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GROWTH OVER THE VERY LONG RUN: IMPLICATIONS OF A SPECIFIC FACTORS MODEL OF ECONOMIC DEVELOPMENT WITH ENDOGENOUS TECHNOLOGICAL CHANGE[1]

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Abstract

We use the two-sector specific factors model, which is known from the theory of international trade, in a growth context to describe major trends of long-run economic development. The endogenous technical progress functions establish the link between the agricultural and the manufacturing sector through the ratio of agricultural to total employment, which is determined by the savings propensities of wage-earners, landlords and capitalists, and by the investment ratio in manufacturing. Without reference to more complicated micro-based models of human capital accumulation highlighting changes in preferences of households and/ or shifts in attitudes of firms towards education, the calibrated two-sector specific factors model can replicate major historical trends and structural turnarounds.

Keywords: Economic growth, technical change, distribution of income, Industrial Revolution

JEL classification: E13, N1, O41

Introduction

In the economically most successful regions (Western Europe and Western Offshoots) the modern era of sustained growth has lasted for more than 150 years. Sustained growth followed an extended period over centuries or even millennia with modest or no growth. Worldwide, according to estimates by Maddison (2007), annual GDP increased on average by 0.02% during the first millennium, by 0.15% during 1000-1500, and by 0.32% between 1500 and 1820. Growth of GDP per capita was negligible from 1-1820.

When it comes to explaining growth over the very long run, the period prior to the Industrial Revolution is usually associated to the name of Thomas Malthus, while the modern era of sustained growth is linked to the work of Robert Solow.[2] Among other things, pre-industrial societies during the “Malthus epoch” were characterised by a high share of labour employed in agriculture. In early societies, nearly the entire labour force was allocated to agriculture, while in modern industrial societies the ratio of agricultural to total labour employment is often as low as 1%. There is ongoing research in explaining this immense socio-economic transformation which took place within roughly 150 years (1750 – 1900). The understanding of this transformation was above all based on the seminal work of Deane and Cole (1969) and later challenged by Crafts (1985), and Crafts and Harley (1992 and 2000).

Another line of research carried out by growth economists is connected with the work of Oded Galor (2005) and (2010) and other contributors to the so called unified growth theory (UGT). The main idea of that approach is to explain the transformation of a long run stationary economy to an economy with sustained growth by stressing the successively increasing importance of human capital formation. A typical example of the type of model used in unified growth theory is Hansen and Prescott (2002), who present a one-good, two-sector model with a special production function for the Malthus sector and a separate production function for the Solow sector: Initially, only the Malthus sector operates, but at

1 We should like to thank G.Rehme and B. Schefold for helpful discussions and suggestions. The errors remain ours.

2 See e.g. Hansen and Prescott (2002).

some point of time the Solow sector comes into play and gradually displaces the Malthus sector. In contrast, our model avoids the dichotomy of the economic development process by using a production model with two distinct production functions, one for agriculture and the other for manufacturing, each with its own technology applying to both epochs. The novelty of our approach is that the Solow era has almost always existed (in a sense, side by side with the Malthus sector), although it became of significant importance rather lately, i.e. since the middle of the 19th century.

As we do not support the dichotomous splitting of economic history into two eras, the Malthus and the Solow phase, we do not need a mechanism arranging for the transition between the former and the latter, which is a major advantage since we do not have to show how the transition was accomplished (by increased human capital formation, by changes of demand due to behavioural changes of households, or by whatever shock-like event) and in particular how the timing of the transition between stagnation and growth evolved.

Instead, long-term economic development is explained by a “mechanical” model, similar to Lucas (1990), in which the ratio of labour to total employment plays a key role. The basic model outlined below is the two-sector specific factors model which is well-known from the economics of international trade, where it helps to clarify the principle of comparative advantage. In our context it also appears to be highly suitable for illustrating central features of economic development, most notably the structural change from agricultural to industrial societies. Variables on which we focus include the wage-rental ratio, the wage-profit ratio, the land-labour ratio, the capital-labour ratio, the relative price of agricultural output, the total factor productivity in agriculture and industry, and the labour productivity in each of the two sectors.

The most important feature of our “mechanical” model relates to the treatment of technical change: We use endogenous technical progress functions in the sectors of agriculture and industry which formally correspond to the type of differential equation introduced by Lucas (1988) for describing human capital accumulation.[3] It is this form of technical progress function that enables us to simulate the gradual decline of agriculture in favour of manufacturing and to draw empirically meaningful conclusions from the specific factors model even beyond 1850. In our view, the specific factors model, in combination with Lucas’ assumption on technological change, provides a much simpler growth theory over the very long run than the type proposed by the recent literature.

Unlike the prominent models of unified growth theory[4], which focus on human capital investment and incorporate individual household maximisation rules, we concentrate on physical capital and describe the demand side of the model using an investment = savings condition in the tradition of Kaldor, with the savings rates of workers, capitalists, and landowners as separate parameters.[5] The driving force on the demand side is the changing distribution of income from rents to profits fostering the accumulation of capital and propelling the structural change from an agricultural to an industrial economy. Together with

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3 The same type of function also appears in Lucas (2009), where he presents a dual economy model on trade and the diffusion of the Industrial Revolution.

4 See the contribution by Galor (2005) in the Handbook of Economic Growth as reference.

5 A micro-based unified growth model would have to distinguish between three types of households, workers, capitalists, and landlords each having specific utility functions, budget constraints, time preference rates, etc. This would lead to enormous complexity, in particular when it comes to aggregation. Also, as recently pointed out by Crafts and Mills (2009), there may be considerable lags between micro-level changes and macro evidence.

endogeneous technical progress on the supply side, our model emphazises the 'structural and social forces' of long run growth. This does not mean that we deny the importance of behavioural changes in explaining major historical trends and structural breaks. Instead, our argument is that these trends and reversals can also be explained by more structural approaches which rely on appropriate concepts of endogenous technical progress and changes in income distribution.[6]

Since the "demographic transition" is not the subject of our analysis, we do not deal with population growth and the trade-off between child quantity and quality,[7] which is an important central feature of many unified growth models.

We called our approach "mechanical" because we are aware of its simplifications and omissions: Firstly, capital (as produced means of production) is not an input in the agricultural production function because that would lead to the problem of aggregating different capital stocks[8] and, in addition, would complicate the model without affecting its major results. Secondly, our model is a pure production model with no explicit consideration of household demand structures, although the level of demand is determined by a savings-equals-investment condition like in the Solow model. Thirdly, we abstract from foreign trade: Our model represents a closed economy or an open economy in which exports and imports are balanced. Again, this may be regarded as an oversimplification but we are convinced that during the 19th century, when the turnaround from economic stagnation to growth occurred, technical progress and capital accumulation played a more important role in determining outputs, wages and profits than, e.g., cheap imports from America or India.[9]

The paper is organised as follows. We start highlighting the empirical trends of variables and indicators playing an important role in the secular movement from era of stagnation to the era of the Industrial Revolution. In the third section we present and explain our two-sector specific factors model. Then, in the fourth section the model parameters are calibrated with given historical data and we try to replicate the empirics of the given historical trends. The last section concludes.

