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Abstract

This paper surveys the asymptotic distributions of three widely used single equation cointegration tests. Particular attention is paid to the case where the regressors are integrated with drift, i.e. at least one of the regressors follows a linear time trend. Even if the regressions are not detrended, the asymptotic critical values are affected by the presence of linear trends in the regressors. Not taking into account this fact leads to tests that are biased towards establishing cointegration too often. The correct limiting distribution theory of regressions without detrending in the presence of integrated regressors with drift is described. Appropriate critical values are readily available from the literature and are simple to use following the tables included here.

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1 Introduction

Many economic time series are not only considered as integrated of order one, I(1), but also display an approximately linear time trend (often after taking logarithms). Nevertheless, most economists prefer regressions of integrated levels of the variables without detrending, see e.g. Watson (1994, p.2895): "*In most (if not all) applications, the cointegrating vector will annihilate both the stochastic and deterministic trend [...].*" Hence, researchers typically run regressions without detrending and use critical values that are designed for regressions without detrending when it comes to cointegration testing. However, these critical values are always simulated under the assumption that the data do not follow linear time trends. Unfortunately, they are not correct if at least one of the integrated regressors has a linear trend (i.e. is integrated with drift).

Since Johansen (1994) it is well known that the presence alone of linear time trends in the data does affect the limiting distributions of multivariate cointegration tests introduced by Johansen (1988), which rely on a multiple equation reduced rank regression. Accordingly, different sets of percentiles have been published and are used depending on the presence or absence of linear trends in the series, see e.g. Osterwald-Lenum (1992) or Johansen (1995). Similarly, in case of single equation cointegration testing "*the deterministic trends in the data affect the limiting distributions of the test statistics whether or not we detrend the data*" (Hansen 1992, p.103). This fact has been established first by Hansen (1992) for the residual-based Dickey-Fuller (DF) test, and further by Hassler (2000, 2001) for the error-correction and the residual-based KPSS test. While the effect of linear trends turns out to be negligible for the residual-based DF test, this is not true for the two latter single equation procedures.

The present article should be helpful for empirical researchers by clarifying the effect of linear time trends on cointegration testing from single equations. The next section briefly states the assumptions on the integrated regressors with or without linear trends and provides some mathematical intuition for the findings presented later on. Section 3 reports the results for the residual-based DF test. The fourth section turns to the residual-based KPSS test, while Section 5 deals with the single equation error-correction test. Correct asymptotic critical values depending on the presence or absence of linear trends are available from the literature. They are collected in tables that should facilitate valid inference for applied work. The final section summarizes the general feature of cointegration testing from single equations in the presence of linear time trends in a non-technical fashion.

2 Integrated processes

Let the typical regressor be denoted by x_t. This is assumed to be a K-dimensional integrated vector process, $x_t = (x_{1,t}, \dots, x_{K,t})'$, defined by

$$x_t = x_{t-1} + m + u_t\,, \quad t = 1, \dots, T\,, \tag{1}$$

where $m = (m_1, \dots, m_K)'$ is a constant vector of length K. The vector of innovation u_t is a zero mean stationary process satisfying a functional central limit theorem for $0 \leq r \leq 1$:

$$T^{-0.5} \sum_{j=1}^{[rT]} u_j \;\Rightarrow\; B(r) \tag{2}$$

with a K-dimensional Brownian motion $B(r)$. It is assumed that the covariance matrix of $B(r)$ is positive definite, which means the process x_t alone is not cointegrated. Obviously, (1) implies by resubstitution

$$x_t = x_0 + m\,t + \sum_{i=1}^{t} u_i\,, \quad t = 1, \ldots, T\,. \tag{3}$$

If $m = (0, \ldots, 0)'$, then x_t is integrated without drift, while $m \neq (0, \ldots, 0)'$ induces a linear time trend in the process, and hence x_t is then called integrated with drift. Using the symbol $O_p(\cdot)$ for order of probability, (2) and (3) yield

$$\begin{aligned} x_t &= x_0 + m\,t + \textstyle\sum_{i=1}^{t} u_i \\ &= O(1) + O(T) + O_p(T^{0.5})\,, \qquad \text{for } m \neq (0, \ldots, 0)'\,. \end{aligned}$$

