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Analysis of Crout, Lu, Cholesky Decomposition, and QR Factorization: A Case Study On The Relationship Between Abiotic (Carbon and Nitrogen) and Biotic (Macrobenthos Diversity) Factors

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Abstract *- Many real world problems can be represented by a system of linear equations, such as in the field of ecology, i.e, the relationship of carbon and nitrogen with macrobenthos diversity. There are many methods to solve linear equations system, then it is necessary to do an analysis of which method is the best so that the user can choose the most efficient method. The methods that will be analyzed are LU, Crout, Cholesky decomposition, and QR factorization. From the calculation of arithmetic operations obtained Cholesky decomposition method is the most efficient method because it has the fewest arithmetic operations. Further, to verify the proposed method we demonstrated simulation with a case study of the relationship between carbon and nitrogen with the macrobenthos diversity based on data from the area of polyculture system and PT. Kayu Lapis Indonesia coastal, Mororejo village subdistrict Kaliwungu district Kendal. From the simulation resultsis obtained that computing time the smallest is the Cholesky decomposition is equal to 1.4664 seconds, which means that the Cholesky decomposition is the most efficient method than the method of LU, Crout decomposition and QR factorization.*

Keywords— *LU Decomposition, Crout decomposition, QR factorization, Cholesky decomposition, Carbon, Nitrogen, Macrobenthos*

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I. INTRODUCTION

In this research will be presented mathematically relationship carbon and nitrogen with macrobenthos diversity through regression models, especially on how to determine the parameters of the multiple linear regression equation. In the process of determining the regression parameter is used least squares method that produces systems of linear equations. Here will be studied how to determine the solutions of the system of linear equations. The system of linear equations can be solved by several methods, such as, the LU decomposition method (Chinchole and Bhadane, 2014), Crout decomposition (Supriyono and Daniel, 2005), QR factorization (Bojanczyk, et al., 1986; Rorres, 2004), and Cholesky Decomposition (Robert and Elizabeth , 1990). Therefore, the assessment needs to be conducted to determine the most efficient method.

One of the applications of mathematics in the field of ecology, in this case represents the relationship between organic matter, carbon and nitrogen to macrobenthos diversity. Animal abundance macrobenthos have a very strong relationship with the content of organic matter in sediments and sedimentary textures (Kinanti *et al*., 2014) This means organic materials such as carbon and nitrogen affect the macrobenthos abundance.

Nitrogen cycle showing a high response to internal feedback mechanism as well as the interdependence of carbon and metal cycles. In contrast, the carbon cycle would appear mainly controlled by the decay of organic matter. Benthic carbon and nitrogen cycle has the potential to greatly affect the fertility level of water in it (Holstein and Wirtz, 2010). Makrobentos react to changes in water quality and analysis makrobentos is one of the most important methods to monitor changes in water quality parameters (Cupsa *et al*., 2010).

Putro (2007) have published about spatial and temporal patterns of the macrobenthic assemblages in relation to environmental variables.

This research will be presented as a mathematical relationship between the carbon and nitrogen to macrobenthos through a system of linear equations. For efficiency, both time and cost will be analyzed Crout, LU, Cholesky decomposation and QR factorization to solve this linear equations system that requires the least mathematical operations. This can be demonstrated through a case study with data from the area of polyculture system and PT. Kayu Lapis Indonesia coastal, Mororejo village subdistrict Kaliwungu district Kendal, to choose the best method that requires the least computation time.

II. MATERIALS AND METHODS

Sampling stations were located at Mororejo village subdistrict Kaliwungu district Kendal. Location I was a cultivated area polyculture system with biota are cultivated in the polyculture ponds, i.e., fish and tiger shrimp that are cultured together with seaweed in coastal waters Mororejo village subdistrict Kaliwungu Kendal district, Central Java. Location II was the coastal area of PT. Kayu Lapis Indonesia Mororejo village subdistrict Kaliwungu district Kendal, Central Java, which is located adjacent to industrial activities as well as direct hit tide. Each location had 3 stations with 3 replicates. Sampling procedures included collecting sediments, fixation, rinsing, sorting, preservation, and identification. Sediment was analysed for sediment grain size, and total organic content.

