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The Influence of Bearing Stiffeners to *Double - Symmetrical I-Section*'s Torsion Stiffness, an Analytical Approach

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ABSTRACT

Bearing Stiffeners are preventing *web-local-buckling* and reinforcing this section for point-loads and shear-forces. This paper discusses *bearing stiffeners'* contribution in enhancing double-symmetric I-sections' torsion capacity. Based on the *Saint Venant's* formula torsion stresses are carried solemnly by the section, neglecting the stiffeners' contribution. However, these stiffeners exhibit significant rotational deformation *with* the I-section in warping, indicating development of internal forces, restraining the warping. Therefore, the negligence of stiffeners' contribution in *Saint Venant's torsion formula* has to be revised.

Torsions within the stiffeners are the *Saint Venant's* and *torsion-shear-stresses* induced by bending. Assuming *out-of-plane* stresses neglected, normal and *bending-shear* torsion stresses are zero, leaving only the *Saint Venant's*. From equilibrium at the *stiffener –to-flange's-joint*, the stiffeners' *natural boundary conditions* equation can be obtained. Their presence leads to a *rotational-torsion* function differentiation along the beam, between stiffeners. But since all points have identical internal torsion forces, the *disturbed differential torsion warping equations* are identical. Using the *geometrical and natural boundary conditions* equation the *mathematical-rotational-torsion-solution* for each field along the beam is obtained.

It can be concluded that the member's *torsion stiffness* increases approaching to *linear* while the increment will approach a *hyperbola* as a function of stiffeners' number and thickness, respectively.

Keywords: stiffeners, double-symmetric I-section, torsion, warping

I. Introduction

In design it is assumed that shear forces acting on an I-section are carried by the web only. When exceeding the web's shear capacity, reinforcement by bearing stiffeners are used so that the resultant shear capacity of stiffener and the web will be greater than or equal to the factored shear force.

In the case of reinforced sections with bearing stiffeners subjected to torsion, the torsion will be carried by *Saint Venant's* section, neglecting the contribution of bearing stiffeners. The actual behavior is that sections carrying a torsion force will undergo rotation and warping, also in the area where the bearing stiffener is present.

The opposite warping of the two flanges will produce an orthogonal rotation of the bearing stiffeners. This bearing stiffener’s rotation proves the presence of an internal torsion force restraining the warping.

This paper will analyze the significance of bearing stiffeners’ contribution to the torsion behavior of I-sections.

2. Methods

For evaluating the bearing stiffener’s contribution in carrying torsion forces, the finite element method can be used. However, this paper will approach the issue analytically

For the case of a section *with* restrained warping solved by the analytical method, two major points has to be explained, i.e.

1. The analytical formulation of the relationship between the stiffeners’ internal forces to the section’s torsion deformation
2. The section’s natural boundaries at the location of stiffeners

For analysis’ purposes the following assumptions were used: the material is perfectly elastic; the behavior is based on *Navier’s* hypothesis; the plates’ thickness are considered small if compared to all other dimensions; overlapping of connected plate-elements is neglected; all the *out-of-plane* stresses are ignore.

Internal forces in the bearing stiffener

Due to warping of the bearing stiffener, the flanges will rotate in an opposite direction as shown in figure 1 (a).

The flange tips will undergo warping of respectively $+\frac{1}{4}bh\phi'$ and $-\frac{1}{4}bh\phi'$. The flange will rotate with an angle of α to its original position.

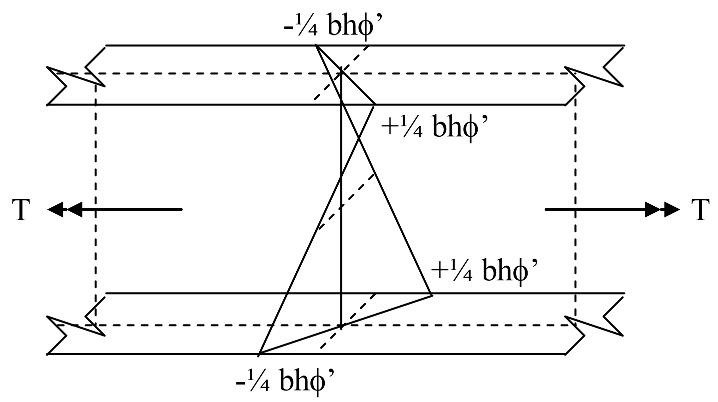
$$\alpha = \frac{\frac{1}{4}bh\phi'}{\frac{1}{2}b} = \frac{1}{2}h\phi', \beta' = \frac{2\alpha}{h} = \frac{h\phi'}{h} = \phi' \qquad (1)$$

Considering the rotation per-unit-length of the element to be *β'*, a *Saint Venant*’s torsion of T_v will act on the bearing stiffener.

