1-1-2016

Network formation and its Impact on Systemic Risk

Selman Erol
University of Pennsylvania, erols@sas.upenn.edu

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Abstract
In the aftermath of the financial crisis of 2008, many policy makers and researchers pointed to the interconnectedness of the financial system as one of the fundamental contributors to systemic risk. The argument is that the linkages between financial institutions served as an amplification mechanism: shocks to smaller parts of the system propagate through the system and result in broad damage to the financial system. In my dissertation, I explore the formation of networks when agents take into account systemic risk.

The dissertation consists of three complementary papers on this topic. The first paper titled ``Network Formation and Systemic Risk'', joint with Professor Rakesh Vohra. We set out the framework and construct a model of endogenous network formation and systemic risk. We find that fundamentally `safer' economies with higher probability of getting good shocks generate higher interconnectedness, which leads to higher systemic risk. This provides network foundations for ``the volatility paradox'' arguing that better fundamentals lead to worse outcomes due to excessive risk taking. Second, the network formed crucially depends on the correlation of shocks to the system. As a consequence, an observer, such as a regulator, facing an interconnected network who is mistaken about the correlation structure of shocks will underestimate the probability of system wide failure. This result relates to the ``dominoes vs. popcorn'' discussion by Edward Lazear. He comments that a fundamental mistake in addressing the crisis was to think that it was ``dominoes'' so that saving one firm would save many others in the line. He continues to argue that it was ``popcorn in a pan'': all firms were exposed to same correlated risks and saving one would not save many others. We complement his discussion by arguing that the same mistake might have been the reason behind why sufficient regulatory precaution was not taken prior to the crisis. The third result is that the networks formed in the model are utilitarian efficient because the risk of contagion is high. This causes firms to minimize contagion by forming dense but isolated clusters that serve as firebreaks. This finding is suggestive that, the worse the contagion, the more the market takes care of it.

In the second paper, titled ``Network Hazard and Bailouts'', I ask how the anticipation of ex-post government bailouts affects network formation. I deploy a significant generalization of the model in the first paper and allow for time-consistent government transfers. I find that the presence of government bailouts introduces a novel channel for moral hazard via its effect on network architecture, which I call ``network hazard''. In the absence of bailouts, firms form sparsely connected small clusters in order to eliminate second-order counterparty risk: expected losses due to defaulting counterparties that default because of their own defaulting counterparties. Bailouts, however, eliminate second-order counterparty risk already. Accordingly, when bailouts are anticipated, the networks formed become more interconnected, and exhibit a core-periphery structure (many firms connected to a smaller number of central firms, which is observed in practice). Interconnectedness within the periphery leads to higher extent of contagion with respect to the networks formed in the absence of intervention. Moreover, solvent core firms serve as a buffer against contagion by increasing the resilience of the many peripheral firms that are connected to the core. However, insolvent core firms serve as an amplifier of contagion since they make peripheral firms less resilient. This implies that in my model, ex-post time-consistent intervention by the government, while ex-ante welfare improving, increases systemic risk and volatility, solely through its effect on the network. A remark is that firms, in my model, do not make riskier individual choices regarding neither their choice of investment risk, nor the number of their counterparties they have. In this sense, network hazard is a genuine form of moral hazard solely through the formation of the detailed network. On another note, the model can also be viewed as a first attempt towards developing a theory of mechanism design with endogenously formed network externalities which might be useful in various other scenarios such as provision of local public goods.

In the final paper, titled “Network Reactions to Banking Regulations”, joint with Professor Guilermo Ordonez, we consider the role of liquidity and capital requirements to alleviate network hazard and systemic risk. In the model, financial firms set up credit lines with each other in order to meet their funding needs on demand. Accordingly, higher liquidity requirements induce firms to form higher interconnectedness in order to be able to find deposits as needed. At a tipping point of liquidity requirements, the network discontinuously jumps in its interconnectedness, which contributes discontinuously to systemic risk. On the other hand, the reaction to capital requirements is smooth. Capital requirements indirectly work as an upper bound in the interconnectedness firms would form. This way, interconnectedness can be effectively reduced to a desired level via capital requirements. Yet capital requirements cannot be used to induce higher interconnectedness. Thusly, in times of credit freeze, capital requirements may not help promote circulation of credit. A conjunction of both liquidity and capital requirements is more effective in promoting desired circulation while reducing systemic risk.

The work in this dissertation suggests that endogenous network architecture is an essential component of the study of financial markets. In particular, network hazard is a genuine form of moral hazard that will be overlooked unless network formation is taken into account, while it has implications about systemic risk. Moreover, this work illustrates how the reaction of networked financial markets to both fundamentals of the economy and to the policy can be non-trivial, featuring non-monotonicity and discontinuity.

Degree Type
Dissertation

Degree Name
Doctor of Philosophy (PhD)

Graduate Group
Economics

First Advisor
Rakesh V. Vohra

Keywords
bailouts, contagion, network formation, regulation, strong stability, systemic risk

Subject Categories
Economic Theory | Finance and Financial Management

NETWORK FORMATION AND ITS IMPACT ON SYSTEMIC RISK

Selman Erol

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2016

Supervisor of Dissertation

____

Rakesh V. Vohra, Professor of Economics

Graduate Group Chairperson

____

Jesús Fernández-Villaverde, Professor of Economics

Dissertation Committee

Camilo García-Jimeno, Assistant Professor of Economics

Guillermo L. Ordoñez, Associate Professor of Economics

Andrew Postlewaite, Professor of Economics

NETWORK FORMATION AND ITS IMPACT ON SYSTEMIC RISK

ACKNOWLEDGEMENTS

I am extremely grateful to God, for everything. I deeply thank my father for his neverending and unconditional support, and my advisor Rakesh Vohra for his guidance, kindness, and patience. It would be impossible without them. My family was always there for me. Selman Sakar, Umut Isik, Kemal Yildiz, and Mustafa Dogan have been the best friends one could ask for. Guillermo Ordonez, Camilo Garcia-Jimeno, and Alp Simsek helped me a great deal in shaping my ideas with their advice, suggestions, time, and kindness. I would also like to thank Andrew Postlewaite, Steven Matthews, Aislinn Bohren, SangMok Lee, Yuichi Yamamoto, George Mailath, David Dillenberger, Mallesh Pai, Dirk Krueger, Hal Cole, Jesus Fernandez-Villaverde, Kelly Quinn, Charmaine Thomas, Isa Hafalir, Bumin Yenmez, Cagri Saglam, Semih Koray, Serhat Dogan, Daniel Neuhann, Ekim Cem Muyan, Murat Alp Celik, Francisco Silva, and many others who would not fit into these pages.

ABSTRACT

NETWORK FORMATION AND ITS IMPACT ON SYSTEMIC RISK

Selman Erol

Rakesh V. Vohra

In the aftermath of the financial crisis of 2008, many policy makers and researchers pointed to the interconnectedness of the financial system as one of the fundamental contributors to systemic risk. The argument is that the linkages between financial institutions served as an amplification mechanism: shocks to smaller parts of the system propagate through the system and result in broad damage to the financial system. In my dissertation, I explore the formation of networks when agents take into account systemic risk.

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the same mistake might have been the reason behind why sufficient regulatory precaution was not taken prior to the crisis. The third result is that the networks formed in the model are utilitarian efficient because the risk of contagion is high. This causes firms to minimize contagion by forming dense but isolated clusters that serve as firebreaks. This finding is suggestive that, the worse the contagion, the more the market takes care of it.

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The work in this dissertation suggests that endogenous network architecture is an essential component of the study of financial markets. In particular, network hazard is a genuine form of moral hazard that will be overlooked unless network formation is taken into account, while it has implications about systemic risk. Moreover, this work illustrates how the reaction of networked financial markets to both fundamentals of the economy and to the policy can be non-trivial, featuring non-monotonicity and discontinuity.

Keywords: network formation, contagion, counterparty risk, systemic risk, bailouts, network hazard, moral hazard, regulation, capital requirements, reserve requirements, phase transition, mechanism design

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Chapter 1

Network Formation and Systemic Risk

This chapter is co-authored with Rakesh V. Vohra.

Abstract

This paper introduces a model of endogenous network formation and systemic risk in OTC markets. A link is a trading opportunity that yields benefits only if the counterparty does not subsequently default. After links are formed, they are subjected to exogenous shocks that are either good or bad. Bad shocks reduce returns from links and incentivize default. Good shocks, the reverse. Defaults triggered by bad shocks might propagate via links. The model yields three insights. First, a higher probability of good shocks generates higher probability of system wide default. Increased interconnectedness in the network offsets the effect of better fundamentals. Second, the network formed critically depends on the correlation of shocks to the links. As a consequence, an outside observer who is mistaken only about the correlation structure of shocks, upon observing a highly interconnected network, will underestimate the probability of system wide default. Third, when the risk of contagion is high, the networks formed in the model are utilitarian efficient.

1.1 Introduction

The awkward chain of events that upset the bankers, began with the collapse of Lehmann Brothers in 2008. Panic spread, the dollar wavered and world markets fell. Interconnectedness of the financial system, some suggested, allowed Lehmann's fall to threaten the stability of the entire system. Thus prompted, scholars have sought to characterize the networks that would allow shocks to one part of the financial network to be spread and amplified. Blume et al. (2013) as well as Vivier-Lirimonty (2006), for example, argue that dense interconnections pave the way to systemic failures. In contrast, Allen and Gale (2000) as well as Freixas et al. (2000), argue that a more interconnected architecture protects the system against contagion because the losses of a distressed institution can be divided among many creditors. With a few exceptions, a common feature of these and other papers (Acemoglu et al. (2013), Eboli (2013), Elliott et al. (2014), Gai et al. (2011), Glasserman and Young (2014)) is an exogenously given network. A node (or subset of them) is subjected to a shock and its propagation studied as the size of the shock varies. Absent are reasons for the presence of links between agents.[1] After all, one could eliminate the possibility of a system-wide failure by never forming links. This is, of course, silly. While every link increases the possibility of contagion, the presence of a link between two agents represents a potentially lucrative joint opportunity. If agents form links anticipating the possibility of system-wide failure, what kinds of networks would they form? In particular, do they form networks that are susceptible to contagion?

In the model we use to answer these questions, agents first form links. The payoff to each party that shares a link is uncertain and depends upon the future realization of a random variable (which we call a shock) and actions taken contingent on the shock. Specifically, there are three stages. In stage one, agents form links which can be interpreted as partnerships or joint ventures. In stage two, each link formed is subjected to

[1] Blume et al. (2013) and Farboodi (2014) are exceptions.

a shock. In stage three, with full knowledge of the realized shocks, each agent decides whether to 'default' or not. The payoff of an agent depends on the action she takes in the third stage as well the actions of her neighbors and the realized shocks. The default decision corresponds to exiting from every partnership formed in stage one. If the only Nash equilibrium of the game in stage three is that everyone defaults, we call that a system wide failure. In our model, default is the result of a 'loss of confidence' rather than simple 'spillover' effects.[2]

In the benchmark version of this model we show that the network formed in stage one is utilitarian efficient. Efficiency is a consequence of the high risk of contagion which forces agents to form isolated clusters that serve as firebreaks. The main source of possible inefficiency, contagion spreading to distant parts of the network, is eliminated by the absence of links between clusters.[3] This is outcome is not entirely obvious because the high risk of contagion might cause agents to form inefficiently few links.

A second contribution is to examine how the probability of system wide failure varies through network formation with a change in the distribution of shocks. In a setting where shocks are independent and binary (good or bad), the probability of system wide failure *increases* with an increase in the probability of a good shock, up to the point at which the formed network becomes a complete graph, i.e. every pair of agents is linked. After this point, systemic risk declines. Intuitively, as partnerships become less risky, agents are encouraged to form more partnerships increasing interconnectedness which increases the probability of system wide failure. This provides network foundations for the volatility paradox proposed by Brunnermeier and Sannikov (2014)

Our final contribution is to show that the structure of the network formed in stage one depends critically on whether the shocks to the links are believed to be correlated

[2]Glasserman and Young (2014) argue that spillover effects have only a limited impact. They suggest that the "mere possibility (rather than the actuality) of a default can lead to a general and widespread decline in valuations....."

[3]Later on we consider various extensions in the strength of contagion and types of agents that lead various other network structures as well.

or independent of each other. When shocks are perfectly correlated, the network formed in stage one is a complete graph. We think this finding relevant to the debate between two theories of financial destruction advanced to explain the 2008 financial crisis. The first, mentioned above, is dubbed the 'domino theory'. The alternative, advocated most prominently by Edward Lazear [4], is dubbed 'popcorn'. Lazear describes it thusly in a 2011 opinion piece in the Wall Street Journal:

> "The popcorn theory emphasizes a different mechanism. When popcorn is made (the old fashioned way), oil and corn kernels are placed in the bottom of a pan, heat is applied and the kernels pop. Were the first kernel to pop removed from the pan, there would be no noticeable difference. The other kernels would pop anyway because of the heat. The fundamental structural cause is the heat, not the fact that one kernel popped, triggering others to follow.
>
> Many who believe that bailouts will solve Europe's problems cite the Sept. 15, 2008 bankruptcy of Lehman Brothers as evidence of what allowing one domino to fall can do to an economy. This is a misreading of the historical record. Our financial crisis was mostly a popcorn phenomenon. At the risk of sounding defensive (I was in the government at the time), I believe that Lehman's downfall was more a result of the factors that weakened our economic structure than the cause of the crisis."

Our model suggests that underlying structural weaknesses (modeled by strong correlations between shocks) and greater interconnectedness can coexist. Therefore, it would be incorrect to highlight the interconnectedness of the system and suggest it *alone* as the cause of instability.

More importantly, this suggests that a mistake in assessing the correlation structure

[4]Chair of the US President's Council of Economic Advisers during the 2007-2008 financial crisis.

of shocks can lead to disproportionately bigger mistakes in assessing the probability of systemwide failure. In the model, a complete network arises for perfectly correlated shocks, the popcorn world, no matter how likely the shocks are to be bad. However, a complete network arises for independent shocks, the dominoes world, only if the shocks are very likely to be good. Therefore, we suggest that Edward Lazear's view might shed light into possible causes for the underestimation the likelihood of a financial crisis.

Our model differs from the prior literature in the following ways. The networks we study are formed endogenously. Babus (2013) also has a model of network formation, but one in which agents share the goal of minimizing the probability of system wide default. In our model agents are concerned with their own expected payoffs and only indirectly with the possibility of system wide failure. Acemoglu et al. (2013) also discusses network formation, within a set of limited alternatives. Zawadowski (2013) models the decision of agents to purchase default insurance on their counter-parties. This can be interpreted as a model of network formation, but it is not a model of an agent choosing a particular counter-party because the counter-parties are fixed. Default insurance serves to change the terms of trade with an existing counter-party. The model in Farboodi (2014) includes network formation with the same solution concept we employ. However, our model is about mutual cross-holdings whereas her model is about directional interbank lending. Furthermore we explicitly characterize all networks formed, and provide detailed comparative statics by determining the exact distribution of defaults. Blume et al. (2013) has networks that form endogenously. However, the risk of a node defaulting is non-strategic and independent of the network formed. In our model, the likelihood of a node defaulting depends on the structure of the network formed.

Another critical difference which allows us to discuss the volatility paradox as well as the popcorn vs. dominoes debate is that we examine the effects of a distribution that generates the shocks rather than the effects of fixed shocks applied to particular nodes. Glasserman and Young (2014) is the only exception we are aware of, but the networks they consider are exogenously given.

In section 1.2, we give a formal description of the model. Section 1.3 characterizes the set of agents that choose to default in stage three for a given realized network and realization of shocks. Section 1.4 uses these results to characterize the structure of the realized networks. Section 1.5 investigates efficiency and systemic risk of the networks formed. Section 1.6 discusses correlated shocks and section 1.7 describes some extensions to the basic model. We propose some future work in Section 1.8.

1.2 The Model

Denote by N a finite set of agents.[5] Each pair of agents in N can form a joint venture. We frequently refer to agents as **nodes** and each potential partnership as a **potential edge**.

A potential edge e, a subset of N with two elements, represents a bilateral contract whose payoff to each party is contingent on some future realized state θ^e and actions that each incident[6] node can take upon realization of θ^e. The set of possible values of θ^e is Θ, a finite set of real numbers.

The model has three stages. In stage one, the stage of network formation, agents, by mutual consent, decide which potential edges to pick. The edges picked are called realized. The set of **realized edges** is denoted E. The corresponding network denoted (N, E), is called a **realized network**.

In stage two, for each realized edge e, θ^e is chosen by nature identically and independently across edges via a distribution ϕ over Θ. We relax the independence assumption in Section 1.6. We denote by $(N, E, \boldsymbol{\theta})$ the realized network and vector of realized θ's.

In stage three, with full knowledge of $(N, E, \boldsymbol{\theta})$ each agent n chooses one of two

[5] We abuse notation by using N to denote the cardinality of the set when appropriate.
[6] A node v is incident to an edge e if $v \in e$.

possible actions called B (business as usual) or D (default), denoted by a_n. Agent n enjoys the sum of payoffs $u_n(a_n, a_m; \theta^{\{n,m\}})$ over all of his neighbors[7] m in (N, E).

We make two assumptions about payoff functions. The first is that if an agent n in (N, E) has degree one and the counter-party defaults, it is the unique best response for agent n to default as well. Formally:

Assumption 1. *$u_n(D, D; \theta) > u_n(B, D; \theta)$ for all n and θ.*

The second assumption is a supermodularity which can be interpreted as a form increasing returns in fulfilling the terms of the partnership.

Assumption 2. *$u_n(D, D; \theta) + u_n(B, B; \theta) > u_n(B, D; \theta) + u_n(D, B; \theta)$ for all n and θ.*

If we focus on a pair of agents (n, m) and denote by e the realized edge between them, the payoff matrix of the game they are engaged in stage three is the following (player n is the row player and m the column player):

	B	D
B	$u_n(B, B; \theta^e)$, $u_m(B, B; \theta^e)$	$u_n(B, D; \theta^e)$, $u_m(D, B; \theta^e)$
D	$u_n(D, B; \theta^e)$, $u_m(B, D; \theta^e)$	$u_n(D, D; \theta^e)$, $u_m(D, D; \theta^e)$

A special case of this game is the coordination game of Carlsson and van Damme (1993) reproduced below that will be considered in section 1.4:

	B	D
B	θ^e, θ^e	$\theta^e - 1$, 0
D	0, $\theta^e - 1$	0, 0

It is clear from this last table that a pair of agents that share a realized edge play a coordination game whose payoffs depend upon the realized state variable θ^e. Following Carlsson and van Damme (1993), the game has a natural interpretation. In stage one

[7] Two distinct nodes v and v' are neighbors if $\{v, v'\} \subset E$. In this case v and v' are also said to be adjacent.

the agents get together to pursue a joint investment. In stage two, θ^e is realized, i.e. new information arrives about the profitability of the project. In stage three, agents are allowed to reassess their decision to continue with the project or not. For other examples of games of this kind and their applications in finance see Morris and Shin (2003).

Two features of the model deserve discussion. First, in contrast to prior literature, shocks, in the form of realized states, apply to edges rather than nodes. In section 1.7 we extend our model to allow for shocks to nodes as well as edges. However, we believe shocks to edges to be of independent interest. An agent's solvency depends on the outcomes of the many investments she has chosen to make. The interesting case is when these investments required coordination with at least one other agent, a joint venture if you will. It is the outcome of this joint venture that will determine whether the participants decide to continue or walk away.

Second, an agent must default on all partnerships or none. While extreme, it is not, we argue, unreasonable. Were an agent free to default on any subset of its partnerships, we could model this by splitting each node in (N, E) into as many copies of itself as its degree.[8] Each copy would be incident to exactly one of the edges that were previously incident to the original node. Thus, our model would easily accommodate this possibility. However, this has the effect of treating a single entity as a collection of independent smaller entities which we think inaccurate. Institutions considering default face liquidity constraints, which restrict, at best, the number of parties they can repay. When a company fails to pay sufficiently many of its creditors, the creditors will force the company into bankruptcy. While entities like countries can indeed selectively default, there is a knock-on effect. Countries that selectively default, have their credit ratings downgraded which raise their borrowing costs for the other activities they are engaged in. Thus, it is entirely reasonable to suppose that the default decisions associated with the edges a node is incident to must be linked. Ours is an extreme, but simple, version of such a linkage.

[8]The degree of a node in a graph is the number of edges incident to it.

1.2.1 Solution concepts

Here we describe the solution concepts to be employed in stages one and three. We begin with stage three as the outcomes in this stage will determine the choices made by agents in stage one.

Agents enter stage three knowing $(N, E, \boldsymbol{\theta})$. With this knowledge, each simultaneously chooses action B or D. We do not allow actions chosen in stage three to be conditioned on what happens in earlier stages. The outcome in stage three is assumed to be a Nash equilibrium. While 'everybody plays D' is a Nash equilibrium, by Assumption 1, it need not be the only one. We focus on the Nash equilibrium in which largest (with respect to set inclusion) set of agents, among all Nash equilibria, play B. Call this the **cooperating equilibrium**. The proposition below shows that the cooperating equilibrium is well-defined and unique, by using rationalizable strategies.

A realized network along with realized states, $(N, E, \boldsymbol{\theta})$, exhibits **system wide failure** if in the cooperating equilibrium of the game all agents in N choose D.[9] In this case, agents can coordinate on nothing but action D. The probability of system wide failure of a realized network is called its **systemic risk**.

Proposition 1. *A cooperating equilibrium is well-defined and unique.*

Proof. Fix $(N, E, \boldsymbol{\theta})$. The profile where all agents in N play D is a Nash equilibrium by Assumption 1. Hence, D is rationalizable for everyone. Let M be the set of agents who have the unique rationalizable action D. For agents in $N \setminus M$, both B and D are rationalizable.

Consider an agent $n \notin M$. B is rationalizable, i.e., B is a best response to some strategy profile, say a_{-n}, of agents $-n$ in which agents in M play D. Let $\Delta(s_{-n})$ be the difference in payoffs for n between playing B and D against strategy profile s_{-n} of $-n$.

[9] This is equivalent to saying that 'everybody plays D' is the only Nash equilibrium.

$\Delta(a_{-n}) \geq 0$ since B is a best reply to a_{-n}.

Now consider the strategy profile b_{-n} of agents $-n$ such that agents in M play D and the rest play B. We will prove that $\Delta(b_{-n}) \geq \Delta(a_{-n})$. In a_{-n}, players in $N \setminus M$ could be playing B or D. Let $K \subseteq N \setminus M$ be those agents who play D in a_{-n} and let Γ_n be the set of neighbors of n in the realized network (N, E). Then $\Delta(b_{-n}) - \Delta(a_{-n}) =$

$$\sum_{k \in K \cap \Gamma_n} \left(u_n \left(B, B; \theta^{\{n,k\}} \right) - u_n \left(D, B; \theta^{\{n,k\}} \right) \right) - \left(u_n \left(B, D; \theta^{\{n,k\}} \right) - u_n \left(D, D; \theta^{\{n,k\}} \right) \right)$$

which is positive by Assumption 2.

As $\Delta(b_{-n}) \geq \Delta(a_{-n}) \geq 0$ it follows that B is a best reply by n to b_{-n}. This argument works for every agent in $N \setminus M$, not just n. Also, recall that D is the unique rationalizable action for agents in M so that D is the unique best reply to any strategy profile in which all agents in M play D. Therefore, a profile where all agents in M play D and all agents in $N \setminus M$ choose B is a Nash equilibrium.

Note that in any Nash equilibrium, everyone in M must play D since it is their unique rationalizable action. Therefore, "M plays D, M^c plays B" is the unique cooperating equilibrium. □

The proof suggests an *equivalent definition* of a cooperating equilibrium: the rationalizable strategy profile in which those who have the unique rationalizable action D play D, while the remainder play B.

Recall that rationalizable actions are those which remain after the iterated elimination of strictly dominated actions. The iteration is as follows. Those agents who have a strictly dominant action D play D. Then, knowing that these agents play D, it becomes strictly dominant to play D for other agents to do so. This iteration stops in a finite number of steps as N is finite. The remaining action profiles are the rationalizable ones, and the cooperating equilibrium is given by the profile in which whoever is not reached in the iteration plays B.

There is a natural analogy between contagion of sequential defaults and rationalizable strategies.[10] First, agents whose incident edges have realized states that cause them to default in any best response, no matter what other players do, default. Then, some agents, knowing that some of their counter-parties will default in any best response, choose to default in any best response. Then some more agents and so on.

In stage one, agents know the distribution by which nature assigns states and the equilibrium selection in stage three. Therefore, they are in a position to evaluate their expected payoff in each possible realized network. Using this knowledge they decide which links to form. Here we describe how the realized network is formed.

Consider a candidate network (N, E) and a coalition of agents $V \subset N$. A **feasible deviation** by V allows agents in V

1. to add any absent edges within V, and
2. to delete any edges incident to at least one vertex in V.

A **profitable deviation** by V is a feasible deviation in which all members of V receive strictly higher expected payoff.[11]

A realized network (N, E) is called **pairwise stable** if there are no profitable deviations by any $V \subset N$ with $|V| \leq 2$ (see Jackson (2010)). G is in the **core**[12] if there are no profitable deviations for any $V \subset N$. We assume that the network formed in stage one is in the core. In the sequel we discuss how our main results change under weaker notions

[10] See Milgrom and Roberts (1990) for more on this. Although not exactly the same, similar algorithms are used in Eisenberg and Noe (2001), Elliott et al. (2014), etc.

[11] The requirement that all agents in a profitable deviation receive strictly higher payoff prevents 'cycling'. To illustrate, consider three nodes $N = \{v_1, v_2, v_3\}$ and $E = \{\{v_1, v_2\}, \{v_2, v_3\}\}$. Suppose v_1 and v_3 deviating to $E' = \{\{v_1, v_3\}, \{v_2, v_3\}\}$, leaves v_1 indifferent and v_3 strictly better off. However, E' is just isomorphic to E and there is no good sense in which v_1 would bother deviating to E'. v_1 and v_2 could very well want to deviate back to E from E'. The same argument applies for E with one element as well. As one can see in this example, precluding 'weak' deviations would be overly restrictive, in particular almost trivially imposing very strong forms of symmetry on any candidate network to be formed.

[12] Farboodi (2014) calls this solution concept group stable as she uses the word to describe something else in that paper. We believe core is an appropriate name for this solution concept, illuminating the resemblance to the cooperative game notion of the core.

of stability.

Our use of the core can be justified as the *strong* Nash equilibrium of a non-cooperative network formation game played between the members of N. Each agent simultaneously proposes to a subset of agents to form an edge. The cost of each proposal is $b > 0$. If a proposal is reciprocated, the corresponding edge is formed. The owners of the edge are refunded b. If a proposal is not reciprocated, b is not refunded and the edge is not formed.

Notice that in any Nash equilibrium of this game, all proposals must be mutual. Consider a *strong* Nash equilibrium of the proposal game. A coalition V can make mutual proposals between themselves to form a missing edge, or undo a proposal by any member which would delete the corresponding edge. Therefore, *strong* Nash equilibria of this game correspond to core networks in the way we have defined it.

1.3 Structure of the Cooperating Equilibrium

For a given $(N, E, \boldsymbol{\theta})$ we characterize the structure of a cooperating equilibrium. In what follows, the following notation will be useful.

Let $\Theta = \{\theta_0, \theta_1, ..., \theta_k\}$ be the set of possible states. For each $v \in N$, let $\Delta_v(\theta) = u_v(B, D; \theta) - u_v(D, D; \theta)$, $\Delta'_v(\theta) = u_v(B, B; \theta) - u_v(D, B; \theta)$; be the gains to v from deviating to B from from D. Denote the vector of these gains by

$$\Delta_v = \big(\Delta_v(\theta_0), \Delta_v(\theta_1), ..., \Delta_v(\theta_k), \Delta'_v(\theta_0), \Delta'_v(\theta_1), ...\Delta'_v(\theta_k)\big) \in \mathbb{R}^{2k+2}.$$

Let $V^c = N \setminus V$ denote the complement of V in N for $V \subset N$. For a given $(N, E, \boldsymbol{\theta})$ let $d(v)$ be the degree of $v \in N$ and $d(v, V, \theta_s)$ be the number of edges in state θ_s which are incident to v and V. Let

$$\pi_s(V|v) = \frac{d(v, V, \theta_s)}{d(v)}$$

be the portion of v's edges that are incident to V and has state θ_s. Denote the vector of these ratios respectively for V^c and V by

$$\pi^v(V) = (\pi_0(V^c|v), \pi_1(V^c|v), ..., \pi_k(V^c|v), \pi_0(V|v), \pi_1(V|v), ..., \pi_k(V|v)) \in \mathbb{R}^{2k+2}.$$

Strictly speaking our notation should depend upon $(N, E, \boldsymbol{\theta})$. However, as these are all fixed in stage three we omit doing so.

Notice that $\Delta_v(\theta) < 0$ and $\Delta_v(\theta) \leq \Delta'_v(\theta)$ for all θ and v (by Assumptions 1 and 2). The following lemma characterizes an agent's best response to the actions of other agents.

Lemma 1. *Consider a $V \subset N$ and $v \in N$. Suppose that agents in $V \setminus \{v\}$ play B, and agents in $(N \setminus V) \setminus \{v\}$ play D. Then D (B) is the unique best reply of v if and only if $\Delta_v \cdot \pi^v(V) < 0$ $(\Delta_v \cdot \pi^v(V) > 0)$.*

Proof. Straightforward. □

Call a $V \subset N$ **strategically cohesive** if for all $v \in V$

$$\Delta_v \cdot \pi^v(V) \geq 0.$$

Proposition 2. *In the cooperating equilibrium, an agent v plays B if and only if there exists a strategically cohesive set V with $v \in V$.*

Proof. (If part) By the 1, $\Delta_v \cdot \pi^v(V) \geq 0$ implies that B is a best reply by v if all players in V play B and others play D. D is rationalizable for every player, therefore, B can never be eliminated for players in V. For all players in V playing B is rationalizable. Hence in the cooperating equilibrium, all of V, in particular v, play B.

