Higher-rank Transmit Beamforming Using Space Time Block Coding

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To my parents

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Abstract

With the rapid development of wireless communications, there has been a massive growth in the number of wireless communications users and progressively more new high data-rate wireless services will emerge. With these developments taking place, wireless spectral resources are becoming much more scarce and precious. As a result, research on spectrally efficient transmission techniques for current and future communication networks attracts considerable interest. As a promising multi-antenna communication technique, transmit beamforming is widely recognized as being able to improve the capacity of wireless systems without requiring additional spectral resources. In conventional (rank-one) beamforming, each user is served by a single beamformer. For certain transmit beamforming applications, the beamforming performance may be poor if the degrees of freedom in the conventional beamformer design become insufficient.

The scope of this thesis is to address the beamforming performance degradation problems induced by the insufficient degrees of freedom in the beamformer design in certain practical scenarios. In this thesis, a fundamentally new idea of higher-rank (>1) transmit beamforming is proposed to improve the beamforming performance. Instead of a single beamformer assigned to each user, multiple beamformers are designed and correspondingly the degrees of freedom in the beamformer design are multiplied, i.e., the increase of the degrees of freedom consists in the increase of the number of design variables. To implement higher-rank beamforming, the central idea is to combine beamforming with different space time block coding (STBC) techniques. Conventionally, STBCs are used to exploit the transmit diversity resulting from the independent fading for different transmit antennas. However, the use of STBCs in the higher-rank beamforming approaches is not for the sake of transmit diversity, but for the sake of design diversity in the sense of degrees of freedom in the beamformer design.

The single-group multicast beamforming problem of broadcasting the same information to all users is firstly considered in the thesis. It is assumed that the transmitter knows the instantaneous channel state information (CSI) which describes the short-term channel conditions of a communication link and can be estimated in modern communication systems. In the conventional approach, a single beamforming weight vector is designed to steer the common information to all users. In the case of a large number of users, the performance of the conventional approach usually degrades severely due to the limited degrees of freedom offered by a single beamformer. In order to mitigate this drawback, a rank-two beamforming approach is proposed in which two independent beamforming weight vectors are designed. In the rank-two beamforming approach, single-group multicast beamforming is combined with the two dimentional Alamouti STBC, and each user is simultaneously served with two Alamouti coded symbols from two beamformers. The degrees of freedom in the beamformer design are doubled and significant performance improvement is achieved.

The multi-group multicast beamforming problem of transmitting the same information to users in the same group while transmitting independent information to users in different groups, is studied next in the thesis, also assuming that instantaneous CSI is available at the transmitter. The rank-two beamforming approach, originally devised for single-group multicasting networks that are free of multiuser interference, is extended to multi-group multicasting networks, where multiuser interference represents a major challenge. By combining multi-group multicast beamforming with Alamouti STBC, two independent beamforming weight vectors are assigned to each user and the degrees of freedom in the beamformer design are doubled resulting in drastically improved beamforming performance.

Then, the multiuser downlink beamforming problem of delivering independent information to different users with additional shaping constraints is investigated in the thesis, also assuming instantaneous CSI at the transmitter. Additional shaping constraints are used to incorporate a variety of requirements in diverse applications. When the number of shaping constraints is large, the degrees of freedom in the beamformer design can be rather deficient. In order to address this problem, a general rank beamforming approach is proposed in which multiuser downlink beamforming is combined with high dimensional ($>$2) real-valued orthogonal space time block coding (OSTBC). In the general rank beamforming approach, the number of beamforming weight vectors for each user and the associated degrees of freedom in the beamformer design are multiplied by up to eight times, which lead to significantly increased flexibility for the beamformer design.

Since instantaneous CSI can be difficult to acquire in certain scenarios, the use of statistical CSI describing the long-term statistical characteristics of the channel can be more practical in these scenarios. The rank-two beamformer designs based on instantaneous CSI can be straightforwardly applied in the case of statistical CSI. However, it is impossible to extend the general rank beamforming approach for the multiuser downlink beamforming problem with additional shaping constraints based on instantaneous CSI to the case of statistical CSI straightforwardly. Therefore, multiuser downlink beamforming with additional shaping constraints using statistical CSI at the transmitter is then studied and an alternative general rank beamforming approach is proposed in the thesis. In the general rank beamforming approach using statistical CSI, multiuser downlink beamforming is combined with quasi-orthogonal space time block coding (QOSTBC). The increased number of beamforming weight vectors and the associated degrees of freedom are much beyond the limits that can be achieved by Alamouti STBC in the beamformer design.

Simulation results demonstrate that the proposed higher-rank transmit beamforming approaches can achieve significantly improved performance as compared to the existing approaches.

Zusammenfassung

Mit der rasanten Entwicklung im Bereich der Funkkommunikation entstand ein ebenso starker Anstieg der Nutzerzahl und zunehmend mehr Dienste, die hohe Datenraten benötigen, werden in Zukunft entstehen. Durch diese Entwicklungen werden die verfügbaren spektralen Ressourcen immer knapper und wertvoller. Übertragungstechniken mit einer hohen spektralen Effizienz sind daher für heutige und zukünftige Kommunikationsnetze ein Forschungsthema von groem Interesse. Sende-Beamforming wurde entwickelt als eine Übertragungstechnik für Mehrantennensysteme, die in der Lage ist die Kapazität eines kabellosen Systems zu erhöhen ohne zusätzliche spektrale Ressourcen zu benötigen. In konventionellem (Rang-Eins-) Beamforming wird jeder Nutzer durch einen einzelnen Beamformer bedient. Für bestimmte Sende-Beamforming Anwendungen kann deren Leistungsfähigkeit begrenzt sein, wenn die Freiheitsgrade für konventionelles Beamformer Design nicht ausreichend sind.

Diese Thesis behandelt Lösungsansätze für die durch geringe Freiheitsgrade entstehende, verringerte Leistungsfähigkeit im Beamformer Design bei praktischen Anwendungen. In der vorliegenden Arbeit schlagen wir eine fundamental neue Idee des Höherer-Rang-Beamforming ($>$1) vor, um dessen Leistungsfähigkeit zu erhöhen. Anstelle eines einzelnen Beamformers für jeden Nutzer werden mehrere Beamformer entwickelt, und durch diese höhere Anzahl an verfügbaren Designparametern steigt gleichermaen die Anzahl der Freiheitsgrade. Der zentrale Ansatz für die Implementierung von Höherer-Rang-Beamforming besteht in der Verwendung verschiedener Raum-Zeit-Block-Codierung (engl. *space time block coding*, STBC) Verfahren. Üblicherweise werden STBCs verwendet um die Sendediversität auszunutzen, welche durch das für verschiedene Sendeantennen unabhängige Kanal-Fading entsteht. Bei der Verwendung von STBCs für Höherer-Rang-Beamforming dient dies jedoch nicht einer Erhöhung der Sendediversität, sondern einer Erhöhung der Freiheitsgrade im Beamformer Design.

Das Einzelgruppen-Multicast-Beamforming Problem, dieselben Informationen an alle Nutzer zu übertragen, wird in dem ersten Teil dieser Thesis betrachtet. Es wird angenom-

men, dass der Sender die aktuellen Kanalinformationen kennt. Diese beschreiben aktuelle Kanaleigenschaften einer Übertragungsstrecke und können in modernen Kommunikationsnetzen mit Hilfe geeigneter Schätzverfahren bestimmt werden. Der konventionelle Ansatz verwendet einen einzelnen Beamforming-Gewichtungsvektor, um die Informationen an alle Nutzer zu übertragen. Für eine groe Anzahl an Nutzern verringert sich jedoch die Leistungsfähigkeit dieser Methode, da ein einzelner Beamformer nur sehr begrenzte Freiheitsgrade ermöglicht. Um diesen Nachteil abzuschwächen stellen wir einen Ansatz basierend auf Rang-Zwei-Beamforming vor, wo zwei unabhängige Beamforming-Gewichtungsvektoren entwickelt werden. In diesem Ansatz wird Einzelgruppen-Multicast-Beamforming mit zweidimensionalem Alamouti STBC kombiniert, und jeder Nutzer wird gleichzeitig mit zwei Alamouti-kodierten Symbolen von zwei Beamformern bedient. Die Freiheitsgrade im Beamformer Design verdoppeln sich dadurch, und eine signifikante Erhöhung der Leistungsfähigkeit wird erreicht.

Das nächste in dieser Thesis behandelte Problem ist Mehrgruppen-Multicast-Beamforming, wo identische Informationen an Nutzer in derselben Gruppe übermittelt werden, aber unterschiedliche Informationen an Nutzer in unterschiedlichen Gruppen. Auch hier wird angenommen, dass aktuelle Kanalinformationen auf der Seite des Senders verfügbar sind. Der Ansatz des Rang-Zwei-Beamforming wurde ursprünglichfür Einzelgruppen-Multicast Netzwerke entwickelt, welche frei von Mehrnutzer-Interferenzen sind, und wird hier für Mehrgruppen-Multicast Netzwerke weiterentwickelt, wo diese ein groes Problem darstellen. Durch eine Kombination von Mehrgruppen-Multicast-Beamforming mit Alamouti STBC werden zwei unabhängige Beamforming-Gewichtungsvektoren jedem Nutzer zugeordnet, was durch die zusätzlichen Freiheitsgrade die Leistungsfähigkeit des Beamformers stark erhöht.

Daraufhin betrachten wir das Mehrnutzer-Downlink-Beamforming Problem, bei dem unabhängige Informationen zu verschiedenen Nutzern mit zusätzlichen Shaping Constraints übertragen werden. Wiederum nehmen wir an, dass aktuelle Kanalinformationen beim Sender verfügbar sind. Die Shaping Constraints repräsentieren verschiedene anwendungsspezifische Anforderungen. Ist deren Anzahl gro, sind die nutzbaren Freiheitsgrade im Beam-

former Design unzureichend. Um dieses Problem zu adressieren stellen wir einen Ansatz für Allgemeiner-Rang-Beamforming vor, bei dem Mehrnutzer-Downlink-Beamforming kombiniert wird mit höherdimensionalem (>2) reellwertigem orthogonalem STBC. Bei diesem Ansatz ist die Anzahl der Beamforming-Gewichtungsvektoren für jeden Nutzer und die entsprechenden Freiheitsgrade bis zu achtfach erhöht. Dies führt zu erheblich höherer Flexibilität beim Beamformer Design.

Da aktuelle Kanalinformationen in manchen Szenarien schwer zu ermitteln sind, kann die Verwendung langfristiger, statistischer Kanaleigenschaften stattdessen praktischer sein. Die Rang-Zwei-Beamformer Designs basierend auf aktuellen Kanalinformationen sind direkt anwendbar auf statistische Kanalinformationen. Es ist jedoch nicht möglich den Ansatz für Allgemeiner-Rang-Beamforming für Mehrnutzer-Downlink-Beamforming mit zusätzlichen Shaping Constraints, welcher auf aktuellen Kanalinformationen basiert, direkt auf statistische Kanalinformationen anzuwenden. Daher wird in dieser Thesis Mehrnutzer-Downlink-Beamforming mit zusätzlichen Shaping Constraints analysiert und ein alternativer Allgemeiner-Rang-Beamforming Ansatz entwickelt. Bei diesem Ansatz wird Mehrgruppen-Downlink-Beamforming kombiniert mit quasi-orthogonalem STBC. Die erhöhte Zahl an Beamforming-Gewichtungsvektoren und entsprechender Freiheitsgrade übertreffen die Grenzen die üblicherweise von Alamouti STBC im Beamformer Design erreicht werden.

Simulationsergebnisse demonstrieren, dass die vorgestellten Methoden für Höherer-Rang-Beamforming wesentlich leistungsfähiger sind, als bisher bekannte Ansätze.

Mathematical Notation

Sets:

$\mathbb{R}^N$	The set of real vectors of length N
$\mathbb{C}^{M \times N}$	$M \times N$ complex matrix (vector)
$\emptyset$	The empty set
$A \cap B$	Intersection of sets A and B
$A \cup B$	Reunion of sets A and B
$x \in A$	Element x belongs to the set A

Vectors and matrices:

$[\cdot]_i$	The i-th row of a matrix
$[\cdot]_{ij}$	Entry in the i-th row and j-th column of a matrix
$\mathrm{rank}(\cdot)$	Rank of a matrix
$\mathrm{diag}\{\cdot\}$	Diagonal matrix formed from the elements in the argument
$\mathbf{I}_N$	$N \times N$ identity matrix
$\mathbf{0}_{M \times N}$	$M \times N$ matrix of zeros
$\mathbf{A} \succeq \mathbf{0}$	Matrix $\mathbf{A}$ is positive semidefinite
$(\cdot)^T$	Transpose
$(\cdot)^*$	Conjugate complex
$(\cdot)^H$	Hermitian (conjugate transpose)
$\mathrm{Tr}\{\cdot\}$	Trace of a square matrix

Norms:

$\|\| \cdot \|\|$	Euclidean (l-2) norm of a vector

Miscellaneous:

$\angle(\cdot)$	Argument of a complex number
$\mathrm{Re}\{\cdot\}$	Real part of a variable
$\mathrm{Im}\{\cdot\}$	Imaginary part of a variable
$\mathrm{E}\{\cdot\}$	Statistical expectation
$\exp(\cdot)$	Exponent of a variable
$\triangleq$	Defined as
$\forall$	For all
$\trianglerighteq_l$	A sign in the set $\{\geq, \leq, =\}$

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Chapter 1

Introduction

1.1 Background

Wireless communications has experienced a phenomenal development in the last few decades not only from the academic research point of view where enormous progress has been made, but also in terms of the huge market size and great impact on the society [1].

Mobile cellular communications is the most widespread radio access application for wireless communications whose development can be divided into generations evolving from the first generation (1G) to the fifth generation (5G) [2]. In 1G, the analog mobile radio systems were used in the 1980s. With the advent of digital technology, the second generation (2G) mobile communication standards and systems were developed. Digital systems in 2G are superior to the analog systems with respect to system capacity, link quality, and addition services such as short message. Moreover, different from the incompatible analog systems employed in different countries in 1G, global system for mobile communications (GSM) in 2G is standardized and has spread all over the world [3]. The success of GSM in 2G motivated the development of the third generation (3G) systems which are the first mobile systems for broadband wireless communication. Based on the wideband code division multiplexing access (CDMA) techniques [4], new applications such as internet browsing and audio/video streaming can be found in 3G communication. Even while 3G networks were still being deployed, the fourth generation (4G) communication has been developed to provide better service quality and boost the system capacity. Nowadays, the long-term evolution (LTE) and LTE-Advanced systems embodying 4G have been deployed and are reaching maturity [5]. In 4G networks, multiple input multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) are the key technologies [6, 7]. To meet the strong demands from the explosive growth of cellular users and the associated potential services, currently the fifth generation (5G) standard is under extensive preliminary investigation and discussion, with e.g., ultra-densification, millimeter wave (mmWave), and massive MIMO being candidate key technologies [8].

Apart from mobile cellular networks, the second important development in wireless communications is the wireless local area networks (WLAN) [2]. The institute of electrical and electronics engineers (IEEE) 802.11 based WLAN represents the most widely deployed WLAN technology. With the migration of critical applications to data networks and the emergence of multimedia applications such as digital audio/video and multimedia games, IEEE 802.11 based WLAN meets different demands of people. Nowadays, WLAN services are widely provided not only at homes and offices but also at restaurants, libraries, and many other locations.

The standardization process of IEEE 802.11 based WLAN originated in the 1990s, and since then several versions 802.11b/a/g/n/ac have been adopted by the mainstream market [9]. 802.11b defined a standard based on the complementary code keying (CCK) mode and became popular at first. It allowed up to 11 Mbps data rate and operated over the 2.4GHz frequency band. Since the data rate of 11 Mbps was not sufficient for many applications, 802.11a based on OFDM became of greater interest. It allowed up to 54 Mbps data rate and operated over a different frequency band of 5 GHz. Later on, 802.11g adapted the same physical layer and media access control specifications as in 802.11a to the frequency band of 2.4 GHz and achieved 54 Mbps data rate [10]. In 2009, the 802.11n standardization process was completed and it offered up to 600 Mbps data rate [11]. The high data rate primarily results from the use of multi-antenna techniques and the use of the increased bandwidth. Recently, the introduction of more advanced multi-antenna transmission techniques has provided additional powerful approaches for boosting the data rate to gigabits per second and leading to the emerging IEEE 802.11ac [12, 13].

While remarkable achievements have been made in the past, the development of wireless communications is still going on very fast and exhibits three major tendencies. First, there will be a massive growth in the number of wireless communications users. Second, there will be a much broader range of wireless products on the market. Third, there will emerge more and more new high data-rate wireless services accounting for various user demands. With these three trends taking place, the wireless spectral resource is becoming more and more scarce and precious because the spectrum available for wireless communications services is limited by nature. Therefore, research on spectrally efficient transmission schemes for current and next generation communication networks is attracting considerable interest. Researchers have investigated various methods to improve the capacity of wireless systems without requiring additional spectral resources. One class of the significant methods is the multi-antenna communication techniques.

In multi-antenna communication systems, there are two prominent techniques to use transmit antenna arrays: transmit beamforming and space time coding [2]. Both techniques can be applied in both MIMO and multi-input single-output (MISO) systems. In this thesis, the applications of transmit beamforming and space time coding techniques are considered and more details are provided in the following subsections.

1.1.1 Transmit Beamforming

Transmit beamforming can be used to transmit signals from an antenna array to a single user or multiple co-channel users simultaneously. It is widely recognized as a promising technique to realize energy- and spectrum-efficient wireless communications and has been included in LTE/LTE-Advanced standards [14] and 802.11n/ac standards [12]. There are typically three types of transmit beamforming scenarios for multiuser services [15], all of which are investigated in this thesis:

- *Single-group multicast beamforming*.

  The same information is delivered to all users.

- *Multi-group multicast beamforming*.

The same information is broadcasted to a selected group of users, but different information is transmitted to different groups of users.

- *Multiuser downlink beamforming (unicast beamforming)*.
  Different information is delivered to different users.

Transmit beamforming aims at boosting the signal power at the desired receiver while decreasing the interference towards the non-intended receivers. This can be achieved by exploiting channel state information (CSI) which describes the channel conditions of a communication link at the transmitter. The signal power at the desired receiver is strengthened by tuning the same signal on each transmit antenna with distinct amplitudes and phases that are delibrately designed, such that the signal components from different antennas can add constructively at the desired receiver. Meanwhile, the interference power at the non-intended receiver is weakened by combining the signal components in a destructive way. The amplitudes and phases of the tuned signals are formulated in the beamforming weight vectors that are designed to yield large inner products with the channel vectors of the desired receivers and small inner products with the channel vectors of the non-intended receivers from the mathematical perspective. If there is line-of-sight (LoS) between the communicating terminals, transmit beamforming can be viewed as forming a beam of signal towards the desired receiver. Therefore, transmit beamforming is more energy-efficient as compared to the omni-directional transmissions and analog directional radiations with directional antennas. Transmit beamforming is also applicable in non-LoS scenarios if the channel knowledge is available at the transmitter by making the multi-path components add constructively or destructively.

The beamformer design for merely maximizing the signal power at the desired receiver can be fairly easy to perform, however, the balance for signal power maximization and interference power minimization at the same time can be difficult to achieve. This results in the following beamformer optimization problems. Typically there exist two related quality of service (QoS) based design formulations for transmit beamforming and both of them are considered in this thesis. One formulation is the problem of minimizing the total transmit power subject to QoS constraints in terms of signal to noise ratio (SNR) (in single-group multicasting) or signal to interference plus noise ratio (SINR) (in unicasting or multi-group multicasting). This formulation is designed for saving the energy consumption costs of network operators while providing QoS assurance to each receiver. The other formulation is the problem of maximizing the minimum SNR or SINR of all intended receivers subject to a total transmit power constraint. This formulation is designed for maximizing the achievable data rate which is determined by the minimum received SNR or SINR, and correspondingly optimizing the user experiences.

1.1.2 Space Time Coding

Space time coding (STC) is devised to exploit the spatial diversity provided by multi-antenna transceivers to improve the diversity gain over the fading channels with the aid of CSI available at the receiver. Unlike transmit beamforming, STC transmission does not require CSI at the transmitter. The diversity gain can be achieved by transmitting multiple redundant copies of a data stream over the independent signal paths between the transmitter and receiver. The signal is transmitted from multiple antennas over multiple consecutive time slots and the encoding process is carried out not only in the time dimension but also in the space dimension. By distributing the transmitted information symbols to both time and space dimension, some replicas of the signal can arrive at the receiver with a better condition than others and thus the downside effects of multi-path fading can be mitigated. Using STC, the achievable data rate and bit error rate (BER) performance can be improved by several orders of magnitude [16, 17].

In 1998, a pioneering work in STC for MIMO wireless channels was proposed in [18], in which two code design criteria have been designed for flat fading channels with coherent receivers, and high-performance space time trellis coding (STTC) techniques have been designed. STTC designed for two to four transmit antennas is well established in slow fading channels. However, STTC suffers from rather high decoding complexity. In the same year, Alamouti proposed a celebrated and powerful space time block coding (STBC) technique for two transmit antennas which improves the quality of the received signal by applying a simple encoding method at the transmitter and linear symbol-by-symbol decoding at the receiver [19]. The decoding complexity of the Alamouti code is much smaller than that of STTC, however, the BER performance is degraded. The Alamouti code inspired extensive research on similar techniques that can be applied for more than two transmit antennas. The authors in [20] proposed orthogonal space time block coding (OSTBC) for more than two transmit antennas which uses the orthogonal design technique at the transmitter side to ensure the full-diversity property and achieves a linear decoding complexity at the receiver side. Despite the advantages of full diversity and low decoding complexity offered by OSTBC, full transmission rate is not possible for OSTBC with complex symbols for more than two transmit antennas [21]. However, OSTBC for real-valued symbols, i.e., real-valued OSTBC, is possible to achieve full rate for two or four or eight transmit antennas [21].

In order to overcome the low transmission rate limitation of OSTBC for more than two antennas, quasi-orthogonal space time block coding (QOSTBC) was proposed in [22] by relaxing the orthogonality property. The QOSTBC in [22] achieves full rate for four transmit antennas, however, partial diversity is obtained and pair-wise decoding is employed at the receiver of which the complexity is higher than that of the symbol-wise decoding. It was shown in [23] and [24] that the full-diversity property can be recovered by performing constellation rotation. Later on, QOSTBC for eight transmit antennas was developed in [25] maintaining the full-rate full-diversity property with pair-wise decoding.

1.2 Thesis Overview and Contributions

In conventional transmit beamforming, each user is served by a single beamformer and each beamformer is designed to bear the signal of a single user. It can also be named as rank-one beamforming. For some transmit beamforming problems, rank-one beamforming can achieve excellent performance. However, for other problems, severe performance degradation may arise due to the limited number of beamformers and the associated insufficient degrees of freedom in the conventional rank-one beamformer design. The extension from a single beamformer to multiple beamformers for each user is not straightforward and it is not an easy task because there exists correlation between beamformers if they bear identical signals. In this thesis, we develop higher-rank (>1) transmit beamforming approaches to address this problem and the central idea is to combine transmit beamforming with different STBC techniques. Unlike conventional STBC, the higher-rank beamforming approach assumes CSI at the transmitter. The use of STBCs in the higher-rank beamforming approaches is not for the sake of transmit diversity, but for the sake of design diversity in the sense of degrees of freedom in the beamformer design. Moreover, most of the wireless communication standards for current and next generation communication networks have defined STBC as well as beamforming or precoding techniques. Therefore, the proposed higher-rank transmit beamforming techniques are applicable in these systems without the need of severe modifications.

This dissertation is organized as follows. Each following chapter considers a different transmit beamforming application and the corresponding higher-rank transmit beamforming techniques are designed. In each chapter, first the related work is introduced and the contribution of the proposed approach is briefly stated. Then, the conventional signal model and problem formulation is revisited. Afterwards, the higher-rank beamforming system model is designed and the corresponding beamformer optimization is carried out. Then, simulation results are provided to demonstrate the proposed approaches. The contributions for each chapter are summarized as follows.

In Chapter 2, the single-group multicast beamforming problem of broadcasting the same signal to all receivers is investigated. It is an interference free problem and the beamformer design is based on the criteria of minimizing the total transmit power subject to SNR constraints for all users. In the case of a large number of users, the performance of the conventional rank-one beamforming methods may degrade severely because of the limited degrees of freedom for designing a single spatially selective beamformer. To deal with this problem, a rank-two beamforming approach is proposed in which beamforming is combined with Alamouti STBC. The degrees of freedom are doubled by introducing two independent beamforming weight vectors that are used to transmit two codewords of the Alamouti code. The proposed rank-two beamforming approach is particularly attractive when it is combined with the use of the semi-definite relaxation (SDR) technique which is a powerful approximation technique capable of transforming many difficult non-convex optimization problems to convex problems that can be solved efficiently. The benefit is that the proposed rank-two beamforming approach is optimal when rank-one or rank-two SDR solutions are obtained, however, the conventional rank-one approaches are only optimal for rank-one solutions. If the rank of the SDR solution is higher than two, a randomization procedure is carried out to generate approximate solutions. Simulation results demonstrate that the proposed rank-two beamforming approach outperforms the rank-one approaches significantly.

In Chapter 3, the multi-group multicast beamforming problem of transmitting independent information to different groups of users is studied. Beamformers are designed according to the criteria of maximizing the minimum SINR of the users in all groups subject to a total transmit power constraint. The rank-two beamforming approach, designed for single-group multicasting networks that are free of multiuser interference, is extended to multi-group multicasting networks, where multiuser interference represents a major challenge. The challenge lies in striking a balance between the signal power maximization and interference power minimization by designing multiple beamforming weight vectors jointly in the rank-two system model. By combining multi-group multicast beamforming with the Alamouti code, the users in each group are served with two beamformers. Due to the orthogonality of the code, the decoding complexity at the receivers is not increased and symbol-by-symbol detection can be performed. The doubled degrees of freedom in the beamformer design lead to significant performance improvement. Besides the SDR based rank-two beamforming approach, a computationally more efficient rank-two beamforming approach is proposed to obtain approximate solutions iteratively by performing sequential convex optimization. Simulation results demonstrate that the proposed rank-two beamforming approaches significantly outperform the existing approaches.

In Chapter 4, the multiuser downlink beamforming problem in the presence of a massive number of arbitrary quadratic shaping constraints is investigated. Additional shaping constraints on the beamformers are used to describe a variety of requirements in diverse applications, e.g., to limit the interference leakage towards neighbouring cells or to guarantee the charging power level at energy harvesting users. Beamformers are designed according to the criteria of minimizing the total transmit power subject to SINR constraints and additional shaping constraints. The massive number of additional shaping constraints result in insufficient degrees of freedom in the beamformer design. In order to increase the degrees of freedom, a general rank beamforming approach is proposed. Extending the rank-two beamforming approach to high dimensional (>2) OSTBC is impractical due to the rate penalty associated with these codes. By applying full-rate real-valued OSTBC in the general rank beamforming approach, up to eight beamformers can be used to deliver the data stream to each user while maintaining the full-rate transmission property. Real-valued OSTBC is employed because the effective channel vector of each user can be adjusted to result in a real vector and thus the orthogonality of the coding matrix is guaranteed. Then, symbol-by-symbol detection can be performed at the receivers and the decoding complexity is not increased as compared to the conventional transmission techniques. The original multi-constraint beamforming problem can be solved using the SDR technique. In contrast to conventional rank-one beamforming approaches in which an optimal beamforming solution can be obtained only when the SDR solution (after rank reduction) exhibits the rank-one property, in the proposed approach optimality is guaranteed when a rank of eight is not exceeded. It can be shown that the proposed approach can incorporate up to 79 additional shaping constraints for which an optimal beamforming solution is guaranteed as compared to a maximum of two additional constraints that bound the conventional rank-one downlink beamforming designs [26, 27]. Simulation results demonstrate the flexibility of the proposed beamformer design.

The rank-two beamformer designs for single-group and multi-group multicasting problems in this thesis are based on the assumption that instantaneous CSI describing the short-term channel conditions is available at the transmitter, and they can be straightforwardly applied to the case when statistical CSI describing the long-term channel characteristics is available at the transmitter. However, the straightforward extension of the general rank beamforming approach assuming instantaneous CSI in Chapter 4 to the case of statistical CSI is impossible. Due to the absence of instantaneous CSI at the transmitter, the orthogonality of the real-valued OSTBC matrix of the equivalent channel can no longer be guaranteed and thus inter-symbol interference is present which leads to performance degradations.

In Chapter 5, an alternative general rank beamforming approach is proposed to solve the multiuser downlink beamforming problem with additional shaping constraints when statistical CSI is available at the transmitter. Beamformers are designed according to the criteria of maximizing the minimum average SINR of users subject to a total transmit power constraint and additional shaping constraints. In the new general rank beamforming approach, beamforming is combined with full-rate QOSTBC. The use of QOSTBC destroys the full-orthogonality structure of the corresponding equivalent channel matrix such that generally maximum-likelihood (ML) pairwise decoding has to be applied for optimal decoding. As an alternative to the pairwise decoding, a simple phase rotation scheme on the beamformers at the transmitter side is proposed to enable simplified symbol-wise decoding. The original beamforming problem is transformed to a semidefinite programming (SDP) problem which can be solved optimally for a massive number of shaping constraints. Simulation results demonstrate a significant performance improvement over the existing approaches.

