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Modular Forms and the Cosmological Constant

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1. Introduction

If string theory is a fundamental theory of nature it must explain the observed vanishing of the cosmological constant. In superstring theory this is expected to be a consequence of spacetime supersymmetry, at least in a flat background [1][2][3]. Since supersymmetry is at best strongly broken in nature, string theory should provide a mechanism for a vanishing cosmological constant which is *independent* of supersymmetry, but no such mechanism is known. Thus, any understanding of the vanishing of the string cosmological constant might prove to be useful in assessing the viability of the string. In this letter we show that if the density on the moduli space $\mathcal{M}_g$ of genus g Riemann surfaces which is associated with the heterotic string [4] satisfies the property of physical factorization, then the vacuum to vacuum amplitude vanishes for the first twenty orders of perturbation theory. Given the truth of a conjecture in algebraic geometry our proof in fact extends to all orders of perturbation theory. The argument involves the theory of modular forms on Teichmüller space and is a generalization of the approach that has recently been used to show that the vacuum amplitude vanishes at two and three loops [5][6][7].

In the following section we will discuss the form of the heterotic string integrand. In section three we show that for $g \leq 20$ the only integrand of this form satisfying modular invariance and factorization is zero. We conclude with a few remarks in section four.

2. The heterotic string

The heterotic string may be described in terms of a local quantum field theory on a super Riemann surface [2][4][8][9][10][11]. The quantum fields may be considered as (a, b) forms on an ordinary Riemann surface, which are locally of the form $f(z,\bar{z})(dz)^a(d\bar{z})^b$. There are 10 scalar fields X^μ, 10 right-moving $(0,\frac{1}{2})$ NSR fermions λ^μ, and 32 left-moving $(\frac{1}{2},0)$ gauge fermions ψ^I. The world-sheet supergeometry is described by a metric (which defines the complex structure) and an anticommuting background gravitino χ, which is a $(1,-\frac{1}{2})$ form. The action is then

$$S = \int_\Sigma \partial X^\mu \bar{\partial} X^\mu + \lambda^\mu \partial \lambda^\mu + \psi^I \bar{\partial} \psi^I + \chi \lambda^\mu \bar{\partial} X^\mu \qquad (2.1)$$

where Σ is the parameter space for the world surface and $\partial, \bar{\partial}$ are the Cauchy-Riemann operators acting on forms [12]. The last term in (2.1) is the coupling

of the matter supercurrent to the background gravitino. As is well-known, this action is invariant under Weyl, Lorentz, coordinate and supergravity transformations.

The vacuum amplitude Z_g is a gauge-fixed integral of the partition function of the above quantum field theory over background two-dimensional supergeometries. The factorization property for the string integrand which we assume below is plausible because Z_g may be derived from the above *local* theory using "ultralocal" measures [13][14][15][16].

After gauge-fixing and integrating over the supermoduli the expression becomes an integral of a density on moduli space $\mathcal{M}$. We denote by ς the holomorphic $3g-3$ form on Teichmüller space T_g, defined so that the vacuum amplitude of the closed, oriented, 26-dimensional bosonic string is [6][17][18][19]

$$Z_g^{bos} = \int_{\mathcal{F}_g} \frac{\varsigma \wedge \bar{\varsigma}}{(det Im\tau)^{13}} \qquad (2.2)$$

where τ is the period matrix and $\mathcal{F}_g \subset T_g$ is a fundamental domain for the mapping class group. Although $det Im\tau$ is only defined on T_g the integrand of (2.2) descends to a density on $\mathcal{M}_g$, that is, it is modular invariant. The vacuum amplitude for the heterotic string may be expressed in terms of zeta as [9][20]

$$Z_g^{het} = \int_{\mathcal{F}_g} \frac{\varsigma \wedge \bar{\varsigma}}{(det Im\tau)^5} \Psi_8 \bar{\xi} \qquad (2.3)$$

where Ψ_8 comes from the gauge fermions and may be expressed in terms of a sum over the theta nullwerte for the surface by

$$\Psi_8 = \left(\sum_\alpha \vartheta_\alpha^8\right)^2 \qquad (2.4)$$

for the $E_8 \times E_8$ string, and

$$\Psi_8 = \sum_\alpha \vartheta_\alpha^{16} \qquad (2.5)$$

for the $Spin(32)/\mathbb{Z}_2$ string. The subscript α labels the half-integral characteristics. Only the even characteristics contribute to the sums.