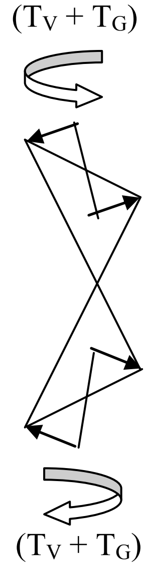
$$T_V = \frac{1}{3}btv^3G\beta' = \frac{1}{3}btv^3G\phi' \qquad (2)$$

Saint Venant’s torsion will induce shear stresses parallel and perpendicular to the flange. Bending shear in the bearing stiffener will produce a torsion force T_G

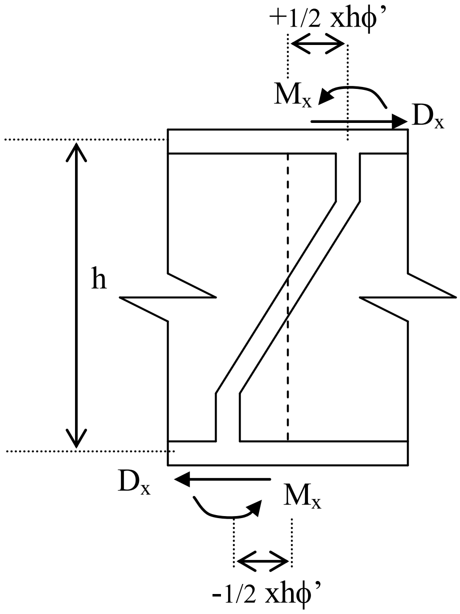
$$T_G = \int_{-b/2}^{+b/2} D_X X dx = \int_{-b/2}^{+b/2} \frac{2M_X}{h} X\,dX = F(L,h,b)\phi' \qquad (3)$$



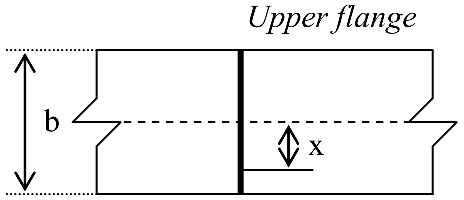
(a) A section in warping



(b) Torsion in the stiffener



(c) Bending and shear in x-section



(d) Position of internal force

Figure 1. Warping and torsion in the bearing stiffener

The bending moment M_x in the bearing stiffener will generate a *normal stress* perpendicular to the flange. Following the assumption that all *out-of-plane* stresses are neglected,

- Normal stresses perpendicular to the flange are ignored, and therefore the bending moment in the stiffeners will be zero and T_G will be vanished.
- The *Saint Venant’s* shear stress perpendicular to the flange will decrease to zero in the direction of the flange’s outer fibres.

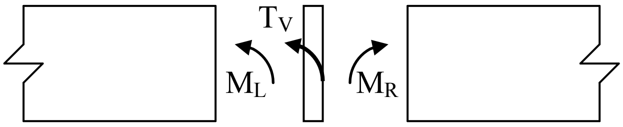


Figure 2. Moment equilibrium at the *stiffener –to-flange’s-joint*

From the flange's moment equilibrium at the position of the stiffener, the following equation (4) can be written.

$$M_R - M_L - T_V = 0, \;\; or \;\; M_R = M_L + T_V \,, or$$

$$EI_S \frac{h}{2} \phi''_R = EI_S \frac{h}{2} \phi''_L + \frac{b}{3} tv^3 G\phi'$$

Since $\phi'_R = \phi'_L = \phi$ the equilibrium equation becomes:

$$EI_S \frac{h}{2} \phi''_R = EI_S \frac{h}{2} \phi''_L + \frac{b}{3} tv^3 G\phi'_L \tag{4}$$

This is the equation of *natural boundary conditions*

3. Analysis

3.1. A beam with one bearing stiffener at mid- point

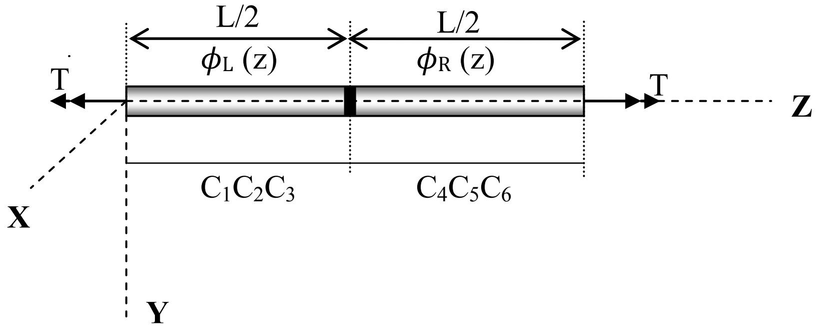


Figure 3. Beam with one bearing stiffener at mid-point

All torsion elements with restrained warping will have the following rotation equation.