This dissertation is based on the following journal and conference publications, which have been published or submitted during the course of my doctoral research:

- X. Wen and M. Pesavento, "Long-term General Rank Multiuser Downlink Beamforming With Shaping Constraints Using QOSTBC," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Mar. 2016.

- K. Law, X. Wen, M. Vu, and M. Pesavento, "General rank multiuser downlink beamforming with shaping constraints using real-valued OSTBC," *IEEE Transactions on Signal Processing*, vol. 63, no. 21, pp. 5758-5771, Nov. 2015.

- K. Law, X. Wen, and M. Pesavento, "General-rank transmit beamforming for multi-group multicasting networks using OSTBC, in *Proc. IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, Jun. 2013, pp. 475-479.

- X. Wen, K. Law, S. Alabed, and M. Pesavento, "Rank-two beamforming for single-group multicasting networks using OSTBC, in *Proc. IEEE Sensor Array and Multi-channel Signal Processing Workshop (SAM)*, Jun. 2012, pp. 69-72.

Chapter 2

Rank-two Transmit Beamforming for Single-group Multicasting

2.1 Introduction

Current and upcoming wireless network standards such as LTE and LTE-Advanced have provisioned the use of multiple antennas at the base station [5, 14]. The flexibility offered by the advanced multi-channel infrastructure and the availability of the downlink CSI facilitate the development of efficient multicast beamforming techniques for multicasting applications such as audio and video streaming with high data traffic. Wireless multicasting is part of the LTE and LTE-Advanced standard defined by the Third Generation Partnership Project (3GPP) that is known as evolved Multimedia Broadcast Multicast Service (eMBMS) [28].

In this chapter, we consider the single-group multicasting problem where the transmitter equipped with multiple antennas broadcasts common information to multiple single-antenna receivers within a certain service area. The transmitter is assumed to have access to instantaneous CSI of all the subscribed users. Instantaneous CSI describes the short-term channel conditions of a communication link and can be estimated using training sequences in both frequency-division duplex (FDD) and time-division duplex (TDD) systems. In FDD systems, by performing downlink training, instantaneous CSI is estimated at the receiver side and fed back to the transmitter. In TDD systems, making use of the reciprocity property of the downlink channel and the uplink channel, uplink training is performed and instantaneous CSI can be estimated at the transmitter directly [29].

2.1.1 Related Work

The single-group multicasting problem has been firstly investigated in the Ph.D. dissertation of Lopez [30], in which the optimization problem of maximizing the sum of the SNR of all users was considered. This problem formulation can be understood as maximizing the average SNR over all users and it results in the principal component computation problem for the optimum beamformer selection. The drawback associated with this design is that the QoS cannot be guaranteed for all users, because the weakest user link determines the common information rate. To address this drawback, two new problem formulations for single-group multicasting were proposed in [31, 32]. One formulation is the problem of minimizing the total transmit power subject to QoS constraints in terms of SNR, which is designed for minimizing the inter-cell interference leakage and saving the energy consumption costs of network operators while providing QoS assurance to each receiver (power minimization problem). The other is the problem of maximizing the minimum SNR of all intended receivers subject to a total transmit power constraint, which is designed for maximizing the common data rate that is determined by the minimum received SNR, and correspondingly improving the user experiences (max-min problem). It has been proven in [32] that both problems are essentially equivalent to each other up to scaling, and they are generally non-convex and NP hard. The approximate solution can be obtained by resorting to the popular SDR technique that is a powerful approximation technique capable of transforming many difficult non-convex optimization problems to convex problems that can be solved efficiently [33]. If a rank-one matrix is obtained, the solution obtained by the SDR approach is optimal. Otherwise, a costly randomization procedure needs to be carried out and due to the approximations involved, the solution is highly suboptimal in general [32]. When the user population is large, high-rank SDR solutions are obtained in general when using the SDR approach proposed in [32] and the approximation quality needs to be improved.

An iterative algorithm for the max-min problem was proposed in [34]. In each iteration, the new weight vector is calculated by updating the previous weight vector with a given step size towards the SNR gradient direction of the receiver with the smallest SNR in the previous iteration. This is followed by a scaling procedure to fulfill the transmit power constraint. When the number of users is large, the algorithm in [34] may achieve better performance in terms of minimum SNR than the SDR approach in [32] and enjoys lower computation complexity, however, the algorithm may not converge and its performance is very sensitive with respect to the initialization weight vector. The work in [34] is outperformed by a similar beamforming algorithm as the one proposed in [35, 36] in which the weight vector that maximizes the average SNR is used for initialization and an adaptive step size is employed. Another line of research based on channel orthogonalization originated from [37] in which channel orthogonalization and a successive orthogonal refinement algorithm similar to [34] was proposed. The drawback is that its performance can be limited by its proposed choice of orthogonalization order and the scaling procedure in the successive orthogonal refinement algorithm. To deal with these drawbacks, a channel orthogonalization method based on QR decomposition [38] has been proposed in [39] by checking various orthogonalization orders and the best one is selected based on the criterion of the minimum total transmit power. Furthermore, the approach in [39] uses an improved non-orthogonal successive local refinement technique as compared to [37]. Recently, another promising approach has been proposed in [40] which develops a second-order cone programming (SOCP) solution to the power minimization problem. The successive linear approximation (SLA) algorithm developed in [40] starts with a feasible beamforming weight vector and the non-convex constraints are linearized around the initialization weight vector using first-order Taylor approximation. The resulting convex optimization problem is solved to obtain the next weight vector, which can be used for linearization for the next iteration subsequently.

The degraded performance associated with the aforementioned rank-one beamforming approaches using a single beamforming weight vector when the user population is large strongly attributes to the fact that the degrees of freedom are insufficient for designing a single spatially selective beamformer. If the number of users is large, the number of design parameters in the single beamforming weight vector is insufficient for meeting the large number of constraints simultaneously.

2.1.2 Contribution

In this chapter, we consider the power minimization problem for single-group multicasting and we propose an OSTBC based approach in which the degrees of freedom are doubled by introducing two independent beamforming weight vectors that are used to transmit two codewords of the Alamouti code [19]. Our proposed beamforming approach is specially attractive when combining with the SDR technique that is conventionally used to compute approximate solutions of the conventional single-group multicast beamforming problem. Unlike the conventional approach in which the performance degradation results from the rank-one approximations involved, in our approach a rank-two approximation is computed from the SDR solution. Therefore, the associated performance degradation from the optimal solution is therefore less severe than in the conventional approach. As shown in the simulation results, our proposed approach achieves better performance in terms of total transmit power than the existing ones while maintaining the same data rate. When the number of users grows large, the improvement becomes more significant.

This chapter is based on my original work that has been published in [41]. The remainder of this chapter is organized as follows. Section 2.2 provides the signal model and formulates the beamforming problem. In Section 2.3, the proposed approach is introduced. Simulation results are provided in Section 2.4 and the summary is made in Section 2.5.

2.2 Conventional Single-group Multicasting

Let us consider a wireless communication system where a base station or access point employing an antenna array of N elements is used to transmit common information to M single-antenna receivers simultaneously. In the single-group multicasting system, a single weight vector is used to steer a beam towards all the receivers in conventional rank-one beamforming approaches. Let us denote $\mathbf{w}$ and s as the $N \times 1$ beamforming weight vector and the zero-mean information symbol with unit power, respectively. Then, the signal received by the i-th receiver is given by

$$y_i = \underbrace{s\mathbf{w}^H\mathbf{h}_i}_{\text{signal}} + \underbrace{n_i}_{\text{noise}} \tag{2.1}$$

where $\mathbf{h}_i$ and n_i denote the $N \times 1$ downlink channel vector of the i-th receiver and the additive white receiver noise with the variance σ_i^2, respectively. By definition, the SNR can be computed as the expected signal power over the noise power. Therefore, based on (2.1), the SNR at the i-th receiver in the conventional rank-one beamforming approach is derived as

$$\mathrm{SNR}_{\mathrm{c},i} \triangleq \frac{\left|\mathbf{w}^H\mathbf{h}_i\right|^2}{\sigma_i^2} \tag{2.2}$$

where ‘c’ refers to the conventional approach. The problem of finding the beamforming weight vector that minimizes the total transmit power subject to the user QoS constraints can be expressed as

$$\begin{aligned} \min_{\mathbf{w}} \quad & \|\mathbf{w}\|^2 \\ \text{s.t.} \quad & \frac{\left|\mathbf{w}^H\mathbf{h}_i\right|^2}{\sigma_i^2} \geq \gamma_i, \quad \forall i = 1, \ldots, M \end{aligned} \tag{2.3}$$

where γ_i denotes the minimum SNR requirement of the i-th user. A particularly popular method for computing approximate solutions of problem (2.3) is the SDR approach [32] in which the transformation $\mathbf{W} \triangleq \mathbf{w}\mathbf{w}^H$ is used. Applying simple trace properties and relaxing the rank-one constraint for $\mathbf{W}$, the problem (2.3) can be relaxed to

$$\begin{aligned} \min_{\mathbf{W}} \quad & \mathrm{tr}\{\mathbf{W}\} \\ \text{s.t.} \quad & \frac{\mathrm{tr}\{\mathbf{W}\mathbf{h}_i\mathbf{h}_i^H\}}{\sigma_i^2} \geq \gamma_i, \; \forall\, i = 1, \ldots, M \\ & \mathbf{W} \succeq 0. \end{aligned} \tag{2.4}$$

The rank of the optimal solution to problem (2.4), denoted by $\mathbf{W}^\star$, is generally greater than one. In this case a generally suboptimal weight vector can be obtained using the random-ization techniques proposed in [32], and the worst-case approximation quality deteriorates linearly with the number of users [42]. Numerical simulations carried out in Section 2.4 further reveal that the probability that the higher-rank solutions are obtained for the SDR in (2.4) increases with the number of QoS constraints in the problem. This increase of the rank of $\mathbf{W}^{\star}$ is associated with a large deviation of the rank-one approximation from the optimal beamforming solution.

Apart from the difficulties emerging from the performance degradation due to the poor SDR approximation in the case of a large number of users, there exists a second effect that is even more prominent. Even in the case that an optimal solution of the NP hard problem (2.3) can be computed, the obtained beamformer generally does not exhibit sufficient spatial selectivity for all users in the system. It is clear that in this case the achievable beamforming gain is limited and the QoS constraints in (2.3) can only be satisfied by increasing the total transmit power. This motivates to introduce a more flexible beamforming design in the following section, in which the degrees of freedom are increased relative to the number of constraints.

2.3 Proposed Approach

The central idea of the proposed approach is to combine the single-group multicasting approach with the concept of OSTBC and to design multiple beamformers instead of a single one for the transmission of the coding matrix. Our approach is applicable for code matrices of arbitrary block size, however, for simplicity of presentation, in this chapter we consider the popular Alamouti code as an example.

2.3.1 Rank-two System Model

In the Alamouti code, two consecutive symbols are jointly encoded. Denote $\mathbf{s} = [s_1, s_2]^T$ as the symbol vector, the corresponding coding matrix is given by

$$\mathcal{X}(\mathbf{s}) \triangleq \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}. \tag{2.5}$$

We consider the case that

$$\begin{cases} \mathbf{x}_1 & \triangleq \; s_1\mathbf{w}_1^H + s_2\mathbf{w}_2^H \\ \mathbf{x}_2 & \triangleq -s_2^*\mathbf{w}_1^H + s_1^*\mathbf{w}_2^H \end{cases} \tag{2.6}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the transmitted signal vectors at the first and second time slot in each block, respectively, and $\mathbf{w}_1$ and $\mathbf{w}_2$ are the $N \times 1$ weight vectors. Assuming the block fading channel model, where the channels $\mathbf{h}_i$ for all $i = 1, \ldots, M$ remain constant over two time slots and the two symbols in each block are uncorrelated with each other, the received signals of the i-th user in the two time slots of each block are given by

$$\begin{bmatrix} y_{i,1} \\ y_{i,2} \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} \mathbf{w}_1^H\mathbf{h}_i \\ \mathbf{w}_2^H\mathbf{h}_i \end{bmatrix} + \begin{bmatrix} n_{i,1} \\ n_{i,2} \end{bmatrix} \tag{2.7}$$

where $n_{i,1}$ and $n_{i,2}$ denote the additive white noise of the i-th user at the first and second time slot, respectively, with the variance σ_i^2. The system model in (2.7) can also be considered as a virtual two-transmit-antenna single-group multicasting system applying the Alamouti code in which the channels between each pair of transmit and receive antennas for the i-th user are given by

$$\begin{cases} \tilde{h}_i(\mathbf{w}_1) \triangleq \mathbf{w}_1^H\mathbf{h}_i \\ \tilde{h}_i(\mathbf{w}_2) \triangleq \mathbf{w}_2^H\mathbf{h}_i \end{cases} \tag{2.8}$$

as illustrated in Fig. 2.1.

Using the equivalent channel representation for space-time block codes [16], the equation (2.7) can be equivalently written as

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{s} + \mathbf{n}_i \tag{2.9}$$

where

$$\mathbf{y}_i \triangleq \begin{bmatrix} y_{i,1} \\ y_{i,2}^* \end{bmatrix}, \; \mathbf{n}_i \triangleq \begin{bmatrix} n_{i,1} \\ n_{i,2}^* \end{bmatrix}, \; \mathbf{H}_i \triangleq \begin{bmatrix} \mathbf{w}_1^H\mathbf{h}_i & \mathbf{w}_2^H\mathbf{h}_i \\ (\mathbf{w}_2^H\mathbf{h}_i)^* & -(\mathbf{w}_1^H\mathbf{h}_i)^* \end{bmatrix}.$$

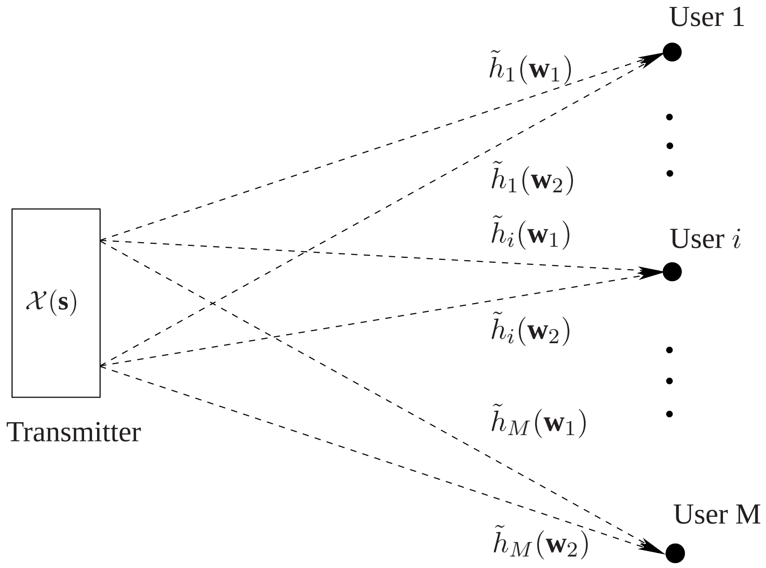

Figure 2.1: Virtual channel formulation of the proposed approach.

By left-multiplying the optimal decoding matrix $\mathbf{H}_i^H$ and dividing by $\left|\mathbf{w}_1^H\mathbf{h}_i\right|^2 + \left|\mathbf{w}_2^H\mathbf{h}_i\right|^2$ on both sides of (2.9), we have

$$\hat{\mathbf{s}} \triangleq \mathbf{s} + \tilde{\mathbf{n}}_i \tag{2.10}$$

where

$$\tilde{\mathbf{n}}_i \triangleq \frac{1}{\left|\mathbf{w}_1^H\mathbf{h}_i\right|^2 + \left|\mathbf{w}_2^H\mathbf{h}_i\right|^2} \begin{bmatrix} (\mathbf{w}_1^H\mathbf{h}_i)^* n_{i,1} + \mathbf{w}_2^H\mathbf{h}_i n_{i,2}^* \\ (\mathbf{w}_2^H\mathbf{h}_i)^* n_{i,1} - \mathbf{w}_1^H\mathbf{h}_i n_{i,2}^* \end{bmatrix}. \tag{2.11}$$

Then, the symbol vector $\mathbf{s}$ can be decoded using a simple linear symbol-wise decoder [19]. Since two symbols are decoded at every two time slots, the data rate of the proposed approach, i.e., on average one symbol per time slot, remains the same as in the conventional uncoded system employing a single beamformer.

2.3.2 Beamformer Optimization

Similar to (2.3), we consider a beamforming approach in which the average total transmit power in each time slot is minimized subject to the QoS constraints for all users, hence

$$\begin{aligned} \min_{\mathbf{w}_1,\mathbf{w}_2} \quad & P \\ \text{s.t.} \quad & \text{SNR}_i \geq \gamma_i,\ \forall\, i = 1, \ldots, M. \end{aligned} \tag{2.12}$$

The total transmit power in the first time slot is given by

$$P_1 \triangleq E\{\mathbf{x}_1^H \mathbf{x}_1\} = \|\mathbf{w}_1\|^2 + \|\mathbf{w}_2\|^2 . \tag{2.13}$$

Due to the orthogonality of the Alamouti code, the power in the second time slot yields the same result. Thus, the total transmit power in each time slot is given by

$$P \triangleq \|\mathbf{w}_1\|^2 + \|\mathbf{w}_2\|^2 . \tag{2.14}$$

According to (2.10), the SNR of s_1 at the i-th user is given by

$$\text{SNR}_i(s_1) \triangleq \frac{\left|\mathbf{w}_1^H \mathbf{h}_i\right|^2 + \left|\mathbf{w}_2^H \mathbf{h}_i\right|^2}{\sigma_i^2}. \tag{2.15}$$

Similarly, the same expression is obtained for $\text{SNR}_i(s_2)$. Thus, the SNR of each symbol at the i-th user is given by

$$\text{SNR}_i \triangleq \frac{\left|\mathbf{w}_1^H \mathbf{h}_i\right|^2 + \left|\mathbf{w}_2^H \mathbf{h}_i\right|^2}{\sigma_i^2}. \tag{2.16}$$

Making use of (2.14) and (2.16), the optimization problem in (2.12) can be rewritten as

$$\begin{aligned} \min_{\mathbf{w}_1,\mathbf{w}_2} \quad & \|\mathbf{w}_1\|^2 + \|\mathbf{w}_2\|^2 \\ \text{s.t.} \quad & \frac{\left|\mathbf{w}_1^H \mathbf{h}_i\right|^2 + \left|\mathbf{w}_2^H \mathbf{h}_i\right|^2}{\sigma_i^2} \geq \gamma_i,\ \forall\, i = 1, \ldots, M. \end{aligned} \tag{2.17}$$

Introducing the substitution $\mathbf{X} \triangleq \mathbf{w}_1\mathbf{w}_1^H + \mathbf{w}_2\mathbf{w}_2^H$, we have

$$\begin{cases} \|\mathbf{w}_1\|^2 + \|\mathbf{w}_2\|^2 = \mathrm{tr}\{\mathbf{X}\} \\ \left|\mathbf{w}_1^H\mathbf{h}_i\right|^2 + \left|\mathbf{w}_2^H\mathbf{h}_i\right|^2 = \mathrm{tr}\{\mathbf{X}\mathbf{h}_i\mathbf{h}_i^H\}. \end{cases} \tag{2.18}$$

By substituting $\mathbf{X}$ and adding the following constraints

$$\begin{cases} \mathbf{X} \succeq \mathbf{0} \\ \mathrm{rank}(\mathbf{X}) \leq 2 \end{cases} \tag{2.19}$$

the problem (2.17) can be equivalently formulated as

$$\begin{aligned} \min_{\mathbf{X}} \quad & \mathrm{tr}\{\mathbf{X}\} \\ \mathrm{s.t.} \quad & \frac{\mathrm{tr}\{\mathbf{X}\mathbf{h}_i\mathbf{h}_i^H\}}{\sigma_i^2} \geq \gamma_i, \quad \forall\, i = 1, \ldots, M \\ & \mathbf{X} \succeq 0, \\ & \mathrm{rank}\{\mathbf{X}\} \leq 2. \end{aligned} \tag{2.20}$$

To solve the problem (2.20), we use the SDR technique by removing the non-convex rank constraint resulting in the relaxed problem

$$\begin{aligned} \min_{\mathbf{X}} \quad & \mathrm{tr}\{\mathbf{X}\} \\ \mathrm{s.t.} \quad & \frac{\mathrm{tr}\{\mathbf{X}\mathbf{h}_i\mathbf{h}_i^H\}}{\sigma_i^2} \geq \gamma_i, \quad \forall\, i = 1, \ldots, M, \\ & \mathbf{X} \succeq 0. \end{aligned} \tag{2.21}$$

Note that the problem (2.21) is identical to the problem (2.4) obtained from the relaxation of the conventional single-group multicasting problem in (2.3), and its solution can be obtained using available convex optimization tools such as CVX [43]. Let $\mathbf{X}^\star$ denote the optimal solution to the problem (2.21). If $\mathbf{X}^\star$ is rank-one, a single weight vector is used to perform transmit beamforming where $\mathbf{w}_1$ is the principal component of $\mathbf{X}^\star$ and $\mathbf{w}_2$ is a zero vector. However, when the number of users is large, the solution is of higher rank in general. If $\mathbf{X}^\star$

is rank-two, $\mathbf{w}_1$ and $\mathbf{w}_2$ can be obtained as the first two principal components of $\mathbf{X}^\star$. We remark that in both cases when $\mathbf{X}^\star$ is either rank-one or rank-two, optimal solutions for $\mathbf{w}_1$ and $\mathbf{w}_2$ to the original problem in (2.17) can be obtained. If the rank is greater than two, the rank reduction techniques proposed in [26] can be applied to obtain a solution with minimal rank. However, in the case that the solution with the minimal rank still exceeds the rank of two, we propose a modified Gaussian randomization technique for rank-two approximations similar to the rank-one approximation method of [32]. Introducing the eigen-decomposition

$$\mathbf{X}^\star = \mathbf{U}\Sigma\mathbf{U}^H \tag{2.22}$$

the idea of the randomization technique is to generate a pair of candidate beamforming vectors for the r-th randomization instance as

$$\begin{cases} \mathbf{w}_{1r} \triangleq \mathbf{U}\Sigma^{1/2}\mathbf{e}_{1r} \\ \mathbf{w}_{2r} \triangleq \mathbf{U}\Sigma^{1/2}\mathbf{e}_{2r} \end{cases} \tag{2.23}$$

where $\mathbf{e}_{1r}$ and $\mathbf{e}_{2r}$ are randomly generated zero-mean complex circular Gaussian vectors with independent and identically distributed (i.i.d.) entries of unit variance.

In the conventional randomization technique of [32] applied to the solution of the SDR of (2.3) given in (2.4), candidates of a single weight vector are generated from the higher-rank solution of the relaxed problem. Afterwards, power scaling is applied to fulfill the QoS constraint of the worst user. However, in our approach, pairs of candidate beamforming vectors are generated and optimal power scaling needs to be performed for each pair of weight vectors. Let p_{1r} and p_{2r} denote the power scaling factors corresponding to $\mathbf{w}_{1r}$ and $\mathbf{w}_{2r}$, respectively. Then, the power scaling problem can be stated as

$$\begin{aligned} \min_{p_{1r},p_{2r}} \quad & p_{1r}\alpha_{1r} + p_{2r}\alpha_{2r} \\ \text{s.t.} \quad & p_{1r}\beta_{1ir} + p_{2r}\beta_{2ir} \geq \gamma_i\sigma_i^2, \quad \forall\, i = 1, \ldots, M, \\ & p_{kr} \geq 0, \quad \forall\, k = 1, 2 \end{aligned} \tag{2.24}$$

where

$$\begin{cases} \alpha_{kr} \triangleq \|\mathbf{w}_{kr}\|^2 \\ \beta_{kir} \triangleq \left|\mathbf{w}_{kr}^H \mathbf{h}_i\right|^2 . \end{cases} \tag{2.25}$$

Then, among all pairs of candidate weight vectors, the one with the lowest total transmit power is chosen as the final solution. Due to the linear programming procedure carried out for each pair of candidate beamformers, the overall computational complexity of this method is comparably high.

Seeking for a low-complexity implementation compared to the linear programming solution introduced above, we propose an alternative suboptimal power scaling procedure in which both weight vectors of each candidate pair are weighted by the same scalar, i.e., $p_{1r} = p_{2r}$.

2.4 Simulation Results

We assume a Rayleigh fading channel with i.i.d. circularly symmetric unit-variance channel coefficients. We also assume without loss of generality that $\sigma_i^2 = 0$dB and $\gamma_i = \gamma$ for all $i = 1, \ldots, M$. All results are averaged over 300 Monte-Carlo runs.

In the simulations, the label 'Method of [32]' stands for the SDR approach in [32], where all three randomization techniques proposed in that reference are used in parallel with 1000 candidate beamformers for each technique according to the specifications in [32]; 'Method of [39]' refers to the channel orthogonalization method of [39]; 'Method of [40]' refers to the SLA algorithm of [40]; 'Proposed (ES)' and 'Proposed (LP)' refer to the proposed equal scaling and linear programming based power scaling approaches, respectively, both employing the Gaussian randomization technique with 1000 pairs of candidate vectors; and 'Lower bound' stands for the total transmit power obtained by solving the relaxation problem (2.4) or (2.21) that may not be achievable.

In the first example, we compare the total transmit power of several methods versus different SNR thresholds when $N = 4$ and $M = 100$. As shown in Fig. 2.2, both 'Proposed

(LP)' and 'Proposed (ES)' outperform the competing methods in terms of the transmit power.

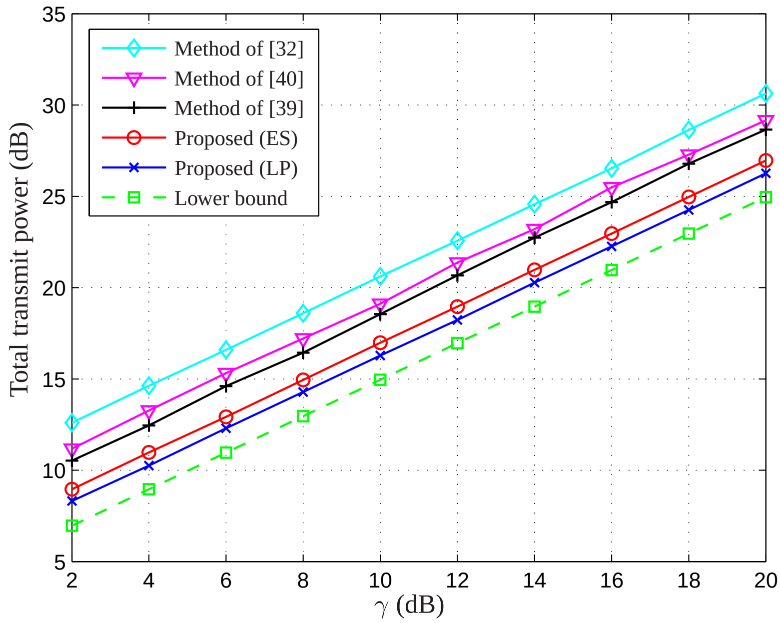

Figure 2.2: Total transmit power vs. SNR thresholds.

In the second example, we compare the different techniques for a varied number of users when $N = 4$ and $\gamma = 10$ dB. Fig. 2.3 and Fig. 2.4 display the histogram of the rank of the optimal solutions $\mathbf{X}^{\star}$ of (2.21) and the total transmit power versus different numbers of users, respectively. When the number of users is small (< 32), in Fig. 2.3, more than 90% of the solutions $\mathbf{X}^{\star}$ are either rank-one or rank-two, in these cases optimal solutions to the problem (2.17) are obtained in 'Proposed (LP)' and 'Proposed (ES)'. In Fig. 2.4, the total transmit power for 'Proposed (LP)' and 'Proposed (ES)' is identical or close to 'Lower bound' and smaller than that of the other techniques. When the number of users is large (> 32), in Fig. 2.3, more than 90% of the solutions exhibit a rank greater than two, in this case approximate solutions are obtained. In Fig. 2.4, 'Proposed (LP)' and 'Proposed (ES)' consume lower power than the others. When the number of users increases, the gap between 'Proposed (LP)' or 'Proposed (ES)' and the known techniques increases as well, which indicates a substantial performance improvement. Since the power reduction benefit that 'Proposed (LP)' achieves over 'Proposed (ES)' is comparably small, the latter technique that offers a significantly reduced complexity may be considered for practical use.

