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Abstract

Guided by game theory we develop a model to explain behavioral equilibria under uncertainty and interaction with the spot market on balancing markets. We offer some insights for the general model and derive explicit solutions for a specific model in which the error distributions and pricing function are given. The most interesting conclusions are the unique existence of an equilibrium and that no participant acts contrary to the aggregate market (either all market participants buy or sell power) and all strategies are, normalized properly, equal (which is rather counter-intuitive). Furthermore the aggregate behavior is a stochastic process varying around its own variance.

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1 Introduction

Industrialized countries need steady and reliable electricity supply. Modeling the electricity market is typically analyzed with either Supply-Function-Equilibria (for examples refer to Klemperer and Meyer (1989), Green and Newbery (1992), Green (1996), Weber and Overbye (1999) or Berry et al. (1999a)), with Cournot-models (e.g. Cardell et al. (1997), Hogan (1997), Hobbs et al. (2000), Boisseleau et al. (2004)) or Stackelberg-models (e.g. Wolf and Smeers (1997) or Chen et al. (2004)). The former is a better fit to the technical realization of many electricity markets where bids are given as supply or cost functions.
The electricity market is a forward market with a single not-storeable good (see Allaz (1992), Allaz and Vila (1993) or Green and Newbery (1992)). The game-theoretic literature usually deals with a two-stage approach with power generators choosing their investment (or, in fact, their capacity) in the first stage and the price in the second (von der Fehr and Harbord (1997), Coq (2002) and Sanin (2005)). Introductions to energy market models can be found in Meibom et al. (2003), Boisseleau et al. (2004), Yao et al. (2004) and Sanin (2005).
We use a simultaneous one-stage approach which is fundamentally different from the usual models and is solely based on the uncertainty of the demand for power and the price on the spot market. This Cournot-Model consists of two phases:

1. In the first phase all participants (henceforth called players) buy and sell energy on the spot market (or wholesale market) at a given price. To simplify analysis we assume that the price is public knowledge.

2. The players cannot act in the second phase. The actual aggregate balance of power is determined as well as the power balance for every player. A transmission system operator (TSO) ensures that demand and supply are in balance. Depending on the aggregate power balance the TSO prices power with a high price if the aggregate market is short on power or a low price otherwise.

   Every player who is short on power is charged with the fixed price (by the TSO) and all players who bought too much power are compensated. This is the balancing power market (or regulating power market).

Three questions immediately arise.

- How much power should each player buy on the spot market?

- How can the TSO regulate the players (which price should be set to suppress strategic power trading)?
- Is there an (unique) equilibrium solution at all?

Regarding the last question Sanin (2005) find such an equilibrium but she used a quite different set of assumptions. In our model this is also true for a realistic case but must not hold in more general cases since the functions of the expected utilities are not convex.
It is not trivial at all that an equilibrium should exist anyway since every player has an incentive to behave contrary to the aggregate market. The price of power will be low if the aggregate market buys too much, and it is therefore better to buy less than the expected demand since the charged price will be low. Similarly there is an incentive to buy too much power if the aggregate market will be low on power, since the compensating price will be high. This suggests that there should be either no nash equilibrium besides buying the expected amount of power or the decision to buy more/less than the expected demand should depend on the variance of the actual demand[1]. It turns out that neither must be the case.
In the case that the expected price on the balancing market equals the spot price, intuition suggests that the power bought by all players should be equal to the expected demand. It turns out that this depends (at least) on the price function of the TSO. Moreover, under reasonable assumptions, the real price on the balancing power market does not matter - only the difference between the minimum and maximum price is relevant.

1.1 Distinction from related literature

We explicitly neglect several characteristics which are commonly part of the widely used models. We do not consider the (double)-auction mechanisms (e.g. Wolfram (1998), Supatgiat et al. (2001), Wolak (2000), Baldick et al. (2004) or Boisseleau et al. (2004)) to calculate prices on either the spot- or the balancing market. We are also not concerned with the technical distribution of power (e.g. Berry et al. (1999b), Kleindorfer et al. (2000) or Willems (2006)) or the behavior of the power generators and therefore neglect any constraints given by

[1]The reasoning could be that players with high variance have an incentive to buy larger amounts of strategic power to insure them against unexpected high prices; this could be exploited, ceteris paribus, by players with low variance.

the generators (as in Berry et al. (1999b), Supatgiat et al. (2001), Yao et al. (2004), Sweeting (2004) or Holmberg (2004)). These are no serious limitations since the price on the balancing market can vary in every round, as long as everything is public knowledge. Additionally we assume that buying strategic power (which means buying intentionally more or less than the expected demand) is not punished. In Germany this is forbidden by law (see for example RWE (2006)) but there has been no actual prosecution in Germany[2] although such behavior is obvious (Rupp, 2003).
The model we present provides a new, straight forward and easy way to verify whether an energy market has reached its equilibrium state and to understand why the aggregate power on the balancing market does not necessarily vary around zero. Additionally we offer a convenient way to see how the prices on the balancing market influence the market behavior. We start with a general setting with risk-neutral players and than proceed to normal distributed forecasting errors and a specific pricing function[3] for the balancing market.