The contribution $\bar{\xi}$ from the right-movers may also be written in terms of a sum over theta characteristics (or spin-structures) as

$$\bar{\xi} = \sum_\alpha \bar{\vartheta}^5 \bar{s}_1 \bar{s}_2 \qquad (2.6)$$

where the functions s_1, s_2 on T_g may be described in terms of determinants and Green functions for $\bar{\partial}$ operators as follows. Let $\mathcal{L}_{a,b}$ denote the bundle of (a,b)-forms, and let $\bar{\partial}_{a,b}$ denote the Cauchy-Riemann operator coupled to this bundle. When $b \neq 0$ this will require a metric. Let ν_i stand for a basis of $2g-2$ zero modes of $\bar{\partial}_{\frac{3}{2},0}$, that is, $\bar{\partial}\nu_i = 0 \quad i = 1, \ldots 2g-2$. By the existence of Poincaré theta series for weight $\frac{3}{2}$ automorphic forms we can choose the basis ν_i globally on T_g in a way that varies holomorphically with the moduli [21]. Using ζ-function regularization, we may define the quantity

$$\left(\frac{det' - \nabla^2}{\int \sqrt{g} det Im\tau}\right)^{11/2} \frac{det\langle \nu_i | \nu_j \rangle}{det \bar{\partial}_{\frac{3}{2},0} \bar{\partial}^{\dagger}_{\frac{3}{2},0}} \tag{2.7}$$

where ∇^2 is the scalar Laplacian. This quantity depends only on the complex structure [22], and is, in fact, the square of a holomorphic function s_1 on T_g [6][17][20][18][19] . Let $\bar{s}_1$ be the complex conjugate of s_1.

The factor $\bar{s}_2$ can be written in terms of correlation functions for insertions of the ghost supercurrent. Denote by $b \in \mathcal{L}_{2,0}$ and $c \in \mathcal{L}_{-1,0}$ the reparametrization ghosts and by $\beta \in \mathcal{L}_{\frac{3}{2},0}$ and $\gamma \in \mathcal{L}_{-\frac{1}{2},0}$ their superpartners. Using the zero modes ν_i we may form the background gravitino $\chi = \sum \varsigma_i \nu_i \in \mathcal{L}_{1,-\frac{1}{2}}$ (we have used a metric here). The ς_i may be thought of as supermoduli [2][9][23]. Then, in terms of the ghost supercurrent $S = \gamma b + \partial c \beta \in \mathcal{L}_{\frac{3}{2},0}$ we may write $\bar{s}_2$ as

$$\bar{s}_2 = \frac{1}{det\langle \nu_i | \nu_j \rangle} \int \prod_{1}^{2g-2} d\varsigma_i \frac{Z(\varsigma)}{Z(0)} \tag{2.8}$$

where $Z(\varsigma)$ is the ghost partition function in the presence of χ

$$Z(\varsigma) \equiv \int d\bar{b} d\bar{c} d\bar{\beta} d\bar{\gamma} e^{-\int \bar{b}\partial\bar{c} + \bar{\beta}\partial\bar{\gamma} + \chi\bar{S}}. \tag{2.9}$$