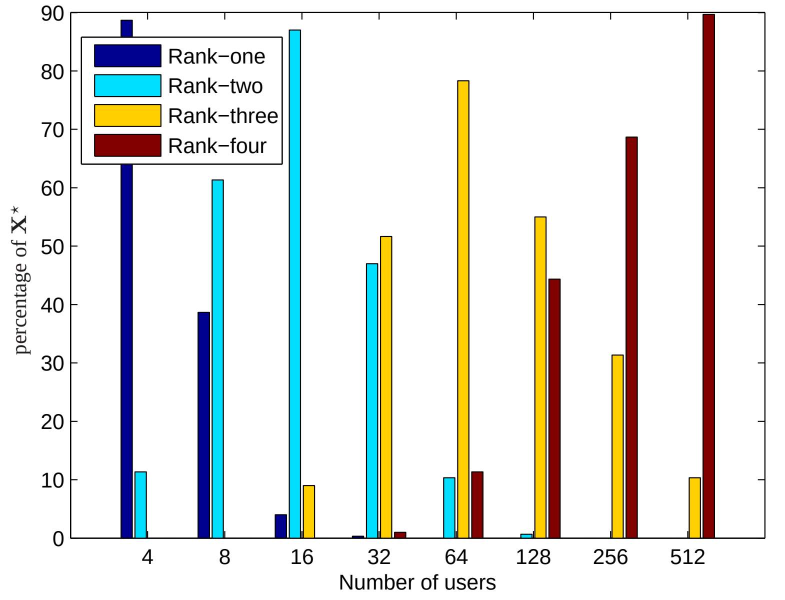

Figure 2.3: Rank percentage of $\mathbf{X}^\star$ vs. number of users.

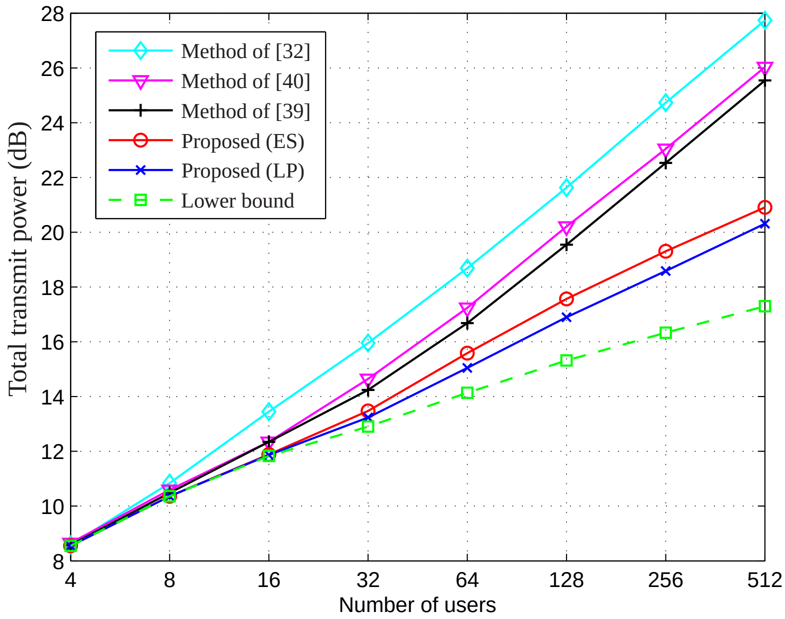

Figure 2.4: Total transmit power vs. number of users.

2.5 Summary

In this chapter, a rank-two beamforming approach in combination with the Alamouti code is developed for single-group multicasting. The proposed approach has been shown to offer substantially better performance than the existing rank-one methods, especially when the number of users is large. As compared to the SDR based rank-one approaches, the computation complexity of the proposed approach is not increased, since identical SDR formulations are obtained for both rank-one and rank-two approaches. When the SDR solution is of rank two, the randomization procedure is avoided in the rank-two approach of which the total computational complexity is decreased as compared to the rank-one approaches. The downlink signaling overhead in the rank-two approach is slightly increased as compared to the rank-one approaches since two individual composite channels are needed for the decoding at each receiver while in the rank-one approaches only one is required. The proposed technique can be generalized to high dimentional (>2) OSTBC to further increase the degrees of freedom for designing spatially selective beamformers. However, this generalization is associated with a reduced transmission rate since full-rate full-diversity OSTBC only exists for the dimension of two [16]. Instead of the high dimensional OSTBC, the full-rate property can be maintained in the high dimensional real-valued OSTBC and QOSTBC, however, the application of these codes in single-group multicasting is impractical for which the reason will be explained in Chapter 4 and Chapter 5.

Chapter 3

Rank-two Transmit Beamforming for Multi-group Multicasting

3.1 Introduction

In multi-group multicast transmit beamforming, independent information is transmitted to different groups and users in the same group receives the same information. As compared to the single-group multicast transmit beamforming that is investigated in Chapter 2 where a single group of users receive the same information, the spectral efficiency of multi-group multicast transmit beamforming can be further improved by serving several groups of co-channel users simultaneously [36, 44–53]. Along with the spectral benefits, the emergence of the multiuser interference becomes a challenging problem in the multi-group multicasting beamformer design.

3.1.1 Related Work

The seminal work on multi-group multicast beamforming [44, 45] dealt with two QoS based problems: the problem of minimizing the total transmit power while satisfying the prescribed minimum SINR requirements of all receivers; and a max-min problem of maximizing the minimum SINR of all users in different groups subject to a total transmit power constraint. Both QoS based beamforming problems have been proven to be NP hard and a SDR based approach was developed in [45] to address the beamforming optimization problems. Rather than the SDR based approach, alternative convex approximation approaches based on SOCP were proposed in [49–51]. The procedure of randomization and power control associated with the SDR based approach is avoided in the SOCP based approaches and iterative algorithms are developed therein. Later on, an iterative inner approximation approach involving sequential convex optimization has been proposed in [52] to solve the max-min multi-group multicasting problem more efficiently. Furthermore, for practical considerations, the constraint of the maximum permitted transmit power level for each antenna is incorporated in the multi-group multicasting beamformer design and the SDR based approach has been developed in [53] following the idea of [45].

As in the single-group multicasting case considered in Chapter 2, when the number of users is large, the flexibility of designing spatially selective beamformers in the conventional adaptive beamforming approaches in [45] and [52] can be rather limited. Therefore, new techniques for improving the beamforming performance are of great practical importance.

3.1.2 Contribution

In this chapter, we apply the rank-two beamforming approach to solve the problem by combining multi-group multicast beamforming with OSTBC. Similar as in conventional beamforming, and different from conventional STBC transmission techniques, we assume that instantaneous CSI of all users is available at the transmitter side. This approach follows the general idea of Chapter 2, which is proposed for single-group multicasting networks where multiuser interference is absent. As compared to the rank-two beamforming approaches in Chapter 2, we consider the multi-group multicasting network where multiuser interference is dominant. In this approach, transmit beamforming is jointly used with Alamouti OSTBC to serve all the users [19]. The users belonging to each group are generally served with up to two beamformers over two consecutive time slots using Alamouti code. Due to the orthogonality of the code, the decoding complexity at the receivers is not increased and symbol-by-symbol detection can be performed. The use of two beamformers per group doubles the degrees of freedom in the beamformer design and offers improved beamforming performance. Interestingly, our QoS based max-min beamforming design results in identical SDR formulations as in the conventional beamforming approach. However, unlike in the conventional rank-one beamforming approach where only rank-one solutions are optimal, here SDR solutions involving a rank smaller or equal to two are proven to be optimal for the original problem. In the case that the SDR solution corresponding to one group exhibits a rank larger than two, a modified randomization technique which is similar to the randomization technique in Chapter 2 is employed to compute the approximate solutions. Furthermore, following the approach of [52], in this chapter we propose an iterative inner approximation technique for rank-two beamforming that is more computationally efficient as compared to the SDR based outer approximation technique. Simulation results show that the proposed rank-two approaches significantly outperform the existing approaches.

This chapter is based on my original work that has been published in [54]. The remainder of this chapter is organized as follows. Section 3.2 introduces the conventional multi-group multicast beamforming problem. In Section 3.3, the modified signal model of the OSTBC based rank-two beamforming is introduced and the modified QoS based max-min rank-two beamformer optimization is performed. Simulation results are provided in Section 3.4 and the summary is made in Section 3.5.

3.2 Conventional Multi-group Multicasting

Consider a wireless communication system where a base station or access point equipped with an antenna array of N elements simultaneously transmits information to M single-antenna users. There are $1 \leq G \leq M$ user groups in total, $\{g_1, \ldots, g_G\}$, where g_k is the index set of the users intended to receive the multicasting stream for the k-th group, and $k \in \mathcal{K}$ where $\mathcal{K} = \{1, \ldots, G\}$. Each user belongs to only one group and decodes the corresponding single data stream. Thus, we have $g_k \cap g_l = \emptyset$ for any $l \neq k$, and $\cup_k g_k = \{1, \ldots, M\}$, treating the symbols of the remaining groups as noise. The multi-group multicasting scenario is illustrated in Fig. 3.1.

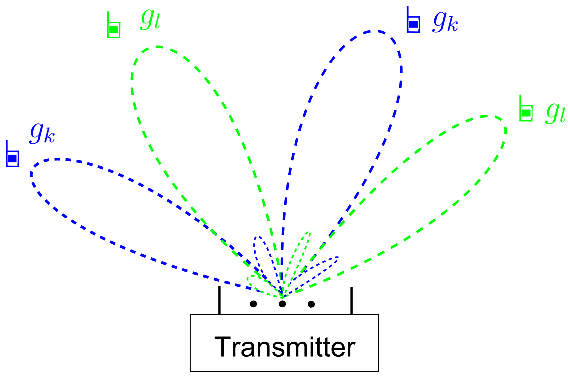

Figure 3.1: Multi-group multicasting

In conventional multi-group multicast beamforming, a single weight vector is designed for each group to transmit information intended for that particular group, thus there are G beamformers in total [45]. Let us denote $\mathbf{w}_k$ and s_k as the $N \times 1$ weight vector that is steered towards the k-th group and the zero-mean mutually statistically independent signal with unit power intended for the k-th group, respectively. The $N \times 1$ transmit signal vector is $\sum_{k=1}^{G} s_k \mathbf{w}_k^*$ and the total transmit power equals $\sum_{k=1}^{G} \|\mathbf{w}_k\|^2$. Then, the signal received by the i-th user in the k-th group is given by [45]

$$y_i = \underbrace{s_k \mathbf{w}_k^H \mathbf{h}_i}_{\text{signal}} + \underbrace{\sum_{l \neq k} s_l \mathbf{w}_l^H \mathbf{h}_i}_{\text{interference}} + \underbrace{n_i}_{\text{noise}} \tag{3.1}$$

where $\mathbf{h}_i$ and n_i denote the $N \times 1$ downlink channel vector and the additive white receiver noise with variance σ_i^2 at the i-th user in the k-th group, respectively. By definition, the SINR can be computed as the expected signal power over the expected interference plus noise power. Therefore, based on (3.1), the SINR of the i-th user belonging to the k-th group in the conventional rank-one beamforming approach is derived as

$$\mathrm{SINR}_{\mathrm{c},i} \triangleq \frac{\left|\mathbf{w}_k^H \mathbf{h}_i\right|^2}{\sum\limits_{l=1, l \neq k}^{G} \left|\mathbf{w}_l^H \mathbf{h}_i\right|^2 + \sigma_i^2} \tag{3.2}$$

where 'c' refers to the conventional approach. As compared to the SNR expression in (2.2) in Chapter 2, the co-channel interference is present in the SINR expression of $\mathrm{SINR}_{\mathrm{c},i}$ in

(3.2). The problem of finding the beamforming weight vectors that maximize the minimum SINR of all users subject to the power constraint $\mathrm{P}_{\max}$ can be formulated as [45]

$$\begin{aligned} \max_{\{\mathbf{w}_k\}_{k=1}^{G}} \quad & \min_{\forall k \in \mathcal{K}} \min_{\forall i \in g_k} \mathrm{SINR}_{\mathrm{c},i} \\ \text{s.t.} \quad & \sum_{k=1}^{G} \|\mathbf{w}_k\|^2 \leq \mathrm{P}_{\max} \end{aligned} \tag{3.3}$$

which can be equivalently written as

$$\begin{aligned} \max_{\{\mathbf{w}_k\}_{k=1}^{G}, t} \quad & t \\ \text{s.t.} \quad & \frac{\left|\mathbf{w}_k^H \mathbf{h}_i\right|^2}{\sum\limits_{l=1, l \neq k}^{G} \left|\mathbf{w}_l^H \mathbf{h}_i\right|^2 + \sigma_i^2} \geq t, \quad \forall i \in g_k \; \forall k \in \mathcal{K} \\ & \sum_{k=1}^{G} \|\mathbf{w}_k\|^2 \leq \mathrm{P}_{\max}. \end{aligned} \tag{3.4}$$

Different from the single-group multicasting scenario investigated in Chapter 2, the power minimization problem and the max-min problem in the multi-group multicasting scenario are not equivalent problems. The power minimization problem here can become infeasible if the number of groups and users is too large and/or the SINR requirements are too stringent and/or the channels of users belonging to different groups are highly correlated. However, the max-min problem of (3.4) is always feasible. Problem (3.4) is proven to be a NP hard problem in [45] and the SDR framework is employed to approximate problem (3.4) by a SDP problem. The Gaussian randomization technique along with the power control involving linear programming in each randomization instance is then applied on the SDR solution to obtain suboptimal feasible rank-one beamforming solutions. As an alternative to the generic SDR technique, a computationally efficient iterative inner approximation technique has been proposed in [52], which in each iteration involves first order Taylor approximation of the originally non-convex constraint set around the feasible solution obtained from the previous iteration.

3.3 Proposed Approach

The central idea of the rank-two beamforming approach proposed in this chapter is to combine multi-group multicast beamforming with the concept of OSTBC based symbol transmission. We apply the Alamouti code which achieves full-rate transmission of one symbol per time slot. In correspondence to the 2×2 Alamouti code matrix that is applied at the transmitter, a pair of weight vectors instead of a single one is used to transmit the data streams to the designated multicasting groups over two consecutive time slots.

3.3.1 Rank-two System Model

Denote $\mathbf{s}_k = [s_{k1}, s_{k2}]^T$ as the symbol vector for the k-th group. In the Alamouti OSTBC, two symbols are transmitted within two time slots. Similar as (2.5) in Chapter 2, the code matrix for $\mathbf{s}_k$ is given by

$$\mathcal{X}(\mathbf{s}_k) \triangleq \begin{bmatrix} s_{k1} & s_{k2} \\ -s_{k2}^* & s_{k1}^* \end{bmatrix}. \tag{3.5}$$

Unlike conventional Alamouti transmission schemes where the code matrix in (3.5) is transmitted from two transmit antennas over two consecutive time slots, here the code is transmitted from all N transmit antennas at the base station or access point using two different beamformers, i.e., $\mathbf{w}_{k1}$ and $\mathbf{w}_{k2}$, which form two virtual antennas over which the code is transmitted. Different from the rank-two beamforming approach in Chapter 2 where a single pair of beamformers are designed to serve all the users, here $\mathbf{w}_{k1}$ and $\mathbf{w}_{k2}$ only serve the k-th group. Defining the beamforming matrix

$$\mathbf{W}_k \triangleq [\mathbf{w}_{k1}, \mathbf{w}_{k2}] \tag{3.6}$$

the transmit signal in each time block is given by $\sum_{k=1}^{G} \mathcal{X}(\mathbf{s}_k)\mathbf{W}_k^H$.

Assuming the block fading channel model, the received signal vector of the i-th user in the k-th group in two consecutive time slots of one transmission block is given by

$$\mathbf{y}_i = \underbrace{\mathcal{X}(\mathbf{s}_k)\mathbf{W}_k^H\mathbf{h}_i}_{\text{signal}} + \underbrace{\sum_{l=1,l\neq k}^{G} \mathcal{X}(\mathbf{s}_l)\mathbf{W}_l^H\mathbf{h}_i}_{\text{interference}} + \underbrace{\mathbf{n}_i}_{\text{noise}} \tag{3.7}$$

where

$$\begin{cases} \mathbf{y}_i \triangleq [y_{i1}, y_{i2}]^T \\ \mathbf{n}_i \triangleq [n_{i1}, n_{i2}]^T \end{cases} \tag{3.8}$$

and y_{ij} and n_{ij} denotes the received signal and the additive white noise of the i-th user at the j-th time slot, respectively. It is clear that (3.7) has a similar structure as (3.1). Using the equivalent channel representation for OSTBC [16], equation (3.7) can be equivalently written as

$$\tilde{\mathbf{y}}_i = \underbrace{\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)\mathbf{s}_k}_{\text{signal}} + \underbrace{\sum_{l=1,l\neq k}^{G} \mathcal{X}(\mathbf{W}_l^H\mathbf{h}_i)\mathbf{s}_l}_{\text{interference}} + \underbrace{\tilde{\mathbf{n}}_i}_{\text{noise}} \tag{3.9}$$

where

$$\begin{cases} \tilde{\mathbf{y}}_i \triangleq [y_{i1}, -y_{i2}^*]^T \\ \tilde{\mathbf{n}}_i \triangleq [n_{i1}, -n_{i2}^*]^T. \end{cases} \tag{3.10}$$

As a common approach to decode the received symbols in OSTBC, the ML decoding problem for detecting the symbols of the i-th user can be formulated as

$$\min_{\mathbf{s}_k \in \mathcal{A}_k} \left\|\tilde{\mathbf{y}}_i - \mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)\mathbf{s}_k\right\|^2 = \min_{\mathbf{s}_k \in \mathcal{A}_k} \left\|\frac{1}{\alpha_i}\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)^H\tilde{\mathbf{y}}_i - \mathbf{s}_k\right\|^2 \tag{3.11}$$

where $\alpha_i \triangleq \mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i$, and $\mathcal{A}_k$ is the vector constellation of $\mathbf{s}_k$. By left-multiplying the decoding matrix $\frac{\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)^H}{\alpha_i}$ on both sides of (3.9), we have

$$\begin{aligned}\hat{\mathbf{s}}_k \triangleq& \frac{1}{\alpha_i}\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)^H\tilde{\mathbf{y}}_i \\ =&\mathbf{s}_k + \underbrace{\frac{1}{\alpha_i}\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)^H(\sum_{l=1,l\neq k}^{G}\mathcal{X}(\mathbf{W}_l^H\mathbf{h}_i)\mathbf{s}_l)}_{\hat{\mathbf{s}}_k^{(I)}} + \underbrace{\frac{1}{\alpha_i}\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)^H\tilde{\mathbf{n}}_i}_{\hat{\mathbf{s}}_k^{(N)}}. \end{aligned} \tag{3.12}$$

Taking into account that the symbols in $\mathbf{s}_k$ are assumed to be statistically independent and making use of the orthogonality property of $\mathcal{X}(\mathbf{W}_k^H\mathbf{h}_i)$, we have

$$\begin{aligned}\mathrm{E}\{\hat{\mathbf{s}}_k\hat{\mathbf{s}}_k^H\} &= \mathbf{I}_2 + \frac{\sum_{l=1,l\neq k}^{G}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}_2 + \frac{\sigma_i^2}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}_2 \\ &= \frac{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i + \sum_{l=1,l\neq k}^{G}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i + \sigma_i^2}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}_2. \end{aligned} \tag{3.13}$$

From the diagonal structure of (3.13), we observe that there exists no inter-symbol interference, thus $\mathbf{s}_k$ can be decoded using a simple linear symbol-wise decoder. According to (3.12), the covariance of the desired signal at the i-th user is

$$\mathrm{E}\{\mathbf{s}_k\mathbf{s}_k^H\} = \mathbf{I}_2, \tag{3.14}$$

the covariance of the interference at the i-th user is

$$\mathrm{E}\{\hat{\mathbf{s}}_k^{(I)}\hat{\mathbf{s}}_k^{(I)H}\} = \frac{\sum_{l=1,l\neq k}^{G}\mathbf{h}_i^H\mathbf{W}_l\mathbf{W}_l^H\mathbf{h}_i}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}_2, \tag{3.15}$$

and the covariance of the noise at the i-th user is

$$\mathrm{E}\{\hat{\mathbf{s}}_k^{(N)}\hat{\mathbf{s}}_k^{(N)H}\} = \frac{\sigma_i^2}{\mathbf{h}_i^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{h}_i}\mathbf{I}_2. \tag{3.16}$$

Based on (3.14), (3.15) and (3.16), the SINR of the i-th user corresponding to symbol s_{k1}

can be written as

$$\mathrm{SINR}_i(s_{k1}) \triangleq \frac{\mathbf{h}_i^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{h}_i}{\sum\limits_{l=1, l\neq k}^{G} \mathbf{h}_i^H \mathbf{W}_l \mathbf{W}_l^H \mathbf{h}_i + \sigma_i^2}. \tag{3.17}$$

Similarly, the same expression is obtained for $\mathrm{SINR}_i(s_{k2})$. Therefore, for the i-th user the SINR is identical for both symbols in each block which is given by

$$\mathrm{SINR}_i \triangleq \frac{\mathbf{h}_i^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{h}_i}{\sum\limits_{l=1, l\neq k}^{G} \mathbf{h}_i^H \mathbf{W}_l \mathbf{W}_l^H \mathbf{h}_i + \sigma_i^2}. \tag{3.18}$$

The total transmit power in the j-th time slot in each block is given by

$$\begin{aligned} P_j \triangleq & \mathrm{E}\{(\sum_{k=1}^{G} [\mathcal{X}(\mathbf{s}_k)]_j \mathbf{W}_k^H)(\sum_{k=1}^{G} [\mathcal{X}(\mathbf{s}_k)]_j \mathbf{W}_k^H)^H)\} \\ = & \mathrm{tr}(\sum_{k=1}^{G} \mathbf{W}_k^H \mathbf{W}_k E\{[\mathcal{X}(\mathbf{s}_k)]_j^H [\mathcal{X}(\mathbf{s}_k)]_j\}) \\ = & \sum_{k=1}^{G} \mathrm{tr}(\mathbf{W}_k \mathbf{W}_k^H) \end{aligned} \tag{3.19}$$

where we make use of the statistical independence of the transmitted symbols among users and the orthogonality of the code matrix. Note that the total transmit power expression in (3.19) is independent of the time index j. Therefore, the total transmit power is identical for all time slots in the OSTBC block and it is given by

$$P \triangleq \sum_{k=1}^{G} \mathrm{tr}(\mathbf{W}_k \mathbf{W}_k^H). \tag{3.20}$$

3.3.2 Beamformer Optimization

We consider a QoS based max-min beamforming approach in which the minimum SINR of all users is maximized subject to the constraint of total transmit power per time slot [45]. We remark that it is practically important and fair to constrain the total transmit power per time slot here because the power constraint in (3.4) in the conventional problem is also restricting the power per time slot. Using (3.18) and (3.20), the beamforming optimization problem can be presented as

$$\begin{aligned}\max_{\{\mathbf{W}_k\}_{k=1}^{G},t} \quad & t \\ \text{s.t.} \quad & \frac{\mathbf{h}_i^H \mathbf{W}_k \mathbf{W}_k^H \mathbf{h}_i}{\sum\limits_{l=1,l\neq k}^{G} \mathbf{h}_i^H \mathbf{W}_l \mathbf{W}_l^H \mathbf{h}_i + \sigma_i^2} \geq t, \quad \forall i \in g_k \, \forall k \in \mathcal{K} \\ & \sum_{k=1}^{G} \operatorname{tr}(\mathbf{W}_k \mathbf{W}_k^H) \leq \mathrm{P}_{\max}. \end{aligned} \tag{3.21}$$

Following the SDR approach, let

$$\mathbf{X}_k \triangleq \mathbf{W}_k \mathbf{W}_k^H = \sum_{j=1}^{2} \mathbf{w}_{kj} \mathbf{w}_{kj}^H, \quad \forall k \in \mathcal{K}. \tag{3.22}$$

By substituting $\mathbf{X}_k$ and adding the following constraints

$$\begin{cases} \mathbf{X}_k \succeq \mathbf{0} \\ \operatorname{rank}(\mathbf{X}_k) \leq 2, \quad \forall k \in \mathcal{K} \end{cases} \tag{3.23}$$

problem (3.21) can be equivalently written as

$$\begin{aligned}\max_{\{\mathbf{X}_k\}_{k=1}^{G},t} \quad & t \\ \text{s.t.} \quad & \frac{\mathbf{h}_i^H \mathbf{X}_k \mathbf{h}_i}{\sum\limits_{l=1,l\neq k}^{G} \mathbf{h}_i^H \mathbf{X}_l \mathbf{h}_i + \sigma_i^2} \geq t, \quad \forall i \in g_k \, \forall k \in \mathcal{K} \\ & \sum_{k=1}^{G} \operatorname{tr}(\mathbf{X}_k) \leq \mathrm{P}_{\max}, \\ & \mathbf{X}_k \succeq 0, \\ & \operatorname{rank}\{\mathbf{X}_\mathrm{k}\} \leq 2, \quad \forall k \in \mathcal{K} \end{aligned} \tag{3.24}$$

where $\mathbf{X}_k \succeq 0$ constrains $\mathbf{X}_k$ to lie in the set of positive semidefinite Hermitian matrices. Substituting the rank-one matrix $\mathbf{X}_k = \mathbf{w}_k \mathbf{w}_k^H$ in the conventional beamforming problem (3.4) and comparing the resulting problem with (3.24), we observe that both problems are identical up to the non-convex rank constraints, i.e., the rank-one constraints in the reformulation of (3.4) and the rank-two constraint in (3.24). As the set of rank-two matrices includes the set of rank-one matrices, we observe that the rank-two beamforming solutions of (3.24) generally yield improved QoS as compared to the rank-one beamforming solutions of (3.4). It follows from the discussion above that the SDR technique applied to both (3.4) and (3.24) results in the same optimization problem given by

$$\max_{\{\mathbf{X}_k\}_{k=1}^G, t} \quad t \tag{3.25a}$$

$$\text{s.t.} \quad \frac{\mathbf{h}_i^H \mathbf{X}_k \mathbf{h}_i}{\sum_{l=1, l\neq k}^{G} \mathbf{h}_i^H \mathbf{X}_l \mathbf{h}_i + \sigma_i^2} \geq t, \quad \forall i \in g_k \; \forall k \in \mathcal{K} \tag{3.25b}$$

$$\sum_{k=1}^{G} \operatorname{tr}(\mathbf{X}_k) \leq \mathrm{P}_{\max}, \tag{3.25c}$$

$$\mathbf{X}_k \succeq 0, \quad \forall k \in \mathcal{K}. \tag{3.25d}$$

Due to the emergence of multiuser interference, the bi-linear constraints appear in (3.25b). Since they are non-linear inequalities, we perform a one-dimensional bisection search over t as in [45] and [55] with the aid of currently available convex optimization tools such as CVX [43]. The computational complexity of the SDP procedure is $\mathcal{O}(G^3N^6 + MGN^2)$ in each bisection search step, which is the same as in the conventional SDR approach in [45]. We remark that due to the difference in the rank constraints, generally the SDR of (3.24) is tighter than that of (3.4).

Denote $\{\mathbf{X}_k^\star\}_{k=1}^G$ as the optimal solution to (3.25), the optimal value associated with it can serve as the upper bound to the original problem (3.21) which is used to evaluate the approximation quality of the proposed approach as shown in the simulation. When $\operatorname{rank}(\mathbf{X}_k^\star) \leq 2$, $\forall k$, the optimal weight vector solutions to the problem (3.21) can be obtained by computing the principal components of $\{\mathbf{X}_k^\star\}_{k=1}^G$ straightforwardly. However, if there exists at least one

$\mathbf{X}_k^\star$ with $\mathrm{rank}(\mathbf{X}_k^\star)>2$, rank reduction techniques proposed in [26] can be applied to reduce the rank of $\mathbf{X}_k^\star$. If after the rank reduction procedure, there still exists at least one $\mathbf{X}_k^\star$ with $\mathrm{rank}(\mathbf{X}_k^\star)>2$, the modified randomization technique is proposed in this chapter for the rank-two case to compute the approximate solutions, which follows the general procedure of the randomization technique proposed in Chapter 2. Note that different from the randomization procedure in Chapter 2, where the rank of a single solution matrix should be considered to decide whether a randomization procedure is necessary to carry out, here multiple ranks of multiple solution matrices $\{\mathbf{X}_k^\star\}_{k=1}^G$ should be considered and the randomization procedure can be necessary even when some $\mathbf{X}_k^\star$ has a rank no greater than two.

In the randomization procedure, let us denote $\mathbf{w}_{k1}^{(r)}$ and $\mathbf{w}_{k2}^{(r)}$ as the candidate weight vectors for $\mathbf{w}_{k1}$ and $\mathbf{w}_{k2}$ in the r-th randomization instance, respectively. If $\mathrm{rank}(\mathbf{X}_k^\star)\leq 2$, $\mathbf{w}_{k1}^{(r)}$ and $\mathbf{w}_{k2}^{(r)}$ are computed as the principal components of $\mathbf{X}_k^\star$; conversely, if $\mathrm{rank}(\mathbf{X}_k^\star)>2$, we first perform an eigen-decomposition on $\mathbf{X}_k^\star$ as

$$\mathbf{X}_k^\star = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{U}_k^H \tag{3.26}$$

then choose

$$\begin{cases} \mathbf{w}_{k1}^{(r)} \triangleq \mathbf{U}_k \boldsymbol{\Sigma}_k^{1/2} \mathbf{e}_{kr1} \\ \mathbf{w}_{k2}^{(r)} \triangleq \mathbf{U}_k \boldsymbol{\Sigma}_k^{1/2} \mathbf{e}_{kr2} \end{cases} \tag{3.27}$$

where $\mathbf{e}_{kr1}$ and $\mathbf{e}_{kr2}$ are the $N \times 1$ vectors containing the realizations of i.i.d. complex circular Gaussian distributed random variables with zero mean and unit variance corresponding to $\mathbf{w}_{k1}^{(r)}$ and $\mathbf{w}_{k2}^{(r)}$, respectively. Then, the global power control procedure over all groups involving bisection search and linear programming is performed to compute a candidate set of the weight vector solutions. Different from the power control procedure in Chapter 2 where a pair of power scaling factors corresponding to two weight vectors are optimized for purely guaranteeing the signal power, here multiple pairs of power scaling factors are optimized and the balance between the signal power and the interference power is considered.

