams”, accepted for publication in the Structural Engineering Journal, ASCE.
[25] Tihin, T. N., and Kuchma, D. A., “Computer-Based Tools for Design by Strut-and-Tie Method: Advances and Challenges”, *ACI Structural Journal*, 99, 5, 2002, pp 586-594.
[26] Tan, K. H., Tong, K., and Tang, C. Y., “Consistent Strut-and-Tie Modelling of Deep Beams with Web Openings”, *Magazine of Concrete Research*, 55, 1, 2003, pp. 65-75.
[27] Guan, H., “Strut-and-tie model of deep beams with web openings: an optimization approach”, *Structural Engineering and Mechanics*, 19, 4, 2005, pp 361-379.
[28] Liang, Q. Q., Xie, Y. M. and Steven, G. P., “Topology Optimization of Strut-and-Tie Models in Reinforced Concrete Structures Using an Evolutionary Procedure”, *ACI Structural Journal*, 97, 2, 2000, pp 322-331.
[29] European Standard EN 1992-1-1:2004, “Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings”, British Standards Institution, 2004, 230p.
[30] AASHTO, *AASHTO LRFD Bridge Specifications*, American Association of State Highway and Transportation Officials, 2nd ed., Washington, DC, 1998.
[31] Canadian CSA Building Code, CAN-A23.3-04, Design of Concrete Structures, Canadian Standards Association. Toronto, 2004.
[32] Kemp, K. O. and Al-Safi, M. T., “An upper Bound Rigid-Plastic Solution for the Shear Failure of Concrete Beams without Shear Reinforcement”, *Magazine of Concrete Research*, 33, 115, 1981, pp 96-102.
[33] Zainai, B. M. and Morley, C. T., “Shear Strength of Reinforced Concrete Wall-Beam Structures: An Improved Solution”, International Conference on solid Mechanics and Structures, Singapore, 1991.
[34] Nielsen, M. P. and Bræstrup, M. W., ‘‘Shear Strength of Prestressed Concrete Beams without Web Reinforcement”, *Magazine of Concrete Research*, 30, 104, 1978, pp 119-128.
[35] Jensen, J. F., ‘‘Discussion of ‘An upper-bound rigid-plastic solution for the shear failure of concrete beams without shear reinforcement’ by K. O. Kemp and M. T. Al-Safi.’’, *Magazine of Concrete Research*, 34, 119, 1982, pp 100–103.
[36] Ashour, A. F. and Morley, C. T., “The Numerical Determination of Shear Failure Mechanisms in Reinforced Concrete Beams”, *The Structural Engineer*, 72, 23 & 24, 1994, pp 395-400.
[37] Ashour, A. F., “Shear Capacity of Reinforced Concrete Deep Beams”, *Journal of Structural Engineering*, ASCE, 126, 9, 2000, pp 1045-1052.
[38] Wang, W., Jiang, D-H and Hsu, C-T T., “Shear Strength of Reinforced Concrete Deep Beams”, *Journal of Structural Engineering*, 119, 8, 1993, pp. 2294-2312.
[39] Ashour, A. F., and Rishi, G., “Tests of Reinforced Concrete Continuous Deep Beams with Web Openings”, *ACI Structural Journal*, 97, 3, 2000, pp. 418-426.
[40] Russo, G., Venir, R. and Pauletta, M., “Reinforced Concrete Deep Beams—Shear Strength Model and Design Formula”, *ACI Structural Journal,* 102, 3, 2005, pp 429-437.
[41] Siao, W. B., “Strut-and-Tie Model for Shear Behavior in Deep Beams and Pile Caps Failing in Diagonal Splitting”, *ACI Structural Journal*, 90, 4, 1993, pp 356-363.
[42] Yang, K. H. and Ashour, A. F., “Code Modelling of Reinforced Concrete Deep Beams”, accepted for publication in the Magazine of Concrete Research.
[43] Cook, W. D. and Mitchell, D., “Studies of Disturbed Regions near Discontinuities in Reinforced Concrete Members”, *ACI Structural Journal,* 95, 2, 1998, pp 206-216.
[44] Yun, Y. M., “Computer Graphics for Nonlinear Strut-and-Tie Model Approach”, *Journal of Computing in Civil Engineering*, ASCE, 14, 2, 2000, pp 127-133.