Two-way analysis of variance was used to compare the results of measurements of physical parameters and the sediment-water chemistry between locations and times of sampling. Data were tested using Kolmogorov-Smirnov test for normal distribution of the data and Levene test for homogeneity of variance. Further post hoc test using Tukey HSD performed to further compare the results of the analysis showed a significant difference ($P<0.05$). Multivariate analysis using the principal component analysis with Euclidean distance performed to describe differences in environmental variability between sampling locations. While multivariate analysis using non-metric multi dimensional scaling using Bray-Curtis similarity was performed to the data macrobenthos to describe the differences between the location and time of sampling.

Furthermore, the data of carbon, nitrogen, and macrobentos will bestudied mathematically through multiple linear regression model. In this case the emphasis on the process of finding the parameters of the regression equation is represented in the form of a system of linear equations. Here will be analyzed Crout, LU, Cholesky decomposition, and QR factorization methods.

III. RESULT AND DISCUSSION

3.1 Analisys of Arithmetic Operations

Comparison of theanalisys of arithmeticoperationsis how much of the necessary arithmetic operations. In this analysis, the division will be combined with the multiplication and addition by subtraction. In this case will be compared LU, Crout, Chelosky decomposition, and QR factorization.

3.1.1 LU Decomposition Method

LU decomposition method can be explained using the following algorithm,

0. Given, matrices A = [a_{ij}], B
1. Find for i = 1, 2, …, n
   $l_{ii} \quad a_{ii}/a_{ii}$
2. Find for i= 2, 3,…, n and j=1, 2, …, n;
   $u_{1j} : \quad a_{1j,}$
   $l_{i1} : \quad a_{i1}/u_{11}$
3. Find for i=3,4,...,n and j=2, 3, ..., n;
   $u_{2j} : \quad a_{2j} - l_{21}u_{1j}$
   $l_{i2} : \quad a_{i2} - l_{i1}u_{12} \ /u_{22}$
4. Find for i = q+1, q+2, ... , n andj p, p 1, …, n;
   $u_{pj} : \quad a_{pj} - \sum^{p}{}_{1}^{1} l_p \ u \ _j$
   $l_i : \quad a_{ip} - \sum \ {}_{1}^{1} l_i \ u \quad /u$
4. Complete the matrix L according to the equation LS = B, to find the value of S.
5. Complete the matrix U according to the equation UZ = S, to find the value of Z.

Explanation of the LU Decomposition algorithm can be seen through the flowchart in the following figure 1.

The total number of arithmetic operations for LU decomposition is,

Addition : $^3 - 4\ ^2 \quad 9 \ - 7$

Multiplication : $^3 - \frac{}{2} \quad \frac{23}{2} - 7$

3.1.2 Crout Decomposition Method

Crout decomposition method can be explained using the followingalgorithm,

0. Given matricesA = [a_{ij}], B
1. Find for i=1,2,3,…,n and j=1,2,…,n;
   $l_{i1} \quad a_{i1}$
   $u_{1j} \quad —$
2. Find for i=1,2,...,n, j =2,3,...,n-1 and k=1,2,...,n;
   $- \sum \ {}_{1}^{1}$
   $\frac{\sum}{}$
3. Complete the matrix L according to the equation LS = B, to find the value of S.
4. Complete the matrix U according to the equation UZ = S, to find the value of Z.

Explanation of theCrout decomposition algorithm can be seen through the flowchart in the following figure 2.