Let $p_{k1}^{(r)}$ and $p_{k2}^{(r)}$ denote the power scaling factors corresponding to $\mathbf{w}_{k1}^{(r)}$ and $\mathbf{w}_{k2}^{(r)}$, respectively. The power control problem can be stated as

$$\begin{aligned}
\max_{\substack{t, p_{kj}^{(r)} \\ \forall j=1,2, \forall k \in \mathcal{K}}} \quad & t \\
\text{s.t.} \quad & \frac{p_{k1}^{(r)} \beta_{k1i}^{(r)} + p_{k2}^{(r)} \beta_{k2i}^{(r)}}{\sum_{l=1, l \neq k}^{G} (p_{l1}^{(r)} \beta_{l1i}^{(r)} + p_{l2}^{(r)} \beta_{l2i}^{(r)}) + \sigma_i^2} \geq t, \ \forall i \in g_k \ \forall k \in \mathcal{K} \\
& \sum_{k=1}^{G} (p_{k1}^{(r)} \alpha_{k1}^{(r)} + p_{k2}^{(r)} \alpha_{k2}^{(r)}) \leq \mathrm{P}_{\max}, \\
& p_{kj}^{(r)} \geq 0, \ \forall \ j = 1, 2, \ \forall k \in \mathcal{K}
\end{aligned} \tag{3.28}$$

where

$$\begin{cases} \alpha_{kj}^{(r)} \triangleq \left\| \mathbf{w}_{kj}^{(r)} \right\|^2 \\ \beta_{kji}^{(r)} \triangleq \left| \mathbf{w}_{kj}^{(r)H} \mathbf{h}_i \right|^2, \forall j = 1, 2, \ \forall i \in g_k, \ \forall k \in \mathcal{K}. \end{cases} \tag{3.29}$$

Among all sets of candidate solutions obtained, the set with the largest SINR value is selected as the final solution.

As an alternative to the SDR based approach, we propose a computationally more efficient approach to obtain approximate solutions iteratively by performing optimization over sequential convex inner approximations, similar as in [52]. Towards this aim, let us consider problem (3.21), which can be written in another form as

$$\begin{aligned}
\max_{\substack{t, \mathbf{w}_{kj} \\ \forall j=1,2, \forall k \in \mathcal{K}}} \quad & t \\
\text{s.t.} \quad & -\left|\mathbf{w}_{k1}^H \mathbf{h}_i\right|^2 - \left|\mathbf{w}_{k2}^H \mathbf{h}_i\right|^2 + t \sum_{l=1, l \neq k}^{G} \left(\left|\mathbf{w}_{l1}^H \mathbf{h}_i\right|^2 + \left|\mathbf{w}_{l2}^H \mathbf{h}_i\right|^2 \right) \\
& + t\sigma_i^2 \leq 0, \ \forall i \in g_k \ \forall k \in \mathcal{K} \\
& \sum_{k=1}^{G} (\|\mathbf{w}_{k1}\|^2 + \|\mathbf{w}_{k2}\|^2) \leq \mathrm{P}_{\max}.
\end{aligned} \tag{3.30}$$

In order to solve the non-convex problem in (3.30), the general idea is to introduce an iterative procedure in which in the $(p+1)$-th iteration, $\mathbf{w}_{kj}$ and t are replaced by $\mathbf{w}_{kj}^{(p)} + \Delta\mathbf{w}_{kj}$ and $t^{(p)} + \Delta t$, $\forall k \in \mathcal{K}$,$\forall j \in \{1,2\}$, where $\mathbf{w}_{kj}^{(p)}$ and $t^{(p)}$ are the beamforming weight vector and the SINR level obtained from the p-th iteration, respectively. By neglecting the non-convex terms $-(\left|\Delta\mathbf{w}_{k1}^H\mathbf{h}_i\right|^2 + \left|\Delta\mathbf{w}_{k2}^H\mathbf{h}_i\right|^2)$ and $\Delta t \sum_{l=1,l\neq k}^{G} (\left|\Delta\mathbf{w}_{l1}^H\mathbf{h}_i\right|^2 + \left|\Delta\mathbf{w}_{l2}^H\mathbf{h}_i\right|^2 - 2\mathrm{Re}\{\Delta\mathbf{w}_{l1}^H\mathbf{h}_i\mathbf{h}_i^H\mathbf{w}_{l1}^{(p)} + \Delta\mathbf{w}_{l2}^H\mathbf{h}_i\mathbf{h}_i^H\mathbf{w}_{l2}^{(p)}\})$ in the SINR constraint in (3.30), the problem in the $(p+1)$-th iteration can be approximated as the following convex problem

$$
\begin{aligned}
\max_{\substack{\Delta t, \Delta\mathbf{w}_{kj} \\ \forall j=1,2, \forall k\in\mathcal{K}}} \quad & \Delta t \\
\text{s.t.} \quad & -\left|\mathbf{w}_{k1}^{(p)H}\mathbf{h}_i\right|^2 - \left|\mathbf{w}_{k2}^{(p)H}\mathbf{h}_i\right|^2 + t^{(p)}\sigma_i^2 \\
& +\Delta t \sum_{l=1,l\neq k}^{G} (\left|\mathbf{w}_{l1}^{(p)H}\mathbf{h}_i\right|^2 + \left|\mathbf{w}_{l2}^{(p)H}\mathbf{h}_i\right|^2) + \Delta t\sigma_i^2 \\
& +t^{(p)} \sum_{l=1,l\neq k}^{G} (\left|(\mathbf{w}_{l1}^{(p)} + \Delta\mathbf{w}_{l1})^H\mathbf{h}_i\right|^2 + \left|(\mathbf{w}_{l2}^{(p)} + \Delta\mathbf{w}_{l2})^H\mathbf{h}_i\right|^2) \leq 0, \\
& \forall i \in g_k \; \forall k \in \mathcal{K} \\
& \sum_{k=1}^{G}(\left\|\mathbf{w}_{k1}^{(p)} + \Delta\mathbf{w}_{k2})\right\|^2 + \left\|\mathbf{w}_{k2}^{(p)} + \Delta\mathbf{w}_{k2})\right\|^2) \leq \mathrm{P}_{\max}.
\end{aligned}
\tag{3.31}
$$

Problem (3.31) can be classified as an inner convex approximation problem. Following from the inner approximation property, this iterative procedure results in a sequence of non-decreasing minimum SINR values. The proposed iterative approximation scheme is initialized with randomly generated weight vectors. With the increase of the iteration p, as soon as the increment of the obtained SINR between two consecutive iterations is below a certain threshold, i.e.,

$$
t^{(p+1)} - t^{(p)} < \epsilon \tag{3.32}
$$

we terminate the iteration. The complexity of the rank-two inner approximation procedure is $\mathcal{O}((M+1)^{1/2}(M+2GN+2)(2GN+1)^2)$, while in the rank-one case as shown in [52] the complexity is $\mathcal{O}((M+1)^{1/2}(M+GN+2)(GN+1)^2)$.

3.4 Simulation Results

We assume Rayleigh fading channels with i.i.d. channel coefficients of unit variance. The noise variance $\sigma_i^2 = 0$dB for all $i = 1, \ldots, M$. We consider the case that $N = 4$, $G = 2$ and $M = 30$ with 15 users in each group. All results are averaged over 300 Monte-Carlo runs.

In our simulation example, we compare the proposed rank-two beamforming approaches with the state-of-the-art approach proposed in [52]. In Fig. 3.2, the worst SINR among all users for different prescribed transmit powers is displayed. Five curves are depicted in Fig. 3.2, where the curve labeled 'SDR upper bound' stands for the upper bound on the SINR provided by the SDR solutions, 'Method of [52]' refers to the inner convex approximation approach for the rank-one beamforming problem with random initialization as proposed in [52], 'Method of [52] with SDR' stands for the rank-one beamforming approach in which SDR is employed in the initialization step and the inner approximation method in [52] is applied only if optimal rank-one solutions are not obtained in the initialization step, 'Proposed (SDR+Randomization)' refers to the proposed SDR based rank-two beamforming approach with 100 randomization instances in each run, and 'Proposed (Inner approx.)' stands for the proposed rank-two beamforming approach with iterative inner approximation. We set the threshold value for iteration termination to $\epsilon = 10^{-4}$. As shown in Fig. 3.2, 'Proposed (Inner approx.)' achieves slightly improved performance as compared to 'Proposed (SDR+Randomization)', and both curves are very close to 'SDR upper bound' and achieve better performance than all the rank-one approaches. This result can further be observed from Fig. 3.3 in which the histogram of the obtained rank of the solution $\{\mathbf{X}_k^\star\}_{k=1}^G$ of problem (3.25) is displayed versus the total transmit power. As shown in Fig. 3.3, rank-two solutions are obtained in most of the considered transmit power values. 'Proposed (SDR+Randomization)' can achieve optimal solutions for both rank-one and rank-two cases, and 'Proposed (Inner approx.)' performs well due to its rank-two approximation. Fig. 3.4 compares the convergence rates of 'Proposed (Inner approx.)' and 'Method of [52]' when $\mathrm{P}_{\max} = 10$dB. We observe that both rates are almost the same, but 'Proposed (Inner approx.)' achieves a better SINR value after the first iteration as compared to 'Method of [52]'.

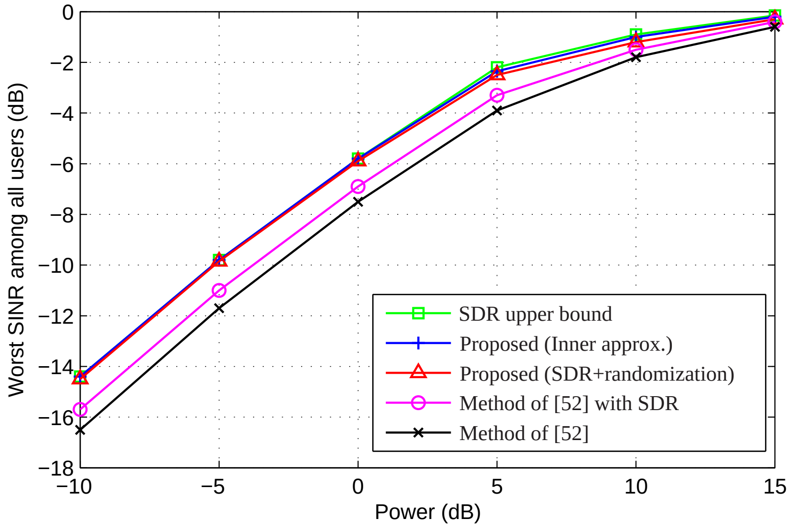

Figure 3.2: Worst SINR vs. total transmit power.

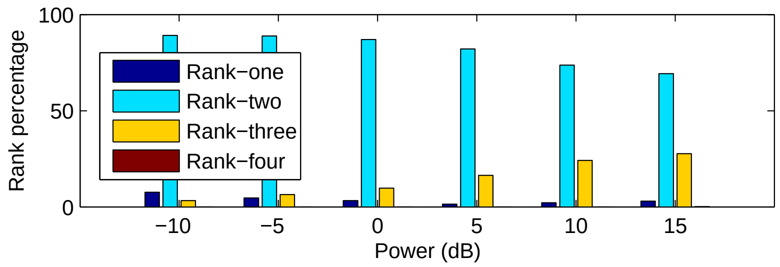

Figure 3.3: Rank percentage of $\{\mathbf{X}_k^\star\}_{k=1}^G$ vs. total transmit power.

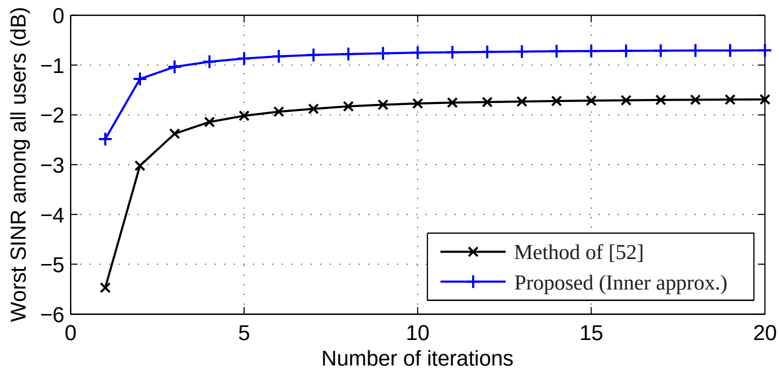

Figure 3.4: Convergence rates of iterative inner approximations.

3.5 Summary

In this chapter, rank-two beamforming approaches are proposed to solve the multi-group multicasting problem. In the SDR based rank-two approach, when the rank of all SDR solutions is smaller or equal to two, optimal solutions can be obtained. The computational complexity of the proposed SDR based approach is not increased as compared to the SDR based rank-one approaches, since SDR formulations are identical for both SDR based rank-one and rank-two approaches. When the largest rank of the SDR solution matrices is two, the randomization procedure is avoided in the rank-two approach resulting in a lower overall computational complexity than the SDR based rank-one approaches. Furthermore, an alternative rank-two iterative inner approximation technique is proposed. Although the computational complexity of the rank-two inner approximation technique is slightly higher than that of the rank-one inner approximation technique, it enjoys lower complexity than the SDR based rank-two technique. The downlink signaling overhead of the proposed rank-two approaches is slightly increased as compared to the rank-one approaches. Similar as single-group multicasting discussed in Chapter 2, the rank-two approaches proposed in this chapter can be extended to combine with high dimensional (>2) OSTBC but with a reduced transmission rate. Moreover, the use of high dimensional full-rate real-valued OSTBC and QOSTBC in multi-group multicasting is also impractical for which the reason will be explained in the following chapters.

Chapter 4

General Rank Downlink Beamforming With Shaping Constraints

4.1 Introduction

As a spectrally efficient multi-antenna technique, multiuser downlink beamforming has been extensively studied in the past few years [15, 29, 55–59]. With the aid of CSI at the transmitter, downlink beamforming can be performed at the base station of cellular networks or access point of WLAN networks to serve multiple co-channel users simultaneously by using spatially selective transmission. It can be considered as a special case for multi-group multicasting that is investigated in Chapter 3 when each group comprises a single user.

4.1.1 Related Work

As a pioneering work in downlink beamforming, the authors in [56] considered the problem of minimizing the total transmit power subject to QoS constraints in terms of the minimum SINR requirements at each user. A particular form of uplink-downlink duality theory was established in [56] and under this framework the downlink beamforming problem was solved using a computationally efficient power iteration algorithm. A similar approach that exploits uplink-downlink duality was proposed in [58], where the downlink beamforming problem of maximizing the minimum SINR among all users subject to a total power constraint was considered.

A different class of approaches was presented in [29, 55, 57] where the downlink beamforming problem was addressed using conic optimization. The authors in [29, 57] solved the beamforming problem by resorting to the concept of SDR and proved that from the rank-relaxed problem, a rank-one solution[1] can always be obtained if the problem is feasible. Moreover, the authors in [55] cast the problem into a computationally efficient standard SOCP for which the corresponding optimality conditions were derived.

All the multiuser downlink beamforming approaches referenced in the previous paragraphs optimize the beamforming weights considering the SINR requirements of the individual users served in the network. In addition to this, supplementary shaping constraints on the beamforming weight vectors can be embedded in the downlink beamforming problem to support a variety of requirements for different applications [60–73]. For example, in hierarchical cellular networks operating under the licensed shared access (LSA) paradigm [74], pico- and femtocell networks co-exist in the same frequency band with the surrounding macrocell [63–65]. Shaping constraints are used at the femtocell base stations to limit the power leakage to the macrocell users [60–62] and the power leakage to concurrent femtocell networks [63–65]. Similarly, in the newly emerging context of physical layer secrecy, shaping constraints are applied to guarantee that the SINRs at the eavesdroppers reside below a given detection threshold such that the confidential information can only be decoded at the intended receiver [66–68]. Recently downlink beamforming has been used for wireless charging in energy harvesting communication networks. In this context, shaping constraints are used to guarantee that the received power at the harvesting nodes is greater than a prescribed threshold to facilitate efficient wireless charging [69–71]. Furthermore, shaping constraints are employed in multiuser downlink networks to limit the interference power leakage to co-channel users, e.g., in neighboring cells [72, 73].

The above mentioned SDR approach lends itself for application in the multi-constraint downlink beamforming problems with a large number of additional shaping constraints [29, 57]. However, if the number of additional shaping constraints is large, the relaxation is

[1] By a rank-one solution we mean that the solution matrices of the SDR problem exhibit the rank-one property.

not guaranteed to be tight and a SDR solution with general rank may be obtained leading to suboptimal or even infeasible beamforming solutions. It was demonstrated in [26, 27] that when the number of additional shaping constraints is upper bounded by two, a rank-one solution can always be found by applying a special rank reduction algorithm [75,76], without losing the optimality of the solution. In other cases, if the solution after rank reduction still exhibits a rank greater than one, a suboptimal beamforming solution can be generated from the SDR solution by using, e.g., randomization techniques [33, 77].

In this chapter, we develop a novel approach to optimally solve the downlink beamforming problem in the case that the number of additional shaping constraints is no greater than 79. We exploit instantaneous CSI knowledge at the transmitter and combine downlink beamforming with full-rate high dimensional real-valued OSTBC to increase the degrees of freedom in the beamformer design. Several works have proposed the idea of combining beamforming with STC [41, 54, 78–87].

In [78], side information in the form of channel estimates was used to design linear beamformers for OSTBC precoded transmission based on a pairwise error probability (PEP) criterion. Two-directional Eigen-beamforming based on channel mean feedback was investigated in [79] using beamforming along with Alamouti coding [19], and the symbol error rate (SER) criterion was employed in the beamformer design. A similar idea was applied in [80] where based on the SER criterion an Eigen-beamformer was designed exploiting the knowledge of channel correlation available at the transmitter. The authors in [81] considered the same problem as in [78], however QOSTBC based beamforming was used instead of OSTBC.

All works of [78–81] considered the single-user MIMO scenario. In the multiuser MIMO scenario, rank-two beamforming approaches have been proposed in Chapter 2 to enhance conventional single-group multicast beamforming in which multiple users are served on the same frequency resource. A similar approach of the rank-two beamforming was developed for single-group multicasting using a relay network in [84, 87]. In Chapter 3, the concept of the rank-two beamforming is extended to solve the multi-group multicast beamforming problem.

By combining Alamouti coding with beamforming, rank-two beamforming approaches that are proposed in Chapter 2 and Chapter 3 outperform the conventional rank-one approaches. However, the drawback associated with these rank-two beamforming approaches is that an optimal solution can only be obtained if the SDR solution exhibits a rank less than or equal to two. Otherwise, an approximate solution is obtained in general. As discussed in Chapter 2, when the rank of the SDR solution is greater than two, high dimensional (>2) OSTBC can be applied to preserve the optimality of the beamforming solution instead of Alamouti coding. However, it is at the expense of a reduced transmission rate associated with these OSTBCs [21].

4.1.2 Contribution

The idea of combining beamforming with OSTBC in this chapter follows the ideas proposed in Chapter 2 and Chapter 3, in which rank-two beamformers are designed in combination with the application of Alamouti coding. In contrast to the rank-two beamformer designs in Chapter 2 and Chapter 3, we consider herein the downlink beamforming problem where each user is designed to be served by multiple beamformers combined with the use of full-rate high dimensional real-valued OSTBC. Real-valued OSTBC is employed in this chapter due to its full-rate property, thus the general rank approach proposed in this chapter achieves full-rate transmission as in the rank-one approaches of [26, 27] and rank-two approaches of Chapter 2 and Chapter 3.

In order to combine downlink beamforming with real-valued OSTBC, the effective channel vector of each user is adjusted to result in a real vector by applying a phase rotation procedure to which the optimal beamforming solution is proven to be invariant. Due to the orthogonality of the real-valued OSTBC, symbol-by-symbol detection can be performed at the receivers and the decoding complexity is not increased as compared to the conventional transmission that does not employ OSTBC. The use of OSTBC results in multiple beamformers at each user and therefore multiplies the degrees of freedom in the beamformer design offering improved beamforming performance. Interestingly, the proposed general rank beamformer design yields the same SDR formulation as in the conventional rank-one beamforming approaches of [26, 27] and the rank-two beamforming approaches of Chapter 2 and Chapter 3, i.e., the beamforming problems after rank relaxation become identical.

In the case that the rank of the SDR solution is greater than one in the conventional rank-one downlink beamforming problem, a rank reduction technique is applied to reduce the rank [26, 27, 88]. Similarly, in the Alamouti coding based beamforming approaches proposed in Chapter 2 and Chapter 3, rank reduction is applied if the SDR solution exhibits a rank greater than two. In our proposed real-valued OSTBC based beamforming approach, the SDR solution after the rank reduction procedure is proven to be optimal for the original problem if all ranks are no greater than eight. In the case that the SDR solution after rank reduction has a rank greater than eight, randomization techniques are applied to compute an approximate solution [33, 77]. Moreover, we analytically prove that in our approach an optimal solution is always attainable if the number of additional shaping constraints does not exceed 79, whereas in the conventional rank-one approach in [26,27] and rank-two approach in Chapter 2 and Chapter 3, the maximal numbers of the shaping constraints are restricted to two and seven, respectively. Simulation results demonstrate the advantage of the proposed general rank beamforming approach.

To sum up, the contributions in this chapter are as follows. We address the problem of optimal QoS based downlink beamforming in the presence of a massive number of arbitrary quadratic shaping constraints by combining linear downlink beamforming with high dimensional real-valued OSTBC exploiting CSI knowledge at the transmitter. The beamformer design in this chapter can be considered as a non-trivial full-rate extension of the Alamouti coding based rank-two beamforming framework of Chapter 2 and Chapter 3 to general rank beamforming supporting up to eight beamformers per user. We analytically prove that in our approach an optimal beamforming solution can always be obtained if the number of additional shaping constraints does not exceed 79.

This chapter is based on my original work that has been published in [89]. The remainder of this chapter is organized as follows. Section 4.2 introduces the signal model and revisits the conventional rank-one downlink beamforming problem. In Section 4.3, the system model corresponding to the real-valued OSTBC based general rank beamforming approach is developed. Section 4.4 formulates the optimal downlink beamforming problem involving real-valued OSTBC and provides the SDR solution. Section 4.5 addresses the problem of computing optimal beamforming vectors from the SDR solution and provides a theoretic analysis regarding the optimality of the proposed downlink beamforming design. Simulation results are carried out in Section 4.6. The summary is made in Section 4.7.

4.2 Conventional Rank-one Beamforming

We consider a wireless communication system where the serving base station or access point equipped with an array of N antennas transmits independent information to M single-antenna receivers. Let s_i denote the information symbol for the i-th receiver with zero mean and unit variance. In conventional (rank-one) beamforming approaches of [15, 26, 27, 29, 55–58], the transmitter sends a superposition of signals $\{s_i\}_{i=1}^{M}$ for the different receivers using the respective $N \times 1$ beamforming vectors $\{\mathbf{w}_i\}_{i=1}^{M}$. The received signal at the i-th single-antenna receiver is then given by [57]

$$y_i = \underbrace{s_i \mathbf{w}_i^H \mathbf{h}_i}_{\text{desired signal}} + \underbrace{\sum_{m=1, m\neq i}^{M} s_m \mathbf{w}_m^H \mathbf{h}_i + n_i}_{\text{interference plus noise}} \tag{4.1}$$

where $\mathbf{h}_i$ and n_i are the $N \times 1$ channel vector containing the flat fading channel conditions and the receiver noise of variance σ_i^2, respectively. The signal model in (4.1) is similar to the multi-group multicasting signal model in (3.1) in Chapter 3. When each group comprises a single user, (3.1) is identical to (4.1). The total transmit power at the base station or access point equals $\sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{w}_i$. By definition, the SINR can be computed as the expected signal power over the expected interference plus noise power. Therefore, based on (4.1), the SINR at the i-th receiver in the conventional rank-one beamforming approach is derived as

$$\text{SINR}_{\text{c},i} \triangleq \frac{|\mathbf{w}_i^H \mathbf{h}_i|^2}{\sum_{m=1, m\neq i}^{M} |\mathbf{w}_m^H \mathbf{h}_i|^2 + \sigma_i^2} \tag{4.2}$$

where 'c' refers to the conventional approach. Note that $\mathrm{SINR}_{\mathrm{c},i}$ in (4.2) is identical to the SINR expression in (3.2) in Chapter 3 if each group consists of a single user. Considering a QoS based beamforming design, we define γ_i as the minimum SINR requirement of the i-th user. Then the extended downlink beamforming problem of minimizing the total transmit power subject to minimum SINR constraints for each user and additional context specific shaping constraints can be formulated as [26, 27]

$$\min_{\{\mathbf{w}_i\}_{i=1}^M} \quad \sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{w}_i \tag{4.3a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{A}_{im} \mathbf{w}_m \trianglerighteq_i b_i, \quad \forall i = 1, \ldots, M \tag{4.3b}$$

$$\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{A}_{lm} \mathbf{w}_m \trianglerighteq_l b_l, \quad \forall l = M+1, \ldots, M+L \tag{4.3c}$$

where (4.3b) represents a well-known reformulation of the SINR constraints with

$$\mathbf{A}_{im} \triangleq \begin{cases} \mathbf{h}_i \mathbf{h}_i^H & m = i, \quad \forall i, m = 1, \ldots, M \\ -\gamma_i \mathbf{h}_i \mathbf{h}_i^H & m \neq i, \quad \forall i, m = 1, \ldots, M \end{cases} \tag{4.4}$$

and

$$b_i \triangleq \gamma_i \sigma_i^2, \ \trianglerighteq_i \triangleq \geq, \quad \forall i = 1, \ldots, M \tag{4.5}$$

and L additional quadratic shaping constraints are formulated in (4.3c) for appropriately chosen (as specified below) $N \times N$ Hermitian matrices $\mathbf{A}_{lm}, \forall l = M+1, \ldots, M+L; \forall m = 1, \ldots, M$, that are not necessarily positive definite, with corresponding thresholds $b_l, \forall l = M+1, ..., M+L$. Note that the shaping constraints in (4.3c) are not for information decoding purpose, while the first M constraints in (4.3b) are for information decoding purpose. Depending on the specific application under consideration, the additional shaping constraints in (4.3c) may take different forms (cf. [26, 27]). Popular example applications which can be formulated under the framework of problem (4.3) are described in Section 4.2.1 and Section 4.2.2.

4.2.1 Positive Semidefinite Shaping Constraints

In the context of cognitive radio networks, $\mathbf{A}_{lm} \triangleq \mathbf{h}_l\mathbf{h}_l^H$, where $\mathbf{h}_l$ denotes the channel vector between the base station and the l-th primary user. In this case, with b_l denoting the upper power threshold at the primary user and choosing $\trianglerighteq_l \triangleq \leq$, the l-th general shaping constraint (4.3c) takes the form

$$\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{A}_{lm} \mathbf{w}_m \leq b_l. \tag{4.6}$$

Therefore, the interference constraint (4.6) is used to guarantee that the power leakage to the primary users is below certain threshold [60–62]. In the context of femtocell networks, $\mathbf{h}_l$ denotes the channel vector between the base station and the l-th concurrent user, and the shaping constraint (4.6) is designed to ensure that the power leakage to concurrent users in coexisting hierarchical networks is below certain threshold [63–65].

In the context of physical layer secrecy networks, in contrast, $\mathbf{h}_l$ denotes the channel vector between the base station and the l-th eavesdropper, and the shaping constraint (4.6) is employed to enforce that the power leakage to eavesdroppers is below certain threshold [66–68].

Similarly, in the context of energy harvesting networks, $\mathbf{h}_l$ denotes the channel vector between the base station and the l-th charging terminal [69–71]. In this case b_l denotes the minimum power threshold to be guaranteed at the charging terminal and $\trianglerighteq_l \triangleq \geq$ is chosen. The shaping constraint (4.3c) can be rewritten as

$$\sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{A}_{lm} \mathbf{w}_m \geq b_l \tag{4.7}$$

with $\mathbf{A}_{lm} \triangleq \mathbf{h}_l\mathbf{h}_l^H$ for $m = 1, \ldots, M$ and $l = M+1, \ldots, M+L$ to ensure efficient wireless charging.

4.2.2 Indefinite Shaping Constraints

Indefinite shaping constraints can be used to perform relaxed nulling, as proposed in [90], to reduce intercell interference in multiuser downlink networks. Let $\mathbf{h}_l$ denote the channel vector from the base station of a given serving cell to a user of a different cell for which the interference shall be limited. Defining $\mathbf{A}_{lm} \triangleq \beta \mathbf{I}_N - \frac{\mathbf{h}_l \mathbf{h}_l^H}{\|\mathbf{h}_l\|^2}$, $b_l = 0$, and choosing β as an appropriate interference threshold parameter [72], the shaping constraint (4.3c) takes the form

$$\mathbf{w}_m^H \mathbf{A}_{lm} \mathbf{w}_m \geq b_l. \tag{4.8}$$

In this design the tolerable interference power induced by the l-th user to the m-th user depends on the spatial signature $\mathbf{h}_l$ of the co-channel user. Besides the applications mentioned above, indefinite shaping constraints can also be used to guarantee a minimum level of path diversity in CDMA systems [72, 73].