This chiral functional integral has local conformal, Lorentz, gravitational, and holomorphic anomalies, all of which are cancelled by the factor $Z(0)$ in the denominator of (2.8) . We may perform the integral over the supermoduli to obtain

$$\bar{s}_2 = \frac{1}{det\langle \nu_i | \nu_j \rangle} \int_{\Sigma \times \cdots \times \Sigma} det_{i,j} \nu_i(w_j) \langle \bar{S}(w_1) \cdots \bar{S}(w_{2g-2}) \rangle \tag{2.10}$$

where the correlation function may be evaluated with Wick's theorem. The two-point function $\langle \bar{b}(x)\bar{c}(y) \rangle \in \mathcal{L}_{0,2}|_x \otimes \mathcal{L}_{0,-1}|_y$ is the kernel of the "inverse"

of $\partial_{0,-1}$. Specifically, if $\bar{b}_i$ denotes a basis of holomorphic $(0,2)$ forms and $\bar{\mu}^i$ denotes a dual basis of $(1,-1)$ forms then

$$\partial \langle \bar{b}(x)\bar{c}(y) \rangle = \delta(x,y) - \bar{b}_i(x)\bar{\mu}^i(y) \tag{2.11}$$

with a similar formula for $\langle \bar{\beta}(x)\bar{\gamma}(y) \rangle$. As $x \to y$ we have $\langle \bar{b}(x)\bar{c}(y) \rangle \sim (\bar{x}-\bar{y})^{-1}$ and $\langle \bar{\beta}(x)\bar{\gamma}(y) \rangle \sim (\bar{x}-\bar{y})^{-1}$. Using the holomorphy of ν_i and the antisymmetry of the determinant one can show that the expression (2.10) is finite.

Unfortunately, (2.10) is not obviously independent of the conformal factor and is not obviously antiholomorphic on T_g. The original expression in [9] was shown to have no conformal anomalies, and (2.10) is simply a rewriting of that expression. So $\bar{s}_2$ is conformally invariant. (A direct proof would be messy.) We will now give a formal argument that $\bar{s}_2$ is indeed antiholomorphic.

Consider the variation of (2.10) under a *holomorphic* Teichmüller deformation. Then $\delta\partial = 0$ and $\delta\bar{\partial} = \mu\partial$ where μ is an infinitesimal Beltrami differential, $\mu \in \mathcal{L}_{-1,1}$ [24][25]. Varying the zero mode equation we find that the basis ν_i changes by

$$\delta\nu_i = -\bar{\partial}^{\dagger} \frac{1}{\bar{\partial}\bar{\partial}^{\dagger}} \mu\partial\nu_i + A_{ij}\nu_j \tag{2.12}$$

where $\bar{\partial} = \bar{\partial}_{\frac{3}{2},0}$ and the matrix A_{ij} describes the difference between a globally defined basis and the locally defined basis whose variation is the first term on the right hand side of (2.12) . [1] Thus

$$\delta(det\langle \nu_i | \nu_j \rangle)^{-1} = -(det\langle \nu_i | \nu_j \rangle)^{-1} TrA \tag{2.13}$$

Furthermore, $\delta(\chi S) = \chi\mu(\partial c)\beta + \delta\chi S$, so if we integrate by parts and use the equations of motion (or, in the path integral version (2.9) , make an upper triangular transformation and use the equations of motion) then

$$\delta \int \prod d\varsigma_i Z(\varsigma) = TrA \int \prod d\varsigma_i Z(\varsigma) \quad . \tag{2.14}$$

Putting together (2.13) and (2.14) we learn that holomorphic Teichmüller deformations of $\bar{s}_2$ are zero, so $\bar{s}_2$ is an antiholomorphic function on T_g, as was to be shown. Thus $\bar{\xi}$, the contribution of the right-movers, is an antiholomorphic function on Teichmüller space. Note, in particular, that all three factors in (2.6) are finite everywhere on T_g.

[1] In deriving (2.12) one should take into account extra variations in the ν_i arising because these have "z-indices". However, for the Bers embedding this variation can be seen to be second order in the local holomorphic coordinates on Teichmüller space.

