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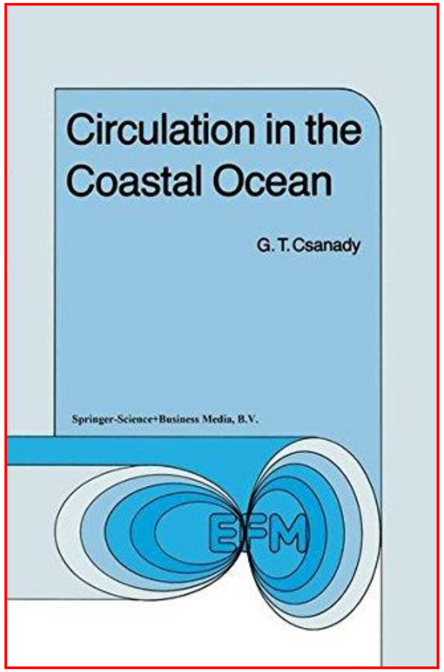
The coastal ocean response
To wind
(part-2)

Reference



Csanady: Chapter 2
"Inertial response to wind"
Sections 2.4, 2.5, 2.7

Wind “setup” (semi-infinite basin) Cross-shore wind

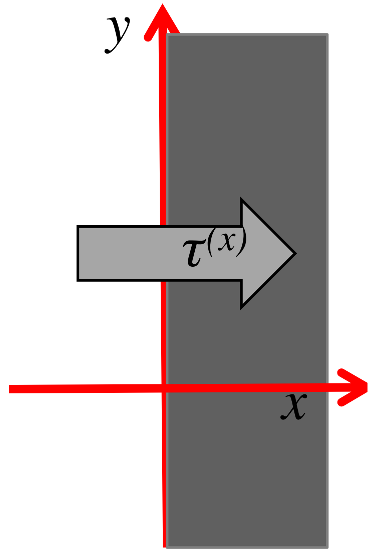
Consider:

Semi-infinite shallow basin with

$x \leq 0$

bounded by a straight infinitely long coast coincident with the y axis.

The basin is forced by a constant wind perpendicular to the coast:



$$\frac{\tau_w^{(x)}}{\rho} = u_*^2 \qquad \tau_w^{(y)} = 0$$

Bottom stress is also neglected

$$\tau_b^{(x)} = \tau_b^{(y)} = 0$$

Under such conditions NO differences in elevation along y can arise. Therefore the transport Equations become:

$$\frac{\partial U}{\partial t} - fV = -c^2 \frac{\partial \eta}{\partial x} + u_*^2$$

$$\frac{\partial V}{\partial t} + fU = 0$$

Long shore transport (V) can be generated ONLY if cross shore (U) transport is existing

Wind "setup" (semi-infinite basin)
Cross-shore wind

Also the equation $c^2 \frac{\partial^2 \eta}{\partial t^2} - c^2 \nabla^2 \eta + f^2 \eta = 0$ (see previous lecture)reduces to ($x \le 0$):

$$c^2 \frac{\partial^2 \eta}{\partial t^2} - c^2 \frac{\partial^2 \eta}{\partial x^2} + f^2 \eta = 0$$

At the coast ($x=0$) The no-Normal transport boundary condition ($U=0$ and then $V=0$) give the static wind setup

$$\boxed{\eta = \frac{u_*^2}{c^2} x}$$

and the above eq. becomes

$$\frac{\partial \eta}{\partial x} = \frac{u_*^2}{c^2}$$

Solving for η allow to solve for the long- (V) and cross- (U) shore transport.
Time independent <u>steady state</u> solutions are:

$$\eta = \frac{u_*^2}{fc} e^{x/R}$$

$$\boxed{R = \frac{c}{f}}$$

$$V = -\frac{u_*^2}{f}(1 - e^{x/R})$$

$$U = 0$$

Wind "setup" (semi-infinite basin) Cross-shore wind

The steady state solution is characterised by:

$V=0$ at $x=0$: No longshore transport at the coast.

$$\frac{\partial \eta}{\partial x} = \frac{u_*^2}{c^2}$$ at $x=0$: no rotation value of the elevation gradient at the coast.

$$V = \frac{u_*^2}{f}$$: Ekman transport far away from the coast

and illustrates how Earth rotation affects the propagation of a pressure signal:

The transition takes places on the scale of the deformation radius R

The establishment of the pressure field is described by the full solution of the Transport equation system (time dependent):

$$c^2 \frac{\partial^2 \eta}{\partial t^2} - c^2 \frac{\partial^2 \eta}{\partial x^2} + f^2 \eta = 0$$

Wind "setup" (semi-infinite basin) Cross-shore wind

An approximate solution valid for *ft>>1* (a long time after the imposition of the wind stress) is:

$$\eta = \frac{u_*^2}{fc}\left[e^{x/R} + \sqrt{\frac{2}{\pi}}\frac{\sin\left(ft - \pi/4\right)}{\sqrt{ft}} - \dots\dots\dots \right]$$

The time dependent solution is formed by the steady state solution plus inertial oscillations of Decaying amplitude.

INERTIAL OSCILLATIONS

Far away from coast and in absence of forcing we can postulate $\eta=0$ and the transport and continuity equations reduce to

$$\frac{\partial U}{\partial t} - fV = 0$$

$$\frac{\partial V}{\partial t} + fU = 0$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

Wind "setup" (semi-infinite basin) Cross-shore wind

$$\frac{\partial U}{\partial t} - fV = 0$$

$$\frac{\partial V}{\partial t} + fU = 0$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

Balance between local Acceleration and Coriolis force

whose solution is:

$$U = U_0 \cos ft$$

$$V = -U_0 \sin ft$$

With U_0= constant and coordinate axes chosen so that $V=0$ at $t=0$.
The interior velocities (local problem) are:

$$u = \frac{U_0}{H} = u_0 \cos ft$$

$$v = \frac{U_0}{H} = -u_0 \sin ft$$

Periodic motion with period $T=2\pi/f$ (half of a pendulum Day).