The paper is organized in the following way: section 2 discusses the model, section 3 includes some results of the most general case when (almost) no assumptions are made about the model parameters, while section 4 and 5 derive the important results for models with normal errors and two specific pricing functions. Section 6 concludes the paper with a short discussion. The bulk of the appendix contains the long proofs of some propositions in section 4 and 5.

2 The Model

Basically the model consists of the stochastic error of each player (difference of predicted and actual power demand of all respective end customers), the strategically bought power on the spot market by each player and the pricing function of the TSO.

- $\mathfrak{X} := (X_1, \dots, X_n)$ is a $\mathbb{R}^n-$valued random variable, whereas X_i is the power demand (of all end customers) of player i

[2] Private correspondence with a manager of a large energy company.

[3] The binary function fits the German balancing market reasonably well. For more information about the German energy market refer to Swider and Weber (2000), Rupp (2003), Meibom et al. (2003) or RWE (2006) for a specific example.

  - $\sigma_i^2 := \text{Var}(X_i), 0 \leq \sigma_i^2 < \infty$
  - $\mu := (\mu_1, \ldots, \mu_n) \in \mathbb{R}^n$ and $\mu_i := \mathbb{E}[X_i]$
  - $X_1, X_2, \ldots, X_n$ are mutually independent and have a density

- $\mathfrak{v} := (v_1, \ldots, v_n) \in \mathbb{R}^n$, whereas v_i is the amount of power strategically bought by player i
- p_s is the price paid on the spot market
- $M = M(\mathfrak{X}, \mu, \mathfrak{v}) := \sum_{i=1}^{n}(X_i - (\mu_i + v_i))$ is the aggregate unsatisfied demand for power
- $p_b = f(M)$ is the price on the balancing market fixed by the TSO, determined with the pricing function $f(M)$ which fulfills the following conditions:
  - $\exists \overline{M} \geq 0 : M \geq \overline{M} \Leftrightarrow f(M) \geq p_s$
  - f is monotonically increasing, non-negative and bounded
  - f is continuous in sections
- $G_i(v_i) = G_i(\mathfrak{X}, \mu, \mathfrak{v}) := (\mu_i + v_i - X_i)f(M) - v_i p_b$ is the monetary gain of player i

All players are fully informed (they know all parameters and functions), all act independently and simultaneously in every round and all players maximize their expected profit. Thus we have to deal with a typical open ended game theoretic problem under full information.
Given all information we have to find all vectors $\mathfrak{v}^*$ which satisfy

$$\mathbb{E}[G_i(\mathfrak{v}^*)] \geq \mathbb{E}[G_i(v_i | v_j = v_j^* \quad \forall j \neq i)] \quad i = 1 \ldots n. \tag{1}$$

Without loss of generality we can set $\mu = 0$ since we can otherwise define $\mathfrak{X}' := \mathfrak{X} - \mu$ and $\mathfrak{v}' := \mathfrak{v} - \mu$. Then $(\mathfrak{v} - \mathfrak{X})$ is distributed as $(\mathfrak{v}' - \mathfrak{X}')$ - the expected demand would simply be adjusted by the strategic amount of power.
Since the global condition given by equation (1) is hard to analyze we restrict ourselves to the commonly used local equilibrium conditions: the first derivative of $\mathbb{E}[G_i(\mathfrak{v})]$ must be zero for all i:

$$\begin{aligned} \frac{\partial}{\partial v_i}\mathbb{E}[G_i(\mathfrak{v})] &= 0 \\ \Leftrightarrow \quad \mathbb{E}\left[(v_i - X_i)\frac{\partial}{\partial v_i} f(M)\right] + \mathbb{E}[f(M)] - p_s &= 0 \\ \Leftrightarrow \quad \mathbb{E}\left[(v_i - X_i)\frac{\partial}{\partial M} f(M)\right] &= \mathbb{E}[f(M)] - p_s. \end{aligned}$$

We can rewrite these equations as

$$\mathbb{E}\left[(v_i - X_i)f'(M)\right] = \mathbb{E}[f(M)] - p_s, \tag{2}$$

$$\mathbb{E}\left[(v_i - X_i)f'(M)\right] = \mathbb{E}\left[(v_j - X_j)f'(M)\right] \qquad \forall i, j. \tag{3}$$

These local conditions imply an equilibrium only for marginal deviations. In the case of non-convex G_i's there might be multiple local equilibria which might be global maxima only for different subsets of players. Henceforth we always refer to local equilibria, unless stated otherwise.

2.1 Intuition

There are several basic concepts which seem to be easily applicable to the model but eventually are not - at least not in the more specific models which are accessible to further analysis.