$$\phi''' - \lambda^2 \phi' = -\frac{T}{EI_W} \tag{5a}$$

Given the general solution as:

$$\phi = C_1 + C_2 cosh\lambda z + C_3 sinh\lambda z + \frac{T}{\lambda^2 EI_W} \tag{5b}$$

The stiffener at mid-point will divide the ϕ into two segments, ϕ_L and ϕ_R each having a different integration constant

$$\phi_L = C_1 + C_2 cosh\lambda z + C_3 sinh\lambda z + \frac{T}{\lambda^2 EI_W} \tag{6a}$$

$$\phi_R = C_4 + C_5 cosh\lambda z + C_6 sinh\lambda z + \frac{T}{\lambda^2 EI_W} \tag{6b}$$

Where: $\lambda = \sqrt{\frac{GJ}{EI_W}}$

From the natural and geometric boundaries for a simply supported beam the *integration-constants* can be calculated.

By substituting the value of z = L into the section's rotation equation we will achieve:

$$\phi = -\frac{\dfrac{TGJ_{PV}}{\lambda^2 EI_W}}{\cosh\lambda\dfrac{L}{2}\left\{\dfrac{EI_S\dfrac{h}{2}\lambda^2}{\cosh\lambda\dfrac{L}{2}} - \dfrac{GJ\lambda}{2\sinh\lambda\dfrac{L}{2}}\right\}} + \frac{TL}{\lambda^2 EI_W} \qquad (7)$$

The first term of the equation represents the bearing stiffener's contribution and the second one is the *Saint Venant's* torsion. It is demonstrated that *one* bearing stiffener at mid-point reduce the section's rotation. It can be seen that the bearing stiffener contributes to the beam's torsion stiffness.

3.2. A beam with multiple bearing stiffeners.

In the case of a beam with *n* number of bearing stiffeners, the beam will be divided in (n+1) fields, each having a different rotation function. Integrating the differential equation 5a, 3(n+1) *integration-constants* for *every* section will be produced. By applying the geometric and natural boundary conditions at the supports and the bearing stiffeners, these 3(n+1) constants can be derived.

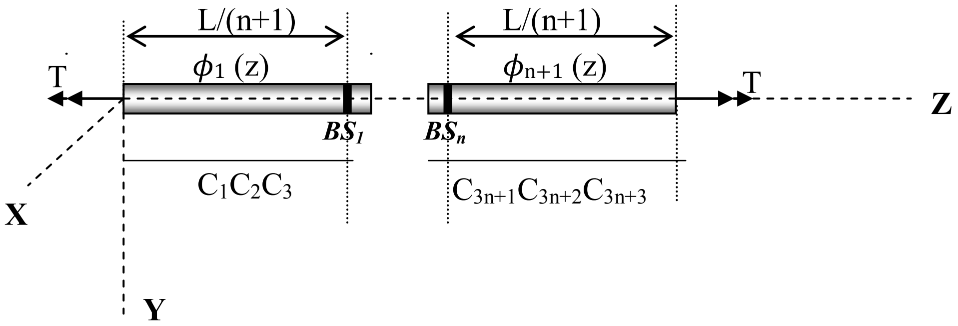


Figure 4. A beam with multiple stiffeners

From analysis a curve as a function of the section's rotation, thickness and number of bearing stiffeners can be developed. The following curves are developed for a *WF 250 x 125* beam.

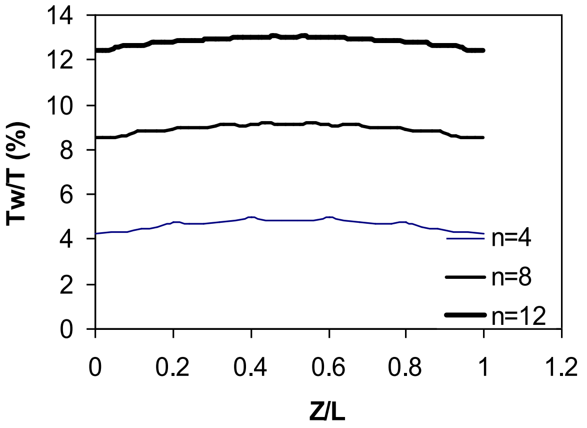


Figure 5. Torsion-warping contribution - stiffener's number relationship for tv/b = 0.048