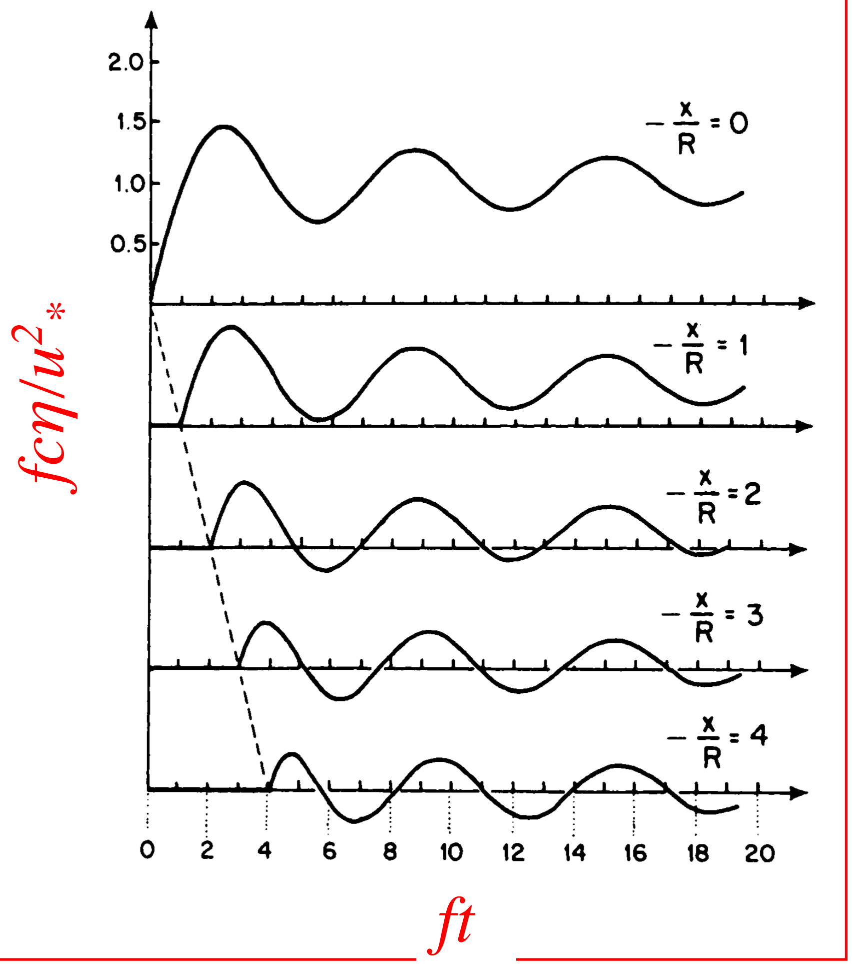
Wind "setup" (semi-infinite basin) Cross-shore wind

$$\eta = \frac{u_*^2}{fc}\left[e^{x/R} + \sqrt{\frac{2}{\pi}}\frac{\sin\left(ft - \pi/4\right)}{\sqrt{ft}} - \dots\dots\dots \right]$$

Inertial oscillation components

Oscillations are slowly decaying.
Decays occur trough dispersal of the waves to infinity

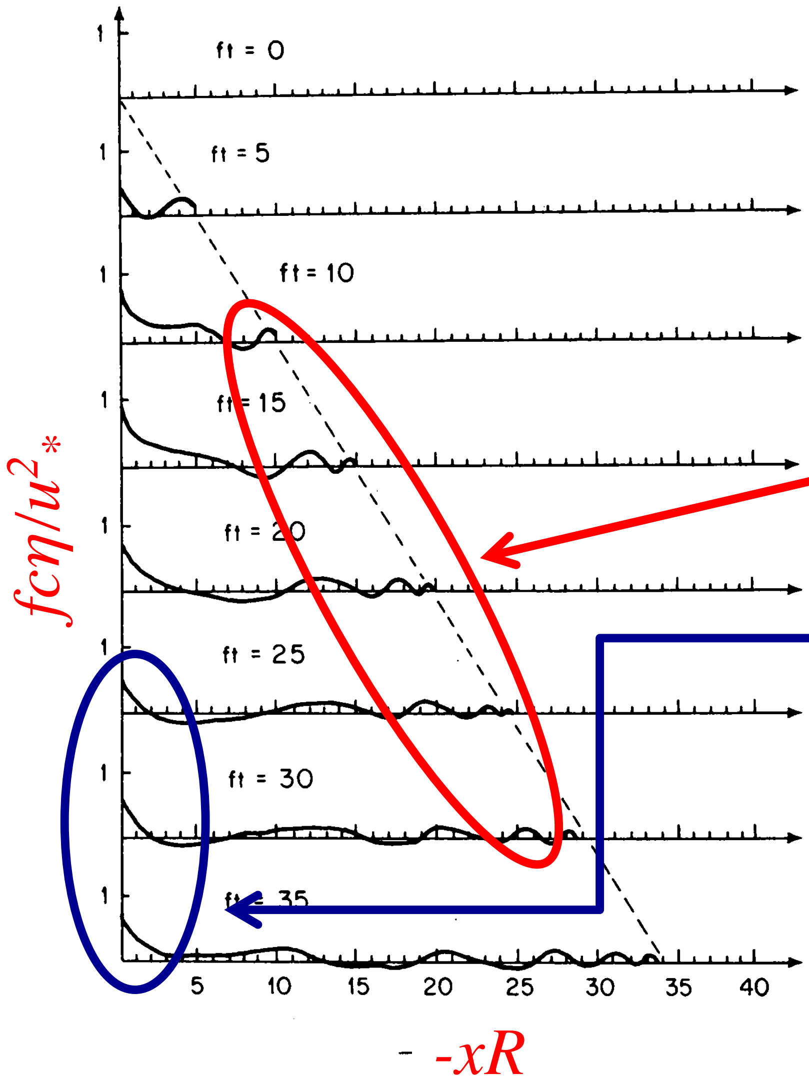
Wind "setup" (semi-infinite basin) Cross-shore wind



Development of setup and oscillations
For the cross-shore wind case.
Plots are for different distances from coast
At different times
Beyond a few radii of deformation
Only oscillations arise

$$\eta = \frac{u_*^2}{fc}\left[e^{x/R} + \sqrt{\frac{2}{\pi}}\frac{\sin\left(ft - \pi/4\right)}{\sqrt{ft}} - \dots\dots\dots\right]$$

Wind “setup” (semi-infinite basin) Cross-shore wind



Development of setup and oscillations
For the cross-shore wind case.
Sea surface elevation at successive times

Oscillations decay while original pulse
Travels to Infinity.