2.1.1 Existence of equilibria

As already mentioned in the introduction, intuition suggests that there should be either no equilibrium at all (except $\mathfrak{v} = 0$) or the signs of the v_i should depend on σ_i^2.

A player should, since he knows the expected amount of power of the aggregate market when all other players strategies are fixed, do the opposite as the aggregate market does: buy more power if the aggregate market is expected to be low on power and buy less otherwise. Only in the case that the aggregate market balance is expected to be zero and the expected price equals the price on the spot market, $\mathfrak{v} = 0$, should be a nash equilibrium. While the latter property holds in the specific models, the former does not.

It would also be intuitive if the sign of each individual would at least depend on the variances of the X_i. Players with large variances should be more concerned to drive the market in one direction to avoid, for example, being charged with high prices on the balancing power market (when the spot price is comparably low). This should be exploitable by players with low variances.

It turns out in the specific models that the strategies of all players share the same sign and, when normalized, are essentially all equal.

2.1.2 Necessary Condition

The necessary conditions expressed in equation (2) imply that every player buys such an amount of energy that the marginal benefit of selling power on the balancing market (RHS) equals the expected reduction of the price (LHS) generated by the marginal increase of power. At first glance it is tempting to assume that $p_s = \mathbb{E}[p_b]$ is a necessary condition for equation (1) to hold[4] - if the price on the spot market equals the expected price on the balancing market there should be no incentive for any player to change his strategy given the strategy of all others.

In the case of $p_s < \mathbb{E}[p_b]$ power on the spot market could be bought at a lower price than the expected price on the balancing power market. Therefore there should be at least one player with an incentive to increase his strategic amount of power since he could sell this marginal increase at an expectedly higher price. In the case of $p_s > \mathbb{E}[p_b]$ there should be at least one player decreasing his strategic amount of power (e.g. selling power on the spot market) in order to regain it at an expectedly lower price on the balancing power market.
It turns out that this intuition must not hold in all cases.

3 The general model

In the most general form of the model we derive only a few results.

Proposition 3.1 *When $\mathfrak{v}^*$ fulfills equation (1) and $X_{n+1} = 0$ a.s., then*

$$\mathbb{E}[f(M)] = p_s \Leftrightarrow (v_1^*, \dots, v_n^*, 0) \text{ also fulfills equation (1) for } n+1.$$

Proof
Remember: $M = \sum_{i=1}^{n}(X_i - v_i)$.
$\Rightarrow$:
$\mathbb{E}[f(M)] = p_s$. Since $\mathbb{E}[f(M - v_{n+1})]$ is monotone in v_{n+1}, for all $w \in \mathbb{R}, w \neq 0$ holds

$$G_{n+1}((v_1^*, \dots, v_n^*, w)) = w(\mathbb{E}[f(M - w)] - p_s) < 0 = G_{n+1}((v_1^*, \dots, v_n^*, 0)).$$

[4]This is a condition concerning the aggregate market and tells us nothing about the individual strategies.

$\Leftarrow$:

Assume $\mathbb{E}[f(M)] \neq p_s$. Since $\mathbb{E}[f(M - v_{n+1})]$ is affine linear in v_{n+1}, there exists a $w \in \mathbb{R}, w \neq 0$ which fulfills

$$G_{n+1}((v_1^*, \ldots, v_n^*, w)) = w(\mathbb{E}[f(M - w)] - p_s) > 0 = G_{n+1}((v_1^*, \ldots, v_n^*, 0)).$$

Therefore $(v_1^*, \ldots, v_n^*, w)$ does not fulfill equation (1) for any $w \neq 0$, whereas $w = 0$ removes player $n + 1$ from the market system.

□

Definition 3.2 (Closed market) *We call a market closed when there is no incentive for other players (who are not forced to participate in the market, e.g. players with zero variance) to join the market*[5]*.*

It sounds reasonable that the market should always be closed since otherwise (up to to an infinite number of) players could join the system. The market is closed anyway when either $\mathfrak{v}^* = 0$ or $G_i(\mathfrak{v}^*) \leq 0, \forall i > n$.

Corollary 3.3 *When the expected price on the balancing power market equals the spot price the former market is closed:* $\mathbb{E}[f(M)] = p_s \Rightarrow$ *the balancing power market is closed.*

4 The specific model

To derive specific results analytically we introduce some further assumptions. We begin with a more general class of pricing function f to subsequently derive afterwards specific results with a binary f. We assume furthermore that all X_i are normally distributed.

[5]Unlike in other markets the TSO must always, at least in theory, satisfy every demand on the balancing market.

