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Abstract

We propose a two-stage procedure for finding realistic benchmarks for nonparametric efficiency analysis. On the first stage the efficient DMUs are figured out by a free disposal hull approach. These benchmarks are directly targeted by directional distance functions and the extent of inefficiency is measured along the direction towards an existing DMU. Two variants for finding the closest or the furthest benchmark are proposed. With this approach there is no need to use linear combinations of existing DMUs as benchmarks which may not be achievable in reality and also no need to accept slacks which are not reflected by the efficiency measure.

JEL classification: C14, D24

Keywords: directional distance functions, targeting, direct benchmarks

1 Introduction

Nonparametric methods for efficiency analysis such as data envelopment analysis (DEA) developed by Charnes et al. (1978) and Banker et al. (1984) as well as free disposal hull (FDH) due to Deprins et al. (1984) and Tulkens (1993) measure the efficiency of decision making units (DMUs) against the benchmark of a piece-wise linear (in the case of DEA) or a step-wise linear (in the case of FDH) frontier function. In both cases one has to choose between input orientation, leading to a point on the frontier function with less input usage at constant output levels, or output orientation, leading to a frontier point with more output at constant input levels. With either orientation these frontier points usually are targets constructed by linear combinations of existing DMUs (in the case of DEA) or targets which are only weakly efficient since they are associated with slacks (in the case of FDH). Those linear combinations need not be achievable for real production processes with slacks as forms of inefficiency which are not reflected in the radial efficiency measure. Only by chance (and with probability zero in the case of continuous inputs and outputs) is a benchmark point on the frontier function identical to an existing efficient DMU.

In the literature, approaches for dealing with this targeting problem are proposed for both oriented models (Coelli (1998), Cherchye and Van Puyenbroeck (2001)) and non-oriented models (Frei and Harker (1999), Golany et al. (1993) and Portela et al. (2003)) to identify the closest targets. In this work we stick to the non-oriented type of models which permit an uneven reduction of inputs jointly with the uneven enhancement of outputs. González and Álvarez (2001) compute input-specific contractions to find the shortest path to the frontier and to identify the relevant benchmarks. Frei and Harker (1999) propose an algorithm to find the shortest projection to the frontier to identify more similar efficient DMUs in terms of input and output quantities. The benchmark point may be an existing DMU but may also be a convex combination of existing DMUs. Portela et al. (2003) is probably closest to the motivation of the approach suggested here. They propose a procedure for identifying the closest targets based on the identification of all efficient facets of the frontier function from FDH (non-convex) as well as DEA (convex) technologies. We deem that our approach is more straightforward and also computationally less demanding. In addition, further restrictions on the inputs and outputs can easily be introduced in the programs discussed subsequently. Other approaches for target setting models require the formulation of preferences in the form of user-supplied weights (Thanassoulis and Dyson (1992), Zhu (1996)). See Thanassoulis et al. (2008, sect. 3.9) for a comprehensive summary.

In this paper we propose an approach based on the concept of directional distance functions (DDF) developed by Chambers et al. (1996) for the direct choice of an efficient benchmark DMU on the frontier function. This target may be the closest existing DMU (which is more realistic in the sense of easier to achieve) or the farthest existing DMU (which leads to a greater efficiency improvement but is more difficult to achieve). The targets are independent of the orientation of the efficiency measurement and are computed by a DDF with endogenously chosen directions. As for DEA and FDH the generalization to an arbitrary number of inputs and outputs is straightforward. More specifically, the procedure is based on two stages with identifying the strongly efficient DMUs (in the sense of Koopmans (1951)) on the FDH frontier on the first stage followed by computing the optimal distances towards these DMUs on the second stage. This approach is related to Briec (2000) with its intention to select the direction towards a strongly efficient part of the frontier function.

The paper proceeds by first explaining the endogenous choice of optimal directions in section 2. This is followed by the presentation of the two-stage approach in section 3. The approach is illustrated and its virtues are discussed with the help of a numerical example in section 4. Section 5 concludes.