(Only if part) Suppose not. Then N is not strategically cohesive (since $v \in N$) and there exists $v_1 \in N$ such that $\Delta_{v_1} \cdot \pi^{v_1}(N) < 0$. Notice that $\pi^{v'}(V') = \pi^{v'}(V'/\{v'\}) = \pi^{v'}(V' \cup \{v'\})$ for any V' and v' since nodes are not adjacent to themselves. Then, $\Delta_{v_1} \cdot \pi^{v_1}(N/\{v_1\}) < 0$. By Lemma 1, v_1's best response to $N/\{v_1\}$ playing B is D. By Assumption 2, v_1's best response to any strategy profile, in particular any strategy profile not eliminated, is D. Thus, v_1 plays D in a cooperating equilibrium. Hence $v_1 \neq v$. Let $N_1 = N/\{v_1\}$. $v \in N_1$. Therefore, by supposition, N_1 is not strategically cohesive. Hence, there exists $v_2 \in N$ such that $\Delta_{v_2} \cdot \pi^{v_2}(N_1) < 0$. Similarly, by Lemma 1 and Assumption 2, v_2's best response to any profile in which $N_1/\{v_2\}$ plays B, in particular any strategy profile not eliminated, is D. Thus v_2 plays D in the cooperating equilibrium, and $v_2 \neq v$. Let $N_2 = N_1/\{v_2\}$. Since N is finite and v plays D in the cooperating equilibrium, we reach a contradiction in a finite number of steps. □

Lemma 2. *If V and V' are both strategically cohesive, then $V \cup V'$ is also strategically cohesive.*

Proof. Consider a $v \in V$. We show that $\Delta_v \cdot [\pi^v(V \cup V') - \pi^v(V)] \geq 0$. In this summation the t'th component is $\Delta_v(\theta_t) \times \left[\pi_t((V \cup V')^c\,|v) - \pi_t(V^c|v)\right]$ and $k + t$'th component is $\Delta'_v(\theta_t) \times [\pi_t((V \cup V')\,|v) - \pi_t(V|v)]$. The terms in the brackets add up to 0. Hence the sum of these two terms is equal to $[\Delta'_v(\theta_t) - \Delta_v(\theta_t)] \times [\pi_t((V \cup V')\,|v) - \pi_t(V|v)] \geq 0$ by Assumption 2.

Therefore, $\Delta_v \cdot \pi^v(V \cup V') = \Delta_v \cdot \pi^v(V) + \Delta_v \cdot [\pi^v(V \cup V') - \pi^v(V)] \geq \Delta_v \cdot \pi^v(V) \geq 0$. □

Call a set $V \subset N$ **maximally cohesive** if it is the largest strategically cohesive set. This is well-defined by Lemma 2.

Proposition 3. *In the cooperating equilibrium, all members of the maximally cohesive set play B, all the others play D.*

Resilience to system wide failure at stage three is determined by the existence of a strategically cohesive set.[13] Strategic cohesiveness is determined by both Δ_v and $\pi^v(V)$. The first captures the effect of payoffs, while the second captures the effect of the structure of the realized network with states. This suggests that the correct ex-post notion of fragility cannot rely on purely network centric measures. Even if one were to look for an appropriate network centric component of a good measure, it would not be measures like too-interconnected-to-fail (which is silent about the neighbors of the neighbors of the too-interconnected node), or degree sequences (which is silent about local structures), but rather cohesiveness which incorporates the idea of a group of nodes reinforcing each other and resisting contagion that began elsewhere.

To separate the effects of network and payoff structure we make some simplifying assumptions and examine their consequences below.

1.3.1 Separating network and payoff effects

We suppose that $\theta < 1$ for all $\theta \in \Theta$ and that payoff functions are the same across agents: $u_v \equiv u$. In particular:

Assumption 3. $u_v(B, B; \theta) = \theta$, $u_v(B, D; \theta) = \theta - 1$, $u_v(D, B; \theta) = u_v(D, D; \theta) = 0$ *(in line with Carlsson and van Damme (1993)).*

For each $V \subset N$ and $v \in N$ let $d(v, V)$ be the number of v's neighbors that are in V. Let

$$\pi(V|v) = \frac{d(v, V)}{d(v)}.$$

Let

$$\pi(v) = (\pi_0(N|v), \pi_1(N|v), ..., \pi_k(N|v)).$$

Given $(N, E, \boldsymbol{\theta})$, a set $V \subset N$ is said to be **ex-post cohesive** if $\pi(V|v) + \theta \cdot \pi(v) \geq 1$ for all $v \in V$. The term $\theta \cdot \pi(v)$ captures v's individual resilience from his payoffs,

[13] One can think of strategic cohesive sets as 'firebreaks'.

$\pi(V|v)$ captures the collective resilience of V as a function of network structure. If V is sufficiently resilient individually and collectively, then it is ex-post cohesive. Notice that under Assumption 3, strategic cohesiveness reduces to ex-post cohesiveness.

For a given $(N, E, \boldsymbol{\theta})$, vertices within an ex-post cohesive set all play B. Thus, they resist default through mutual 'support'. To illustrate, suppose a vertex v's incident edges all have states that are negative valued. Then, $1 - \theta \cdot \pi(v) > 1$ so that v cannot be part of any ex-post cohesive set. Thus, v defaults for sure. As another example, suppose all elements of Θ are positive. Then, the maximally cohesive set would be N itself for *any* possible case of $(N, E, \boldsymbol{\theta})$. Thus, in any realization, all agents play B. Similarly, if all states in Θ were negative, the maximally cohesive set would be the empty set. In every realization all agents would choose D, i.e., there would be certainty of system wide failure.

p-cohesiveness

Ex-post cohesiveness is closely related to p-cohesiveness introduced in Morris (2000). The significance and relevance of p-cohesiveness is further illuminated in Glasserman and Young (2014). Given $p \in \mathbb{R}$, a set V is ***p*-cohesive** if for all $v \in V$, $\pi(V|v) \geq p$. p-cohesiveness imposes a uniform bound on the number of neighbors each vertex in V has in V. Ex-post cohesiveness imposes heterogeneous bounds on the same quantity that depend solely on the realized characteristics of v, particularly how v's edge states are distributed.[14] Notice that if Θ was a singleton, say $\{\theta_0\}$, ex-post cohesiveness would be equivalent to $(1 - \theta_0)$-cohesiveness.

p-cohesiveness is an ex-ante concept relying only on the structure of (V, E). Ex-post cohesiveness, as its name suggests, is an ex-post concept that depends on $(N, E, \boldsymbol{\theta})$. To illustrate, consider a realized edge with a "bad state" $\theta < 0$ in which $\Delta_v(\theta)$ and $\Delta'_v(\theta)$ are

[14]Ex-post cohesiveness can trivially be applied to situation in which edges have heterogeneous volumes.

very small. The presence of such an edge would help a set 'containing' the edge become "more" p-cohesive, however it makes it "less" ex-post cohesive. In this sense, lack of strategic cohesiveness is the appropriate ex-post notion of fragility taking into account the variety in states, while lack of p-cohesiveness is possibly an appropriate ex-ante notion of fragility when the states of edges are not yet realized.

1.3.2 Two states

We introduce a further simplification, $|\Theta| = 2$, with one state being positive and the other negative. This will be convenient for the analysis of the network formation stage and is sufficient to capture most of the essential intuition.

Assumption 4. $\Theta = \{\theta_0, \theta_1\}$, $\theta_1 < 0 < \theta_0$.

In addition:

Assumption 5. $0 < \theta_0 < min\{\frac{1}{N-1}, \frac{-\theta_1}{N-2}\}$.

Assumption 5 ensures that the maximum possible sum of gains from trade scale linearly with N. Another way to interpret this is that the system as a whole cannot withstand bad shocks that make up a fraction of more than $1/N$ of all edges. This assumption simplifies contagion dynamics and buys us great technical convenience in the benchmark model as we will see in the next proposition. We relax this assumption later on.

A **path** between two nodes v_0 and v_{k+1} is a sequence of nodes $v_0, v_1, ..., v_k, v_{k+1}$ such that $\{v_i, v_{i+1}\} \subset E$ for all $i = 0, 1, ..., k$. Two nodes are connected nodes if there is path between them. A subset V of nodes is a connected set if any two elements of V are connected by a path that is resides entirely in V. $V \subset N$ **maximally** connected if V is connected and there is no strict superset of V that is connected.

Proposition 4. *Fix* $(N, E, \boldsymbol{\theta})$. *A set* $V \subset N$ *of nodes is ex-post cohesive if and only if it is (ex-ante) maximally connected and (ex-post) all edges with endpoints in* V *have state* θ_0.

Proof. Choose any $V \subset N$ and any $v \in V$. Observe that $\pi(V|v) = 1$ if and only if all of v's neighbors are in V. Otherwise $\pi(V|v) \leq 1 - \frac{1}{N-1}$. Also, $\theta \cdot \pi(v) = \theta_0$ if and only if all edges of v are θ_0. Otherwise $\theta \cdot \pi(v) \leq \frac{N-1}{N}\theta_0 + \frac{1}{N}\theta_1 < 0$. Note that $1 - \frac{1}{N-1} + \theta_0 < 1$ and $1 + \frac{N-1}{N}\theta_0 + \frac{1}{N}\theta_1 < 1$. Therefore, $\pi(V|v) + \theta \cdot \pi(v) \geq 1$ if only if both $\pi(V|v) = 1$ and $\theta \cdot \pi(v) = \theta_0$ hold. Equivalently, V is ex-post cohesive if and only if for any $v \in V$ all of v's neighbors are in V and all edges incident to v are in state θ_0. □

In the cooperating equilibrium, an agent defaults even if only one edge in the agent's maximally connected component is in the bad state. This is a consequence of the strong contagion embedded in Assumption 5. The condition $0 < \theta_0 < \frac{-\theta_1}{N-2}$ ensures that anyone incident to at least one bad edge defaults. The condition $0 < \theta_0 < \frac{1}{N-1}$ ensures that anyone who has at least one defaulting neighbor also defaults. In a later section, we relax this assumption and discuss its consequences.

1.4 Network Formation

In this section we characterize the set of core networks under the assumptions stated previously. We show that a core network consists of a collection of node disjoint complete subgraphs[15]. By forming into complete subgraphs agents increase the benefits they enjoy from partnerships. However, the complete subgraphs formed are limited in size[16] and order[17], and are disjoint. In this way agents ensure that a default in one portion of the realized network does not spread to the entire network. This extreme structure is a consequence of the spareness of our model. However, it suggests that more generally we should expect to see collections of densely connected clusters that are themselves sparsely connected to each other. Blume et al. (2013) have a similar finding in their paper.

We first need to determine an agent's expected payoff in various realized networks.

[15]A graph (N', E') is a subgraph of (N, E) if $N' \subset N$ and $E' \subset E$.
[16]The size of a subgraph or a subset of edges is the number of edges in it.
[17]The order of a subgraph or a subset of nodes is the number of nodes in it.

Recall that nature determines states identically and independently across edges. Let α be the probability that an edge has state θ_0 and $1-\alpha$ be the probability that it has state θ_1. Consider $v \in N$ and suppose that in a realized network, v has degree d and the maximally connected component that contains v has e edges. By virtue of Proposition 4 we need only consider the case where everyone in the maximally connected component defaults or no one does. The probability that every node in the relevant component defaults is $1-\alpha^e$. In this case, v gets 0. The probability that no one in the relevant component defaults is α^e. In this case, v gets $d\theta_0$. So v's expected payoff in stage two is $d\alpha^e\theta_0$. Using this, we can find what happens in stage one.

Being pairwise stable, henceforth stable, is a necessary condition for being a core network. We first identify conditions on stable networks, then move onto core networks.

1.4.1 Stable networks

Lemma 3. *Any stable network consists of disjoint complete subgraphs.*

Proof. Suppose, for a contradiction, a stable network with two non-adjacent nodes v' and v'' in the same connected component. Take a path $v' = v_1$, v_2,... $v_t = v''$ between v' and v''. Insert the edge $\{v', v''\}$ and delete $\{v', v_2\}$, as well as $\{v_{t-1}, v''\}$. The degrees of v' and v are unchanged but the number of edges in the component that contains them strictly decreases. Hence, this is a profitable pairwise deviation by v' and v'' which contradicts stability. Therefore, in any stable network all nodes within the same connected component are adjacent, which completes the proof. □

The orders of these complete subgraphs are not arbitrary. Let $U(d) := d\alpha^{(0.5)d(d+1)}$, and $d^* = \arg\max_{d \in \mathbb{N}} U(d)$. For generic α, d^* is well defined. Note that $U(d)$ is strictly increasing in $d \in \mathbb{N}$ up to d^*, and strictly decreasing after d^*. Further, d^* is an increasing step function of α. Let $h^* \geq d^*$ be the largest integer h such that $U(1) \leq U(h)$. Let

$h^{**} \leq d^*$ be the largest integer such that $\frac{1}{\alpha} \leq \frac{h+1}{h}\alpha^{(0.5)h(h+1)} = \frac{U(h+1)}{h\alpha^h}$.

Proposition 5. *Any network that consists of disjoint complete subgraphs, each with order between $h^{**}+1$ and h^*+1, is stable. Call these **uniform stable networks.***[18]

Proof. Consider a uniform-stable network and suppose that there is a profitable bilateral deviation by two nodes. Take one of them, let her have degree d, and let her have $e = d(d+1)/2$ edges in her complete subgraph. Suppose that in the bilateral profitable deviation she deletes x of her incident edges in her complete subgraph, and adds $t \in \{0,1\}$ new edges.

If $x = d$, her payoff is at most $\alpha = U(1) \leq U(d)$ (since $1 \leq d \leq h^*$) which cannot be a profitable deviation. So $x < d$, which means she is still incident to $e - x$ edges in her old component. Then her payoff is at most $(d - x + t)\alpha^{e-x+t}$. If $t - x \leq 0$, this is less than $d\alpha^e$ since $y\alpha^y$ is strictly increasing up to k^* in $y \in \mathbb{N}$ and $h \leq k^*$. Then $t - x > 0$, which is possible only when $t = 1$ and $x = 0$. This is true for the other deviator as well. Therefore, these two deviators keep all their previous edges and connect to each other with a new edge.

Let the other deviator have degree d'. Without loss of generality, let $d \leq d'$. Then, the deviator with the smaller degree has her payoff moved from $d\alpha^{(0.5)d(d+1)}$ to $(d+1)\alpha^{1+(0.5)d(d+1)+(0.5)d'(d'+1)}$ which is less than or equal to $(d+1)\alpha^{1+d(d+1)}$. This being a profitable deviation immediately implies $d < h^{**}$, which is a contradiction. □

1.4.2 Core networks

Lemma 4. *If a network is in the core, it consists of a collection of disjoint complete subgraphs, all but one of order (d^*+1). The remaining complete subgraph is of order at most d^*+1.*

[18] This is close to a complete characterization of all stable networks in the following sense. Any complete subgraph in any stable network has to be of order at most h^*+1. Moreover, there can be at most one complete subgraph with order less than $h^{**}+1$. The bound on the smallest order depends on what the second smallest order is, and is more involved to characterize.

Proof. By Lemma 3 a core network (if it exists) is composed of disjoint complete subgraphs. The payoff to an agent in a $(d+1)$-complete subgraph is $U(d) = d\alpha^{(0.5)d(d+1)}\theta_0$. This is strictly increasing up to d^*.

First, no complete subgraph can have order $d+1 > d^*+1$ in the realized network. Otherwise, d^*+1 members could deviate by forming a (d^*+1)-complete subgraph and cutting all other edges. This would be a strict improvement since d^* is the unique maximizer of $U(d)$.

Second, there cannot be two complete subgraphs of order $d+1 < d^*+1$. Suppose not. Let there be $d'+1$ nodes all together in these two complete subgraphs. Then $min\{d'+1, d^*+1\}$ nodes would have a profitable deviation by forming an isolated complete subgraphs since $U(d)$ is increasing in d up to d^*. □

A realized network in the core necessarily consists of a collection of complete subgraphs of order d^*+1 and one 'left-over' complete subgraph with order different from (d^*+1). To avoid having to deal with the 'left-over' we make a parity assumption about N. For the remainder of the analysis we assume $N \equiv 0\,(mod\, d^*+1)$. In fact, without this assumption, the core may be empty. To see why, assume that the 'left-over' complete subgraph is of order 1. This single left-over agent would like to have any edge rather than having none, and any other agent would be happy to form that edge since that extra edge does not carry excess risk. We would expect a pairwise deviation which would contradict the stability. However, even in this case, $N-1$ agents don't have a deviation among themselves without using the single left over node. In section 1.7 we consider solution concepts other than core or stability as well.

Theorem 1. *For $N \equiv 0\,(mod\, d^*+1)$, the core is non-empty, unique (up to permutations) and consists of disjoint (d^*+1)-complete subgraphs.*

Proof. Assuming non-emptiness of the core and the parity assumption, Lemma 4 suffices to yield uniqueness once we have existence. It remains to show that a realized network

$G = (N, E)$ consisting of disjoint complete subgraphs $C_1, C_2, ..., C_k$ all of order $(d^* + 1)$ (for k such that $N = k\,(d^* + 1)$) is a core network.

For any profitable deviation by V' from G to G', define $\phi(V', G')$ as the number of edges between V' and N/V' in G'. Let the minimum of ϕ be attained at (V^*, G^*).

Consider G^*. Take a node $v' \in V^*$ that is adjacent to N/V^*. Suppose that there exists $v'' \in V^*$ such that v' is connected but not adjacent to. Cut one edge connecting v' to N/V^* and join the missing edge between v' and v''. This new graph, say G'', is also a profitable deviation by V^* from G. This is because when we move from G^* to G'', the degrees of all nodes in V^* weakly increase, and their component sizes weakly decreases. However, $\phi(V^*, G'') < \phi(V^*, G^*)$, which is a contradiction. Therefore, any node in V^* that is connected to v' is adjacent to it. The same holds for any node that is adjacent to N/V^*.

Take a node in V^* with minimal degree, say v with degree d. Let $d' \geq 0$ be the number of v's neighbors in N/V^*. Suppose $d' \geq 1$. By the last paragraph, a node in V^* that is connected to a neighbor of v can only be a neighbor of v. Therefore, any neighbor of v in V^* has at most $d - d'$ neighbors in V^*, hence at least $d' \geq 1$ neighbors in N/V^*. So by the last paragraph, v and his $d - d'$ neighbors in V^* are all adjacent to each other, forming $(0.5)(d - d' + 1)(d - d')$ edges. Each of them have at least d' edges to N/V^*, so that makes $d'(d - d' + 1)$ edges. Finally, since nodes in N/V^* have not deviated from G and are connected to each other, they are all adjacent to each other, forming $(0.5)d'(d' - 1)$ edges. Therefore, in v's maximally connected component, there are at least $(0.5)d(d + 1)$ edges, so that his payoff is at most $U(d)$. Now suppose $d' = 0$. Then all v's d neighbors are in V^*, hence all have degree at least d. Then again, v's component has at least $d(d + 1)/2$ edges, so that his payoff is at most $U(d)$. In both cases, v's payoff in G^* is at most $U(d) \leq U(d^*)$; contradiction with profitable deviation from G. □

Theorem 2. *For $N < d^* + 1$, the unique core network is the N-complete subgraph.*

Proof. Recall that $d\alpha^{(0.5)d(d+1)}$ is increasing in d up to $d^* > N$. The remainder of the proof follows the proof of Theorem 1 by replacing $d^* + 1$ with N. We omit the details. □

1.5 Efficiency and Systemic Risk

In this section we define what it means for a network to be efficient and show that a network is efficient if and only if it is in the core. The other stable networks are inefficient, which suggests that some inefficiencies in observed networks stem from the inability of large groups to coordinate.

We also identify another source of inefficiency by relaxing the assumptions governing the strength of contagion. When bad shocks are highly contagious, any expected externality that a node imposes on others turns back on itself, and is naturally internalized. On the other hand, when bad shocks are weakly contagious, agents don't need to consider anyone other than their immediate neighbors. As a consequence, they don't internalize their externalities which leads to excess connectivity and inefficiency.

We further show that systemic risk in the efficient/core network increases as the probability α of a good shock increases. This follows the safety belt argument: as the economy gets safer, agents form networks with higher systemic risk. This intuition, however, may change with different notions of systemic risk.

1.5.1 Efficiency

The efficient network

Call a realized network (N, E) **efficient** if it maximizes the sum of expected payoffs of agents among all realized networks. Consider a connected subgraph with e edges. A node in the subgraph with degree d enjoys an expected payoff of $d\alpha^e\theta_0$. Therefore, the sum of payoffs of nodes within the graph is $2e\alpha^e\theta_0$. Here we use the well known fact that the sum of degrees is twice the number of edges. It follows then, that the problem of finding an efficient network devolves into two parts: how to partition nodes into maximally connected components, and how many edges to put into each component.

Let $k^* = \arg\max_{y \in \mathbb{N}} y\alpha^y$. For generic α this is well defined.[19] Note that $y\alpha^y$ is strictly increasing in $y \in \mathbb{N}$ up to k^* and strictly decreasing after k^*. Note also that when maximizing $y\alpha^y$ over the non-negative reals, the maximum occurs at a number $y^* = -\frac{1}{log(\alpha)}$ where $\alpha^{y^*} = \mathbf{e}^{-1}$. Here $\mathbf{e}$ is Euler's constant and y^* lies in the interval $(\frac{\alpha}{1-\alpha}, \frac{1}{1-\alpha})$.

Theorem 3. *If $N \equiv 0 \,(mod\, d^* + 1)$, a network is efficient if and only if it is in the core.*

Proof. Recall that $U(x) = x\alpha^{(0.5)x(x+1)}$. Let $\mathcal{U} = \{u \in \mathbb{R} \mid u = U(x)$ for some $x \in \mathbb{N}\}$. The maximum of $\mathcal{U}$ is achieved, uniquely, at $x = d^*$. Let $\bar{u} = U(d^*)$. Notice that this is the average payoff at the core network. We will prove that the average is strictly less in any other network.

Consider an efficient network G and suppose it to be made up of a collection of disjoint connected components: $C^1, C^2, C^3, \ldots$. Consider component C^i and suppose it has q_i edges. The total payoff of C^i scales with $2q_i\alpha^{q_i}$. If $q_i \neq k^*$ we can improve total payoff by deleting or adding (if not complete graph) edges to C^i. Therefore, we can assume that $q_i = k^*$, or that C^i is complete.

Let r_i be the largest integer such that $r_i(r_i-1)/2 < q_i \leq r_i(r_i+1)/2$. Let w_i be such that $q_i = r_i(r_i-1)/2 + w_i$, where $1 \leq w_i \leq r_i$. Note that there must be at least $r_i + 1$ nodes in C^i.

Case 1: $1 \leq w_i \leq \frac{r_i - 1}{2}$.

The average degree of nodes in C^i is at most $\frac{2k^*}{r_i+1} = \frac{r_i(r_i-1)+2w_i}{r_i+1} \leq r_i - 1$. Note that $k^* = q_i \geq (r_i-1)r_i/2 + 1$. Hence the average payoff per node is at most $(r_i-1)\alpha^{k^*} <$

[19] For α such that $(1-\alpha)^{-1}$ is integral, there are two integers in the arg max: $\frac{\alpha}{1-\alpha}$ and $\frac{1}{1-\alpha}$. In other cases, the arg max is unique: it is the unique integer in the open interval $(\frac{\alpha}{1-\alpha}, \frac{1}{1-\alpha})$, i.e. $\lfloor \frac{1}{1-\alpha} \rfloor$.

$(r_i - 1)\alpha^{\frac{(r_i-1)r_i}{2}} \leq \bar{u}$. So the average payoff is strictly less than $\bar{u}$.

Case 2: $r_i - 1 \geq w_i \geq \frac{r_i}{2}$.

Since $w_i < r_i$, $k^* = q_i \leq r_i(r_i+1)/2 - 1$. The average degree of nodes in C^i is at most $\frac{2k^*}{r_i+1} \leq \frac{r_i(r_i+1)-2}{r_i+1} \leq r_i - \frac{2}{r_i+1}$. Note that $k^* = q_i = (r_i-1)r_i/2 + w_i \geq r_i^2/2$. Hence the average payoff per node is at most $\left(r_i - \frac{2}{r_i+1}\right)\alpha^{r_i^2/2}$. Now we show that this is strictly less than $(r_i-1)\alpha^{(r_i^2-r_i)/2} = U(r_i-1)$. That is equivalent to showing that $\alpha < \left(\frac{r_i+1}{r_i+2}\right)^{2/r_i}$. Recall that k^* is the unique integer between $\alpha/(1-\alpha)$ and $1/(1-\alpha)$. Therefore, $\alpha \leq 1 - \frac{1}{k^*+1} \leq 1 - \frac{2}{r_i(r_i+1)}$. Hence, it suffices to verify that

$$1 - \frac{2}{r_i(r_i+1)} < \left(\frac{r_i+1}{r_i+2}\right)^{\frac{2}{r_i}} \iff \left(\frac{r_i+1}{r_i+2}\right)^{\frac{2}{r_i}} > \frac{(r_i+2)(r_i-1)}{(r_i)(r_i+1)}$$

$$\iff (r_i+2)\log\left(1 - \frac{1}{r_i+2}\right) > r_i \log\left(1 - \frac{1}{r_i}\right)$$

which is true since the function $f(x) = x\log(1-\frac{1}{x})$ is strictly increasing. Therefore, the average payoff is strictly less than $U(r_i-1) \leq U(d^*) = \bar{u}$.

Case 3: $w_i = r_i$. (This covers the case in which C^i is complete as well.)

Then the average payoff per node is less than $U(r_i) \leq \bar{u}$, and the inequality is strict unless C^i is a $(d^*+1)-$complete graph. □

All stable networks other than the core network are, thus, inefficient.[20] This suggests that some inefficiencies that arise in the observed networks may stem from the inability of large groups to coordinate at the network formation stage.

In order to focus on the benchmark case we economize on the proofs of other results by sketching lengthy ones or omitting entirely, proofs similar to previous proofs, in the

[20] Blume et al. (2013) find that their stable networks are not efficient. However, their notion of efficient is a worst-case one, very different from the one employed here. Farboodi (2014) also finds that formed networks are inefficient, despite having the core as the solution concept.

paper. The fundamental techniques we use are contained in the proofs provided thus far.

Relaxing the strength of contagion

In this subsection only, we relax the assumption governing the strength of contagion to provide better intuition for why agents may or may not form efficient networks.

In Assumption 5, $\theta_1 + (N-2)\theta_0 < 0$ ensures anyone incident to an edge subject to a bad shock defaults, whatever his degree $d \leq N-1$ is. This allows a single bad shock to start a contagion, and we keep this unchanged here. The condition $(\theta_0 - 1) + (N-2)\theta_0 < 0$ ensures that a node, even when all incident edges are good, has to default if at least one neighbor defaults, whatever his degree $d \leq N-1$ is. This governs the spread of contagion, and we relax this condition here.

First note that under $\theta_1 + (N-2)\theta_0 < 0$, a realized network is Nash [21] only if the degrees of all nodes are less than or equal to k^*.

If $2(\theta_0 - 1) + (N-3)\theta_0 < 0$ that would mean, a node incident to a bad edge, and has degree exactly N, defaults if she has two defaulting neighbors. But, it is unlikely for relatively large N that any node will have degree N since in any Nash, hence stable, hence core network, all degrees must be less than or equal to k^*. What is actually relevant for a node with degree d is $2(\theta_0 - 1) + (d-3)\theta_0 < 0$, hence we could safely relax the assumption by many degrees, especially for large N.

For this reason, we consider the other extreme, as a way of retarding contagion: if all a node's incident edges are good, she defaults only when all her neighbors default. As long as one neighbor does not default, she does not default either. Formally: $(N-2)(\theta_0 - 1) + \theta_0 > 0$.

In this case, the expected payoff of an agent who has degree d, and whose neighbors

[21] A network in which no node has a profitable unilateral deviation

have degrees $n_1, n_2, ..., n_d$ is

$$\frac{1}{\alpha}\alpha^d\left(\alpha^{n_1}+\alpha^{n_2}+...+\alpha^{n_d}\right).$$

Define $k^{**} := argmax_{d\in\mathbb{N}} d\alpha^{2d-2}$. Note that $\frac{k^*}{2}-1 \leq k^{**} \leq \frac{k^*}{2}+1$.

Proposition 6. *A network is efficient if and only if it is k^{**}-regular.*

Proof. See appendix. □

Proposition 7. *Consider any stable network. There are at most $\left(\frac{k^*+1}{2}\right)^2$ many nodes with degree different than k^*-1. The remainder have degree k^*-1. In this sense, any sufficiently large stable network is almost k^*-1 regular, hence inefficient.*

Proof. See appendix. □

Note that stable networks exhibit almost double the efficient level of degree per node. In this sense, there is excess interconnection at any stable network when the risk of contagion is low. There are other properties of stable networks which are not of first order importance, thus omitted.[22]

Proposition 8. *If $N \geq \left(\frac{k^*}{2}+1\right)^2$ and $\alpha > \frac{2}{\mathbf{e}}$, the core is empty.*

Proof. See appendix. □

Recall that $k^* = \lfloor\frac{1}{1-\alpha}\rfloor$. When α is such that $k^* < \frac{1}{1-\alpha}-\alpha$, there are no stable networks for large N. For α such that $\frac{1}{1-\alpha}-\alpha < k^* < \frac{1}{1-\alpha}$, we have the following.

Proposition 9. *If $N \equiv 0\,(mod\, k^*)$ and α such that $\frac{1}{1-\alpha}-\alpha < k^* < \frac{1}{1-\alpha}$, then, a network that consists of disjoint complete subgraphs of order k^* is stable.*

Proof. See appendix. □

[22]For example, nodes with degrees other than k^*-1 are in close proximity to each other.

When contagion is very strong, any externality imposed on another at any distance, comes back to 'bite one.' The strength of contagion ensures nodes internalize their externalities. Hence, they form efficient structures, in the form of complete subgraphs. When contagion is very weak, nodes no longer internalize the externalities they impose on others. Therefore, efficiency is lost. This highlights the risk of contagion (conditional on it being initiated) as a source of *efficiency* (but not necessarily higher welfare with respect to the weak contagion case), rather than inefficiency, in our main result.

Comparative Statics

We return to the benchmark model with strong contagion, and provide some comparative statics on efficiency.