4.1 f as the error-function

Let f be given by

$$f(M) = f(M, a, p_h, p_l) \quad := \quad \frac{(p_h - p_l)}{2}(\mathrm{erf}(aM) + 1) + p_l. \tag{4}$$

The error-function $\mathrm{erf}(x)$ is by definition $\frac{2}{\sqrt{\pi}} \int\limits_0^x e^{-t^2} dt$. The parameter a controls the slope around $M = 0$. More details are given in the appendix A.1.
To simplify notation we define

$$V := \sum_{k=1}^{n} v_k \text{ and } S^2 := \sum_{k=1}^{n} \sigma_k^2 \text{ as well as } V_i := V - v_i \text{ and } S_i^2 := S^2 - \sigma_i^2.$$

Thus we can write $\mathbb{E}[M] = -V$ and $\mathrm{Var}(M) = S^2$, hence

$$M \sim \mathcal{N}(-V, S^2).$$

4.2 Equilibria

Necessary conditions for the nash equilibrium (equation (1)) to hold for a $\mathfrak{v}^*$ are that the first derivatives of the G_i are zero and the second derivatives are negative. It is then sufficient to show that the local maximum attained is the global maximum for each player and the relationship between all v_i^* is unique. In the specific model we derive the necessary conditions which are sufficient for local equilibria.

Theorem 4.1 *Let all X_i be normally distributed and f as given in equation (4). The (not necessarily unique) local nash equilibrium, given by equations (2) and (3), is determined by the following conditions*

$$\begin{aligned} v_j^* &= \frac{2na^2\sigma_j^2 + 1}{2na^2\sigma_1^2 + 1} v_1^* \quad , j = 2, \ldots, n, \\ \mathbb{E}[f(M^*)] - p_s &= \mathbb{E}\left[(v_1^* - X_1) f'(M^*)\right] \end{aligned}$$

There are either one or two local equilibria.

The proof is given in appendix A.1. The theorem establishes the conditions for local equilibriums. In such a situation no player will deviate from his current strategy by any small amount. However, we emphasize that the possibility that a player may achieve a higher benefit, if he deviates by a large amount, is not ruled out in general.

Corollary 4.1 *Some interesting properties immediately follow from the theorem for local equilibria:*

- *All v_i^* have the same sign.*
- $\frac{v_j^*}{2na^2\sigma_j^2+1} = \frac{v_i^*}{2na^2\sigma_i^2+1}$. *Properly normalized, all v_i^* are equal.*
- *Either all v_i^* are zero or none.*

Whether the market system is closed is not obvious. The derived properties are not sufficient to answer this question. However it would not be unreasonable to suppose that the expected benefits are negative in the local equilibriums which would be sufficient for a closed market.

Corollary 4.2 *For constant a we have*

$$\mathfrak{v}^* = \overrightarrow{0} \Leftrightarrow \mathbb{E}[f(M)] = p_s = \frac{p_h + p_l}{2}.$$

This is also true for $n \to \infty$.

Proof

With the optimal $\mathfrak{v}^*$ the left side of equation (2) is without loss of generality

$$\frac{(p_h - p_l)v_1 a(4n^2a^2\sigma_1^2 + 1)}{\sqrt{2a^2S^2+1}\sqrt{\pi}(2na^2\sigma_1^2+1)} e^{-\frac{a^2n^2(2a^2S^2+1)}{2na^2\sigma_1^2+1}v_1^2}.$$

Therefore the left side is zero iff $\mathfrak{v}^* = 0$ for every fixed n as well as for $n \to \infty$. Thus $\mathfrak{v}^* = 0$ implies $\mathbb{E}[f(M)] = p_s$ while $\mathbb{E}[f(M)]$ evaluates to $\frac{p_h+p_l}{2}$.

$\mathbb{E}[f(M)]$ is a strictly monotonic continuous function and therefore a unique v_1 exists such that $\mathbb{E}[f(M)] = p_s$. Since on the one hand $V^* = 0 \Rightarrow \mathbb{E}[f(M)] = \frac{p_h+p_l}{2}$ and, additionally, all v_i^* share the same sign, all v_i^* must be zero.

□

Example

Let us assume that we have 10 identical players with $\sigma^2 = 1, a = 1, p_h = 1, p_l = 0$ and $p_s = 0.75$. Obviously, $v_i^* = v_j^* \quad \forall i, j$. Since the price on the spotmarket is high compared to the neutral expected price $p_b = 0.5$, we expect any equilibrium v_i to be negative. The second condition of theorem 4.1 is satisfied for $v_i \approx -0.21751$. Since the analytical form is not easily accessible for evaluation (neither exact nor numerical), we performed some simulations which indicate that this $\mathfrak{v}^*$ is the global maximum for all players and the expected benefits are negative. Thus this seems to be a true nash equilibrium as defined by equation (1) and is minimizing the losses of all players. Since there is no expected benefit for any additional player (the new equilibrium for $n = 11, \sigma_{11} = 0$ also yields negative benefits) the market is closed.