2 Optimal Directions

Instead of sticking to a pure input-orientation or a pure output-orientation as in the cases of DEA and FDH the approach of directional distance functions as introduced by Chambers et al. (1996) allows for a more flexible specification of the direction in which inefficiency is measured. This approach is based within nonparametric efficiency measurement with the technology set $T = \{(\boldsymbol{x}, \boldsymbol{y}) \in \mathbb{R}_+^{p+q} : \boldsymbol{x} \geq \mathbf{0} \text{ can produce } \boldsymbol{y} \geq \mathbf{0}\}$ defining the set of feasible input-output combinations with the input quantities collected in the p-vector $\boldsymbol{x}$ and the output quantities in the q-vector $\boldsymbol{y}$. The directional distance function $DDF(\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{g}_x, \boldsymbol{g}_y) = \sup\{\delta \geq 0 : (\boldsymbol{x} - \delta \boldsymbol{g}_x, \boldsymbol{y} + \delta \boldsymbol{g}_y) \in T\}$ measures the distance towards the boundary of the technology set from the input-output point $(\boldsymbol{x}, \boldsymbol{y})$ in the direction $\boldsymbol{g}_x \geq \mathbf{0}$, $\boldsymbol{g}_y \geq \mathbf{0}$.

To compute the DDF with real data let the $p \times n$ matrix $\boldsymbol{X}$ contain the data for the p inputs of the n DMUs with the ith column $\boldsymbol{x}_i$ comprising the input values of DMU i and likewise the $q \times n$ matrix $\boldsymbol{Y}$ contain the data for the q outputs with the ith column $\boldsymbol{y}_i$ comprising the output values of DMU i. Then the DDF can be computed as the solution of the linear programming problem for an exogenously given vector of directions $(\boldsymbol{g}_x', \boldsymbol{g}_y')' \geq \mathbf{0}$ as stated in (1):

$$\begin{array}{lrclcr}
\max\limits_{\delta,\boldsymbol{\lambda}} & & & \delta & & \\
\text{s.t.} & \boldsymbol{x}_i & - & \delta \boldsymbol{g}_x & \geq & \boldsymbol{X\lambda} \\
 & \boldsymbol{y}_i & + & \delta \boldsymbol{g}_y & \leq & \boldsymbol{Y\lambda} \\
 & & & \mathbf{1}'\boldsymbol{\lambda} & = & 1 \\
 & & & \boldsymbol{\lambda} & \geq & \mathbf{0}
\end{array} \tag{1}$$

The constraint $\mathbf{1}'\boldsymbol{\lambda} = 1$ (with $\mathbf{1}$ as a conformable vector of ones) allows for variable returns to scale.

In a proposal to endogenize the computation of the direction vector we adopt the idea of Hampf and Krüger (2015) developed in an environmental efficiency context to specify:[1]

$$\begin{array}{lrcllr}
\max\limits_{\delta,\boldsymbol{\alpha}_x,\boldsymbol{\alpha}_y,\boldsymbol{\lambda}} & & & \delta & & \\
\text{s.t.} & \boldsymbol{x}_i & - & \delta \boldsymbol{\alpha}_x \odot \boldsymbol{x}_i & \geq & \boldsymbol{X\lambda} \\
 & \boldsymbol{y}_i & + & \delta \boldsymbol{\alpha}_y \odot \boldsymbol{y}_i & \leq & \boldsymbol{Y\lambda} \\
 & & & \mathbf{1}'\boldsymbol{\alpha}_x + \mathbf{1}'\boldsymbol{\alpha}_y & = & 1 \\
 & & & \mathbf{1}'\boldsymbol{\lambda} & = & 1 \\
 & & & \boldsymbol{\lambda}, \boldsymbol{\alpha}_x, \boldsymbol{\alpha}_y & \geq & \mathbf{0}
\end{array} \tag{2}$$

[1] See also Färe et al. (2013) for a related proposal to compute endogenous directions in the case of a slacks-based directional measure.