In other words, the component model (3) consists of a bounded starting value (denoted by $O(1)$), of a linear trend growing with order T (denoted by $O(T)$), and of a stochastic component of order $T^{0.5}$. Hence, the the stochastic trend is of lower order than the linear trend, and this is the reason for the results presented in the next sections. Consider first the scalar case $K = 1$: Clearly, the linear trend $m\,t$ of order T grows faster than $T^{0.5}$ and hence dominates the stochastic one. Therefore, in the long run x_t will behave like a scalar linear time trend. In case $K > 1$, there are K common stochastic trends, $\sum_{i=1}^{t} u_i$, of order $T^{0.5}$ (because there is no cointegration among the K components of the process), and **one** common linear trend, $m\,t$, of order T. Again, the linear trend will hence dominate asymptotically one of the K stochastic trends. This is the intuition, why x_t from (3) for $m \neq (0, \ldots, 0)'$ behaves in the long run like a K-dimensional vector of 1 linear trend and $K - 1$ stochastic trends. This intuition will be reflected in the results of the following three sections.

3 Residual-based Dickey-Fuller test

Engle and Granger (1987) suggested to apply the test for the null hypothesis of a unit root by Dickey and Fuller (1979) to regression residuals. The null hypothesis is that the scalar y_t and the K-vector x_t are not cointegrated,

$$y_t = a + b' \, x_t + z_t \, , \quad z_t = z_{t-1} + v_t \, , \quad t = 1, \ldots, T \, , \tag{4}$$

where z_t is a scalar integrated process driven by v_t with similar properties like u_t, see Section 2. The residuals are computed from an ordinary least squares (OLS) regression without detrending,

$$y_t = \widehat{a} + \widehat{b}' \, x_t + \widehat{z}_t \, , \quad t = 1, \ldots, T \, .$$

Then, the auxiliary DF regression is run by OLS (denoted now with tildes),

$$\Delta \widehat{z}_t = \widetilde{c} \, \widehat{z}_{t-1} + \sum_{i=1}^{p} \widetilde{d}_i \, \Delta \widehat{z}_{t-i} + \widetilde{res}_t \, ,$$

where $\Delta = 1 - L$ represents the usual difference operator, and p is chosen so that the auxiliary residuals $\widetilde{res}_t$ are approximately white noise. The test statistic is simply a studentized version of $\widetilde{c}$,

$$\tau_\mu = \frac{\widetilde{c} - 0}{s.e.(\widetilde{c})} \, .$$

The residual-based DF test is one-sided. The null hypothesis of no cointegration, $c = 0$, is rejected for too small (i.e. too negative) values.

Analogously the test may be performed with residuals from a detrended regression,

$$y_t = \widehat{\alpha} + \widehat{\beta}' \, x_t + \widehat{\delta} \, t + \widehat{\zeta}_t \, , \quad t = 1, \ldots, T \, ,$$

$$\Delta \widehat{\zeta}_t = \widetilde{\gamma} \, \widehat{\zeta}_{t-1} + \sum_{i=1}^{p} \widetilde{\delta}_i \, \Delta \widehat{\zeta}_{t-i} + \widetilde{res}_t \, ,$$

$$\tau_\tau = \frac{\widetilde{\gamma} - 0}{s.e.(\widetilde{\gamma})} \, .$$

The resulting limiting distributions are summarized next, where $\xrightarrow{d}$ is the symbol for convergence in distribution.

