Multiple Revision on Horn Belief Bases

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Abstract. In logic programming, Horn clauses play a basic role, and in many logical constructs their consideration is important. In this paper we study the multiple revision of a belief base where the underlying logic is composed by Horn clauses. The main difficulties as to restricting to the Horn fragment for revision operators by a single sentence are analyzed, and general results are presented about *multiple revision* operators on belief bases. We define *prioritized multiple revision* operators under a more restricted logic than classical propositional logic, i.e. *Horn logic*. We propose a set of postulates and representation theorems for each operation. This work is relevant for multiple revision in areas that employ Horn clauses, such as logic programming and deductive databases applications.

Keywords: Horn Belief Revision, Belief Base, Multiple Change.

1 Introduction

1.1 Motivation

Belief revision studies the process by which a rational agent changes its current beliefs due to the arrival of new information. The best known method so as to carry out this change of belief in a rational agent is the AGM paradigm (see Alchourrón, C. et al. [1]). In recent works, generalizations of this framework, which considers the new information (epistemic input) as a set of logical sentences, were presented. In them, we see the importance of considering multiple change operators because they offer the possibility of taking incoming information as separate pieces of information that could be treated differently during the process of change (see Fuhrmann, A. et al. [7], Falappa, M. et al. [6]).

In recent years there have been several studies that show a significant effort in defining operations of contraction and revision AGM style operating under Horn logic (see Wassermann, R. et al. [14], Zhuang, Z. [15], Valdez, N. et al. [13]). From these studies it is noted that there are some differences between classical AGM revision and Horn revision. These differences occur from the weakened expressibility of Horn clause theories. In reality, the problem is not in the Horn revision specifically. Rather, it is considered that when we focus on Horn theories, belief change operators are not interdefinable, or rather, not easily interdefinable (see Zhuang, Z. et al. [16]). Belief change under Horn logic is important for several reasons. Firstly, many artificial intelligence systems are expressed in Horn clause language, and have also found widespread use in database theory, in areas such as logic programming, truth maintenance systems and deductive databases.

In this paper, we consider a type of multiple change under the language (expressively weak) of Horn clauses. We focus on belief revision, where new information is consistently incorporated into *the belief base* of the agent. We treat prioritized multiple change operators, where it is assumed that the new beliefs should be fully accepted. Every sentence of this input set is simultaneously processed in the revision[3]. As the main contribution of this paper, we characterize Horn prioritized multiple revision through a set of postulates. In each case, we present different constructions via kernel change and partial meet change techniques.

1.2 Preliminaries

We present here the terminology that we will use in the rest of the paper. We consider a propositional language $\mathcal{L}$, on a set of literals $\mathbf{P} = \{a, b, \ldots\}$, with semantics of a standard theoretical model. We adopt a propositional languaje $\mathcal{L}$ with a complete set of boolean connectives: $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$. The symbol $\top$ represents a tautology o *truth* and the symbol $\perp$ represents a contradiction or *falsum*. Lowercase Greek characters $\alpha, \delta, \ldots$ denote formulas and uppercase Latin characters $A, B, C, \ldots$ denote sets of formulas. The characters γ and σ are reserved to represent the functions of selection and incision to change operators, respectively. We use a consequence operation Cn that takes each set of sentences to obtain another set of sentences and satisfies the standard properties of Tarski.

We make extensive use of the theory of Horn logic. A Horn clause is a clause with at most one positive literal. A Horn formula is a conjunction of Horn clauses. A Horn theory is a set of Horn formulas. The Horn language $\mathcal{L}_H$ is a restriction $\mathcal{L}$ for Horn formulas. The Horn logic obtained $\mathcal{L}_H$ has the same semantics as propositional logic obtained from $\mathcal{L}$, but restricted to Horn formulas.

The remainder of this paper is organized as follows: Section 2 revision basic notions are given, presenting classic single revision and then multiple revision. We conclude this section by introducing beliefs change with a Horn logic, and related works that have been carried out in this area reasoning. In Section 3 a representation of results is presented for multiple revision in Horn fragment. This extension is completed defining two operators of Horn prioritized multiple revision: Horn prioritized partial meet multiple revision operator, and Horn prioritized kernel multiple revision operator. In Section 3, we present the conclusions and possible research lines for future work.