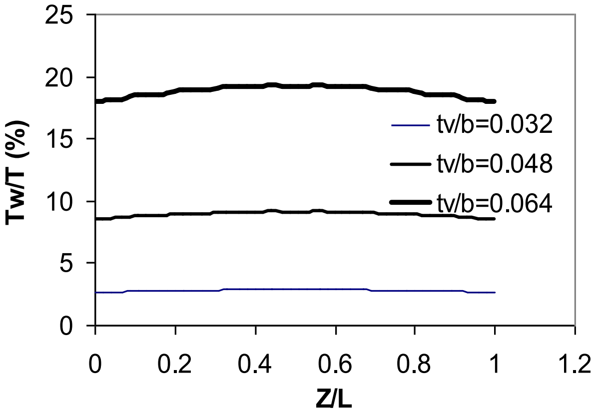


Figure 6. Torsion-warping contribution - BS thickness relationship for n = 8

The torsion-warping contribution in carrying torsion as a function number of stiffeners is shown in figure 5, while the contribution of BS thickness can be seen in figure 6. The horizontal axis is the ratio between *the stiffener's positions to the beam's length*, and the vertical axis represents the *torsion-warping to torsion-force ratio*. The influence of the torsion-warping is more significant approaching mid-point, and increases approaching the position of stiffener.

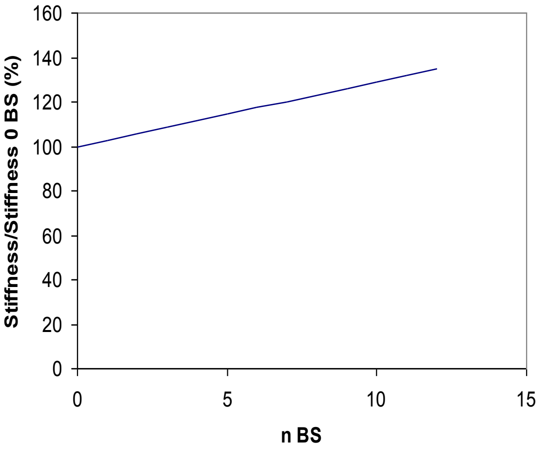


Figure 7. Torsion-stiffness ratio - n relationship for tv/b=0.064

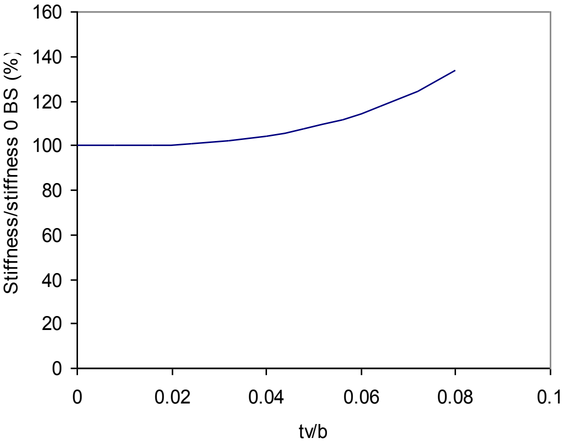


Figure 8. Torsion-stiffness ratio - BS thickness relationship for n = 6 stiffeners

The torsion-stiffness ratio increase for multiple stiffeners with a ratio tv/b = 0.064 is shown in figure 7, while figure 8 shows the torsion-stiffness ratio increase as a function of bearing stiffener's tickness.

4. Result and Conclusion

From the analytical analysis of a *WF 250 x 125* section having equally distributed bearing stiffeners along the beam, the following can be concluded;

1. The torsion-warping contribution increases toward mid-point of the beam
2. The torsion-warping contribution increases in the direction of the bearing stiffeners
3. The member's *torsion stiffness* increases approaching to *linear* as a function of the stiffeners' number n
4. The member's *torsion stiffness* increment will approach a *hyperbola* as a function of stiffeners' thickness

5. References

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Nomenclature (Notation)

ϕ : Angle of section rotation to its reference position
T : Torsion Moment
E : Modulus of Elasticity
G : Modulus of Rigidity
L : Length of beam
BS : Bearing Stiffener
b : Width of flange
h : Distance from the centroid of the upper flange to the bottom flange of an I section
t_v : Vertical bearing stiffeners thickness
α : Angle of flange section rotation at bearing stiffener
β : Angle of bearing stiffeners rotation
I_S : Flange's moment of inertia with respect to the section's week axis
I_W : Warping constant
T_G : Stiffeners torsion moment resulting from shear bending
Tv : St. Venant orsion moment of bearing stiffeners
Tw : Torsion warping moment of section
J_{PV} : Torsion constant of bearing stiffener
J : Torsion constant of section
Z : Variable of longitudinal axis