■

5 The binary model

A specific sub-model of the model presented in section 4, which can be analyzed in more detail, is attained by taking the limits $a \to \infty$, which transforms the pricing function into an indicator function

$$f(M) = \begin{cases} p_h & \text{if } M \geq 0 \\ p_l & \text{otherwise.} \end{cases} \tag{5}$$

All previous results can now be easily reevaluated - analyzing G in the binary model beforehand had been impractical. A binary pricing function is not a bad approximation, as can be seen in RWE (2006) or Rupp (2003), in the the case of the RWE-balancing market in Germany.

Theorem 5.1 *Let all X_i be normally distributed, f be given as in equation (5) and Φ^{-1} be the inverse of the normal distribution function. The unique global nash equilibrium, given by equation (1), is determined by*

$$v_i^* = \frac{\sigma_i^2}{S}\Phi^{-1}\left(\frac{p_h - p_s}{p_h - p_l}\right). \tag{6}$$

The proof is given in appendix A.3.

Corollary 5.1 (The market process) *Let $\lambda := \Phi^{-1}([p_h - p_z]/[p_h - p_n])$ and $X = \sum_{i=1}^n X_i$ then summation over all players leads to the following process characterization:*

$$\boxed{M = -\lambda\sqrt{Var(X)} + X.}$$

Thus the demand of power on the balancing market is a stochastic process driven by two different components: the variance of the actual demand of all players, the distance from the spotmarket price to the upper price cap and the price bandwith on the balancing market.
When the prices do not vary (i.e. λ is a constant) the process M solely varies around its own variance (with the mean $-\lambda\sqrt{\text{Var}(X)}$ and the noise term X). Thus, given a negative λ, the process is driven upwards (downwards) if the variance of X increases (decreases). Naturally the variance of M is the same as the variance of X. This is a curious result because such processes are not commonly found in practice.

In contrast to the previous setting, the following corollary is proven to hold:

Corollary 5.2 $\mathbb{E}[f(M)] = p_s$ *and therefore the market system is closed.*

The proof is part of the proof of theorem 5.1. The corollary from the previous section can be transferred one-to-one.

Corollary 5.3

- *All* v_i^* *have the same sign.*
- $\frac{v_j^*}{\sigma_j^2} = \frac{v_i^*}{\sigma_i^2}$. *Properly normalized all* v_i^* *are equal.*
- *Either all* v_i^* *are zero or none.*

The binary pricing function also implies

Corollary 5.4 *Only the span between the high and low price cap is relevant - the absolute prices do not matter. Without loss of generality the TSO can set* $p_l = 0$ *and influence the market by varying the high price* p_h.

6 Conclusion

Motivated by the balancing power market system we have developed a one-stage game-theoretic model to describe optimal behavior of rational players on balancing markets. In such a system players buy power on a spot market and then the unsatisfied power demand is traded automatically on the balancing power market at a given price (depending on the aggregate demand of power of all customers).

We explicitly incorporated the uncertainty of the players in regard to the power demand of their customers and the price on the spot market as the driving forces to determine the optimal strategy of each individual player. Besides some general results, we derive an explicit characterization for the unique global nash equilibrium and the properties of the aggregate market in the case of a binary pricing function.

We emphasize three important properties of the binary case: in equilibrium all players follow the same strategy (discerned only by the variances of their forecast error), the expected price on the balancing market will be the same as on the spot market (a different approach by Sanin (2005) also leads to identical prices on both markets) and the behavior of the aggregate market is a stochastic process varying about its own variance.

The existence of such an equilibrium is not trivial. On the one hand it may depend, in theory, on the specific model (a similar continuous pricing function leads to two local equilibria - but it is not ruled out that the solution is a global equilibrium). Furthermore, it is not obvious that an equilibrium should exist anyway - at first glance, switching the strategy given the fixed strategy of all other players should result in a higher benefit.

We think that the benefit of every player in the specific models will always be negative in the equilibrium but have not proven this claim because the equations are too complicated to be solved. The behavior of the players could then be interpreted as an insurance against the uncertainty of their forecast errors. The higher the variance and the larger the difference between the price of the spot market and the mean price on the balancing power market, the more the player must buy to be on the safe side that the worst case (buying very expensive balancing power when the price on the spot market was low or selling power at a very low price when the price was high) is avoided in favor of the less expensive situation (selling power on the balancing market at a very low price when the price on the spot market was low or buying power at a very high price when the price was high).