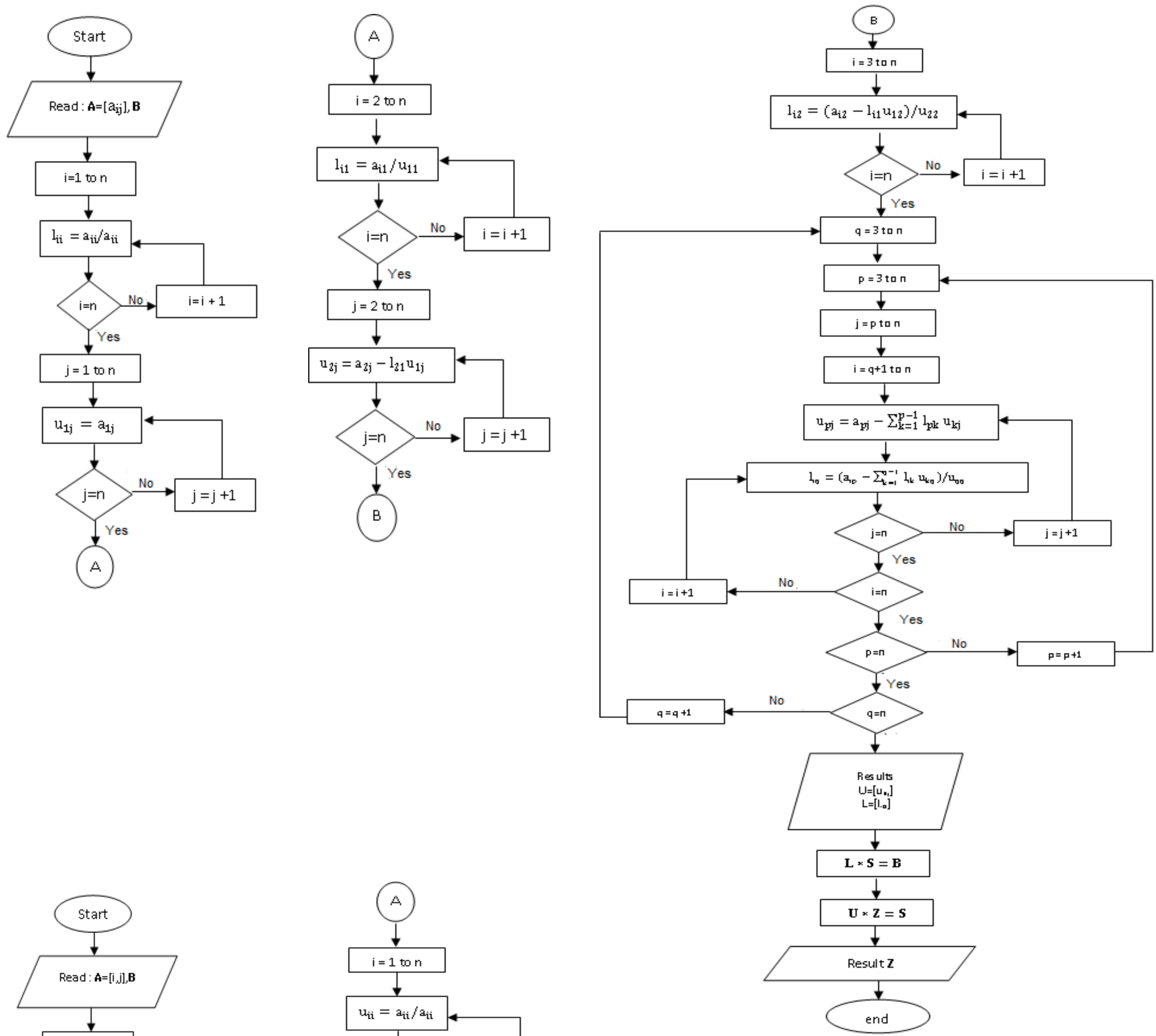

Figure 1. Flowchart LU Decomposition

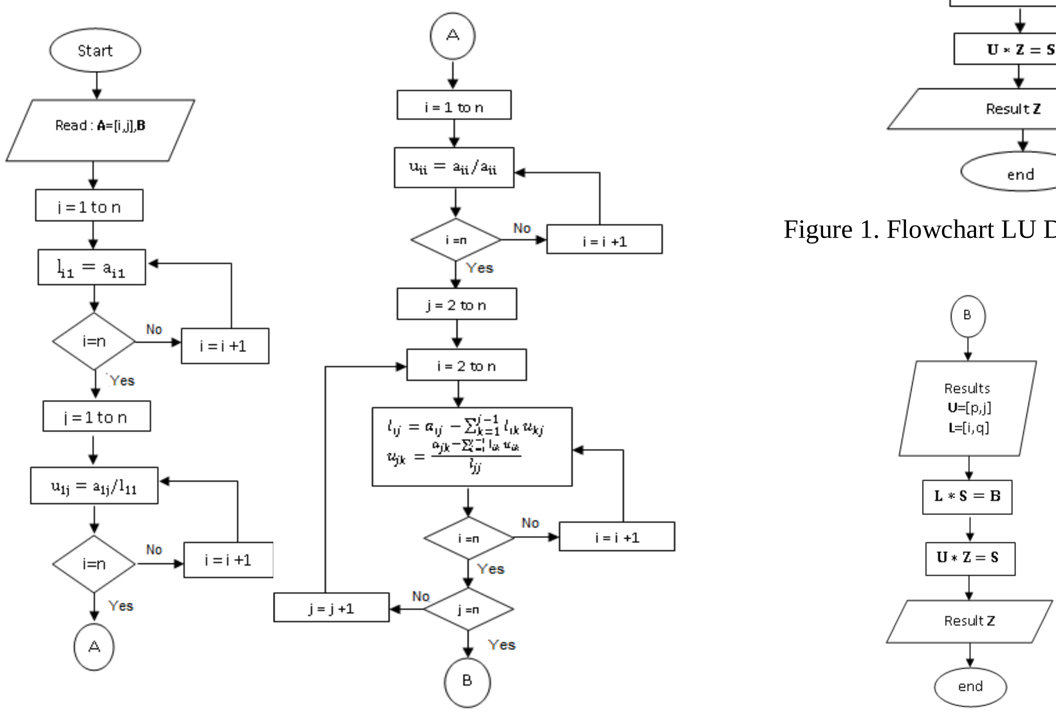

Figure 2. Flowchart Crout decomposition

The total number of arithmetic operations for Crout decomposition is,

Addtion : $^3 - 2\ ^2\ \ 2 - 1$

Multiplication : $3 - \frac{3}{2} \quad \frac{}{2} - 1$

3.1.3 QR factorization method

QR factorization method can be explained using the following algorithm,

0. Given, matrices A = $[a_{ij}]$, B
1. Find for j = 1,2, 3, ...,n;

$$\frac{p \quad y}{p \quad y}$$

$$\frac{\langle \ , \ \rangle \quad \langle \ , \ \rangle \quad \cdots \langle \quad , \quad \rangle}{\langle \ , \ \rangle \quad \langle \ , \ \rangle \quad \cdots \langle \quad , \quad \rangle}$$

2. Find for i = 1, 2, 3, ...,n and j = 1, 2, 3, ...,n;
   r_{ij}
3. Complete the appropriate matrix equation Q, from equation QS = B, to find the value of S.
4. Complete the matrix R according to the equation RZ = S, to find the value of Z.

Explanation of the QR factorization algorithm can be seen through the flowchart in the following figure,