Table 1 Normalised load capacity λ of deep beams predicted by mechanism analysis.

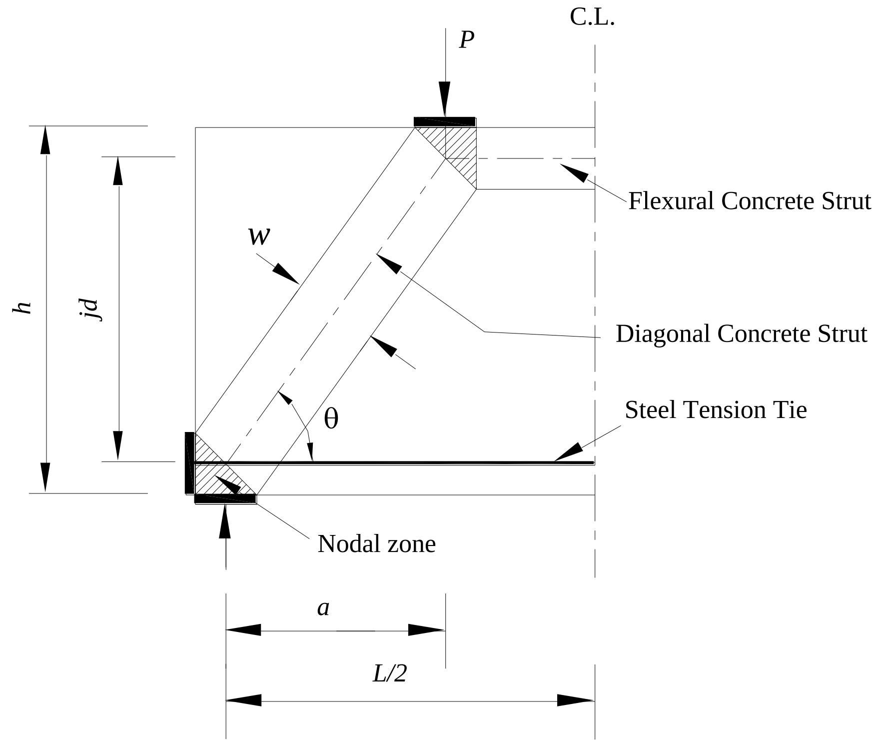
Source	Deep beam details	Mechanism	Normalised load capacity, $\lambda = P/(bhf_c^{'})$
[37]	Simply supported deep beams	Fig. 4	$\lambda = \frac{1}{X_{ic}}\left[\frac{\nu}{2h}L_c r(1-\sin\alpha)+\sum_{i=1}^{N_{\phi v}}\phi_{vi}\left\|X_i - X_{ic}\right\|+\sum_{j=1}^{N_{\phi h}}\phi_{hj}\left\|Y_j - Y_{ic}\right\|\right]$
[6, 24]	Continuous deep beams	Fig. 5	$\lambda = \frac{1}{L/2}\left[\frac{\nu}{2h}L_c r(1-\sin\alpha)+\sum_{i=1}^{N_{\phi v}}\phi_{vi}\left\|X_i - X_{ic}\right\|+\sum_{j=1}^{N_{\phi h}}\phi_{hj}\left\|Y_j - Y_{ic}\right\|\right]$
[10]	Deep beams with fixed end supports	Fig. 6	$\lambda = \frac{1}{L/2}\left[\frac{\nu}{2h}[L_c r(1-\sin\alpha)+Y_{ic}^2]+\sum_{i=1}^{N_{\phi v}}\phi_{vi}\left\|X_i - X_{ic}\right\|+\sum_{j=1}^{N_{\phi h}}\phi_{hj}\left\|Y_j - Y_{ic}\right\|+\sum_{k=1}^{N_{\phi h1}}\phi_{hk}\left\|Y_k - Y_{ic}\right\|\right]$
[6, 10, 24]	Pure translation mechanism	Fig. 7	$\lambda = \nu\left[\sqrt{1+(\frac{a}{h})^2}-\frac{a}{h}\right]+2\psi_v\frac{a}{h}$
[39]	Continuous deep beams with web openings	Fig. 8	$\lambda = \frac{1}{L/2}\left[\frac{\nu}{2h}[L_1 r_1(1-\sin\alpha_1)+L_2 r_2(1-\sin\alpha_2)]+\sum_{i=1}^{N_{\phi v}}\phi_{vi}\left\|X_i - X_{ic}\right\|+\sum_{j=1}^{N_{\phi h}}\phi_{hj}\left\|Y_j - Y_{ic}\right\|\right]$