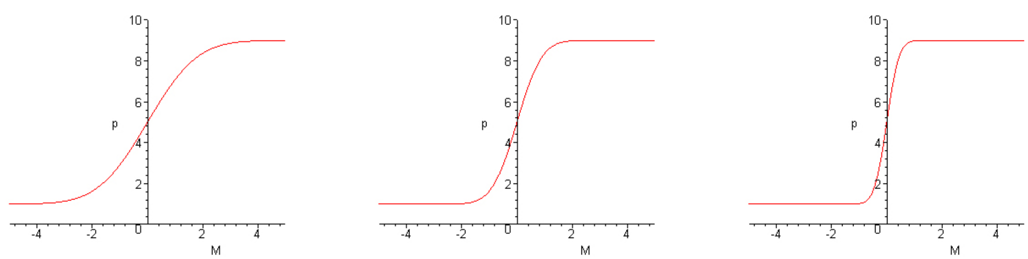
There are still some questions left unanswered. We think, but have in no way proven, that the benefit of the equilibria in the specific models is always negative. We only considered pure strategies. Mixed strategies will always lead to nash equilibria (which may not be different than the equilibria we found) but will probably be difficult to analyze analytically. Moreover, in the case of binary pricing functions the results may hold regardless of the kind of distribution of the error terms (as long as they have continuous densities). Although we have reasons to doubt, other pricing functions may lead to different results. It would also be interesting to incorporate stepwise pricing functions or a covariance structure (removing the independence assumption) or to answer the question whether there is any model specification which allows differently signed[6] optimal strategies.

In an upcoming paper we apply this model to the German balancing power market of the RWE zone and show that the data can be applied to the binary model and the behavior of the aggregate market can be replicated as soon as the prices of the spotmarket (taken from the EEX, the European Power Exchange stock market, EEX (2006)) become similar to the mean prices of the balancing power market (which has in fact been the case for the last two years 2004 and 2005).

A Appendix

A.1 The price function $f(M)$

The following graphs show f with $p_h = 9$, $p_l = 1$ and $a = 0.5, 1$ and 2:



This function fulfills all conditions given in section 2 and is infinitely often differentiable

The first and second derivatives are

$$\frac{\partial}{\partial M} f(M) = \frac{a(p_h - p_l)}{\sqrt{\pi}} e^{-(aM)^2} \text{ and } \frac{\partial^2}{\partial M^2} f(M) = \frac{-2a^3(p_h - p_l)M}{\sqrt{\pi}} e^{-(aM)^2}.$$

This function does not fit very well for realistic prices where M is near zero. Prices near the mean value of the high and low price caps are only rarely seen in Germany. Nevertheless, this function is still important since its main purpose is to analytically tackle the binary case.

[6] Equilibria in which some market participants buy and some sell strategic power.

A.2 Proof of Theorem 4.1

Proof Equation (2) is the second condition in the theorem. The first condition refers to equation (3) and can be written explicitly and simplified:

$$
\begin{aligned}
\forall i,j: \quad & \frac{1}{\sigma_i\sqrt{2\pi}}\int_{-\infty}^{\infty}(v_i-x)e^{-\frac{x^2}{2\sigma_i^2}}\int_{-\infty}^{\infty}\frac{a(p_h-p_l)}{\sqrt{\pi}}e^{-(a(x+y-V))^2}\frac{e^{-\frac{y^2}{2S_i^2}}}{S_i\sqrt{2\pi}}dydx \\
= \quad & \frac{1}{\sigma_j\sqrt{2\pi}}\int_{-\infty}^{\infty}(v_j-x)e^{-\frac{x^2}{2\sigma_j^2}}\int_{-\infty}^{\infty}\frac{a(p_h-p_l)}{\sqrt{\pi}}e^{-(a(x+y-V))^2}\frac{e^{-\frac{y^2}{2S_j^2}}}{S_j\sqrt{2\pi}}dydx \\
\Leftrightarrow \quad & \frac{(p_h-p_l)(v_i+2a^2v_iS_i^2+2a^2v_i\sigma_i^2-2a^2\sigma_i^2V)ae^{-\frac{V^2a^2}{1+2a^2S_i^2+2a^2\sigma_i^2}}}{(1+2a^2S_i^2+2a^2\sigma_i^2)^{\frac{3}{2}}\sqrt{\pi}} \\
= \quad & \frac{(p_h-p_l)(v_j+2a^2v_jS_j^2+2a^2v_j\sigma_j^2-2a^2\sigma_j^2V)ae^{-\frac{V^2a^2}{1+2a^2S_j^2+2a^2\sigma_j^2}}}{(1+2a^2S_j^2+2a^2\sigma_j^2)^{\frac{3}{2}}\sqrt{\pi}}.
\end{aligned}
$$

Since by definition $2a^2S_k^2+2a^2\sigma_k^2=2a^2S^2$ for all k, we can rewrite these relationships to

$$
\begin{aligned}
\forall i,j: \quad v_i+2a^2v_iS^2-2a^2\sigma_i^2(V_j+v_j) &= v_j+2a^2v_jS^2-2a^2\sigma_j^2(V_j+v_j) \\
\Leftrightarrow \quad \forall i,j: \quad \frac{v_i+2a^2v_iS^2-2a^2V_j(\sigma_i^2-\sigma_j^2)}{1+2a^2S^2+2a^2(\sigma_i^2-\sigma_j^2)} &= v_j. \qquad (7)
\end{aligned}
$$

In the next steps we show that the v_i^* given in the theorem is a solution for these relationships. Since the equations (3) form a linear equation-system with n unknown parameters and $n-1$ independent equations the derived relationship is unique[7] (in other words: every solution has one degree of freedom). We then proceed to show that the second derivative has two roots. Since $\mathbb{E}[G_i]$ tends to $-\infty$ as v_i tends to $\pm\infty$ and is continuous there exist two local maxima and one local minima[8]. Therefore, the last free v_1 is determined by the largest local maxima.