where '$\odot$' denotes the (direct) Hadamard product. Here, the direction vectors $\boldsymbol{\alpha}_x \odot \boldsymbol{x}_i$ and $\boldsymbol{\alpha}_y \odot \boldsymbol{y}_i$ are computed along with δ to find the direction of the maximum distance towards the boundary of the technology set which is given by the frontier function. The additional constraint $\mathbf{1}'\boldsymbol{\alpha}_x + \mathbf{1}'\boldsymbol{\alpha}_y = 1$ permits the identification of δ as a units-invariant measure. The solution value for δ is equal to zero for efficient DMUs and larger than zero for inefficient DMUs, thus measuring the extent of *in*efficiency.

The optimization problem (2) is nonlinear but can be easily transformed to a linear programming problem by substituting $\boldsymbol{\gamma}_x = \delta\boldsymbol{\alpha}_x$ and $\boldsymbol{\gamma}_y = \delta\boldsymbol{\alpha}_y$ which leads to the linear program (3)

$$
\begin{array}{llcll}
\max\limits_{\boldsymbol{\gamma}_x,\boldsymbol{\gamma}_y,\boldsymbol{\lambda}} & \mathbf{1}'\boldsymbol{\gamma}_x & + & \mathbf{1}'\boldsymbol{\gamma}_y & \\
\text{s.t.} & \boldsymbol{x}_i & - & \boldsymbol{\gamma}_x \odot \boldsymbol{x}_i & \geq \boldsymbol{X\lambda} \\
 & \boldsymbol{y}_i & + & \boldsymbol{\gamma}_y \odot \boldsymbol{y}_i & \leq \boldsymbol{Y\lambda} \\
 & & & \mathbf{1}'\boldsymbol{\lambda} & = 1 \\
 & & & \boldsymbol{\lambda}, \boldsymbol{\gamma}_x, \boldsymbol{\gamma}_y & \geq \mathbf{0}
\end{array}
\tag{3}
$$

In this program the objective function is now $\mathbf{1}'\boldsymbol{\gamma}_x + \mathbf{1}'\boldsymbol{\gamma}_y = \delta(\mathbf{1}'\boldsymbol{\alpha}_x + \mathbf{1}'\boldsymbol{\alpha}_y)$ which is equal to δ once the constraint $\mathbf{1}'\boldsymbol{\alpha}_x + \mathbf{1}'\boldsymbol{\alpha}_y = 1$ in (2) is invoked. For practical purposes program (3) can be solved by the ordinary simplex algorithm and the solution values for δ, $\boldsymbol{\alpha}_x$ and $\boldsymbol{\alpha}_y$ can be backed out from $\boldsymbol{\gamma}_x$ and $\boldsymbol{\gamma}_y$ by $\delta = \mathbf{1}'\boldsymbol{\gamma}_x + \mathbf{1}'\boldsymbol{\gamma}_y$, $\boldsymbol{\alpha}_x = \boldsymbol{\gamma}_x/\delta$ and $\boldsymbol{\alpha}_y = \boldsymbol{\gamma}_y/\delta$. Note that the particular specification of the direction vectors here also lets the efficiency measure be invariant with respect to units of measurement.

3 Direct Targeting

Our two-stage approach uses variants of DDF with optimal direction choice on both stages. On the first stage, DDF is used to identify the strongly efficient DMUs on the FDH frontier. This subset of DMUs is used on the second stage for directly targeting a strongly efficient DMU as a benchmark. As we will explain in more detail in the discussion of the numerical example in the next section neither DEA (because of DMU D in the example) nor FDH (because of DMU E there) is sufficient on the first stage.