Note that the total payoff in a network which consists of disjoint complete subgraphs of order $d+1$ is $N \times U(d)$. The figures below illustrate the differences in connectivity and efficiency between core and stable networks. [23]

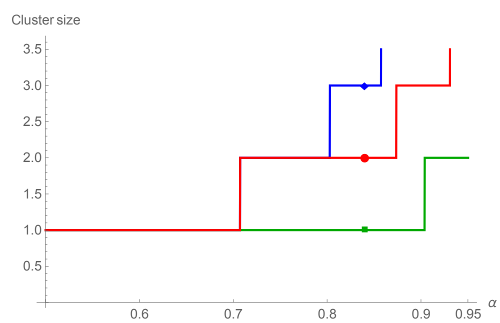


Figure 1: Figure 1(a): For $0.5 < \alpha < 0.9$

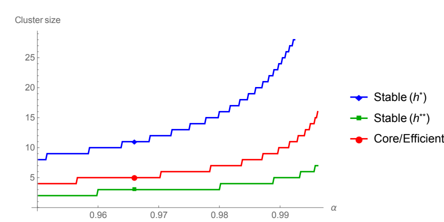


Figure 2: Figure 1(b): For $\alpha > 0.9$

Figure 3: Cluster Sizes of Stable and Core Networks vs. α

[23] We plot the properties of the the most and the least interconnected uniform-stable networks, the ones with cluster size h^* and h^{**}.

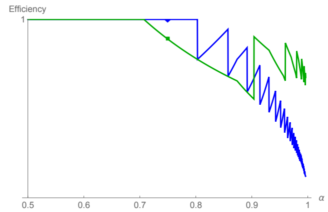


Figure 4: Figure 2(a): Efficiency in Stable Networks vs. α

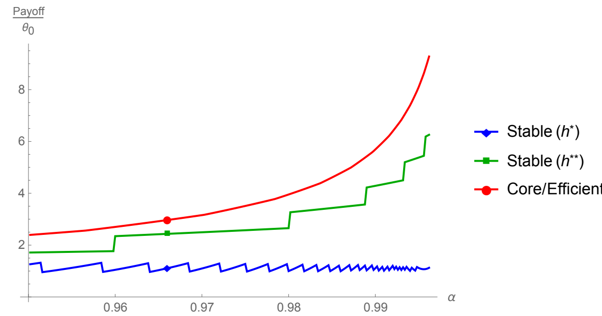


Figure 5: Figure 2(b): Payoff Per Node in Stable and Core/Efficient Networks vs. α

Figure 6: Payoffs and Efficiency in Stable Networks

1.5.2 Systemic risk

Systemic risk at the core/efficient network

Fix $N \equiv 0\,(mod\, d^* + 1)$ and consider the core network. Recall that all nodes of a maximal complete subgraph play D if at least one of the edges in the complete subgraph is in a 'bad' state; otherwise they all choose action B. The probability that any node/all nodes in a maximal complete subgraph chooses D is $1 - \alpha^{(0.5)d^*(d^*+1)}$. Hence, the probability that everybody defaults, i.e. systemic risk, is

$$\left(1 - \alpha^{(0.5)d^*(d^*+1)}\right)^{\frac{N}{d^*+1}}.$$

For fixed α, the above expression is increasing in $d^* < N$. An increase in d^* leads to fewer but larger complete subgraphs. Thus, for fixed α higher interconnectedness translates into higher systemic risk. For fixed d^*, the expressiong decreases in α. However it is not apriori clear whether systemic risk increases or decreases with a change in α. Note that as α increases, the core consists of fewer but larger clusters. As one can see in Figure 3, it turns out, systemic risk of the core/efficient network increases with α. In our model, d^* (weakly) increases with α. It increases at such a rate that systemic risk of the core/efficient network also increases with α.[24] This is displayed in Figure 3.

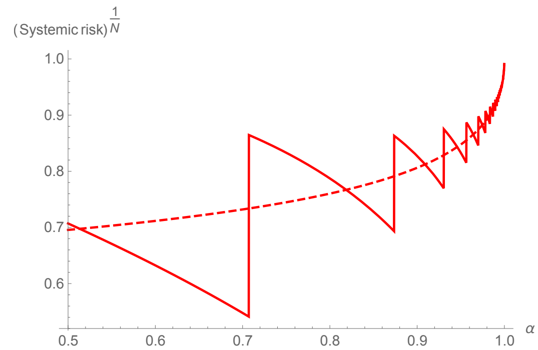


Figure 7: Systemic Risk of the Core Network vs. α

Intuitively, as the economy gets fundamentally safer, agents form much larger clusters. That is in their individual interest and furthermore the outcome is efficient. However, the risk from interconnectedness dominates the safety from α, and this results in *increased* systemic risk: catastrophic events become more frequent. Note that once α becomes too large and hits $\left(\frac{N-1}{N}\right)^{\frac{1}{N}}$, d^* becomes N and the clusters cannot get any larger. Hence the systemic risk cannot get any larger and it starts decreasing again.

Figures 4 below show how the expected number of defaults, $N \times \left(1 - \alpha^{(0.5)d^*(d^*+1)}\right)$ varies with α.

[24]Since d^* is a step function of α, in intervals where d^* stays constant the probability decreases. However, this is an artifact of discreteness. When α hits $\left(\frac{d-1}{d}\right)^{\frac{1}{d}}$, d^* jumps from $d-1$ to d. If one considers these jumping points of α, the probability is increasing. In order to clarify further, recall the definition of $d^* = argmax_{d \in \mathbb{N}} d\alpha^{(0.5)d(d+1)}$. For a "smooth version" of d^* as a function of α, a real number $d^* = argmax_{d \in \mathbb{R}} d\alpha^{(0.5)d(d+1)}$, the probability is strictly increasing.

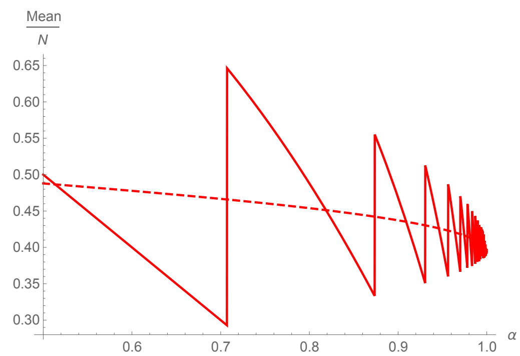


Figure 8: Mean of the Number of Defaults at Core Network vs. α

We can actually pin down the exact distribution of the number of nodes that default. Given α, the number of maximal complete subgraphs that fail is k with probability

$$\binom{\frac{N}{d^*+1}}{k}\left(1-\alpha^{(0.5)d^*(d^*+1)}\right)^k\left(\alpha^{(0.5)d^*(d^*+1)}\right)^{\frac{N}{d^*+1}-k}.$$

This is also the probability that $(d^*+1)k$ agents default and the rest do not. For $N=100$, Figure 5 illustrates the distribution.

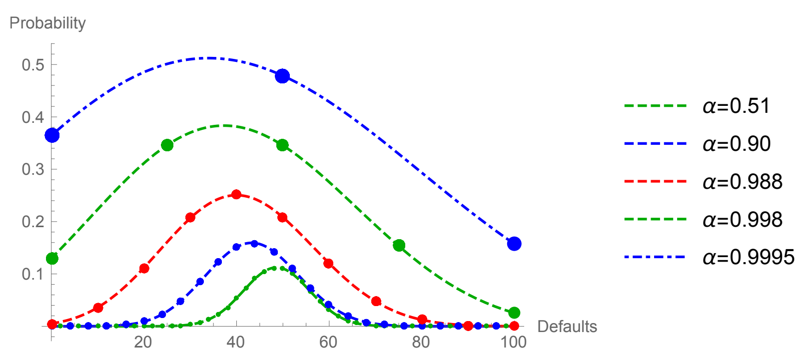


Figure 9: Probability Distribution of the Number of Defaults at Core Network (For $N=100$)

There is no first order stochastic dominance order among these distributions. However, the distributions with larger α's second order stochastically dominate those with smaller α's.

Systemic risk at stable and core/efficient networks

Next, we compare the systemic risk of stable networks with core/efficient networks. Call uniform stable networks whose maximal complete subgraphs all have order larger than or equal to $d^* + 1$ be called **upper-uniform stable** networks, and those with all maximal complete subgraphs having order smaller than $d^* + 1$ be called **lower-uniform stable** networks.

Proposition 10. *Take $N \equiv 0 \,(mod\, d^* + 1)$. Upper-uniform (lower-uniform) stable networks have higher (lower) systemic risk than the core/efficient network.*

Proof. Recall that $\left(1 - \alpha^{(0.5)x(x+1)}\right)^{1/x}$ is increasing in x. Take any complete subgraph with order $d + 1 \geq d^* + 1$.

$$1 - \alpha^{(0.5)d(d+1)} = \left(1 - \alpha^{(0.5)d(d+1)}\right)^{(d+1)/(d+1)} \geq \left(1 - \alpha^{(0.5)d^*(d^*+1)}\right)^{(d+1)/(d^*+1)}.$$

Let $d_t + 1$'s be the orders of maximally complete subgraphs of a upper-uniform stable network. Then

$$\prod_t \left(1 - \alpha^{(0.5)d_t(d_t+1)}\right) \geq \left(1 - \alpha^{(0.5)d^*(d^*+1)}\right)^{\frac{1}{d^*+1}\sum d_t+1} = \left(1 - \alpha^{(0.5)d^*(d^*+1)}\right)^{\frac{N}{d^*+1}}.$$

The case for lower-uniform stable networks have the similar proof. □

Figure 6 illustrates the difference in systemic risk between stable and core networks for various values of α.

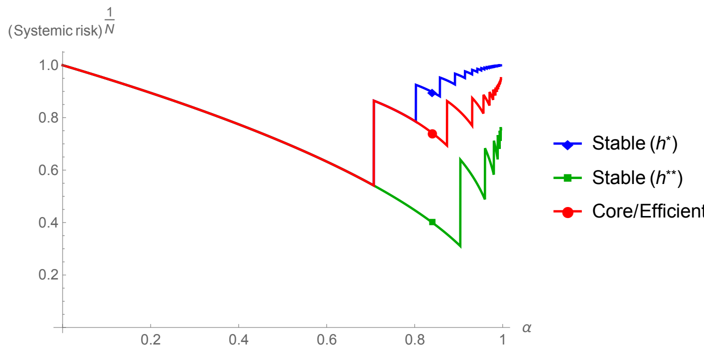


Figure 10: Systemic Risk in Stable and Core Networks vs. α

These findings suggest that some inefficiencies in observed networks may stem from the inability of parties to coordinate. However, systemic risk of these inefficient networks can be more or less than that of the core network. Thus, systemic risk is not a good indicator of inefficiency. The frequency of catastrophic events can be more or less at inefficient networks than the efficient network.

1.6 Correlation

We noted earlier a debate about whether interconnectedness of nodes is a significant contributor to systemic risk. An alternative theory is that the risk faced is via common exposures, i.e., popcorn. Observed outcomes might be similar in both scenarios but the dynamics can be significantly different.

We model the popcorn story as perfect correlation in states of edges through ϕ. Thus, ϕ is such that with probability σ *all* edges have state θ_0, with probability $1 - \sigma$ *all* edges are in state θ_1.

There is no change in the analysis of stage three. As for stage one, now there is no risk of contagion.

Theorem 4. *Under 'popcorn', the unique core (and unique stable) network is the complete graph* *on N nodes, denoted K_N.*

Proof. In any given realized network, if all states are θ_0 then everybody play B and if all states are θ_1 then everybody play D. The payoff of an agent with d edges is $d\theta_0$ or 0 respectively. Thus, the expected payoff of each agent is $d\sigma\theta_0$. Then, it is clear that in a core (or stable) network there cannot be any missing edges because that would lead to a profitable pairwise deviation. The only candidate is K_N which is as clearly in the core. □

When agents anticipate common exposures (popcorn) rather than contagion, they form highly interconnected networks in order to reap the benefits of trade.

In an independent shocks world, the probability that everybody defaults in K_N is $1 - \alpha^N$, which is the highest systemic risk that any network can achieve in this world. However, K_N is as safe as all the other possible realized networks in the correlated shocks world. This highlights the importance of identifying the shock structure before investigating a given network. A specific network and a particular shock structure might very well be incompatible.

1.6.1 More general correlation

Perfect correlation and complete independence are two extremes. Here we extend the benchmark model to allow for a correlation structure that is in between. With some probability the economy operates as 'normal' and edges are subject to their own idiosyncratic shocks, while with complementary probability a common exposure to risk is realized and all edges have bad states. Formally, with probability $1 - \sigma$ all edges are θ_1, while with probability σ all states of edges are i.i.d.: θ_0 with probability α and θ_1 with probability $1 - \alpha$. Notice that '$\sigma = 1$, $\alpha > 0$' is the extreme case of 'independence with α being the probability of an edge being in a good state'. The case '$\alpha = 0$, $\sigma < 1$' is the extreme case of 'perfect correlation with σ being the probability of all edges being in a good state'.

In this setting, the expected payoff of an agent is $d\alpha^e\sigma\theta_0$. Clearly, the identical analysis in section 4 goes through for any σ. Notice that as α tends to 1, d^* diverges to ∞. For some $\bar{\alpha} < 1$, $\alpha > \bar{\alpha}$ implies that $d^* > N$. Then, by Theorem 2, the unique core is K_N. This illustrates that Theorem 4 is not an anomaly due to perfect correlation. In fact, it is a corollary of Theorem 2; the same result holds for sufficiently strong correlation not just perfect correlation.

1.7 Extensions

We summarize three variations to our model to illustrate robustness of our results. The first considers weaker notions of network formation. The second allows for shocks to nodes in addition to edge shocks. Lastly, we consider different forms of asymmetries between nodes and see how the results are altered.

1.7.1 Weaker notions of network formation

The results above about the core assume the ability of any coalition to get together and 'block'. Networks that survive weaker notions of blocking are also of interest. Two natural candidates are Nash networks and stable networks. The first preclude deviations by single nodes only, while the second by pairs only. All core networks are pairwise stable, and all pairwise stable networks are Nash networks.

Robustness to unilateral deviations is too permissive. Most (permutation classes of) graphs with degree less than k^* are Nash networks. This is because no node can add an edge in a feasible Nash deviation. As for deleting edges, for graphs that are sufficiently well connected a unilateral deletion will not reduce the cluster size very much. Hence, agents are not going to delete edges since they already have less than k^* edges. We have already studied stability before in the benchmark model.

Here we consider the middle ground between the core and stable networks. Call a network (N, E) ***t*-stable** if no coalition of size t or less has a profitable deviation. Notice that N-stable is equivalent to the core, and 2-stable is equivalent to stable.

Proposition 11. *For any $t \geq d^* + 1$, the unique t-stable network is the core.*

Keeping in mind that we typically think of $d^* + 1$ as being relatively small with respect to N, this proposition shows us that the results in the paper don't need the full power of the core that precludes profitable deviations by any coalition. A restriction on relatively small sized coalitions is sufficient. The next theorem concerns $t \leq d^*$.

Proposition 12. *Take any $t \leq d^*$. Let $h^*(t) \geq d^*$ be the largest integer such that $U(t) \leq U(h^*(t))$. Any network that consists of disjoint complete subgraphs, each with order between $d^* + 1$ and $h^*(t) + 1$, is t-stable. Call these **upper-uniform t-stable** networks.*

Notice that as $t \leq d^*$ gets smaller, upper-uniform t-stable networks become similar to upper-uniform stable networks. As $t \leq d^*$ gets larger, $h^*(t)$ approaches $d^* + 1$, so that upper-uniform t-stable networks become closer to core networks. After d^*, for $t \geq d^* + 1$ the only t-stable network is the core itself (the upper-uniform $(d^* + 1)$-stable network). These results bridge the gap between the core and stability.

As t gets larger, t-stable-complete networks become more efficient in a sense. Networks are subjected to further constraints by precluding deviations by larger coalitions, and the remaining set of networks get closer to the efficient/core networks, increasing the efficiency. Similarly, systemic risk of upper-uniform t-stable networks decline with larger values of t.

1.7.2 Node shocks

We now consider shocks to individual nodes. There are two ways to think about such shocks. The first is an idiosyncratic shock that affects an institution without any direct effect to any other institution, such as liquidity shocks. The second is one in which the financial sector has ties with the real sector and these ties are subject to shocks as well. In the model, each node (financial institution) is incident to an (imaginary) edge outside of the network. The shock to this edge is effectively an idiosyncratic shock to the node itself.

These shocks can be correlated but we consider the case of independent node-shocks only. Formally, after stage two has ended and before we move on to stage three, each 'imaginary' edge independently defaults with probability $1 - \beta$ or proceed as **normal** with probability β.

In stage three, ex-post cohesive sets are maximally connected sets all of whose edges are in state θ_0 and nodes are normal. In this case members of such a set play B and get θ_0 for each edge they have. Otherwise they play D and get 0.

In stage two, the expected payoff of a node with degree d in a maximally connected component with e edges and f nodes has payoff, $d\alpha^e \beta^f \theta_0$.

As for stage one, the earlier results apply. A core network will consist of disjoint complete subgraphs. Let $d^{**} := \arg\max_{d \in \mathbb{N}} d\alpha^{(0.5)d(d+1)} \beta^{d+1}$. Theorems and comparative statics concerning the core apply with d^* replaced by d^{**}.

Note that d^{**} is smaller than d^*. This tells us that when agents are exposed to new types of risks, which effectively increases their overall risk, they form less interconnected networks.

1.7.3 Different Types of Agents

The ex-ante symmetry of agents leads to symmetric realized network as well. Here, we allow one agent to differ from the others in its exposure to risks from states of edges.

This one agent, named C, has a utility function which does not depend on the state of its incident edges. In particular, for some fixed $p \in (0, 1)$, $u_C(B, B; \theta) = p$, $u_C(B, D; \theta) =$

$p - 1$, $u_C(D, B; \theta) = u_C(D, D; \theta) = 0$ for every θ. On the other hand, the other agents enjoy the same payoffs as in the benchmark model from all their incident edges, except the edges with C. The payoffs associated with edges incident with C have the form: $u(B, B; \theta) = \theta + \varepsilon$, $u(B, D; \theta) = \theta - 1$, $u(D, B; \theta) = u(D, D; \theta) = 0$.[25] In particular, the game played on the edges of C is given by

	B	D
B	$p, \theta + \varepsilon$	$p - 1, 0$
D	$0, \theta - 1$	$0, 0$

For technical convenience, we take p such that $\frac{1}{1-p}$ is an integer: $s^* := \frac{1}{1-p} \in \mathbb{N}$, and $p \geq \alpha^* := \alpha^{(0.5)d^*(d^*+1)}$.[26] Subsequently we will provide an interpretation of agent C as a 'lender'.

Call a set of nodes not containing C a **group** if these nodes are connected without using paths going through C. If a group is connected to C, call it a C**-group**, otherwise an NC**-group**. If C defaults, everybody in all C-groups default in any strategy profile that survives iterated dominance. If strictly more than p portion of C's neighbors play D, node C's only best response is to play D. If at most fraction p of C's neighbors play D, then B is a best response of C to the belief that the remaining nodes play B. Therefore, the unique cooperating equilibrium is given by: 1) all NC-groups behaving as in the benchmark case, 2) if more than p portion of C's neighbors have at least one bad edge in their group, all C-groups and C play D, 3) if more than or equal to $1 - p$ portion of C's neighbors have all good edges in their group, then those groups and C play B, the other C-groups play D.

Proposition 13. *Any stable network consists of some complete subgraphs each containing vertex C but are otherwise disjoint, and some other disjoint complete subgraphs.*

[25] ε can be thought of as a robustness or selection tool. Without this slight perturbation, indifferences lead to many candidates for core which are less intuitive than the unique candidate for the core with this perturbation. We don't provide explicit bounds on ε but it can be chosen to be bounded away from 0 as N diverges to infinity.

[26] It is easy to check that $\alpha^* > 0.5$, indeed very close to 0.6 independently of α.

Proof. See appendix. □

Thus, C becomes a central node with many clusters around it, which are still internally densely connected. The number of attached clusters can be large at stable networks, so that C serves as a channel through which contagion might spread from one cluster to the other. In this sense, this 'favored' node becomes too central and contributes excessively to systemic risk.

Proposition 14. *Take N such that $N > 1 + (d^* + 1)s^*$. Any core network consists of exactly s^* many complete subgraphs of order $d^* + 1$ that include C and are otherwise disjoint, and some isolated disjoint complete subgraphs of order $d^* + 1$, and possibly one more left-over isolated complete subgraph of order less than $d^* + 1$.*

Proof. See appendix. □

In stable networks, there can be many complete subgraphs, possibly more than s^* many, that include C. However, in the core, there are at most s^* complete subgraphs that contain C. When s^* or fewer complete subgraphs contain C, a contagion that starts at some complete subgraph cannot cause C to default. In fact, even if all but one of the complete subgraphs that contain C defaults it is still a best response for C not to default. If, however, there are at least $s^* + 1$ complete subgraphs containing C, if all but one default, then, C will default. Thus, no complete subgraph will want to connect to C once C is contained in too many complete subgraphs as this would increase the risk of contagion from other complete subgraphs.

The comparison of stable and core networks here reinforces the previous intuition that the inability of large groups to coordinate leads to inefficiencies. Moreover, we see here that the number of firms matter for the global properties of the network. In an economy where there are a few firms, the result resembles networks with highly interconnected central nodes. However, if the number of firms keeps growing, while the number of risk free nodes remain bounded, the network is going to look more and more like the core in the benchmark model.

Borrowing and lending

Here we illustrate how C can be interpreted as a lender. Every investment, in the benchmark case, requires two partners. Now, suppose that the agents can undertake these ventures solo only if they can find outside funding. Node C represents this outside funding source. No other node can serve in this role. Without borrowing from C, agents must form partnerships for the investments.

An investment undertaken by a single agent n with the backing of C will involve two funding rounds, at the amounts $x \geq 1$ and $y > 0$ respectively. After the initial investment x, C and n are informed what the stochastic gross return R will be on the investment. Execution requires a second stage infusion of y. Lending x involves risk and requires a gross rate of return $r > 1$ determined exogenously. Lending y is optional and decided after R is observed. This is riskless and the gross rate of return on y is 1.

An edge between n and C represents a decision by C to extend to n the initial amount of x. After the edge is formed, x is a sunk cost for C. After R is determined by nature in the second stage, both C and n must decide whether to continue with the project.

If both C and n choose to continue (this will correspond to action B), C lends n the extra y and the investment is complete. Node n obtains R and pays C back $rx + y$. Hence the payoff to C is $rx + y - x - y = (r-1)x$ and to n is $R - rx - y$.

If C chooses to continue (action B) but n defaults (play D), then C does not give y, and n does not return the initial x. The payoffs to C in this case is $-x$ and to n is 0.

If C chooses to stop (action D), but n chooses B, n pays C back rx which he owes (C uses these funds to pay its other debts and still defaults), but does not obtain y, and hence cannot complete the project. Therefore, the payoffs are 0 for C and $-rx$ for n. If both play D, both get 0. The game form is given by

	B	D
B	$rx - x, R - rx - y$	$-x, 0$
D	$0, -rx$	$0, 0$

Define p to be $1 - \frac{1}{r}$. Since all edges of C have the same payoff structure, his payoffs can be scaled for normalization. Multiply C's payoffs by $\frac{1-p}{x}$. Assume that the uncertainty in R is tied to the state of the edge θ in the form $R = \varepsilon + y + rx + \theta$. Then the game form on the edges of C becomes:

	B	D
B	$p, \theta + \varepsilon$	$p - 1, 0$
D	$0, -c$	$0, 0$

Here $c > 1$. This is identical to the extension outlined above, modulo c. Notice this does not effect our results as long as $c > 1 - \theta$ for all θ, which is true. The interest rate r could be determined endogenously via $\frac{1}{1-p^*}$ where p^* is the endogenous probability of default for n. That is beyond the scope of this paper.

Other forms of asymmetry

There can be many forms of asymmetries between nodes and edges. For example α's could be different. Indeed, if all α's are in an interval (α_0^2, α_0) for some $\alpha_0 \in (0, 1)$, then stable networks still consist of disjoint complete subgraphs.

Alternatively, consider the benchmark model with node shocks with differing individual default probabilities.

Proposition 15. *If there is one firm with a different node shock probability, say $\beta' > \beta$, everything follows similarly. The core exists and is unique and consists of disjoint cliques of order $d^* + 1$ for appropriate modularity of N.*

If there are several groups of people such that each group has number of people divisible by $d^* + 1$ and members of each group have the same β among themselves, possibly different across groups, then there is assortative matching in the core: 'safer' firms cluster with 'safer' firms from top to bottom.

1.8 Future Work

The model we introduce is tractable and rich. We have considered some extensions, and many more important extensions are possible. We list some of them here.

A major extension is allowing for government intervention in the contagion and/or network formation stages. Would the anticipation of government intervention be harmful due to moral hazard costs, or would the ex-post gains from intervention outweigh moral hazard costs ? Should there be caps on the ability of a government to intervene ? What are the welfare implications of specific policies ? Furthermore, government reputation can be considered when the model is cast into a dynamic framework.

As we have illustrated in the asymmetry section, borrowing and lending can be incorporated into the model and endogenous prices can be tractably determined.

Another important but difficult extension is introducing asymmetric information. For example in stage three, nodes could be modeled to know the states of their incident edges but not the rest. It is important to see the what happens in that case, yet it is significantly harder to solve for technical reasons.

In the network formation stage, we have introduced a proposal game to micro-found the solution concepts. The agents could have started off with an existing status-quo network, and build extra edges on top of the the existing ones. It would be interesting to see how this will alter the resulting network. Furthermore, one can think of a dynamic proposal game to see whether first-movers tend to become too central.

Recall that the maximal cohesive sets protect themselves from contagion, and this result is independent of the particular coordination game later embedded. Network formation is driven by the utility functions, and it is important to see what other utility functions, symmetric or asymmetric among agents, lead to. Some that are of particular interest would be those that resemble borrowing and lending correspondences.

Other extensions can include allowing for more than two actions; allowing for moderate strength of contagion; allowing for heterogeneous volumes of edges; allowing for bilateral transfers between neighbors and allowing for different forms of correlations of shocks.

1.9 Conclusion

In our model, rational agents who anticipate the possibility of system wide failure during network formation, guard against it by segregating themselves into densely connected clusters that are sparsely connected to each other. As the economy gets fundamentally safer, they organize into larger clusters which results in an increase in systemic risk.

Whether the networks formed efficiently trade-off the benefits of surplus generation against systemic risk depends on two factors. First is the ability of agents to coordinate among themselves during network formation. If the networks formed are robust to bilateral deviations only, they are inefficient. If robust to deviations by relatively larger subsets, they are fully efficient. Second, is the infectiousness of counter-party risk, which serves as a natural mechanism for agents to internalize externalities. With strong contagion, agents recognize they are in the same boat during network formation.

Our model highlights that assessing the vulnerability of a network to system wide failure cannot be done in ignorance of the beliefs of agents who formed that network. Efficient markets generate structures that are safe under the correct specification of shocks, which will appear fragile under the wrong specification of the shock structure. Thus, mistakes in policy can arise from a misspecification in the correlation of risks.

Asymmetries between firms can lead to the emergence of ‘central’ institutions. However, it does not follow that they are ‘too-big’ or ‘too-interconnected’ if the networks formed are in the core. If the networks are robust to bilateral deviations only, then, there can be excess interconnectedness around these central institutions which can generate an excessive risk of contagion. However, in a large enough economy, these central groups become marginal and isolated.

Chapter 2

Network Hazard and Bailouts

Abstract

I introduce a model of contagion with endogenous network formation and strategic default, in which a government intervenes to stop contagion. The anticipation of government bailouts introduces a novel channel for moral hazard via its effect on network architecture. In the absence of bailouts, the network formed consists of small clusters that are sparsely connected. When bailouts are anticipated, firms in my model do not make riskier individual choices. Instead, they form networks that are more interconnected, exhibiting a core-periphery structure (wherein many firms are connected to a smaller number of central firms). Interconnectedness within the periphery increases spillovers. Core firms serve as a buffer when solvent and an amplifier when insolvent. Thus, in my model, ex-post time-consistent intervention by the government improves ex-ante welfare but it increases systemic risk and volatility through its effect on network formation. This paper can be seen as a first attempt at introducing a theory of mechanism design with endogenous network externalities.

2.1 Introduction

The financial crisis of 2008 alerted many to the risk that the failure of a few individual financial institutions might, through the interconnectedness of the financial system, damage the economy as a whole. Such systemic risk can be ameliorated ex-ante using regulatory tools, yet the inability of government to credibly commit to not intervening suggests that an ex-post response, in the form of bailouts, is unavoidable. Bailouts of failing institutions are criticized because they encourage excessive risk taking by individual institutions. Excessive risk taking may trigger cascading failures, but explanations of what generates the underlying interconnectedness are lacking. In this paper, I argue that the anticipation of bailouts influences the formation of networks among financial institutions, creating a novel form of moral hazard: 'network hazard.' I exhibit a model in which the anticipation of bailouts has two main effects. First, it loosens the market discipline and generates more interconnectedness. Second, it leads to the emergence of systemically important financial institutions, and this produces core-periphery networks.[1] As a consequence, systemic risk, volatility, and ex-ante welfare all increase.

My model has three stages. Stage one is the network formation stage. Firms[2] form links with each other by mutual consent. A pair of firms that have a link are called counterparties of each other. A link in its most general form represents a mutually beneficial trading opportunity.[3] However, benefits from trade are realized only if neither party subsequently reneges; in the model doing so is called default. Stage two is the intervention stage. Each firm receives an idiosyncratic exogenous shock, good or bad. Shocks capture the fundamental productivity of firms: a firm that experiences a good shock is called a good firm, while one that experiences a bad shock is called a bad firm. When shocks occur the government intervenes. Stage three is the contagion stage. A firm can take one of two actions: continue or default. A firm that defaults receives a payoff, an outside option, independent of the actions of its counterparties. A firm that continues receives a payoff contingent upon the actions of its counterparties, its own action, and its shock. More links yield more potential benefits, but these are offset by the costs imposed by defaulting counterparties. (For a visualization of timing of events, see Figure 11 for the

[1] Core-periphery architecture is widely observed in practice. See, among many others, Vuillemey and Breton (2014), and Craig and Von Peter (2014).

[2] To emphasize the wide number of interpretations of the model I refer to agents as firms rather than as financial institutions. See Section 2.7.1.