We have to show that

$$
\frac{v_i+2a^2v_iS^2-2a^2V_j(\sigma_i^2-\sigma_j^2)}{1+2a^2S^2+2a^2(\sigma_i^2-\sigma_j^2)}=\frac{2na^2\sigma_j^2+1}{2na^2\sigma_i^2+1}v_i
$$

[7] Players with identical variances just reduce the number of equations.

[8] As is shown later, these three local extremals collapse to one when the parameter a or n tends to infinity.

holds:

$$
\begin{aligned}
& \frac{v_i + 2a^2 v_i S^2 - 2a^2 V_j(\sigma_i^2 - \sigma_j^2)}{1 + 2a^2 S^2 + 2a^2(\sigma_i^2 - \sigma_j^2)} \\
= \;& \frac{v_i + 2a^2 v_i S^2}{1 + 2a^2 S^2 + 2a^2(\sigma_i^2 - \sigma_j^2)} - \frac{\frac{2a^2 v_i}{2na^2\sigma_i^2+1}\left(\sum\limits_{\substack{k=1\\k\neq j}}^{n} 2na^2\sigma_k^2 + 1\right)(\sigma_i^2 - \sigma_j^2)}{1 + 2a^2 S^2 + 2a^2(\sigma_i^2 - \sigma_j^2)} \\
= \;& \frac{(v_i + 2a^2 v_i S^2)(2na^2\sigma_i^2 + 1) - 2a^2 v_i(\sigma_i^2 - \sigma_j^2)(2na^2 S_j^2 + n - 1)}{(1 + 2a^2 S^2 + 2a^2[\sigma_i^2 - \sigma_j^2])(2na^2\sigma_i^2 + 1)} \\
= \;& \frac{v_i}{2na^2\sigma_i^2 + 1}\left(\frac{(1 + 2a^2 S^2)(2na^2\sigma_i^2 + 1) - 2a^2(\sigma_i^2 - \sigma_j^2)(2na^2 S_j^2 + n - 1)}{1 + 2a^2 S^2 + 2a^2(\sigma_i^2 - \sigma_j^2)}\right) \\
= \;& \frac{v_i}{2na^2\sigma_i^2 + 1}\left(\frac{(1 + 2a^2 S^2)(1 + 2na^2\sigma_j^2) + (4na^4\sigma_j^2 + 2a^2)(\sigma_i^2 - \sigma_j^2)}{1 + 2a^2 S^2 + 2a^2(\sigma_i^2 - \sigma_j^2)}\right) \\
= \;& \frac{v_i}{2na^2\sigma_i^2 + 1}\left(\frac{(1 + 2a^2 S^2)(1 + 2na^2\sigma_j^2) + (1 + 2na^2\sigma_j^2)2a^2(\sigma_i^2 - \sigma_j^2)}{1 + 2a^2 S^2 + 2a^2(\sigma_i^2 - \sigma_j^2)}\right) \\
= \;& \frac{2na^2\sigma_j^2 + 1}{2na^2\sigma_i^2 + 1} v_i
\end{aligned}
$$

The second derivative is

$$
\begin{aligned}
& \frac{\partial^2}{\partial v_i^2}\mathbb{E}[G_i(v)] \\
= \;& \frac{\partial}{\partial v_i}\left(\mathbb{E}\left[(v_i - X_i)\frac{\partial}{\partial v_i} f(M)\right] + \mathbb{E}[f(M)]\right) \\
= \;& \frac{\partial}{\partial v_i}\left(-\mathbb{E}\left[(v_i - X_i)\frac{\partial}{\partial M} f(M)\right]\right) - \frac{\partial}{\partial M}\mathbb{E}[f(M)] \\
= \;& \mathbb{E}\left[(v_i - X_i)\frac{\partial^2}{M^2} f(M)\right] - 2\mathbb{E}\left[\frac{\partial}{\partial M} f(M)\right] \\
= \;& \int\limits_{-\infty}^{\infty} \frac{(v_i - x)}{\sqrt{2\pi\sigma_i^2}} e^{\frac{-x^2}{2\sigma_i^2}} \int\limits_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi S_i^2}} e^{\frac{-y^2}{2S_i^2}} \frac{-2a^3(p_h - p_l)(y + x - V)}{\sqrt{\pi}} e^{-a^2(y+x-V)^2} dy dx \\
& -2\int\limits_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi S^2}} e^{\frac{-x^2}{2S^2}} \frac{a(p_h - p_l)}{\sqrt{\pi}} e^{-a^2(x-V)^2} dx \\
= \;& -2(p_h - p_l)a\frac{-e^{-\frac{a^2V^2}{1+2a^2S^2}}}{(1 + 2a^2S^2)^{5/2}\sqrt{\pi}}\Big[a^4(4S^4 - 2S^2Vv_i - 2S^2\sigma_i^2 + 2V^2\sigma_i^2) \\
& + a^2(4S^2 - Vv_i - \sigma_i^2) + 1\Big]
\end{aligned}
$$