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[3] We do not consider the partial acceptance or disjunctive acceptance of new incoming beliefs (Falappa, M. et al. [6]).

2 Background

2.1 Multiple Revision

The term *multiple revision* is used to refer to revision operations which allow simultaneous revision by more than one sentence. They must be distinguished from *repeated* or *iterated* revision, i.e., the application of two or more revisions in a sequence. Delgrande and Yi [3] developed a general framework for belief revision called *parallel belief revision*. They present a *basic approach*, where they develop the minimal conditions for revision by a set of formulas. In the basic approach, they perform an adaptation of the AGM postulates for revision by a set of formulas, which in turn are similar to the postulates given in [12], adapted for belief states. It is shown in [12] that Groves [9] representation theorem can be generalized for revision by a set of formulas.

Given that we deal with a finite language, the systems of spheres of Grove's construction are interdefinable with faithful rankings, and so the representation theorem of [10] can also be generalized. A revision operator $\otimes$ satisfies basic and supplementary postulates for revision $(K \otimes 1) - (K \otimes 8)$ iff there exists a faithful ranking $\preceq_K$ for an arbitrary belief state K, such that for any set of sentences S: $(K \otimes S) = T(min(Mod(S), \preceq_K))$ [3].

Falappa et al. [6] presented various types of constructions to characterize multiple belief change (syntactic point of view). Here, we consider two of those kinds of constructions resulting from generalization techniques of revision on belief bases.

2.2 Belief change with Horn Logic

In the AGM model, an underlying logic is assumed that is at least as expressive as propositional logic and wherein a set of beliefs is represented by its logical closure. For several reasons work with sets deductively closed is not attractive from a computational point of view. Many systems operate under nonclassical logic, so the AGM model cannot be directly applied. Preferably, Horn revision theory does not diverge too much from the theory of regular revision, as the same intuitions guide both inquiries. A Horn revision operator will be a function which maps Horn knowledge bases and Horn formulas to Horn knowledge bases.

Delgrande and Peppas [4] present the main difficulties in restricting to the Horn fragment, (from a semantic point of view), as well as what can be done to overcome them. They consider Katsuno and Mendelzon's representation theorems [10] and wonder if this result representation which connects the revision postulates with faithful assignments still holds in the Horn fragment, and if any concrete operators exist for the Horn fragment.

3 Horn Multiple Revision

In this section, we develop multiple revision of Horn belief base and propose new postulates of rationality that are adopted from the postulates for multiple revision of Falappa et al. [6]. In these postulates, prevailing three fundamental principles: the new information (set of sentences) must appear at the Horn belief base revised, the revised base must be consistent and the operation of multiple revision have to change beliefs as least as possible. Let HB be (for its acronym in English: **H**orn belief **B**ase) a Horn belief base, A, and B set of Horn clauses, and $*$ an operator of Horn multiple revision that satisfies the following postulates:

($HB*1$) **Inclusion** $HB*A \subseteq HB \cup A$.
($HB*2$) **Success** $A \subseteq HB*A$.
($HB*3$) **Weak Success** If A is consistent then $A \subseteq HB*A$.
($HB*4$) **Relative Success** $A \subseteq HB*A$ or $HB*A = HB$.
($HB*5$) **Consistency** If A es consistent then $HB*A$ is consistent.
($HB*6$) **Vacuity 1** If A is inconsistent then $HB*A = HB$.
($HB*7$) **Vacuity 2** If $HB \cup A \nvdash \perp$ then $HB*A = HB \cup A$.
($HB*8$) **Uniformity 1** Given A and B two consistent sets, for all subset X of HB, if $(X \cup A) \vdash \perp$ if and only if $(X \cup B) \vdash \perp$ then $HB \backslash (HB*A) = HB \backslash (HB*B)$.
($HB*9$) **Uniformity 2** Given A and B two consistent sets, for all subset X of HB, if $(X \cup A) \vdash \perp$ if and only if $(X \cup B) \vdash \perp$ then $HB \cap (HB*A) = HB \cap (HB*B)$.
($HB*10$) **Relevance** If $\alpha \in HB \backslash (HB*A)$ then there is a set C such that $HB*A \subseteq C \subseteq (HB \cup A)$, C is consistent with A but $C \cup \{\alpha\}$ is inconsistent with A.
($HB*11$) **Core-Retainment** If $\alpha \in HB \backslash (HB*A)$ then there is a set C such that $C \subseteq (HB \cup A)$, C is consistent with A but $C \cup \{\alpha\}$ is inconsistent with A.

