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Contents

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FIRE ENGINEERING

The Lateral Torsional Buckling of I Beams with Cross Beams

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Abstract

High flexural-rigid cross beams at the joints of a main beam can function as restraining members to prevent rotation during lateral torsional buckling (LTB). With less flexural-rigid members, joint rotation results in a decrease of the main beam's LTB critical moment. This paper elucidates the cross beam's minimum flexural rigidity to prevent the main beam's joint rotation during torsional buckling. It is assumed that material behaves elastically, the beam's web doesn't undergo distortions, and shear forces effects are neglected. The cross-beam's flexural rigidity is represented by a spiral spring. Under buckling, this spring produces a torque moment, proportional to the joint rotation. The torque will disturb the main beam's LTB equation system. By adjusting the spring constant, the joint rotation is minimized, thus reducing the torque's disturbing effect within the equation. By neglecting this effect, the LTB equations at all fields of main beam are identical to the general buckling equations for constant moments. For *n* cross beams, 4 (*n*+1) integration constants are resulted. By applying the boundary conditions at beam ends and utilizing the geometry and natural boundary conditions at the joints, 4 (*n*+1) homogeneous equations for the integration constant are obtained. By conducting the trial and error method, the critical moment resulting in a zero determinant for the homogeneous equation coefficient matrix is acquired. Then, the LTB first-mode deformation shape can be drawn. The analysis shows that to achieve the critical moment, a main beam having a cross beam located at mid span needs the most optimum (smallest) flexural rigidity of cross beam than when it is in other locations ($\neq L/2$). Observing the first-mode shape, for a certain spring constant value, the rotation at the joint will approach zero.

1. Introduction

The lateral torsional buckling (LTB) moment of a beam is expressed in Eq. (1) (Salmon and Johnson, 1996). C_b of beam with constant moment is equal to 1. The behavior of a main beam having cross beam is different from those without cross beam. A cross beam contributes to restrain the beam during the process of LTB. Such restraint will then increase the critical moment of the main beam. Due to lack of regulation to deal with LTB on main beam having a cross beam, engineers often ignore the influence of cross beam. This will lead to a larger dimension of the main beam designed.

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$$M_{cr} = C_b \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y I_w} \tag{1}$$

In this paper, the influence of cross beam on LTB critical moment of the main beam having a constant moment is examined. To determine LTB critical moment of a main beam having cross beam, finite difference and Rayleigh-Ritz methods are generally applied by solving 2 variables using trial and error process (Chajes, 1970). For finite difference method, the two variables are the numbers of main beam's segments and the LTB critical moment. Whereas for Rayleigh-Ritz method, the numbers of sinus functions series and the LTB critical moment are the two variables to iterate. In this study, an approach is implemented by reducing the variable numbers from two to one, substituting an exact solution into the LTB differential equation of a disturbed segment from a cross beam moment. Thus, the trial and error process needs to be done for only one variable which is the LTB critical moment. This approach has proven to successfully converge for cases of beam with vertical stiffener (Tudjono, 2005) and thin rectangular beam with cross beam (Tudjono *et al.*, 2011) and significantly reduce the computational time.

2. Problem

Two main problems will be discussed herein the paper:

i. How much is the influence of flexural rigidity of a cross beam on the rise of LTB critical moment?
ii. How much is the minimum flexural rigidity of a cross beam to steadily hold the main beam (zero rotation) during the LTB? When this parameter is known, in the design process, engineers can decide whether the effect of cross beam(s) needs to be taken into account when calculating the critical moment.

3. Methodology

3.1 Scope of Study

The analysis is carried out with assumptions as follows:

- Material is under elastic condition
- Only flexural stiffness is accounted for, that the cross beam is assumed as a spiral spring with no shear force. Thus, the spiral spring's constant is around $\frac{2\,EI_{CB}}{L_{CB}}$ to $\frac{6\,EI_{CB}}{L_{CB}}$.
- Beam's web does not undergo distortions
- The main beam is I beam with 2 symmetrical axes.
- Spiral spring works on the centre of gravity of the main beam cross section.