[3] Other interpretations of the network are discussed in Section 2.7.1.

benchmark model with the absence of intervention and Figure 22 for the full model with the presence of intervention.)

In the model, a bad firm's dominant action is default and receipt of the outside option. Given that a defaulting firm imposes costs on its counterparties, a good firm with sufficiently many defaulting counterparties may find it iteratively dominant to default so as to enjoy the outside option. Thus, default decisions triggered by bad shocks might propagate through the entire network. Foreseeing this "contagion," the government intervenes at the end of stage two. Specifically, the government commits to a transfer policy that is conditional on the actions taken by firms in stage three, and this transfer policy maximizes ex-post welfare. Government cannot commit to a transfer policy prior to stage two. The difference between the absence and presence of intervention in the network formed can be explained as follows.

In the absence of the anticipation of intervention, a firm prefers that its counterparties are counterparties only of each other. This is so because in the model benefits come from links with immediate counterparties, and counterparties of counterparties typically harm a firm in expectation.[4] A firm that does not have a "second-order counterparty" limits its exposure to second-order counterparty risk: the risk that it incurs losses due to defaults by good counterparties that default because of their own defaulting counterparties. This force generates a market discipline that leads uniquely to the formation of dense clusters that are isolated from each other, and this network structure eliminates second-order counterparty risk.

To illuminate the main effects of the anticipation of intervention on the network formed, consider as a starting point a baseline case of government intervention. In this case, good firms contribute to welfare by continuing and bad firms reduce welfare by

[4]This is one main difference with other models, including models of intermediation in which links serve to channel funds across a long chain of borrowing and lending firms. My model also can be interpreted as one of intermediation in which links are borrowing partnerships a la Afonso and Lagos (2015) and Farboodi (2015), but funds cannot travel farther than one link. See Section 2.7.1.

continuing. Bailouts are costless and the government is not restricted by a budget or by any other form of ex-ante commitment. Therefore, the government optimally induces good firms to continue and bad firms to default.[5] In return, each firm knows that its good counterparties are going to continue even if these good counterparties have many bad counterparties of their own. Thus, second-order counterparty risk is eliminated as a byproduct of optimal intervention. This loosens the market discipline because firms no longer concern themselves with the counterparties of their counterparties. The significant effect of bailouts on the network topology emerges because each firm anticipates that its counterparties can get bailed-out and not because each anticipates that itself will be bailed-out. The elimination of second-order counterparty risk has two main effects on the induced network topology and systemic risk.

The first effect arises across ex-ante identical firms. Because firms no longer concern themselves with second-order counterparty risk, the isolated clusters that form in the absence of intervention dissolve, and an interconnected network emerges (See Figure 24 for a visualization). But bad firms do not get bailed-out under the optimal policy.[6] Consequently, each good firm still incurs losses because bad counterparties still default. If a good firm has too many bad counterparties and thus is forced into default, the government steps in and bails-out the good firm. However, to induce the good firm to continue the government offers it the smallest transfers possible, and so the good firm is indifferent between defaulting or not. In other words, a firm gains nothing when it is bailed-out, and the risk that it incurs losses due to defaults by bad counterparties (first-order counterparty risk) remains unaltered. As a consequence, during network formation, firms do not overconnect or underconnect: instead, each firm has the exact same number of counterparties

[5]In Sections 2.5 and 2.6 I consider the costs of bailouts, restrictions that allow governments to bailout only systemically important firms, budget restrictions on government, and other notions of welfare. I also demonstrate that the main effects of bailouts that arise in the baseline case are enhanced under these extensions.

[6]Rochet and Tirole (1996b), too, discuss this type of direct assistance to good firms that face failure due to counterparties (as opposed to indirect assistance rendered through bad counterparties). In Sections 2.5 and 2.6 I examine in detail why factors such as bailout costs and budget constraints that render indirect assistance to good firms through bad counterparties are optimal.

whether or not intervention occurs. That said, the network becomes more interconnected when clusters dissolve. As the network becomes more interconnected, and when each firm has the same number of counterparties, the extent of potential contagion increases. The threat of system wide default also increases, but the government intervenes to stop contagion. Compared to no intervention, ex-ante welfare is higher under interventions that involve bailouts, and with bailouts systemic risk increases. In other words, the number of firms that face indirect default[7] and get bailed-out under intervention is larger than the number of firms that face indirect default and do, in fact, default in the absence of intervention.

The second effect arises across firms that are not identical ex-ante. Such differences arise because some firms have less equity than others or some firms specialize in different sectors.[8] Under heterogeneity, some firms typically have a greater appetite for counterparties than others. In the absence of bailouts, such high demanding firms are unable to convince low demanding firms to become counterparties. This occurs because high demanding firms would have too many counterparties, which would increase second-order counterparty risk for their low demanding counterparties. When bailouts eliminate second-order counterparty risk, high demanding firms are welcome to become counterparties with all firms. In this manner, high demanding firms become central to the network. The network exhibits a core-periphery structure because of bailouts: high demanding central firms make up the core of the network and low demanding firms make up the periphery (See Figure 29 for a visualization). Because of the firms at the core of the network the counterparty risks faced by peripheral firms are correlated. In return, when a sufficient number of core firms default after experiencing bad shocks, peripheral firms become less resilient; that is, a few bad shocks to the peripheral counterparties of a peripheral good firm will cause the latter to default. Thus, the core serves as an amplifier

[7] An indirect default refers to a default decision by a good firm which suffers sufficiently many counterparty losses.

[8] As explained in Section 2.7.1, firms can have ex-ante differences for many reasons. For example, some banks are located at money centers that have access to many investors while others are small deposit-collecting banks.

of contagion across the periphery. When a sufficient number of core firms experience good shocks and then either continue or get bailed-out, peripheral firms become more resilient. Only a large number of bad shocks to the peripheral counterparties of a peripheral good firm will force the latter to default. Thus, the core serves as a buffer against contagion. The formation of a core-periphery structure makes very bad and very good outcomes more likely, and this generates volatility.

The force most responsible for network hazard is the elimination of second-order counterparty risk. Network hazard is a genuine source of moral hazard. Consider a scenario in which, during network formation, each firm can individually choose between two risk levels: safe investments or high risk/high return investments. Firms exploit this choice, and when they do it affects how the network is formed because it alters the number of each firms' counterparties. However, firms make the identical risk choices whether or not intervention is available. Moreover, firms that anticipate bailouts do not overconnect or underconnect; instead, their networks become more interconnected. When bailouts are available heterogeneous firms continue to form core-periphery networks. This is so because in the model, firms offered bailouts are indifferent about whether or not to default. Accordingly, firms do not benefit from overconnection or underconnection, nor will they benefit from choosing riskier investments in face of bailouts. Network hazard is a genuine form of moral hazard that emerges only when a network is formed. This is true even when firms are not incentivized to choose riskier individual investments.

Other extensions of the baseline case are worth mentioning. Under some extensions, incentives to form a core-periphery network are particularly strong. Consider a core-periphery network that has high demanding firms at the core and low demanding firms at the periphery. If a sufficient number of core firms suffer bad shocks, many peripheral firms will be forced into default. If bailouts are costly, if there is a budget constraint on the government, or if the government is committed to bailing-out only systemically important firms, the government in some cases might bailout the bad core firms in order to indirectly support troubled good firms at the periphery. This would be an alternative to bailing out an excessive number of peripheral firms. As a consequence, the ex-ante payoff to a peripheral firm would increase because it would now have counterparties (core firms) that would be bailed-out even if they suffered bad shocks. In a core-periphery structure this possibility reduces the first-order counterparty risk of peripheral firms, and this in turn increases their incentives to maintain a core-periphery network. Note that these arguments do not require ex-ante heterogeneity of firm types. Indeed, when bailouts are costly, even ex-ante identical firms that have the same demand for counterparties can in response to bailouts form a core-periphery network. That is, core-periphery is not an artifact of firm heterogeneity.

Under these same extensions, incentives to form an interconnected network across identical firms also become stronger. If the network is interconnected (if it does not consist of isolated clusters), some good firms might benefit from the bailouts of bad counterparties that are optimally executed for the sake of other good firms. This indirect assistance to a good firm that is not facing default increases its payoff. Accordingly, firms have higher incentives to make their network more interconnected in order to force the government to execute more bailouts and benefit from such indirect support even when the support is not needed to avoid default.

Related literature: A voluminous literature examines moral hazard in a variety of contexts, including banking (Chari and Kehoe (2013), Cordella and Yeyati (2003), Freixas (1999), Holmstrom and Tirole (1997), Keister (2010), Mailath and Mester (1994), and many others), yet this literature contains very little discussion of network formation. In examinations of bailouts and systemic risk, authors such as Caballero and Simsek (2013), Elliott, Golub, and Jackson (2014), Freixas, Parigi, and Rochet (2000), Gaballo and Zetlin-Jones (2015), Leitner (2005), and Rochet and Tirole (1996b) analyze networks of bilateral exposures. But in these analyses, which contrast with mine, network architecture is not endogenously formed by firms and moral hazard arises from individual choices about excessive risk taking, bankers' decision to shirk, lack of monitoring among banks, etc.

A few recent papers such as Acharya and Yorulmazer (2007), Acharya (2009), and Farhi and Tirole (2012) propose, on the basis of correlations of investment risks, that moral hazard arises from collective behavior. This important form of interconnection does not constitute a network of bilateral relationships formed through mutual consent. Such a correlation of risk generates systemic risk, but it does not do so via contagion through a network of bilateral linkages. I examine how bailouts affect both incentives for forming bilateral links and collective incentives that shape interconnections, and how this process affects systemic risk and welfare.

There is also a large and growing literature that examines systemic risk and networks. Early contributors include Allen and Gale (2000), Eisenberg and Noe (2001), Kiyotaki and Moore (1997), Rochet and Tirole (1996b), and in more recent years, Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015b), Elliott, Golub, and Jackson (2014), Glasserman and Young (2014), and others.[9] These papers examine contagion within fixed networks. Other scholars, among them Drakopoulos, Ozdaglar, and Tsitsiklis (2015a), Freixas, Parigi, and Rochet (2000), and Minca and Sulem (2014) examine the problem of how to stop contagion in exogenous networks.[10] Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015c), Cabrales, Gottardi, and Vega-Redondo (2014), Elliott and Hazell (2015), Erol and Vohra (2014), Farboodi (2015), Goldstein and Pauzner (2004) and others[11] study the formation of networks by agents who take systemic risk into account. In contrast to mine, these studies do not

[9] An unfortunately incomplete list is Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015a), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010), Allen, Babus, and Carletti (2012), Amini and Minca (2014), Blume et al. (2011), Bookstaber et al. (2015), Caballero and Simsek (2013), Eboli (2013), Elliott, Golub, and Jackson (2014), Freixas, Parigi, and Rochet (2000), Gai and Kapadia (2010), Gai et al. (2011), Gale and Kariv (2007), Gottardi, Gale, and Cabrales (2015), Glover and Richards-Shubik (2014), Gofman (2011), Gofman (2014), Kiyotaki and Moore (2002), Lim, Ozdaglar, and Teytelboym (2015), Vivier-Lirimonty (2006).

[10] Other similar papers are Amin, Minca, and Sulem (2014), Drakopoulos, Ozdaglar, and Tsitsiklis (2015b), and Motter (2004). There is also another less related branch of papers examining mitigation of systemic risk by ex-ante regulation. Rochet and Tirole (1996a) can be seen as an example, comparing the efficacy of different payment systems.

[11] Such as Babus (2013), Babus and Hu (2015), Blume et al. (2013), Chang and Zhang (2015), Condorelli and Galeotti (2015), Kiyotaki and Moore (1997), Lagunoff and Schreft (2001), Moore (2011), Wang (2014), Zawadowski (2013).

consider the possibility that the anticipation of ex-post government intervention might affect the network. My paper contributes to this literature by investigating how the anticipation of ex-post bailouts affects endogenous networks and systemic risk.

My model most closely resembles one proposed by Erol and Vohra (2014). Indeed, I substantially generalize their model to allow for arbitrary levels of exposures (strength of contagion), more general payoff functions, heterogenous firms, incomplete information, and government intervention. In technical terms, my network formation theorem examines the formation of strongly stable networks[12], wherein the payoffs to agents within each network are derived from a semi-anonymous graphical game with complementarities.[13] Moreover, the structure of the network formed and the extent of systemic risk on the resulting network features a phase transition property in the number of firms (See Figure 19 for a visualization). To the best of my knowledge, this is the first phase transition result in the number of players for endogenously formed networks. As for the case of intervention, the model can be seen as a first attempt towards developing a theory of mechanism design that has endogenously determined network externalities at an ex-ante stage.

Structure: Section 2 introduces the benchmark model. Section 3 studies networks formed in the absence of intervention. Section 4 examines the baseline case of government intervention, and it introduces the concepts of induced interconnectedness and core-periphery. Section 5 examines the robustness of the induced architecture and Section 6 examines extensions. Section 7 discusses various interpretations of the model as well as future research, and Section 8 concludes. Each section ends with remarks that summarize its core messages.

[12] I discuss strongly stable networks in Section 2.2.4. For more on various notions of network formation, see Bala and Goyal (2000), Bloch and Dutta (2011), Bloch and Jackson (2006), Dutta, Ghosal, and Ray (2005), Dutta and Mutuswami (1997), Fleiner, Janko, Tamura, and Teytelboym (2015), Galeotti, Goyal, and Kamphorst (2006), Goyal and Vega-Redondo (2005), Jackson and Van den Nouweland (2005), Jackson and Watts (2002), Jackson and Wolinsky (1996), Ray and Vohra (2015), Shahrivar and Sundaram (2015), Tarbush and Teytelboym (2015) and Teytelboym (2013).

[13] See Jackson (2010) for definitions of these technical terms.

2.2 Benchmark model

I introduce the benchmark model with complete information and no government intervention.

2.2.1 Environment

Let $N = \{n_1, n_2, ..., n_k\}$ be a set of k **firms**.[14] Each firm $n_i \in N$ has a **type** $\gamma_i \in \Gamma$, where Γ is a finite set.[15] There are three stages.

In **stage one**, the network formation stage, firms form bilateral relationships, called **links**, by mutual consent. The details can be found in Section 2.2.4. If two firms n_i and n_j decide to form a link, the link formed is denoted $e_{ij} = e_{ji} = \{n_i, n_j\}$, and the resulting set of links is denoted $E \subset [N]^2$. (N, E) is the realized **network**. If $e_{ij} \in E$, n_i and n_j are called **counterparties**. Given (N, E), $N_i = \{n_j : e_{ij} \in E\}$ denotes the set of counterparties of n_i, and $d_i = |N_i|$ the **degree** of n_i.[16]

In **stage two**, firms receive **shocks**. Each firm independently gets a good shock G with probability $\alpha \in (0, 1)$, or a bad shock B with probability $1 - \alpha$. $\theta_i \in \{G, B\}$ denotes the realized shock to firm n_i.

In **stage three**, the contagion stage, each firm can choose to continue business and fulfill all obligations, or not continue via a default option. The decision to **continue** is denoted C and the decision to **default** is denoted D. Firm n_i's action in stage three is denoted $a_i \in \{C, D\}$.

Upon termination of stage three, each firm n_i receives a **payoff** depending on its type

[14] In the remainder of the paper, definitions are inline and boldfaced.

[15] Types determine the payoff function of each firm. These differences can arise due to many reasons including equity level, specialization, access to investment opportunities, access to depositors, business model, geographic location, location specific regulatory restrictions,...

[16] For ease of notation I drop the E subscript from N_i and d_i.

γ_i, its degree in the realized network d_i, its shock θ_i, its action a_i, and the number of its counterparties that default (or fail) $f_i = \left|\{j \in N_i : a_j = D\}\right|$.[17] Formally, the payoff of firm n_i is denoted U_i and is given by

$$U_i(\vec{a}, \vec{\theta}, E, \vec{\gamma}) = P\left(a_i, f_i, d_i, \theta_i, \gamma_i\right)$$

where $P(a, f, d, \theta, \gamma) : \{C, D\} \times \mathbb{N} \times \mathbb{N} \times \{G, B\} \times \Gamma \to \mathbb{R}$.

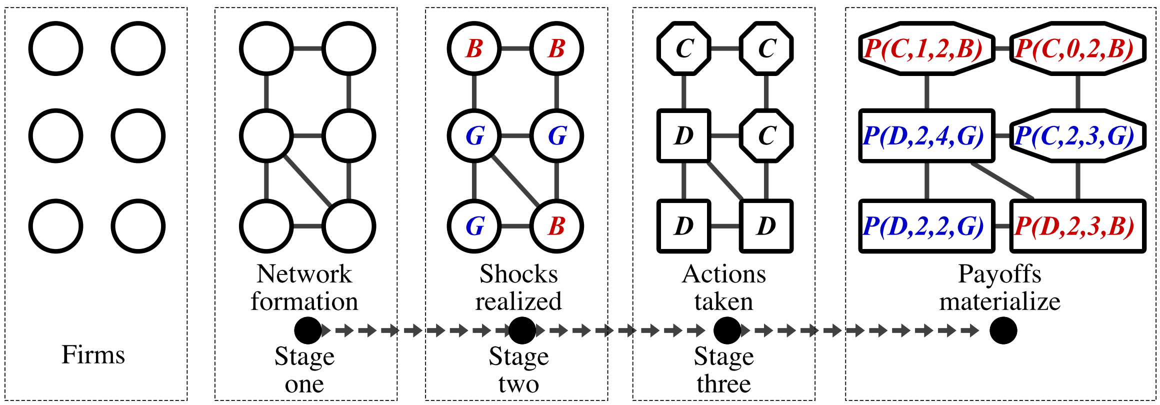


Figure 11: Timing of events in the benchmark model (Illustration for 6 firms with homogenous types: $\gamma_i = \gamma$ for all n_i so γ notation is dropped in the figure for simplicity.)

2.2.2 Assumptions

Assumption 6. *For any d, θ, γ; $P(C, f, d, \theta, \gamma)$ is strictly decreasing in f, and $P(D, f, d, \theta, \gamma)$ is constant in f.*[18]

P being decreasing in f for $a = C$ captures the idea that a defaulting firm causes costs to its counterparties that continue business. The costs need not be additive. On the other hand, P being constant in f for $a = D$ means that default can be seen as walking away

[17] In fact, the payoff depends on the action profile of counterparties. The names and types of counterparties do not matter so that the payoff can be written as a function of d_i and f_i only instead of the whole action profile of counterparties.

[18] Throughout the paper, assumptions that are maintained from the point they are stated have numbers. Assumptions that are invoked as needed have names rather than numbers. This is to make it easier to recall the meaning and effect of each particular assumption. Other inline assumptions are particular to the subsection they appear in.

from obligations with an outside option which does not depend on the number of one's counterparties that default.

Under Assumption 6, for any (d, θ, γ), $P(a, f, d, \theta, \gamma)$ is submodular in (a, f).[19] In return, for any $(\vec{\theta}, E, \vec{\gamma})$, $U_i(\vec{a}, \vec{\theta}, E, \vec{\gamma})$ is supermodular in $\vec{a} = (a_1, a_2, ..., a_k)$. Therefore, the game in stage three is a supermodular game.

Assumption 7. *For any d, γ;*
$P(C, 0, d, B, \gamma) < P(D, -, d, B, \gamma)$ and
$P(C, 0, d, G, \gamma) > P(D, -, d, G, \gamma)$.

Assumption 7 allows one to interpret B as a large bad shock and G as a good shock. The first condition in Assumption 7 ensures that it is strictly dominant for any firm with a bad shock to default. Otherwise, "contagion" never starts. The second condition in Assumption 7 ensures that a firm with a good shock continues if all of its counterparties continue. Otherwise every firm always default in any equilibrium. Note that there is no assumption on how many defaulting counterparties will force a firm into default. That is, any level for the "strength of contagion" is allowed.

An example of a function P that satisfies both Assumptions 6 and 7 is given as follows. $P(C, f, d, G, \gamma) = (d+1) - c_\gamma \times f$, $P(C, f, d, B, \gamma) = -(d+1) - c_\gamma \times f$, $P(D, f, d, \theta, \gamma) = 0$ where $c_\gamma > 0$ for all $\gamma \in \Gamma$.

2.2.3 Interpretation of the model

Each link, in its most general form, represents a mutually beneficial trading opportunity, such as a joint project, between the counterparties involved. However, benefits realize in full only if neither party reneges, called default in the model. Moreover, firms cannot selectively default on their counterparties. That is, a firm either maintains all its obliga-

[19]The order on $\{C, D\}$ is one in which C is the higher action and D is the lower action. The order for $f \in \mathbb{N}$ is the regular increasing order on $\mathbb{N}$.

tions to all counterparties, or breaks all obligations. In the following, this assumption is without loss of generality. A firm optimally chooses to default on all or none even if allowed to selectively default.

There are various interpretations the model which I elaborate on in Section 2.7.1. Here I present a lead example for the reader who wishes a concrete setting to keep in mind.

Lead example: Each firm has a specialization.[20] Each link is a joint project that requires the expertise and effort of both counterparties to succeed. Kickstarting each project initially costs each counterparty 1 unit. Each firm borrows these initial funds from outside the system in stage one, with interest rate r due in stage three.[21,22] These loans are directly invested into the projects. Each project requires costly supervision by both counterparties. In stage two, each firm receives an idiosyncratic shock that determines their cost of supervision.[23] A firm with a bad shock has cost per project $\tilde{c}(B,\gamma)$, and a firm with a good shock has cost per project $\tilde{c}(G,\gamma)$. Upon observing the shocks, each firm decides to continue or default. Projects which have both counterparties continuing yields safe return R to each counterparty. Projects which have at least one defaulting counterparty fails.[24] Assume that $\tilde{c}(B,\gamma) > R > r \geq 1$ and $R - r > \tilde{c}(G,\gamma) \geq 0$.

This way, a firm that continues, which has d projects (hence counterparties) out of which f many has failed, receives $R \times (d-f)$ from projects and incurs $\tilde{c} \times (d-f)$ cost of effort. It further pays its loans rd. Thus, its payoff is $P(C,f,d,\theta,\gamma) = (R - \tilde{c}(\theta,\gamma) - r) \times d - (R - \tilde{c}(\theta,\gamma)) \times f$. On the other hand, a firm that defaults has no return from projects,

[20] The specialization is not necessarily the type γ.

[21] For example, a bank borrows from depositors, a real sector firm borrows from banking sector.

[22] r can also be thought of as payments to employees due the returns from projects in stage three.

[23] This can directly be a cost of effort, or some change in the prices of the inputs that the firm buys for producing its specialized product that is needed for the project to succeed.

[24] This is also without loss of generality. A continuing counterparty of a defaulting firm, by incurring an extra cost $c^* > R$, can finish the project and get $2R$. For simplicity I assume that the project fails since it needs a specialized input of the counterparty that cannot be replaced.

cannot pay its loans back, and gets $P(D, -, d, \theta, \gamma) = -\varepsilon$.[25,26]

Now I describe a diversification example. The reader may skip directly to Section 2.2.4 without loss of understanding.

Diversification example: Each firm has a proprietary project and a non-proprietary project. A proprietary project has high management costs, so that other firms do not buy parts of the proprietary project due to its high moral hazard costs. On the other hand, non-proprietary projects have low management costs, so that other firms may find it beneficial to buy shares of non-proprietary projects.

The uncertainty regarding a proprietary project is resolved in stage two. The uncertainty regarding a non-proprietary project is resolved at the end of stage three. Once two firms sell each other shares of their non-proprietary projects, a link is formed between the two firms. The rationale for this exchange is diversification against the risk in stage three. If the non-proprietary projects of a firm yield low returns, it may be unable to pay for its liabilities. Accordingly, it may have to liquidate some other assets at discounted prices, leading to liquidation costs. By selling each other shares of their non-proprietary projects, firms increase the likelihood that their liquid assets (returns from projects) remain above their liabilities. This, in expectation, reduces liquidation costs. Below are examples of balance sheets that illustrate this situation.

[25] $\varepsilon > 0$ is an arbitrarily small number to ensure that firms with degree 0 continue. It is not essential for anything in the model, and $\varepsilon = 0$ is equally fine in technical terms.

[26] Consider the case in which firms are allowed to selectively default. Clearly, each firm defaults on projects in which the counterparty defaults. Suppose that a firm continues with d^+ projects, and defaults on $d - d^+$, where $0 \leq d^+ \leq d - f$. $P(C, f, d, \theta, \gamma) = (R - \tilde{c}(\theta, \gamma)) \times d^+ - rd$, so the firm always chooses $d^* = d - f$ or 0. $d^+ = d - f$ corresponds to action C and $d^+ = 0$ corresponds to action D.

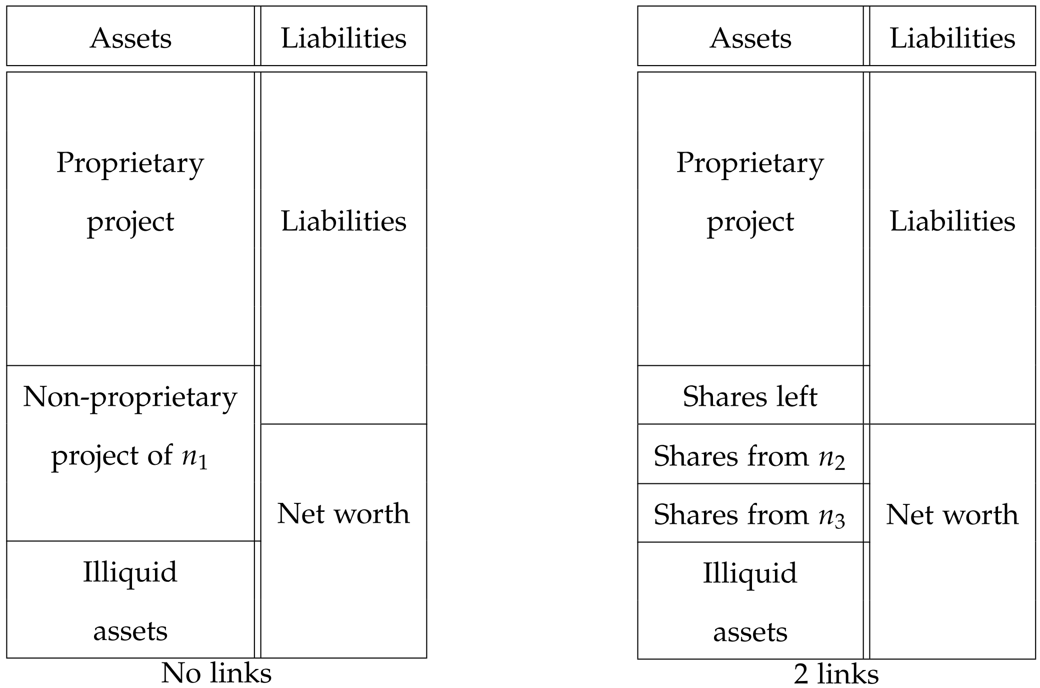


Figure 12: Balance sheet of firm n_1 in stage one

Consider a firm n_1. Each project of n_1 returns the value depicted in the first balance sheet if it is a successful project. If unsuccessful, a project returns 0. If firm n_1's proprietary project is unsuccessful ($\theta_1 = B$) and returns 0, n_1's net worth is negative and it defaults. Suppose that n_1's proprietary project was successful ($\theta_1 = G$).

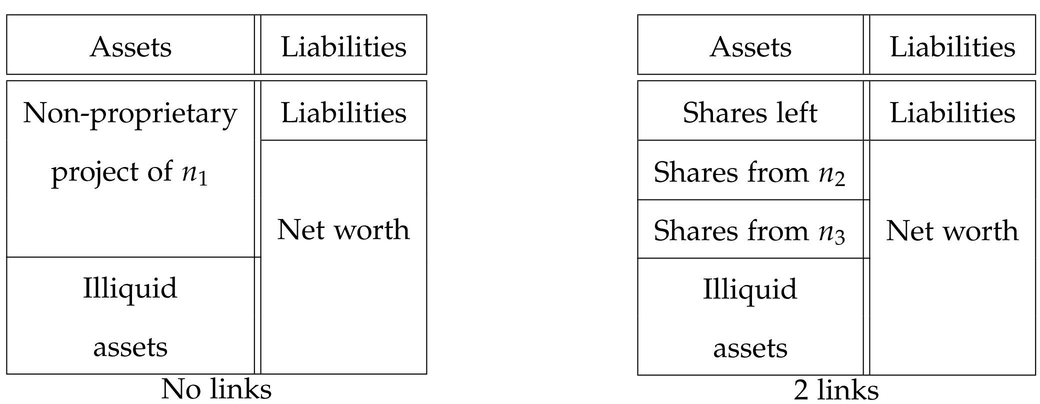


Figure 13: Balance sheet of firm n_1 in stage three conditional on $\theta_1 = G$

If n_1 has no links, as illustrated in the first balance sheet, and if firm n_1's non-proprietary project fails and returns 0 at the end of stage three, n_1 must liquidate the illiquid assets at a cost. If n_1's non-proprietary project succeeds, n_1 can pay for its liabilities. The expectation of this final payoff over the returns from the non-proprietary investment gives n_1's payoff $P(C, 0, 0, G, \gamma)$. However if firm n_1 has two links, with firms n_2 and n_3 as depicted in the second balance sheet, unless all three projects of n_1, n_2, and n_3 fail, firm n_1 does not incur the liquidation cost of illiquid assets. The expectation of the final payoff over the returns from the non-proprietary investment is now $P(C, 2, 0, G, \gamma)$, which is larger than $P(C, 0, 0, G, \gamma)$ due to reduced expected losses from liquidation. This way firms diversify against the risk of getting low returns from non-proprietary projects and having to liquidate illiquid assets at discounted prices in order to pay for liabilities. Accordingly, each link brings some diversification benefit to a firm.

However, links also bring some potential costs to a firm depending on the default decisions of its counterparties. If a firm defaults, the projects it originated fail. Therefore, if a firm continues and some of its counterparties default, the shares of the defaulting counterparties' non-proprietary projects return 0 for sure. Accordingly, the continuing firm incurs losses since it now has only some portion of the returns from the project it originated. In the first balance sheet, n_1, in expectation over the returns from its non-proprietary project, has payoff $P(C, 0, 0, G, \gamma)$. In the second balance sheet, if firms n_2 and n_3 default, their projects fail and n_1 receives nothing back from the corresponding shares. Therefore, n_1 incurs some direct costs. If n_1 continues, it can get at most half of the full value of its non-proprietary project. Its payoff in expectation over returns from its non-proprietary project is then $P(C, 2, 2, G, \gamma)$. Now n_1 may find an orderly default in stage two optimal for early liquidation of illiquid assets instead of risking fire sales in stage three. This example is further elaborated in Section 2.7.1.