4.2.3 Semidefinite Relaxation

In this subsection we briefly revisit the SDR approach that is widely used to approximately solve the beamforming problem of form (4.3). The power minimization problem (4.3) is a quadratically constrained quadratic programming (QCQP) problem which is NP hard in general [33]. Denote $\mathbf{X}_i \triangleq \mathbf{w}_i \mathbf{w}_i^H$, problem (4.3) can be rewritten as

$$\begin{aligned}
\min_{\{\mathbf{X}_i\}_{i=1}^M} \quad & \sum_{i=1}^M \mathrm{Tr}(\mathbf{X}_i) \\
\text{s.t.} \quad & \sum_{m=1}^M \mathrm{Tr}(\mathbf{A}_{lm}\mathbf{X}_m) \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, M+L \\
& \mathbf{X}_i \succeq \mathbf{0}, \;\; \mathrm{rank}(\mathbf{X}_i) = 1, \quad \forall i = 1, \ldots, M.
\end{aligned} \tag{4.9}$$

The SDR technique can be employed to solve the convex relaxation of problem (4.9) by removing the rank constraints [29, 57]. Since the SDR solution is not of rank one in general, rank reduction techniques are applied to obtain a solution to problem (4.9) with a reduced rank [26, 27], see also Section 4.5.1. However, in the case that a rank-one solution does not exist, an approximate solution can be computed from the SDR solution using, e.g., the popular randomization procedures as used in [77] and [33].

4.3 General Rank Beamforming

The central idea of combining optimal downlink beamforming with the concept of real-valued OSTBC proposed in this work follows the general framework of Chapter 2 and Chapter 3 in which rank-two beamformers are designed by combining beamforming with Alamouti coding and making use of the CSI available at the transmitter. As compared to the rank-two approaches, we employ full-rate real-valued OSTBC to further increase the degrees of freedom in the beamformer design which grow linearly with the size of the code. Extending the rank-two beamforming approach to high dimensional (>2) OSTBC has previously been discussed in Chapter 2 and Chapter 3 as impractical due to the rate penalty associated with these codes. By applying real-valued OSTBC at the transmitter, multiple beamformers can be used to deliver the data stream to each user while maintaining the full-rate transmission property.

4.3.1 Full-rate Real-valued OSTBC

Let $\mathcal{X}(\mathbf{u})$ be a $K \times K$ real-valued OSTBC matrix given by [21]

$$\mathcal{X}(\mathbf{u}) = \sum_{k=1}^{K} u_k \mathbf{C}_k \tag{4.10}$$

where K is the number of symbols per block, $\mathbf{u} \triangleq [u_1, \ldots, u_K]^T$ is an arbitrary $K \times 1$ real vector and $\mathbf{C}_k$ is a $K \times K$ real code coefficient matrix. Per definition the OSTBC matrix $\mathcal{X}(\mathbf{u})$ satisfies the orthogonality property

$$\mathcal{X}^H(\mathbf{u})\mathcal{X}(\mathbf{u}) = \mathcal{X}(\mathbf{u})\mathcal{X}^H(\mathbf{u}) = \|\mathbf{u}\|_2^2 \mathbf{I}_K \tag{4.11}$$

which will be used in the following subsection. In this chapter, we only consider real-valued OSTBC matrices with $K = 1, 2, 4$ or 8 which are the only possible sizes to achieve full rate [21]. We note that, for a complex symbol vector $\mathbf{u}$, the orthogonality property in (4.11) can only be satisfied if $K \leq 2$ [16, 17]. Examples for real-valued OSTBC matrices are

$$\mathcal{X}([u_1, u_2]^T) \triangleq \begin{bmatrix} u_1 & u_2 \\ -u_2 & u_1 \end{bmatrix}, \tag{4.12}$$

$$\mathcal{X}([u_1, u_2, u_3, u_4]^T) \triangleq \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \\ -u_2 & u_1 & -u_4 & u_3 \\ -u_3 & u_4 & u_1 & -u_2 \\ -u_4 & -u_3 & u_2 & u_1 \end{bmatrix}, \tag{4.13}$$

and

$$\mathcal{X}([u_1, \dots, u_8]^T) \triangleq \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 \\ -u_2 & u_1 & u_4 & -u_3 & u_6 & -u_5 & -u_8 & u_7 \\ -u_3 & -u_4 & u_1 & u_2 & u_7 & u_8 & -u_5 & -u_6 \\ -u_4 & u_3 & -u_2 & u_1 & u_8 & -u_7 & u_6 & -u_5 \\ -u_5 & -u_6 & -u_7 & -u_8 & u_1 & u_2 & u_3 & u_4 \\ -u_6 & u_5 & -u_8 & u_7 & -u_2 & u_1 & -u_4 & u_3 \\ -u_7 & u_8 & u_5 & -u_6 & -u_3 & u_4 & u_1 & -u_2 \\ -u_8 & -u_7 & u_6 & u_5 & -u_4 & -u_3 & u_2 & u_1 \end{bmatrix}. \tag{4.14}$$

4.3.2 General Rank System Model

Denote $\mathbf{s}_i \triangleq [s_{i1}, \dots, s_{iK}]^T$ as the $K \times 1$ complex symbol vector for the i-th user with $K \leq N$ and $K \in \{1, 2, 4, 8\}$, i.e., in correspondence with the dimension of the real-valued OSTBC matrices in (4.12)-(4.14). In this work, we employ the real-valued OSTBC structure $\mathcal{X}(\cdot)$ given in (4.10) on the complex symbol vector $\mathbf{s}_i$. Instead of weighting each symbol by a beamforming vector as in (4.1), a code matrix $\mathcal{X}(\mathbf{s}_i)$ is transmitted for each user applying K beamformers of length N, denoted as $\mathbf{w}_{i1}, \dots, \mathbf{w}_{iK}$. In this case, taking a slightly different perspective, each of the K beams can be regarded as a virtual antenna from which the OSTBC is transmitted. In our scenario we consider a block fading channel model where the channels remain constant over K time slots. The received signal y_{ik} at the i-th user in the k-th time slot is given by

$$y_{ik} = \sum_{m=1}^{M}\sum_{k'=1}^{K}[\mathcal{X}(\mathbf{s}_m)]_{kk'}\mathbf{w}_{mk'}^H\mathbf{h}_i + n_{ik} \tag{4.15}$$

where n_{ik} is the noise of the i-th user in the k-th time slot. In a compact matrix notation, the received signal vector $\mathbf{y}_i \triangleq [y_{i1}, \ldots, y_{iK}]^T$ at the i-th user within the transmission period of K time slots is given by

$$\begin{aligned}\mathbf{y}_i &= \sum_{m=1}^{M}\mathcal{X}(\mathbf{s}_m)\mathbf{W}_m^H\mathbf{h}_i + \mathbf{n}_i \\ &= \underbrace{\mathcal{X}(\mathbf{s}_i)\mathbf{W}_i^H\mathbf{h}_i}_{\text{desired signal}} + \underbrace{\sum_{m=1, m\neq i}^{M}\mathcal{X}(\mathbf{s}_m)\mathbf{W}_m^H\mathbf{h}_i + \mathbf{n}_i}_{\text{interference plus noise}}\end{aligned} \tag{4.16}$$

where

$$\mathbf{W}_i \triangleq [\mathbf{w}_{i1}, \ldots, \mathbf{w}_{iK}], \quad K \in \{1, 2, 4, 8\} \tag{4.17}$$

is the beamforming matrix, and $\mathbf{n}_i \triangleq [n_{i1}, \ldots, n_{iK}]^T$. Note that the equivalent system model in (4.16) shares a similar form as (3.7) in Chapter 3, however, the coding matrix and the number of beamforming weight vectors for each user are different. We assume that the noise vector $\mathbf{n}_i$ at the i-th receiver is zero mean spatially and temporally white circular complex Gaussian with covariance matrix $\sigma_i^2\mathbf{I}_K$. The above system model can be reformulated in the following equivalent form [16]

$$\begin{aligned}\tilde{\mathbf{y}}_i &= \sum_{m=1}^{M} \mathcal{X}(\mathbf{W}_m^H \mathbf{h}_i)\mathbf{s}_i + \tilde{\mathbf{n}}_i \\ &= \mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i)\mathbf{s}_i + \underbrace{\sum_{m=1, m\neq i}^{M} \mathcal{X}(\mathbf{W}_m^H \mathbf{h}_i)\mathbf{s}_m}_{\tilde{\mathbf{i}}_i} + \tilde{\mathbf{n}}_i \qquad (4.18)\end{aligned}$$

where

$$\tilde{\mathbf{y}}_i \triangleq \Big[y_{i1}, -y_{i2}, \ldots, -y_{iK}\Big]^T, \qquad (4.19)$$

$$\tilde{\mathbf{i}}_i \triangleq \sum_{m=1, m\neq i}^{M} \mathcal{X}(\mathbf{W}_m^H \mathbf{h}_i)\, \mathbf{s}_m, \qquad (4.20)$$

$$\tilde{\mathbf{n}}_i \triangleq \Big[n_{i1}, -n_{i2}, \ldots, -n_{iK}\Big]^T. \qquad (4.21)$$

Note that (4.18) has a similar form as (3.9) in Chapter 3. In order to implement full-rate transmission and symbol-wise decoding for each user, the code matrix $\mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i)$ has to exhibit the orthogonality property (4.11). This however requires that the virtual channel vectors $\{\mathbf{W}_i^H \mathbf{h}_i\}_{i=1}^{M}$ become real-valued, i.e., the condition

$$\mathbf{W}_i^H \mathbf{h}_i \in \mathbb{R}^K, \quad \forall i = 1, \ldots, M \qquad (4.22)$$

holds. We remark that in general $\mathbf{W}_m^H \mathbf{h}_i$ is not real-valued for $m \neq i$, and thus $\mathcal{X}(\mathbf{W}_m^H \mathbf{h}_i)$ does not necessarily satisfy the orthogonal property in (4.11). In the following, we derive explicit expressions for the SINR of the symbols received at the destinations under the assumption that condition (4.22) is satisfied and that signal user detection is applied at the receivers.

For an orthogonal matrix $\mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i)$, i.e., with $\mathbf{W}_i^H \mathbf{h}_i$ satisfying (4.22), the transmitted symbol vector can be equalized as

$$\begin{aligned}\hat{\mathbf{s}}_i &= \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^2}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i)\tilde{\mathbf{y}}_i \\ &= \mathbf{s}_i + \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^2}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i)(\tilde{\mathbf{i}}_i + \tilde{\mathbf{n}}_i). \end{aligned} \tag{4.23}$$

Based on (4.23), the covariance matrix of the received interference contained in $\hat{\mathbf{s}}_i$ is given by

$$\begin{aligned}\mathbf{R}_i^{(\mathrm{I})} &= \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^4}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i)\mathrm{E}\{\tilde{\mathbf{i}}_i\tilde{\mathbf{i}}_i^H\}\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i) \\ &= \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^4}[\sum_{m=1,m\neq i}^{M}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i)\mathcal{X}(\mathbf{W}_m^H\mathbf{h}_i)\times \\ &\quad \mathcal{X}^H(\mathbf{W}_m^H\mathbf{h}_i)\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i)] \end{aligned} \tag{4.24}$$

and the covariance matrix of the noise in $\hat{\mathbf{s}}_i$ is given by

$$\begin{aligned}\mathbf{R}_i^{(\mathrm{N})} &= \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^4}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i)\mathrm{E}\{\tilde{\mathbf{n}}_i\tilde{\mathbf{n}}_i^H\}\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i) \\ &= \frac{\sigma_i^2}{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^2}\mathbf{I}_K. \end{aligned} \tag{4.25}$$

In the rank-two beamforming approach in multi-group multicasting in Chapter 3, the orthogonality property of the Alamouti coding matrix can be straightforwardly applied to facilitate the calculation of the interference power. However, this is not the case for the general rank beamforming approach, because $\mathcal{X}(\mathbf{W}_m^H\mathbf{h}_i)$ does not necessarily satisfy the orthogonal property for $m \neq i$ which results in the complicated structure of $\mathbf{R}_i^{(\mathrm{I})}$ in (4.24). In order to compute the interference power based on (4.24), we introduce the following lemma.

Lemma 4.1. *Assume that $\boldsymbol{\psi}$ and $\boldsymbol{\omega}$ are a real and a complex vector both with the dimension $K\times 1$, respectively. Let $\boldsymbol{\Phi} \triangleq \mathcal{X}^H(\boldsymbol{\psi})\mathcal{X}(\boldsymbol{\omega})\mathcal{X}^H(\boldsymbol{\omega})\mathcal{X}(\boldsymbol{\psi})$ where $\mathcal{X}(\cdot)$ is a $K\times K$ real-valued OSTBC structure that fulfils (4.11). Then*

$$[\boldsymbol{\Phi}]_{kk} = \|\boldsymbol{\psi}\|_2^2\|\boldsymbol{\omega}\|_2^2 \quad \forall k = 1,\dots,K \tag{4.26}$$

where $[\mathbf{\Phi}]_{kk}$ is the k-th diagonal element of the matrix $\mathbf{\Phi}$.

Proof 4.1. Let $\mathcal{X}(\boldsymbol{\omega}) \triangleq \mathbf{\Omega}_1 + j\mathbf{\Omega}_2$ where $\mathbf{\Omega}_1$ and $\mathbf{\Omega}_2$ are real orthogonal matrices from the definition of $\mathcal{X}(\boldsymbol{\omega})$. Then

$$\begin{aligned} \mathcal{X}(\boldsymbol{\omega})\mathcal{X}^H(\boldsymbol{\omega}) &= (\mathbf{\Omega}_1\mathbf{\Omega}_1^T + \mathbf{\Omega}_2\mathbf{\Omega}_2^T) + j(\mathbf{\Omega}_2\mathbf{\Omega}_1^T - \mathbf{\Omega}_1\mathbf{\Omega}_2^T) \\ &= \|\boldsymbol{\omega}\|_2^2\mathbf{I}_R + j(\mathbf{\Omega}_2\mathbf{\Omega}_1^T - \mathbf{\Omega}_1\mathbf{\Omega}_2^T). \end{aligned} \tag{4.27}$$

Hence

$$\begin{aligned} \mathbf{\Phi} &= \|\boldsymbol{\omega}\|_2^2\mathcal{X}^H(\boldsymbol{\psi})\mathcal{X}(\boldsymbol{\psi}) + j\mathcal{X}^H(\boldsymbol{\psi})(\mathbf{\Omega}_2\mathbf{\Omega}_1^T - \mathbf{\Omega}_1\mathbf{\Omega}_2^T)\mathcal{X}(\boldsymbol{\psi}) \\ &= \|\boldsymbol{\psi}\|_2^2\|\boldsymbol{\omega}\|_2^2\mathbf{I}_K + j\mathcal{X}^H(\boldsymbol{\psi})(\mathbf{\Omega}_2\mathbf{\Omega}_1^T - \mathbf{\Omega}_1\mathbf{\Omega}_2^T)\mathcal{X}(\boldsymbol{\psi}). \end{aligned} \tag{4.28}$$

Since $\mathbf{\Phi}$ is a Hermitian matrix and $\mathcal{X}(\boldsymbol{\psi})$ is a real matrix, $\mathcal{X}^H(\boldsymbol{\psi})(\mathbf{\Omega}_2\mathbf{\Omega}_1^T - \mathbf{\Omega}_1\mathbf{\Omega}_2^T)\mathcal{X}(\boldsymbol{\psi})$ is a skew symmetric matrix, i.e., its elements on the main diagonal are zero. Then the equation (4.26) holds. □

Substituting the real-valued vector $\boldsymbol{\psi} = \mathbf{W}_i^H\mathbf{h}_i$ and complex vector $\boldsymbol{\omega} = \mathbf{W}_m^H\mathbf{h}_i$ in (4.24) according to (4.22), and applying Lemma 4.1, the interference power of the i-th user in the k-th time slot can be expressed as

$$[\mathbf{R}_i^{(\mathrm{I})}]_{kk} = \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^2} \sum_{m=1,m\neq i}^{M} \|\mathbf{W}_m^H\mathbf{h}_i\|_2^2. \tag{4.29}$$

With (4.25) and (4.29), the SINR corresponding to symbol s_{ik} is given by

$$\begin{aligned} \mathrm{SINR}(s_{ik}) &\triangleq \frac{\mathrm{E}\{s_{ik}s_{ik}^*\}}{[\mathbf{R}_i^{(\mathrm{I})}]_{kk} + [\mathbf{R}_i^{(\mathrm{N})}]_{kk}} \\ &= \frac{\|\mathbf{W}_i^H\mathbf{h}_i\|_2^2}{\sum\limits_{m=1,m\neq i}^{M} \|\mathbf{W}_m^H\mathbf{h}_i\|_2^2 + \sigma_i^2}. \end{aligned} \tag{4.30}$$

Note that the SINR expression in (4.30) is independent of the time index k. Therefore, the

SINR for the i-th user is identical for all symbols in the OSTBC block and it is given by

$$\mathrm{SINR}_i \triangleq \frac{\|\mathbf{W}_i^H \mathbf{h}_i\|_2^2}{\sum\limits_{m=1, m\neq i}^{M} \|\mathbf{W}_m^H \mathbf{h}_i\|_2^2 + \sigma_i^2} \tag{4.31}$$

which exhibits a similar structure as the SINR expression in (3.17) in Chapter 3. For simplicity of presentation, the SINR constraints in the general rank approach can be written in a similar form as in the rank-one beamforming approach of (4.3b), i.e.,

$$\sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{im} \mathbf{W}_m \mathbf{W}_m^H) \trianglerighteq_i b_i \quad \forall i = 1, \ldots, M \tag{4.32}$$

where $\mathbf{A}_{im}$ is defined in (4.4).

Since each symbol appears only once in each row of the code matrix $\mathcal{X}(\mathbf{s}_i)$, cf. (4.12)-(4.14), the transmit power towards the i-th user in the k-th time slot can be computed as

$$\begin{aligned} P_{ik} &= \mathrm{E}\{\mathbf{e}_k^H \mathcal{X}(\mathbf{s}_i) \mathbf{W}_i^H \mathbf{W}_i \mathcal{X}^H(\mathbf{s}_i) \mathbf{e}_k\} \\ &= \mathrm{Tr}(\mathbf{W}_i \mathrm{E}\{\mathcal{X}^H(\mathbf{s}_i) \mathbf{e}_k \mathbf{e}_k^H \mathcal{X}(\mathbf{s}_i)\} \mathbf{W}_i^H) \\ &= \mathrm{Tr}(\mathbf{W}_i \mathbf{W}_i^H) \end{aligned} \tag{4.33}$$

where $\mathbf{e}_k$ is the k-th column of the $N \times N$ identity matrix. Similarly we observe that the transmit power P_{ik} is identical in all K time slots. Let $P_i = P_{ik}$ represent the transmit power towards the i-th user in each time slot. Then the total transmit power in each time slot amounts to

$$\sum_{i=1}^{M} P_i = \sum_{i=1}^{M} \mathrm{Tr}(\mathbf{W}_i \mathbf{W}_i^H). \tag{4.34}$$

With multiple beamformers designed for each user instead of a single one, the additional shaping constraints in (4.3c) can be expressed as

$$\begin{aligned}
& \sum_{m=1}^{M}\sum_{k=1}^{K} \mathbf{w}_{mk}^H \mathbf{A}_{lm} \mathbf{w}_{mk} \\
= \quad & \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{W}_m^H \mathbf{A}_{lm} \mathbf{W}_m) \\
= \quad & \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{lm} \mathbf{W}_m \mathbf{W}_m^H) \trianglerighteq_l b_l \quad \forall l = M+1, \ldots, M+L.
\end{aligned} \tag{4.35}$$

4.4 The Power Minimization Problem

In this section, we consider the problem of minimizing the total transmit power per time slot subject to SINR constraints at each user and additional shaping constraints on the beamformers. Taking into account that according to (4.22) the virtual channel vectors $\{\mathbf{W}_i^H \mathbf{h}_i\}_{i=1}^M$ must be real-valued in order to satisfy the orthogonality property for simple decoding, the optimization problem is formulated in the following form

$$\min_{\{\mathbf{W}_i\}_{i=1}^M} \quad \sum_{i=1}^{M} \mathrm{Tr}(\mathbf{W}_i \mathbf{W}_i^H) \tag{4.36a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{lm} \mathbf{W}_m \mathbf{W}_m^H) \trianglerighteq_l b_l \quad \forall l = 1, \ldots, M+L \tag{4.36b}$$

$$\mathbf{W}_i^H \mathbf{h}_i \in \mathbb{R}^K, \quad \forall i = 1, \ldots, M. \tag{4.36c}$$

We remark that as a special case, the Alamouti code can be employed in our proposed scheme without the need of imposing the constraint (4.36c), since the Alamouti code satisfies the orthogonality property of (4.11) for an arbitrary complex vector $\mathbf{u}$ while achieving full rate. In this case the proposed scheme becomes similar to the rank-two schemes proposed in Chapter 2 and Chapter 3.

4.4.1 Phase Rotation Invariance Property

To solve the problem (4.36), we first consider a relaxed problem of (4.36) by removing the constraints (4.36c)

$$\min_{\{\mathbf{W}_i\}_{i=1}^M} \quad \sum_{i=1}^{M} \mathrm{Tr}(\mathbf{W}_i \mathbf{W}_i^H) \tag{4.37a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{lm} \mathbf{W}_m \mathbf{W}_m^H) \trianglerighteq_l b_l \quad \forall l = 1, \ldots, M+L. \tag{4.37b}$$

Let $\mathbf{W}_i^\star \triangleq \left[\mathbf{w}_{i1}^\star, \ldots, \mathbf{w}_{iK}^\star\right]$ for all $i = 1, \ldots, M$ denote an optimal solution of (4.37). Then we can perform the phase rotation on $\{\mathbf{W}_i^\star\}_{i=1}^M$ according to

$$\mathbf{W}_i'^\star \triangleq \mathbf{W}_i^\star \boldsymbol{\Theta}_i \quad \forall i = 1, \ldots, M \tag{4.38}$$

where the diagonal matrix $\boldsymbol{\Theta}_i$ is given by

$$\boldsymbol{\Theta}_i \triangleq \mathrm{diag}\{\exp(j\angle(\mathbf{w}_{i1}^{\star H}\mathbf{h}_i)), \ldots, \exp(j\angle(\mathbf{w}_{iK}^{\star H}\mathbf{h}_i))\}. \tag{4.39}$$

Since $\{\mathbf{W}_i'^\star\}_{i=1}^M$ satisfies all the constraints in (4.36), including constraint (4.36c), it is a feasible solution to the unrelaxed problem (4.36). As the total transmit power associated with $\{\mathbf{W}_i'^\star\}_{i=1}^M$ is the same as that associated with the optimal solution $\{\mathbf{W}_i^\star\}_{i=1}^M$, we conclude that $\{\mathbf{W}_i'^\star\}_{i=1}^M$ is an optimal solution to the original problem (4.36). In other words, relaxing the real-valued requirements expressed in constraints (4.36c) in the beamforming problem (4.36) results in an equivalent problem. An optimal solution for the original problem (4.36) can always be computed from the solution of the relaxed problem (4.37) by applying the phase rotation proposed in (4.38). Therefore, without loss of generality, we can solve (4.37) for solving (4.36). We remark that real-valued OSTBC can be applied in downlink beamforming since the virtual channel vectors for all the users can be adjusted to be real vectors by performing phase rotation. However, real-valued OSTBC cannot be applied in single-group multicasting in Chapter 2 and multi-group multicasting in Chapter 3 following the same way, because in both applications multiple users are served by a common beamforming matrix on which the phase rotation can only adjust one virtual channel vector to be a real vector.

4.4.2 SDR Approach

Let us define the variable transformation

$$\mathbf{X}_i \triangleq \mathbf{W}_i \mathbf{W}_i^H, \quad \forall i = 1, \ldots, M. \tag{4.40}$$

The transformation in (4.40) is similar to (3.22) in Chapter 3, the difference lies in the dimension of the beamforming matrices. Substituting $\mathbf{X}_i$ in (4.36) and adding the constraints

$$\begin{cases} \mathbf{X}_i \succeq \mathbf{0} \\ \text{rank}(\mathbf{X}_i) \leq K, \quad \forall i = 1, \ldots, M \end{cases} \tag{4.41}$$

to ensure that the transformation (4.40) exists, problem (4.37) converts to a rank constrained SDP problem

$$\min_{\{\mathbf{X}_i\}_{i=1}^M} \quad \sum_{i=1}^{M} \text{Tr}(\mathbf{X}_i) \tag{4.42a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \text{Tr}(\mathbf{A}_{lm}\mathbf{X}_m) \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, M+L \tag{4.42b}$$

$$\mathbf{X}_i \succeq \mathbf{0}, \tag{4.42c}$$

$$\text{rank}(\mathbf{X}_i) \leq K, \quad \forall i = 1, \ldots, M. \tag{4.42d}$$

We remark that problem (4.42) is identical to problem (4.9) except for the rank constraint. While in the latter problem the optimization variable $\mathbf{X}_i$ is restricted to the set of rank-one matrices, in our proposed formulation (4.42) the rank of the matrix must not exceed K. This shows that the feasible set of our proposed beamforming approach is greater than that of the conventional one.

Since the rank constraints in (4.42d) are non-convex, we employ the SDR approach [57] to obtain a relaxed convex optimization problem in which the rank constraints in (4.42d) are omitted,

$$\min_{\{\mathbf{X}_i\}_{i=1}^M} \quad \sum_{i=1}^{M} \mathrm{Tr}(\mathbf{X}_i) \tag{4.43a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{lm}\mathbf{X}_m) \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, M+L \tag{4.43b}$$

$$\mathbf{X}_i \succeq \mathbf{0}, \quad \forall i = 1, \ldots, M. \tag{4.43c}$$

For later reference, we also provide the Lagrange dual problem of (4.43) which has the following form [26]

$$\begin{aligned} \max_{\{\eta_l\}_{l=1}^{M+L}} \quad & \sum_{l=1}^{M+L} \eta_l b_l \\ \text{s.t.} \quad & \mathbf{Z}_i = \mathbf{I} - \sum_{l=1}^{M+L} \eta_l \mathbf{A}_{li} \succeq \mathbf{0} \quad \forall i = 1, \ldots, M \\ & \eta_l \trianglerighteq_l^* 0 \quad \forall l = 1, \ldots, M+L \end{aligned} \tag{4.44}$$

where

$$\trianglerighteq_l^* \triangleq \begin{cases} \geq & \text{if } \trianglerighteq_l \text{ is } \geq \\ \text{unrestricted}^2 & \text{if } \trianglerighteq_l \text{ is } = \\ \leq & \text{if } \trianglerighteq_l \text{ is } \leq \end{cases} . \tag{4.45}$$

Note that, according to our previous observation, problem (4.43) is identical to the SDR of the rank-one beamforming problem (4.9). Therefore, the computational complexity of solving the general rank beamforming problem does not differ from that of the rank-one and rank-two schemes discussed in Chapter 2 and Chapter 3. This is due to the observation that the computational complexity of the proposed general rank approach mainly consists in solving (4.43), which is the same as in the rank-one and rank-two approaches. Problem (4.43) belongs to the class of separable SDP problems [26, 27] and can be solved efficiently using solvers such as CVX [43, 91]. Denote $\{\mathbf{X}_i^\star\}_{i=1}^M$ as an optimal solution to the problem (4.43). Then we can apply the rank reduction algorithm proposed in [26] and [27] with the

[2] i.e., the constraint is omitted.

input $\{\mathbf{X}_i^\star\}_{i=1}^M$ to reduce the rank of the optimal solution. A detailed description of the rank reduction procedure for general rank matrices is provided in Section 4.5.1.

4.5 Beamforming Matrices Generation

In this section, we derive a sufficient condition on the maximum number of shaping constraints under which a solution to (4.36) can always be obtained from the SDR solution. In this context, we adapt the rank reduction algorithm of [26], [27] with modified stopping criteria for its application in general rank beamforming. Then we address the issue of determining the smallest code dimension K for all downlink users based on the output of the rank reduction procedure. In the case that a SDR solution after the rank reduction procedure has a rank greater than eight, a randomization procedure is proposed to obtain a suboptimal solution to the problem (4.36).

4.5.1 Rank Reduction Procedure

The rank reduction procedure for general separable SDP of form (4.9) has been proposed in [26, 27] for the rank-one beamforming problem. It is effective to reduce the rank of the output solution to one in solving the optimal beamforming problem when its SDP relaxation always exists a rank-one solution. However, when the rank reduction procedure in [24] is applied in the general rank beamforming problem, it may stop and return a higher-rank solution even though the rank can be further reduced. By employing a modified stopping criteria, the rank reduction procedure is applied in our approach to compute a solution whose rank cannot be further reduced from any optimal solution of (4.43).