These postulates are adaptations of similar postulates from multiple revision [6].

3.1 Horn Prioritized Multiple Revision

The purpose of this paper is to extend to the Horn logic the two classes of operators of prioritized multiple revision. Here also, for the construction of operators we will consider the generalization techniques of revision of classical belief base.

Horn Prioritized Multiple Partial Meet Revision

We begin by to obtain the operator of Horn prioritized multiple partial meet revision, but we need first to define the set of consistent remains A-consistent-remainders.

Definition 1. *Let HB be and Horn belief base, and A consistent set of Horn clauses. The set of A-*consistent-remainders *of HB, noted by $HB \perp_{\top} A$, is the set of sets X such that:*

1. *$X \subseteq HB$.*
2. *$X \cup A$ is consistent.*
3. *For any X' such that $X \subset X' \subseteq HB$ then $X' \cup A$ is inconsistent.*

That is, $HB \perp_{\top} A$ is the set of maximal HB-subsets consistent with A.

The revision by a set of Horn clauses is based on the concept of A-consistent-remainders. In order to complete the construction, we must define a selection function that selects the best consistent remainders.

Definition 2. *Let HB be and Horn belief base. γ is and* consolidated selection function *for HB if and only if, for all Horn clauses set A:*

1. *If $HB\underline{\perp}_{\top}A \neq \varnothing$ then $\varnothing \neq \gamma(HB\underline{\perp}_{\top}A) \subseteq HB\underline{\perp}_{\top}A$.*
2. *If $HB\underline{\perp}_{\top}A = \varnothing$ then $\gamma(HB\underline{\perp}_{\top}A) = \{HB\}$.*

Observation 1 *Let HB be Horn belief base and A and consistent set of Horn clauses. Suppose that $\alpha \in HB$ and $\alpha \in A$, then $\alpha \in X$ for all $X \in HB\underline{\perp}_{\top}A$ and, therefore, $\alpha \in \bigcap(HB\underline{\perp}_{\top}A)$.*

Proof. Follows from the result of (Falappa et al. [6]).

From the above observation and Definition **2** it follows that all the Horn sentences of $HB \cap A$ are *'protected'* due to that they are included in the intersection of any collection of remainders. That is, a consolidated selection function selects a subsets of the set $HB\underline{\perp}_{\top}A$ whose elements all contain the set $HB \cap A$.

Definition 3. *Let HB be and Horn belief base and γ consolidated selection function for HB. The* Horn prioritized multiple partial meet revision *on HB that is generated by γ is the operator $*_\gamma$ such that, for all set of Horn clauses A:*

$$HB *_\gamma A = \begin{cases} \bigcap \gamma(HB\underline{\perp}_{\top}A) \cup A & \text{if } A \text{ is consistent} \\ HB & \text{otherwise} \end{cases}$$

An operator $*$ is and Horn prioritized multiple partial meet revision on HB if and only if there is and consolidated selection function γ for HB such that for all sets A, $HB * A = HB *_\gamma A$.

Example 1. Let $HB = \{p, q, q \to r, r \to s, q \wedge s \to t\}$ y $A = \{\neg p, \neg t\}$. Then we have $HB\underline{\perp}_{\top}A = \{\{q, q \to r, r \to s\}, \{q \to r, r \to s, q \wedge s \to t\}\}$. We have three possible outcomes for the consolidated selection function and operators Horn prioritized multiple partial meet revision associated.

$\gamma_1(HB\underline{\perp}_{\top}A) = \{q, q \to r, r \to s\}$ y $HB *_{\gamma_1} A = \{q, q \to r, r \to s, \neg p, \neg t\}$.
$\gamma_2(HB\underline{\perp}_{\top}A) = \{q \to r, r \to s, q \wedge s \to t\}$ y $HB *_{\gamma_2} A = \{q \to r, r \to s, q \wedge s \to t, \neg p, \neg t\}$.
$\gamma_3(HB\underline{\perp}_{\top}A) = \{q \to r, r \to s\}$ y $HB *_{\gamma_3} A = \{q \to r, r \to s, \neg p, \neg t\}$.