2 Empirical trends

As mentioned above, we address the secular movements of the following variables: the wage-rental ratio ($\omega_{L/X}$), the wage-profit ratio ($\omega_{L/K}$), the land-labour ratio (x), the capital-labour ratio (k), the relative price of agricultural output (P), the total factor productivity in agriculture (A) and industry (M), and the respective labour productivities (y_a,y_m). O'Rourke and Williamson (2005) compiled "raw data"[10] from various sources on five of these variables

6 As Crafts and Mills (2009) put it, "it may be inappropriate to focus on positive feedbacks between population growth and technological progress as the key to escape from the Malthusian era rather than considering alternative models of technological change that draw on ideas from endogenous growth theory in the large rather than unified growth theory in particular." (p. 92)

7 In modern societies this trade-off has ultimately been resolved in favour of fewer, though less better educated, children by means of human capital investment.

8 This was one of the crucial questions of the Cambridge-Cambridge controversies in the 1960s and 1970s.

9 See also Allen (2009).

10 The data are often the result of trend extrapolations or regressions.

covering the two consecutive periods 1500 – 1840 and 1840 – 1936 of the British economy. The data show the following long-term trends:

- The land-labour ratio (x) fell over both periods.
- The wage-rental ratio ($\omega_{L/X}$) declined between 1500 and 1840/50 but rose thereafter.
- The relative price of agricultural output (P) steeply increased during the period 1500 – 1840, and moderately decreased during the second period.
- Industrial productivity (y_m) fluctuated highly in the first period, with no clear trend becoming apparent, but surged during the period 1840 – 1936.
- Total factor productivity in agriculture (A) rose during both periods.

Attempting to explain the above trends, O'Rourke and Williamson (2005) hinted to the specific factors model, but did not provide a rigorous analysis.[11] Instead, they presented econometric estimates which support their key message that major structural breaks in production and distribution after 1840 can be explained by the opening up of the European economy to international trade, which coincided with the Industrial Revolution. The signs of their linear regressions are displayed in Table 1.[12, 13]

Table 1. Signs of linear regressions in O'Rourke and Williamson (2005)

	Period	1500-1750		1842-1936
Regressions		dependent variables		
		P	$\omega_{L/X}$	$\omega_{L/X}$
	x	–	+	
independent variables	A		–	
	y_m		+	+
	P			–

In section 4, we will test our model as well as O'Rourke and Williamson's (2005) econometric approach in a calibration exercise.

3 The Basic Model

In the two-sector specific factors model two commodities are produced: agricultural (= food) products using land and labour and manufactured (= non-food) goods using capital and labour. Hence, land is specific to the agricultural sector while capital is specific to the manufacturing sector. We assume that both consumption goods *and* capital goods are produced in the manufacturing sector.

The variables used in the model (see appendix for summary) should all include an index t for time. By convention, however, t is omitted unless it is needed for clarification.

11 There are no functions spelled out, and the list of variables considered is incomplete.

12 Since, as a consequence of global market integration, the relative price of agriculture changes its character from a formerly dependent to an independent variable, the regression structure is different for the second period 1842-1936: The wage-rent ratio (dependent variable) becomes a linear function of industrial productivity and the relative price of agricultural output (independent variables).

13 The generally high statistical quality of the estimates does not come as a surprise if fitted data is regressed, see fn 6.

Production functions

The production functions are of the Cobb-Douglas type and given by

$$Y_a = A\, L_a^{\alpha}\, X^{1-\alpha}, \;\; 0 < \alpha < 1 \tag{1}$$

$$Y_m = M\, L_m^{\beta}\, K^{1-\beta}, \; 0 < \beta < 1 \tag{2}$$

or in per capita form:

$$y_a = A\, x_a^{1-\alpha} \equiv c_a \tag{3}$$

$$y_m = M\, k_m^{1-\beta} \equiv c_m + i_m, \tag{4}$$

where Y_a, Y_m denotes the output of agriculture and manufacturing, respectively. A, M are the time-dependent levels of technology in the two sectors. L_a and L_m stand for the quantity of labour employed in agriculture and manufacturing. X is the total quantity of land, while x_a is the land-labour ratio in agriculture. K represents the quantity of capital and k_m the capital-labour ratio in manufacturing. y_a $(\equiv c_a)$ and y_m $(\equiv c_m + i_m)$ are the output-labour ratios in agriculture and manufacturing, respectively. c_a (c_m) denotes consumption of the agricultural (manufacturing) output per unit of labour employed in agriculture (manufacturing). Note that $y_m \equiv c_m + i_m$ is an identity and not a market-clearing condition.

Full employment/ full capacity

Employment in agriculture (L_a) and manufacturing (L_m) is equal to total labour supply (L).[14] Hence:

$$L_a + L_m = L \tag{5}$$

Using ρ as the symbol indicating the share of agricultural labour in total labour supply (= demand), one can also write:

$$x = \rho\, x_a \tag{6}$$

and

$$k = (1 - \rho)\, k_m, \tag{7}$$

with x as the overall land-labour ratio and k as the economy-wide capital-labour ratio.

Endogenous technical progress

In models dealing with economic growth over the very long run, there are various possibilties to introduce endogenous technical progress. Generalizing Matsuyama's (1992) approach, one possibility of describing the development of the technology level in agriculture and manufacturing could be:

$$A_{t+1} - A_t = \dot{A} = \gamma Y_a \tag{8}$$

$$M_{t+1} - M_t = \dot{M} = \varepsilon Y_m \tag{9}$$

where $(\gamma, \varepsilon > 0)$.

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[14] Clark (2001) explains why the assumption of full employment for agricultural labourers is reasonable, at least after 1640.

According to (8) and (9) technical knowledge in agriculture and manufacturing would accumulate as a side-effect of productive experience in the two sectors. This approach has the major disadvantage that, when integrated in a model of long-term development, explosive growth cannot be excluded, unless additional restrictions exist, as shown by Strulik and Weisdorf (2008).

The same problem would arise, at least with unlimited population growth and full employment, if technological change in the two sectors were proportional to the respective size of labour employment, as proposed by Kremer (1993) in his one-sector model:

$$\frac{A_{t+1}-A_t}{A_t} = \frac{\dot{A}}{A} = \lambda L_a \qquad (10)$$

$$\frac{M_{t+1}-M_t}{M_t} = \frac{\dot{M}}{M} = \mu L_m \qquad (11)$$

$(\lambda, \mu > 0)$.