Let $\{\mathbf{X}_i^\star\}_{i=1}^M$ denote a solution of the SDR problem (4.43) and $(\{\eta_l^\star\}_{l=1}^{M+L}, \{\mathbf{Z}_i^\star\}_{i=1}^M)$ the corresponding solution of its dual problem (4.44). The rank reduction algorithm successively reduces the rank of the solution $\{\mathbf{X}_i^\star\}_{i=1}^M$ as follows. Introducing the factorization $\mathbf{X}_i^\star \triangleq \mathbf{Q}_i\mathbf{Q}_i^H$ where $\text{rank}(\mathbf{X}_i^\star) = \text{rank}(\mathbf{Q}_i) = K_i$. Starting from the given solution, the algorithm solves the following homogeneous system of equations corresponding to (4.43b)

$$\sum_{m=1}^{M} \mathrm{Tr}(\mathbf{Q}_m^H \mathbf{A}_{lm} \mathbf{Q}_m \mathbf{\Delta}_m) = 0 \quad \forall l = 1, \ldots, M+L \tag{4.46}$$

where $\mathbf{\Delta}_m \in \mathbb{C}^{K_m \times K_m}$ represents an unknown arbitrary Hermitian matrix. The number of real unknowns in (4.46) equals $\sum_{i=1}^{M} \mathrm{rank}^2(\mathbf{X}_i^\star)$, whereas the number of equations in (4.46) is $M+L$. Hence (4.46) must admit a nontrivial solution when the following inequality [26, 27]

$$\sum_{i=1}^{M} \mathrm{rank}^2(\mathbf{X}_i^\star) = \sum_{i=i}^{M} K_i^2 \leq M+L \tag{4.47}$$

is violated. A solution $\{\tilde{\mathbf{X}}_i^\star\}_{i=1}^M$ that exhibits a reduced rank can then be computed as

$$\tilde{\mathbf{X}}_i^\star = \mathbf{Q}_i(\mathbf{I} - \frac{1}{\delta_{\max}} \mathbf{\Delta}_i)\mathbf{Q}_i^H \quad \forall i = 1, \ldots, M \tag{4.48}$$

where $\delta_{\max}$ is the largest eigenvalue of all the matrices of $\{\mathbf{\Delta}_i\}_{i=1}^M$ that satisfy (4.46). It is simple to show that $\{\tilde{\mathbf{X}}_i^\star\}_{i=1}^M$ computed in (4.48) also satisfies the Karush-Kuhn-Tucker (KKT) conditions [26]

i) *Primary feasibility*: $\sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{lm}\tilde{\mathbf{X}}_m^\star) \trianglerighteq_l b_l$, $\tilde{\mathbf{X}}_m^\star \succeq \mathbf{0}$

ii) *Complementary slackness* $\mathrm{Tr}(\tilde{\mathbf{X}}_i^\star \mathbf{Z}_i^\star) = 0$.

Note that the Lagrangian multipliers $\{\eta_l^\star\}_{l=1}^{M+L}$, $\{\mathbf{Z}_i^\star\}_{i=1}^M$ and the optimal value of the dual problem are not changed with the rank reduction procedure. Thus the zero gradient condition and the dual feasibility condition are always satisfied. Furthermore we observe that from the complementary slackness conditions, the Lagrangian multipliers for $\{\tilde{\mathbf{X}}_i^\star\}_{i=1}^M$ are the same as the multipliers for $\{\mathbf{X}_i^\star\}_{i=1}^M$. Hence $\{\tilde{\mathbf{X}}_i^\star\}_{i=1}^M$ is optimal for the relaxed problem (4.43). Note that due to the rank reduction step in (4.48), at least one of the solution matrices (i.e., the solution $\tilde{\mathbf{X}}_i^\star$ corresponding to the homogeneous solution $\mathbf{\Delta}_i$ that exhibits the largest eigenvalue) has its rank reduced, while the ranks of the other solution matrices do not increase. This implies that the left hand side of (4.47) reduces. The above steps are repeated by assigning $\{\tilde{\mathbf{X}}_i^\star\}_{i=1}^M$ to be a new input of the algorithm until the inequality (4.47) is fulfilled. However, in our approach the rank reduction procedure is stopped after the maximum number of iterations *max_iter* is reached. The modified stopping criteria ensures that the ranks of $\{\tilde{\mathbf{X}}_i^\star\}_{i=1}^M$ cannot be further reduced when *max_iter* is set as

$$\textit{max\_iter} = \sum_{i=1}^{M} \text{rank}(\mathbf{X}_i^\star) - M \tag{4.49}$$

which always ensures that in the ultimate case a rank-one solution can be obtained. It represents the maximum number of iterations that can be carried out before the internal exit criteria of the algorithm must apply. Note that at each iteration a homogeneous system of linear equations is solved, and one matrix decomposition and one singular value decomposition need to be carried out. However, compared with the optimization problem (4.43), the operation cost of the iterations is negligible. The rank reduction procedure is summarized in Algorithm 4.1.

Input $\{\mathbf{X}_i^\star\}_{i=1}^M$ an optimal solution to the problem (4.43),
  $\{\mathbf{A}_{lm}\}_{m=1,\ldots,M;l=1,\ldots,M+L}$,
  max_iter maximum number of iterations;
Output $\{\mathbf{X}_i^\star\}_{i=1}^M$ such that the rank of any of the matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$
  cannot be further reduced;
while Number of iterations $\leq$ *max_iter* **do**
  Decompose $\mathbf{X}_i^\star = \mathbf{Q}_i \mathbf{Q}_i^H \quad \forall i = 1, \ldots, M$;
  Find a non-zero solution of the equation (4.46);
  if (4.46) does not admit a nontrivial solution **then**
    break
  else
    Let $\delta_{\max} \triangleq \max_{\substack{1 \leq l \leq K_m \\ 1 \leq i \leq M}} \{|\delta_{li}|\}$ where δ_{li} is the l-th eigenvalue of $\mathbf{\Delta}_i$;
    Set $\mathbf{X}_i^\star = \mathbf{Q}_i(\mathbf{I} - \frac{1}{\delta_{\max}}\mathbf{\Delta}_i)\mathbf{Q}_i^H \quad \forall i = 1, \ldots, M$;
  end if
end while

Algorithm 4.1: Rank reduction procedure

4.5.2 Number of Additional Shaping Constraints

Next, we derive conditions on the number of additional shaping constraints and the code dimension K of the real-valued OSTBC for which optimal beamforming solution can always be obtained. These conditions are stated by the following lemma.

Lemma 4.2. *Assume that the relaxed problem* (4.43) *and its dual* (4.44) *are solvable*[3] *and that the condition*

$$L \leq (K+1)^2 - 2 \tag{4.50}$$

is satisfied, then there always exists an optimal solution $\mathbf{X}_i^\star$ *for problem* (4.43) *with* $\mathrm{rank}(\mathbf{X}_i^\star)$ $\leq K$ *for all* $i = 1, \ldots, M$.

Proof 4.2. We follow a similar line of argument as in [26] and prove Lemma 4.2 by contradiction. Assume that (4.50) is satisfied and there exists a matrix $\mathbf{X}_j^\star$ with $\mathrm{rank}(\mathbf{X}_j^\star) > K$ for some j such that the matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$ satisfy (4.47). We observe that none of the matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$ are zero matrices, as otherwise at least one of the SINR constraints in (4.32) would be violated due to the positive semidefiniteness of $\{\mathbf{X}_i^\star\}_{i=1}^M$ and the definition of $\{\mathbf{A}_{im}\}_{i,m=1}^M$ in (4.4). Hence all the matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$ must have a rank greater than or equal to one. Then

$$\sum_{i=1}^{M} \text{rank}^2(\mathbf{X}_i^\star) \overset{(a)}{\geq} M - 1 + (K+1)^2 \overset{(b)}{>} M + L \tag{4.51}$$

where strict equality holds in "(a)" if and only if there are $M-1$ rank-one matrices in $\{\mathbf{X}_i^\star\}_{i=1}^M$ and the last matrix has rank $K+1$, and the strict inequality in "(b)" follows from (4.50). The inequality (4.51) however contradicts our assumption that (4.47) is fulfilled. Hence all the matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$ must have ranks less than or equal to K. We conclude that the maximum number of additional shaping constraints L for which a rank less than or equal to K can be obtained is given by

$$L = (K+1)^2 - 2. \tag{4.52}$$

□

[3]"solvable" means that a bounded optimal value of the optimization problem can be obtained [26].

Lemma 4.2 indicates that we can always find an optimal solution to problem (4.36) by using the SDR approach and the rank reduction procedure described in Algorithm 4.1 if condition (4.52) is satisfied. From (4.52), we can calculate the maximum numbers of additional shaping constraints for different choices of $K \in \{1, 2, 4, 8\}$ as listed in Table 4.1 such that an optimal solution to problem (4.36) can always be obtained. We observe from Table 4.1 that our proposed scheme can accommodate a maximum number of 79 additional shaping constraints which corresponds to the choice of the code dimension $K = 8$.

Number of beamformers per user K	Number of additional shaping constraints
1	2
2	7
4	23
8	79

Table 4.1: Number of additional shaping constraints

Since a smaller code size of the real-valued OSTBC matrix results in a shorter decoding latency at the receiver side, we seek to obtain the smallest value of K for all downlink users based on the output of the rank reduction procedure in Algorithm 4.1. If the updated $\{\mathbf{X}_i^\star\}_{i=1}^M$ after the rank reduction procedure satisfies $\mathrm{rank}(\mathbf{X}_i^\star) \leq 8$ for all $i = 1, \ldots, M$, then the smallest number K is chosen from $K \in \{1, 2, 4, 8\}$ such that

$$K \geq \mathrm{rank}(\mathbf{X}_i^\star) \quad \forall i = 1, \ldots, M. \tag{4.53}$$

Note that when $\mathrm{rank}(\mathbf{X}_i^\star) < K$, e.g., $\mathrm{rank}(\mathbf{X}_i^\star) = 3$ and $K = 4$, $\mathrm{rank}(\mathbf{X}_i^\star)$ beamformers are used to transmit K symbols in K time slots using the $K \times K$ real-valued OSTBC. The corresponding beamforming matrices are then obtained as

$$\mathbf{W}_i^\star = [\mathbf{Q}_i,\, \mathbf{0}_{N\times(K-\mathrm{rank}(\mathbf{X}_i^\star))}] \quad \forall i = 1, \ldots, M \tag{4.54}$$

where $\mathbf{X}_i^\star = \mathbf{Q}_i\mathbf{Q}_i^H$ with $\mathbf{Q}_i \in \mathbb{C}^{N\times\mathrm{rank}(\mathbf{X}_i^\star)}$.

4.5.3 General Rank Randomization Procedure

In the case that (4.50) is violated and if at least one of the matrices in $\{\mathbf{X}_i^\star\}_{i=1}^M$, after rank reduction, exhibits a rank greater than eight, then the following randomization technique which involves a linear power control problem can be applied to generate a feasible but generally suboptimal beamforming solution for problem (4.36). Note that in practice this randomization procedure may not be relevant as the number of constraints is already very large for which optimal rank-eight solution matrices are obtained. The randomization procedure is introduced in this chapter for completeness.

Similar as the randomization procedure proposed in Chapter 2 and Chapter 3, let us decompose the matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$ into

$$\mathbf{X}_i^\star = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{U}_i^H \quad \forall i = 1, \ldots, M. \tag{4.55}$$

The corresponding beamforming matrices $\{\bar{\mathbf{W}}_i\}_{i=1}^M$ are then randomly generated as

$$\bar{\mathbf{W}}_i \triangleq [\bar{\mathbf{w}}_{i1}, \bar{\mathbf{w}}_{i2}, \ldots, \bar{\mathbf{w}}_{i8}] = \mathbf{U}_i \mathbf{\Sigma}_i^{1/2} \mathbf{\Lambda}_i \quad \forall i = 1, \ldots, M \tag{4.56}$$

where $\mathbf{\Lambda}_i$ is the $N \times 8$ matrix whose elements are drawn from an i.i.d. complex circular Gaussian distribution with zero mean and unit variance. Note that the instances of the beamforming matrices $\{\bar{\mathbf{W}}_i\}_{i=1}^M$ generated in (4.56) are generally not feasible for problem (4.37), because the randomization procedure is invoked only when the number of constraints is very large and it can be very difficult for randomly generated samples to satisfy all of the constraints. In order to compute a feasible solution with spatial characteristics corresponding to $\{\bar{\mathbf{W}}_i\}_{i=1}^M$, a power control problem is solved. Let $\sqrt{p_{ij}}$ be the power scaling factors corresponding to the beamformers $\bar{\mathbf{w}}_{ij}$ for all $i = 1, \ldots, M$ and $j = 1, \ldots, 8$. Define $\rho_{ij} \triangleq \mathrm{Tr}(\bar{\mathbf{w}}_{ij}\bar{\mathbf{w}}_{ij}^H)$ and $\zeta_{lij} \triangleq \mathrm{Tr}(\mathbf{A}_{li}\bar{\mathbf{w}}_{ij}\bar{\mathbf{w}}_{ij}^H)$, then the power control problem can be formulated as

$$
\begin{aligned}
\min_{\substack{p_{ij}, i=1,\ldots,M \\ j=1,\ldots,8}} \quad & \sum_{i=1}^{M} \sum_{j=1}^{8} p_{ij} \rho_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{M} \sum_{j=1}^{8} p_{ij} \zeta_{lij} \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, M + L
\end{aligned} \tag{4.57}
$$

which is a linear programming problem. The randomization procedure is summarized in Algorithm 4.2, where $\mathbf{P}_i \triangleq \text{diag}\{\sqrt{p_{i1}}, \ldots, \sqrt{p_{i8}}\}$ is the power scaling matrix corresponding to the i-th beamforming matrix.

Input $\{\mathbf{X}_i^\star\}_{i=1}^M$ with ranks greater than 8 for some i,
    N_{rand} number of iterations,
    P_{opt} optimal value of the power control problem;
Output $\{\bar{\mathbf{W}}_i \mathbf{P}_i\}_{i=1}^M$ beamforming matrices of the problem (4.43);
Set $K = 8$, $P_{\text{opt}} = +\text{Inf}$;
for $k = 1$ to N_{rand} **do**
  Obtain $\bar{\mathbf{W}}_i$ according to (4.56);
  Solve the power control problem (4.57);
  if The optimal value of (4.57) is less than P_{opt} **then**
    Set P_{opt} to be equal to the optimal value and
    store the scaled beamforming matrices $\{\bar{\mathbf{W}}_i \mathbf{P}_i\}_{i=1}^M$;
  else
    Discard the matrices $\{\bar{\mathbf{W}}_i\}_{i=1}^M$;
  end if
end for

Algorithm 4.2: Randomization procedure

With the rank reduction procedure in Algorithm 4.1 and the randomization procedure in Algorithm 4.2, a solution to problem (4.36) can be computed following the procedure summarized in Algorithm 4.3.

4.6 Simulations

Four simulation examples are provided to demonstrate the performance of our proposed downlink beamforming scheme with a large number of additional shaping constraints of

Input $\{\mathbf{X}_i^\star\}_{i=1}^M$ an optimal solution to the problem (4.43);
Output $\{\mathbf{W}_i^\star\}_{i=1}^M$ beamforming matrices of the problem (4.36),
K number of beamformers per user;
if rank($\mathbf{X}_i^*$) > 1 for some i **then**
Apply Algorithm 4.1 to obtain the rank-reduced matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$;
end if
if rank($\mathbf{X}_i^\star$) $\leq 8\ \forall i = 1, \dots, M$ **then**
Choose K to be the smallest number out of $\{1, 2, 4, 8\}$
such that rank($\mathbf{X}_i^*$) $\leq K \quad \forall i = 1, \dots, M$;
Decompose $\{\mathbf{X}_i^\star\}_{i=1}^M$ to obtain $\{\mathbf{W}_i^\star\}_{i=1}^M$ using (4.54);
else
Apply Algorithm 4.2 to obtain suboptimal beamforming matrices $\{\mathbf{W}_i^\star\}_{i=1}^M$;
end if
Rotate matrices $\{\mathbf{W}_i^\star\}_{i=1}^M$ if necessary according to (4.22).

Algorithm 4.3: Summary

different types. The base station is equipped with a uniform linear array (ULA), and the transmit antennas are spaced half wavelength apart. The noise powers of the downlink users in all examples are assumed to be $\sigma_i^2 = -10$dB for all $i = 1, \dots, M$. We also declare that rank($\mathbf{X}_m^\star$) $= \xi$ if the $(\xi + 1)$-th largest eigenvalue is smaller than 0.01% of the sum of all eigenvalues.

4.6.1 Example 1

In the first example, we consider the design of downlink beamformers with external wireless charging terminals. The number of antennas at the base station is 12 $(N = 12)$. Considering a LoS transmission scenario, three downlink users $(M = 3)$ connected to the base station are located at directions $\theta_1 = -5°$, $\theta_2 = 10°$ and $\theta_3 = 25°$ relative to the array broadside of the serving base station. There are 22 charging terminals, which are centered around

$$\vartheta_{4,\dots,14} = [-80°, -75°, -70°, -65°, -60°, -55°, -45°, -35°, -25°, -8°, -2°] \tag{4.58}$$

and

$$\vartheta_{15,\ldots,25} = [12^\circ, 18^\circ, 35^\circ, 45^\circ, 50^\circ, 55^\circ, 60^\circ, 65^\circ, 70^\circ, 75^\circ, 80^\circ] \tag{4.59}$$

relative to the serving base station under consideration. For all downlink users and charging terminals, the spatial signatures are modeled as

$$\mathbf{h}(\theta_m) = \left[1, e^{j\pi\sin(\theta_m)}, \ldots, e^{j\pi(N-1)\sin(\theta_m)}\right]^T \quad \forall m = 1, \ldots, 25 \tag{4.60}$$

i.e., the path loss of all downlink users and charging terminals is identical [26]. To make our simulation results more meaningful, we randomly vary the locations of the downlink users and the charging terminals in different Monte-Carlo runs, i.e., the angles of departure at the base station are simulated as

$$\theta_m \;\; = \vartheta_m + \Delta\theta_m \quad \forall m = 1, \ldots, 25 \tag{4.61}$$

where the random variations $\Delta\theta_m$ are drawn from a uniformly distributed within the interval $[-0.25^\circ, 0.25^\circ]$. We use the additional shaping constraints in (4.43b) to ensure predefined charging power levels at the l-th charging terminal in each time slot where $\mathbf{A}_{lm} = \mathbf{h}(\theta_l)\mathbf{h}^H(\theta_l)$ for all $m = 1, \ldots, M$ and $l = 4, \ldots, 25$. We set the minimum power threshold b_l to be 5dB for each charging terminal and $\trianglerighteq_l = \geq$. The SINR targets γ_i at the individual downlink users are varied between 0dB and 10dB. The simulation results are averaged over 300 Monte-Carlo runs. In each run, the number of randomization instances is set to 300 for all approaches if necessary. Specifically, for the rank-one beamforming approach, the randomization procedure needs to be performed if the rank of at least one solution matrix of the relaxed problem (4.43) after rank reduction exceeds one. For the rank-two beamforming approach, the randomization procedure is carried out if the rank is larger than two. For the general rank beamforming approach, the randomization procedure should be used if the rank is larger than eight.

The ranks of the solution matrices of the relaxed problem (4.43) after the rank reduction procedure are plotted in Fig. 4.1. According to the reduced rank property provided in

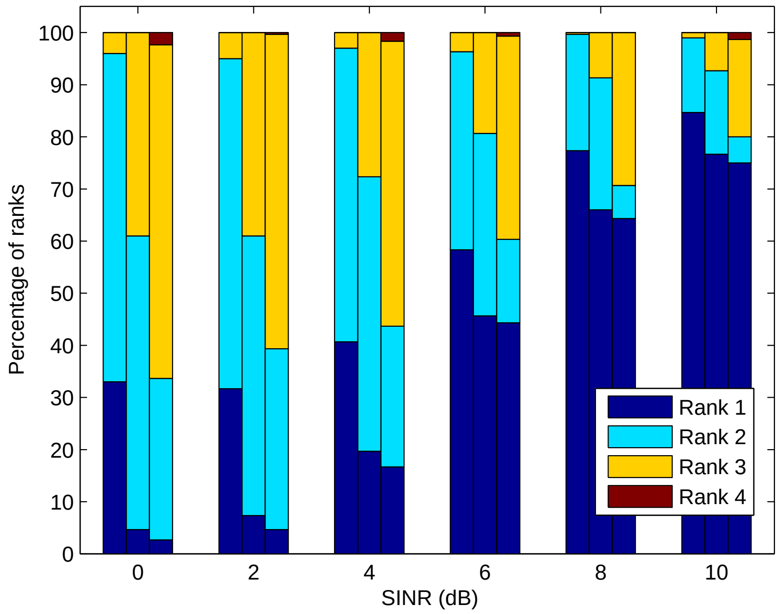

Figure 4.1: The ranks of the matrices $\mathbf{X}_1^\star$ (left bar), $\mathbf{X}_2^\star$ (middle bar), $\mathbf{X}_3^\star$ (right bar) after the rank reduction procedure.

Table 4.2, the code dimension is selected as $K = 4$. It can be analytically proven from a power scaling argument that problem (4.43), in the case of power charging constraints, is always feasible for all approaches. In Fig. 4.2, we display the total transmit power per time slot at the base station versus the SINR for different approaches. As shown in Fig. 4.2, the proposed general rank beamforming approach outperforms the competing approaches in terms of transmit power. In the low SINR region, the gap between the rank-one and rank-two approaches and the proposed approach is large, because as shown in Fig. 4.1, most of the solution matrices are of high rank (≥ 2) and thus the suboptimal randomization approximation is performed in the rank-one and rank-two approaches. In the high SINR region, the gaps between different approaches decrease because as shown in Fig. 4.1, the percentage of rank-one solution matrices increases which results in an increased number of optimal solutions for all approaches.

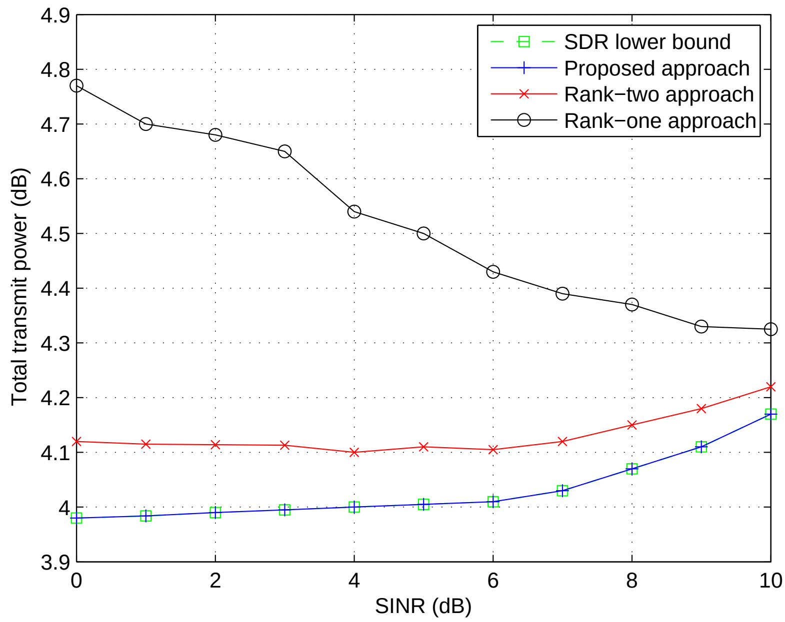

Figure 4.2: Total transmit power per time slot at the base station.

4.6.2 Example 2

In the second example, we consider the downlink beamformer design according to problem (4.43) for beam pattern (BP) with smooth and flat sidelobes to reduce the interference to co-channel users. We assume that in our simulation scenario the base station consists of 18 antennas $(N = 18)$. In this simulation the locations of three downlink users $(M = 3)$ are the same as in the previous example, i.e., $\theta_1 = -5°$, $\theta_2 = 10°$ and $\theta_3 = 25°$. The SINR thresholds of downlink users are set to $\gamma_i = 10$dB. Moreover, we assume that nineteen co-channel users connected to a neighboring base station are present in the scenario, which are located at

$$\mu_{1,\dots,19} = [-89.375°, -80°, -70.625°, -61.25°, -51.875°,$$
$$-42.5°, -33.125°, -23.75°, -14.375°, 2°, 3°, 17°, 18°,$$
$$34.375°, 43.75°, 53.125°, 62.5°, 71.875°, 81.25°]. \tag{4.62}$$

The channel propagation model is the same as defined in (4.60). The interference power at the direction μ_j relative to the base station in each time slot can be written as

$$f(\mu_j) = \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{(j+3)m}\mathbf{X}_m) \tag{4.63}$$

where $\mathbf{A}_{(j+3)m} = \mathbf{h}(\mu_j)\mathbf{h}^H(\mu_j)$ for all $m = 1, \ldots, 3$ and $j = 1, \ldots, 19$. In our beamformer design, the interference power is upper bounded by $b_{j+3} = -10\text{dB}$ and $\trianglerighteq_{j+3} = \leq$ for all $j = 1, \ldots, 19$. In addition to these constraints, we guarantee that the interference power at the direction μ_l attains a local minimum value by adding interference derivative constraints, i.e., the interference in the vicinity of the constraint directions remains approximately constant if

$$\begin{aligned} -\epsilon_a \leq \frac{df(\mu_j)}{d\mu_j} \leq \epsilon_a \quad \text{and} \quad \frac{d^2 f(\mu_j)}{d\mu_j^2} > 0 \\ \forall j = 1, \ldots, 19 \end{aligned} \tag{4.64}$$

where the threshold is set to $\epsilon_a = 10^{-5}$,

$$\frac{df(\mu_{\bar{j}})}{d\mu_{\bar{j}}} = \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{(j+3)m}\mathbf{X}_m), \quad \forall j = 20, \ldots, 38 \tag{4.65}$$

and

$$\frac{d^2 f(\mu_{\bar{j}})}{d\mu_{\bar{j}}^2} = \sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{(j+3)m}\mathbf{X}_m), \quad \forall j = 58, \ldots, 76 \tag{4.66}$$

are satisfied, for all $m = 1, \ldots, M$,

$$\mathbf{A}_{(j+3)m} = \begin{cases} \frac{d\mathbf{h}(\mu_{\bar{j}})}{d\mu_{\bar{j}}}\mathbf{h}^H(\mu_{\bar{j}}) + \mathbf{h}(\mu_{\bar{j}})\frac{d\mathbf{h}^H(\mu_{\bar{j}})}{d\mu_{\bar{j}}}, & \forall j = 20, \ldots, 38 \\ \frac{d\mathbf{h}(\mu_{\bar{j}})}{d\mu_{\bar{j}}}\mathbf{h}^H(\mu_{\bar{j}}) + \mathbf{h}(\mu_{\bar{j}})\frac{d\mathbf{h}^H(\mu_{\bar{j}})}{d\mu_{\bar{j}}}, & \forall j = 39, \ldots, 57 \\ \mathbf{h}(\mu_{\bar{j}})\frac{d^2\mathbf{h}^H(\mu_{\bar{j}})}{d\mu_{\bar{j}}^2} + \frac{d^2\mathbf{h}(\mu_{\bar{j}})}{d\mu_{\bar{j}}^2}\mathbf{h}^H(\mu_{\bar{j}}) + \\ 2\frac{d\mathbf{h}(\mu_{\bar{j}})}{d\mu_{\bar{j}}}\frac{d\mathbf{h}^H(\mu_{\bar{j}})}{d\mu_{\bar{j}}}, & \forall j = 58, \ldots, 76 \end{cases} \tag{4.67}$$

$$b_{j+3} = \begin{cases} \epsilon_a, & \forall j = 20, \ldots, 38 \\ -\epsilon_a, & \forall j = 39, \ldots, 57 \\ 0, & \forall j = 58, \ldots, 76 \end{cases} \tag{4.68}$$

$$\trianglerighteq_{j+3} = \begin{cases} \leq, & \forall j = 20, \ldots, 38 \\ \geq, & \forall j = 39, \ldots, 57 \\ \geq, & \forall j = 58, \ldots, 76 \end{cases} \tag{4.69}$$

with $\bar{j} \triangleq j \mod 19$, i.e., the remainder of j divided by 19. The received sum power at direction θ relative to the base station, referred to as the sum BP, is defined as

$$\sum_{m=1}^{M} \|\mathbf{h}(\theta)\mathbf{W}_m^\star\|_2^2 \tag{4.70}$$

where $\mathbf{W}_m^\star$ is the rank reduced solution given in (4.54).

The BPs are presented in Fig. 4.3, and the rank properties of the solution matrices are provided in Table 4.2. Note that there is a total number of 76 additional shaping constraints in this simulation. According to Lemma 4.2, we can find an optimal solution to the optimization problem (4.43) with the rank less than or equal to 8 by using the rank reduction procedure discussed in Algorithm 4.1. Based on the results in Table 4.2, we select the code dimension $K = 4$.