A wake of inertial oscillations is left behind the
Pulse.

Once the wake decays the static wind setup
remains but only at distances of order R from
The coast

$$\eta = \frac{u_*^2}{fc}\left[e^{x/R} + \sqrt{\frac{2}{\pi}}\frac{\sin\left(ft - \pi/4\right)}{\sqrt{ft}} - \ldots\ldots\ldots \right]$$

Wind "setup" (semi-infinite basin) Cross-shore wind

$$\frac{\partial U}{\partial t} - fV = -c^2 \frac{\partial \eta}{\partial x} + u_*^2$$

$$\frac{\partial V}{\partial t} + fU = 0$$

Long shore transport (V) can be generated ONLY if cross shore (U) transport is existing

At the coast:

$U=0$ at all times

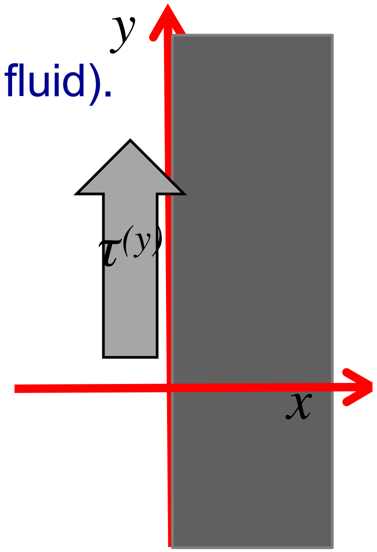
Then no long shore transport can develop

At distance of order R transient cross-shore transport Appears at times of order f^{-1}, allowing the establishment Of steady state along shore transport.

Wind "setup" (semi-infinite basin) Along-shore wind

Without Earth Rotation (trivial case)

Fluid accelerate downwind, no pressure field generated (no coast "damming" the fluid).



With Earth rotation

Generation of cross stream Coriolis force balanced by pressure field leading to Coastal sea Level changes.

Consider:

Semi-infinite shallow basin with

$x \leq 0$

bounded by a straight infinitely long coast coincident with the y axis.

The basin is forced by a constant wind parallel to the coast:

$$\frac{\tau_w^{(x)}}{\rho} = 0 \qquad \frac{\tau_w^{(y)}}{\rho} = u_*^2$$

Bottom stress is also neglected

$$\tau_b^{(x)} = \tau_b^{(y)} = 0$$

Wind "setup" (semi-infinite basin) Along-shore wind

Then the transport equations become:

$$\frac{\partial U}{\partial t} - fV = -c^2 \frac{\partial \eta}{\partial x}$$

$$\frac{\partial V}{\partial t} + fU = u_*^2$$

Again along y no pressure

Gradient can develop

At the coast the $(x=0)$ the boundary condition $U=0$ yields:

$$fV = c^2 \frac{\partial \eta}{\partial x} \qquad \frac{\partial V}{\partial t} = u_*^2$$

$$\frac{\partial \eta}{\partial x} = \frac{f}{c^2} V \qquad V = u_*^2 t$$

$$\frac{\partial \eta}{\partial x} = ft \frac{u_*^2}{c^2}$$

For the cross shore Wind case it was:

$$\frac{\partial \eta}{\partial x} = \frac{u_*^2}{c^2}$$

- At the boundary (ONLY!!!) fluid accelerates downwind, as in the trivial no rotation case.
- The developing coastal current is in geostrophic balance with the developing pressure field.

Wind "setup" (semi-infinite basin) Along-shore wind

The equation:

$$c^2 \frac{\partial^2 \eta}{\partial t^2} - c^2 \frac{\partial^2 \eta}{\partial x^2} + f^2 \eta = 0$$

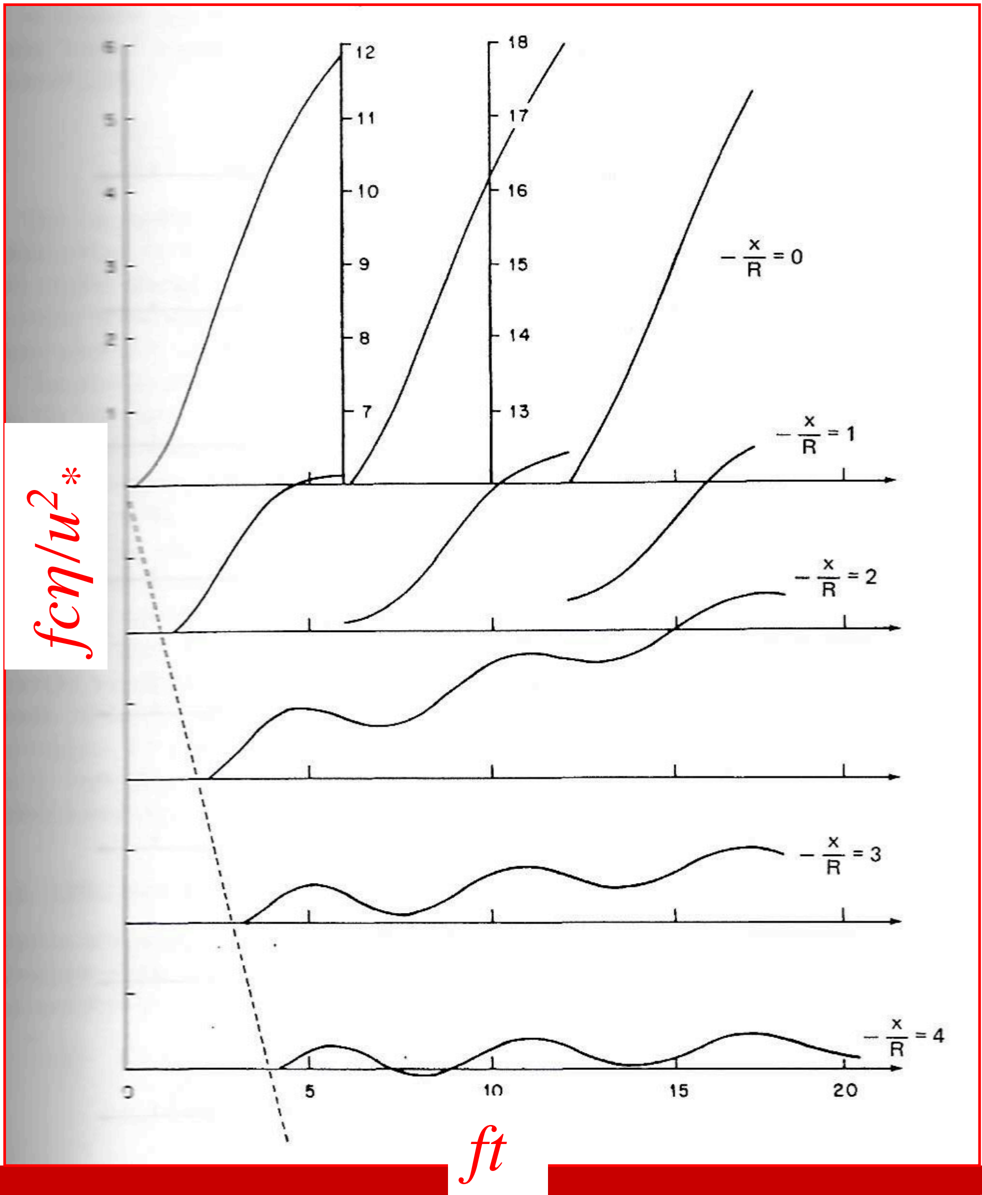
Used for the cross-shore wind can still be used in order to solve for η