3.2 Numerical Approach

3.2.1 Beam Without Cross Beam

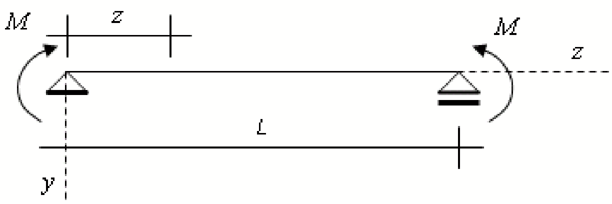

Figure 1: Beam having a constant moment

The differential elementary equation of a beam having a constant moment as shown in Fig. 1 is given as follows:

$$EIw\,\beta'''' - GJ\,\beta'' - \frac{M^2}{EI_y}\,\beta = 0 \quad (2)$$

The general solution of this differential equation is:

$$\beta = C_1 \sin \underline{a}z + C_2 \cos \underline{a}z + C_3\, e^{\underline{b}z} + C_4\, e^{-\underline{b}z} \quad (3)$$

$$\underline{a} = \sqrt{\sqrt{\left(\frac{GJ}{2EI_W}\right)^2 + \frac{M^2}{E^2 I_y I_W}} - \frac{GJ}{2EI_W}} \qquad \underline{b} = \sqrt{\sqrt{\left(\frac{GJ}{2EI_W}\right)^2 + \frac{M^2}{E^2 I_y I_W}} + \frac{GJ}{2EI_W}}$$

3.2.2 Beam with Cross Beam

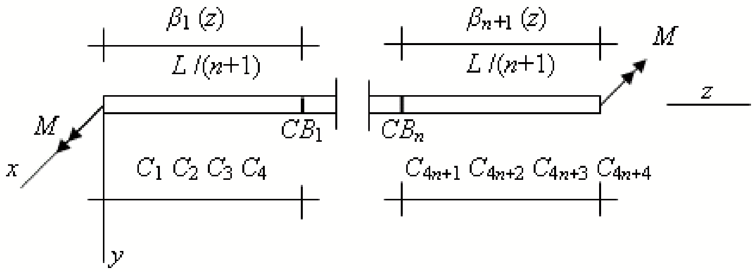

Figure 2: Beam with *n* cross beams

The presence of cross beam as in Fig. 2 yields a disturbance to Eq. (2). Weak axis moment will be disturbed by i^{th} cross beam moment of $K\beta_i \frac{du}{dz}$ as shown in Fig. 3. The influence of this disturbance will be lesser when β_i approaches 0. When the cross beam flexural rigidity is sufficient, the cross section will not rotate as $\beta_i = 0$.

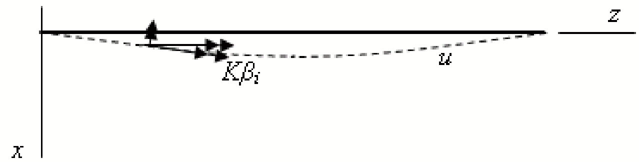

Figure 3: Moment disturbance on strong axis of main beam due to cross beam

And also, when β_i is very small, the results from numerical analysis will approach those from exact solution. In this case, the interference of such disturbance in moment analysis is ignored. In the critical condition, beam in between cross beams has similar differential equation to that of without cross beam. However, the solutions are different in terms of constants of integration. Each beam has 4 constants of integration. Thus, for *n* cross beams, the total of constants of integration will be 4 (*n*+1). By applying 4 boundary conditions at both ends and 4 at *n* cross beams, 4 (*n*+1) homogeneous equations will be obtained. The equations are derived using Eqs. (3) to (6). The LTB critical moment can then be obtained from a determinant of homogeneous equations' coefficients matrix which equals to zero. The homogeneous equations are governed from natural and geometrical boundary conditions, both of which are described as follows.