Proposition 1: *Under the null hypothesis (4) of no cointegration and with* $x_t = (x_{1,t}, \ldots, x_{K,t})'$ *from (1) it holds as* $T \to \infty$*:*

a) $\tau_\mu \ \xrightarrow{d}\ \mathcal{DF}_\mu(K)$ *for* $m = (0, \ldots, 0)'$,

b) $\tau_\tau \ \xrightarrow{d}\ \mathcal{DF}_\tau(K)$ *for any* m,

c) $\tau_\mu \ \xrightarrow{d}\ \mathcal{DF}_\tau(K-1)$ *for* $m \neq (0, \ldots, 0)'$.

Here, $\mathcal{DF}_\tau(K)$ and $\mathcal{DF}_\mu(K)$ stand for the limiting distributions given by Phillips and Ouliaris (1990) in terms of K-dimensional, detrended and demeaned standard Brownian motions, respectively. The result c) is due to Hansen (1992) and reproduces the intuition provided at the end of Section 2: If at least one of the K integrated regressors displays a linear trend and the regression is run without detrending, then the asymptotic theory of a detrended regression with $K-1$ integrated regressors applies. If in particular $K = 1$ in c), then $\mathcal{DF}_\tau(0)$ abbreviates the limiting distribution of the detrended DF test applied not to residuals but to directly observed series, cf. Dickey and Fuller (1979) and Phillips and Perron (1988),

$$\Delta y_t = \widetilde{\alpha} + \widetilde{\delta}\, t + \widetilde{\gamma}\, y_{t-1} + \sum_{i=1}^{p} \widetilde{\delta}_i\, \Delta y_{t-i} + \widetilde{res}_t \, ,$$

$$\frac{\widetilde{\gamma} - 0}{s.e.(\widetilde{\gamma})} \ \xrightarrow{d}\ \mathcal{DF}_\tau(0) \, .$$

Table 1: Comparison of 10% critical values from Proposition 1

K	1	2	3	4	5
$\mathcal{DF}_\mu(K)$	-3.0462	-3.4518	-3.8110	-4.1327	-4.4242
$\mathcal{DF}_\tau(K)$	-3.4959	-3.8344	-4.1474	-4.4345	-4.6999
$\mathcal{DF}_\tau(K-1)$	-3.1279	-3.4959	-3.8344	-4.1474	-4.4345

It is interesting to compare the different asymptotic critical values. The ones used most often in practice are those by MacKinnon (1991), which are reproduced for the 10% level in Table 1. From this table we observe that

i) $\mathcal{DF}_\tau(K)$ is shifted to the left relative to $\mathcal{DF}_\mu(K)$,

ii) $\mathcal{DF}_\tau(K-1)$ is shifted slightly to the left relative to $\mathcal{DF}_\mu(K)$,

iii) the difference between $\mathcal{DF}_\tau(K-1)$ and $\mathcal{DF}_\mu(K)$ is small and decreases as K grows.

Observations ii) and iii) imply together with Proposition 1: If there are linear trends in our regressors but regressions are performed without detrending, then the standard use of $\mathcal{DF}_\mu(K)$ instead of the correct $\mathcal{DF}_\tau(K-1)$ distribution results in rejecting the null hypothesis of no cointegration too often; but this overrejection is very small and may be considered as negligible in practice. Unfortunately, for other single equation cointegration tests this is not true.

4 Residual-based KPSS test

Kwiatkowski, Phillips, Schmidt and Shin (1992) proposed as a counterpart to the DF procedure a test for the null hypothesis of stationary time series against the alternative of integratedness. Applied to regression residuals this KPSS test has the null hypothesis that y_t and the K-vector x_t are cointegrated,

$$y_t = a + b'\, x_t + e_t\,, \quad t = 1, \ldots, T\,, \tag{5}$$

where e_t is again a scalar stationary process with zero mean satisfying a functional central limit theorem. The idea of a residual-based KPSS cointegration test was suggested independently by Harris and Inder (1994), Leybourne and McCabe (1994) and most rigorously by Shin (1994), see also McCabe, Leybourne and Shin (1997). In particular, Shin (1994) stresses that the KPSS test applied to OLS residuals will not in general lead to limiting distributions independent of nuisance parameters. To guarantee limits free of nuisance parameters, so-called efficient modifications of OLS are required, e.g. fully modified OLS by Phillips and Hansen (1990) and Hansen (1992), or dynamic OLS by Saikkonen (1991) and Stock and Watson (1993), or the canonical cointegrating regression by Park (1992); technical details are omitted here.