Analytically more appropriate to a dual economy where, in relative terms, one sector shrinks in favour of the other would be postulating that technical change is proportional to the degree of industrialization, measured by the ratio of agricultural to total employment (ρ_t) instead of the level of employment or output:

$$\frac{A_{t+1}-A_t}{A_t} = \frac{\dot{A}}{A} = \lambda \frac{L_a}{L} = \lambda \rho_t \qquad (12)$$

$$\frac{M_{t+1}-M_t}{M_t} = \frac{\dot{M}}{M} = \mu \frac{L_m}{L} = \mu\,(1-\rho_t) \qquad (13)$$

with λ, $\mu > 0$ and the solutions:

$$A_t = e^{\lambda \int \rho_t \, dt}$$

$$M_t = e^{\mu \int (1-\rho_t)\, dt}$$

Note that both A_t and M_t are monotone increasing functions of t, with A_t having a maximum at $\rho = 0$ and M_t having a minimum at $\rho = 1$.

The advantage of (12) and (13) is straightforward: Since $0 \leq \rho \leq 1$, λ and μ represent the maximal growth rates of the technology level in agriculture and manufacturing, respectively. In the extreme case of $\rho = 1$, the economy is 100% agriculture-based, whereas in the other extreme, $\rho = 0$, we have a purely industrial economy.[15] In between these extremes, the agricultural and the industrial sector *co*-exist, with a given percentage increase (decrease) in M_t (A_t) entailing the same development of industrialization, no matter what level of M_t (A_t) has been attained. See also Figure 1.

15 Equation (13) is formally identical to the Uzawa-Rosen formulation of human capital accumulation adopted by Lucas (1988), see p. 19.

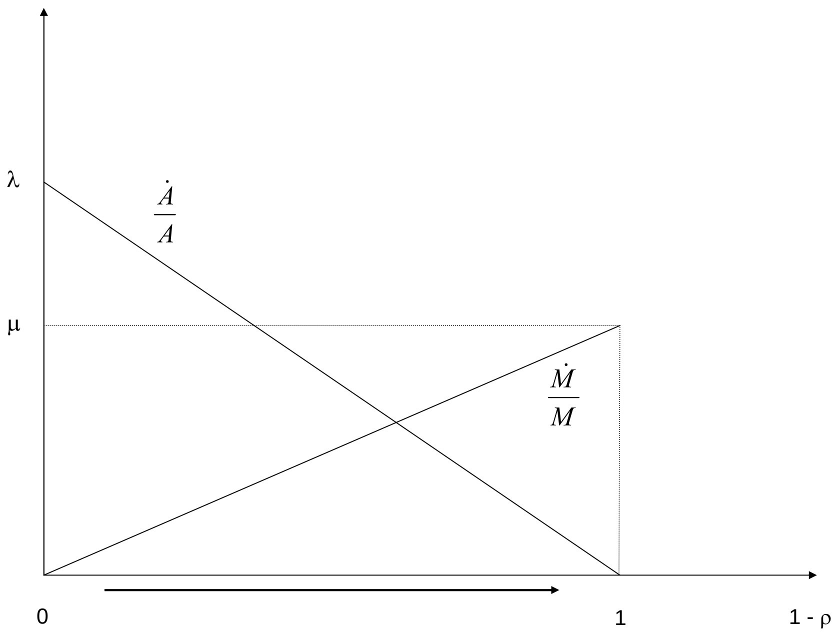

Figure 1: Technical progess in agriculture and manufacturing (The arrow indicates that the economy is moving from $\rho = 1$ to $\rho = 0$.)

<u>Efficiency conditions</u>

Under the assumption of competitive markets for labour, land, and capital, the standard efficiency conditions require

$$\left.\begin{aligned} &\partial Y_a/\partial L_a = W/P, \\ &\partial Y_m/\partial L_m = W, \\ &\partial Y_a/\partial X = R_X/P, \\ &\partial Y_m/\partial K = R_K, \end{aligned}\right\} \qquad (14)$$

where W is the (nominal) wage rate of homogeneous labour, P the relative price of agricultural in terms of manufacturing output, R_X the rental rate of land, and R_K the rental rate of capital.

Taking account of (1) - (4) gives for the derivatives in (13):

$$\left.\begin{aligned} &W/P = A\ \alpha\, x_a^{1-\alpha} \\ &W = M\ \beta\, k_m^{1-\beta} \\ &R_X/P = A\ (1 - \alpha)\ x_a^{-\alpha} \\ &R_K = M\ (1- \beta)\ k_m^{-\beta} \end{aligned}\right\} \qquad (15)$$

Therefore, after inserting (13) into the factor-price ratios between labour and land ($\omega_{L/X}$) and between labour and capital ($\omega_{L/K}$)

$$\omega_{L/X} = \frac{W/P}{R_x/P} = \frac{\partial Y_a/\partial L_a}{\partial Y_a/\partial X_a}$$

and

$$\omega_{L/K} = \frac{W}{R_K} = \frac{\partial Y_m / \partial L_m}{\partial Y_m / \partial K_m},$$

two expressions eventually result for the specific factors model which relate $\omega_{L/X}$ to the land-labour ratio (x), and $\omega_{L/K}$ to the capital intensity (k):

$$\omega_{L/X} = \frac{\alpha}{1-\alpha} x_a = \frac{\alpha}{1-\alpha} \frac{x}{\rho} \tag{16}$$

$$\omega_{L/K} = \frac{\beta}{1-\beta} k_m = \frac{\beta}{1-\beta} \frac{k}{1-\rho} \tag{17}$$

Apart from the extreme cases, $\rho = 1$ and $\rho = 0$, equations (16) and (17) are not as simple as in a static one-good world, where the positive correlation between $\omega_{L/X}$ (or $\omega_{L/K}$) and x (or k) can be explained by diminishing returns in agriculture (or manufacturing). Instead, in the dynamic two-sector specific factors model the development of $\omega_{L/X}$ (and $\omega_{L/K}$) is also influenced by ρ, the share of agricultural employment in total employment.

The factor price ratio between labour and land

Differentiating the second expression in (16) with respect to time gives:

$$\dot{\omega}_{L/X} = \frac{\alpha\, x}{(1-\alpha)\,\rho}\left(\frac{\dot{x}}{x} - \frac{\dot{\rho}}{\rho}\right) = \omega_{L/X}\left(\frac{\dot{x}}{x} - \frac{\dot{\rho}}{\rho}\right) \tag{18}$$

or

$$\frac{\dot{\omega}_{L/X}}{\omega_{L/X}} = \frac{\dot{x}}{x} - \frac{\dot{\rho}}{\rho} = \frac{\dot{X}}{X} - \frac{\dot{L}_a}{L_a} \tag{19}$$

As land under cultivation has remained fairly stationary since the 12^{th} century,[16] equation (19) may be simplified further:

$$\frac{\dot{\omega}_{L/X}}{\omega_{L/X}} = -\frac{\dot{L}_a}{L_a}, \tag{20}$$

from which follows:

$$\omega_{L/X,t} = \pi\, L_{a,t}^{-1} \tag{21}$$

with $\pi = \alpha/(1-\alpha)\, X$ = constant, see equation (16).