Note: $\lambda = (P/bhf_c^{'})$ = normalised load capacity; $\phi_{vi} = (A_{vi}f_{yi}/bhf_c^{'})$ and $\phi_{hj} = (A_{hj}f_{yj}/bhf_c^{'})$ = vertical and horizontal reinforcement ratios for individual bars crossing the yield line, respectively; A_{vi}, X_i and f_{yi} = area, horizontal coordinate, and yield strength of the vertical bar i crossing the yield line, respectively; A_{hj}, Y_j and f_{yj} = area, vertical coordinate, and yield strength of the horizontal bar j crossing the yield line, respectively; $N_{\phi v}$ and $N_{\phi h}$ = number of vertical and horizontal bars crossing the hyperbolic yield line; the subscript k is used to identify the horizontal reinforcement of total number $N_{\phi h1}$ crossing the flexural yield line in case of deep beams with fixed end supports; $\psi_v = (A_{vi}f_{yi}/bs_v f_c^{'})$ = smeared intensity of the vertical shear reinforcement, and s_v = vertical web reinforcement spacing.. The rest of geometrical notations are defined in Figs. 4 to 8.

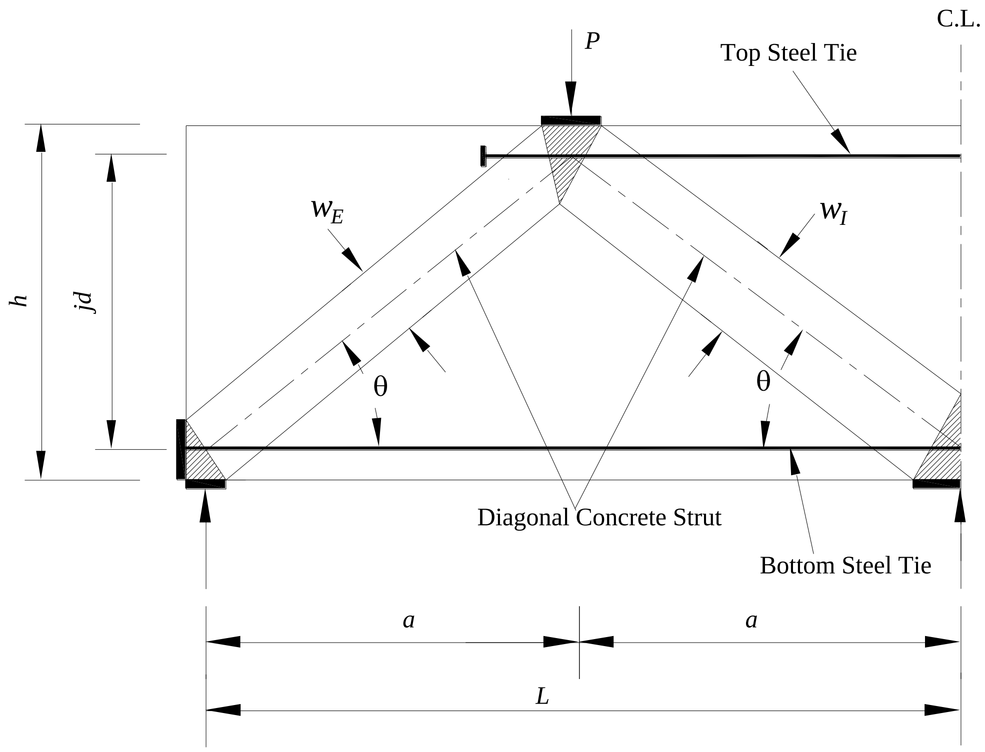
Table 2 Comparisons between experimental and predicted load capacities.