2.2.4 Solution concepts

In stage three, firms play a supermodular game given the realized network and shocks. The solution concept is the **cooperating equilibrium**: it is the Nash equilibrium in which, any firm which can play C in at least one Nash equilibrium, plays C. Due to supermodularity of the game in stage three, this equilibrium notion is well-defined. Supermodularity of $\{U_i\}_{i \in N}$, via Topkis' Theorem, implies that the best-responses are increasing in others' actions. In return, by Tarski's Theorem, the set of Nash equilibria is a complete lattice. The cooperating equilibrium is the unique highest element of the lattice of Nash equilibria.[27]

The cooperating equilibrium can be obtained in two ways. The first is by iterating the myopic best-response dynamics starting with the 'everyone plays C' action profile.[28] The second way, which is subtly different, is to apply iterated elimination of strictly dominated strategies.[29] In both cases, the constructed sequence of action profiles reaches and stops at the cooperating equilibrium. Following the latter, an alternative definition of the cooperating equilibrium can be given via iterated elimination of strictly dominated strategies. 'The rationalizable strategy profile in which all firms play the highest action they can rationalize' is identical to cooperating equilibrium. This has a natural **contagion** interpretation. Call firms that receive a bad shock, **bad firms** and firms that receive a good shock, **good firms**. Bad firms, are insolvent and find it strictly dominant to default on their obligations. Call these **direct defaults**. After some bad firms default in this way, some good firms who are counterparties with sufficiently many defaulting firms also become 'troubled', and find continuing iteratively strictly dominated, and so on. Call these **indirect defaults**. Contagion stops when no further firm finds it iteratively strictly dominated to continue business. Iterated elimination resembles contagion black-boxed

[27]See Vives (1990) for more on how complementarities generate a lattice structure on the set of Nash equilibria. See Milgrom and Shannon (1994) for more on supermodular games.

[28]This is standard. A similar algorithm is considered in Vives (1990), Eisenberg and Noe (2001), Elliott et al. (2014), Morris (2000), Goyal and Vega-Redondo (2005), and others.

[29]This link between rationalizability and the extreme points of the lattice is introduced in Milgrom and Roberts (1990).

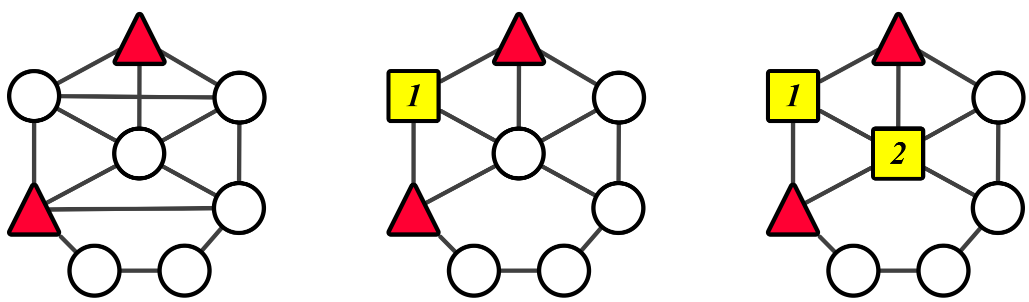
into a single period. Below is an illustration of how contagion works.

Note. *For simplicity, examples (not results) in the paper use an additively separable form given by*

$$P(C, f, d, G, \gamma) = u(d, \gamma) - c(f, \gamma); \quad P(C, f, d, B, \gamma) = -r - c(f, \gamma); \quad P(D, \cdot) = 0,$$

where $u, c : \mathbb{N}_0 \times \Gamma \to \mathbb{R}_+$ *and* c *is strictly increasing in* f*. A good shock brings revenue given by* u*. Returns from a bad shock is* $-r < 0$*. Counterparty losses are subtracted from revenue. Default gives a safe outside option normalized to* 0*. Henceforth I present only the functions* u *and* c *in the examples, not* P *as a whole.*

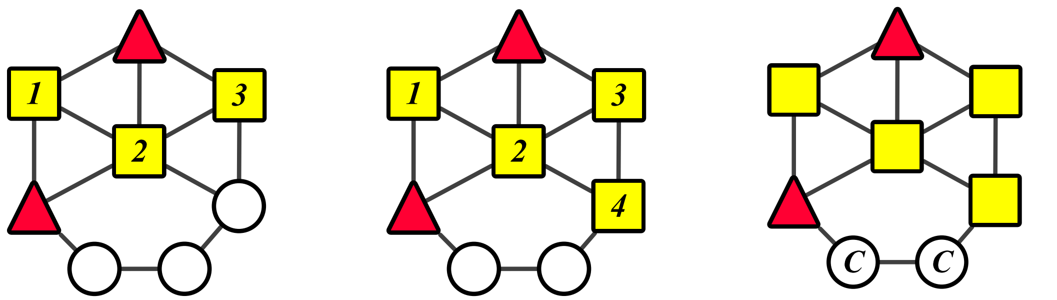
Example 1. $u(d, \gamma) = d$, $c(f, \gamma) = 2f$*. In this example, a firm with degree* d *defaults once it has strictly more than* $d/2$ *defaulting counterparties. The figure below illustrates how defaults propagate through the system, and how cooperating equilibrium can be obtained via iterated elimination of strictly dominated strategies.*



Red triangle: B shock. White circle: G shock. Red triangles default directly.

First wave of defaults: $2 > \frac{3}{2}$. Yellow square "1" defaults indirectly.

Second wave of defaults: $3 > \frac{5}{2}$. Yellow square "2" defaults indirectly.



Third wave of defaults: $2 > \frac{3}{2}$. Yellow square "3" defaults indirectly.

Fourth wave of defaults: $2 > \frac{3}{2}$. Yellow square "4" defaults indirectly.

White circles: $1 \leq \frac{2}{2}, 1 \leq \frac{2}{2}$ Cooperating equilibrium: White circles C, others: D.

Figure 14: Illustration of contagion and cooperating equilibrium

Throughout the paper, I refer to losses due to bad counterparties as **first-order counterparty losses**. Losses due to defaulting good counterparties who default due to their bad counterparties is dubbed **second-order counterparty loss**. Higher order counterparty losses are defined analogously. Expected counterparty losses of a specified order is called **counterparty risk** of that order.[30] If a firm faces no counterparty risk of order t, then it faces no counterparty risk of order $t' > t$ either.

In **stage one**, firms evaluate a network according to their expected payoffs in the cooperating equilibrium in stage three. Firms form the network as follows. Consider a candidate network (N, E) and a subset N' of firms. A **feasible deviation** by N' from E is one in which N' can simultaneously add any missing links within N', cut any existing links within N', cut any of the links between N' and N/N'.

[30]Note that since iterated elimination of strictly dominated strategies reaches the set of rationalizable strategy profiles independent of the order of elimination, one needs to be careful about the higher orders in losses. The specific order I employ is that all strategies that can be eliminated in one iteration are eliminated all at once.

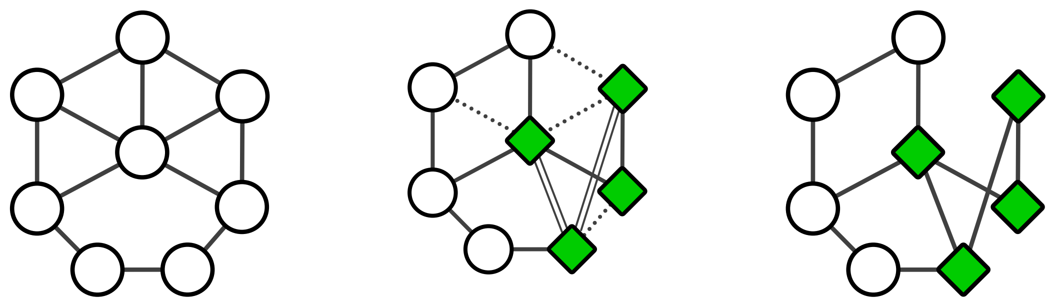


Figure 15: A feasible deviation

A **profitable deviation** by N' from E is a feasible deviation in which the resulting network yields strictly higher expected payoff to every member of N'. A **Pareto profitable deviation** by N' from E is a feasible deviation in which the resulting network yields weakly higher expected payoff to every member of N', and strictly higher payoff to at least one member of N'. A network (N, E) is **strongly stable** if there are no subsets of N with a profitable deviation from E. A network (N, E) is **Pareto strongly stable** if there are no subsets of N with a Pareto profitable deviation from E.[31]

In the model, the advantage of Pareto strong stability is that it gives uniqueness of the network formed, but existence requires some divisibility assumptions on the number of firms, solely to avoid integer problems. Strong stability yields existence without divisibility assumptions on the number of firms, but leaves some small room for multiplicity. Since I aim to compare the absence of government intervention with its presence, I find uniqueness more important. Therefore, I take Pareto strong stability as my main solution concept but provide some results for strong stability as well.

[31]Strong stability here follows Dutta and Mutuswami (1997). They establish the link of this concept to strong Nash equilibria. Pareto strong stability here is called strong stability in Jackson and Van den Nouweland (2005). They tie this solution concept to core. Farboodi (2015) uses strong stability, under the name group stability. Erol and Vohra (2014) also use strong stability under the name core networks.

Strongly stable networks correspond to strong Nash equilibria of an underlying proposal game. See Erol and Vohra (2014) for details of the proposal game. Therefore, strong stability results that follow can be thought of as *characterizing strong Nash equilibria* of a network formation game. See Dutta and Mutuswami (1997) for more on the relation between strong Nash equilibria and strongly stable networks.

Pairwise stable networks[32] while widely used in the literature are abundant in my setup. Moreover, strong stability guarantees *Pareto efficiency* of the network formed, given the behavior in stage three. It is government's task to fix the inefficiencies in stage three.

2.3 Absence of government intervention

In this section, I characterize the networks that are formed in the absence of government intervention, and examine various measures of systemic risk.

2.3.1 Preliminaries

Consider the difference in payoff from continuing or defaulting for a good firm:

$$\Delta P(f, d, \gamma) = P(C, f, d, G, \gamma) - P(D, -, d, G, \gamma).$$

By Assumptions 1 and 2, $\Delta P(0, d, \gamma) > 0$, and $\Delta P(f, d, \gamma)$ is decreasing in f for any given (d, γ). Define the **resilience** of a γ-type good firm with degree d as $R(d, \gamma) := max\{f \leq d : \Delta P(f, d, \gamma) \geq 0\}$. $R(d, \gamma)$ is the maximum number of counterparty defaults that a good firm of type γ can absorb before finding it optimal to default. For example, $R(d, \gamma) = d$ means that no counterparties can force a good firm with type γ and degree d into default. The following simple conditions characterize the best response of n_i in stage three for any given $(a_{-i}, E, \vec{\theta}, \vec{\gamma})$:

- If $\theta_i = B$, then $a_i = D$.
- If $\theta_i = G$, then;
  - If among N_i, more than or equal to $R(d_i, \gamma_i)$ many firms play D, then $a_i = D$.
  - If among N_i, less than or equal to $R(d_i, \gamma_i)$ many firms play D, then $a_i = C$.

The exact characterization of the cooperating equilibrium depends on the structure of

[32] Networks that don't have a profitable deviation by any pairs or singletons of firms.

(N, E). In order to state the main theorems, it suffices to find the cooperating equilibrium payoffs for a specific configuration. A star-shaped network is one in which one node, called the center, is adjacent to all other nodes, and all other nodes are adjacent to only the center node. Suppose that (N, E) has a subnetwork disjoint from all other vertices, which is star-shaped. Let n_i be the center of the star with d_i leaves.[33] If the center firm n_i gets a good shock, and it has less than or equal to $R(d_i, \gamma_i)$ many bad counterparties, then in the cooperating equilibrium n_i continues whereas good counterparties continue and bad counterparties default. If more than $R(d_i, \gamma_i)$ counterparties get bad shocks, then n_i defaults. Therefore, the expected payoff of n_i at the center of a disjoint star subnetwork is given by

$$V(d, \gamma) = \mathbb{E}_\theta \left[\max\left\{P(C, \left|\{j \in N_i : \theta_j = B\}\right|, d_i, \theta_i, \gamma_i), P(D, -, d_i, \theta_i, \gamma_i)\right\}\right]$$

$$= \mathbb{E}_{\theta_i}\left[P(D, -, d_i, \theta_i, \gamma_i)\right] + \alpha \times \mathbb{E}_{\theta_{-i}}\left[\max\left\{\Delta P(\left|\{j \in N_i : \theta_j = B\}\right|, d_i, \gamma_i), 0\right\}\right].$$

Proposition 16. *In any network in which a γ-type firm has degree d, its expected payoff is at most $V(d, \gamma)$.*

Proof. Consider any E and take any firm n_i. The distribution of the number of defaulting counterparties of n_i in the cooperating equilibrium first-order-stochastically dominates the distribution of the number of directly defaulting counterparties of n_i due to potential spillovers. The latter equals the distribution of the total number of defaulting counterparties of n_i if n_i were at the center of a disjoint star with d_i leaves, because there is no second-order counterparty risk for n_i in the star configuration. The second term in the expression $V(d_i, \gamma_i)$, $\max\{P(C, f_i, d_i, G, \gamma_i), P(D, -, d_i, G, \gamma_i)\}$, is a decreasing function of f_i. Since the expectation of a decreasing function decreases with respect to first order stochastic dominance, n_i gets at most $V(d_i, \gamma_i)$. □

Therefore, a disjoint star subnetwork is an 'ideal' configuration for the center of the

[33] The leaves of a star network are all nodes except the center.

star conditional on its degree, in the sense that it cannot achieve a higher expected payoff in any other network in which it has the same degree. Thusly, call $V(d,\gamma)$ the **γ-ideal payoff conditional on degree d**. Also consider the best degree conditional on being at the center of a star subnetwork in a network of m firms: $d^*(m,\gamma) := argmax_{d<m} V(d,\gamma)$.[34] Call $d^*(m,\gamma)$ the **γ-ideal degree among m firms**, $d^*(m,\gamma)+1$ the **γ-ideal order among m firms**, and the expected payoff $V(d^*(m,\gamma),\gamma)$ the **γ-ideal payoff among m firms**. Note that $d^*(m,\gamma)$ is a weakly increasing function of m.

The next result states that a clique[35] with firms of equal or higher resilience is another ideal configuration for a firm.

Proposition 17. *Consider a clique with $d+1$ firms which is not connected to any other vertices. Consider a firm n_i in this clique. If all firms in the clique have same or higher resilience than n_i, then n_i achieves the γ_i-ideal payoff conditional on degree d.*

Proof. If $f \leq R(d,\gamma_i)$ many firms are bad in the clique, all the good firms in the clique can rationalize continuing: when they all continue, continuing is a best reply. This is because they have the same or higher resilience. Bad firms cannot rationalize continuing so they always default. Thus from the viewpoint of any single firm n_i, if it gets a good shock, and $f \leq R(d,\gamma_i)$ many firms get bad shocks, in the cooperating equilibrium it continues and incurs losses due to f bad counterparties since all other good firms continue as well. If n_i gets a good shock, but $f > R(d,\gamma_i)$, then it defaults and gets the fixed outside option for the good firms. If it gets a bad shock, it gets the fixed outside option for the bad firms. Thus, its payoff is identical to $V(d,\gamma_i)$. □

[34]I assume that P is such that V admits no indifferences over integers. I also assume that a good firm is never indifferent between default or continue: $P(C,f,d,G,\gamma) \neq P(D,-,d,G,\gamma)$ for any f. These are already true for generic P. The purpose is to rule out some cumbersome and unintuitive cases of indifference that would unnecessarily make the analysis messier.

In any case, for the sake of completeness, the following makes sure that these are satisfied. For some $\tilde{P}$ taking values in $\mathbb{Q}$ and some small $\epsilon_1,\epsilon_2 \in \mathbb{R}^+\backslash\mathbb{Q}$, $P(D,-,d,\theta,\gamma) \equiv \tilde{P}(D,-,d,\theta,\gamma) - \epsilon_1(d+1)$ and $P(C,f,d,\theta,\gamma) = \tilde{P}(C,f,d,\theta,\gamma) - \epsilon_1\epsilon_2(d+1)$ for all f,d,θ,γ.

[35]A clique is a network in which all nodes are adjacent to each other.

Finally, define the set of **safe γ-counterparty degrees**

$$S(\gamma) := \{d \in \mathbb{N}_0 : R(d, \gamma) \geq d - 1\}$$

. This is the set of degrees such that having a γ-type counterparty of such degree does not carry any second-order counterparty risk. Consider a good firm n_i, and consider a counterparty n_j of n_i with degree d_j. If n_i finds it optimal to default directly, or indirectly due to losses from firms other than n_j, then n_i already gets a fixed outside option and does not worry about n_j's action. Otherwise, even if all other $d_j - 1$ counterparties of n_j default, resilience of n_j, $R(d_j, \gamma_j)$, is still sufficiently large for n_j to continue if n_i continues. Thus, conditional on n_i being a good firm, having a partner n_j with a safe γ_j-counterparty degree does not bring more counterparty risk to n_i than what n_j already brings individually as first-order counterparty risk. That is, a counterparty with a safe counterparty degree has the highest resilience a firm could possibly need in its counterparties.

Proposition 18. *Consider two counterparties n_i, n_j that both achieve their ideal expected payoffs conditional on their degrees. Then, either*

- *they both have unsafe counterparty degrees, their set of their counterparties are identical except each other, and they have the same resilience, or*
- *they both have safe counterparty degrees.*

Proof. For any $n_x, n_y \in N$, let $d_{xy} = |N_x \cap N_y|$. Take an arbitrary E, and a firm n_x with degree d_x. Take any counterparty $n_y \in N_x$. Suppose that $min\{R(d_x), d_{xy}\} + (d_y - d_{xy} - 1) > R(d_y)$. Then in the event that $min\{R(d_x), d_{xy}\}$ many firms in $N_x \cap N_y$ and all the $d_y - d_{xy} - 1$ many firms in $(N_y \backslash \{n_x\}) \backslash N_x$ get bad shocks, and all else get good shocks, n_y would default, and that would cause n_x to incur a non-zero loss on top of the direct costs from bad partners. That is, there is second-order counterparty risk for n_x through n_y. Due to the existence of such a positive probability event, conditional on the event that both n_x and n_y are good, and less than $R(d_x)$ many counterparties of n_x are bad, the distribution of the number of defaulting counterparties of n_x in (N, E) first order stochastically dominates the same distribution in the case when n_x were at the center of a star with d_x leaves, i.e. no second-order counterparty risk case. Hence, n_x's expected payoff is strictly less than $V(d_x, \gamma_x)$. That is, if n_x achieves $V(d_x, \gamma_x)$, for all counterparties n_y of n_x, $min\left\{R(d_x), d_{xy}\right\} + \left(d_y - d_{xy} - 1\right) \leq R(d_y)$ is satisfied.

Since both n_i and n_j achieve their ideal payoffs conditional their degrees, the inequality is satisfied for both. $min\left\{R(d_i), d_{ij}\right\} + d_j - d_{ij} - 1 \leq R(d_j)$ and $min\left\{R(d_j), d_{ij}\right\} + d_i - d_{ij} - 1 \leq R(d_i)$.

If one of them, say n_i has a safe counterparty degree, $R(d_i, \gamma_i) \geq d_i - 1 \geq d_{ij}$, so that $min\left\{R(d_i, \gamma_i), d_{ij}\right\} = d_{ij}$. Then the inequality becomes $d_j - 1 \leq R(d_j)$. Thus, the other n_j must also have a safe counterparty degree.

Consider the case in which both have unsafe counterparty degrees. $d_j \notin S(\gamma_j)$ so $R(d_j, \gamma_j) < d_j - 1$. Then $min\left\{R(d_i, \gamma_i), d_{ij}\right\} < d_{ij}$. That implies $min\left\{R(d_i, \gamma_i), d_{ij}\right\} = R(d_i, \gamma_i)$ so that $R(d_i, \gamma_i) + d_j - d_{ij} - 1 \leq R(d_j, \gamma_j)$. Similarly if $d_j \notin S(\gamma_j)$, $R(d_j, \gamma_j) + d_i - d_{ij} - 1 \leq R(d_i, \gamma_i)$. Add both up to get $d_i + d_j \leq 2(d_{ij} + 1)$. That implies that $d_i = d_j = d_{ij} + 1$, which in turn implies that $N_i \backslash \{n_j\} = N_j \backslash \{n_i\}$. Put that back into the inequalities to get $R(d_i, \gamma_i) = R(d_j, \gamma_j)$. $\square$

The only way two counterparties with unsafe counterparty degrees get their ideal payoff conditional on their degrees is that none of them increases the second-order counterparty risk of the other. This is only possible if they have exactly the same counterparties and resilience. Another remark is that a firm with a safe counterparty degree cannot achieve its ideal payoff conditional on its degree if it has any counterparty with an unsafe counterparty degree.

Corollary 1. *Take any component.*[36] *All firms in the component achieve their ideal payoffs given their degrees if and only if either*

[36]Two nodes are connected if one can be reached from the other in a sequence of adjacent nodes. A subnetwork is connected if any two nodes in it are connected. A component is a maximally connected subnetwork: it is connected and if any other node is added to the subnetwork it is not connected anymore.

• *they all have unsafe counterparty degrees, the component is a clique (hence all have same degree), and they all have the same resilience, or*

• *they all have safe counterparty degrees.*

Note that this corollary does not state anything about what the ideal degrees are. This is solely conditional on given degrees. In the next subsection, I pin down the networks firms form using Propositions 16, 17, and 18. Then I investigate measures of systemic risk.

2.3.2 Strongly stable networks

Homogenous firms: Here I consider the case when all firms are of type γ. Therefore, suppress the dependence on γ in the notation for simplicity until further notice. By Propositions 16 and 17, a disjoint clique is an ideal configuration for all firms in it. The idea is that a disjoint clique eliminates any second-order counterparty risk for members, because all counterparties of a firm's counterparties are its counterparties, and there are no second-order counterparties. The reason is partly that, any 'second-order counterparty risk' in a clique is already accounted for in the first-order counterparty risk since all counterparties of a firm's counterparties are already its counterparties in the clique. Indeed, as pointed out, a firm with degree d can achieve $V(d)$ only if it can eliminate second-order counterparty risk completely. Propositions 16, 17, and 18 lead the way to the main theorems of the section without government.

Theorem 5. ***(Pareto strongly stable networks)*** *Let $d^* = d^*(k) + 1$. The set of Pareto strongly stable networks is as follows.*

• *If $d^* \notin S$, and k is divisible by $d^* + 1$: $\frac{k}{d^*+1}$ many disjoint cliques of order*[37] *$d^* + 1$.*

• *If $d^* \in S$ and kd^* is an even number: any d^*-regular*[38] *network.*

• *Otherwise, Pareto strongly stable networks do not exist.*[39]

[37] Order of a subnetwork is the number of nodes in it.
[38] A network is d-regular if all nodes have degree d.
[39] Non-existence is due to cycles of deviations that arise solely due to integer problems.

Proof. If there is any firm who is not achieving $V(d^*)$ payoff, the ideal payoff among k firms, then this firm, and d^* other firms could deviate to forming a disjoint clique of order d^*+1 and all get $V(d^*)$. This would be a Pareto improvement. Hence, in any Pareto strongly stable network, all firms must achieve $V(d^*)$. The only way this is possible is as follows. First, all firms must have degree d^*. Also, by Propositions 1, 2, 3, if d^* is an unsafe counterparty degree, network must be in disjoint cliques, which is only possible when k is divisible by d^*+1. If d^* is a safe counterparty degree, network must be any d^*-regular structure, which is possible only when kd^* is even. In these configurations, all firms get their ideal payoffs among k firms, so there are no Pareto profitable deviations. □

Pareto strongly stable networks may not exist due to integer problems. However, strongly stable networks always exist. Stating the set of strongly stable networks requires some additional notation. Construct a sequence iteratively as follows. Set $n_0 = k$. For $t \geq 1$, as long as $d^*(k_t) \notin S$, set $k_t = k_{t-1} - d^*(k_{t-1}) - 1 \geq 0$. Let k_κ be the last element of the sequence: $d^*(k_\kappa) \in S$. That is, find the ideal degree among the remaining number of firms, and separate that many plus one firms aside. Iterate, and stop when ideal degree is a safe counterparty degree.

Theorem 6. ***(Strongly stable networks)***

•***(Existence)*** *The following is a strongly stable network: There are κ disjoint cliques with orders $d^*(k_{t-1})+1$, for $t = 1, 2, ..., \kappa$, and another disjoint residual subnetwork which is almost-$d^*(k_\kappa)$-regular[40] among the k_κ remaining nodes.*

•***(Almost uniqueness)*** *In any strongly stable network, there are κ disjoint cliques with orders $d^*(k_{t-1})+1$ nodes, for $t = 1, 2, ..., \kappa$. The remaining k_κ nodes constitute an approximately-$d^*(k_\kappa)$-regular[41] network.[42]*

[40] A network is almost-d-regular if all nodes, except at most one of them, have degree d and the possible residual node has degree 0. An almost-d-regular network always exists among $d+1$ or more nodes.

[41] A network is approximately-d-regular if all nodes, except at most d of them, have degree d.

[42] Concerning the remaining k_κ nodes, more can be said on the structure of the subnetwork using Erdos-Gallai Theorem. If the degree sequence of the remaining k_κ firms is given by x_1,...,x_κ, then the sequence $d^*(k_\kappa) - x_k$,...,$d^*(k_\kappa) - x_1$ cannot be a graphic sequence.

A graphic sequence is sequence of integers such that there is a simple graph whose node degrees are given by the sequence. Erdos-Gallai Theorem provides a necessary and sufficient condition for a sequence being

Proof. (Existence) As I stated before, by Propositions 1 and 2, being part of a clique with order $d^*(k_0)+1$ gives the highest payoff any configuration can achieve for a firm among a network of k_0 firms. Therefore, nodes in the clique with order $d^*(k_0)+1$ have no incentive to deviate to any other network. The argument can be applied iteratively for the κ cliques. As for the remaining almost-$d^*(k_\kappa)$-regular part, all nodes have degree $d^*(k_\kappa) \in S$ (except possibly one which is not connected to anyone). That is, all these remaining nodes (except the singleton) have safe counterparty degrees. Then there is no second-order counterparty risk and two good counterparties are sufficient for each other to rationalize continuing. Hence for any firm (except the singleton) has $V(d^*(k_\kappa))$ expected payoff, which is the highest any can achieve among k_κ people. If there is a singleton left-over firm with degree 0, it cannot convince anyone to deviate either, because everyone else is already getting their maximum possible payoff among people they could convince to deviate.

(Almost uniqueness) Take any strongly stable network. Let $d^* = d^*(k_0)$. First consider $d^* \notin S$. If all nodes have strictly less than $V(d^*)$ expected payoff, d^*+1 of them can deviate to a (d^*+1)-clique and improve. Hence, there is at least one firm who gets $V(d^*)$ payoff, say n_{i_0}. Then $d_{i_0} = d^* \notin S$.

For any counterparty of n_{i_0} which gets $V(d^*)$, say n_j, it must be that $d_j = d^* \notin S$. By Proposition 3, $N_{i_0} \backslash \{n_j\} = N_j \backslash \{n_{i_0}\}$. Let $N_0 = N_{i_0} \cup \{n_i\}$. Thus all firms in N_0 which get $V(d^*)$ are connected to all other firms in N_0, and none else.

Consider firms in N_0 that get less than $V(d^*)$, say N_1. Suppose that $N_1 \neq \emptyset$. Consider the deviation by N_1 in which they keep all existing edges with N_0, they connect all of the missing edges in N_1, and they cut all edges they have with N_0^C. After this deviation, N_0 becomes a (d^*+1)-clique and all firms get $V(d^*)$ so that all the deviators in N_1 get strictly better off. Therefore, $N_1 = \emptyset$, so that N_0 is already a (d^*+1)-clique.

All in all, in any strongly stable network of k_0 nodes, if $d^*(k_0) \notin S$, there exists a disjoint clique of order $d^*(k_0)+1$. Now the same arguments can be repeated for firms in

graphic.

the remaining $k_1 = k_0 - d^*(k_0) - 1$ nodes. Then among those, there must be a clique with $d^*(k_1) + 1$ nodes, then $d^*(k_2) + 1$ nodes.... as long as $d^*(k_t) \notin S$.

When $d^*(k_\kappa) \in S$ first time in the sequence, for the remaining k_κ people, among them there cannot be $d^*(k_\kappa) + 1$ or more people that have degree other than $d^*(k_\kappa)$ because then $d^*(k_\kappa) + 1$ many would deviate and form a clique, and get $V(d^*(k_\kappa))$.

The tighter condition mentioned in the footnote is also necessary. If the sequence $d^*(k_\kappa) - x_1$,...,$d^*(k_\kappa) - x_\kappa$ is graphic, then an appropriate isomorphism of the graph with this particular degree sequence can be joined with the existing remainder, so that all deviators increase their degree to $d^*(k_\kappa)$. This way, all firms achieve their ideal payoffs among k_κ firms, so that all deviators get strictly better off. □

Heterogenous firms: Now consider heterogenous firms again. Let $\Gamma = \{\gamma^1, ..., \gamma^g\}$. For any number of firms $m \in \mathbb{N}$ and any two types $\gamma, \gamma' \in \Gamma$, if their ideal degree among m firms, and the resulting resiliences are the same, say that γ and γ' are ***m*-similar**: $d^*(m, \gamma) = d^*(m, \gamma')$ and $R(d^*(m, \gamma), \gamma) = R(d^*(m, \gamma'), \gamma')$. Notice that m-similarity is an equivalence relation. Consider k firms in N, and the equivalence classes induced by k-similarity. Index the equivalence classes by ι. Let k^ι be the number of firms in equivalence class ι. For an equivalence class ι, denote the ideal degree and induced resilience of the class with $d^{*\iota} = d^*(k, \gamma)$ and $R^{*\iota} = R(d^*(k, \gamma), \gamma)$, where γ is an element of the equivalence class. If for an equivalence class ι, the ideal degree among k firms is a safe counterparty degree, $R^{*\iota} \geq d^{*\iota} - 1$, call this class a **safe class**, otherwise **unsafe class**.