3. Modular forms

As in the bosonic case, the various factors in the heterotic string integrand (2.3) are only defined on Teichmüller space. It is only the entire expression which is modular invariant, that is, free of global diffeomorphism anomalies on the world sheet [26]. Under a symplectic modular transformation the period matrix transforms by

$$\tau \to \tilde{\tau} = (A\tau + B)(C\tau + D)^{-1} \tag{3.1}$$

and

$$detIm\tau \to |det(C\tau + D)|^{-2} detIm\tau \quad . \tag{3.2}$$

Using the transformation properties of theta functions one can show that

$$\Psi_8(\tilde{\tau}) = \left(det(C\tau + D)\right)^8 \Psi_8(\tau) \quad . \tag{3.3}$$

Therefore, we also have [2]

$$\xi(\tilde{\tau}) = \left(detC\tau + D)\right)^8 \xi(\tau) \quad . \tag{3.4}$$

In the previous section we showed that ξ is finite and holomorphic on T_g, so it is a modular form on T_g of weight eight. It was pointed out in [5][6][7] for $g = 2$ and in [6][7] for $g = 3$ that a consequence of the work of Igusa [27] is that modular forms on T_g for $g = 2, 3$ of weight eight are unique, up to an overall constant. (In particular, for $g = 2, 3$ the two expressions for Ψ_8 in (2.4) and (2.5) differ only by a constant.) Thus we must have $\xi = \kappa\Psi_8$ for some constant κ. If $\kappa \neq 0$ we obtain the partition function of a 26-dimensional *bosonic* string theory in which 16 coordinates have been split into independent left- and right-movers and compactified on independent maximal tori of $E_8 \times E_8$ or $Spin(32)/Z_2$ [28].

It was argued in [5] that the choice $\kappa \neq 0$ is incompatible with the factorization requirements explained below because of the known vanishing of the one loop dilaton tadpole in the heterotic string. Thus we must choose $\kappa = 0$ at two loops. Since the dilaton tadpole is proportional to the vacuum graph we also have $\kappa = 0$ at three loops, by the same argument. Thus, if we knew that weight eight modular forms on T_g were unique for $g \geq 4$ we could show that Z_g vanishes to all orders by induction. Unfortunately, the analog of Igusa's theorem is not known for $g \geq 4$ but we can instead use the factorization hypothesis to argue that $Z_g = 0$ for $g \leq 20$.

We first describe the factorization hypothesis. Consider the behavior of the string integrand on the boundary of moduli space. For a description of this boundary see [29][5][16][19], and references therein. In particular, we will model the boundary by the plumbing fixture $zw = t$. On the boundary component Δ_i for $i > 0$ (fig.1) we may expand

$$\varsigma = dt dm_1 dm_2 (a_0 t^{\alpha} + a_1 t^{\alpha+1} + \cdots) \tag{3.5}$$

where $\alpha = -2$ and m_1, m_2 describe the moduli of curves genus i and $g - i$ with distinguished points (that is, the universal curves of genus i and $g - i$) [3]. The first part of the factorization hypothesis states that the coefficients are of the form $a_k(m_1, m_2) = a_k^{(1)}(m_1) a_k^{(2)}(m_2)$. Similarly we may expand

$$\varsigma\xi = dt dm_1 dm_2 (b_0 t^{\beta} + b_2 t^{\beta+1} + \cdots) \tag{3.6}$$

From the definition of the theta function and the behavior of τ on Δ_i we also find the expansion

$$\Psi_8 = \Psi_8(m_1)\Psi_8(m_2) + \cdots \tag{3.7}$$

so that the asymptotic behavior of the integral is given by

$$\sum_{k,l=0}^{\infty} \int_{\mathcal{F}_i} dm_1 \frac{\tilde{a}_k^{(1)}(m_1)\bar{b}_l^{(1)}(m_1)}{(detIm\tau_1)^5} \int dt \wedge d\bar{t}\, t^{\alpha+k}\, \bar{t}^{\beta+l} \int_{\mathcal{F}_{g-i}} dm_2 \frac{\tilde{a}_k^{(2)}(m_2)\bar{b}_l^{(2)}(m_2)}{(detIm\tau_2)^5} \tag{3.8}$$

where τ_1 and τ_2 are the period matrices of the curves of genus i and $g - i$. The coefficients $\tilde{a}$ in (3.8) differ from those in (3.5) because of the higher order terms in (3.7) .