An approximate solution valid for $ft>>1$ is

$$\eta = \frac{u_*^2}{fc}\left[(ft)e^{x/R} - \sqrt{\frac{2}{\pi}}\frac{\sin(ft-\pi/4)}{\sqrt{ft}} + \ldots\ldots\ldots\right]$$

That describes a pressure field with inertial oscillations of decaying amplitude (as for the cross-shore wind case, BUT, differently from the previous case the NON oscillatory part (confined to a coastal band of scale width R) **increase** linearly with time

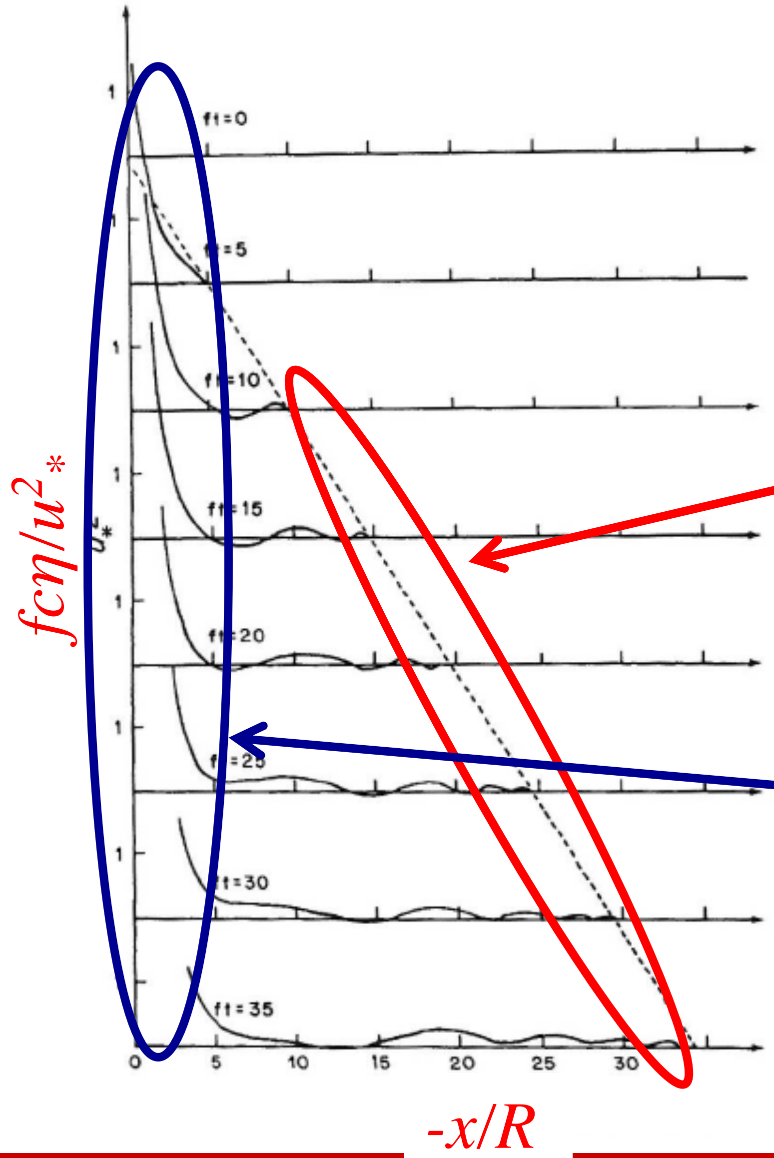
Wind "setup" (semi-infinite basin)
Along-shore wind



Development of the surface elevation Field at different distances fro coast.

Close to the coast the sea level keep rising

Wind "setup" (semi-infinite basin) Along-shore wind



Spatial structure of the pressure field
At different times.