Source	Supporting condition	No. of beams	μ^*	σ^*	ρ^*
(a) Strut-and-tie models					
[20]	Simple	123	1.15	0.16	0.14
[21]	Simple	175	1.40	0.31	0.22
[40]	Simple	240	1.00	0.19	0.19
[41]	Simple	51	1.04	0.12	0.12
[22]	Simple	233	0.76	0.29	0.38
[42]	Simple	448	1.31	0.47	0.36
[24]	Continuous	75	1.12	0.37	0.33
(b) Mechanism approach					
[38]	Simple	64	1.02	0.13	0.13
[37]	Simple	172	1.02	0.20	0.20
[24]	Continuous	75	1.07	0.14	0.13

* μ, σ and ρ stand for the mean, standard deviation and coefficient of variation of predicted and experimental capacities. All deep beams considered in the above investigations were solid, without web openings.



(a) Simply supported beams



(b) Continuous deep beams

Fig. 1 Qualitative strut-and-tie models for reinforced concrete deep beams.

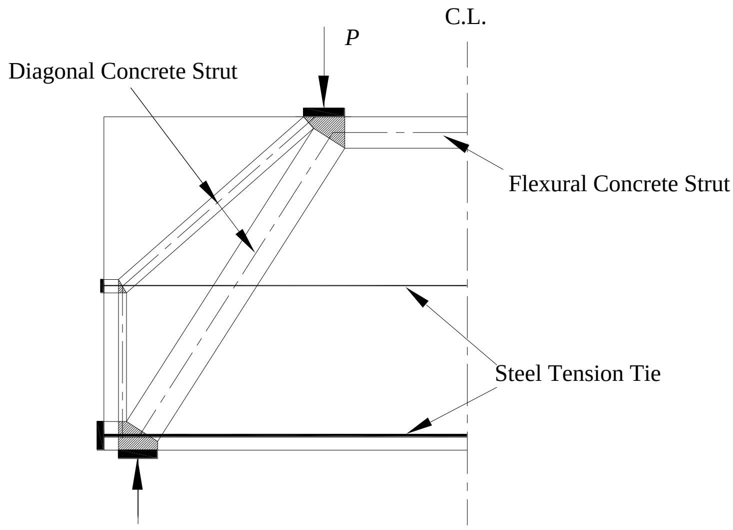


Fig. 2 Strut-and-tie model for deep beam with horizontal web reinforcement.