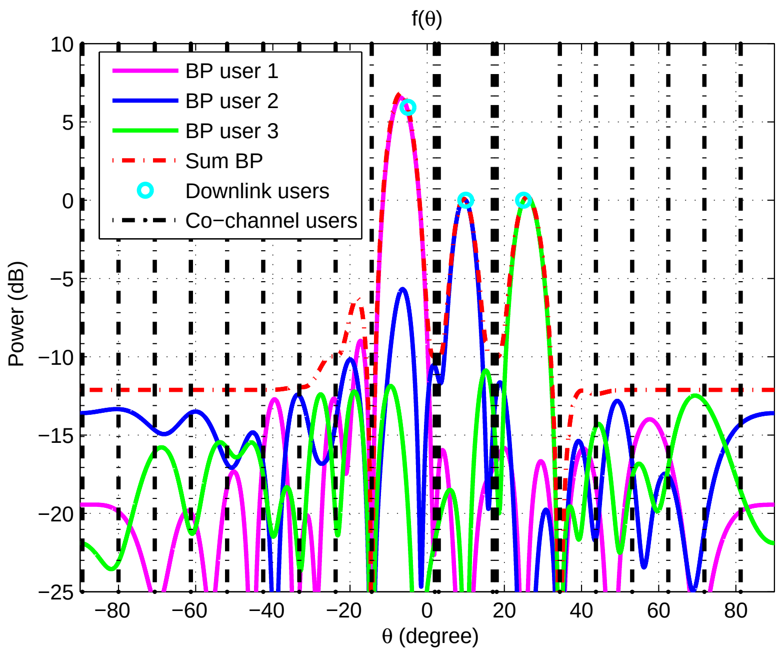

Figure 4.3: User BPs and sum BP with smoothed and suppressed sidelobes.

	$\mathbf{X}_1^\star$	$\mathbf{X}_2^\star$	$\mathbf{X}_3^\star$
Original rank in (4.43)	14	15	15
Reduced rank	2	3	4

Table 4.2: Rank property before and after applying rank reduction algorithm.

As shown in Fig. 4.3, the proposed approach is capable of coping with a large number of additional shaping constraints. Furthermore, as listed in Table 4.2, the ranks of the solution matrices have been significantly reduced which demonstrate the effectiveness of the rank reduction procedure.

4.6.3 Example 3

The same scenario as in Example 2 is considered to perform a comparison between our proposed approach with the conventional rank-one and rank-two approaches. All location parameters remain unchanged. Furthermore, we assume that all angles of departures are also subject to variations in different Monte-Carlo runs, which are defined in the same way as in Example 1. The required SINRs γ_i at the downlink users are uniformly varied between 0dB

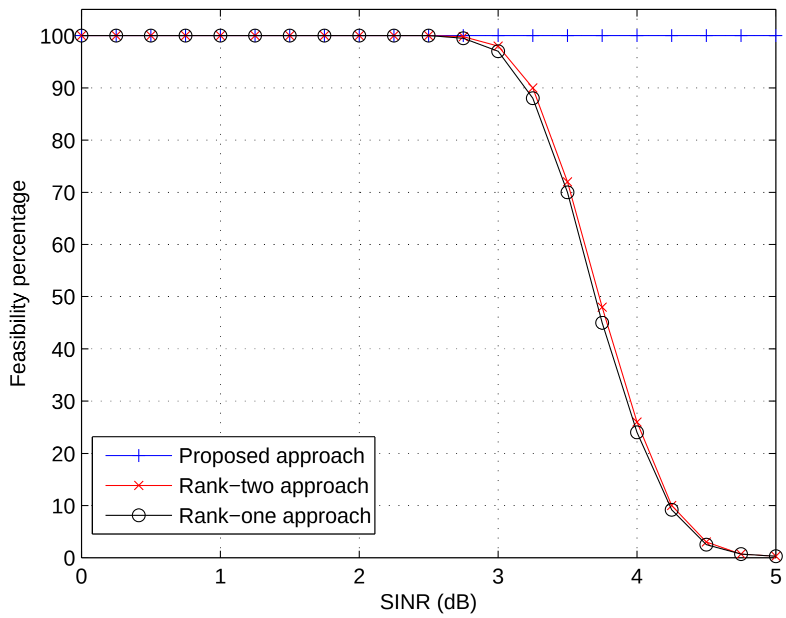

Figure 4.4: The feasibility percentage of all approaches.

and 5dB. The results are averaged over 300 independent Monte-Carlo runs and the number of randomization instances in each run is set to 100 for all approaches if necessary. The feasibility percentage of all approaches is displayed in Fig. 4.4. From Fig. 4.4, we observe that the proposed approach is always feasible for different SINR thresholds. In contrast to this, the feasibility of the rank-one and rank-two approaches decreases with increasing SINR thresholds. This demonstrates that our proposed approach has a wider feasibility range compared to existing approaches. The ranks of the solution matrices of the relaxed problem (4.43) after the rank reduction procedure are plotted in Fig. 4.5. As shown in Fig. 4.4, when $\gamma_i < 3$dB, all three approaches are feasible. This is due to the fact that in this case, as shown in Fig. 4.5, rank-one solutions are obtained for all approaches. In other words, optimal solutions are obtained for all approaches and thus the performance obtained from all approaches is identical. Therefore, when $\gamma_i < 3$dB, the code dimension for our proposed method is chosen as $K = 1$. In contrast to this when $\gamma_i \geq 3$dB we observe from Fig. 4.5 that the rank of the optimal solutions takes different values in the range between one and five. Thus in contrast to the rank-one and rank-two beamforming approaches if a rank larger than two is obtained, our proposed approach retains the optimality property and yields feasible solutions

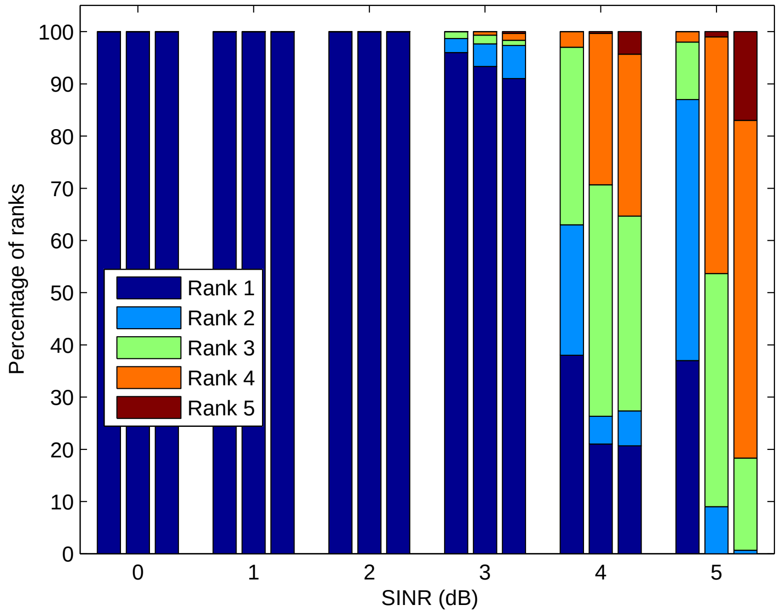

Figure 4.5: The ranks of the matrices $\mathbf{X}_1^\star$ (left bar), $\mathbf{X}_2^\star$ (middle bar), $\mathbf{X}_3^\star$ (right bar) after the rank reduction procedure.

while the competing approaches yield suboptimal solutions or even become infeasible for $\gamma_i \geq$ 3dB.

4.6.4 Example 4

The aim of the fourth example is to demonstrate the interference power suppression at each co-channel user to a fraction of its maximum value. In this example the concept of relaxed nulling is used to formulate the additional (indefinite) shaping constraints for interference power limitation [72]. The base station under consideration is equipped with a ULA of 15 antennas that are spaced half wavelength apart $(N = 15)$. Three downlink users served by the base station are located at $\theta_1 = -15°$, $\theta_2 = 5°$ and $\theta_3 = 25°$ relative to the base station. We assume that twenty two co-channel users served by neighboring base stations are present in our scenario which are located at the same position as in (4.58) and (4.59). We set the SINR thresholds to the same value as in Example 2. Similarly, the spatial signatures are modeled according to (4.60). We limit the interference power to the coexisting users by the following constraints

$$\text{Tr}(\mathbf{h}(\theta_j)\mathbf{h}(\theta_j)^H\mathbf{X}_i) \leq \beta\|\mathbf{h}(\theta_j)\|_2^2\text{Tr}(\mathbf{X}_i)$$
$$\forall i = 1,\ldots,3, \quad \forall j = 4,\ldots,25 \tag{4.71}$$

where $\beta \ll 1$ is an interference constraint parameter. The above constraints can be reformulated into the form of (4.43b) where, for all $\tilde{m}, m = 1,\ldots,3$,

$$\mathbf{A}_{(22(\tilde{m}-1)+j)m} = \begin{cases} \beta\|\mathbf{h}(\theta_j)\|_2^2\mathbf{I} - \mathbf{h}(\theta_j)\mathbf{h}(\theta_j)^H, & \tilde{m} = m \\ \mathbf{0}, & \tilde{m} \neq m \end{cases} \tag{4.72}$$

$$b_{n+3} = 0, \tag{4.73}$$

$$\trianglerighteq_{n+3} = \geq, \quad \forall n = 1,\ldots,66; \forall j = 4,\ldots,25. \tag{4.74}$$

We note that the matrix $\mathbf{A}_{lm}$ is either zero or indefinite for all $l = 4,\ldots,69, m = 1,\ldots,3$ and there is a total number of 66 additional shaping constraints in this simulation. In the simulation, β is chosen to be 0.5%. In this example, the code dimension K in the proposed approach is chosen as $K = 4$ because the rank of optimal solutions takes different values in the range between two and four. As shown in Fig. 4.6, the interference power at the locations of the coexisting users is limited to a reasonable level.

4.7 Summary

In this chapter, we propose a general rank beamforming approach for the multiuser downlink beamforming problem with additional shaping constraints. The general rank approach increases the degrees of freedom in the beamformer design by using high dimensional full-rate real-valued OSTBC. In our proposed approach, an optimal solution can be obtained when the ranks of all SDR solution matrices are less than or equal to eight after the rank reduction procedure. Moreover, in our scheme an optimal solution for the original problem can be found when the number of additional shaping constraints is less than or equal to 79. The range of applications for our proposed beamforming scheme is hence much wider than

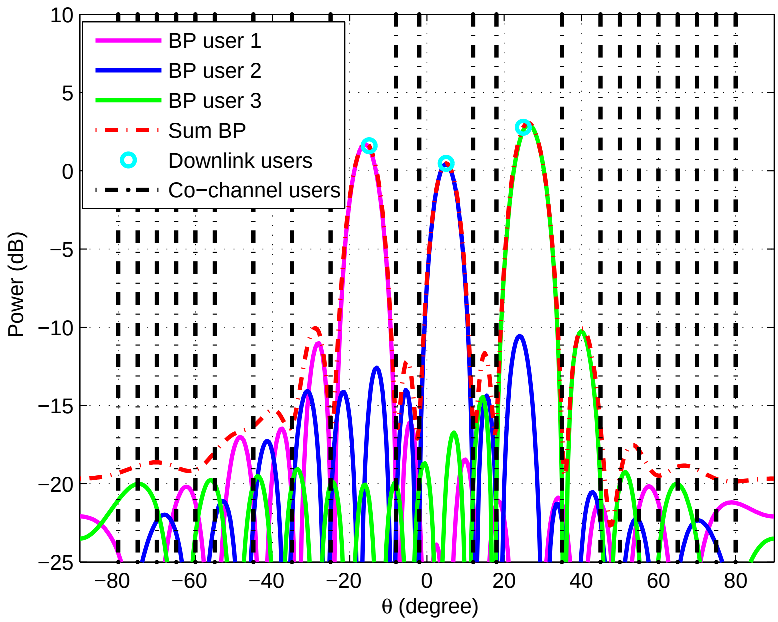

Figure 4.6: User BPs and sum BP of downlink beamforming problem subject to interference power constraints.

that of the conventional rank-one and rank-two approaches. Although our proposed general rank beamforming approach is associated with a slight increase in the signaling overhead as compared to the rank-one and rank-two beamforming approaches, in general the computational complexity of the general rank approach is lower than that of the rank-one and rank-two beamforming approaches, because SDR formulations are identical for all approaches whereas the randomization procedure is avoided in the general rank approach if the rank of all SDR solutions is no greater than eight.

Chapter 5

Long-term General Rank Downlink Beamforming With Shaping Constraints

5.1 Introduction

The rank-one beamforming problem of minimizing the total transmit power subject to SINR constraints and additional shaping constraints has been investigated in [26, 27, 72, 73]. As a massive number of constraints is incorporated, the degrees of freedom in the rank-one beamformer design can be rather deficient which may cause the optimization problem either to be infeasible or be difficult to solve optimally. To increase the degrees of freedom in the beamformer design, a general rank beamforming approach is proposed in Chapter 4 which combines beamforming with full-rate high dimensional real-valued OSTBC and it outperforms the conventional rank-one approaches and rank-two approaches proposed in Chapter 2 and Chapter 3. The general rank beamforming approach in Chapter 4 is designed based on the assumption that instantaneous CSI is available at the transmitter. However, instantaneous CSI can be difficult to acquire in practical cases. In FDD systems, instantaneous CSI needs to be fed back from the users to the base station for each frequency band resulting in a prohibitive signaling overhead especially in fast fading scenarios [29, 58]. Since statistical CSI describing the long-term channel characteristics, e.g., covariance based CSI, changes at a significantly lower rate as compared to the instantaneous CSI, only infrequent feedback from users is required. Therefore, the use of statistical CSI is generally more practical.

In this chapter, we propose a non-trivial extension of the general rank beamforming approach proposed in Chapter 4 to the case when covariance based CSI is available at the transmitter. We consider the problem of maximizing the minimum SINR among all users while satisfying the total transmit power constraint and additional shaping constraints. The key problem associated with the general rank beamforming approach in Chapter 4, when it is applied in the case of covariance based CSI, is that due to the absence of instantaneous CSI at the transmitter, the orthogonality of the code matrix of the equivalent channel can not be guaranteed and thus inter-symbol interference is present which results in performance degradation in terms of significantly increased SER. To address this issue, a new general rank beamforming approach is developed in this chapter to solve the downlink beamforming problem by combining downlink beamforming with full-rate QOSTBC. Instead of the real-valued OSTBC employed in Chapter 4, QOSTBC is used in this chapter because the inter-symbol interference in QOSTBC induced by the orthogonality loss of the coding matrix can be made much smaller than that in the real-valued OSTBC. A new phase rotation procedure on beamformers associated with QOSTBC is designed to ensure that the average inter-symbol interference is eliminated and correspondingly a simple symbol-wise decoder is developed for QOSTBC. In our proposed QOSTBC based general rank beamforming approach, the original beamforming problem is transformed to a convex optimization problem using SDR which can be solved efficiently. The SDR solution after the rank reduction procedure is optimal for the original problem if all SDR solution matrices do not exhibit a rank larger than eight, which can be guaranteed if the number of additional shaping constraint does not exceed 79, cf. Chapter 4.

This chapter is based on my original work that has been submitted in [92]. The remainder of this chapter is organized as follows. Section 5.2 introduces the signal model and revisits the conventional rank-one downlink beamforming problem. In Section 5.3, the system model corresponding to the QOSTBC based general rank beamforming approach is developed. Section 5.4 formulates and solves the optimal downlink beamforming problem. Simulation results are displayed in Section 5.5 and a summary is made in Section 5.6.

5.2 Conventional Rank-one Beamforming

Let us consider a wireless communication system where a base station or access point equipped with an antenna array of N elements simultaneously communicates independent information symbols to M single-antenna receivers. We assume that the channels are random, and covariance based CSI is available at the transmitter. The information symbol intended for the i-th receiver is denoted as s_i with zero mean and unit variance. Then, the signals $\{s_i\}_{i=1}^{M}$ are steered to different receivers in a spatially separated way using the respective $N \times 1$ beamforming vectors $\{\mathbf{w}_i\}_{i=1}^{M}$. The received signal at the i-th receiver is then given by [29]

$$y_i = \underbrace{s_i \mathbf{w}_i^H \mathbf{h}_i(t)}_{\text{desired signal}} + \underbrace{\sum_{m=1, m\neq i}^{M} s_m \mathbf{w}_m^H \mathbf{h}_i(t) + n_i}_{\text{interference plus noise}} \tag{5.1}$$

where $\mathbf{h}_i(t)$ and n_i are the $N \times 1$ time-varying channel vector and complex circularly white Gaussian noise with variance σ_i^2 of the i-th receiver, respectively. Note that the signal model in (5.1) which is identical to the signal model in (4.1) in Chapter 4 is provided here for the reading convenience. By definition, the SINR can be computed as the expected signal power over the expected interference plus noise power. Therefore, based on (5.1), the long-term average SINR at the i-th receiver in the conventional rank-one beamforming approach is derived as

$$\text{SINR}_{\text{c},i} \triangleq \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\sum\limits_{m=1, m\neq i}^{M} \mathbf{w}_m^H \mathbf{R}_i \mathbf{w}_m + \sigma_i^2} \tag{5.2}$$

where 'c' refers to the conventional approach and $\mathbf{R}_i = \text{E}\{\mathbf{h}_i(t)\mathbf{h}_i^H(t)\}$ [15]. Note that if $\mathbf{R}_i = \mathbf{h}_i(t)\mathbf{h}_i^H(t)$, the SINR expression in (5.2) is identical to that in (4.2) in Chapter 4. The total transmit power at the base station equals $\sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{w}_i$. Then, the problem of finding the weight vectors that maximize the minimum average SINR of all users subject to the total transmit power constraint $\mathrm{P}_{\max}$ and additional shaping constraints can be formulated as

$$\begin{aligned}
\max_{\{\mathbf{w}_i\}_{i=1}^M} \quad & \min_{i=1,\ldots,M} \mathrm{SINR}_{\mathrm{c},i} && \text{(5.3a)}\\
\text{s.t.} \quad & \sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{w}_i \leq \mathrm{P}_{\max} && \text{(5.3b)}\\
& \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{A}_{lm} \mathbf{w}_m \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, L && \text{(5.3c)}
\end{aligned}$$

which can be equivalently written as

$$\begin{aligned}
\max_{\{\mathbf{w}_i\}_{i=1}^M, t} \quad & t && \text{(5.4a)}\\
\text{s.t.} \quad & \mathrm{SINR}_{\mathrm{c},i} \geq t, \quad \forall i = 1, \ldots, M && \text{(5.4b)}\\
& \sum_{i=1}^{M} \mathbf{w}_i^H \mathbf{w}_i \leq \mathrm{P}_{\max} && \text{(5.4c)}\\
& \sum_{m=1}^{M} \mathbf{w}_m^H \mathbf{A}_{lm} \mathbf{w}_m \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, L && \text{(5.4d)}
\end{aligned}$$

where L additional shaping constraints are formulated in (5.4d) for appropriately chosen $N \times N$ Hermitian and possibly indefinite matrices $\mathbf{A}_{lm}$ with corresponding thresholds b_l. The additional shaping constraints in (5.4d) can be constructed for different applications as described in Section 4.2.1 and Section 4.2.2 in Chapter 4.

As compared to the power minimization problem considered in Chapter 4, the max-min problem is considered in this chapter because the SER comparison is carried out in the simulation and it is fair to compare the SER under the same total transmit power budget for different approaches. Note that the max-min problem in (5.4) may be infeasible due to the additional shaping constraints, however, the max-min multi-group multicast beamforming problem of (3.4) in Chapter 3 and the max-min downlink beamforming problem without additional shaping constraints are always feasible. Similar as the power minimization problem of (4.3) in Chapter 4, problem (5.4) is a non-convex QCQP problem and can be approximated by a SDP problem using the SDR technique [33, 91].

5.3 General Rank Beamforming

The central idea of combining downlink beamforming with QOSTBC in this chapter follows the general framework of Chapter 2, 3 and 4 in which beamformers are designed by combining beamforming with OSTBC. When applying the general rank beamforming using real-valued OSTBC in the downlink beamforming problem with additional shaping constraints as proposed in Chapter 4, the effective channel vectors have to be adjusted to real-valued vectors by specific phase rotations on beamformers to ensure that the corresponding coding matrix becomes orthogonal such that symbol-by-symbol decoding can be performed. The phase rotation procedure in the real-valued OSTBC is based on instantaneous CSI available at the transmitter, thus it cannot be applied in the problem considered in this chapter since only covariance based CSI is assumed to be available at the transmitter. Meanwhile, the SINR expression for the real-valued OSTBC case can be difficult to obtain. In this chapter, we apply QOSTBC and a new phase rotation procedure is designed to eliminate the average inter-symbol interference such that symbol-by-symbol decoding can be used at the receivers.

5.3.1 Full-rate QOSTBC

Full-rate orthogonal codes with complex symbol constellations in its code matrix are impossible to be obtained for systems with more than two transmit antennas. To design full-rate codes, QOSTBC is proposed in which the strict requirement of full orthogonality of the code matrix is slightly relaxed [16, 17]. Correspondingly, the simple symbol-by-symbol decoding property is lost. However, pairs of symbols can optimally be decoded independently for 4×4 and 8×8 in QOSTBC [25]. Examples of the 4×4 and 8×8 QOSTBC matrix are as follows

$$\mathcal{X}([s_1, s_2, s_3, s_4]^T) \triangleq \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}. \tag{5.5}$$

and

$$\mathcal{X}([s_1,\dots,s_8]^T) \triangleq \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ -s_2^* & s_1^* & s_4^* & -s_3^* & s_6^* & -s_5^* & -s_8^* & s_7^* \\ -s_3 & -s_4 & s_1 & s_2 & s_7 & s_8 & -s_5 & -s_6 \\ -s_4^* & s_3^* & -s_2^* & s_1^* & s_8^* & -s_7^* & s_6^* & -s_5^* \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 & s_2 & s_3 & s_4 \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 & s_1 & -s_2 \\ -s_8^* & -s_7^* & s_6^* & s_5^* & -s_4^* & -s_3^* & s_2^* & s_1^* \end{bmatrix}. \tag{5.6}$$

5.3.2 General Rank System Model

Denote $\mathbf{s}_i \triangleq [s_{i1},\dots,s_{iK}]^T$ as the $K \times 1$ complex symbol vector for the i-th user with $K \leq N$ and $K \in \{4, 8\}$ in accordance with the dimension of the QOSTBC matrices. Instead of weighting each symbol by a single beamforming vector as in (5.1), a QOSTBC matrix $\mathcal{X}(\mathbf{s}_i)$ is transmitted for each user with the help of K beamformers of length N, denoted as $\mathbf{w}_{i1},\dots,\mathbf{w}_{iK}$. In this case, each of the K beams can be regarded as a virtual antenna from which the QOSTBC matrix is transmitted. In our scenario we consider a block fading channel model where the channels remain constant over K time slots. The received signal y_{ik} at the i-th user in the k-th time slot is given by

$$y_{ik} = \sum_{m=1}^{M}\sum_{k'=1}^{K}[\mathcal{X}(\mathbf{s}_m)]_{kk'}\mathbf{w}_{mk'}^H\mathbf{h}_i(t) + n_{ik} \tag{5.7}$$

where n_{ik} is the noise of the i-th user in the k-th time slot. The received signal vector $\mathbf{y}_i \triangleq [y_{i1},\dots,y_{iK}]^T$ at the i-th user within the transmission period of K time slots can be written in a matrix form as

$$\begin{aligned} \mathbf{y}_i &= \sum_{m=1}^{M}\mathcal{X}(\mathbf{s}_m)\mathbf{W}_m^H\mathbf{h}_i(t) + \mathbf{n}_i \\ &= \underbrace{\mathcal{X}(\mathbf{s}_i)\mathbf{W}_i^H\mathbf{h}_i(t)}_{\text{desired signal}} + \underbrace{\sum_{m=1,m\neq i}^{M}\mathcal{X}(\mathbf{s}_m)\mathbf{W}_m^H\mathbf{h}_i(t) + \mathbf{n}_i}_{\text{interference plus noise}} \end{aligned} \tag{5.8}$$

where

$$\mathbf{W}_i \triangleq [\mathbf{w}_{i1}, \ldots, \mathbf{w}_{iK}] \tag{5.9}$$

is the beamforming matrix, and the noise vector

$$\mathbf{n}_i \triangleq [n_{i1}, \ldots, n_{iK}]^T. \tag{5.10}$$

The above system model can be reformulated in the following equivalent form [16]

$$\tilde{\mathbf{y}}_i = \mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i(t))\mathbf{s}_i + \tilde{\mathbf{i}}_i + \tilde{\mathbf{n}}_i \tag{5.11}$$

where $\mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i(t))$ denotes the quasi-orthogonal equivalent channel matrix and

$$\begin{aligned} \tilde{\mathbf{y}}_i &\triangleq \Big[y_{i1}, -y_{i2}, \ldots, -y_{iK}\Big]^T, && (5.12)\\ \tilde{\mathbf{i}}_i &\triangleq \sum_{m=1, m\neq i}^{M} \mathcal{X}(\mathbf{W}_m^H \mathbf{h}_i(t))\, \mathbf{s}_m, && (5.13)\\ \tilde{\mathbf{n}}_i &\triangleq \Big[n_{i1}, -n_{i2}, \ldots, -n_{iK}\Big]^T. && (5.14) \end{aligned}$$

Employing the 4×4 QOSTBC matrix in (5.5) and multiplying $\frac{\mathcal{X}^H(\mathbf{W}_i^H \mathbf{h}_i(t))}{\|\mathbf{W}_i^H \mathbf{h}_i(t)\|_2^2}$ on both sides of (5.11), we have

$$\begin{aligned} \hat{\mathbf{s}}_i &\triangleq \frac{1}{\|\mathbf{W}_i^H \mathbf{h}_i(t)\|_2^2} \mathcal{X}^H(\mathbf{W}_i^H \mathbf{h}_i(t))\tilde{\mathbf{y}}_i \\ &= \mathbf{G}_i \mathbf{s}_i + \frac{1}{\|\mathbf{W}_i^H \mathbf{h}_i\|_2^2} \mathcal{X}^H(\mathbf{W}_i^H \mathbf{h}_i)(\tilde{\mathbf{i}}_i + \tilde{\mathbf{n}}_i) && (5.15) \end{aligned}$$

where

$$\mathbf{G}_i \triangleq \frac{\mathcal{X}^H(\mathbf{W}_i^H \mathbf{h}_i)\mathcal{X}(\mathbf{W}_i^H \mathbf{h}_i)}{\|\mathbf{W}_i^H \mathbf{h}_i(t)\|_2^2} = \begin{bmatrix} 1 & 0 & -g_i & 0 \\ 0 & 1 & 0 & g_i \\ g_i & 0 & 1 & 0 \\ 0 & -g_i & 0 & 1 \end{bmatrix}, \tag{5.16}$$

$$g_i \triangleq \frac{2\mathrm{Im}\{\mathbf{w}_{i1}^H \mathbf{h}_i(t)\mathbf{h}_i(t)^H \mathbf{w}_{i3} - \mathbf{w}_{i2}^H \mathbf{h}_i(t)\mathbf{h}_i^H(t)\mathbf{w}_{i4}\}}{\|\mathbf{W}_i^H \mathbf{h}_i(t)\|_2^2} j, \tag{5.17}$$

and $j = \sqrt{-1}$. We observe in (5.16) that g_i and $-g_i$ represent inter-symbol interference terms for $\hat{\mathbf{s}}_i$. Due to the quasi-orthogonal property of the equivalent channel matrix as in (5.16), pairwise ML detection is the optimum detection for information symbols transmitted with QOSTBC. However, it is associated with a decoding complexity increase as compared to symbol-wise decoding [16]. To enhance the characteristics of the equivalent MIMO channel in (5.11) and reduce the decoding complexity by enabling simple symbol-by-symbol detection, we design the beamforming matrices $\mathbf{W}_i$ such that the quasi-orthogonal equivalent channel matrix is further orthogonalized. The orthogonalization of (5.16) requires knowledge of instantaneous CSI, i.e., $\mathbf{h}_i(t)$, which is not known at the transmitter. Therefore, here we consider the average inter-symbol interference power defined as

$$\bar{g}_i \triangleq \mathrm{E}\{g_i\} = \frac{2\mathrm{Im}\{\mathbf{w}_{i1}^H \mathbf{R}_i \mathbf{w}_{i3} - \mathbf{w}_{i2}^H \mathbf{R}_i \mathbf{w}_{i4}\}}{\mathrm{Tr}(\mathbf{W}_i^H \mathbf{R}_i \mathbf{W}_i)} j. \tag{5.18}$$

In order to achieve the best decoding performance, the average inter-symbol interference in $\hat{\mathbf{s}}_i$ should be adjusted to null, i.e.,

$$|\bar{g}_i|^2 = 0. \tag{5.19}$$

For a given beamformer $\mathbf{W}_i^\star \triangleq \left[\mathbf{w}_{i1}^\star, \ldots, \mathbf{w}_{iK}^\star\right]$, a sufficient but not necessary condition for satisfying (5.19) is

$$\begin{cases} \mathrm{Im}\{\mathbf{w}_{i1}^{\star H} \mathbf{R}_i \mathbf{w}_{i3}^\star\} = 0 \\ \mathrm{Im}\{\mathbf{w}_{i2}^{\star H} \mathbf{R}_i \mathbf{w}_{i4}^\star\} = 0. \end{cases} \tag{5.20}$$

To satisfy (5.20), phase rotation can be performed on beamformer $\mathbf{W}_i^\star$ in various ways, e.g.,

$$\begin{cases} \mathbf{w}_{i1}'^\star \triangleq \mathbf{w}_{i1}^\star \exp(j\angle(\mathbf{w}_{i1}^{\star H}\mathbf{R}_i\mathbf{w}_{i3}^\star)) \\ \mathbf{w}_{i2}'^\star \triangleq \mathbf{w}_{i2}^\star \exp(j\angle(\mathbf{w}_{i2}^{\star H}\mathbf{R}_i\mathbf{w}_{i4}^\star)) \\ \mathbf{w}_{i3}'^\star \triangleq \mathbf{w}_{i3}^\star \\ \mathbf{w}_{i4}'^\star \triangleq \mathbf{w}_{i4}^\star. \end{cases} \tag{5.21}$$

We remark that QOSTBC cannot be applied in single-group multicasting in Chapter 2 and multi-group multicasting in Chapter 3 in a similar way as in downlink beamforming, because in both applications multiple users are served by a common beamforming matrix on which the phase rotation can only eliminate the inter-symbol interference of one user.