Decaying pressure pulse penetrate the fluid

Close to the coast the surface elevation
Increase and outgrows the setup determined by
The cross-shore wind case

Wind "setup" (semi-infinite basin) Along-shore wind

$$\eta = \frac{u_*^2}{fc}\left[(ft) e^{x/R} - \sqrt{\frac{2}{\pi}} \frac{\sin(ft - \pi/4)}{\sqrt{ft}} + \dots\dots\dots \right]$$

The transport components associated with the non-oscillatory part of the solution for η are:

$$U = \frac{u_*^2}{f}\left(1 - e^{x/R}\right)$$

$$V = u_*^2 t e^{x/R}$$

N.B. The non oscillatory part of the solution is significant for the coastal band of scale width R

V: in geostrophic balance with the pressure field $fV = c^2 \dfrac{\partial \eta}{\partial x}$

U: significant except very close to the coast $\left| x / R << 1 \right|$ and NOT in geostrophic balance (no long shore pressure gradient)

Wind "setup" (semi-infinite basin) Along-shore wind

The dynamical role of the non oscillatory U transport can be understood by observing again the Transport equation

$$\frac{\partial V}{\partial t} = u_*^2 - fU$$

Near the coast ($U \cong 0$): the wind simply accelerate the fluid downwind. $\left(V = u_*^2 t\right)$

Far away from the coast we have:

$$U \cong \frac{u_*^2}{f}$$

Ekman transport associated with Longshore wind

And consequently $V=0$ (no longshore transport develops)

In between the gradual change of U provides the transition.

Wind "setup" (semi-infinite basin) Along-shore wind

Some more insight on the mechanism of sea level rise/fall due to longshore wind.

The solution:

$$V = u_*^2 t e^{x/R}$$

Implies:

$$\frac{\partial V}{\partial y} = 0$$

Therefore the continuity equation for the case analysed so far is: $\dfrac{\partial U}{\partial x} = -\dfrac{\partial \eta}{\partial t}$

With $\boxed{x = 0} \longrightarrow 0 < U < \dfrac{u_*^2}{f} \longleftarrow \boxed{|x/R >> 1|}$

The implied change $\dfrac{\partial U}{\partial x}$ in between REQUIRES sea level rise/fall according to the Continuity equation.

To compute sea level changes at the coast the offshore value of U and the spatial scale R of its changes must be known.

Wind “setup” (semi-infinite basin) Along-shore wind

Moreover:

Looking back at the boundary condition:

$$\frac{\partial \eta}{\partial x} = ft\frac{u_*^2}{c^2}$$

$$V = u_*^2 t$$

Close to the shore (x→0) the cross-shore “Ekman” tranport is missing (U≅0)

Then:

Sea level change from the pressure field geostrophic adjustment to the developing long shore current.

Coastal sea level changes accompany the development of longshore currents due NOT only To wind (e.g. to coastal fresh water input)

Wind "setup" (semi-infinite basin) The local solution

Consider the linearised equation of motion:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} + \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial u}{\partial z}\right)$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} + \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial v}{\partial z}\right)$$

With $\frac{A_v}{\rho_0}$ = constant below/above the surface/bottom wall layers

Sea level gradients $\frac{\partial \eta}{\partial x}$ $\frac{\partial \eta}{\partial y}$ (depth independent forcing term) provided by solving the global problem.

The correspondent solutions of the local problem $u(z, t)$, $v(z,t)$ may be written as:

$$u = u_1(t) + u_2(z,t) \qquad v = v_1(t) + v_2(z,t)$$

Wind "setup" (semi-infinite basin)
The local solution

$$u\text{-}= u_1(t)+u_2(z,t) \qquad v = v_1(t)+v_2(z,t)$$

$u_1(t), v_1(t)$ is the solution of the linearised equation of motion with the stress terms deleted.

$$\frac{\partial u_1}{\partial t} - f v_1 = -g\frac{\partial \eta}{\partial x}$$

$$\frac{\partial v_1}{\partial t} + f u_1 = -g\frac{\partial \eta}{\partial y}$$

The above are O.D.E (ordinary differential equations) solvable with the initial conditions:

$$u_1=v_1=0 \ (t=0)$$

The solutions describe the "pressure field induced" component of the flow.

Subtracting the above equations from the linearised equation of motions, yields the depth-dependent (frictionally induced) velocity components

$$\frac{\partial u_2}{\partial t} - f v_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial u_2}{\partial z}\right)$$

$$\frac{\partial v_2}{\partial t} + f u_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial v_2}{\partial z}\right)$$

Wind "setup" (semi-infinite basin)
The local solution

$$\frac{\partial u_2}{\partial t} - f v_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial u_2}{\partial z}\right)$$

$$\frac{\partial v_2}{\partial t} + f u_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial v_2}{\partial z}\right)$$

The above equation can be solved with initial conditions:

$$u_2=v_2=0\ (t=0)$$

And surface/bottom stress boundary conditions:

$$A_v \left.\frac{\partial u_2}{\partial z}\right|_{z=\eta} = \tau_w^{(x)} \qquad A_v \left.\frac{\partial v_2}{\partial z}\right|_{z=\eta} = \tau_w^{(y)}$$

$$A_v \left.\frac{\partial u_2}{\partial z}\right|_{z=-H} = \tau_b^{(x)} \qquad A_v \left.\frac{\partial v_2}{\partial z}\right|_{z=-H} = \tau_b^{(y)}$$

Wind "setup" (semi-infinite basin) The local solution

$$\frac{\partial u_2}{\partial t} - f v_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial u_2}{\partial z}\right) \qquad A_v \left.\frac{\partial u_2}{\partial z}\right|_{z=\eta} = \tau_w^{(x)} \qquad A_v \left.\frac{\partial v_2}{\partial z}\right|_{z=\eta} = \tau_w^{(y)}$$

$$\frac{\partial v_2}{\partial t} + f u_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial v_2}{\partial z}\right) \qquad A_v \left.\frac{\partial u_2}{\partial z}\right|_{z=-H} = \tau_b^{(x)} \qquad A_v \left.\frac{\partial v_2}{\partial z}\right|_{z=-H} = \tau_b^{(y)}$$

Assuming A_v= constant, the above system with the specified boundary conditions Specify the description of a time–dependent Ekman layer evolution.

N.B. the steady state solution is the "classic" Ekman problem solution.

The time dependent solution adds, to the steady state one (steady state Ekman layer), inertial oscillations propagating surface→downward.