Boundary conditions at both ends:

- for $z = 0$ $\beta = 0, \beta'' = 0$
- for $z = L$ $\beta = 0, \beta'' = 0$

Boundary conditions on cross beams:

- Derivative and functional continuity : $\beta_L = \beta_R$, $\beta'_L = \beta'_R$
- Moment balance : $GJ\,\beta'_L - EIw\beta'''_L = GJ\,\beta'_R - EIw\beta'''_R + K\,\beta_R$
- Flexural web balance : $\beta''_L = \beta''_R$

Where K is cross beam's flexural rigidity. When the homogeneous equations are described in more detail in conjunction with cross beams, constants of integrations can be obtained as follows:

2 boundary conditions at left end:

$$C_2 + C_3 + C_4 = 0$$
$$-\underline{a}^2 C_2 + \underline{b}^2 C_3 + \underline{b}^2 C_4 = 0 \qquad (4)$$

4 boundary conditions on i^{th} cross beam:

- $C_{(1+4i)} \sin \underline{a}z_i + C_{(2+4i)} \cos \underline{a}z_i + C_{(3+4i)} e^{\underline{b}zi} + C_{(4+4i)} e^{-\underline{b}zi} =$
  $C_{(1+4(i-1))} \sin \underline{a}z_i + C_{(2+4(i-1))} \cos \underline{a}z_i + C_{(3+4(i-1))} e^{\underline{b}zi} + C_{(4+4(i-1))} e^{-\underline{b}zi}$
- $\underline{a}C_{(1+4i)} \cos \underline{a}z_i - C_{(2+4i)} \sin \underline{a}z_i + \underline{b}C_{(3+4i)} e^{\underline{b}zi} - \underline{b}C_{(4+4i)} e^{-\underline{b}zi} =$
  $\underline{a}C_{(1+4(i-1))} \cos \underline{a}z_i - \underline{a}C_{(2+4(i-1))} \sin \underline{a}z_i + \underline{b}C_{(3+4(i-1))} e^{\underline{b}zi} - \underline{b}C_{4+4(i-1))} e^{-\underline{b}zi}$
- $(GJ\underline{a} - EIw\underline{a}^3)C_{(1+4i)} \cos \underline{a}z_i - (GJ\underline{a} - EIw\underline{a}^3)C_{(2+4i)} \sin \underline{a}z_i + (GJ\underline{b} + EIw\underline{b}^3)C_{(3+4i)} e^{\underline{b}zi} - (GJ\underline{b} + EIw\underline{b}^3)C_{(4+4i)} e^{-\underline{b}zi} + KC_{(1+4i)} \sin \underline{a}z_i + KC_{(2+4i)} \cos \underline{a}z_i + KC_{(3+4i)} e^{\underline{b}zi} + KC_{(4+4i)} e^{\underline{b}zi} = (GJ\underline{a} - EIw\underline{a}^3)\, C_{(1+4(i-1))} \cos \underline{a}z_i - (GJ\underline{a} - EIw\underline{a}^3)\, C_{(2+4(i-1))} \sin \underline{a}z_i + (GJ\underline{b} + EIw\underline{b}^3)\, C_{(3+4(i-1))} e^{\underline{b}zi} - (GJ\underline{b} + EIw\underline{b}^3)\, C_{(4+4(i-1))} e^{-\underline{b}zi}$
- $-\underline{a}^2 C_{(1+4i)} \sin \underline{a}z_i - \underline{a}^2 C_{(2+4i)} \cos \underline{a}z_i + \underline{b}^2 C_{(3+4i)} e^{\underline{b}zi} + \underline{b}^2 C_{(4+4i)} e^{-\underline{b}zi} =$
  $-\underline{a}^2 C_{(1+4(i-1))} \sin \underline{a}z_i - \underline{a}^2 C_{(2+4(i-1))} \cos \underline{a}z_i + \underline{b}^2 C_{(3+4(i-1))} e^{\underline{b}zi} + \underline{b}^2 C_{4+4(i-1))} e^{-\underline{b}zi}$ (5)

2 boundary conditions at right end:

$$C_{(1+4n)} \sin \underline{a}L + C_{(2+4n))} \cos \underline{a}L + C_{(3+4n)} e^{\underline{b}L} + C_{(4+4n)} e^{-\underline{b}L} = 0$$
$$-\underline{a}^2 C_{(1+4n)} \sin \underline{a}L - \underline{a}^2 C_{(2+4n)} \cos \underline{a}L + \underline{b}^2 C_{(3+4n)} e^{\underline{b}L} + \underline{b}^2 C_{(4+4n)} e^{-\underline{b}L} = 0 \qquad (6)$$

$\underline{a}$ dan $\underline{b}$ values are function of moment. By applying trial and error method, the LTB critical moment is obtained. It is when a determinant of homogeneous equations' coefficients matrix which equals to zero.