On the first stage, we establish the efficient DMUs by computing the mixed-integer programs (4) for each DMU $i \in \{1, ..., n\}$

$$
\begin{array}{llcll}
\max\limits_{\boldsymbol{\gamma}_x,\boldsymbol{\gamma}_y,\boldsymbol{\lambda}} & \mathbf{1}'\boldsymbol{\gamma}_x & + & \mathbf{1}'\boldsymbol{\gamma}_y & \\
\text{s.t.} & \boldsymbol{x}_i & - & \boldsymbol{\gamma}_x \odot \boldsymbol{x}_i & \geq \boldsymbol{X\lambda} \\
 & \boldsymbol{y}_i & + & \boldsymbol{\gamma}_y \odot \boldsymbol{y}_i & \leq \boldsymbol{Y\lambda} \\
 & & & \mathbf{1}'\boldsymbol{\lambda} & = 1 \\
 & & & \lambda_j,\ j = 1, ..., n & \in \{0, 1\} \\
 & & & \boldsymbol{\gamma}_x, \boldsymbol{\gamma}_y & \geq \mathbf{0}
\end{array}
\tag{4}
$$

Program (4) is just program 3 with the additional binary restrictions for the λ-factors. DMUs with $\mathbf{1}'\boldsymbol{\gamma}_x + \mathbf{1}'\boldsymbol{\gamma}_y = 0$ are strongly efficient since the endogenous choice of the direction vector eliminates the possibility of weakly efficient DMUs which would be possible in the FDH approach. These strongly efficient DMUs are recorded and serve as the benchmarks for the other DMUs. Which of these benchmarks is chosen for a particular DMU i is determined on the second stage. If we could measure all inputs and outputs as strictly continuous variables weakly efficient observations would occur with probability zero and FDH would be sufficient on the first stage. But in usual real data situations we are not in this position.

On the second stage, the mixed-integer linear programs are computed for each DMU $i \in \{1, ..., n\}$

$$
\begin{array}{llcllcr}
\max\,|\min\limits_{\boldsymbol{\gamma}_x, \boldsymbol{\gamma}_y, \boldsymbol{\lambda}} & \mathbf{1}'\boldsymbol{\gamma}_x & + & \mathbf{1}'\boldsymbol{\gamma}_y & & & \\
\text{s.t.} & \boldsymbol{x}_i & - & \boldsymbol{\gamma}_x \odot \boldsymbol{x}_i & = & \bar{\boldsymbol{X}}\boldsymbol{\lambda} \\
 & \boldsymbol{y}_i & + & \boldsymbol{\gamma}_y \odot \boldsymbol{y}_i & = & \bar{\boldsymbol{Y}}\boldsymbol{\lambda} \\
 & & & \mathbf{1}'\boldsymbol{\lambda} & = & 1 \\
 & & & \lambda_j,\, j = 1, ..., n & \in & \{0, 1\} \\
 & & & \boldsymbol{\gamma}_x, \boldsymbol{\gamma}_y & \geq & \mathbf{0}
\end{array}
\tag{5}
$$

where the matrices $\bar{\boldsymbol{X}}$ and $\bar{\boldsymbol{Y}}$ are the reduced matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$ containing only the strongly efficient observations (i.e. those columns with solution value of zero for the target function on the first stage).

Program (5) is a further modification of the linear DDF program with endogenous directions (3). The equality restrictions for the inputs and the outputs, together with the restrictions on the λ values, guarantee that one and only one strongly efficient DMU is directly targeted as the benchmark for the efficiency measurement for the DMU i under consideration. The restrictions furthermore assure that the solutions for $\boldsymbol{\gamma}_x$ and $\boldsymbol{\gamma}_y$ lead to permissible directions in the sense of reducing inputs and increasing outputs.

The measure of inefficiency is computed as $\delta = \mathbf{1}'\boldsymbol{\gamma}_x + \mathbf{1}'\boldsymbol{\gamma}_y$ with the respective solution values for DMU i and has the usual interpretation as a DDF efficiency measure. It is equal to zero for the efficient DMUs and the larger for a larger distance towards the chosen benchmark DMU.