In order to understand the frictional properties, the equations above are vertically integrated to obtain a degenerated form of the transport equations:

U_2, V_2 : **depth integrated**
Velocities u_2, v_2

$$\frac{\partial U_2}{\partial t} - f V_2 = \frac{1}{\rho_0}\left(\tau_w^{(x)} - \tau_b^{(x)}\right)$$

$$\frac{\partial V_2}{\partial t} + f U_2 = \frac{1}{\rho_0}\left(\tau_w^{(y)} - \tau_b^{(y)}\right)$$

Wind "setup" (semi-infinite basin) The local solution

$$\frac{\partial U_2}{\partial t} - fV_2 = \frac{1}{\rho_0}\left(\tau_w^{(x)} - \tau_b^{(x)}\right)$$

$$\frac{\partial V_2}{\partial t} + fU_2 = \frac{1}{\rho_0}\left(\tau_w^{(y)} - \tau_b^{(y)}\right)$$

Example: bottom stress neglected and only wind forcing a longshore wind stress:

$$\frac{\tau_w^{(y)}}{\rho_0} = u_*^2$$

the solution is:

$$U_2 = \frac{u_*^2}{f}\left[1 - \cos\left(ft\right)\right]$$

$$V_2 = \frac{u_*^2}{f}\sin\left(ft\right)$$

Wind "setup" (semi-infinite basin) The local solution

$$U_2 = \frac{u_*^2}{f}\left[1 - \cos(ft)\right]$$

$$V_2 = \frac{u_*^2}{f}\sin(ft)$$

A superposition of Ekman transport (long shore wind stress case)

$$U = \frac{u_*^2}{f}$$

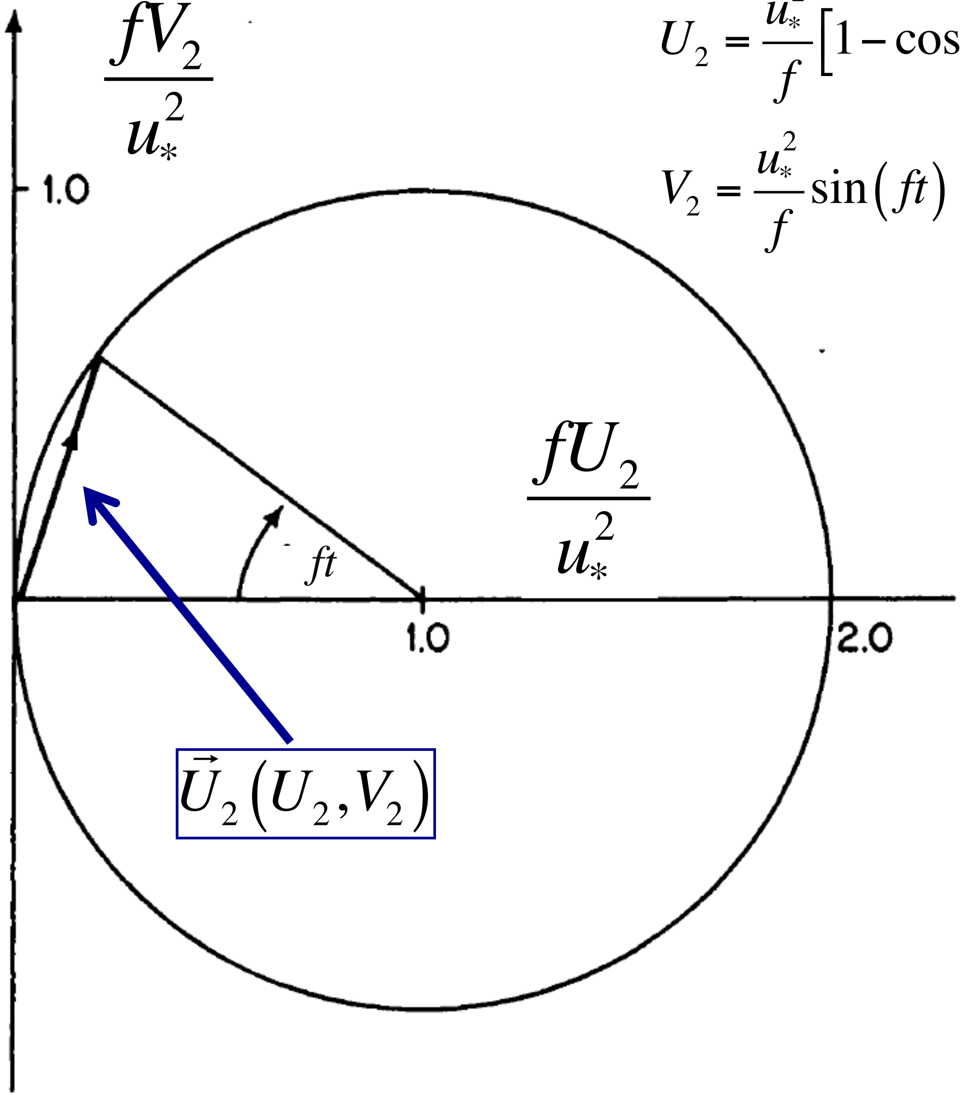
$$V = 0$$

And inertial oscillation

$$U = U_0 \cos(ft)$$

$$V = V_0 \sin(ft)$$

Wind "setup" (semi-infinite basin)
The local solution



$$U_2 = \frac{u_*^2}{f}\left[1-\cos(ft)\right]$$

$$V_2 = \frac{u_*^2}{f}\sin(ft)$$

Hodograph of frictional contribution U_2
To total transport vector $\vec{U}_2(U_2,V_2)$

With positive wind stress along y. Inertial frequency oscillations originate directional changes between $\pi/2$ and $-\pi/2$, while Magnitude changes between 0 and twice the Ekman transport
The local solution consist of a steady state Ekman spiral and of inertial oscillations.

Steady state Ekman spiral: "classic solution ofthe Ekman problem: spiral extending in depth to D_E (Ekman depth)'

Inertial oscillations progressively extending in depth and (after "long" time) becoming inperceptible as the momentum become distributed over a deep water column.