4. Analysis Results

4.1 n Uniformly Distributed Cross Beams

Analyses were performed on main beam having uniformly distributed 1, 2 and 3 cross beams. For a case with 1 cross beam, it is located at mid span. For 2 cross beams, the locations are at $L/3$ and $2L/3$. And for 3 cross beams, the locations are at $L/4$, $L/2$ and $3L/4$. The trial and error method is carried out from smaller to greater number of the beam's flexural rigidity. There will be a point whereby the LTB critical moment is stable at one value as the flexural rigidity increases. The critical moments from the aforementioned analysis are then compared to those obtained from Eq. (1) with beam's length equals to $L/(n+1)$ and presented in Table 1.

Table 1: Critical moment of IWF 250×125 beam with cross beam(s)

Cross beam (n)	$\frac{KL}{EI_y}$ minimum	M_{cr} real (kNm)	M_{cr} formula $L/(n+1)$ (kNm)
1	0.7099	106.9239	106.9215
2	1.9127	213.9804	213.9803
3	4.1770	362.4113	362.4085

The results from the analysis of main beam with cross beams that have variable flexural rigidity are presented in Fig. 4. M_{cro} is the critical moment of main beam without cross beam and CB denotes cross beam.

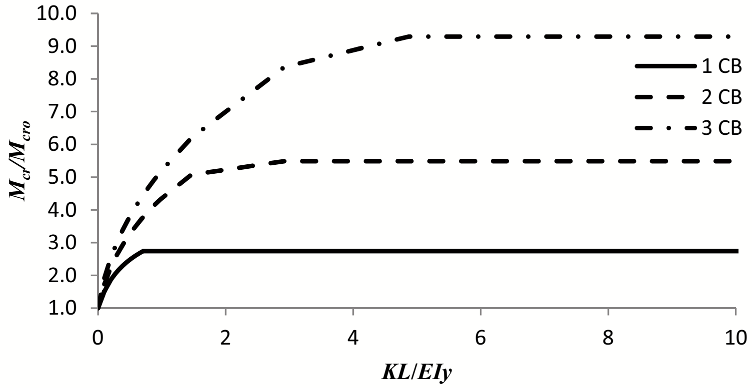

Figure 4: Critical moment for various *n* cross beams' rigidity

4.2 One Cross Beam

Beam with a cross beam located at $L/2$ and $L/3$ from left end are analyzed and compared as shown in Fig. 5.

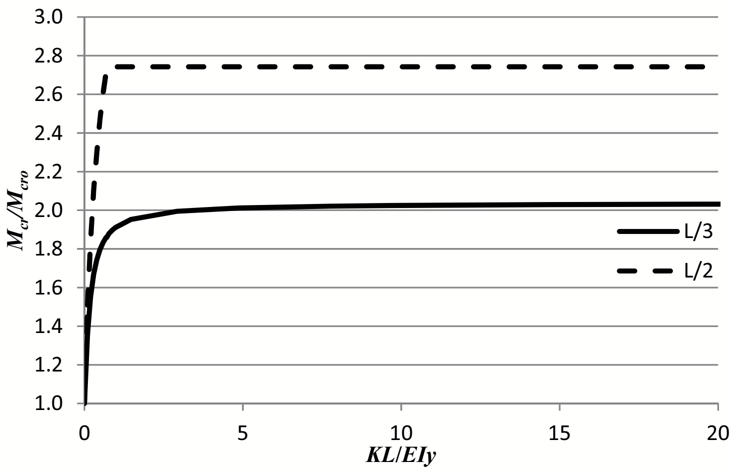

Figure 5: Critical moment on 1 cross beam of different location

The analysis shows that the critical moment's convergence rate of a case whereby the cross beam is at *L/2* is faster than that of *L/3*. The critical moment of IWF 250×125 beam with a length of *L* having the cross beam at *L/3* with *KL/EIy* = 48.6869 is 79.3488 kNm, yet it is not a stable one. Such critical moment is in between the critical moment for a main beam with a length equal to 2*L/3* (= 68.3842 kNm) and a length equal to *L/3* (= 213.9960 kNm). The first mode shape of the main beam with 1 cross beam located at *L/2* and *L/3* from the left end are shown in Fig. 6.