Theorem 1. *For each Horn belief base HB, $*$ is and operator* Horn prioritized multiple partial meet revision *if and only if satisfies $(HB*1)$, $(HB*3)$, $(HB*5)$, $(HB*6)$, $(HB*9)$ and $(HB*10)$.*

Proof. **- Postulates to Construction** Let HB be a set of Horn sentences and $*$ and operator that satisfies the postulates $(HB*1)$, $(HB*3)$, $(HB*5)$, $(HB*6)$, $(HB*9)$, and $(HB*10)$. We must show that $*$ is and Horn prioritized multiple partial meet revision.

Part A:

1. γ is a well defined function. Suppose that $HB\underline{\perp}_{\top}A = HB\underline{\perp}_{\top}B$ for A and B two consistent sets. It follows from $(HB*9)$ that $HB \cap (HB * A) = HB \cap (HB * B)$. Therefore, $\gamma(HB \cap (HB * A)) = \gamma(HB \cap (HB * B))$.
2. If $HB\underline{\perp}_{\top}A = \varnothing$ then $\gamma(HB\underline{\perp}_{\top}A) = \{HB\}$ from the definition of γ.

3. If $HB \perp\!\!\!\perp_{\top} A \neq \varnothing$ we must show that $\varnothing \neq \gamma(HB \perp\!\!\!\perp_{\top} A) \subseteq HB \perp\!\!\!\perp_{\top} A$. It follows from the definition of γ that $\gamma(HB \perp\!\!\!\perp_{\top} A) \subseteq HB \perp\!\!\!\perp_{\top} A$. Now we prove that $\gamma(HB \perp\!\!\!\perp_{\top} A) \neq \varnothing$. By $(HB * 3)$ it holds that $A \subseteq HB * A$ and, consequently $(HB \cap (HB * A)) \cup A \subseteq HB * A$. On the other hand, it follows from $(HB * 5)$ that $HB * A \nvdash \perp$ and, therefore, we can conclude that $(HB \cap HB * A) \cup A \nvdash \perp$. Hence, there must be some set $X \in HB \perp\!\!\!\perp_{\top} A$ such that $HB \cap HB * A \subseteq X$. Thus $\gamma(HB \perp\!\!\!\perp_{\top} A) \neq \varnothing$.

Part B: $*_\gamma$ is equal to $*$, that is, $HB *_\gamma A = HB * A$, for any set A. Suppose that A is consistent.

a Let $\alpha \in HB * A$. We must show that $\alpha \in HB *_\gamma A$. We have two cases:
  - $\alpha \in A$ then it is trivial that $\alpha \in HB *_\gamma A$.
  - $\alpha \notin A$. By $(HB * 1)$ it holds that $HB * A \subseteq HB \cup A$ and we can conclude that $\alpha \in HB$. Hence, $\alpha \in HB \cap HB * A$. Therefore, it follows from the definition of γ that $\alpha \in X$ for all $X \in \gamma(HB \perp\!\!\!\perp_{\top} A)$.

b Let $\alpha \notin HB * A$. We must show that $\alpha \notin HB *_\gamma A$. If $\alpha \notin HB * A$ we need to find some $X \in \gamma(HB \perp\!\!\!\perp_{\top} A)$ such that $\alpha \notin X$. We have two cases:
  - $\alpha \in HB$. By $(HB * 10)$ there exists some C such that $HB * A \subseteq C \subseteq (HB \cup A)$, $C \cup A \nvdash \perp$ and $C \cup A \cup \{\alpha\} \vdash \perp$. Then we may infer that $\alpha \notin A$ and $\alpha \notin C$. Let $T = HB \cap C$. Then $\alpha \notin T$, $(HB \cap (HB * A)) \subseteq T \subseteq HB$, $T \cup A \nvdash \perp$ and $T \cup A \cup \{\alpha\} \vdash \perp$. Then we may extend T to a maximal subset X of HB consistent with A such that $(HB \cap (HB * A)) \subseteq X$ and $\alpha \notin X$. Therefore, $X \in \gamma(HB \perp\!\!\!\perp_{\top} A)$ and $\alpha \notin X$.
  - $\alpha \notin HB$. Then no set in $HB \perp\!\!\!\perp_{\top} A$ will contain α.