With (21) one can easily describe the dramatic reversal of long-run trends in the ratio of wages to land rents seen in the 19^{th} century. As illustrated in Figure 2, agricultural employment reached its maximum in approximately 1850. At this date, the parallel development of the wage-rental ratio and the land-labour ratio was broken. The phase of

16 According to Postan (1966), p. 548, crop acreage has not increased significantly since 1066. Of course, new land was gained for agriculture, but much of it was marginal and, hence, did not earn rent.

declining wages to land rents ended to give way to the new era of relative improvement in workers' living conditions compared to landowners.[17]

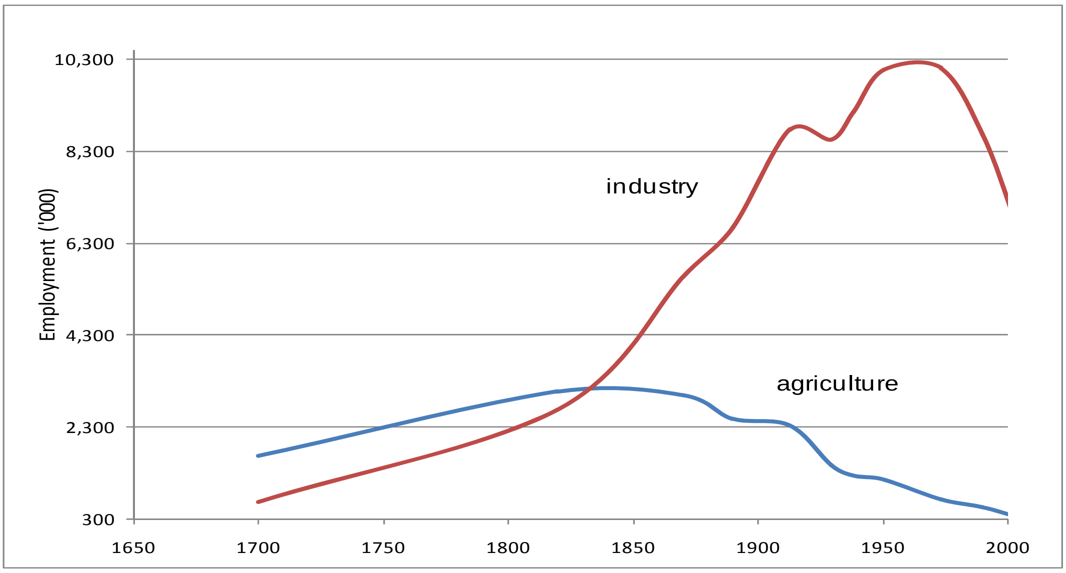

Figure 2: Agricultural and industrial employment in the UK [source of data: Maddison (2007), p. 384]

The entries in Table 2 illustrate the development of ρ, which had come down from a level of 70%-80% before 1700. As noted by Voth (2003), the British agricultural labour share in the 19th century was unusually low in comparison to other countries; e.g. in the USA (Japan) it was still as high as 70% in 1820 (1870). Note that between 1820 and 1870, i.e. the period when the dramatic structural turnaround took place, ρ dropped by 40%.

Table 2. Agricultural labour share in the UK, 1700-2003 (in %)

UK	1700	1820	1870	1890	1913	1929	1950	1973	2003
ρ	56	37.6	22.7	16.1	11.7	7.7	5.1	2.9	1.2
%- change p.a.		-0.33%	-1.00%	-1.70%	-1.38%	-2.58%	-1.21%	-2.42%	-4.21%

Source of data: Maddison (2007), p. 76 and p. 384

The factor price ratio between labour and capital

After differentiating equation (17) with respect to time, we get:[18]

$$\frac{\dot{\omega}_{L/K}}{\omega_{L/K}} = \frac{\dot{k}}{k} + \frac{\dot{\rho}}{1-\rho} = \frac{\dot{k}}{k} + \frac{\dot{\rho}}{\rho}\frac{\rho}{(1-\rho)} \qquad (22)$$

In our specific factors model, the growth rate of the factor price ratio between labour and capital is determined by capital accumulation and the structural term on the right-hand side of (22), which measures the gradual transition from an agricultural to an industrial economy.

17 The Chow test statistics by O'Rourke and Williamson (2005), p. 14, suggest that the structural break occurred in 1838.

18 The secular movements of the capital intensity and the factor price ratio between labour and capital are not covered by O'Rourke and Williamson's (2005) econometric approach.

Since $\dot{\rho} < 0$, and $(1 - \rho) > 0$, this term is negative and equation (22) shows that capital accumulation is a *sine qua non* for improving the distribution of income in favour of workers: In early phases of economic development, when a country is dominated by agriculture, capital accumulation is not sufficient to balance the negative structural effect, hence the living conditions of workers stagnate or even decline.

As illustrated in Figure 3, the development of the gross stock of domestic reproducible fixed assets was rather flat during the pre-industrial phase of the British economy, but gained momentum as of 1780 due to massive investments in residential housing as a consequence of accelerating urbanisation, huge investments in factory buildings and equipment, as well as in infrastructure, in particular the railways after 1830.

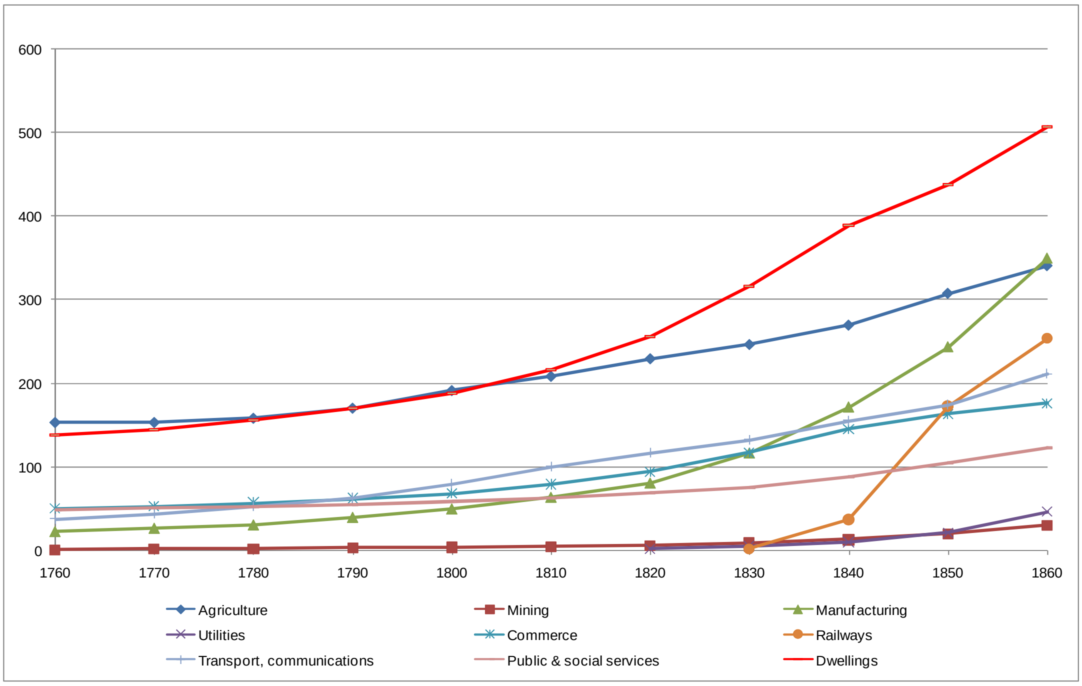

Figure 3: Gross stock of domestic reproducible fixed assets (£ million at constant prices), Great Britain, 1760-1860 [source of data: Feinstein and Pollard (1988)]. Note that in 1850-60 the British capital stock made up 94% of the capital stock in the entire U.K.