Suppose that for each safe class ι, k^ι is divisible by $d^{*\iota} + 1$, and for each unsafe class ι, $d^{*\iota} k^\iota$ is an even number.

Proposition 19. ***(Pareto strongly stable networks)*** *The following is the set of Pareto strongly stable networks. Disjoint cliques of k-similar unsafe classes with their ideal order among k firms,* $d^{*\iota} + 1$, *and a disjoint subnetwork of safe classes, in which each has their ideal degree among k* *firms.*[43,44,45]

Proof. Similar to Theorem 1. □

Note that the cliques can have different orders since members of separate equivalence classes may demand various degrees. This result illustrates that network formation theorems are not a artifacts of symmetry of firms, they are rather consequences of matching and sorting.

Proposition 19 does not exhaust all possibilities for Pareto strong stability. Under divisibility conditions on the numbers of each type, different than those in the proposition, there could still exist Pareto strongly stable networks, which is not the case in Theorem 6.

As for strongly stable networks, construct a sequence in the following way. $k_0 = k$. Pick any type γ^{t_1}, let $k_1 = k_0 - d^*(k_0, \gamma^{t_1}) - 1$. Pick any type γ^{t_2} (it can be the same with γ^{t_1}), let $k_2 = k_1 - d^*(k_1, \gamma^{t_2})$,... At any step κ, if for all types $\gamma \in \Gamma$, $k_\kappa \in S(\gamma)$ or the number of k_κ-similar types of γ are less than $d^*(k_\kappa, \gamma) + 1$, stop. Call each such sequence $k_0, ..., k_\kappa$ a feasible sequence.

Proposition 20. *(**Strongly stable networks, necessary condition**) Any strongly stable network satisfies the following. There exists a feasible sequence $\{k_t\}_{t=0}^{\kappa}$ such that, in the network there are κ disjoint cliques which consist of $k_{t-1} - k_t$ many k_{t-1}-similar nodes, for $t = 1, 2, ..., \kappa$, and another disjoint subgraph with k_κ nodes.*

Proof. Similar to necessity part of Theorem 2. □

When there is heterogeneity, the remainder term is problematic due to integer prob-

[43] In the "safe" part of the network, firms can also become counterparties with other classes with respect to k-similarity since they all have safe counterparty degrees.

[44] Such a remainder subnetwork exists: an example is disjoint cliques of ideal order. It can be any other configuration with the same degree sequence.

[45] Under these divisibility conditions, strongly stable networks that are not Pareto strongly stable can be different only in the remainder subnetwork by at most $\tilde{d}$ firms where $\tilde{d}$ is the largest ideal degree among k_κ firms across firms in the remainder.

lems that arise. If the partition induced by the equivalence classes on Γ with respect to k_κ-similarity is not the trivial partition with one element, then the sorting argument fails. Firms, whose ideal degrees among the remainder k_κ firms are unsafe counterparty degrees, are not able to achieve their ideal payoff among the remaining k_κ firms anymore. Thus sorting trick does not work any further. This may lead to non-existence of strongly stable networks. However, if there are appropriate numbers of firms from each type in N, so that integer problems do not arise in the remainder, existence and uniqueness is restored already for Pareto strongly stable networks.

2.3.3 Illustrations and phase transition

Here I mainly focus on how the network topology and resulting systemic risk evolves as the number of firms increase. The function V encodes the changes in the network topology. The limit behavior of V dictates a particular structure for all networks above a certain size. However, the transition to large networks from small networks can be erratic. In particular, for relatively small numbers of firms, the network topology can exhibit discontinuous changes, a phase transition, when one more firm is added to the economy. I use homogenous types for illustrations, so drop γ from notation for now.

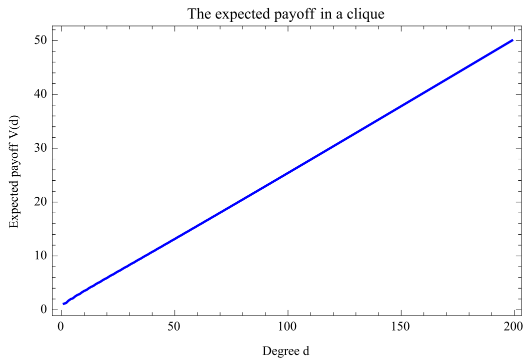
Large networks: Recall that $d^*(m)$ is weakly increasing in m. Let

$$d^* = \limsup_{m\to\infty} d^*(m).$$

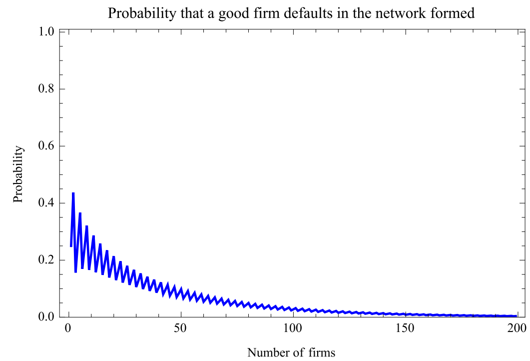
If $V(d)$ has a global maximizer, it is $d^* < \infty$. Otherwise $d^* = \infty$.

Corollary 2. ***(Large networks)*** *If $d^* < \infty$, for $k > d^*$, Pareto strongly stable networks are d^*-regular, in cliques or arbitrary configurations depending on resilience $R(d^*)$. The network is sparse. If $d^* = \infty$, Pareto strongly stable networks are complete for infinitely many k. The network is dense.*

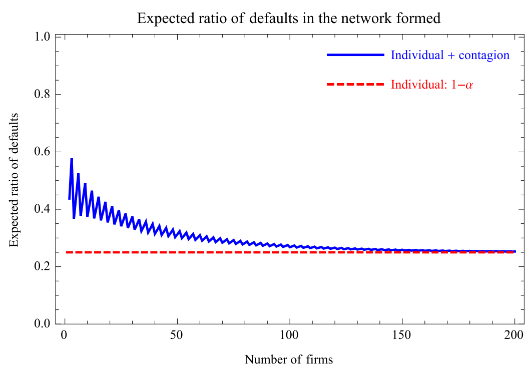
Example 2. $\alpha = 0.75$; $u(d) = d$, $c(f) = 3f$.



Plot of $V(d)$; Complete network K_k formed



Counterparty risk vanishes



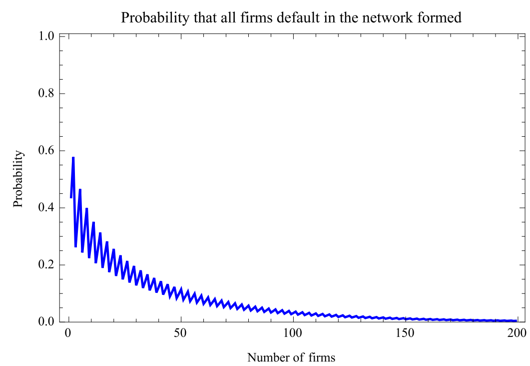
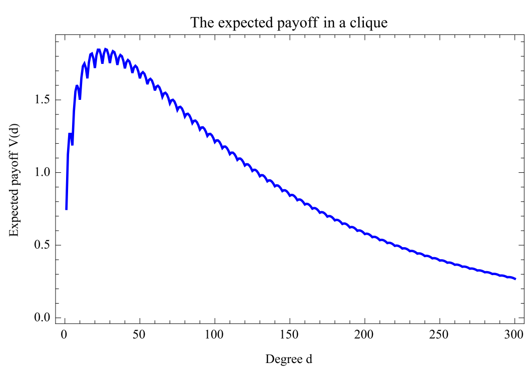
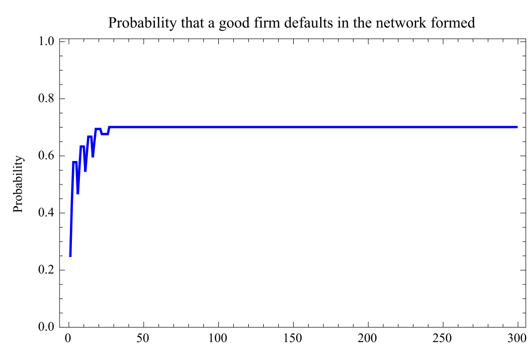


Figure 16: Measures of systemic risk for Example 2

Example 3. $\alpha = 0.75$; $u(d) = d$, $c(f) = 5f$.



Plot of $V(d)$; Disjoint cliques of order 28 formed



Counterparty risk persists

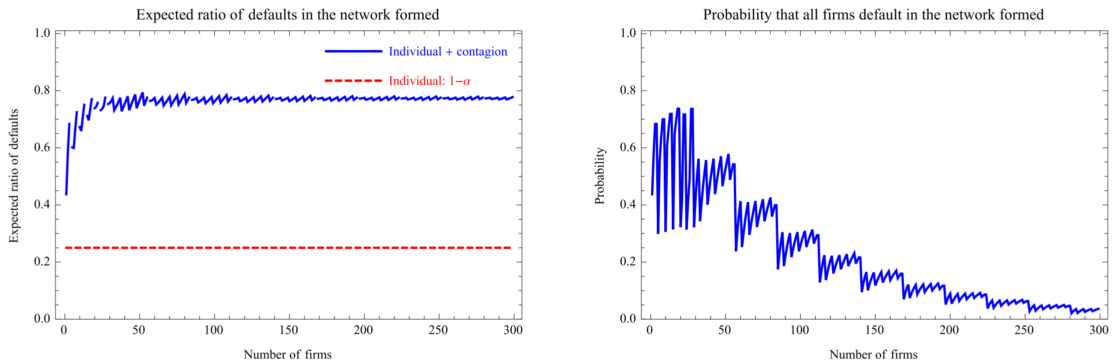


Figure 17: Measures of systemic risk for Example 3

As the reader might have already noticed, there is a relationship between the long term behavior and the comparison between expected cost of a single edge vs. gain from a single. Consider the specification in examples: additively separable payoffs in d and f. Suppose that $u : \mathbb{R}^+ \to \mathbb{R}^+$ is $\mathcal{C}^2$, increasing, concave, whereas $c : \mathbb{R}^+ \to \mathbb{R}^+$ is $\mathcal{C}^2$, strictly increasing, and convex. Let $l = \lim_{d\to\infty} \frac{u(d)}{c(d(1-\alpha))}$. Notice that if $l < 1$, then V goes to 0 as $k \to \infty$, so V has a global maximizer, say $d^* + 1$. Hence Pareto strongly stable networks consist of $(d^* + 1)$-cliques for $k > d^*$. If $l > 1$, V is unbounded. Hence Pareto strongly stable networks are complete for infinitely many k.

The limit of the rate of return from having more edges and the expected cost of these edges (modulo contagion costs which is eliminated by the clique structure) determine whether the network grows unboundedly or not. For low expected rates of return from having counterparties, in order to prevent contagion becoming almost certain, firms persist in isolating clusters, and contagion persists in the limit at a bounded rate. For higher rates of return, the one clique, complete network, keeps growing since contagion diminishes in the limit due to high rate of return.

Small networks: Recall that $d^*(m)$ is weakly increasing in m. Hence, the size of the cliques formed never decrease when new firms are added to the economy. Here I look into the rate at which the size increases with m. That is, as more firms are added, would the cliques grow smoothly, or would there be an abrupt jump in the size ? The significance of this question is as follows. When the economy is growing in the sense that the number of firms is increasing, if the network topology changes radically after a threshold number of firms leading to a jump in systemic risk, this may call for network related policy measures as a function of the size of the economy with regards to the number of firms.

Corollary 3. ***(Phase transition)*** *If $V(d)$ has a local maximum which is not a global maximum the network topology exhibits phase transition in the number of firms. Formally, for some k, the order of cliques in the network increases by more than the number of firms added to the economy. For any k for which such a jump happens, the network actually jumps to a complete network K_{k+1}.*

This situation can occur for various reasons regarding the fundamentals. One possibility is that benefits are 'more convex' than costs, but costs are relatively large for small degrees.

Example 4. $\alpha = 0.75$; $u(d) = d^{1.2}$, $c(f) = 15f$.

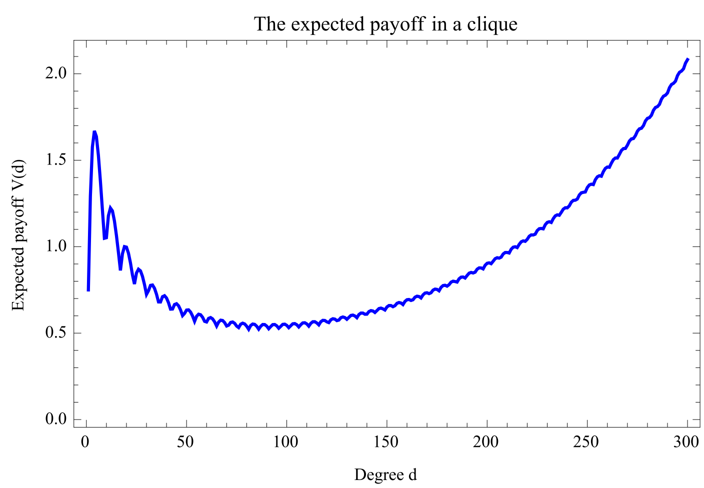


Figure 18: $V(d)$ for Example 4

Here in this example, the network exhibits a phase transition. For $k \leq 5$, $d^*(k) = k - 1$ so a complete network is formed. For $6 \leq k \leq 276$, $d^*(k) = 4$ and there are as many cliques of order 5 as possible, and possibly a residual subnetwork.[46] For $k \geq 277$, $d^*(k) = k - 1$ and a complete network is formed.

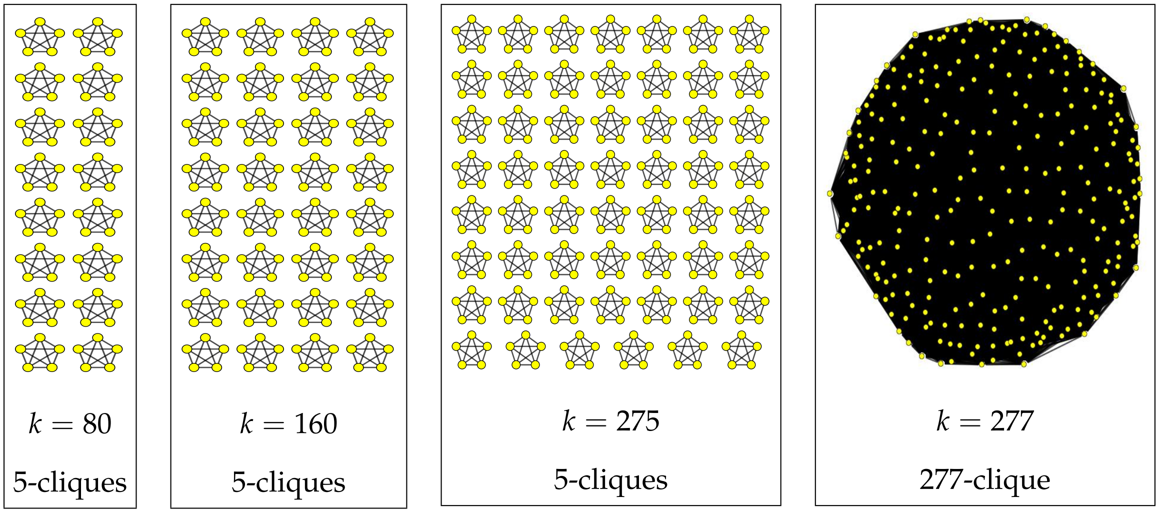
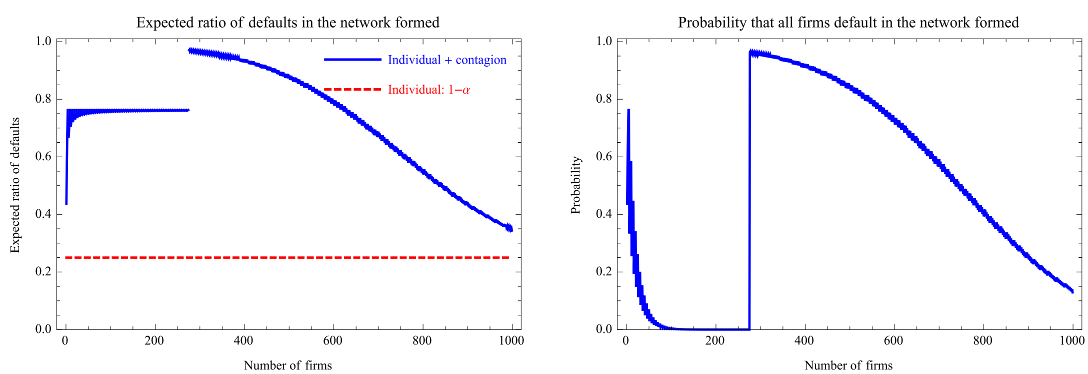


Figure 19: Example 4; Phase transition of strongly stable networks: $k = 80, 160, 275, 277$

The phase transition of the network architecture causes a radical jump in systemic risk, the probability of systemwide failure.



Expected ratio of defaults　　Probability that all firms default

Figure 20: Measures of systemic risk for Example 4

Corollary 4. *If $V(d)$ does not have a local maximum which is not a global maximum, the network topology changes smoothly in the number of firms. Formally, the order of cliques in the network*

[46] These are strongly stable networks. Pareto strongly stable networks exists only for k divisible by 5 if k is between 6 and 276.

never increases by more than the number of firms added to the economy.

Then, it is important for economic intuition to pin down conditions on fundamentals that determine when V can have a local maximum that is not a global maximum. This situation occurs typically when costs dominate benefits for small degrees, yet benefits can catch up for large degrees. In this case, firms in a "small economy" prefer to have some counterparties in order to have any small expected payoff even if it is very unlikely. However, they refrain from having more counterparties since that makes losses more likely. The potential losses are already very large compared to benefits so that very few counterparty losses would already force the firm into default. Once some positive expected return is ensured, the highest priority is to reduce the likelihood of counterparty losses. As soon as economy is "large" enough that potential benefits finally catch up with expected costs, firms form a giant cluster.

Technically, this would happen if P, as a function of d or f, has derivative[47] which is not monotonic, such as an elliptic curve. For P that is concave or convex in d or f, benefits being "more convex" than costs results in local maxima for smaller values, but is later superseded after a threshold since benefits are "more convex".

Another situation in which such a jump may happen is when firms have some 'equity' which does not depend their degree. In this case, firms fear the risk losing the equity if they incur too many counterparty losses. Hence, as long as expected benefits are not large enough, firms do not form any links. When the economy is large enough so that the cost of losing the equity is small compared to the opportunity cost of not forming a giant cluster, firms form all links possible. That is, even if costs are more 'convex' than benefits, due to the equity, there is more to lose, hence firms wait for connecting until it becomes sufficiently beneficial to do so.

I have illustrated via examples how systemic risk changes with the network topology.

[47] The derivative of an appropriately selected continuously differentiable interpolation of P

Simulations verify that clique structure consisting of cliques with order $d+1$ indeed minimizes certain measures of systemic risk across all d-regular networks. Such a result would formalize the idea that payoff maximizing firms, as a byproduct, minimize some certain measures of systemic risk.

Conjecture 5. *Take any d, and any k which is divisible by $d+1$. Consider the class of d-regular networks. The expected number of first-order indirect defaults (that is the good firms that default due to their bad counterparties), and the probability that all firms default, are both minimized when the network consists of cliques of order $d+1$ (hence total number of defaults is also minimized since all defaults in a clique are of first order).*

Summarizing remarks of the section:

Remark 1: Jointly achieving ideal payoffs conditional on a given level of (degree) connectivity is a matter of completely eliminating second-order counterparty risk from the component. If not all firms are at naturally safe levels of connectivity for their counterparties, then the only way to eliminate second-order counterparty risk is to form an isolated dense cluster, which is possible only if all firms have identical connectivity and resilience.

Remark 2: Each firm first evaluates the optimal level of first-order counterparty risk exposure it individually prefers. Then firms collectively implement these levels in dense isolated clusters. Cluster structure eliminates second-order counterparty risk and locally minimize contagion. Thusly, firms maximize their payoff.

Remark 3: Even in the absence of intervention, the endogenous network topology may feature a discontinuous phase transition as the economy gets larger. The transition may lead to abrupt changes in systemic risk.

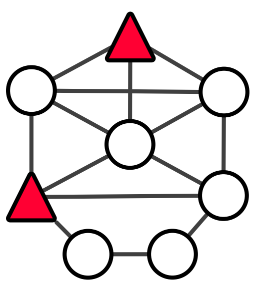
Remark 4: For large numbers of firms, if the expected rate of return from having one more counterparty is positive, firms form complete network and first-order counterparty risk diminishes. However, if the expected rate of return is negative, first-order counterparty risk, hence 'firebreaks', persist.

2.4 Presence of government intervention

2.4.1 Preliminaries

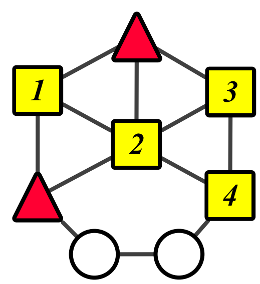
It is natural to think that fundamentally good firms with high counterparty losses could continue operations in the market instead of defaulting. Hence a good firm being forced into indirect default due to counterparties may cause some welfare loss. Government, then, may wish to step in and save good firms to increase welfare. The following example illustrates this possibility in the context of my model.

Recall Example 1: $u(d) = d$, $c(f) = 2f$. 2 firms get bad shocks, and 6 get good shocks as depicted in the lefthand network. The network on the right illustrates the order in which firms default during contagion.



Initial shocks.

Red triangle: B. White circle: G.



Propagation of defaults in waves.

Indirect defaults in order: yellow squares.

Figure 21: Stopping contagion

In this example, if government could convince the first wave of indirect defaults, labelled by "1", to continue instead of defaulting, contagion would stop. Only bad firms would fail and all good firms would survive. Four good firms would be saved.

Intervention: Consider a government that has the ability to intervene in the market at the end of stage two. Government announces and commits to a **transfer scheme**, think capital injections, $\{T_i(\vec{a}\,|\,\vec{\theta}, E, \vec{\gamma})\}_{i\in N} \geq 0$. Here T_i describes the amount of transfer to firm n_i. The intervention is time-consistent in the sense that the government cannot commit to an intervention policy before stage two. However, it can commit from the end of stage two onwards. For simplicity, drop the notation $(\vec{\theta}, E, \vec{\gamma})$ since those are all set when the government intervenes.

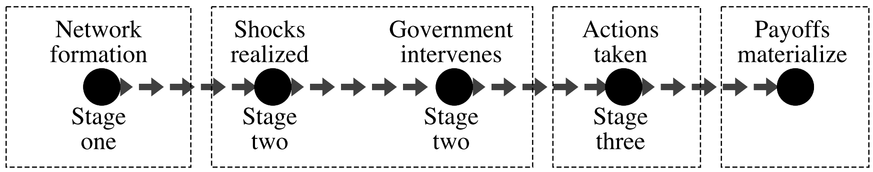


Figure 22: Timing of events

In general T_i depends on the entire action profile $\vec{a}$. But then, the induced game in stage three may not be supermodular anymore. Hence it is necessary to restate the definition of cooperating equilibrium in a way consistent with the original definition. For now, I assume that T_i depends only on a_i. This way the induced game is supermodular and cooperating equilibrium is well defined. In Section 2.6.4, I define the appropriate generalized cooperating equilibrium and show that the optimal policy (yet to be defined) has the property that T_i depends only on a_i. I skip that for now in order not to complicate the exposition.

A transfer to n_i who would otherwise default, which makes sure that it continues in the induced cooperating equilibrium can be interpreted as bailing-out firm n_i. Formally, T is said to **bailout** n_i if

1. $T_i(C) > T_i(D) = 0$,
2. in the cooperating equilibrium induced by T, $a_i = C$,
3. for any T' with $T'_{-i} \equiv T_{-i}$, $T'_i(D) = 0$, $T'_i(C) < T_i(C)$, in the cooperating equilibrium induced by T', $a_i = D$.[48]

[48]This definition of bailout does not allow for 'overcompensating' a firm for continuing. A more general definition is fine as well. It would only require introducing more definitions later on to distinguish between

Notion of welfare: Each firm n_i generates some welfare given by $W(a_i, d_i, \theta_i, \gamma_i) - \chi f_i$, for some constant $\chi \geq 0$. Total welfare is given by the sum over all firms. Welfare W_i is not necessarily the same with payoff of firm n_i, which is natural in this context. I assume that good firms increase welfare by continuing:

Assumption 8. *$W(C, d, G, \gamma) > W(D, d, G, \gamma)$ for all d, γ.*

Depending on the interpretation of the model appropriate notion of welfare can have more structure. In the **baseline case** of government intervention, bailouts are costless, the government is not restricted by a budget constraint, there is no incomplete information, bad firms reduce welfare by continuing as opposed to defaulting ($W(D, d, B, \gamma) > W(C, d, B, \gamma)$ for all d, γ), and the government is not restricted by an ex-ante commitment to bailing-out only systemically important firms. Moreover counterparty losses do not enter welfare significantly, that is $\chi = 0$.[49] First, the welfare loss from a default decision of a firm n_i is already accounted for in W_i, and there is no need to double-count the loss in welfare generated by each counterparty of n_i. Second, I want to abstract away from the incentives of the government to subsidize firms that do not face the risk of default. This way the focus is on contagion. All of these specifications of the baseline case are relaxed one by one in Sections 2.5 and 2.6. Indeed, the insights obtained in the baseline case get stronger under these generalizations.

Optimal policy: An ex-post welfare maximizing transfer scheme which uses minimal transfers when indifferent is called an **optimal policy**. Let N^B and N^G denote the set of bad and good firms. Also, $N_i^B = N_i \cap N^B$, $N_i^G = N_i \cap N^G$ denote the bad and good counterparties of n_i.

Proposition 21. *The unique optimal policy T^* is given by*

$$T_i^*(a_i) = \begin{cases} P(D, -, d_i, G, \gamma_i) - P(C, |N_i^L|, d_i, G, \gamma_i) & \text{if } \theta_i = G,\ a_i = C,\ R(d_i) < |N_i^L| \\ 0 & \text{otherwise.} \end{cases}$$

overcompensating and non-overcompensating bailouts.

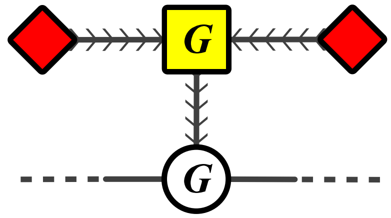
[49]Everything goes through for sufficiently small $\chi > 0$ as well.

Proof. The optimal policy must induce all good firms to continue and all bad firms to default. Then the question is how to implement this using minimal transfers. Notice that under the desired action profile, without transfers, a good firm n_i has payoff $P(C, |N_i^B|, d_i, G, \gamma_i)$. In order for n_i to continue, counterparty losses from bad firms, who are not getting bailed-out, must be compensated at least to an extent that makes n_i indifferent between defaulting or note. That is the transfer to n_i must be at least $\max\left\{0\,,\, P\left(D, -, d_i, G, \gamma_i\right) - P(C, \left|N_i^B\right|, d_i, G, \gamma_i)\right\}$. Otherwise, n_i cannot rationalize continuing.

What remains is to show that this transfer profile indeed induces the desired action profile. Consider an good firm n_i. If it believes that all good firms continue, and n_i receives T_i^*, then its payoff from continuing is greater than or equal to $P\left(D, -, d_i, G, \gamma_i\right)$. Hence C is a best response. Therefore, if all good firms receive T^*, then they can all rationalize continuing by believing that all good firms continue. Hence, under T^*, in the cooperating equilibrium, all good firms continue and all bad firms default. □

This type of bailout policy does not overcompensate firms. It makes the bailed-out firm indifferent between defaulting or not. Hence, the bailed-out firm is only as good as it defaulted. The firm's 'initial decision makers' or 'previous owners' receive nothing extra as a result of being bailed-out. Therefore, first-order counterparty risk is unchanged in response to bailouts.

The main channel through which government intervention affects network formation is the second-order counterparty risk. Since government cannot commit to the transfer policy in stage one, firms, during network formation, know that $T^*(\cdot \mid \vec{\theta}, E, \vec{\gamma})$ will be implemented in stage three. As a byproduct of the optimal policy, in the presence of intervention, each firm knows that any good counterparty is, one way or the other, going to continue. This eliminates second-order counterparty risk.



Byproduct of optimal intervention: second-order counterparty risk is eliminated. 'White circle' does not face any risk of 'yellow square' indirectly defaulting due to 'red diamonds'.

Figure 23: The main channel for intervention's effect on the network

Therefore, firms are not concerned with the counterparties of their counterparties any further, although they are still concerned with the shocks of their immediate counterparties. Below I explore how this channel manifests itself in the networks formed.[50]

2.4.2 Government-induced interconnectedness

Here I introduce the first effect of government intervention on the network topology. The anticipation of intervention eliminates second-order counterparty risk, and causes the cliques to dissolve. The network becomes interconnected. For the purpose of illustrating this effect it is sufficient to consider homogenous firms.

Assumption (Homogeneity). *$\gamma_i = \gamma$ for all $n_i \in N$.*

'Homogeneity' is maintained in Section 2.4.2. Thus I drop the γ notation until Section 2.4.3.

Theorem 7. *If $kd^*(k)$ is an even number, the set of Pareto stable networks is the set of $d^*(k)$-regular networks, otherwise, empty set.*[51]

[50]In Sections 2.5 and 2.6 I consider various extensions on top of the baseline case. Typically, if there are further constraints on the government, the optimal policy may reduce first-order counterparty risk as well as eliminating second-order counterparty risk. In return, all insights of the baseline case are maintained, and further effects on the network topology emerge.

[51]The clique structure is typically not Pareto strongly stable due to the residual in the presence of intervention. It is Pareto strongly stable if and only if $d^*(k) + 1$ divides k.

For any k, any almost-$d^*(k)$-regular network is strongly stable. Any strongly stable network is approximately-$d^*(k)$-regular.