$(H * 6)$ covers the limit case in with A is inconsistent.

- **Construction to Postulates** Let $*_\gamma$ be is and operator Horn prioritized multiple partial meet revision on HB that is generated by γ, being γ an arbitrary consolidated selection function for HB. we must show that $*_\gamma$ satisfies the postulates $(HB * 1)$, $(HB * 3)$, $(HB * 5)$, $(HB * 6)$, $(HB * 9)$, and $(HB * 10)$.

  $(HB * 1)$ As during the revision is obtained $X \subseteq HB$ being $X \in HB \perp\!\!\!\perp_{\top} A$, then this postulate is trivially shown.

  $(HB * 3)$ Suppose that A is consistent. By definition $A \subseteq HB *_\gamma A$, then it is trivially shown.

  $(HB * 5)$ Suppose that A is consistent. If $HB \perp\!\!\!\perp_{\top} A \neq \varnothing$ then by definition every $X \in HB \perp\!\!\!\perp_{\top} A$ is consistent with A. Therefore, the intersection of any subset of $HB \perp\!\!\!\perp_{\top} A$ is consistent with A. We conclude that $HB *_\gamma A$ is consistent.

  $(HB * 6)$ Trivial by definition.

  $(HB * 9)$ Given A y B two consistent sets. We must show that $HB \cap (\bigcap \gamma(HB \perp\!\!\!\perp_{\top} A) \cup A) = HB \cap (\bigcap \gamma(HB \perp\!\!\!\perp_{\top} B) \cup B)$. Now, observing that: $HB \cap (\bigcap \gamma(HB \perp\!\!\!\perp_{\top} A) \cup A) = (HB \cap (\bigcap(\gamma(HB \perp\!\!\!\perp_{\top} A))) \cup (HB \cap A) = (\bigcap \gamma(HB \perp\!\!\!\perp_{\top} A)) \cup (HB \cap A) = \bigcap \gamma(HB \perp\!\!\!\perp_{\top} A)$, where the last equality is due to the fact that $HB \cap A \subseteq X$ for all $X \in HB \perp\!\!\!\perp_{\top} A$ (see Observation **1**) and, consequently, $HB \cap A \subseteq \bigcap \gamma(HB \perp\!\!\!\perp_{\top} A)$. Analogously: $HB \cap (\bigcap \gamma(HB \perp\!\!\!\perp_{\top} B) \cup B) = \bigcap \gamma(HB \perp\!\!\!\perp_{\top} B)$. Since γ is a well defined function then $\gamma(HB\perp_{\top}A) = \gamma(HB\perp_{\top}B)$ (attentive to $\gamma(HB\perp_{\top}A) = \gamma(HB\perp_{\top}B)$ continue our assumption that for all subsets X de HB, $(X \cup A) \vdash \bot$ if and only if $(X \cup B) \vdash \bot$). Hence, $\bigcap\gamma(HB\perp_{\top}A) = \bigcap\gamma(HB\perp_{\top}B)$. Therefore: $HB \cap (\bigcap\gamma(HB\perp_{\top}A) \cup A) = HB \cap (\bigcap\gamma(HB\perp_{\top}B) \cup B)$.

$(HB * 10)$ Suppose that $HB\perp_{\top}A \neq \varnothing$. Give $\beta \in HB$ and $\beta \notin HB *_{\gamma} A$. It means there is some $X \in HB\perp_{\top}A$ such that $\beta \notin X$. Hence, there is some X such that $\beta \notin X$, $X \cup A$ is consistent but $X \cup A \cup \beta$ is inconsistent. Now, suppose that $HB\perp_{\top}A = \varnothing$ in this case A is inconsistent. By definition, $HB *_{\gamma} A = HB$ and relevance is vacuously satisfied.

Horn Prioritized Multiple Kernel Revision

Now let's get, the operator Horn prioritized multiple kernel revision, which is based on sets A-inconsistent-kernels.

Definition 4. *Let HB be and Horn belief base, and A consistent set of sentences of Horn clauses. The set of* A-inconsistent-kernels *of HB, noted by $HB\perp\!\!\perp_{\bot}A$, is the set of sets X such that:*

1. *$X \subseteq HB$.*
2. *$X \cup A$ is inconsistent.*
3. *For any X' such that $X' \subset X \subseteq HB$ then $X' \cup A$ is consistent.*

That is, $HB\perp\!\!\perp_{\bot}A$ is the set of minimal HB-subsets inconsistent with A.