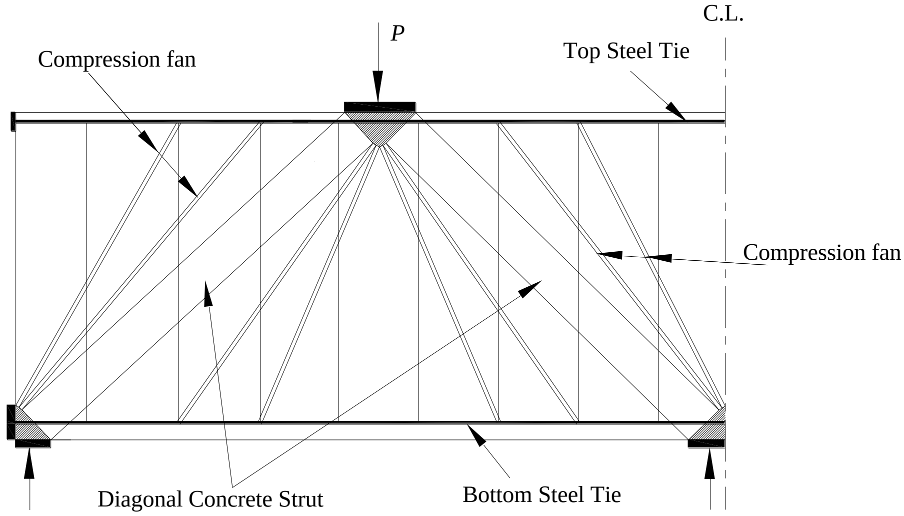


Fig. 3 Strut-and-tie model for continuous deep beam with vertical web reinforcement.

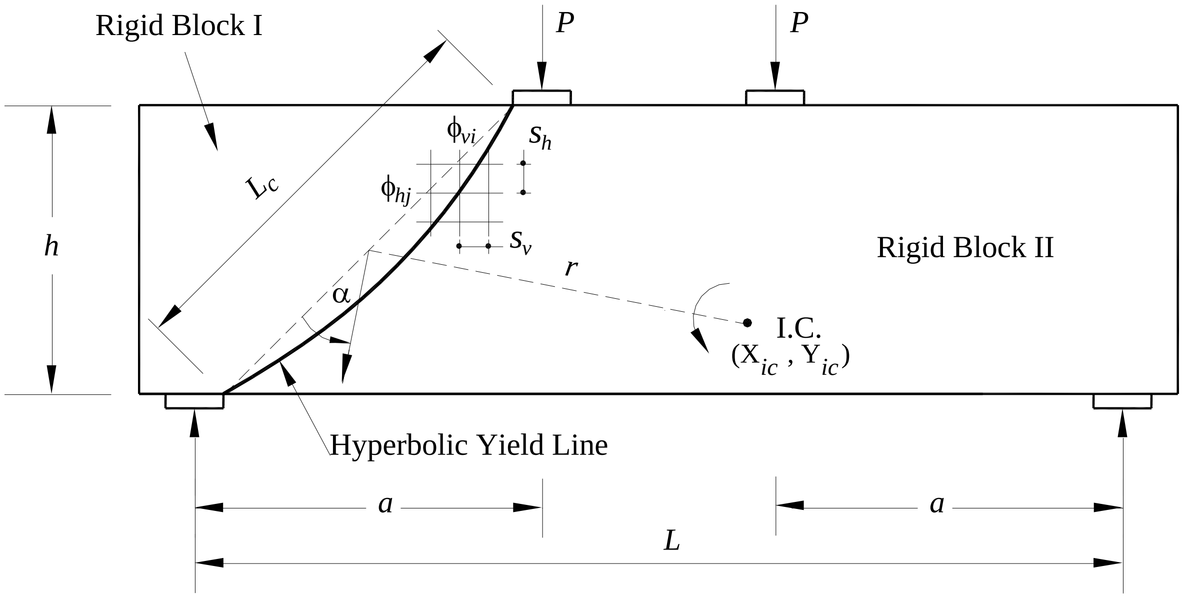


Fig. 4 Un-symmetrical failure mechanism of simply supported deep beams.