Let tildes signify any of those (or other) estimators that are efficient in the sense of Saikkonen (1991). The regression without detrending is given by

$$y_t = \widetilde{a} + \widetilde{b}'\, x_t + \widetilde{e}_t\,, \quad t = 1, \ldots, T\,.$$

The KPSS test builds on cumulation of the squared partial sum of residuals. The test statistic is

$$\eta_\mu = \frac{T^{-2}}{\widehat{\omega}_e^2} \sum_{t=1}^{T} \left(\sum_{i=1}^{t} \widetilde{e}_i \right)^2 \,,$$

where $\widehat{\omega}_e^2$ is a consistent spectral estimator of the long-run variance of e_t,

$$\omega_e^2 = \sum_{j=-\infty}^{\infty} E\left(e_t\, e_{t+j}\right)\,.$$

Most often this long-run variance is estimated by means of the simple Bartlett window popularized by Newey and West (1987) in econometrics; again technical details are of no interest here. The null hypothesis of cointegration is rejected for too large values of η_μ.

A detrended regression results in a test statistic η_τ of completely analogous structure, only that it builds on detrended efficient residuals:

$$y_t = \widetilde{\alpha} + \widetilde{\beta}'\, x_t + \widetilde{\delta}\, t + \widetilde{\varepsilon}_t\,, \quad t = 1, \ldots, T\,,$$

$$\eta_\tau = \frac{T^{-2}}{\widehat{\omega}_\varepsilon^2} \sum_{t=1}^{T} \left(\sum_{i=1}^{t} \widetilde{\varepsilon}_i \right)^2 .$$

The resulting limiting distributions are summarized next.

Proposition 2: *Under the null hypothesis (5) of cointegration and with* $x_t = (x_{1,t}, \ldots, x_{K,t})'$ *from (1) it holds as* $T \to \infty$*:*

a) $\eta_\mu \overset{d}{\to} \mathcal{KPSS}_\mu(K)$ *for* $m = (0, \ldots, 0)'$,

b) $\eta_\tau \overset{d}{\to} \mathcal{KPSS}_\tau(K)$ *for any* m,

c) $\eta_\mu \overset{d}{\to} \mathcal{KPSS}_\tau(K-1)$ *for* $m \neq (0, \ldots, 0)'$.

Here, $\mathcal{KPSS}_\tau(K)$ and $\mathcal{KPSS}_\mu(K)$ abbreviate the asymptotic distributions derived by Shin (1994) in terms of K-dimensional standard Brownian motions. The result c) has recently been given in Hassler (2001) and can be restated in simple words as follows: If at least one of the K integrated regressors displays a linear trend but the regression is computed without detrending, then the limiting distribution of a detrended regression with only $K-1$ integrated regressors arises. Similarly as in Proposition 1: If $K = 1$ in c), then $\mathcal{KPSS}_\tau(0)$ denotes the limiting distribution of the detrended KPSS

Table 2: Comparison of 10% critical values from Proposition 2

K	1	2	3	4	5
$\mathcal{KPSS}_\mu(K)$	0.231	0.163	0.121	0.094	0.075
$\mathcal{KPSS}_\tau(K)$	0.097	0.081	0.069	0.056	0.050
$\mathcal{KPSS}_\tau(K-1)$	0.119	0.097	0.081	0.069	0.056

test applied to an observed series y_t instead of to residuals, see Kwiatkowski et al. (1992),

$$y_t = \widetilde{\alpha} + \widetilde{\delta}\, t + \widetilde{y}_t\,, \quad t = 1, \ldots, T\,,$$

$$\frac{T^{-2}}{\widehat{\omega}_y^2} \sum_{t=1}^{T} \left(\sum_{i=1}^{t} \widetilde{y}_i \right)^2 \overset{d}{\to} \mathcal{KPSS}_\tau(0)\,.$$