In order to complete the construction, we must define an incision function that cuts in every inconsistent kernel.

Definition 5. *Let HB be a Horn belief base. σ is a consolidated incision function for HB if and only if, for all consistent set of Horn clauses A:*

1. *$\sigma(HB\perp\!\!\perp_{\bot}A) \subseteq \bigcup HB\perp\!\!\perp_{\bot}A$.*
2. *If $X \in HB\perp\!\!\perp_{\bot}A$ then $X \cap (\sigma(HB\perp\!\!\perp_{\bot}A)) \neq \varnothing$.*

Observation 2 *Let HB be a Horn belief base and A consistent set of Horn clauses. Suppose that $\alpha \in HB$ and $\alpha \in A$, then $\alpha \notin \bigcup(HB\perp\!\!\perp_{\bot}A)$ and , therefore, $\alpha \cap \bigcup(HB\perp\!\!\perp_{\bot}A) = \varnothing$.*

Proof. His demonstration is similar in procedure to the observation of **1**.

From the above observation and Definition **5** it follows that all the Horn sentences of A are *'protected'* it the sense that they can not be considered for removing by the consolidated incision function. That is, a consolidated incision function selects among the sentences of $HB\backslash A$ that make $HB \cup A$ inconsistent.

Definition 6. *Let HB be and Horn belief base and σ a consolidated incision function for HB. The* Horn prioritized multiple kernel revision *on HB that is generated by σ is the operator $*_{\sigma}$ such that, for all set of Horn clauses A:*

$$HB *_{\sigma} A = \begin{cases} (HB\backslash\sigma(HB\perp\!\!\perp_{\bot}A)) \cup A & \text{if } A \text{ is consistent} \\ HB & \text{otherwise} \end{cases}$$

An operator $*$ is a Horn prioritized multiple kernel revision for HB if and only if there is a consolidated incision function σ for HB such that for all sets A, $HB * A = HB *_{\sigma} A$.

Example 2. Let $HB = \{p, q, q \rightarrow r, r \rightarrow s, q \wedge s \rightarrow t\}$ and $A = \{\neg p, \neg t\}$. Then we have $HB {\perp\!\!\!\perp}_{\perp} A = \{\{p\}, \{q, q \rightarrow r, r \rightarrow s, q \wedge s \rightarrow t\}\}$. Some of the possible outcomes for the consolidated incision function and operators Horn prioritized multiple kernel revision associated are:

$\sigma_1(HB {\perp\!\!\!\perp}_{\perp} A) = \{p, q \wedge s \rightarrow t\}$ and $HB *_{\sigma_1} A = \{q \wedge s \rightarrow t, \neg p, \neg t\}$.
$\sigma_2(HB {\perp\!\!\!\perp}_{\perp} A) = \{p, q, q \wedge s \rightarrow t\}$ and $HB *_{\sigma_2} A = \{q, q \rightarrow r, r \rightarrow s, \neg p, \neg t\}$.
$\sigma_3(HB {\perp\!\!\!\perp}_{\perp} A) = \{p, q\}$ and $HB *_{\sigma_3} A = \{q \rightarrow r, r \rightarrow s, q \wedge s \rightarrow t, \neg p, \neg t\}$.
$\sigma_4(HB {\perp\!\!\!\perp}_{\perp} A) = \{p, q, q \rightarrow r\}$ and $HB *_{\sigma_4} A = \{r \rightarrow s, q \wedge s \rightarrow t, \neg p, \neg t\}$.

Theorem 2. *For each Horn belief base HB, $*$ is and operator* Horn prioritized multiple kernel revision *if and only if satisfies* $(HB * 1)$, $(HB * 3)$, $(HB * 5)$, $(HB * 6)$, $(HB * 8)$, *y* $(HB * 11)$.

Proof. **- Postulates to Construction** Let HB be Horn belief base and $*$ an operator that satisfies the postulates $(HB*1)$, $(HB*3)$, $(HB*5)$, $(HB*6)$, $(HB * 8)$, y $(HB * 11)$. We must show that $*$ is a Horn prioritized multiple kernel revision.