Proof. Now that firms know T^* will be implemented in stage three, they know that all good firms continue and all bad firms default. Then their expected payoffs are given directly by V function independent of the topology of the network. In an almost-$d^*(k)$-regular network, all firms except possibly one is getting the maximum possible payoff. Hence, there cannot be any deviations: any almost-$d^*(k)$-regular network is strongly stable. Now take any strongly stable network. There cannot be $d^*(k) + 1$ or more people with degree other than $d^*(k)$ because then they would deviate and form a clique, and achieve $V(d^*(k))$. Hence any strongly stable network is approximately-$d^*(k)$-regular. As for Pareto strongly stable networks, even if one firms has less than $V(d^*(k))$ expected payoff, this firm and $d^*(k)$ other could deviate and form an isolated clique. It would not hurt the other deviators, but strictly benefit the first firm. Thus all firms must be achieving $V(d^*(k))$ in any Pareto strongly stable network, so that all firms must have degree $d^*(k)$. This is possible only if $d^*(k)k$ is even. It is clear that any such network is Pareto strongly stable. □

After second-order counterparty risk is eliminated by government, practically, all degrees become safe counterparty degrees even if they don't belong to S. Therefore, cliques dissolve into an interconnected network. However, first-order counterparty risk is not altered. Any bad firm still defaults, hence imposes same costs on its counterparties. This way firms do not overconnect or underconnect, they only make the network more interconnected.

Example 6. $\alpha = 0.75$, $u(d) = d$, $c(f) = 6f$, $k = 40$.

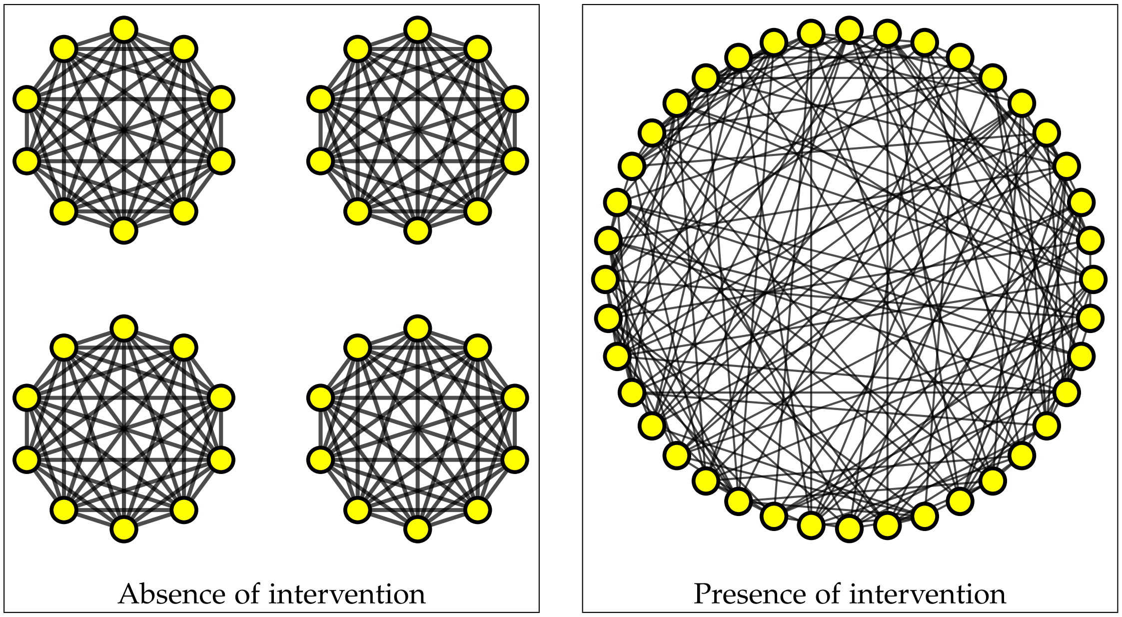


Figure 24: Network topology for Example 6

Welfare and systemic risk: Let $d^* = limsup_{m \to \infty} d^*(m)$. If V does not have a global maximizer, $d^* = \infty$. In that case, for infinitely many k, complete network K_k is formed. For such k, government intervention has no effect on the network formed. Moreover, since in a clique (a complete network is also a clique) all indirect defaults are of first order, the indirect defaults in the absence of intervention is one-to-one with bailouts in the presence of intervention. Therefore, quite trivially, government intervention increases ex-ante welfare and does not change systemic risk. If $R(k-1) \neq k-1$ welfare gain is strictly positive since the probability of at least one indirect default is positive.

Suppose that $d^* < \infty$. Suppose that kd^* is even.[52] If $R(d^*) \geq d^* - 1$, again, government intervention has no effect on the network formed. d^*-regular networks are formed in both cases. Welfare increases and systemic risk is not affected. Moreover, if $R(d^*) \neq d^*$, the welfare gain is strictly positive.

The non-trivial and most relevant case is when $d^* < \infty$ and $R(d^*) < d^* - 1$. Suppose

[52]Strongly stable networks have small remainder terms. For neat comparison I focus on Pareto strongly stable networks, which are essentially unique. That requires the divisibility assumption.

that k is divisible by $d^* + 1$. In this case, absence of intervention leads to cliques of order $d^* + 1$ whereas presence of intervention leads to d^*-regular networks.

Proposition 22. *Government intervention strictly increases ex-ante welfare.*

Proof. When there are no bailouts, every now and then some good firms default due to sufficiently many counterparty failures. This leads to a loss of welfare by $W(C, d^*, G) - W(D, d^*, G) > 0$ for each such firm. When there are bailouts, since all firms have the same degree d^*, the only difference in welfare is the sum of terms $W(C, d^*, G) - W(D, d^*, G) > 0$, which proves that welfare increases. Since $R(d^*) \neq d^*$, there is a positive probability of an indirect default, so that welfare increases strictly. □

Note. *The clustering coefficient*[53] *in the absence of intervention is larger than the clustering coefficient in the presence of intervention. Because, a clique structure gives the maximum clustering possible,* 1.

Now I illustrate how various measures of systemic risk change with the anticipation of bailouts with another simpler example.

Example 7. $\alpha = 0.75$, $u(d) = d$, $c(f) = 7f$, $k = 20$.

In this example, $V(d)$ *is globally maximized at* $d^* = 3$. *Resilience is* $R(d^*) = 0$, *so it takes one counterparty to force a firm into default.* $R(d^*) < d^* - 1$ *so firms form cliques of order* 4 *in the absence of intervention. In the presence of intervention, they form a* 3*-regular network.*

[53] The number of triangles in the network divided by the total number of possible triangles. Global and average clustering coefficients are equal since the graph is regular.

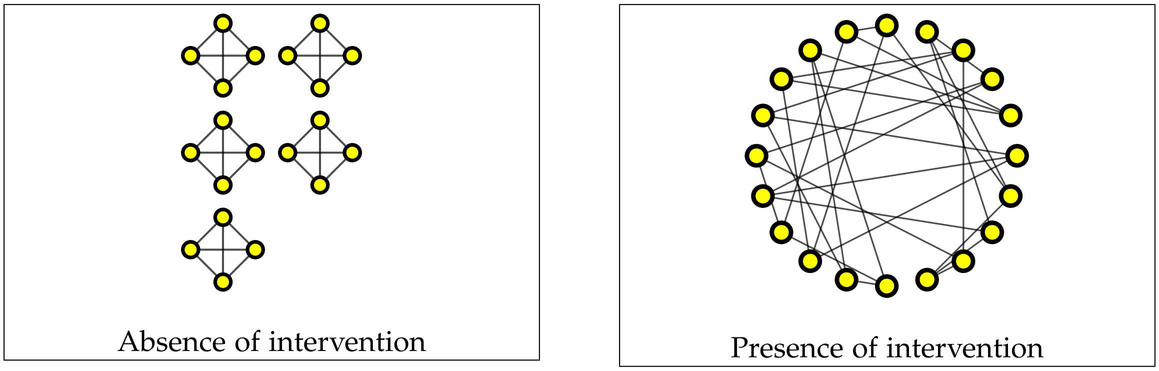
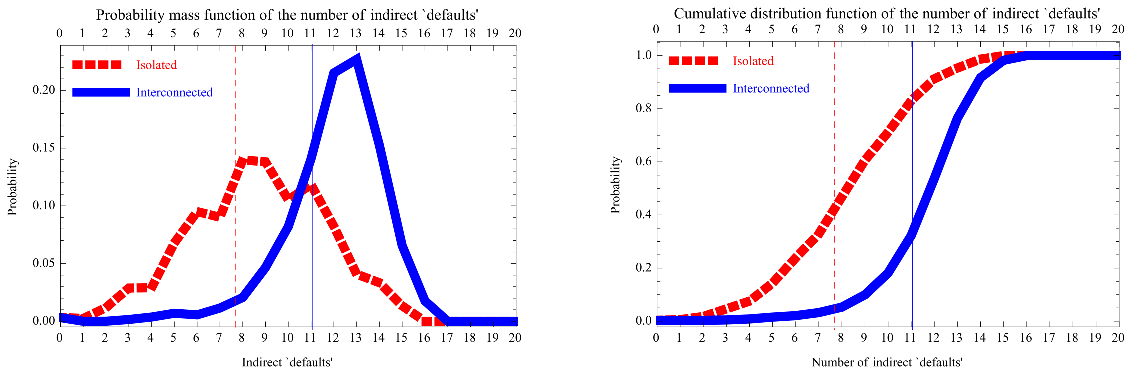


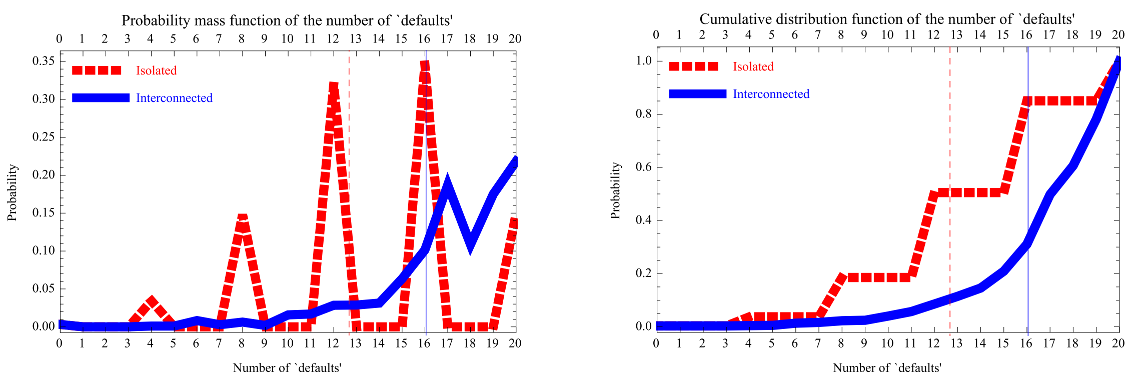
Figure 25: Network topology for Example 7

Note that in the figures below, indirect 'default' refers to bailed-out firms for the interconnected network in presence of intervention, whereas total number of 'defaults' refers to direct defaults plus bailouts. The systemic risk comparison is number of indirect defaults in the benchmark case vs. bailouts in the intervention case (firms that would default indirectly if it wasn't for bailouts). Notice that with cliques, all indirect defaults are of first order. In an interconnected network, even though first-order indirect defaults are not necessarily all indirect defaults, second-order defaults never realize since government bails-out all first-order indirect defaults. Thus, all bailouts are of first-order indirect defaults under government intervention as well. These two magnitudes are being compared: first-order (hence all) indirect defaults in absence of intervention vs. first-order (hence all) bailouts in the presence of intervention. Red dashed line labeled 'isolated' represents the clique structure that is formed in the benchmark case. Blue solid line labeled 'interconnected' represents the presence of intervention.



Probability mass function　　　　Cumulative distribution function

Figure 26: Indirect defaults in the absence of intervention vs. Bailouts in the presence of intervention for Example 7



Probability mass function　　　　Cumulative distribution function

Figure 27: Total defaults in the absence of intervention vs. Bailouts and Defaults in the presence of intervention for Example 7

An immediate corollary of Conjecture 1, which simulations verify, would be the following.

Conjecture 8. *Government intervention increases systemic risk:*

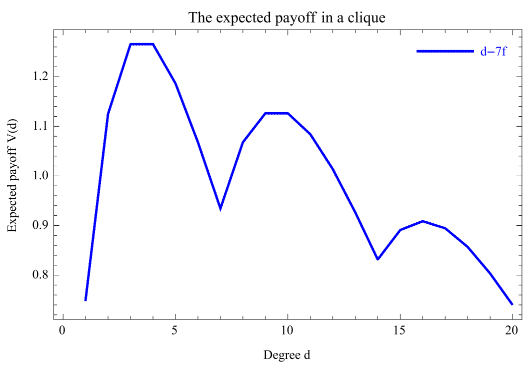
1) *expected number of bailouts in the presence of intervention is larger than the expected number of indirect defaults in the absence of intervention,*

2) *probability that all firms either default or get bailed-out in the presence of intervention is larger than the probability that all firms default in the absence of intervention.*

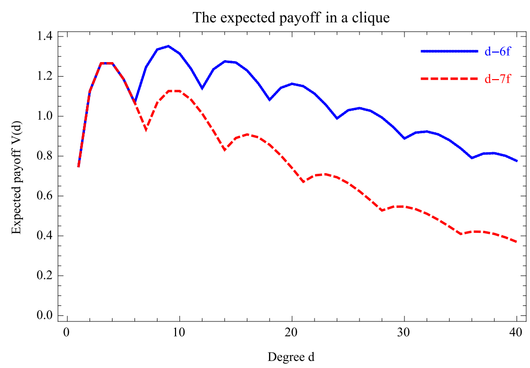
2.4.3 Government-induced systemic importance

Now I introduce the second effect of government intervention. When there is heterogeneity in firm types γ, typically different types have different ideal degrees. Without loss of generality, there will be some firms with higher ideal degree among k firms. Below is an example of how difference in γ can alter P and generate heterogeneity in demand.

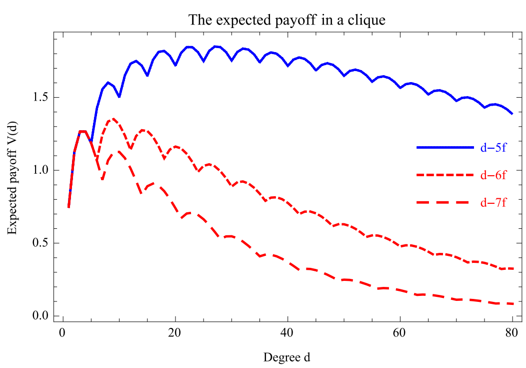
Example 9. $\alpha = 0.75$. $\Gamma = \{\gamma^1, \gamma^2, \gamma^3, \gamma^4\}$ *is given as follows. For all* $\gamma \in \Gamma$, $u(d, \gamma) = d$. *Moreover,* $c(f, \gamma^1) = 7f$, $c(f, \gamma^2) = 6f$, $c(f, \gamma^3) = 5f$, $c(f, \gamma^4) = 4f$.



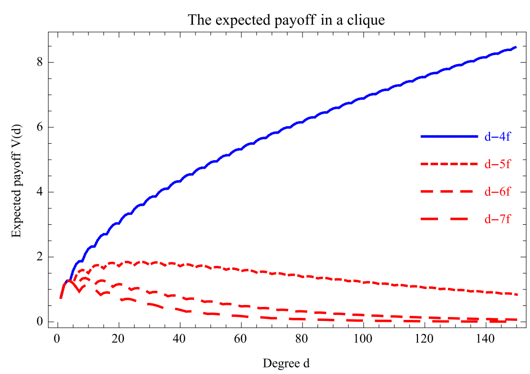
$V(d, \gamma^1)$



$V(d, \gamma^1)$ and $V(d, \gamma^2)$



$V(d, \gamma^1)$, $V(d, \gamma^2)$, and $V(d, \gamma^3)$



$V(d, \gamma^1)$, $V(d, \gamma^2)$, $V(d, \gamma^3)$, and $V(d, \gamma^4)$

Figure 28: Heterogeneity in demand for Example 9

If the types that demand higher degree demand too high of a degree, there would not be sufficiently many firms with k-similar types, so that these firms would not be able to convince other types to become counterparties. The reason is that the types that demand higher degrees, if not very resilient, would bring on second-order counterparty risk to the other types that prefer lower degree. However, in the presence of government intervention, second-order order counterparty risk is eliminated so that other types would also be willing to become counterparties. This way, types with high ideal degrees among k firms become central to the network, and the network typically becomes a core-periphery network. In order to illustrate this effect, it is sufficient to have two types. Arguments can be generalized to more than two types. Suppose that there is one with high demand for having counterparties, the other with low demand for counterparties. Again for simplicity of statements of results I assume away integer problems and the cases in which ideal degrees are safe counterparty degrees. Results can be generalized to non-divisibility and unsafe counterparty degree cases analogous to the benchmark case.

Assumption (Minimal heterogeneity). *Two types:* $\Gamma = \{L, H\}$*. There are* k_L *many* L *firms and* k_H *many* H *firms.*

Demand heterogeneity: $V(d, L)$ *is maximized at* d_L^* *on* $[0, k]$*.* $V(d, H)$ *is increasing on* $[k_H, k]$*.*

Non-trivial resilience: $R(d_L^*, L) < d_L^* - 1$*, and* $R(d, H) < d - 1$ *for all* $k - 1 \geq d \geq k_H$*.*

No integer problems: $d_L^* + 1$ *divides* k_L *and* $(d_L - k_H)k_L \geq 0$ *is an even integer.*

'Minimal heterogeneity' is maintained in Section 2.4.3. Call L the low (counterparty demanding) type and H the high (counterparty demanding) type since $d^*(m, H) = m - 1$ for all $m \in [k_H, k]$.

Theorem 8. *In the absence of intervention, the unique Pareto strongly stable network is given by disjoint cliques of order* $d_L^* + 1$ *of low types, and another disjoint clique of order* k_H *of high types. In the presence of intervention, the unique Pareto strongly stable network structure is given by core-periphery: high types (core) are counterparties with all firms, and low types (periphery) form a* $(d_L - k_H)$*-regular subnetwork among each other aside of their links with high types.*

Proof. Consider any Pareto strongly stable network. Similar to before, all low types must achieve their L-ideal payoffs. Recall the proof of Proposition 3. A low type can achieve its L-ideal payoff only if its all counterparties are also low type, or high type with a safe counterparty degree. But, a high type having a safe counterparty degree means its degree is less than k_H. That is, any high type who has a low type counterparty must have degree less than k_H. These high types, then, are getting less than their H-ideal payoff among k_H firms. The other high types, those that are not adjacent to any low type firms are also getting less than or equal to their ideal degree among k_H firms. Then, all high types can deviate to form an isolated clique, which Pareto improves all of them. Therefore, no low type can have any high type counterparty. Then the only Pareto strongly stable option among themselves, as in the benchmark case, is disjoint cliques since d_L^* is not a safe counterparty degree. For high types, also, a clique of k_H firms is the only Pareto strongly stable configuration among themselves. What remains is to show that this is indeed Pareto strongly stable. Similar to the idea outlined here, if there was a deviation involving any low type, it must be as good as its ideal payoff, which would make sure any high type that connects with must get strictly worse off due to since it must reduce its degree below $k_H - 1$. There is no Pareto deviation among high types by themselves either. Thus the network is Pareto strongly stable.

Under the anticipation of intervention, all firms are, via intervention, safe counterparties with regards to second-order counterparty risk. Then in any of the described core-periphery networks, all firms are getting their ideal payoffs among k firms. Then there is no Pareto profitable deviation. Then in any Pareto strongly stable network, all firms still must get their ideal payoffs among k firms, which implies that all high types must have degree $k - 1$, which then implies that all low types must have degree $d_L - k_H$ among themselves. □

Example 10. *Continue with Example 9, with* $L = \gamma^2$ *and* $H = \gamma^4$*. That is,* $u(d, L) = d$*,* $c(d, L) = 6f$*, and* $u(d, H) = d$*,* $c(d, H) = 4f$*.* $k_L = 50$ *and* $k_H = 6$*.*

Under this specification, $d_L^* = 9$ *and* $R(d_L^*, L) = 1$*.* $V(d, H)$ *is increasing and* $R(d, H) =$

$\left\lfloor \frac{d}{4} \right\rfloor$. *In the absence of intervention, network consists of* 5 *cliques of low types with order* 10*, and one clique of high types with order* 6*. With government, network consists of one core of* 6 *high types, each connected to everyone. Low types are peripheral; each have* 3 *links with other low types.*

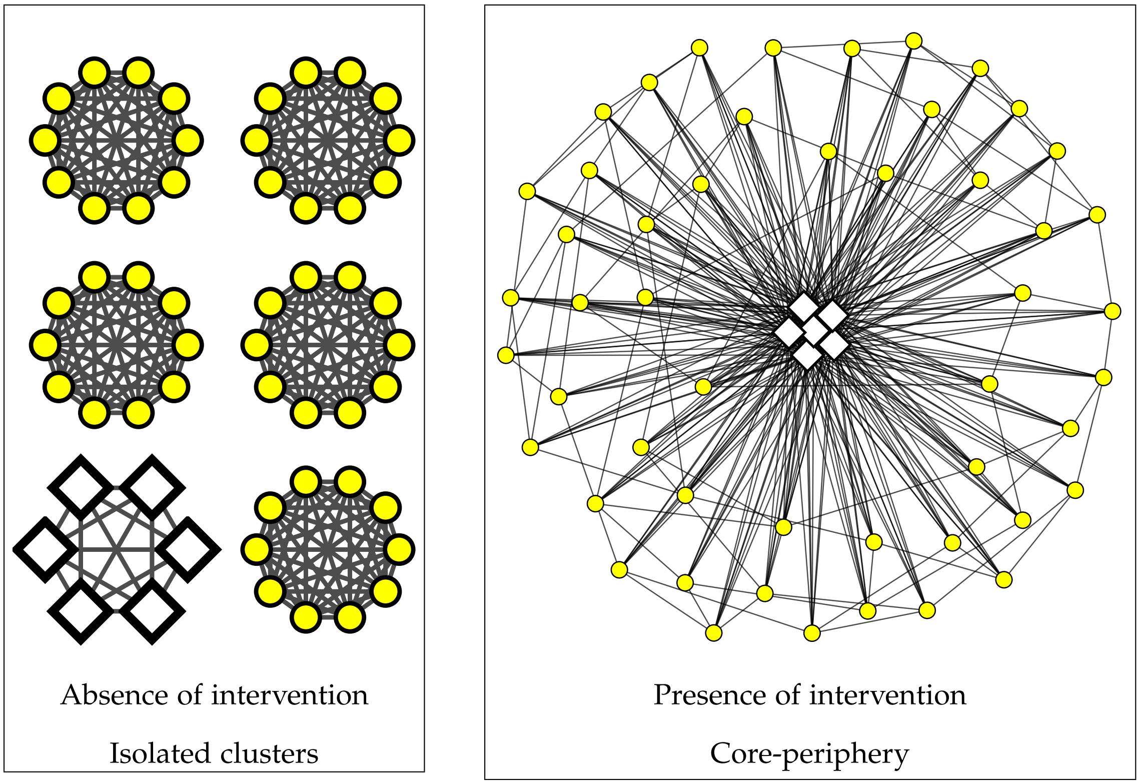


Figure 29: Core-periphery formed as a consequence of intervention, Example 10

Welfare and systemic risk: There is a change in systemic risk due to the change in the architecture. Now it takes 13 out of 50 low firms to get bad shocks to force the core firms into default. In that case, however, government saves all 6 core firms, and all is under control. This is why low types don't mind connecting with high types at the core even when the core has very large degree. However, if 2 core firms out of 6 get bad shocks, that forces all 50 peripheral firms into default. Government now bails-out all good firms out of 50 peripheral firms. For low type firms, ex-ante it does not matter which 9 firms they have links with; it is the same first-order counterparty risk. Government takes care of second-order counterparty risk, so counterparties can very well be central high types.

The following simpler example illustrates such effects on measures on systemic risk.[54]

Example 11. *Continue with Example 9.* $L = \gamma^1$ *and* $H = \gamma^4$. $k_L = 16$ *and* $k_H = 2$.

That is, $u(d, L) = d$, $c(f, L) = 7f$ *and* $u(d, H) = d$, $c(f, H) = 4f$. *In this case,* $d_L^* = 3$ *and* $R(d_L^*, L) = 0$.

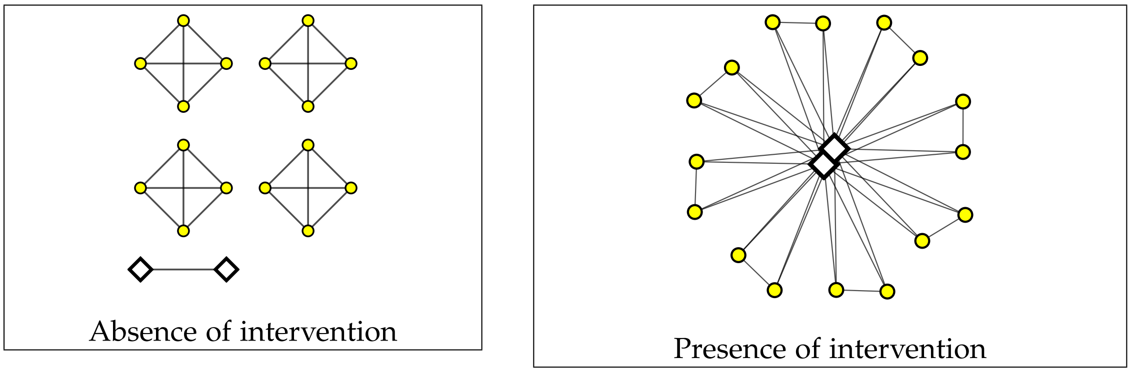


Figure 30: Core-periphery formed as a consequence of intervention, Example 11

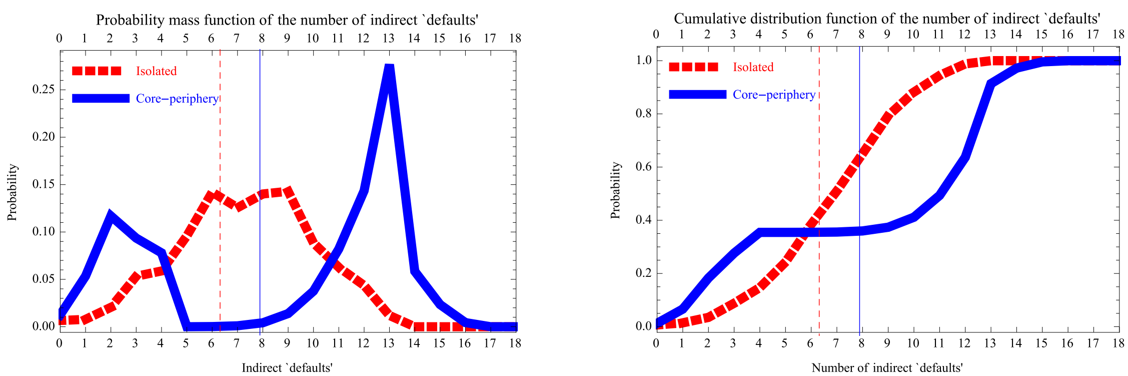


Figure 31: Indirect defaults in the absence of intervention vs. Bailouts in the presence of intervention for Example Example 11

[54]Previous example is computationally challenging to find the exact probability distribution of the number defaults.

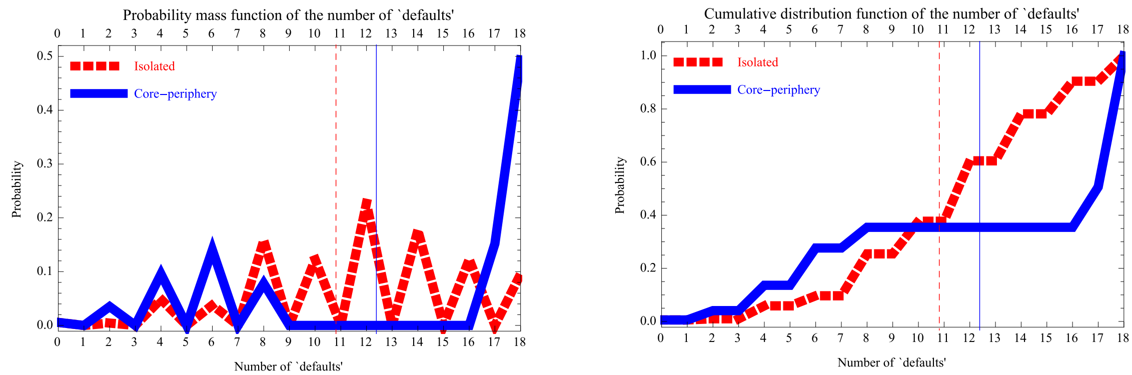


Figure 32: Total defaults in the absence of intervention vs. Bailouts and Defaults in the presence of intervention for Example 11

Core-periphery structure, makes the 'very good' and 'very bad' outcomes more likely. Very bad outcome is that core gets bad shocks, and amplifies the contagion among periphery causing most peripheral firms into indirect defaults, hence bailouts. Very good outcome is that the core gets good shocks, and mitigates the contagion among periphery by contributing to resilience of the peripheral firms.

Ex-ante welfare comparison now depends also on how the degree of high types enter welfare: $W(a_i, d_i, \theta_i, H)$*. If welfare is hurt significantly when high types operate at high degree levels, i.e.* $W(a_i, d_i, \theta_i, H)$ *is decreasing 'sharply' in* d_i*, ex-ante welfare could be hurt. But this would be an ad-hoc situation. If* $W(a_i, d_i, \theta_i, H)$ *is not decreasing, ex-ante welfare would be increased both for inducing H firms to operate at socially better capacity and making sure good firms continue as it was the case for interconnectedness. In this sense, core-periphery may hold high benefits for welfare even though it increases volatility.*

Discussion of 'Minimal heterogeneity': *Note that 'Minimal heterogeneity' rules out the case the cases in which* d_L^* *is an L-safe counterparty degree (*$R(d_L^*, L) \geq d_L^* - 1$*) and* $k-1$ *is an H-safe counterparty degree (*$R(k-1, H) \geq k-2$*). In these 4 cases, one has to further consider the sub-cases in which* $R(d_L^*, L) = d_L^*$ *and\or* $R(k-1, H) = k-1$ *to see whether the welfare gain is strict or not. It leads to 16 cases, cumbersome notation, and long statements for results so I skip these cases. Essentially, interconnectedness and systemic importance can be de-coupled using the* *resilience of L and H types. Resilience of L captures the change in the interconnectedness across L firms. Resilience of H captures the change in the systemic importance of H firms. However, when resilience of these types are extremely high, there is essentially not a big change in systemic risk since absence of intervention also features little contagion due to high resilience. One exception is that, if resilience of H types is extremely high whereas L types are not extremely resilient, even in the absence of intervention, core-periphery would be formed featuring clusters of L firms in the periphery. There would be volatility due to the bad shocks to H firms.*

A final remark is that $d_L^* \geq k_H$ is not essential for the strongly stable networks. When $d_L^* < k_H$, there is no Pareto strongly stable network due to cycles of deviations. However, under $d_L^* < k_H$ all networks in which L firms are counterparties with only H firms, and all H firms are counterparties with each other on top of their links with L firms is strongly stable. That is, it is not critical that the periphery has links among each other which requires $d_L^* \geq k_H$. Even when that is not feasible due to $d_L^* < k_H$, there are strongly stable networks that feature core-periphery structure whereas core-periphery is not strongly stable in the absence of intervention.