Table 2 compares asymptotic percentiles at the 10% level taken from Shin (1994), and from Kwiatkowski et al. (1992) for $\mathcal{KPSS}_\tau(0)$. The most striking observation is: The critical values of $\mathcal{KPSS}_\mu(K)$ are considerably larger than those of $\mathcal{KPSS}_\tau(K-1)$ (e.g. twice as big for $K = 1$). Hence, Proposition 2 has a drastic consequence. If there is a linear trend in any of the regressors but efficient regressions are run without detrending (i.e. η_μ is computed), then the common use of $\mathcal{KPSS}_\mu(K)$ will reject the null hypothesis of cointegration far less often than the use of the correct distribution $\mathcal{KPSS}_\tau(K-1)$. In other words, η_μ applied with $\mathcal{KPSS}_\mu(K)$ instead of $\mathcal{KPSS}_\tau(K-1)$ in the presence of linear trends results in a conservative test with little power confirming the null of cointegration too often.

5 Error-correction test

The last single equation cointegration test considered here was proposed by Banerjee, Dolado and Mestre (1998). It has the null hypothesis (4) that the scalar y_t and x_t are not cointegrated. But the test does not rely on residuals from a static regression but rather on the fact that cointegration is equivalent to the existence of an error-correction mechanism,

$$\Delta y_t = a + c\,(y_{t-1} - \lambda' x_{t-1}) + b'\,\Delta x_t + \text{ lagged differences } + e_t\,,$$

where lagged differences may be included to obtain errors e_t free of serial correlation. The test assumes exogeneity of x_t in the sense that in case of cointegration, Δx_t does not adjust to past equilibrium deviations. Under this assumption, if y_t and the vector x_t of length K are cointegrated with vector λ, then the adjustment parameter c is negative, $c < 0$. Or the other way round, if $c = 0$ in this error-correction equation, then y_t and x_t are not cointegrated.

This reasoning justifies the following OLS regression without detrending (where $\pi = -c\,\lambda$):

$$\Delta y_t = \widehat{a} + \widehat{c}\,y_{t-1} + \widehat{\pi}'\,x_{t-1} + \widehat{b}'\,\Delta x_t^{(k)} + \text{ lagged differences } + \widehat{e}_t\,.$$

The test statistic is the usual t ratio testing for $c = 0$,

$$EC_\mu = t_c = \frac{\widehat{c} - 0}{s.e.(\widehat{c})}\,.$$

Under appropriate assumptions on the exogeneity of x_t the limiting distribution is free of nuisance parameters but of course not Gaussian. The null hypothesis of no cointegration is rejected for too negative values of EC_μ. A detrended test relies simply on a detrended regression:

$$\Delta y_t = \widehat{\alpha} + \widehat{\delta}\,t + \widehat{\gamma}\,y_{t-1} + \widehat{\phi}'\,x_{t-1} + \widehat{\beta}'\,\Delta x_t + \text{ lagged differences } + \widehat{\varepsilon}_t\,,$$

$$EC_\tau = t_\gamma = \frac{\widehat{\gamma} - 0}{s.e.(\widehat{\gamma})}\,.$$

The corresponding asymptotic theory is characterized next.

Proposition 3: *Under the null hypothesis (4) of no cointegration and with $x_t = (x_{1,t}, \ldots, x_{K,t})'$ from (1) it holds as $T \to \infty$:*

a) $EC_\mu \overset{d}{\to} \mathcal{BDM}_\mu(K)$ *for* $m = (0, \ldots, 0)'$,

b) $EC_\tau \overset{d}{\to} \mathcal{BDM}_\tau(K)$ *for any* m,

c) $EC_\mu \overset{d}{\to} \mathcal{BDM}_\tau(K-1)$ *for* $m \neq (0, \ldots, 0)'$ *where* $\mathcal{BDM}_\tau(0) = \mathcal{DF}_\tau(0)$.