<u>The relative price of agricultural output</u>

From the above efficiency conditions the relative price of agricultural output can easily be derived. Since

$$W = \partial Y_m / \partial L_m = P\ \partial Y_a / \partial L_a = M\,\beta\, k_m^{1-\beta} = P\,A\,\alpha\, x_a^{1-\alpha} \qquad (23)$$

we get:

$$P = \frac{M\,\beta\, k_m^{1-\beta}}{A\,\alpha\, x_a^{1-\alpha}} = \frac{\beta\, y_m}{\alpha\, y_a} \qquad (24)$$

The price of the agricultural product, relative to the manufacturing good, is inversely proportional to the sectoral labour productivities. P declines (rises) if the growth rate of agricultural labour productivity exceeds (falls short of) the growth rate of labour productivity in the manufacturing sector. In the latter case, gains in living standards would occur primarily through the falling relative price of manufactured goods, as described by Broadberry and Gupta (2006) for the pre-1800 period.

P roughly doubled between 1500-1600, based on the original data by Phelps Brown and Hopkins (1957), partly because agricultural productivity fell by 24% as a result of strong population pressure.[19] Between 1600 and 1700 agricultural productivity recovered – it increased by 51% – , and *P* remained more or less constant, indicating that the productivity jump in agriculture must have been accompanied by a similar productivity gain in manufacturing.[20] Finally, *P* rose by 40% between 1700 and 1800, based on the original figures from Schumpeter (1938).[21] At the same time, agricultural productivity increased by 24% suggesting an even stronger manufacturing productivity rise. Hence, the specific factors model supports the finding of economic historians (such as Crafts and Harley, 1992) that the Industrial Revolution did not appear as a sudden shock or "big bang", but had long been developed before its "official" beginning in 1760.

Sectoral productivity growth

Taking the natural logarithm of the intensive form of the production functions (3)-(4) and differentiating with respect to time yields under consideration of (6) and (8)-(9):

$$\frac{\dot{y}_a}{y_a} = \frac{\dot{A}}{A} + (1-\alpha)\frac{\dot{x}_a}{x_a} = \lambda\rho + (1-\alpha)\left(\frac{\dot{x}}{x} - \frac{\dot{\rho}}{\rho}\right) \qquad (25)$$

$$\frac{\dot{y}_m}{y_m} = \frac{\dot{M}}{M} + (1-\beta)\frac{\dot{k}_m}{k_m} = \mu(1-\rho) + (1-\beta)\left(\frac{\dot{k}}{k} + \frac{\dot{\rho}}{\rho}\frac{\rho}{1-\rho}\right) \qquad (26)$$

If the land-labour ratio declines with the same speed as the fraction of agricultural to total employment, i.e. if the expression in brackets on the right-hand side of (25) is zero, then agricultural productivity growth is only attributable to technical change; otherwise an additional structural effect becomes involved. As we know from the argument above, see equation (19), the structural break occurred by approximately 1850. Prior to that date, the growth rate of x had exceeded that of ρ, with the consequence that the total factor productivity growth in agriculture, measured by $\lambda\rho$, had surpassed the growth rate of agricultural output per worker in agriculture. Since, over a period of 300 years, output per worker in agriculture had risen by 43%,[22] total factor productivity in agriculture must have increased by an even higher rate, if one sticks to the assumptions of our mechanical model. However, at same point in the 19th century, this trend reversed and total factor productivity growth in agriculture fell short of growth of output per worker in agriculture.

19 The figures on agricultural productivity are from Allen (2000).

20 O'Rourke and Williamson (2005) combine data from Phelps Brown and Hopkins (1957) with data from Schumpeter (1938) which are not totally consistent; e.g., for the period 1660-1700 P increased according to the former while it decreased according to the latter.

21 If prices of consumption goods exclusive of cereals are considered, the price development of the latter was not significantly different from the development of producers' good prices.

22 According to figures by Allen (2000), British output per worker in agriculture declined by 24% from 1500 until 1600, increased by 51% during 1600 and 1700, followed by a rise of 34% between 1700 and 1750 and a decline of 7% from 1750 until 1800.

Mutatis mutandis, the above way of reasoning can be applied to the manufacturing sector, where in the early phase of economic development its growth performance had been "*too far away*" from its maximal potential, so to speak: Total factor productivity growth, measured by $\mu\ (1 - \rho)$, had remained below manufacturing output growth per worker, before the former gained momentum during the Industrial Revolution and eventually surpassed the latter.

Saving and investment

With the growth equations in the previous section defining the growth of agricultural output per worker and of manufacturing output per worker, the description of the growth process is not yet complete. What is still missing is the development of capital accumulation. To complete the picture, we refer to a central building block of standard two-sector growth models in the tradition of Hahn (1965), the $S = I$ condition.

Under the assumption that the gross savings propensities out of wages (s_w), profits (s_c) and rents (s_x) are nonnegative parameters in the range between 0 and 1 (not necessarily constant), total gross savings in the economy are:

$$s_w\, W L + s_c\, R_K\, K + s_x\, R_X\, X = S$$

Gross investment is defined as

$$I \quad = \quad \dot{K} + \delta\, K$$

In equilibrium with $S = I$, after making use of (6)-(7) and (14), the following expression holds:

$$i_m = \frac{I}{L_m} = \frac{\beta\, M\, k_m^{\;1-\beta} \left[s_w + s_c \dfrac{1-\beta}{\beta}(1-\rho) + s_x \dfrac{1-\alpha}{\alpha}\rho \right]}{1-\rho} \tag{27}$$

Note that $M\, k_m^{\;1\text{-}\beta}$ is manufacturing output per worker or y_m in our notation.