Wind "setup" (semi-infinite basin) The local solution

Pressure field induced interior velocities

$$\frac{\partial u_1}{\partial t} - f v_1 = -g \frac{\partial \eta}{\partial x}$$

$$\frac{\partial v_1}{\partial t} + f u_1 = -g \frac{\partial \eta}{\partial y}$$

The global problem solved so far yielded the TOTAL depth integrated tranport U, V.
The splitting of the solution in two parts

$$u = u_1(t) + u_2(z,t) \qquad v = v_1(t) + v_2(z,t)$$

Applies also to the transports so that it is possible to write

$$u_1 = \frac{1}{H}\left(U - U_2\right)$$

$$v_1 = \frac{1}{H}\left(V - V_2\right)$$

Wind "setup" (semi-infinite basin)
The local solution

$$u_1 = \frac{1}{H}\left(U - U_2\right)$$

$$v_1 = \frac{1}{H}\left(V - V_2\right)$$

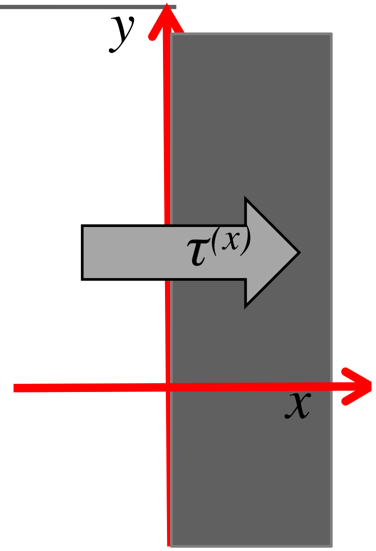
Therefore the local problem solution can be carried out in two steps (after solution of global problem):

1) Determining the frictionally induced interior velocity $u_2(z,t)$ and $v_2(z,t)$.

$u_2(t)$ and $v_2(t)$ can be found by solving the Ekman layer evolution problem. With inertial oscillations
Neglected this reduces to solving the steady state ekman problem.

2) The pressure field induced velocities $u_1(t)$ and $v_1(t)$ are then calculated from the above equations.
Neglecting inertial oscillations this reduced to subtraction of the Ekman transport from the total transport

Wind “setup” (semi-infinite basin)
The local solution



Local solution for the cross-shore wind case.

With bottom stress neglected we solve

$$\frac{\partial u_2}{\partial t} - f v_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial u_2}{\partial z}\right) \qquad \frac{\partial v_2}{\partial t} + f u_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial v_2}{\partial z}\right)$$

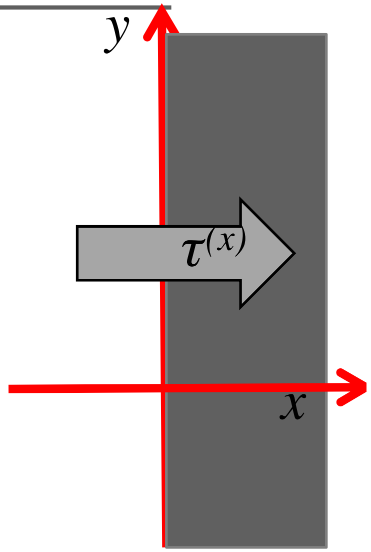
Neglecting inertial oscillations in both the global and local problem the non oscillatory solution applies.

The global solution was: $\eta = \frac{u_*^2}{fc} e^{x/R} \qquad U = 0 \qquad V = -\frac{u_*^2}{f}\left(1 - e^{x/R}\right)$

While the non oscillatory “Global” solution for the frictionally induced flow computed according to $\frac{\partial U_2}{\partial t} - f V_2 = u_*^2 \qquad \frac{\partial V_2}{\partial t} + f U_2 = 0$ is:

$$V_2 = -\frac{u_*^2}{f}$$ ←Ekman transport and $$U_2 = 0$$

Wind “setup” (semi-infinite basin) The local solution



Local solution for the cross-shore wind case.

The pressure field induced velocity is

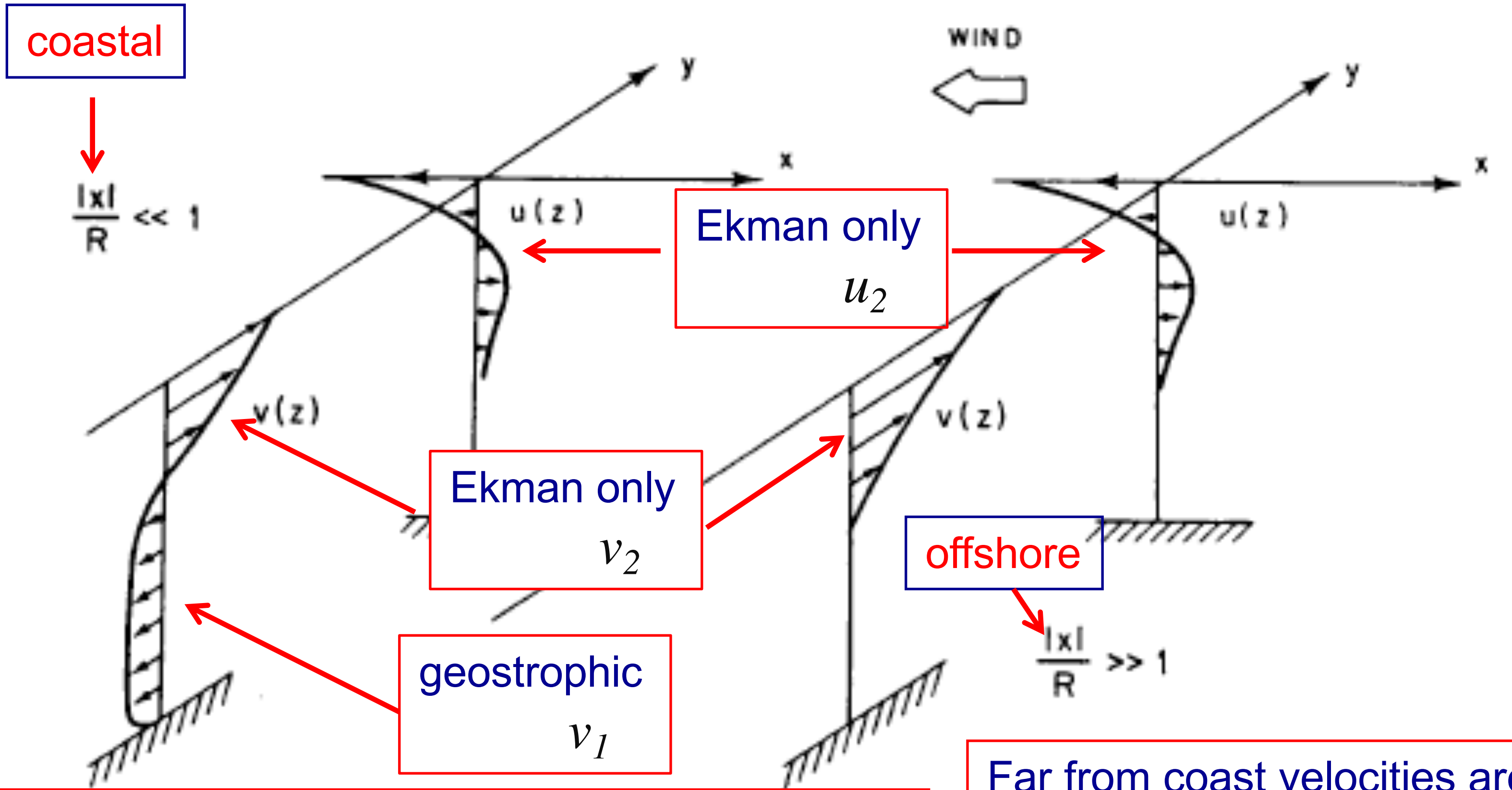
$$u_1 = 0 \qquad\qquad v_1 = \frac{u_*^2}{fH} e^{x/R}$$