Obviously, $\mathcal{BDM}_\tau(K)$ and $\mathcal{BDM}_\mu(K)$ denote the limiting distributions depending only on K that are given by Banerjee, Dolado and Mestre (1998) in case of (not) detrending. Result c) was recently proven in Hassler (2000), where $\mathcal{DF}_\tau(0)$ is described after Proposition 1. The meaning of result c) is again: If at least one of the K integrated variables x_t follows a linear trend but the error-correction regression is run without detrending, then the limit theory of a detrended error-correction regression with only $K-1$ integrated variables applies; if in particular $K = 1$, then $\mathcal{BDM}_\tau(0)$ reduces to the limiting distribution of the usual detrended DF test.

Finally, Table 3 contains asymptotic critical values at the 10% level taken from Banerjee, Dolado and Mestre (1998) (and from MacKinnon (1991) for $\mathcal{BDM}_\tau(0) = \mathcal{DF}_\tau(0)$). The interesting observation is: The critical values of $\mathcal{BDM}_\mu(K)$ are clearly larger than those of $\mathcal{BDM}_\tau(K-1)$ (the difference being roughly 0.2 for all K). As we reject for too small values this means:

Table 3: Comparison of 10% critical values from Proposition 3

K	1	2	3	4	5
$\mathcal{BDM}_\mu(K)$	-2.89	-3.19	-3.42	-3.66	-3.82
$\mathcal{BDM}_\tau(K)$	-3.39	-3.62	-3.82	-4.00	-4.18
$\mathcal{BDM}_\tau(K-1)$	-3.13	-3.39	-3.62	-3.82	-4.00

If there is a linear trend in any component of $x_t = (x_{1,t}, \dots, x_{K,t})'$ but error-correction regressions are computed without detrending, then the usual reliance on $\mathcal{BDM}_\mu(K)$ will reject the null hypothesis of no cointegration more often than the claimed level of significance The use of $\mathcal{BDM}_\tau(K-1)$ provides correct significance levels asymptotically. Put differently, EC_μ applied with $\mathcal{BDM}_\mu(K)$ instead of $\mathcal{BDM}_\tau(K-1)$ in the presence of linear trends results in a oversized test that does not control the probability of a type I error and establishes cointegration too often.

6 Summary

In this paper we survey the limiting distributions of three widely used single equation cointegration tests in the presence of linear time trends: the residual-based Dickey-Fuller test, the residual-based KPSS test, and the error-correction test. Those tests try to discriminate between the absence or presence of cointegration between a scalar time series y_t and a K-dimensional vector x_t, where the latter is not cointegrated by assumption. Particular attention is paid to the case where the regressor x_t is integrated with drift, i.e. at least one of the K components of $x_t = (x_{1,t}, \dots, x_{K,t})'$ follows a linear trend

The following properties hold true for all three tests:

(i) If the regressions include time as a linear regressor (detrended regressions), then detrended asymptotic distributions (denoted by subscript τ) depending on K arise, no matter whether x_t has a drift or not.

(ii) If the regressions do not include time as a linear regressor (regressions without detrending) ...,

a) ... **and** if x_t does not contain a drift, then demeaned asymptotic distributions (denoted by subscript μ) depending on K arise,

b) ... **but** if the K-vector x_t does contain a drift, then detrended asymptotic distributions from (i) for $K - 1$ arise (i.e. critical values from a) are inappropriate),

c) ... but if x_t does contain a drift **and** the wrong critical values from a) are applied, then the tests are biased towards finding cointegration.

The tables included in the paper should make it simple for empirical workers to apply the correct critical values in case (ii) of not detrending.

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