Similarly, on Δ_0 (fig.2), where $\tau_{11} = \frac{i}{2\pi} log t^{-1} \to i\infty$, we may describe the moduli and period matrix of the remaining curve of genus $g - 1$ with two distinguished points by m_1 and τ_1. Then, the modular invariant proper time is

$$s = detIm\tau / detIm\tau_1 = Im\tau_{11} + \cdots \tag{3.9}$$

[2] Since there are no global anomalies ξ is invariant under the subgroup (known as the Torelli group) of the mapping class group which preserves a canonical homology basis. We may therefore consider ξ as a function on the image of T_g in the Siegel upper half plane.

[3] The case $i = 1$ is special because of the extra automorphism of the torus.

and the asymptotic behavior of the integral is

$$\sum_{k,l=0}^{\infty} \int \frac{dt \wedge d\bar{t}}{s^5} t^{\alpha+k} \bar{t}^{\beta+l} \int_{\mathcal{T}_{g-1}} \frac{dm_1}{(detIm\tau_1)^5} a_k(m_1) \bar{b}_l(m_1) \tag{3.10}$$

where the expansion coefficients are defined as before.

As discussed in [5][16] these expansions have natural physical interpretations in terms of particles propagating along the tubes which are becoming long and skinny (see fig. 1,2). The phase integral of t forces $\alpha + k = \beta + l = n$ and corresponds to the $L_0 - \tilde{L}_0$ projection, while $4\pi n$ is the mass squared of the particle. The second part of the factorisation hypothesis is that the expansion coefficients give the appropriate one- and two- point functions in (3.8) and (3.10) respectively.

In the heterotic string the lowest mass particle should be at $n = 0$ for the right-moving sector, which means $\beta = 0$ in (3.6) . Thus ξ must have a second order zero on *all* the boundary components Δ_i of $\mathcal{M}$. We will now show that this condition is incompatible with the other properties of ξ, unless ξ is identically zero. Note that we do not need the full strength of the factorisation hypothesis as we have stated it, we only need to know the order of vanishing of the leading coefficient. We now state our main result. Let $\overline{\mathcal{M}}$ be the compactified moduli space of stable curves.

Theorem: There is no nonzero modular form of weight eight with second order zeroes on $\overline{\mathcal{M}} - \mathcal{M}$ for $g \leq 20$.

Proof: The transformation law (3.4) of the modular form Ψ_8 on T_g may be considered as defining a line bundle $\mathcal{E}$ on $\mathcal{M}$ which extends in a natural way to $\overline{\mathcal{M}}$. Let η be a weight eight modular form with second order zeroes on $\overline{\mathcal{M}} - \mathcal{M}$, then η/Ψ_8 is a meromorphic function on $\overline{\mathcal{M}}$. Thus, η is also a section of $\mathcal{E}$. Let the divisor class of $\mathcal{E}$ be $a\lambda - b\delta$ where λ is the Hodge line bundle, and $\delta = \sum_{i=0}^{[g/2]} \delta_i$, where δ_i is the class defined by the divisor Δ_i. We will show below that in fact $a = 8$ and $b = 0$. Let us first derive a consequence of this. If $div\eta = D + 2\Delta$ where $D \subset \mathcal{M}_g$ is an effective divisor [4] then

$$D \sim 8\lambda - 2\delta \tag{3.11}$$

[4] Actually, we must take the closure of D in $\mathcal{M}$.

so the class $8\lambda - 2\delta$ is an effective divisor class, and therefore must lie in the *effective cone*, which may be described as follows. The region of the (x, y) plane where the bundle $y\lambda - x\delta$ has a power which admits a holomorphic section is called the effective cone, and may be depicted as in fig.3. The slope S_g of the effective cone has been investigated by algebraic geometers [30][31]. Using the recent estimates of [31] one can show that $S_g > 4$ for $g \leq 20$. Thus, for $g \leq 20$ no such form η exists.