The instantaneous change in the capital intensity of the economy

$$\dot{k} = i_m\,(1-\rho) - (\delta + n)\, k \tag{28}$$

may therefore be written as: [23]

$$\dot{k} = \beta\, M \left(\frac{k}{1-\rho} \right)^{1-\beta} \left[s_w + s_c \frac{1-\beta}{\beta}(1-\rho) + s_x \frac{1-\alpha}{\alpha}\rho \right] \; - \; (\delta + n)\, k \tag{29}$$

More generally, for a given development of population and given parameter values of $\alpha, \beta, s_w, s_c, s_x$, and δ, the change in the capital-labour ratio is a function of k, ρ, and μ:[24]

$$\dot{k} = \dot{k}\,(k, \rho, \mu) \tag{30}$$

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[23] In the extreme case of $\rho = 0$, $k_m = k$ so that (27) takes the form of the basic Solow model.

[24] Since M is a function of μ; see equation (11).

<u>The ratio of agricultural to total employment as endogenous variable</u>

After inserting (4) into (27), ρ becomes a function of the savings propensities, the distributional shares (α and β) and the proportion of capital goods production in the manufacturing sector:

$$\rho = \frac{i_m / y_m - [\beta s_w + (1-\beta) s_c]}{i_m / y_m - \beta \left(\frac{1-\beta}{\beta} s_c - \frac{1-\alpha}{\alpha} s_x \right)} \tag{31}$$

For given parameter values of α and β, the structural transformation of the economy is driven by the savings propensities of workers (wages), capitalists (profits) and landlords (rent), and by the allocation of manufacturing output between investment and consumption goods, i.e. by the composition of demand in the manufacturing sector. Note that in this model, the investment ratio i_m/y_m is taken as given; it could be endogenized by means of an appropriate theory of investment demand. However this would be a rather ambitious project.

During the "Malthusian epoch", bondsmen in the countryside and workers in the cities did not save; their incomes stayed at subsistence level. Capitalists were more or less engaged in trade or in the putting-out-system, not in industry, which did not exist at that time. Landlords did not save significantly,[25] their style of living was "consumption-oriented". Landlords scorned work and mocked the merchant's thriftiness. Rent was spent on luxurious castles and palaces, clothes and banquets. Rather small parts of rent were saved and, if so, were mainly used to maintain buildings (stables, sheds, and palaces) and the stock of cattle. In England serfdom was abolished earlier than on the European continent, which compelled landlords to rent their land to farmers. These farmers (yeomen and commoners) introduced capitalistic norms into farming because they understood that they had to maximise the difference between revenue and cost (= rent plus wages). Additionally, they became interested in technical knowledge for improving the fertility of soil for crop enhancement. One innovation was the transition from the three-field system to the crop rotation system, which increased the cultivated area by one third.

Following Horrell and Humphries (1992) and Horrell (1996) that workers did not save and all of the savings came from landlords and capitalists, equation (31) is reduced to:

$$\rho = \frac{z}{z + q s_x} = \underset{\quad (+) \quad\;\; (-) \;\; (-)}{\varphi(i_m / y_m,\ s_c, s_x)} \tag{32}$$

with

$z \equiv i_m/y_m - (1\text{-}\beta)\, s_c$

and

$q \equiv \beta\,(1\text{-}\alpha)/\alpha > 0.$

[25] According to Hilton (1962), savings rates of medieval estates were less than 5%.

As can easily be shown, ρ is an increasing, concave function of i_m/y_m (alternatively, a decreasing, convex function of c_m/y_m) and a declining, convex function of s_c and s_x: In early phases of economic development, when agriculture dominated, the manufacturing sector mainly produced capital goods, in particular tools and intermediate goods for production. Later on, consumption goods became more and more important, in particular during the "Industrious Revolution", which preceded the Industrial Revolution, when a broad class of modern consumers with new tastes for luxuries and semi-luxuries emerged, see de Vries (2008). The fraction of labour employed in agriculture declined, which was reinforced by an increase in the savings propensity of land owners and capitalists. As a consequence, both the increasing relative importance of non-agricultural consumption goods and the higher growth of savings by landlords and capitalists contributed to an exodus of agricultural workers, leaving the sector with their technological know-how.[26] These workers found jobs in new industries, e.g. metal refining and fabrication of durable and non-durable consumer goods, in and around rapidly expanding urban centres.

4 Model Calibration[27]

To receive an endogenous determination of the agricultural labour ratio ρ, we have to model the law of motion for the development of total employment as well as the law of motion for the investment ratio in the manufacturing sector exogenously.[28] For total employment we assume that

$$L_t = L_{t-1}^{\theta_L}; \quad 1700 \leq t \leq 1920,$$

with $\theta_L = 1.0205$. The starting value for L is given by 2.998 million, respectively, based on estimates by Maddison. The investment ratio in manufacturing is assumed to follow the below-mentioned law of motion:

$$\left(\frac{i_m}{y_m}\right)_t = \left(\frac{i_m}{y_m}\right)_{t-1}^{\theta_m}; \quad 1700 \leq t \leq 1920,$$

where $\theta_m = 0.956$ and the starting value for 1700 is given by 3.34% based on the data by Feinstein (1988) and Broadberry and van Leeuwen (2008).

The development of total employment as well as the investment ratio in the manufacturing sector implied by the above laws of motion, with the parameters and starting values as indicated, are shown in Figure 5 below. The parameter choice of the exponents for the corresponding laws of motion ensure a close fit of the exogenously implemented variables L and i_m/y_m over time to real data.

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26 Note that this effect is cushioned by the productivity gain in agriculture stemming from a smaller labour force employed on more fertile land, see equation (23).

27 The simulations were carried out using MATLAB 7.1.

28 From a methodological point of view, the calibration model is similar to the one applied by Strulik and Weisdorf (2008), chapter 4. Strulik and Weisdorf calibrate their model such that it approximates the peak of population growth as well as the slowdown of total factor productivity. In contrast, we choose parameters such that the development of total employment and the *de facto* investment rate in manufacturing over the simulated period (1700-1920) is approximated as close as possible.