The benchmark determined in this way can be the furthest efficient DMU when program (5) is solved as a maximization problem or the closest efficient DMU when it is solved as a minimization problem, indicated by the shortcut notation 'max | min' in program (5). This is indicated by the shortcut notation max | min in program 5. The furthest benchmark is associated with a large efficiency improvement but is probably much more difficult to reach in practice. The closest benchmark is more likely to be achievable but may only be associated with a small efficiency improvement. The two-stage procedure outlined above is only required if minimization is chosen on the second stage.[2]

In related work, Briec (2000) also covers the binary constraint on the λ-values as one possibility for the admissible set of aggregation weights. However, Briec does not explicitly point out the practical consequences for the selection of a direct benchmark. This gap is filled here. In addition, Briec only treats the maximization case, whereas we show here that minimization also makes sense in this setting.

[2]Note that in the case of maximization on the second stage, we obtain the same solutions as on the first stage. This is quite natural here, since $\bar{\boldsymbol{X}}$ and $\bar{\boldsymbol{Y}}$ are just contain those observations which are found efficient on the first stage.

4 Numerical Example

Now we turn to the illustration of the two-stage approach by means of a numerical example with 7 DMUs named A to G which use a single input to produce a single output. The input quantities are given by $\boldsymbol{X} = (2, 4, 8, 3, 2, 7, 9)$ and the output quantities by $\boldsymbol{Y} = (2, 6, 8, 3, 1, 3, 7)$.[3]

Figure 1: DEA and FDH Projections

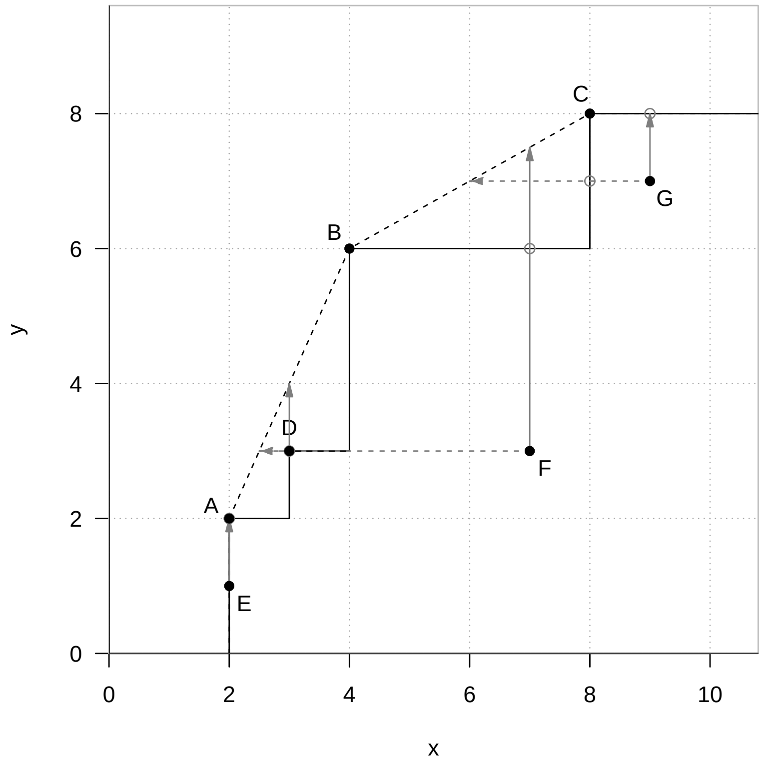


Figure 1 shows the positions of the DMUs as well as the FDH frontier function as the solid step-wise line and the DEA frontier as the dashed extensions between DMUs A, B and C, originating from the convexity assumption. The solid gray vertical arrows show the output-oriented directions for the efficiency measurement towards the DEA frontier with circles indicating the intersections with the FDH frontier. Likewise, the dashed gray horizontal arrows show the input-oriented directions. It is evident that the benchmark points on the frontier functions are either linear combinations of the efficient DMUs or are associated with slacks. The numerical results for the efficiency measures and the associated λ-values are reported in panels I and II of table 1 for DEA and FDH, respectively.