We now compute a. We may put a natural norm on λ by defining the norm of a local section $\omega_1 \wedge \cdots \wedge \omega_g$ constructed from a basis of canonically normalised Abelian differentials by

$$\| \omega_1 \wedge \cdots \wedge \omega_g \|^2 = det(\omega_i|\omega_j) = detIm\tau \quad . \tag{3.12}$$

We can put a norm on modular forms of weight k by defining

$$\| s \|^2 = (detIm\tau)^k |s|^2 \quad . \tag{3.13}$$

Thus, for $k = 8$ we have

$$c_1(\mathcal{E}) = 8\frac{i}{2\pi}\partial\bar{\partial} log detIm\tau = 8c_1(\lambda) \quad . \tag{3.14}$$

Finally, we may argue that $b = 0$ using techniques employed in [30] . It is shown in [30] that the boundary of $\mathcal{M}$ can be filled with curves whose intersection number with Δ is negative, and whose intersection number with λ is zero [5] . A typical such curve in Δ_i may be described as follows. Attach a smooth curve of genus i at a point p to a smooth curve of genus $g - i$ at a point q. Holding q fixed let p range over the curve of genus i. This family of curves is itself a curve in Δ_i, and it can be shown that it has the intersection numbers stated above. There is a simlilar construction for curves in Δ_0. If the divisor class of $\mathcal{E}$ were of the form $8\lambda - b\delta$ with b nonzero, then any divisor in that class would have a nontrivial intersection number with the curves mentioned above. On the other hand, by (3.7) we see that Ψ_8 is either nonzero or vanishes identically on the entire curve, the latter case holding only for curves lying within a codimension one subvariety of the *boundary*. In the first case the intersection number of the

[5] The intersection number of a curve with a divisor is the first Chern class of the restriction to that curve of the line bundle defined by the divisor.

curve with $div\Psi_8$ is clearly zero. Thus the divisor of Ψ_8 certainly does not have the required property, and hence $b = 0$. ■

It follows from the above theorem and the previous physical arguments that the vacuum amplitude Z_g must vanish for $g \leq 20$. Clearly the method of proof could extend to all orders of perturbation theory if S_g were known to be larger than four. In fact, all known effective divisors have a slope $S_g \geq 6 + \frac{12}{g+1}$, and it is natural to conjecture that $S_g > 6$ at every genus. An argument of this kind might even apply in the infinite genus setting of [16] . Unfortunately, we cannot turn the argument around to say that the expected vanishing of Z_g to all orders of string perturbation theory implies a lower bound on S_g, since a weight eight modular form with second order zeroes need not correspond to the string integrand. However, it has been argued in [16] that the fundamental principles of string theory are encoded in the analytic and factorization properties of the string integrand. Thus, we could use such a modular form to *define* a new string theory. This theory would presumably be identical to the heterotic string for $g \leq 20$ and differ thereafter. Such a bizarre possibility is so unlikely from the physical point of view, that one is lead to believe that $S_g > 4$ for all genus.

4. Conclusion

The expansions (3.6) and (3.7) have analogues on the multiple intersections of the boundary components. Thus we may use the factorization hypothesis to obtain information about N point functions as suggested in [16] . The vanishing of the string integrand point by point on moduli space does *not* imply that the N-point functions vanish, but rather that the sum of the N-point functions over particles with specified values of L_0 and $\tilde{L}_0$ must vanish. It is commonly believed that the only divergences in superstring amplitudes come from the dilaton one-point function [32]. Without explicitly constructing these amplitudes and investigating their divergences we cannot claim to have proven finiteness, but we believe our results give very strong evidence in favor of finiteness.