Based on (5.15), the covariance matrix of the received multiuser interference contained in $\hat{\mathbf{s}}_i$ is given by

$$\begin{aligned} \mathbf{C}_i^{(\mathrm{I})} &\triangleq \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^4}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i(t))\mathrm{E}\{\tilde{\mathbf{i}}_i\tilde{\mathbf{i}}_i^H\}\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i(t)) \\ &= \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^4}[\sum_{m=1,m\neq i}^{M}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i(t))\mathcal{X}(\mathbf{W}_m^H\mathbf{h}_i(t)) \times \\ &\quad \mathcal{X}^H(\mathbf{W}_m^H\mathbf{h}_i(t))\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i(t))]. \end{aligned} \tag{5.22}$$

Applying Lemma 4.1 in Chapter 4, the average multiuser interference power of the i-th user in the k-th time slot can be expressed as

$$[\mathbf{C}_i^{(\mathrm{I})}]_{kk} \triangleq E\{\frac{\sum_{m=1,m\neq i}^{M}\|\mathbf{W}_m^H\mathbf{h}_i(t)\|_2^2}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^2}\} = \frac{\sum_{m=1,m\neq i}^{M}\mathrm{Tr}(\mathbf{W}_m^H\mathbf{R}_i\mathbf{W}_m)}{\mathrm{Tr}(\mathbf{W}_i^H\mathbf{R}_i\mathbf{W}_i)}. \tag{5.23}$$

Based on (5.15), the covariance matrix of the noise in $\hat{\mathbf{s}}_i$ is given by

$$\begin{aligned} \mathbf{C}_i^{(\mathrm{N})} &\triangleq \frac{1}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^4}\mathcal{X}^H(\mathbf{W}_i^H\mathbf{h}_i(t))\mathrm{E}\{\tilde{\mathbf{n}}_i\tilde{\mathbf{n}}_i^H\}\mathcal{X}(\mathbf{W}_i^H\mathbf{h}_i(t)) \\ &= \frac{\sigma_i^2}{\|\mathbf{W}_i^H\mathbf{h}_i(t)\|_2^2}\mathbf{I}_K. \end{aligned} \tag{5.24}$$

The average noise power of the i-th user in the k-th time slot can be expressed as

$$[\mathbf{C}_i^{(\mathrm{N})}]_{kk} \triangleq \mathrm{E}\{\frac{\sigma_i^2}{\|\mathbf{W}_i^H \mathbf{h}_i(t)\|_2^2}\} = \frac{\sigma_i^2}{\mathrm{Tr}(\mathbf{W}_i^H \mathbf{R}_i \mathbf{W}_i)}. \tag{5.25}$$

Then, the average SINR corresponding to symbol s_{ik} in the proposed general rank beamforming approach is given by

$$\begin{aligned} \mathrm{SINR}(s_{ik}) &\triangleq \frac{\mathrm{E}\{s_{ik} s_{ik}^*\}}{|\bar{g}_i|^2 \,\mathrm{E}\{s_{ik'} s_{ik'}^*\} + [\mathbf{C}_i^{(\mathrm{I})}]_{kk} + [\mathbf{C}_i^{(\mathrm{N})}]_{kk}} \\ &= \frac{\mathrm{Tr}(\mathbf{W}_i^H \mathbf{R}_i \mathbf{W}_i)}{\sum\limits_{m=1, m\neq i}^{M} \mathrm{Tr}(\mathbf{W}_m^H \mathbf{R}_i \mathbf{W}_m) + \sigma_i^2}, \end{aligned} \tag{5.26}$$

where k' is the index number of the entry g_i or $-g_i$ in the k-th row of G_i in (5.16). Note that the designed average orthogonality property resulting from (5.19) is used in deriving $\mathrm{SINR}(s_{ik})$ which is different from the SINR derivation in (4.30) in Chapter 4. Since the expression of $\mathrm{SINR}(s_{ik})$ in (5.26) is independent of the time index k, SINR_i, the average SINR for the i-th user, is identical for all symbols in the QOSTBC block which is given by

$$\mathrm{SINR}_i \triangleq \frac{\mathrm{Tr}(\mathbf{W}_i^H \mathbf{R}_i \mathbf{W}_i)}{\sum\limits_{m=1, m\neq i}^{M} \mathrm{Tr}(\mathbf{W}_m^H \mathbf{R}_i \mathbf{W}_m) + \sigma_i^2}. \tag{5.27}$$

The total transmit power in each time slot equals $\sum\limits_{i=1}^{M} \mathrm{Tr}(\mathbf{W}_i \mathbf{W}_i^H)$ which can be computed in a similar way as in Chapter 4. With multiple beamformers designed for each user, the additional shaping constraints in (5.4d) can be expressed as

$$\sum_{m=1}^{M} \mathrm{Tr}(\mathbf{A}_{lm} \mathbf{W}_m \mathbf{W}_m^H) \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, L. \tag{5.28}$$

5.4 Beamformer Optimization

The optimization problem of maximizing the minimum average SINR in (5.26) of all users subject to the power constraint and additional shaping constraints can be formulated as

$$\max_{\{\mathbf{W}_i\}_{i=1}^M, t} \quad \min_{i=1,\dots,M} \text{SINR}_i \tag{5.29a}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} \text{Tr}(\mathbf{W}_i \mathbf{W}_i^H) \leq \text{P}_{\max} \tag{5.29b}$$

$$\sum_{m=1}^{M} \text{Tr}(\mathbf{A}_{lm} \mathbf{W}_m \mathbf{W}_m^H) \trianglerighteq_l b_l, \quad \forall l = 1, \dots, L \tag{5.29c}$$

which can be equivalently written as

$$\max_{\{\mathbf{W}_i\}_{i=1}^M, t} \quad t \tag{5.30a}$$

$$\text{s.t.} \quad \frac{\text{Tr}(\mathbf{W}_i^H \mathbf{R}_i \mathbf{W}_i)}{\sum\limits_{m=1, m\neq i}^{M} \text{Tr}(\mathbf{W}_m^H \mathbf{R}_i \mathbf{W}_m) + \sigma_i^2} \geq t, \quad \forall i = 1, \dots, M \tag{5.30b}$$

$$\sum_{i=1}^{M} \text{Tr}(\mathbf{W}_i \mathbf{W}_i^H) \leq \text{P}_{\max} \tag{5.30c}$$

$$\sum_{m=1}^{M} \text{Tr}(\mathbf{A}_{lm} \mathbf{W}_m \mathbf{W}_m^H) \trianglerighteq_l b_l, \quad \forall l = 1, \dots, L. \tag{5.30d}$$

To solve problem (5.30), let us employ the SDR approach similar to that employed in Chapter 4 and define the variable transformation as follows

$$\mathbf{X}_i \triangleq \mathbf{W}_i \mathbf{W}_i^H, \quad \forall i = 1, \dots, M. \tag{5.31}$$

By substituting $\mathbf{X}_i$ and adding the following constraints

$$\begin{cases} \mathbf{X}_i \succeq \mathbf{0} \\ \text{rank}(\mathbf{X}_i) \leq K, \quad \forall i = 1, \dots, M \end{cases} \tag{5.32}$$

to guarantee that the transformation (5.31) exists, problem (5.30) converts to a rank constrained problem

$$\max_{\{\mathbf{X}_i\}_{i=1}^M, t} \quad t \tag{5.33a}$$

$$\text{s.t.} \quad \frac{\text{Tr}(\mathbf{X}_i \mathbf{R}_i)}{\sum\limits_{m=1, m\neq i}^{M} \text{Tr}(\mathbf{X}_m \mathbf{R}_i) + \sigma_i^2} \geq t, \quad \forall i = 1, \ldots, M \tag{5.33b}$$

$$\sum_{i=1}^{M} \text{Tr}(\mathbf{X}_i) \leq \text{P}_{\max} \tag{5.33c}$$

$$\sum_{m=1}^{M} \text{Tr}(\mathbf{A}_{lm} \mathbf{X}_m) \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, L \tag{5.33d}$$

$$\mathbf{X}_i \succeq \mathbf{0}, \quad \forall i = 1, \ldots, M \tag{5.33e}$$

$$\text{rank}(\mathbf{X}_i) \leq K, \quad \forall i = 1, \ldots, M. \tag{5.33f}$$

Following the SDR approach, the rank constraints of (5.33f) are removed, and a relaxed optimization problem is obtained as

$$\max_{\{\mathbf{X}_i\}_{i=1}^M, t} \quad t \tag{5.34a}$$

$$\text{s.t.} \quad \frac{\text{Tr}(\mathbf{X}_i \mathbf{R}_i)}{\sum\limits_{m=1, m\neq i}^{M} \text{Tr}(\mathbf{X}_m \mathbf{R}_i) + \sigma_i^2} \geq t, \quad \forall i = 1, \ldots, M \tag{5.34b}$$

$$\sum_{i=1}^{M} \text{Tr}(\mathbf{X}_i) \leq \text{P}_{\max} \tag{5.34c}$$

$$\sum_{m=1}^{M} \text{Tr}(\mathbf{A}_{lm} \mathbf{X}_m) \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, L \tag{5.34d}$$

$$\mathbf{X}_i \succeq \mathbf{0}, \quad \forall i = 1, \ldots, M. \tag{5.34e}$$

As compared to the rank-constrained problem of (4.43) in Chapter 4 where the total transmit power is the objective function, in problem (5.34) the total transmit power is included in the constraint. Moreover, the SINR constraints in (4.43) are linear constraints, however, the constraints in (5.34b) are bi-linear constraints. Therefore, we perform a one-dimensional bisection search over t to solve the problem (5.34) efficiently as in Chapter 3. Denote $\{\mathbf{X}_i^\star\}_{i=1}^M$

as an optimal solution to problem (5.34). Then we apply the rank reduction algorithm proposed in Chapter 4 with the input $\{\mathbf{X}_i^\star\}_{i=1}^M$ to reduce the rank of the optimal solution. If the rank-reduced solution set $\{\mathbf{X}_i^\star\}_{i=1}^M$ obtained after the rank reduction procedure satisfies $4 < \max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) \le 8$, we choose K=8; if $\max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) \le 4$, we choose K=4. If $\max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) = 2$, the proposed approach is equivalent to the rank-two approach proposed in Chapter 2 and Chapter 3. If $\max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star)$=1, the proposed approach is equivalent to the rank-one approach. The corresponding beamforming matrices are calculated by eigenvalue decomposition on $\{\mathbf{X}_i^\star\}_{i=1}^M$ followed by the proposed phase rotation procedure as defined in (5.21). In the case that $\max\limits_{1\le i\le M} \operatorname{rank}(\mathbf{X}_i^\star) > 8$, we choose K=8 and the following randomization procedure can be used to obtain a suboptimal solution to problem (5.30). Similar as in Chapter 4, the randomization procedure may not be relevant in practice since the number of constraints is already very large for which optimal rank-eight solution matrices can be obtained.

Similar as the randomization procedure in Chapter 4, let us first decompose the matrices $\{\mathbf{X}_i^\star\}_{i=1}^M$ as $\mathbf{X}_i^\star = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{U}_i^H$. Then, the corresponding beamforming matrices $\{\bar{\mathbf{W}}_i\}_{i=1}^M$ for one randomization instance are calculated according to

$$\bar{\mathbf{W}}_i \triangleq [\bar{\mathbf{w}}_{i1}, \bar{\mathbf{w}}_{i2}, \ldots, \bar{\mathbf{w}}_{i8}] = \mathbf{U}_i \mathbf{\Sigma}_i^{1/2} \mathbf{\Lambda}_i \quad \forall i = 1, \ldots, M \tag{5.35}$$

where $\mathbf{\Lambda}_i$ is the randomly generated $N \times 8$ matrix whose elements are drawn from an i.i.d. complex circular Gaussian distribution with zero mean and unit variance. Similar as in Chapter 4, the beamforming matrices $\{\bar{\mathbf{W}}_i\}_{i=1}^M$ in (5.35) are generally infeasible for problem (5.30) since it is very difficult for the randomly generated instances to fulfill a massive number of constraints at the same time. In order to obtain a feasible solution with spatial characteristics corresponding to $\{\bar{\mathbf{W}}_i\}_{i=1}^M$, a power control problem needs to be solved. Denote $\sqrt{p_{ij}}$ as the power control scaling factors corresponding to the beamformers $\bar{\mathbf{w}}_{ij}$ for all

$i = 1, \ldots, M$ and $j = 1, \ldots, 8$. Further define

$$\begin{cases} \rho_{ij} \triangleq \mathrm{Tr}(\bar{\mathbf{w}}_{ij}\bar{\mathbf{w}}_{ij}^H) \\ \upsilon_{qij} \triangleq \mathrm{Tr}(\mathbf{R}_q\bar{\mathbf{w}}_{ij}\bar{\mathbf{w}}_{ij}^H) \\ \zeta_{lij} \triangleq \mathrm{Tr}(\mathbf{A}_{li}\bar{\mathbf{w}}_{ij}\bar{\mathbf{w}}_{ij}^H) \end{cases} \tag{5.36}$$

then the power allocation problem can be formulated as

$$\max_{\substack{t, p_{ij}, i=1,\ldots,M \\ j=1,\ldots,8}} \quad t \tag{5.37a}$$

$$\text{s.t.} \quad \frac{\sum_{j=1}^{8} p_{ij}\upsilon_{iij}}{\sum_{m=1, m\neq i}^{M}\sum_{j=1}^{8} p_{mj}\upsilon_{imj} + \sigma_i^2} \geq t, \quad \forall i = 1, \ldots, M \tag{5.37b}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{8} p_{ij}\rho_{ij} \leq \mathrm{P}_{\max} \tag{5.37c}$$

$$\sum_{i=1}^{M}\sum_{j=1}^{8} p_{ij}\zeta_{lij} \trianglerighteq_l b_l, \quad \forall l = 1, \ldots, L. \tag{5.37d}$$

Different from the power control problem of (4.57) in Chapter 4, the total transmit power is included in the constraints of problem (5.37) and the global power control procedure involving bisection search and linear programming is performed. Then, among all sets of the candidate beamforming matrices after the power scaling, the one with the largest SINR value is chosen as the final solution.

Similar as the general rank beamforming approach in Chapter 4, each user is served with up to eight beamformers in the proposed general rank beamforming approach, and a maximum number of 79 additional shaping constraints can be accommodated for which an optimal solution can be obtained.

5.5 Simulations

In the simulation, we consider the downlink beamformer design that limits the interference to co-channel users which is similar to Example 2 in Section 4.6.2 and Example 3 in Section 4.6.3 of Chapter 4. The difference is that here the long-term covariance based CSI is used, and the optimization problems are always feasible for all beamforming approaches in the simulation.

The base station is equipped with a ULA of N=15 antennas spaced half a wavelength apart. There are three downlink users located at θ_1=$-7°$, θ_2=10° and θ_3=27° relative to the array broadside. The downlink users are assumed to be surrounded by a large number of local scatterers corresponding to the same angular spread of σ_θ for all users, as seen from the base station. The channel covariance matrices $\{\mathbf{R}_i\}_{i=1}^3$ are calculated as [57, 93]

$$[\mathbf{R}_i]_{kl} = \exp(j\pi(k-l)\sin\theta_i)\exp(-\frac{(\pi(k-l)\sigma_\theta\cos\theta_i)^2}{2}) \quad \forall i = 1,\ldots,3. \tag{5.38}$$

Moreover, there are 19 co-channel users connected to a neighboring base station which are located at

$$\begin{aligned}\mu_{1,\ldots,19} = [&-89.375°, -80°, -70.625°, -61.25°, -51.875°,\\ &-42.5°, -33.125°, -30°, -23.75°, -15°, 2°, 18°,\\ &36°, 43.75°, 49°, 53.125°, 62.5°, 71.875°, 81.25°].\end{aligned} \tag{5.39}$$

The interference power at the direction μ_l in each time slot $f(\mu_l) = \sum_{m=1}^{3} \mathrm{Tr}(\mathbf{h}_{\mu_l}\mathbf{h}_{\mu_l}^H\mathbf{X}_m)$ is upper bounded by $b_l = -3$dB, and $\mathbf{h}_{\mu_l}$ is the channel vector corresponding to the direction μ_l. In addition to these constraints, the interference derivative constraints are also taken into account, i.e., $-\epsilon_a \leq \frac{df(\mu_l)}{d\mu_l} \leq \epsilon_a$ and $\frac{d^2f(\mu_l)}{d\mu_l^2} > 0$ for all $l = 1,\ldots,19$ where the threshold is set to $\epsilon_a = 10^{-5}$, and $\frac{df(\mu_l)}{d\mu_l}$ and $\frac{d^2f(\mu_l)}{d\mu_l^2}$ are computed in the same way as in Example 2 in Section 4.6.2 of Chapter 4. With these derivative constraints, the interference power at the direction μ_l is ensured to obtain a local minimum value and the interference in the vicinity of the constraint directions remains approximately constant.

We assume σ_i^2=-10dB for all $i = 1, \ldots, 3$ and $\mathrm{P}_{\max}$=0dB. The results are averaged over 300 independent Monte-Carlo runs in which all angles of departures are subject to variations defined in the same way as in Example 1 in Section 4.6.1 of Chapter 4. In each run, 200 instantaneous channel realizations are generated for each downlink user obeying the distribution corresponding to $\mathbf{R}_i$, and 100 symbols are transmitted within each instantaneous channel realization. The number of randomization samples in each run is set to 300 for all approaches if necessary and QPSK modulation is used.

In this example, we compare the proposed approach with the existing ones. The code dimension K in the proposed approach is chosen as $K = 4$ since $2 < \max_{1 \leq i \leq M} \mathrm{rank}(\mathbf{X}_i^\star) \leq 4$. In Fig. 5.1, the worst SINR for different spread angles is displayed. As shown in Fig. 5.1, the proposed approach achieves much higher SINR than that of the rank-one and rank-two approaches which is zero for all spread angles, i.e., the problem is infeasible for rank-one and rank-two approaches in practice. In Fig. 5.2, the worst-user SER for different spread angles

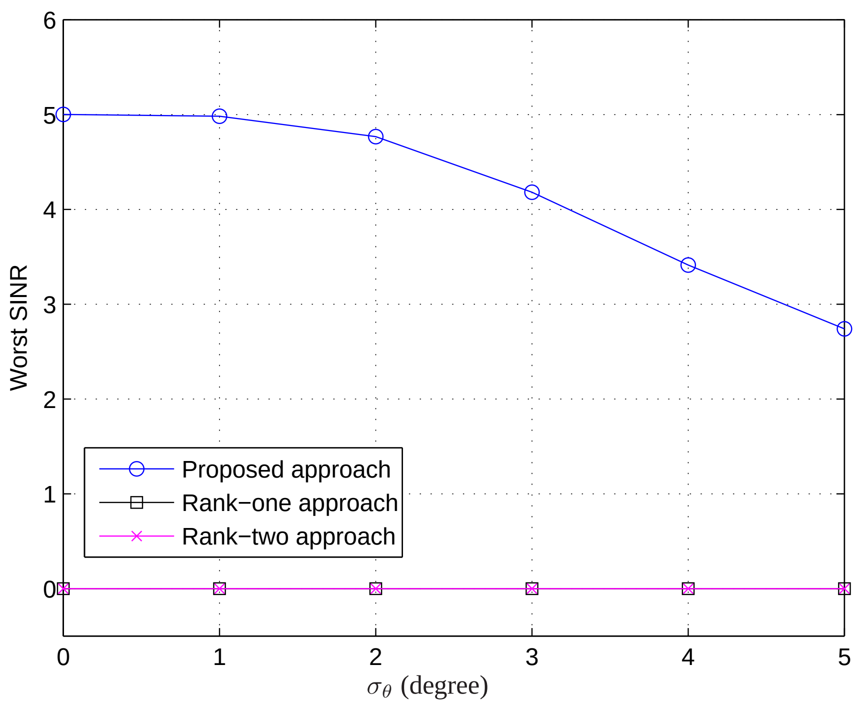

Figure 5.1: Worst SINR versus varying spread angles

is displayed. In the legend of Fig. 5.2, ‘GR’ refers to the general rank approach; ‘qs’ and ‘rl’ refer to the use of QOSTBC and real-valued OSTBC, respectively; ‘PCR’ refers to the phase rotation when $\mathbf{R}_i$ is approximated by its principal component $\mathbf{h}_i^{(p)}$ and the phases of beamformers are rotated to fulfill $\mathrm{Im}\{\mathbf{W}_i^H \mathbf{h}_i^{(p)}\} = 0$ for all $i = 1, \ldots, 3$ as in Chapter 4; ‘PR’, ‘RR’ and ‘AR’ refer to the proposed phase rotation in (5.21), random phase rotation, and the phase rotation of using instantaneous CSI which is an ideal case, respectively; ‘SW’ and ‘ML’ refer to symbol-wise and ML decoder, respectively. As shown in Fig. 5.2, QOSTBC based beamforming approaches achieve much better performance than real-valued OSTBC based beamforming approaches. ‘GR (qs PR ML)’ achieves only slightly worse performance than ‘GR (qs AR SW)’ which serves as the unachievable lower bound, and is better than all other approaches. ‘GR (qs PR SW)’ achieves better performance than all other symbol-wise decoders.

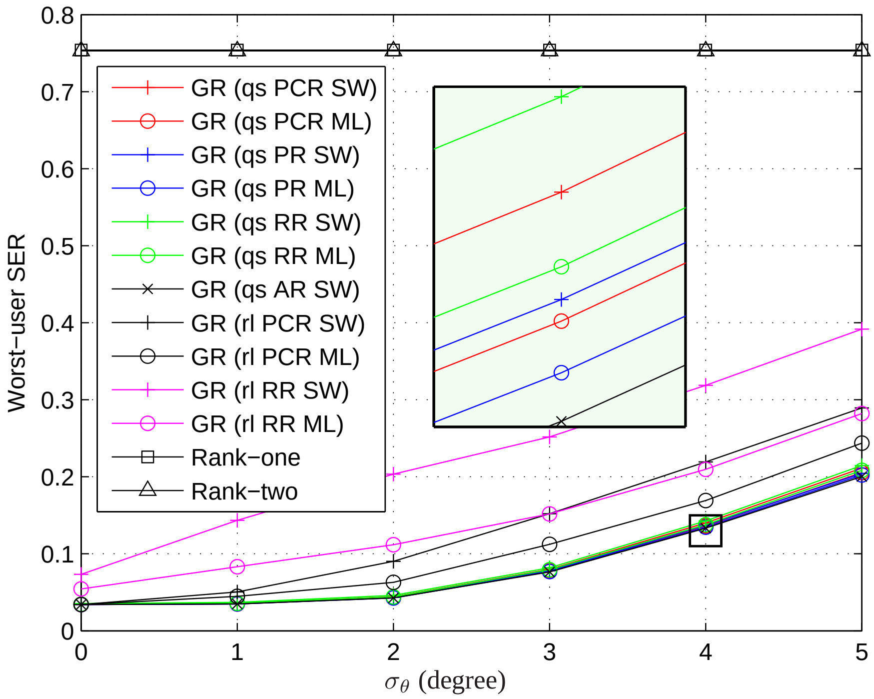

Figure 5.2: Worst-user SER versus varying spread angles

5.6 Summary

In this chapter, we propose a general rank beamforming approach for the multiuser downlink beamforming problem with additional shaping constraints exploiting covariance based CSI at the transmitter. The proposed general rank beamforming approach increases the degrees of freedom in the beamformer design by using QOSTBC. Besides the pairwise decoding for QOSTBC, a phase rotation procedure on beamformers is proposed to enable simplified symbol-wise decoding. The proposed general rank beamforming approach significantly outperforms the conventional rank-one and rank-two approaches and real-valued OSTBC based general rank approach. Similar as the general rank approach in Chapter 4, despite the signaling overhead is slightly increased, the computational complexity of the general rank approach in this chapter is lower than that of the rank-one and rank-two approaches in general.

Chapter 6

Conclusions and Outlook

Transmit beamforming is widely recognized as a promising technique to realize energy- and spectrum-efficient wireless communications. In certain transmit beamforming scenarios, the degrees of freedom in the beamformer design can be insufficient and severe performance degradation is caused due to the limited number of beamformers in the conventional rank-one transmit beamforming approach. In this dissertation, we develop higher-rank transmit beamforming approaches to address this problem by combing beamforming with different STBCs and four practical transmit beamforming problems are investigated.

The rank-two transmit beamforming approach in combination with the Alamouti code is firstly developed for single-group multicasting in Chapter 2. The proposed approach doubles the degrees of freedom in the beamformer design and offers substantially better performance than the rank-one methods. When the number of users is large, the improvement is more significant.

The rank-two transmit beamforming approach is then applied to solve the multi-group multicasting problem in Chapter 3. Besides the SDR based rank-two technique, an alternative rank-two iterative inner approximation technique is proposed as well. The proposed rank-two beamforming approaches can achieve better performance than the conventional rank-one beamforming approaches.

In single-group multicasting and multi-group multicasting, besides the Alamouti code, high dimensional OSTBCs can be used to further increase the degrees of freedom in the beamformer design. However, it is associated with the rate penalty disadvantage. The use of high dimensional real-valued OSTBC in both applications are also impractical because multiple users are served by a common beamforming matrix thus the effective channel vector of only one user can be adjusted to be real to facilitate symbol-wise decoding. The similar problem occurs in the use of QOSTBC as well. Therefore, the beamformer design with significantly increased degrees of freedom while maintaining the full-rate and simple decoding property is still an open problem for single-group multicasting and multi-group multicasting.

Besides the rank-two beamforming approaches, a general rank transmit beamforming approach is devised in Chapter 4 for the multiuser downlink beamforming problem with additional shaping constraints. The general rank beamforming approach can multiply the degrees of freedom in the beamformer design by up to eight times with the use of high dimensional full-rate real-valued OSTBC. Our proposed general rank beamforming framework exhibits an underlying optimization problem structure that is similar to that of the conventional rank-one and rank-two beamforming approaches. The extension of the proposed general rank approach to the case of imperfect instantaneous CSI is an important open research problem. Robust beamforming approaches need to be developed which can benefit greatly from the huge increase in the degrees of freedom offered by the proposed approach if the orthogonality losses of the equivalent channel matrix at the decoder due to the imperfect instantaneous CSI at the transmitter can be overcome.

Another general rank transmit beamforming approach is proposed in Chapter 5 for the multiuser downlink beamforming problem with additional shaping constraints based on covariance based CSI at the transmitter. By using QOSTBC, the general rank beamforming approach in Chapter 5 significantly increases the degrees of freedom in the beamformer design. The robust design based on imperfect covariance based CSI can be an interesting research problem.

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List of Abbreviations

1G	First generation
2G	Second generation
3G	Third generation
3GPP	Third generation partnership project
4G	Fourth generation
5G	Fifth generation
BP	Beam pattern
CCK	Complementary code keying
CDMA	Code division multiplexing access
CSI	Channel state information
eMBMS	Evolved multimedia broadcast multicast service
FDD	Frequency-division duplex
IEEE	Institute of electrical and electronics engineers
i.i.d.	Independent and identically distributed
KKT	Karush-Kuhn-Tucker
LoS	Line-of-sight
LTE	Long-term evolution
LTE-A	Long-term evolution advanced
LSA	Licensed shared access
max	Maximize
MIMO	Multiple input and multiple output
MISO	Multiple input and single output

min	Minimize
ML	Maximum likelihood
mmWave	Millimeter wave
NP hard	Non-deterministic polynomial-time hard
OFDM	Orthogonal frequency division multiplexing
OSTBC	Orthogonal space-time block coding
PEP	Pairwise error probability
PSD	Positive semidefinite
QCQP	Quadratically constrained quadratic program/programming
QoS	Quality of service
QOSTBC	Quasi-orthogonal Space-time Block Coding
QPSK	Quadrature phase-shift keying
SDMA	Space-division multiple access
SDP	Semidefinite program/programming
SDR	Semidefinite relaxation
SER	Symbol error rate
BER	Bit error rate
SINR	Signal to interference plus noise ratio
SLA	Successive linear approximation
SNR	Signal to noise ratio
SOCP	Second-order cone program/programming
s.t.	Subject to
TDD	Time-division duplex
ULA	Uniform linear array

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