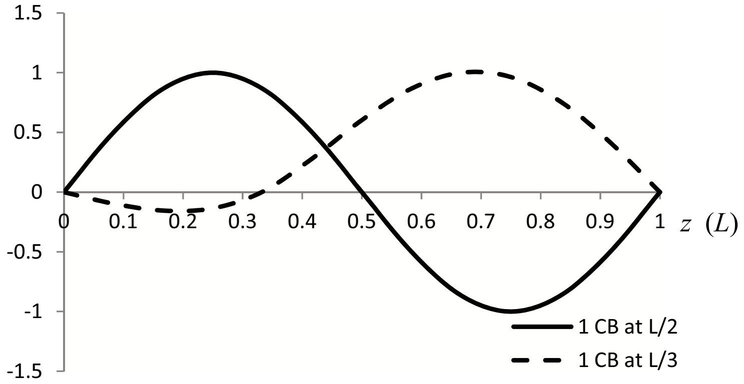

Figure 6: The first Mode Shape of LTB with 1 cross beam

The LTB modes shapes of beam with 2 cross beams uniformly distributed for various value of *KL/EIy* are shown in Fig. 7.

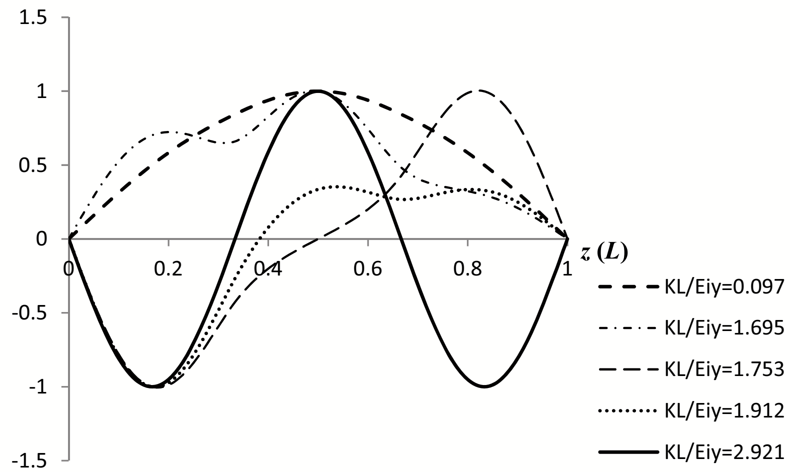

Figure 7: The Modes Shapes of LTB with 2 cross beams

5. Conclusion

From the analysis results, conclusions can be drawn as follows:

i. The approach has proven to successfully converge for cases of beam with cross beam(s).

ii. The analysis shows that to achieve LTB critical moment on a case of uniformly distributed *n* cross beams, the higher the *n* value (numbers of cross beams), the higher the minimum flexural rigidity of cross beam is needed.
iii. In the analysis using only 1 cross beam, the positioning of cross beam at mid span provides faster convergence rate than other locations due to its smaller flexural rigidity.
iv. When only 1 cross beam is installed, the positioning of cross beam at mid span shows the most optimum (largest) critical moment.

6. References

[1] Chajes, A (1970), *Principles of Structural Stability Theory*, Prentice Hall, New Jersey.

[2] Salmon, Charles G and Johnson, John E (1996), *Steel Structures Design and Behavior*, Harper Collins College Publishers, Wisconsin.

[3] Sri Tudjono (2005), *Study of The Influence of Vertical stiffeners to Lateral Torsional Buckling of I Section Steel Beams*, Doctoral Thesis, Technology Institute of Bandung, Indonesia.

[4] Sri Tudjono, Windu Partono and Joko Purnomo (2011), *Analysis on The Contribution of Cross Beam to A Torsional Buckling of Thin Rectangular Beam Section*, *in Proc. of the 3st International Conference of European Asian Civil Engineering Forum*, Yogyakarta, Indonesia.