One interesting problem is to see whether arguments of this kind will apply for backgrounds other than flat space. Toroidal compactifications only modify the string integrand by a modular invariant multiplicative factor (the invariant norm squared of a theta function) and therefore do not affect the argument. Of more interest are nonsupersymmetric backgrounds. Perhaps the simplest possibility to investigate is an orbifold compactification. The relevant modular

forms will then only be modular forms under the subgroup of the modular group which preserves the boundary conditions. In general, the space of modular forms under such subgroups grows rapidly with the index of the subgroup so we cannot expect any general results, but this line of argument might prove useful in some specific examples. A speculation in the spirit of our proof may be found in [16] where it is suggested that analytic arguments combined with some information on the hermitian metric on the primary field bundle can be used to show the vanishing of the cosmological constant.

We have argued that the cosmological constant in the heterotic string vanishes for $g \leq 20$ without making explicit use of spacetime supersymmetry. It is natural to ask if the nonexistence of certain kinds of modular forms is merely an obscure consequence of supersymmetry. The connection with supersymmetry does not seem to be obvious, and it is a logical possibility that mathematical theorems on modular forms reflect some other mechanism for the vanishing cosmological constant. If this is so, this physical mechanism might be relevant to what is observed in nature.

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Fig. 1: Δ_i

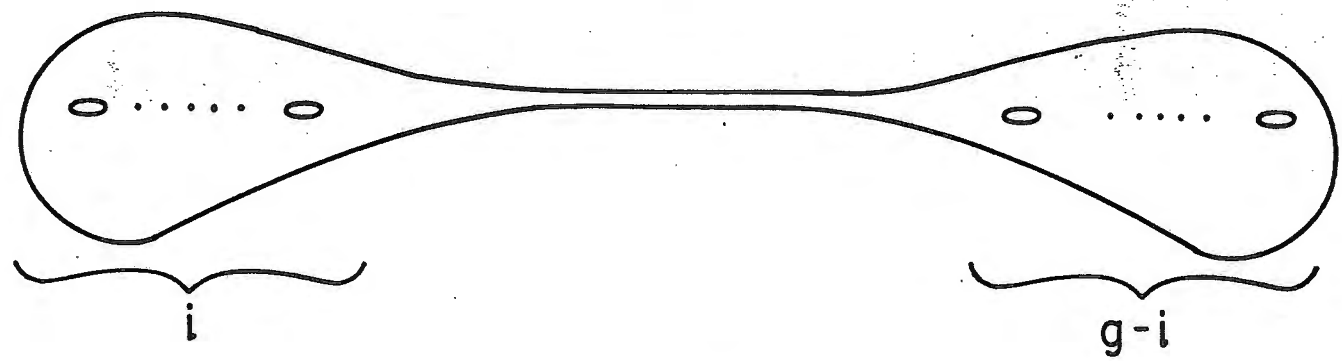

Fig. 2: Δ_0

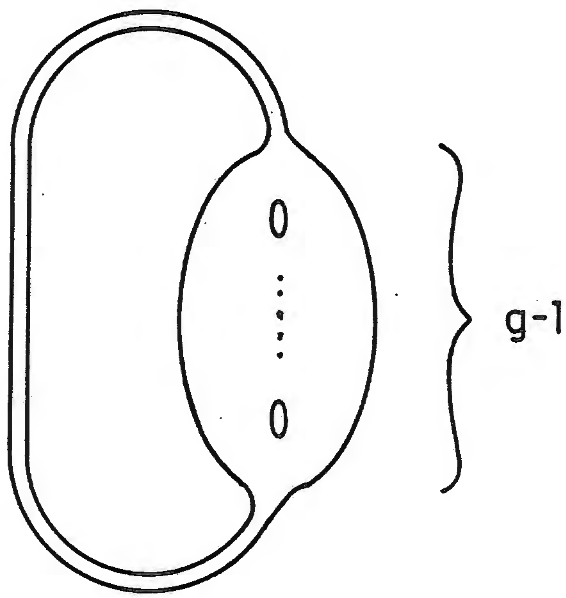

Fig. 3