Part A:

1. σ is a well defined function. Suppose that $HB {\perp\!\!\!\perp}_{\perp} A = HB {\perp\!\!\!\perp}_{\perp} B$ for A and B two consistent sets. It follows from $(HB * 8)$ que $HB \backslash (HB * A) = HB \backslash (HB * B)$. Therefore, $\sigma(HB {\perp\!\!\!\perp}_{\perp} A) = \sigma(HB {\perp\!\!\!\perp}_{\perp} B)$.
2. $\sigma(HB {\perp\!\!\!\perp}_{\perp} A) \subseteq \bigcup HB {\perp\!\!\!\perp}_{\perp} A$. If $\alpha \in \sigma(HB {\perp\!\!\!\perp}_{\perp} A)$ then $\alpha \in \bigcup (HB {\perp\!\!\!\perp}_{\perp} A)$ and then $\alpha \in HB \backslash HB * A$. By $(HB * 11)$ there is any some C such that $C \subseteq (HB \cup A)$, $C \cup A \nvdash \bot$ but $C \cup A \cup \{\alpha\} \vdash \bot$. Let X be an arbitrary element $C \cup \{\alpha\} {\perp\!\!\!\perp}_{\perp} A$ where $C \cup \{\alpha\} {\perp\!\!\!\perp}_{\perp} A \neq \varnothing$). Then X such that $X \subseteq C \cup \{\alpha\}$, $X \cup A \vdash \bot$ and, for any X' such that $X' \subset X$, it holds that $X' \cup A \nvdash \bot$. Means $\alpha \in X$ and that $X \cap A = \varnothing$. As $X \subseteq C \cup \{\alpha\}$ and $C \subseteq (HB \cup A)$ we obtain that $X \subseteq HB$. Hence, $X \in HB {\perp\!\!\!\perp}_{\perp} A$ and, since $\alpha \in X$, we can conclude that $\alpha \in \bigcup (HB {\perp\!\!\!\perp}_{\perp} A)$.
3. Let $X \in HB {\perp\!\!\!\perp}_{\perp} A$. We need to show that $X \cap \sigma(HB {\perp\!\!\!\perp}_{\perp} A) \neq \varnothing$. Suppose that A is consistent. By $(HB * 5)$ then $HB * A$ is consistent. Since X is inconsistent with A by $(HB * 3)$ then $X \nsubseteq HB * A$. Therefore, there is some $\beta \in X$ and $\beta \notin HB * A$, which means that $\beta \in \sigma(HB {\perp\!\!\!\perp}_{\perp} A)$.

Parte B: $*_\sigma$ is equal to $*$, that is, $HB *_\sigma A = HB * A$, for any set A. Suppose that A is consistent.

a Let $\alpha \in HB * A$. It follows from the definition of $\sigma(HB {\perp\!\!\!\perp}_{\perp} A)$ que $\alpha \notin \sigma(HB {\perp\!\!\!\perp}_{\perp} A)$. By $(HB * 1)$ it holds that $HB * A \subseteq HB \cup A$ and we can conclude that $\alpha \in HB \cup A$. Therefore, by definition of $*_\sigma$, $\alpha \in HB *_\sigma A$.

b Let $\alpha \notin HB * A$. We must show that $\alpha \notin HB *_\sigma A$. By $(HB * 3)$ we have that $\alpha \notin A$. Since by definition $HB *_\sigma A$ is equal to $(HB \backslash \sigma(HB {\perp\!\!\!\perp}_{\perp} A)) \cup A$ then it only remains to show that $\alpha \notin HB \backslash \sigma(HB {\perp\!\!\!\perp}_{\perp} A)$. To do that we consider two cases:
- $\alpha \notin HB$. Then it follows trivially that $\alpha \notin HB \backslash \sigma(HB {\perp\!\!\!\perp}_{\perp} A)$.
- $\alpha \in HB$. Then it follows from definition of $\sigma(HB {\perp\!\!\!\perp}_{\perp} A)$ that $\alpha \in \sigma(HB {\perp\!\!\!\perp}_{\perp} A)$ and, therefore, $\alpha \in HB \backslash \sigma(HB {\perp\!\!\!\perp}_{\perp} A)$.

$(HB * 6)$ covers the limit case in with A is inconsistent.