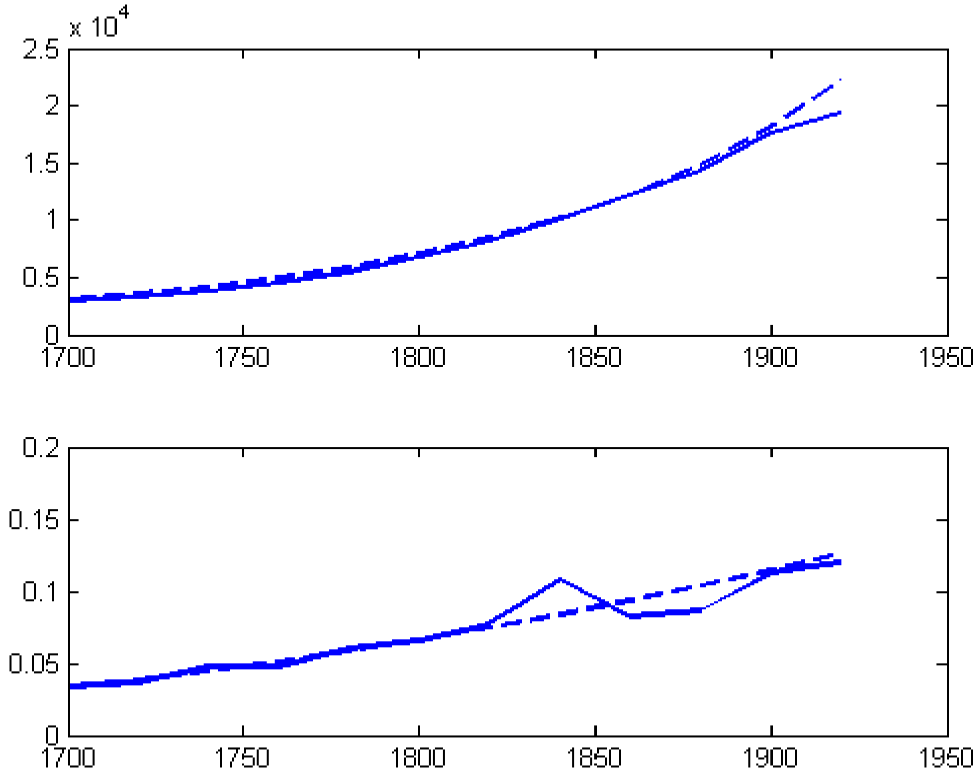

Figure 5: Development of L and i_m/y_m during 1700 – 1920. *Solid lines*: observed data; *dashed lines*: model outputs

We make use of the above assumption on savings that, on average, workers did not save. The savings rates of landlords and capitalists are implemented exogenously such that the fit of the exogenous laws of motion is close to reasonable data by various researchers, e.g., Williamson (1991) or Allen (2009).[29]

For the endogenous part of the model we set the following parameters:[30] $\alpha = 0.4$, $\beta = 0.65$, $\lambda = 0.2$, $\mu = 0.09$, $\delta = 0.05$. Starting values of the technology levels are $M_0 = 8.5$ and $A_0 = 0.677$. The area under cultivation is fixed at an average of 18 million hectares, based on the data by Mitchell and Deane (1962).

Simulation estimates for the wage-rent ratio ($\omega_{L/X}$), the share of agricultural labour in total labour supply (ρ), agricultural productivity (y_a), manufacturing productivity (y_m), and the capital intensity outside of agriculture (k_m) are obtained from equations (3), (4), (21), (24), (29), and (31), taking (8) and (9) into account.[31]

29 In fact, our estimates are somewhat lower, since our model abstracts from agricultural investment. Note that the latter remained rather flat during 1851-1920, while, e.g., investment in manufacturing increased by the factor 8 in the same period. See Feinstein (1988), p. 444-445.

30 In 1700 total land rents held a share of 40% in net farm outputs, see Clark (2001). Since our model ignores capital payments in agriculture which were in the range of 20%, we use a broader definition of land rents including profits, resulting in a 60% share of land rents and hence a 40% share of farm wages as a rough estimate for α. The parameter setting of $\beta = 0.65$ corresponds to the wage share typical of modern economies. λ, μ, A_0, and M_0 are derived from data included in Clark (2001) and Crafts and Harley (1992).

31 Since consistent data on the relative price of agricultural products in terms of non-agricultural goods and services are not available for the period 1700 – 1920, we do not simulate P.

The characterization of the model needs some qualification when comparing the simulation outputs with historical data. Until now we have used the standard terminology of two-sector models, distinguishing between “agriculture” and “manufacturing”. However, such a distinction is not adequate from an empirical point of view if there are other significant sectors in addition to the latter, as was the case for the British economy between 1700 and 1920. To handle the observed data consistently, we stick to the two-sector model, but call the second sector “non-agriculture” instead of “manufacturing”, because otherwise the labour shares in the two sectors would not sum up to unity and “capital” would cover the stock of domestic reproducible fixed assets in manufacturing only.

The development of $\omega_{L/X}$ and ρ is plotted in Figure 6, again with solid lines for point observations in intervals of 20 years and dashed lines for simulation outputs.

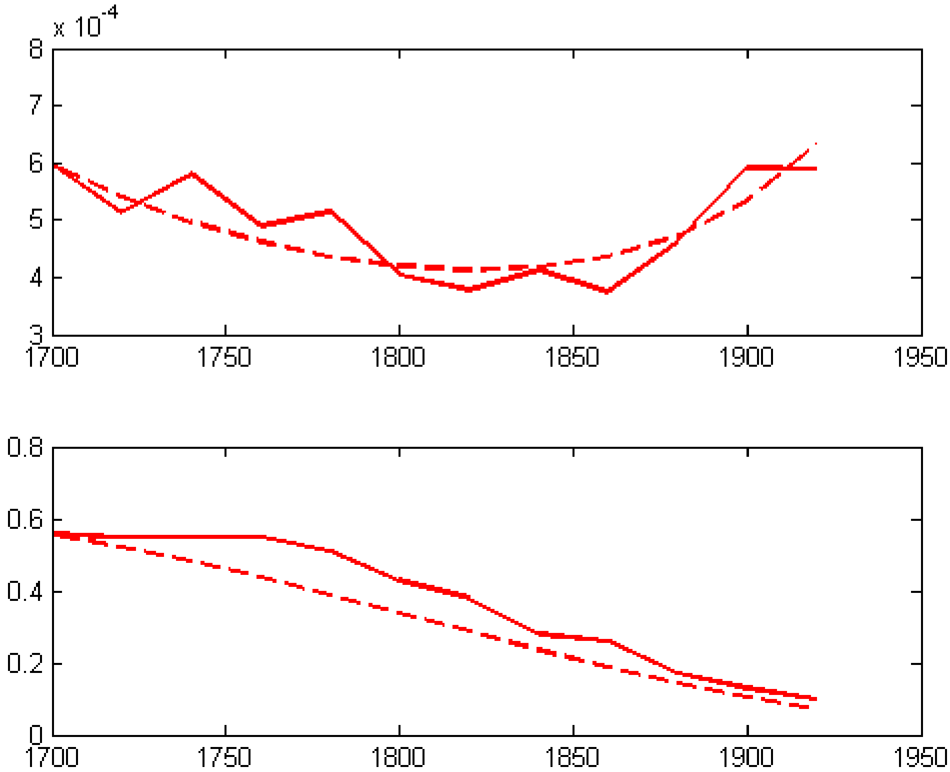

Figure 6: Development of $\omega_{L/X}$ and ρ during 1700 – 1920. *Solid lines*: observed data; *dashed lines*: model outputs

As Figure 6 indicates, the model replicates the history of $\omega_{L/X}$ during 1700 – 1920 quite adequately. In particular, it passes the fundamental test of showing the turnaround of the wage-rent ratio around 1850. The model also reproduces the downward movement of ρ satisfactorily.

Actual and simulated values of y_a, y_m, and k_m are plotted in Figure 7. The output-labour ratios in agriculture and the remaining economy are tracked quite closely; the simulation result of k_m is excellent. Note that the anaemic development of the capital intensity ended around 1800, with a dramatic upheaval in subsequent years, in particular after 1850.