Considering DMU F as an example we see that this DMU is compared to a combination of A and B in the case of an input-oriented DEA analysis and with a combination of B and C in the case of an output-oriented DEA analysis. The circled projection points on the FDH frontier show that F has a relatively large input slack with respect to its benchmark

[3]The computations are programmed in R using the package 'lpSolve' for the mixed-integer linear programs and the package 'Benchmarking' (accompanying Bogetoft and Otto (2011)) for the preparation of the figures.

Table 1: Numerical Results

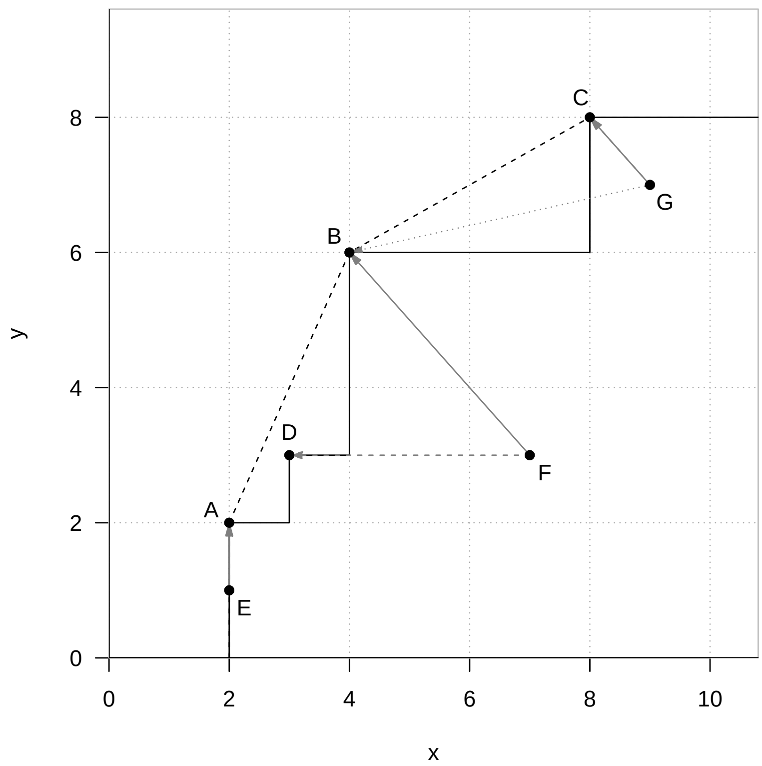
	A	B	C	D	E	F	G	A	B	C	D	E	F	G
Panel I:	DEA (output oriented)							DEA (input oriented)						
θ_{DEA}	1.000	1.000	1.000	1.333	2.000	2.500	1.143	1.000	1.000	1.000	0.833	1.000	0.357	0.667
λ_{A}	1	0	0	0.500	1	0	0	1	0	0	0.750	1	0.750	0
λ_{B}	0	1	0	0.500	0	0.250	0	0	1	0	0.250	0	0.250	0.500
λ_{C}	0	0	1	0	0	0.750	1	0	0	1	0	0	0	0.500
Panel II:	FDH (output oriented)							FDH (input oriented)						
θ_{FDH}	1.000	1.000	1.000	1.000	2.000	2.000	1.143	1.000	1.000	1.000	1.000	1.000	0.429	0.889
λ_{A}	1	0	0	0	1	0	0	1	0	0	0	1	0	0
λ_{B}	0	1	0	0	0	1	0	0	1	0	0	0	0	0
λ_{C}	0	0	1	0	0	0	1	0	0	1	0	0	0	1
λ_{D}	0	0	0	1	0	0	0	0	0	0	1	0	1	0
Panel III:	DDF (maximum distance)							DDF (minimum distance)						
$\delta_{\text{Stage 1}}$	0.000	0.000	0.000	0.000	1.000	1.429	0.254							
$\delta_{\text{Stage 2}}$	0.000	0.000	0.000	0.000	1.000	1.429	0.254	0.000	0.000	0.000	0.000	1.000	0.571	0.254
α_x	–	–	–	–	0.000	0.300	0.438	–	–	–	–	0.000	1.000	0.438
α_y	–	–	–	–	1.000	0.700	0.563	–	–	–	–	1.000	0.000	0.563
λ_{A}	1	0	0	0	1	0	0	1	0	0	0	1	0	0
λ_{B}	0	1	0	0	0	1	0	0	1	0	0	0	0	0
λ_{C}	0	0	1	0	0	0	1	0	0	1	0	0	0	1
λ_{D}	0	0	0	1	0	0	0	0	0	0	1	0	1	0