- **Construction to Postulates** Let σ be an arbitrary consolidated incision function for HB and $*_\sigma$ be the Horn prioritized multiple kernel revision on HB that is generated by σ. Then, for all sets A we must show that $*_\sigma$ satisfies the postulates $(HB * 1)$, $(HB * 3)$, $(HB * 5)$, $(HB * 6)$, $(HB * 8)$, and $(HB * 11)$.
  $(HB * 1)$ Trivial by definition.
  $(HB * 3)$ Suppose that A is consistent. By definition $A \subseteq HB *_\sigma A$, then the proof is trivial.
  $(HB * 5)$ Suppose that A is consistent. Since, $\sigma(HB \perp\!\!\!\perp_{\perp} A)$ cuts every subset of HB inconsistent with A then $HB \backslash \sigma(HB \perp\!\!\!\perp_{\perp} A)$ is consistent with A.
  $(HB * 6)$ Trivial by definition.
  $(HB * 8)$ Given A and B two consistent sets. Suppose that for all subset X of H, $(X \cup A) \vdash \bot$ if and only if $(X \cup B) \vdash \bot$. Since σ is a well defined function then $HB \perp\!\!\!\perp_{\perp} A = HB \perp\!\!\!\perp_{\perp} B$, then $\sigma(HB \perp\!\!\!\perp_{\perp} A) = \sigma(HB \perp\!\!\!\perp_{\perp} B)$. Therefore, $HB \backslash (HB *_\sigma A) = HB \backslash (HB *_\sigma B)$.
  $(HB * 11)$ Let $\beta \in HB$ and $\beta \notin HB *_\sigma A$. Means $HB *_\sigma A \neq HB$ and, by definition $\beta \notin (HB \backslash \sigma(HB \perp\!\!\!\perp_{\perp} A)) \cup A$ y $\beta \in HB$, we can conclude that $\beta \in \sigma(HB \perp\!\!\!\perp_{\perp} A)$. By definition $\sigma(HB \perp\!\!\!\perp_{\perp} A) \subseteq \bigcup HB \perp\!\!\!\perp_{\perp} A$, and it follows that there is some $X \in HB \perp\!\!\!\perp_{\perp} A$ such that $\beta \in X$. X is a minimal HB-subset inconsistent with A. Let $Y = X \backslash \{\beta\}$. Then Y is such that $Y \subset X \subseteq HB \subseteq HB \cup A$, Y is consistent with A but $Y \cup \{\beta\}$ is inconsistent with A.

4 Conclusions and Future Work

In this paper we have explored the multiple revision on belief bases regarding Horn theories. One of our conclusions, tells us that there are some differences between classical AGM revision and Horn revision. These differences occur due to the weakened expressibility of Horn clause theories. Actually, the problem is not in the Horn revision specifically. Rather, it is considered that when we focus on Horn theories, belief change operators are not interdefinable, or rather, not easily interdefinable (see Zhuang, Z. et al. [16]). Preferably, the theory of Horn revision does not diverge too much from the theory of regular revision, as the same intuitions guide both inquiries.

On the other hand, if we focus on belief revision from a semantic point of view, we find some difficulties when restricting to the Horn fragment. This is because only the sets of interpretations closed under intersection can be represented in the Horn fragment. The problem this creates for belief revision is that the *standard model-based operators* might produce results that are not in the fragment. Its solution is to consider a condition *Horn compliant* limiting the allowable preorders for the generation of the model-based Horn revision and the addition of an *Acyc* postulate which restricts the cycles in their *faithful rankings* (for further reading see Delgrande and Peppas [4]).

Here, our research has a syntactic approach and it allowed us to realize the extent of prioritized multiple revision operators to Horn logic, preserving their intuitions according to its postulates and constructive models. The main contribution of our research is the construction of operators of prioritized revision under Horn logic. They are based on maximal consistent remainders (for Horn prioritized multiple partial meet revision) and minimal inconsistent kernels (for Horn prioritized multiple kernel revision). We have proposed a set of postulates and some relations among them. Also, we have presented a representation theorem for each operator. As future work, we will study merge operators or symmetric change operators in Horn fragments. Our aim is to find axiomatic characterizations for each case (partial meet and kernel). Subsequently, we want to investigate the selective multiple revision in Horn theory, that is, revisions where only a subset of the epistemic input is accepted.

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