Without loss of generality we set $i = 1$ and use $v_j = \frac{2na^2\sigma_j^2+1}{2na^2\sigma_1^2+1}v_1$ and $V = \frac{2na^2S^2+n}{2na^2\sigma_1^2+1}v_1$. The second derivative then has the following two roots:

$$\pm\frac{2na^2\sigma_1^2+1}{a\sqrt{2na^2S^2+n}}\sqrt{2a^2S^2-a^2\sigma_1^2+1}.$$

Therefore for every a and n there are two $\mathfrak{v}^*$ which satisfy the local equilibrium conditions - the final vector is then given by the larger $\mathbb{E}[G_1(\mathfrak{v}^*)]$.

□

A.3 Proof of theorem 5.1

Proof

1. Theorem 4.1 and equation (3) lead to

$$v_j^* = \lim_{a\to\infty}\frac{2na^2\sigma_j^2+1}{2na^2\sigma_i^2+1}v_i^* = \frac{\sigma_j^2}{\sigma_i^2}v_i^* \text{ and therefore } V_i^* = \sum_{j=1,j\neq i}^{n} v_j^* = \frac{v_i^*}{\sigma_i^2}S_i^2.$$

2. Now condition (2) can be expressed as

$$\begin{aligned}
&\lim_{a\to\infty}\mathbb{E}[(v_i^*-X)f'(M)] = \mathbb{E}[f(M)]-p_s\\
\Leftrightarrow\ &\lim_{a\to\infty}\frac{(p_h-p_l)(v_i^*+2a^2v_i^*S_i^2-2a^2\sigma_i^2V_i^*)ae^{-\frac{V^{*2}a^2}{1+2a^2S^2}}}{(1+2a^2S^2)^{\frac{3}{2}}\sqrt{\pi}} = \mathbb{E}[f(M)]-p_s\\
\Rightarrow\ &\frac{(p_h-p_l)(v_i^*S_i^2-\sigma_i^2V_i^*)e^{-\frac{V^{*2}}{2S^2}}}{\sqrt{2\pi}S^3} = \mathbb{E}[f(M)]-p_s\\
\Leftrightarrow\ &\frac{(p_h-p_l)(v_i^*S_i^2-\sigma_i^2\frac{v_i^*}{\sigma_i^2}S_i^2)e^{-\frac{V^{*2}}{2S^2}}}{\sqrt{2\pi}S^3} = \mathbb{E}[f(M)]-p_s\\
\Leftrightarrow\ &0 = \mathbb{E}[f(M)]-p_s
\end{aligned}$$

   Therefore we have $\mathbb{E}[f(M)] = p_s$ in any equilibrium.

3. $V^* = v_i^*\frac{S^2}{\sigma_i^2}$. The vector $\mathfrak{v}^*$ is unique, since the only free variable v_i^* has a a

single solution in equation (2):

$$
\begin{aligned}
& \mathbb{E}[f(M)] = p_s \\
\Leftrightarrow\ & P(M \geq 0)p_h + [1 - P(M \geq 0)]p_l = p_s \\
\Leftrightarrow\ & P(M \geq 0)(p_h - p_l) = (p_s - p_l) \\
\Leftrightarrow\ & P(X \geq V^*) = \frac{p_s - p_l}{p_h - p_l} \\
\Leftrightarrow\ & P(X < V^*) = \frac{p_h - p_s}{p_h - p_l} \\
\Leftrightarrow\ & P\left(X < v_i^* \frac{S^2}{\sigma_i^2}\right) = \frac{p_h - p_s}{p_h - p_l} \\
\Leftrightarrow\ & v_i^* \frac{S}{\sigma_i^2} = \Phi^{-1}\left(\frac{p_h - p_s}{p_h - p_l}\right) \\
\Leftrightarrow\ & v_i^* = \frac{\sigma_i^2}{S}\Phi^{-1}\left(\frac{p_h - p_s}{p_h - p_l}\right)
\end{aligned}
$$

Therefore we have a single unique extremal which has to be a global maximum. Plugging V into the second derivative from the appendix A.2 and taking the limit[9] $a \to \infty$ the second derivative now becomes

$$
-\frac{(p_h - p_l)(2S^2 - \sigma_1^2)}{\sqrt{2\pi}S^3} e^{-\frac{S^2 v_1^2}{2\sigma_1^4}} < 0.
$$

□

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[9]The limit $n \to \infty$ also leads to a strictly negative second derivation.

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