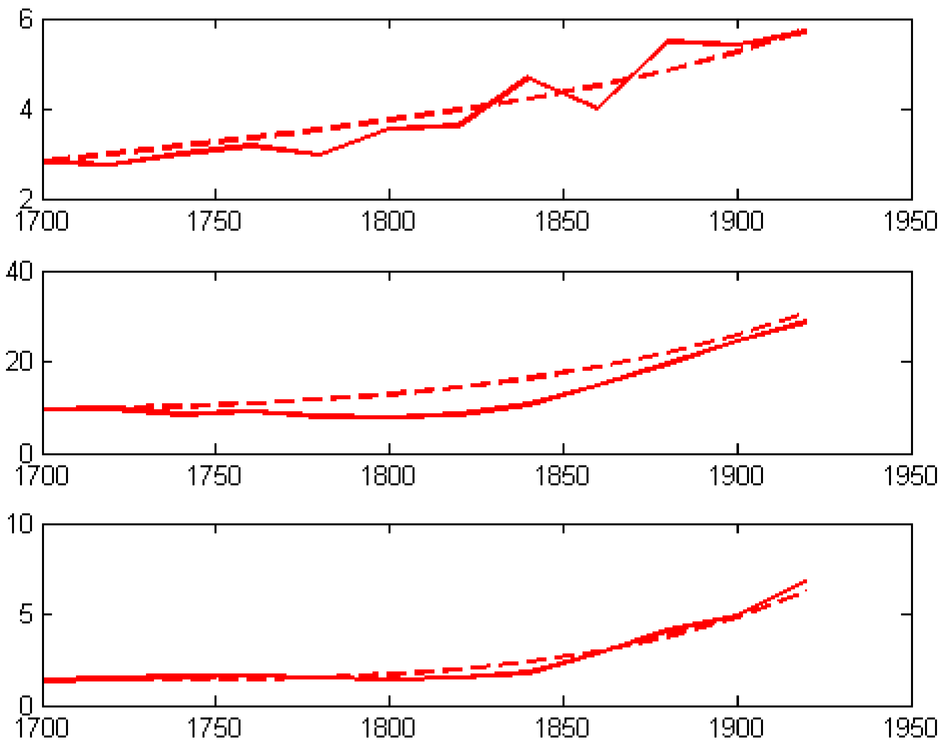

Figure 7: Development of y_a, y_m, and k_m during 1700 – 1920. *Solid lines*: observed data; *dashed lines*: model outputs

5 Conclusions

In this paper we have presented a "mechanical" growth model composed of three building blocks: (a) a two-sector specific factors model covering the sectors agriculture and manufacturing; (b) endogenous technical progress functions for the two sectors, which introduce dynamics into the specific factors model; and (c) a Kaldorian savings function which distinguishes between savings by workers, capitalists, and landlords.

Despite its simplicity, the model can describe major empirical trends in economic development over the very long run: It highlights (a) the parallel development of the land-labour ratio and the wage – land rent ratio until 1850; (b) the reversal of this relationship in the 19th century; and (c) the transformation of the U.K. from an economy based on agriculture to one dominated by industry – a creeping process which had gradually got under way but which massively accelerated after the Industrial Revolution. This transformation becomes manifest in the development of the ratio of agricultural employment to total employment, which is a key variable of the model since it plays an important role in the development of the distribution of income, in the long-term trend of productivity growth, and in the process of capital accumulation.

The model calibration confirms also results of O'Rourke and Williamson (2005) who described the long-term behaviour of the land-labour ratio, the wage-rent ratio, the relative price of agricultural output, industrial labour productivity, and of the total factor productivity in agriculture. In addition, what the model brings to light is a structural break in the development of agricultural labour productivity at some point between 1830 and 1850. Data shortages do not allow the simulation of the relative price of agricultural products in terms of non-agricultural goods and services.

In comparison to competing unified growth models, the above model is more conventional in two respects: It is a closed economy model which does not deal with the effects of global market integration, and it models physical instead of human capital accumulation. This does not mean that the growing openness to international trade and the behavioural changes in parents' preferences with regard to their offspring ("child quantity - quality trade-off") is denied. Instead, we simply confirm the more classical view that technical progress, physical capital accumulation and the role of income distribution were the driving forces of economic development during the Industrial Revolution.

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Appendix I (Notation of variables)

Y_a:	output of agricultural good
Y_m:	output of manufactured good
P:	price of agricultural good relative to manufactured good ($P = P_a; P_m \equiv 1$)
L_a:	quantity of labour employed in agriculture
L_m:	quantity of labour employed in manufacturing
X:	total quantity of land under cultivation
K:	total quantity of capital
x:	land-labour ratio $= X/L$
k:	capital-labour ratio $= K/L$
x_a:	land-labour ratio in agriculture $= X/L_a$
k_m:	capital-labour ratio in manufacturing $= K/L_m$
i_m:	investment per unit of labour employed in manufacturing
c_a:	consumption of agricultural goods per unit of labour employed in agriculture
c_m:	consumption of manufactured goods per labour unit employed in manufacturing
A:	level of technology in agriculture
M:	level of technology in manufacturing
W:	wage rate of homogenous labour
R_X:	rental rate of land
R_K:	rental rate of capital
$\omega_{L/X}$:	factor-price ratio between labour and land
$\omega_{L/K}$:	factor-price ratio between labour and capital
y_j:	output-labour ratio in sector j, $j = a, m$
n:	growth rate of labour force
ρ:	ratio of labour employed in agriculture $= L_a/L$
δ:	rate of depreciation
s_w, s_c, s_x:	gross savings propensities of wage-earners, capitalists, and landlords
$\alpha, \beta, \lambda, \mu$:	constant parameters

Appendix II (Statistical data)

Employment (in agriculture and non-agriculture)

1700-1840: Maddison (2007); missing data geometrically interpolated

1860-1920: Feinstein (1972); employment in non-agriculture adjusted by an index which describes the development of annual working hours

GDP (total and sectoral)

1700-1840: Broadberry and van Leewen (2008)

1860-1920: Data for 1700-1840 prolonged by growth rates derived from Feinstein (1972); Y_m comprises industry, transport/ communications, distribution and other services

Capital

K_m means total gross stock of capital minus agricultural fixed assets minus dwellings.

1700-1760: Linear adjustment of 1760-80 data by Feinstein and Pollard (1988); Great Britain only

1760-1850: Feinstein and Pollard (1988); Great Britain only

1850-1920: Feinstein and Pollard (1988); U.K.

Investment

I_m is defined as total gross domestic captial formation excluding agricultural and residential investment. Source of data as for capital.

Wage-rent ratio

Decadal averages from Clark (2001); England and Wales only

Total factor productivity in agriculture

Decadal averages form Clark (2001), England and Wales only