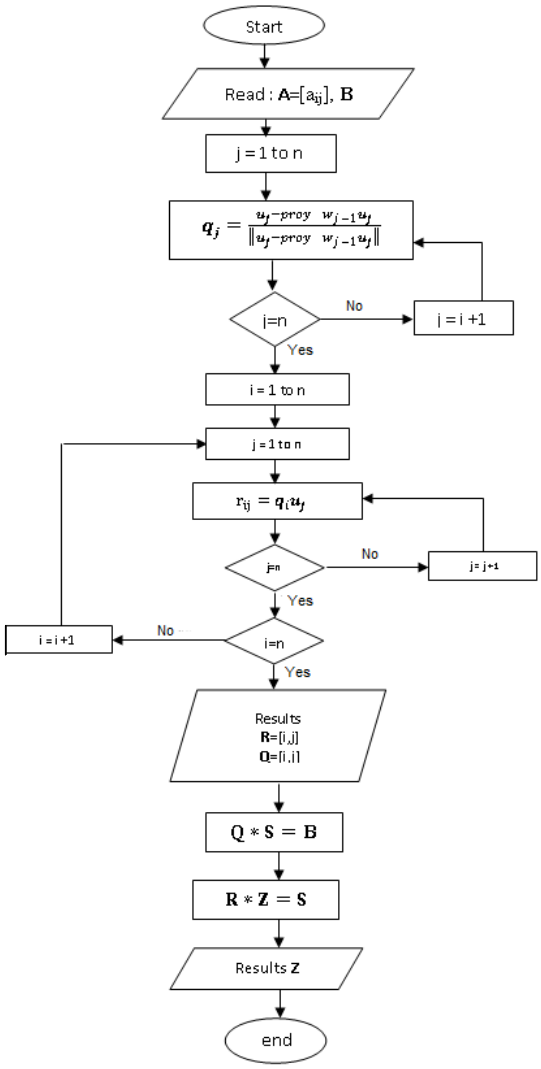

Figure 3. Flowchart QR factorization

The total number of arithmetic operations for the QR factorization is,

Addition : $\frac{}{2} \quad 2 \ ^2 - \frac{3}{2}$

Multiplication : $\frac{}{2} \quad \frac{}{2}$

3.1.4 Cholesky Decomposition Method

Cholesky decomposition method can be explained using the following algorithm,

0. Given, matrices**A** = $[a_{ij}]$, **B**
1. Find for i = 1 and j =1;
   $l_{11} \quad \sqrt{a_{11}}$
2. Find for i=2,3,...,n;
   $l_{2i} \quad a_{i1}/l_{11}$
3. Find for i=2,3,...,n;
   $l_{ii} \quad \overline{a_{ii} - \sum^{i}{}_{1}^{1} l_i^2}$
4. Find for i = 2, 3, ...,n and p = i+1, i+2, ...,n;
   $l_{pi} \quad a_{pi} - \sum^{i}{}_{1}^{1} l_i \ l_p \ \ / \ l_{ii}$
5. Complete the matrix L according to the equation LS = B, to find the value of S.
6. Complete the appropriate matrix L^T from the equation $L^T Z$ = S, to find the value of Z.

Explanation of theCholesky decomposition algorithm can be seen through the flowchart in the following figure,

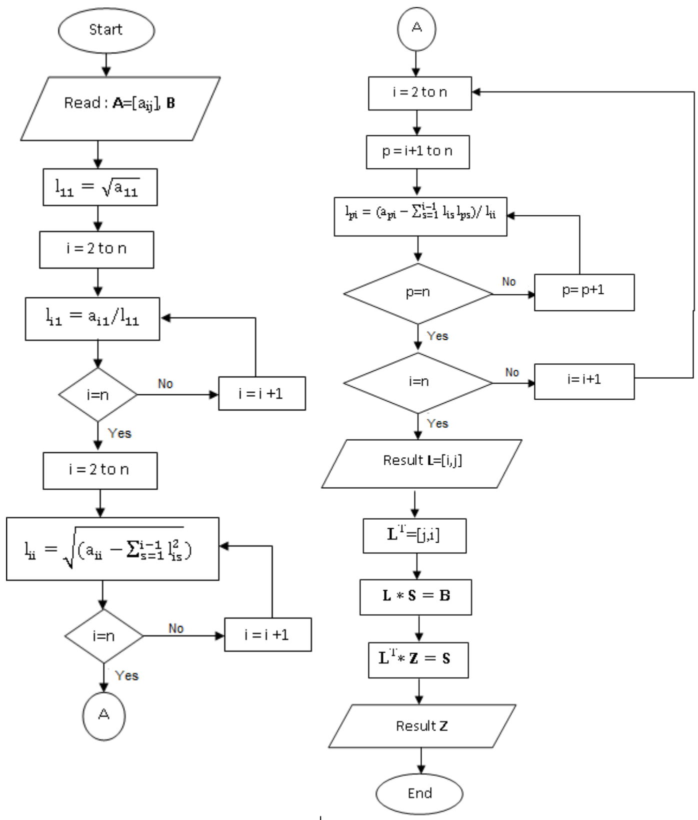

Figure 4. Flowchart Cholesky Decomposition

The total number of arithmetic operations for the Cholesky decomposition is,

Addition : $\frac{}{2} - \frac{}{2}$

Multiplication : $\frac{}{2} \quad \frac{}{2}$

4.2. SIMULATION RESULTS

The data used in this paper is the measurement data of carbon, nitrogen and makrobentos based on research data that has been done Putro, et. al (2014). Regression model based on the data of carbon, nitrogen and makrobentos sequential type to forms defined variables X_1, X_2 and Y. The next step, substitute the data of carbon and nitrogen into the systems of linear equations (Montgomery and Elizabeth, 1992) as follows,