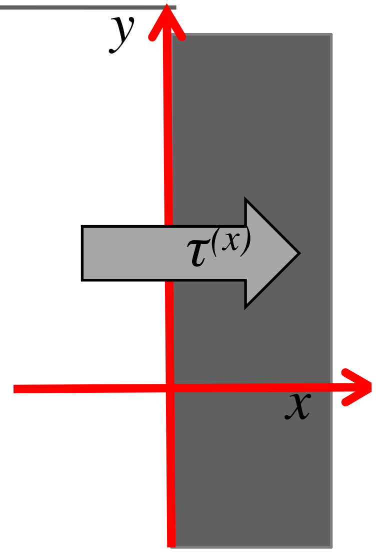
The frictionally induced velocity profiles u_2 and v_2 are obtained (neglecting inertial oscillation) by solving a “classical” steady state surface Ekman layer profile.

(see eq. 5.6 in Pinardi’s notes)

Wind "setup" (semi-infinite basin)
The local solution

Local solution for the cross-shore wind case.



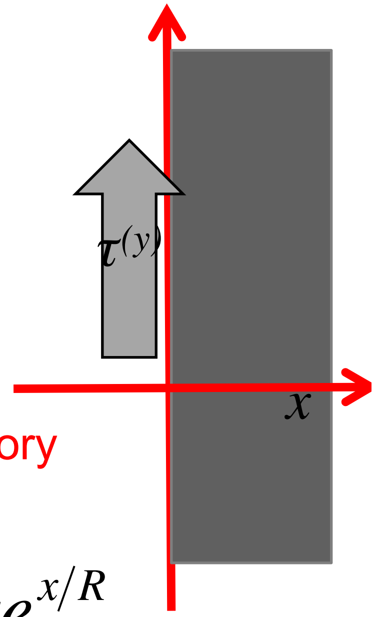


Close to coast geostrophic longshore transport EXACTLY balance the the longshore Ekman Transport, so that the cross-shore Coriolis force is zero.

Far from coast velocities are significant ONLY
In the Ekman layer

Wind "setup" (semi-infinite basin)
The local solution

Local solution for the along-shore wind case.



With bottom stress neglected we solve (again)

$$\frac{\partial u_2}{\partial t} - f v_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial u_2}{\partial z}\right) \qquad \frac{\partial v_2}{\partial t} + f u_2 = \frac{\partial}{\partial z}\left(\frac{A_v}{\rho_0}\frac{\partial v_2}{\partial z}\right)$$

Neglecting inertial oscillations in both the global and local problem the non oscillatory solution applies (again).

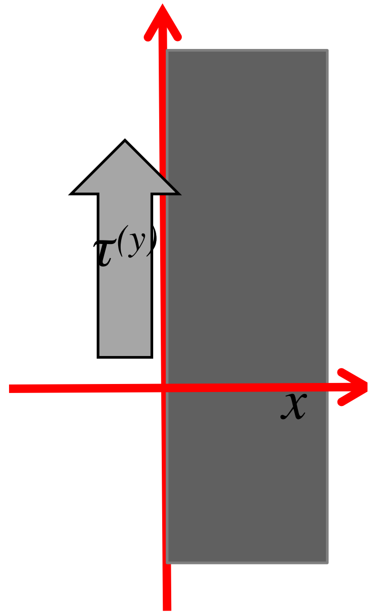
The global solution was $\quad \eta = \frac{u_*^2 t}{fc} e^{x/R} \qquad U = \frac{u_*^2}{f}\left(1 - e^{x/R}\right) \qquad V = u_*^2 t e^{x/R}$

While the non oscillatory "Global" solution for the frictionally induced flow computed according

to $\quad \frac{\partial U_2}{\partial t} - f V_2 = 0 \quad \frac{\partial V_2}{\partial t} + f U_2 = u_*^2 \quad$ is:

$V_2 = 0 \qquad$ Ekman transport $\longrightarrow U_2 = -\frac{u_*^2}{f}$

Wind "setup" (semi-infinite basin)
The local solution



Local solution for the along-shore wind case.

The pressure field induced velocity is

$$u_1 = -\frac{u_*^2}{fH} e^{x/R} \qquad v_1 = \frac{u_*^2 t}{H} e^{x/R}$$

The frictionally induced velocity profiles u_2 and v_2 are obtained (again) (neglecting inertial oscillation) by solving a "classical" steady state surface Ekman layer profile.

(see eq. 5.6 in Pinardi's notes)

Wind "setup" (semi-infinite basin) The local solution

Local solution for the along-shore wind case.