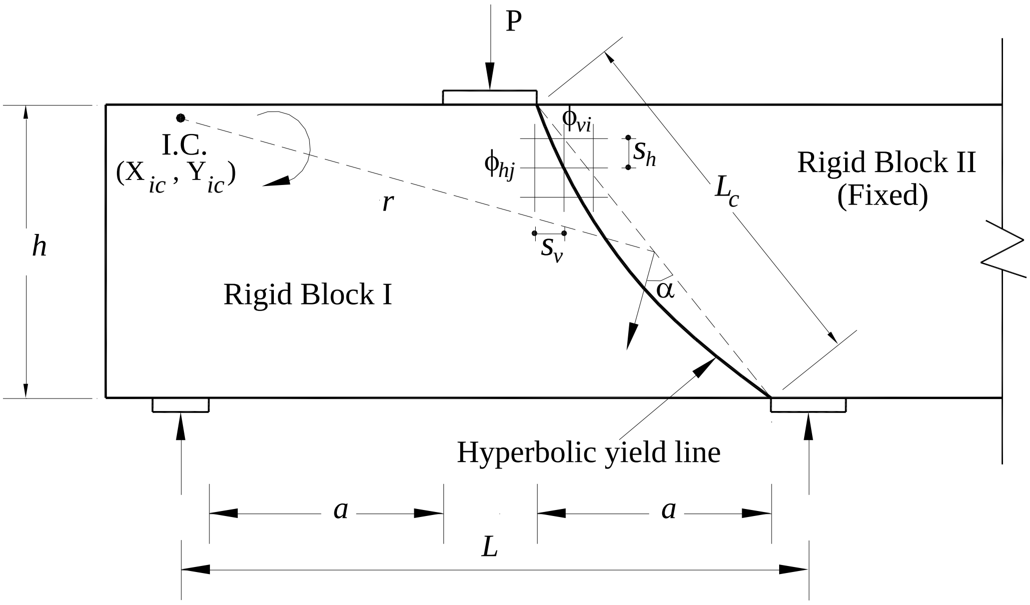


Fig. 5 Failure mechanism of end span continuous deep beams.

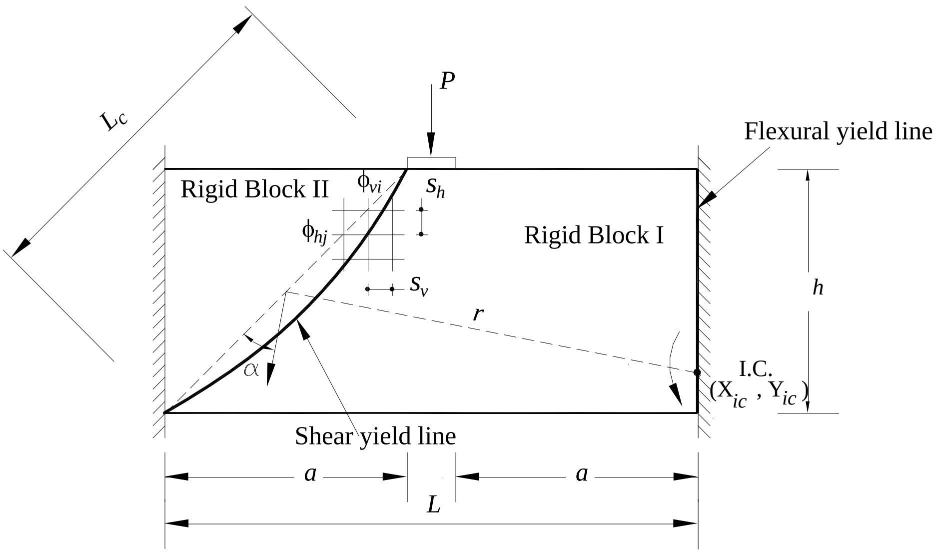


Fig. 6 Failure mechanisms (flexure and shear) of deep beams with fixed end supports.