1 1 1 / 1 2 2 / 1
1 1 / 1 2 1 1 / 1 1 2 2 / 1 1
1 2 / 1 1 2 1 / 1 2 2 2 / 1 2

Further, we find the systems of linear equations in the form,

$$36 \quad + 72{,}49 \quad _1 + 32{,}15 \ _2 = 21{,}33$$
$$72{,}49 \quad + 164{,}17 \ _1 + 67{,}28 \ _2 = 49{,}88$$
$$32{,}15 \quad + 67{,}28 \ _1 + 29{,}28 \ _2 = 20{,}15 \qquad (3.1)$$

Rewrite the equation (3.1) in the matrix form,

$$\begin{matrix} 36 & 72{,}49 & 32{,}15 \\ 72{,}49 & 164{,}17 & 67{,}28 \\ 32{,}15 & 67{,}28 & 29{,}28 \end{matrix} \quad \begin{matrix} \\ _1 \\ _2 \end{matrix} \quad \begin{matrix} 21{,}33 \\ 49{,}88 \\ 20{,}15 \end{matrix} .$$

Completion of the linear equation (4.1) will be calculated using the method of LU, Crout decomposition, QR factorization, and Cholesky Decomposition. Furthermore, all four methods are compared and will be analized the most efficient method.

3.2.1 Simulation of the LU decomposition method

Finding the value of the regression model parameters can be calculated by the method of LU decomposition. Completion the systems of linear equations using LU decomposition is as follows:

1. Form the matrix A into matrices L and U matrices, suppose the above matrix with AZ = B,

$$\begin{matrix} 36 & 72{,}49 & 32{,}15 \\ 72{,}49 & 164{,}17 & 67{,}28 \\ 32{,}15 & 67{,}28 & 29{,}28 \end{matrix} , \quad \begin{matrix} \\ _1 \\ _2 \end{matrix} , \quad \begin{matrix} 21{,}33 \\ 49{,}88 \\ 20{,}15 \end{matrix}$$

2. We can find the matrices L and U,

$$\begin{matrix} 36 & 72{,}49 & 32{,}15 \\ 0 & 18{,}2033 & 2{,}5424 \\ 0 & 0 & 0{,}2132 \end{matrix}$$
$$\begin{matrix} 1 & 0 & 0 \\ 2{,}0136 & 1 & 0 \\ 0{,}8931 & 0{,}1397 & 1 \end{matrix}$$

3. Obtain the value of matrix Z from equation LS = B

$$\begin{matrix} 1 & 0 & 0 \\ 2{,}0136 & 1 & 0 \\ 0{,}8931 & 0{,}1397 & 1 \end{matrix} \quad \begin{matrix} _1 \\ _2 \\ _3 \end{matrix} \quad \begin{matrix} 21{,}33 \\ 49{,}88 \\ 20{,}15 \end{matrix}$$

then, with foreward substitution, we find

$$\begin{matrix} 21{,}33 \\ 6{,}9297 \\ 0{,}1333 \end{matrix}$$

4. Look for the values of parameter from equation, UZ = S,

$$\begin{matrix} 36 & 72{,}49 & 32{,}15 \\ 0 & 18{,}2033 & 2{,}5424 \\ 0 & 0 & 0{,}2132 \end{matrix} \quad \begin{matrix} \\ _1 \\ _2 \end{matrix} \quad \begin{matrix} 21{,}33 \\ 6{,}9297 \\ 0{,}1333 \end{matrix}$$

Further, with backward substitution, we obtain

$$\begin{matrix} -0{,}5566 \\ 0{,}2934 \\ 0{,}6252 \end{matrix}$$

So that we have the regression model as follows,

$$-0{,}5566 \quad 0{,}2934 \ _1 \quad 0{,}6252 \ _2$$

Retrieved computing time for LU Decomposition method with Matlab R2008a,*cputime* = 1,5600 seconds.