Note: $\delta_{\text{Stage 1}}$ is the same for both minimum and maximum distance variants, '–' indicates indeterminate solutions for the efficient DMUs.

B in an output-oriented analysis whereas no slack appears in an input-oriented analysis where the benchmark is D. An input-oriented DEA analysis of DMU G shows that this is compared to a fifty-fifty mixture of B and C while G is only compared to C with output orientation and in this case has an additional input slack. In the case of a FDH analysis of G slacks would arise with both orientations. As can be easily imagined from adding further inefficient DMUs to the figure that in most occasions linear combinations or points associated with slacks will be chosen as benchmark points by DEA and FDH.

A rather special case is DMU D which is inefficient when evaluated against die DEA frontier while being part of the FDH frontier. Cases like D are the reason for the reliance on the FDH frontier since there is no existing DMU in the north-west of D which could be used as benchmark for direct targeting. The second special case is DMU E which is positioned as a weakly efficient point on the vertical part of the frontier function. Here, an input-oriented FDH analysis would result in E being indicated as efficient but having an output slack with respect to A. This particular case shows that simply using FDH on the first stage would not be sufficient for selecting the strongly efficient DMUs as benchmark candidates for the second stage of the procedure.[4]

Figure 2: DDF Direct Targeting



The results of the second stage of the two-step procedure for the numerical example are depicted in figure 2. In the figure, the dashed arrows indicate the direction to the closest DMU (min in program (5)) and the solid arrows the direction to the furthest DMU (max in program (5)).

In the case of the inefficient DMU F there is DMU D acting as the closest and B as the furthest benchmark DMU. Thus, the minimum distance is in the direction to D

[4]I am grateful to Benny Hampf for spotting this possibility and indicating its solution.

whereas the maximum distance points to B. For DMU G there is only one DMU eligible as a benchmark and this is DMU C for both minimization and maximization. Here, the direction to B would be associated with a decreasing input but also a decreasing output quantity (as indicated by the dotted arrow) and therefore is not permissible. The weakly inefficient DMU E gets assigned A as its unique benchmark.

The numerical results for the two-stage direct targeting approach are shown in panel III of table 1 for the maximization on the left side and for the minimization on the right side. Here, the directions are presented in form of α-values adding up to unity which are backed out from the solution for the γ-values and the inefficiency measure by $\alpha_x = \gamma_x/\delta$ and $\alpha_y = \gamma_y/\delta$ (with $\delta = \gamma_x + \gamma_y$). From the solution values for λ out of $\{0, 1\}$ we can directly identify the respective benchmark DMUs. We also observe that the inefficiency measures on the first stage are exactly equal to those obtained by maximization on the second stage as explained above.

5 Concluding Remarks

In this paper we develop a two-stage procedure for selecting suitable existing strongly efficient DMUs as benchmarks for the efficiency evaluation of inefficient DMUs. The procedure uses a specific variant of DDF with endogenous direction choice on the first stage for finding the strongly efficient DMUs and then computes optimal directions for targeting the closest or furthest DMU with the aid of another modified DDF model on the second stage. This procedure can improve the decision support derived from the results of a nonparametric efficiency analysis since the benchmark points chosen in this way are real in the sense of being neither constructed from linear combinations nor are associated with slacks. Thus, both forms of technical inefficiency are jointly reflected by the (in)efficiency measure relative to the benchmark of an existing DMU.

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