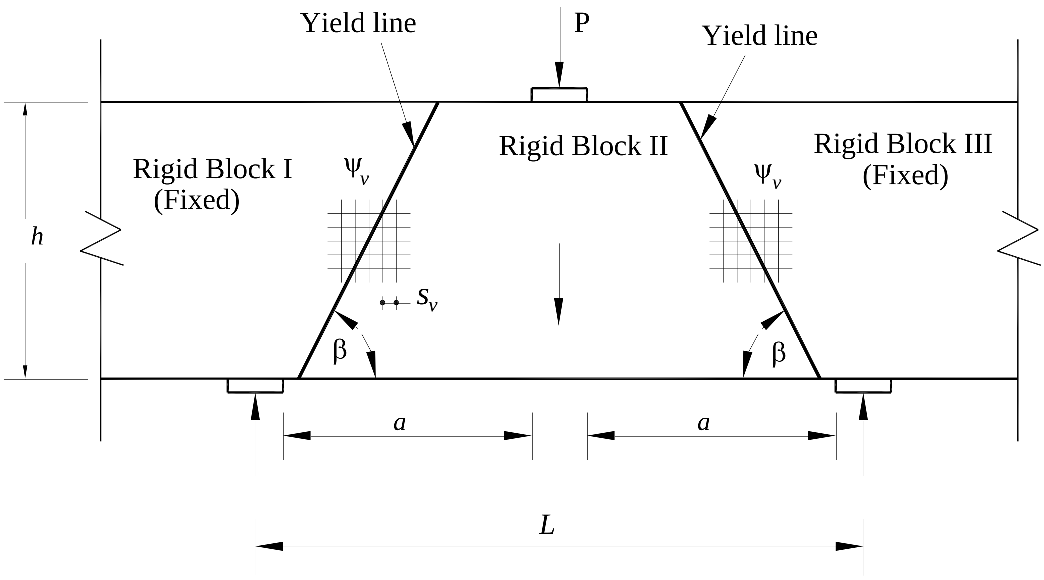


Fig. 7 Diagonal splitting (pure translation) failure mechanism.

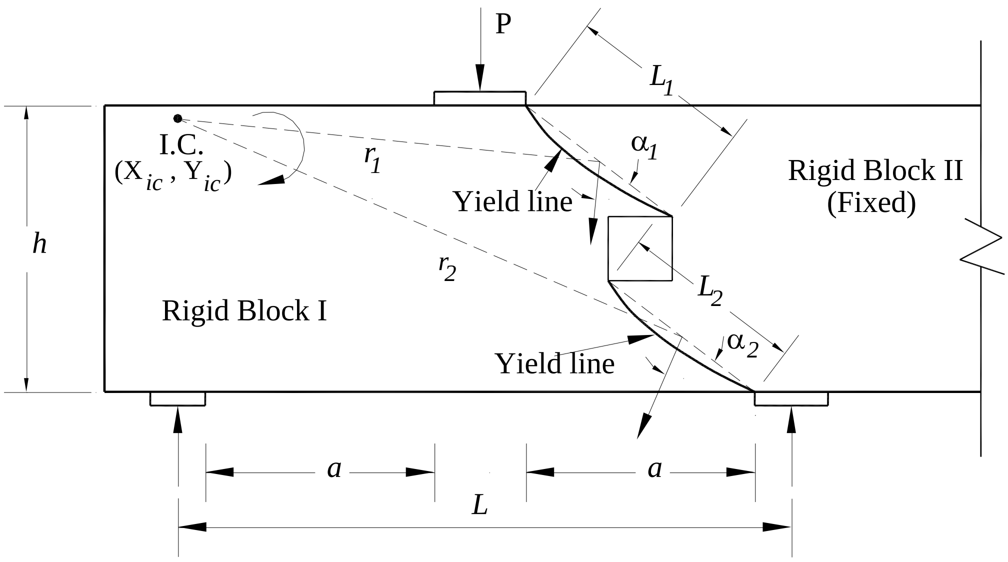


Fig. 8 Failure mechanism of continuous deep beams having web openings within Interior shear span.