3.2.2 Simulation of the Crout decomposition method

Finding the value of the regression model parameters can be calculated by the method of Crout decomposition. Completion the systems of linear equations using Crout decomposition is as follows:

1. Form the matrix A into matrices L and U matrix, suppose the above matrix with AZ = B,

$$\begin{matrix} 36 & 72{,}49 & 32{,}15 \\ 72{,}49 & 164{,}17 & 67{,}28 \\ 32{,}15 & 67{,}28 & 29{,}28 \end{matrix} , \quad \begin{matrix} \\ _1 \\ _2 \end{matrix} , \quad \begin{matrix} 21{,}33 \\ 49{,}88 \\ 20{,}15 \end{matrix}$$

2. Then, find matrixL = (l_{ij}) and U = (u_{ij}) as follows,

$$_{11} \quad 36$$
$$_{21} \quad 72{,}49$$
$$_{31} \quad 32{,}15$$
$$_{12} \quad _{12}/\ _{11} = 2{,}01361$$
$$_{13} \quad _{13}/\ _{11} = 0{,}893056$$
$$_{22} \quad _{22} - \ _{21}\ _{12} \quad 18{,}20341$$
$$_{23} \quad _{23} - \ _{21}\ _{13}\ /\ _{22} \quad -0{,}28621$$
$$_{32} \quad _{32} - \ _{31}\ _{12} \quad 2{,}54244$$
$$_{33} \quad _{33} - \ _{31}\ _{13} - \ _{32}\ _{23} \quad 1{,}29592$$

We can obtain,

$$\begin{matrix} 1 & 2{,}01361 & 0{,}893056 \\ 0 & 1 & -0{,}28621 \\ 0 & 0 & 1 \end{matrix}$$
$$\begin{matrix} 36 & 0 & 0 \\ 72{,}49 & 18{,}20341 & 0 \\ 32{,}15 & 2{,}54244 & 1{,}29592 \end{matrix}$$

3. Find Sfrom equationLS = B

$$\begin{matrix} 36 & 0 & 0 \\ 72{,}49 & 18{,}20341 & 0 \\ 32{,}15 & 2{,}54244 & 1{,}29592 \end{matrix} \quad \begin{matrix} _1 \\ _2 \\ _3 \end{matrix} \quad \begin{matrix} 21{,}33 \\ 49{,}88 \\ 20{,}15 \end{matrix}$$

With forward substitution, can be found,

$$\begin{matrix} 0{,}5925 \\ 0{,}3807 \\ 0{,}1028 \end{matrix}$$

4. Find parameter values from equationUZ= S

$$\begin{matrix} 1 & 2{,}01361 & 0{,}893056 \\ 0 & 1 & -0{,}28621 \\ 0 & 0 & 1 \end{matrix} \quad \begin{matrix} \\ _1 \\ _2 \end{matrix} \quad \begin{matrix} 0{,}5925 \\ 0{,}3807 \\ 0{,}1028 \end{matrix}$$

With backward substitution, can be found,

$$\begin{matrix} -0{,}3251 \\ 0{,}4101 \\ 0{,}1028 \end{matrix}$$

We have a regression model that representing the relasionship carbon and nitrogen with macrobethos diversity,

$$-0{,}3251 \quad 0{,}4101 \ _1 \quad 0{,}1028 \ _2$$

Retrieved computing time for Chelosky Decomposition method with Matlab R2008a, *cputime* = 2,2932 seconds

3.2.3 Simulation Method QR Factorization

Finding the value of the regression model parameters can be calculated by the method of QR factorization. Completion the systems of linear equations using QR factorization is as follows:

1. Form the matrix A into matrices Q and R matrices, we let the matrix above with AZ = B,

36	72,49	32,15			21,33
72,49	164,17	67,28 ,		1 ,	49,88
32,15	67,28	29,28		2	20,15

2. We can find matrices R and Q by using Matlab R2008a,

− 87,0886	− 191,453	− 80,101
0	− 8,8838	− 1,1592
0	0	0,1806

− 0,4134	0,7487	− 0,5182
− 0,8324	− 0,5414	− 0,1183
− 0,3692	0,3825	0,847

3. Obtain matrix S based on equation QS =B,

− 0,4134	0,7487	− 0,5182	1	21,33
− 0,8324	− 0,5414	− 0,1183	2	49,88
− 0,3692	0,3825	0,847	3	20,15

Then we find,

− 57,7745
− 3,3309
0,1129

4. Further, look for the values of parameter from equation R Z = S

− 87,0886	− 191,453	− 80,101		− 57,7745
0	− 8,8838	− 1,1592	1	− 3,3309
0	0	0,1806	2	0,1129

− 0,5566
0,2934
0,6252

We have a regression model that representing the relasionship carbon and nitrogen with macrobethos diversity, with parameter founded by QR factorization, as follows,

− 0,5566 0,2934 1 0,6252 2

Retrieved computing time for Chelosky Decomposition method with Matlab R2008a, *cputime* = 1,5756 seconds

3.2.4 Simulation of the Cholesky Decomposition Method

Finding the value of the regression model parameters can be calculated by the method of Cholesky decomposition. Completion of linear equations system by using the Cholesky decomposition is as follows:

1. Form the matrix with AZ= B,

36	72,49	32,15		21,33
72,49	164,17	67,28 ,	1 ,	49,88
32,15	67,28	29,28	2	20,15

2. Test whether the matrix A is symmetric and positive definite. From calculations using Matlab R2008a note that the matrix A is symmetric and positive definite.
3. Then find the value L =(l_{ij}),

6	0	0
12,0817	4,2665	0
5,3583	0,5959	0,4617

4. Find matrix S from equation LS = B

6	0	0	1	21,33
12,0817	4,2665	0	2	49,88
5,3583	0,5959	0,4617	3	20,15

We have ,

3,555
1,6242
0,2887

5. Further, look for the value of parameter from equation $L^TZ = S$

6	12,0817	5,3583		3,555
0	4,2665	0,5959	1	1,6242
0	0	0,4617	2	0,2887

We find matrix,

− 0,5566
0,2934
0,6252

So, the solution to the regression model parameter values obtained are,{(− 0,5566 , 0,2934 , 0,6252 $)^T$} and the regression model can be represented by

− 0,5566 0,2934 1 0,6252 2

Retrieved computing time for Chelosky Decomposition method with Matlab R2008a, cputime = 1.4664 seconds

Furthermore, comparison of methods studied LU decomposition, Crout decomposition, QR factorization and Chelosky decomposition on arithmetic operations and computing time to solve linear equations system (4.1)that obtained from cputime with Matlab R2008a using a computer with processor specs: Intel (R) Atom (TM) CPU N470 @ 1,83GHz 1.83, memory: 1.99 GB, the operating system: Windows 7 in the following table,

Table 1. Comparison of arithmetic operations

Method	Arithmetic Operations		*Cputime* (seconds)
	The sum of add	The sum of Multiplication	
LU decomposition	3 − 4 2 9 − 7	3 − 2 23/2 − 7	1,5600
Crout decomposition	3 − 2 2 2 − 1	3 − 3/2 2 − 1	2,2932
QR factorization	2 2 2 − 3/2	2 2	1,5756
Chelosky decomposition	2 − 2	2 2	1,4664

From Table 1. indicates that the most efficient method is the method of Chelosky decomposition because it has the fewest arithmetic operations. This is also verified from the calculation to determine the parameters of the regression model the relationship between carbon and nitrogen to macrobenthos diversity which calculated using Matlab R2008a. We obtained the least of cputime is the Cholesky decomposition of 1.4664 seconds, which means that the Cholesky decomposition is the most efficient method.

IV. CONLUSIONS

Based on the discussion in the previous section we concluded that the result of arithmetic operations and simulation calculations using the method of LUdecomposition, Crout decomposition, QR factorization and Cholesky decomposition method that the most efficient method is Chelosky decomposition. This is verified by the Cholesky decomposition method has the fewest arithmetic operations that is as much 2 − 2 addition and 2 2 multiplication.

From the simulation results to solve a system of linear equations that representing the relationship between carbon and nitrogen with the macrobenthos diversity obtained that thesmallest computing time is the Cholesky decomposition i.e., 1.4664 seconds, which means that the Cholesky decomposition is the most efficient method compared by the LU, Crout decomposition, and QR factorization method. From the regression model, we have the positive parameter this implies that abiotic factors (carbon and nitrogen) influence positively on the macrobenthos diversity, owing to the source of carbon and nitrogen as food resoueces of them.

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