2.4.4 Individual risk behavior

For now assume *'Homogeneity'*. Since first-order counterparty risk is not altered, firms do not overconnect or underconnect. So there is a sense in which firms do not make riskier individual choices. Here I try to formalize this idea further and argue that if firms were allowed to choose the risk level of their investments, i.e. alter the distribution of shocks with their choices, government intervention would not cause any change in their choices of risk level, but they would still make the network interconnected.

Suppose that firms can choose to invest in regular projects or high risk/high return projects. Formally, firms now have payoff multipliers ζ and risk parameters α, which are choices. Firm n_i receives $\theta_i = G/B$ with probability $\alpha_i/1 - \alpha_i$, and receives a payoff

$\zeta_i \times P(a_i, f_i, d_i, \theta_i)$. Normalize the outside options to 0: $P(D, -, d, \theta) = 0$ for all d, θ.[55]

A firm n_i can choose $(\alpha_i, \zeta_i) \in \{(\alpha^s, \zeta^s), (\alpha^r, \zeta^r)\}$ where $\alpha^s > \alpha^r$ and $\zeta^s < \zeta^r$. (α^s, ζ^s) is the regular and safer project, whereas (α^r, ζ^r) is the high risk/high return project. The choice of project type is simultaneous with network formation. Alter the definition of the network to incorporate the risk choices, and the definition of feasible deviation to allow for each deviating firm to choose a different risk level as well. In the realized network, call firms that chose safer project **safe firms**, and the others **risky firms**.

Given E, let d_i^s denote the number of safe counterparties of n_i and d_i^r denote the number of risky counterparties of n_i. Let $\mathbf{f}(x; y, p, z, q)$ denote the probability that out of y trials with a p-coin and z trials with a q-coin, the total number of heads is x.

For given risk profile $\vec{\alpha}$, n firm n_i's expected payoff at the center of a star is

$$V(d_i^s, d_i^r, \alpha_i, \zeta_i) = \alpha_i \zeta_i \times \mathbb{E}_{f_i} \left[\max \left\{ P\left(C, f_i, d_i, H\right) , 0 \right\} \right],$$

where $f_i \sim \mathbf{f}(\cdot; d_i^s, 1-\alpha^s, d_i^r, 1-\alpha^r)$.

For $x \in \{s, r\}$, denote $\tilde{V}^x(d) := \mathbb{E}_f \left[\max \left\{ P\left(C, f, d, H\right) , 0 \right\} \right]$ for $f \sim \mathbf{f}(\cdot; d, 1-\alpha^x)$. Let $d^{x*} := argmax_d \tilde{V}^x(d)$ and $\tilde{V}^{x*} = \tilde{V}^x(d^{x*})$. The expected payoff of a firm n_i at the center of a star with d_i leaves, with all having the same risk choice $x \in \{s, r\}$, is $V^x(d_i, \alpha_i, \zeta_i) = \alpha_i \zeta_i \tilde{V}^x(d_i)$. Assume away divisibility issues: $d^{x*} + 1$ divides k for both $x \in \{s, r\}$, and k is even. Assume that there is second-order counterparty risk at desired interconnectedness: $R(d^{x*}) < d^{x*} - 1$ for both $x \in \{s, r\}$.

Proposition 23. *In the absence / presence of intervention:*

- *If $\alpha^s \zeta^s > \alpha^r \zeta^r$, all firms choose s. Pareto strongly stable networks are: disjoint cliques of order $d^{s*} + 1$ / d^{s*}-regular networks.*
- *If $\alpha^s \zeta^s \tilde{V}^{s*} < \alpha^r \zeta^r \tilde{V}^{r*}$, all firms choose r. Pareto strongly stable networks are: disjoint cliques of*

[55] Investment risk can be thought of as the risk in the cost $\tilde{c}$ of inputs/supervision in the lead example, or directly a risk in the rate of return R.

order $d^{r*}+1$ / d^{r*}-regular networks.

- *If $\alpha^s\zeta^s < \alpha^r\zeta^r$ but $\alpha^s\zeta^s\tilde{V}^{s*} > \alpha^r\zeta^r\tilde{V}^{r*}$, there does not exist any strongly stable network in either case.*

Proof. Define $\tilde{V}(d_i^s, d_i^r, \alpha_i, \zeta_i) = \mathbb{E}_{f_i}\left[\max\left\{P\left(C, f_i, d_i, H\right), 0\right\}\right]$ where $f_i \sim \mathbf{f}(\cdot; d_i^s, 1-\alpha^s, d_i^r, 1-\alpha^r)$. $\mathbf{F}(\cdot; y, p, d-y, q)$ first order stochastically dominates $\mathbf{F}(\cdot; y', p, d-y', q)$ if $y > y'$ and $p > q$. $\max\left\{P\left(C, f, d, H\right), 0\right\}$ is a decreasing function of f. It decreases strictly at $f = 0$. So $\tilde{V}(t, d-t, \alpha, \zeta)$ is strictly increasing in t. Then, $\tilde{V}^{r*} = \tilde{V}(0, d^{r*}, \alpha, \zeta) < \tilde{V}(d^{r*}, 0, \alpha, \zeta) \leq max_d \tilde{V}(d, 0, \alpha, \zeta) = \tilde{V}^{s*}$.

First part: If $\alpha^s\zeta^s > \alpha^r\zeta^r$, in any network, any r firm would deviate unilaterally to s. Thus in any (Pareto) strongly stable network, all firms choose s. If all firms choose s, the only candidates are, as established in benchmark/baseline cases, ($d^{s*}+1$)-cliques/d^{s*}-regular networks. All firms get $\alpha^s\zeta^s\tilde{V}^{s*}$ expected payoff. These are (Pareto) strongly stable. Suppose that there is a profitable deviation. Since $\alpha^s\zeta^s > \alpha^r\zeta^r$ and $\tilde{V}(t, d-t, \alpha, \zeta)$ is strictly increasing in t, the same network deviation with all deviators choosing s is also a profitable deviation. After this new deviation, all firms still choose s. Then this must be a profitable deviation from the benchmark/baseline case, which does not exist.

Second part: Since $\alpha^s\zeta^s < \alpha^r\zeta^r$, in any network, any s firm would unilaterally deviate to r. Thus in any (Pareto) strongly stable network, all firms choose r. If all firms choose r, the only candidates are, as established in benchmark/baseline cases, ($d^{r*}+1$)-cliques/d^{r*}-regular networks. Now that $\alpha^s\zeta^s\tilde{V}^{s*} < \alpha^r\zeta^r\tilde{V}^{r*}$, these networks are (Pareto) strongly stable. All firms get $\alpha^r\zeta^r\tilde{V}^{r*}$ expected payoff. Suppose that there is a profitable deviation. If there are no deviators that chose s in the deviation, the network deviation would be a profitable deviation in the benchmark/baseline case, which does not exist. Take any deviator that chose s. Suppose it has d^s/d^r many s/r counterparties after the deviation. Then its payoff is at most $V(d^s, d^r, \alpha^s, \zeta^s) \leq V(d^s + d^r, 0, \alpha^s, \zeta^s) \leq \alpha^s\zeta^s\tilde{V}^{s*} < \alpha^r\zeta^r\tilde{V}^{r*}$. Contradiction.

Third part: Since $\alpha^s\zeta^s < \alpha^r\zeta^r$, in any network, any s firm would unilaterally deviate to r. Thus in any (Pareto) strongly stable network, all firms choose r. But then $d^{*s}+1$ many firms would get together and form an isolated clique, and all choose s. This would improve their payoff since $\alpha^s\zeta^s\tilde{V}^{s*} > \alpha^r\zeta^r\tilde{V}^{r*}$. □

The ability to choose the risk level is indeed exploited by firms. This choice also affects network architecture by altering the degrees firms have. However, anticipation of government intervention does not lead to any change in the choice of risk or degree, but just serve to interconnect the network.

It is straightforward to extend this proposition to *'Minimal heterogeneity'* following the same proof. However, it requires cumbersome notation since there are now 9 cases instead of 3 cases. I skip formalizing this to save notation and space.

Summarizing remarks of the section:

Remark 5: In the absence of government intervention, each firm prefers that the counterparties of its counterparties are only among its own counterparties. This way, it is not exposed to any second-order counterparty risk. This force creates a market discipline guiding firms towards forming clusters, effectively serving as firebreaks. In the presence of intervention, however, second-order counterparty risk is readily eliminated as a byproduct of optimal intervention. Therefore, cliques dissolve into an interconnected network. On the other hand, firms still face the full extent of first-order counterparty risk. Hence no firm over-connects or under-connects. They just make the network more interconnected.

Remark 6: Time-consistent bailouts are welfare improving since they do not alter firms' incentive to take on extra first-order counterparty risk while they save firms that suffer high counterparty losses. However, the network becomes more interconnected and susceptible to contagion since firms no longer worry about second or higher counterparty risk coming from distant parts of the network.

Remark 7: Anticipation of government intervention makes some small number of firms more central through the anticipation that they will be bailed-out. The network formed features core-periphery structure as a consequence of anticipation of bailouts. This endogenously correlates the counterparty risk of peripheral firms, which make up the majority of the economy. If sufficiently many core firms get bad shocks, there would be need for an unusually high number of bailouts among the peripheral firms. If sufficiently many core firms get good shocks, core serves as a buffer. Hence good and bad events become more likely. Therefore, bailouts generate volatility through its effect on the network architecture.

Remark 8: Since bailouts do not overcompensate firms, government intervention in my model does not lead to moral hazard in investment decisions: all firms choose the same risk behavior individually and same number of projects. Decision makers lose everything even if their firm is bailed-out, and are only as good as defaulting. As a result there is no incentive to alter risk decisions. However, there is network hazard: second-order counterparty risk is eliminated and firms interconnect the network, leading to increased systemic risk. Also, under heterogeneity, firms form a core-periphery network without changing their risk choices in response to bailouts.

2.5 Robustness

In the presence of intervention high types strictly prefer the core-periphery over the clique structure. As for low type firms, they are indifferent between connecting with high types or low types in the baseline case. Also, homogenous types are indifferent between an interconnected network and the clique structure in the baseline case. Then a question of robustness arises. In this section I argue that the induced interconnectedness and core-periphery are indeed the robust outcomes. Under slight variations of the model, all firms under heterogeneity strictly prefer the core-periphery network, and all firms under homogeneity strictly prefer making the network more interconnected. The variations I consider are relaxations of the baseline case of government intervention.

Counterparty losses in welfare and subsidy via bailouts: The parameter χ can be large or small depending on the interpretation. In the baseline case, I have abstracted away from the incentives of government to subsidize firms that do not face the risk of default so that the focus has been on contagion. Besides that, the effect of a default decision by a firm n_i on total welfare was already accounted for in W_i. In some other interpretations considered in Section 2.2.3, links represent transfers between counterparties which firms use for outside investments. This was another reason for χ to be small. For the sake of completeness I consider large χ as well.

Costs of bailing-out each firm: Since transfers go back to some households, bailouts possibly do not, ex-post, hurt aggregate welfare. Some scholar argue that there are indeed significant ex-post costs associated with bailouts. These costs can take various forms. It can be a lumpsum cost of passing a bill from congress. It can be a proportional cost through the distortionary taxes imposed to fund bailouts, which alter labor choice and reduce welfare. One more possibility is the cost of managing wide scale bailouts, which would be proportional to the number of bailouts. I consider a middle case between all: there is a fixed cost to bailing out each firm n_i, $\lambda(d_i, \theta_i, \gamma_i) \geq 0$.

Budget constraint: There can be limitations on the amount of transfers rather than the number of bailouts as observed in practice. I consider a budget constraint. b, the budget is maximum amount of transfer government can execute.

Incompletely informed government: One serious problem with bailouts at time of urgently needed intervention is informational imperfections. Generally, it is costly and timely to assess which firms need bailouts and the amount of transfers they need. I leave serious treatment of information acquisition to future work. I consider two cases: government observes all shocks, or observes no shocks.

Productive bad firms: The shocks in the lead example are shocks to the fundamental productivity of firms via their costs. However, in other contexts, a shock could simply be a liquidity shock, like a bank-run with none or very little relation to underlying fundamentals. In that case, it might be natural to think that bailing-out bad firms could also improve welfare in and of itself.

Assumption (Baseline government)**.** *No subsidy to solvent firms via bailouts:* $\chi = 0$*.*
Costless bailouts: $\lambda \equiv 0$*.*
Unrestricted budget: $b = \infty$*.*
Bad firms reduce welfare: $W(D, d, B, \gamma) > W(C, d, B, \gamma)$ *for all* d, γ*.*
Government observes all shocks $\vec{\theta}$*.*

'Baseline government' was maintained in Section 2.4. Now I relax the components *of* *'Baseline government'* one by one.

2.5.1 Core-periphery: subsidy aspect of bailouts

• *'Minimal heterogeneity'* is maintained. *'Baseline government'* is relaxed to allow for $\chi > 0$.

In this subsection, I argue that when the reduction of counterparty losses could improve welfare aside of its effect on default decisions, core-periphery result strengthens. Now, a firm with sufficiently high degree is likely to be bailed-out even if it is a bad firm, in order for the government to benefit from welfare improvement in the reduction of the losses of its counterparties. This can be seen analogous to debt guarantees. High type firms willing to have high degree are now valuable counterparties since they get bailed-out even if they get bad shocks. In return, low type firms strictly prefer becoming counterparties with high types in order reduce first-order counterparty risk.

For simplicity, assume that $W(D, d, B, \gamma) - W(C, d, B, \gamma)$ does not depend on γ or d, say it is equal to a constant δ. If it were intrinsically more important that high types continue than low types, then all the following still holds. Similarly if as long as $\{W(D,d,B,\gamma) - W(C,d,B,\gamma)\} /d$ is decreasing in d everything goes through.

Government still finds it optimal that all good firms continue. Moreover, if a bad firm has degree larger than δ/χ, by bailing this firm out, government looses δ, yet gains χd from counterparties of the bad firm. Thus the optimal policy involves bailing out all bad firms with degree larger than δ/χ, and all the good firms that face default due to first-order losses from bad firms of degree less than δ/χ. Call firms with degree larger than δ/χ **preferred counterparties** and such degrees **preferred counterparty degrees**. Suppose that $\delta/\chi < k-1$ so this situation can indeed occur. Also assume that $d_L^* < \delta/\chi$ so that low types are not automatically willing to have preferred counterparty degrees and face a tradeoff.

High types now are likely to become preferred counterparties. The reason is that their ideal degree among k firms is $k-1$ which makes sure they get bailed-out even when they get bad shocks. This way, low types would like to be counterparties with high types. Yet, it may still be possible that low types enjoy such benefits among each other by becoming counterparties of each other at preferred counterparty degrees. Therefore, it is not immediate that low types would strictly prefer being counterparties with high types.

Assume that $P(C,f,d,G,\gamma)$ is submodular in (f,d) and $P(D,-,d,G,\gamma)$ is increasing in d. Define $V(d,s,\gamma) = E_f\left[max_a\left\{P(a, max\{f-s,0\},d,\theta,\gamma)\right\}\right]$ where $f \sim \boldsymbol{f}(\cdot,d,1-\alpha)$, the binomial distribution with a $(1-\alpha)$-coin. Also $d^{**}(s,\gamma) := argmax_{d<k}V(d,s,\gamma)$.

Proposition 24. *If $\delta/\chi > d^{**}(k-1,L)$, in any strongly stable network all high type firms are counterparties with all firms. As for the subnetwork across low types:*

1) *An almost-($d^{**}(k_H,L) - k_H$)-regular network among each other gives a strongly stable network.*

2) In any strongly stable network, this subnetwork must be approximately-($d^{**}(k_H,L) - k_H$)-regular.

Proof. By the assumptions on P, $V(d,s,\gamma)$ is supermodular in (d,s). Hence, $d^{**}(s,\gamma)$ is increasing in s. In particular, $d^{**}(k-1,L) > d^{**}(k_H,L) > d^{**}(0,L) = d_L^* > k_H$.

Note that the optimal policy always makes sure all good firms continue. Then the payoffs of firms are indeed given by the $V(d,s,\gamma)$ function. Consider any strongly stable network. Suppose that there is a low type firm with preferred counterparty degree d. Suppose that s of its counterparties have preferred counterparty degrees. Since $d > \delta/\chi > d^{**}(k-1,L) > d^{**}(s,L)$. Then it would individually deviate by cutting $d - d^{**}(s,L)$; starting with non-preferred counterparties. Thus in any strongly stable network, all low types have non-preferred counterparty degrees.

For high types, since $V(d,0,H) = V(d,H)$ is increasing in d and $V(d,s,H)$ is supermodular in (d,s), $V(d,s,H)$ is also increasing in d for any fixed s. Thus, high types are happy to become counterparties with any counterparty, preferred or non-preferred. Since in any strongly stable network all low types are non-preferred the only preferred counterparties that low types can have are potentially the high types while high types are happy to become counterparties. Thus in any strongly stable network, all low types must be counterparties with all high types. Otherwise a low type would cut one link with a low type and add with the high type.

The rest follows similar ideas with proofs presented before. □

2.5.2 Core-periphery: restriction to bailing-out only systemically important firms

• *'Minimal heterogeneity'* and *'Baseline government'* are maintained.

In practice, there might be legal restrictions on what kinds of firms can be bailed-out. One example is to bailout only systemically important firms. Here I argue that such a commitment might actually reinforce incentives for connecting with these systemically important firms. The idea is that if government cannot bailout good but "small" firms facing failure due to bad and "large" firms, it may have to bailout these bad and "large" firms. Ex-ante, that would mean, connecting with "large" firms reduces first-order counterparty risk since they may get bailed-out even when they get bad shocks.

Suppose that there is a threshold degree $\bar{d}$ such that a firm n_i with $d_i < \bar{d}$ cannot be bailed-out. Again, there are two types as described before. Also assume that $k-1 \geq \bar{d} > d^{**}(k-1, L)$ and k_L is divisible by $d_L^* - k_H + 1$.

Proposition 25. *The unique strongly stable network is given as follows. All high firms are connected to all firms, and all low firms, aside of their links with high firms, form disjoint cliques of order $d_L^* - k_H + 1$ among each other. This is also Pareto strongly stable. All low firms receive strictly higher payoff than the case in which they formed their own cliques of order $d_L^* + 1$.*

Proof. Since $\bar{d} > d^{**}(k-1, L)$, a low type firm is never going to have a degree higher than $\bar{d}$. Then in any configuration, it can achieve at most the payoff it gets as if it had k_H many counterparties with degree larger than $\bar{d}$, and on top of that, there is no second-order counterparty risk. This payoff is achieved in this configuration. Moreover, note that, in any other configuration, either it would have less counterparties with degree larger than $\bar{d}$, or among peripheral firms, it would not be in cliques. The sorting argument of the main theorem and Proposition 3 implies that the only strong stable configuration among low types is that they form cliques of order $d_L^* - k_H + 1$ among each other, while they are all connected to all high types, who are always happy to connect with low types anyway. □

2.5.3 Core-perihery with homogenous firms: costly bailouts

• *'Homogeneity'* is maintained. *'Baseline government'* is relaxed to allow for $\lambda > 0$.

In this subsection I illustrate how core-periphery structure is not an artifact of heterogeneity. The idea is as follows. When there are costs of bailing-out each firm, and the cost of bailing-out a bad firm is not significantly higher than that of bailing-out a good firm, government, in some instances, may find it optimal to bailout central bad firms instead of bailing-out its all too many good counterparties. Then if firms do not generate a topology that makes some firms more critical than others, typically, only the good firms would be bailed-out at an amount that makes them indifferent. If some firms arise to be critical, which would optimally get bailed-out even if they are bad firms, then the firms that are counterparties with the critical firms benefit from a reduction in first-order counterparty risk. All in all, some firms may want to downsize to become the periphery, in order to have a 'safe' core, leading to a core-periphery structure. I illustrate this possibility with an example. For the example that follows, suppose that cost and welfare does not depend on degree: $\lambda_\theta \equiv \lambda(d, \theta)$, $\Delta W_\theta \equiv W(C, d, \theta) - W(D, d, \theta)$ for all θ.

Example 12. *$k = 4$. $\alpha = 0.7$, $u(d) = \sqrt{d}$, $c(f) = (1.4) \times f$. $3\lambda_G > \lambda_B - \Delta W_B > 2\lambda_G$, $\Delta W_G - \lambda_G > 0$.*

Under this specification, $V(d)$ is maximized at 3. So in the absence of intervention, complete network K_4 is formed. In the presence of intervention, if bailing-out bad firms was too costly, so that $\lambda_B - \Delta W_B > 3\lambda_G$, only good firms would ever be bailed-out and again K_4 would be formed. But now that $3\lambda_G > \lambda_B - \Delta W_B$, in some instance, it may be the optimal choice to bailout a bad firm. In particular consider a star with 3 leaves.

Note that, $R(1) = 0$. If the center of the star gets a bad shock, and all leaves get good shocks, all leaves are facing indirect default. Without intervention, all will default. Then government may bailout each good leaf and increase welfare by $3(\Delta W_G - \lambda_G) > 0$. Or bailout the center, which induces leaves to continue as well, to increase welfare by $(\Delta W_B - \lambda_B) + 3\Delta W_G > 3(\Delta W_G - \lambda_G)$. Therefore, in case of the star network, if center gets a bad shock and leaves get good shocks, government bails-out the center.

Also, $R(2) = R(3) = 1$ and $\lambda_B - \Delta W_B > 2\lambda_G$. This means that there is no other situation among 4 firms that requires bailing-out a bad firm. In all other configurations and realizations, only and all good firms are bailed-out.

Therefore, the payoff of the center is $V(3)$ indeed. But now, the leaves enjoy the possibility that a bad counterparty gets bailed-out, hence reduce first-order counterparty risk. The payoff of the leaves are $\alpha(\alpha + (1 - \alpha)\alpha^2)(u(1) - c(0))$ which is strictly larger than $V(3)$. Hence, the star *network (core-periphery) is the unique strongly stable network among* 4 *identical firms.*

2.5.4 Interconnectedness: budget restrictions

• *'Homogeneity'* is maintained. *'Baseline government'* is relaxed to allow for $b < \infty$.

I argue in this subsection that limitations on transfers proportional to the amount, such as a budget constraint, may have adverse effects. Firms may now strictly prefer an interconnected network. This way, they can reduce first-order counterparty risk by benefiting from bailouts of bad counterparties which are executed for other good firms instead of themselves. For this purpose, I stick to homogenous firms.

When the budget is large enough, government makes sure bad firms default in order to avoid their unproductive operations. When the budget is possibly not large enough to bailout all indirect defaults, government is not able to achieve the 'first best' anyways. Then depending on the welfare cost of a bad firm continuing business, and the network structure, it may become optimal to bailout bad firms in order to reduce counterparty losses and indirectly help good firms stay in business.

For example consider a star network with d leaves. Suppose that the center gets a bad shock and the leaves get good shocks, but are all forced into default due to losses from the center. If the budget were to allow bailing-out all good firms on the leaves, government would do that. When budget is limited, it may very well become optimal to bailout the center only in order to achieve the second best: the welfare would be reduced by the cost of bad center staying in business, but all the leaves would continue as well.

For technical reasons, it is significantly harder to solve this problem for general parameter specifications. How many and which of bad and good firms should be bailed-out is difficult to find in closed form for an arbitrary network. Finding the network formed is even harder. In order to illuminate the effect of the necessity of bailing-out bad firms due to budget restrictions or costs, I make a simplifying but extreme assumption to focus on the case in which it is too 'expensive' to bailout good firms compared to bailing out bad firms, but it is still optimal to induce good firms to continue. That is, bad firms are bailed-out for the sake of good firms. For all d, d', f

$$|P(D, -, d, G) - P(C, f, d, G)| > b > (k-1) \times \{P(D, -, d', B) - P(C, 0, d', B)\}.$$

Also, for all d, d',

$$W(C, d, G) - W(D, d, G) > (d-1) \times [W(D, d', B) - W(C, d', B)].$$

Under this assumption, government **1)** cannot bailout any good firms, **2)** can bailout all bad firms, **3)** wants to bailout minimum number of bad firms to induce all good firms to continue.

Let $d+1 \in \mathbb{N}$ be called a **non-trivial order** if $R(d) \notin \{0, d\}$. Notice that if $R(d) = 0$, then any bad counterparty of a good firm with degree d would be bailed-out (and possibly some bad firms that are connected at a longer distance). Hence degree d is completely safe with respect to counterparty losses from bad firms. If $R(d) = d$, then no good firm with degree d can ever fail due to any counterparty losses anyway. If a firm has degree d where $d+1$ is trivial, then it is indifferent between any topology for the rest of the network. For nontrivial orders, the following holds.

Theorem 9. *In any Pareto strongly stable network, there exists at most one disjoint clique with a non-trivial order.*

If bailouts are anonymous[56]*, in any strongly stable network, there exists at most one disjoint clique with a non-trivial order and the members of the clique have strictly worse expected payoff than any other firm (except at most one firm) in the network.*

Proof. Fix an optimal policy T. Within a clique of order $d+1$, if $R(d)$ or less firms get

[56] Anonymity only means playing a mixed strategy when there are indifferences. It is natural to think of bailouts as not treating identities too differently and picking one name with probability 1 out of all other equally well options with other names.

bad shocks, government does not intervene; all good firms continue and all bad firms default. If $f \geq R(d)+1$ or more firms get bad shocks, then government bails-out exactly $f - R(d)$ many bad firms. It can be any mixture over $f - R(d)$ element subsets of f many bad firms. Thus the following is the payoff of a firm in a clique with order $d+1$:

$$\tilde{V}(d) := \mathbb{E}_\theta \left[\max \left\{ P(C, \min\{|\{n_j : \theta_j = B\}|, R(d)\}, d, G) \, , \, P(D, -, d, \theta_i) \right\} \right].$$

Take any non-trivial $d+1$. Consider the following subgraph of $d+3$ firms: a disjoint $d-$regular subgraph which has its complement given by a Hamilton cycle among the $d+3$ firms. Now I show that each $d+3$ firms get strictly more than $\tilde{V}(d)$ in this configuration. Enumerate the firms along the Hamilton cycle as $n^1, n^2, ..., n^{d+3}$. Take a firm, say n^1. If n^1 gets L, its payoff is $P(D, -, d, L)$ in any case. If it gets a good shock, and $f \geq R(d)+1$ many of its counterparties get bad shocks, then government bails-out at least $f - R(d)$ of its counterparties, so that its payoff is at least $P(C, R(d), d, G) > P(C, f, d, G)$. If it gets a god shock and $f \leq R(d)$ many of its counterparties get bad shocks, then government may or not may not bailout some bad counterparties, and in any case, n^1's payoff is larger than $P(C, f, d, G) > P(C, R(d), d, G)$. Therefore, conditional on any event, n^1's payoff is at least the same, so the expected payoff is at least $\tilde{V}(d)$. Then it suffices to find a positive probability event under which n^1 gets strictly higher payoff than before.

Consider the event that n^1 gets G; $n^2, n^3, ..., n^{R(d)+2}$ get B; $n^{R(d)+3}, ..., n^{d+3}$ get G. Under this shock profile, only $R(d)$ many counterparties of n^1 get B: $n^3, ..., n^{R(d)+2}$. However, n^{d+2} has exactly $R(d)+1$ many counterparties that got B: $n^2, n^3, ..., n^{R(d)+2}$. Due to optimality, exactly one of $n^2, n^3, ..., n^{R(d)+2}$ must be bailed-out so that n^{d+2} can play C. Since d is non-trivial, $R(d)+2 \geq 3$. Government is indifferent between bailing-out n^2 and $n^{R(d)+2}$ so by anonymity of the bailouts, government cannot play a pure strategy that bails-out n^2. Then there is a positive probability that one of $n^3, ..., n^{R(d)+2}$ gets bailed-out. This event gives n^1 at least $P(C, R(d)-1, d, G)$, which is strictly larger than $P(C, R(d), d, G)$. Hence, n^1's payoff is strictly improved. (Without anonymity, government can play a pure strategy. If n^2 is bailed-out, $n^{R(d)+3}$ gets strictly better off. If $n^{R(d)+2}$ gets bailed-out, n^1 gets strictly better off. If one of $n^3, ..., n^{R(d)+1}$ gets bailed-out, both n^1 and $n^{R(d)+3}$ get strictly better off. Hence, this is a Pareto improvement on top of $\tilde{V}(d)$ payoffs.)

Now suppose that there are two non-trivial cliques, say with orders $d+1$ and $d'+1$. W.l.o.g. let $\tilde{V}(d) \geq \tilde{V}(d')$. Then two firms from $(d'+1)$-clique and all firms from the $(d+1)$-clique can form the previously mentioned configuration and get strictly better off. (Without anonymity, get a Pareto improvement). Contradiction: there can be at most one non-trivial clique. Furthermore, there cannot be two other firms in the network that both have less than or equal to $\tilde{V}(d)$ payoff due to the same argument. □

2.5.5 Interconnectedness: costs of forming links

- *'Homogeneity'* and *'Baseline government'* are maintained.

Note that the payoff function P encodes the costs of forming links as well. Here I illustrate how small additional costs of forming links can serve to break indifferences, and indeed make an interconnected network selected out by firms over the clique structure in the presence of bailouts. Suppose that there are very small additional ex-ante costs of links to the firms. Each link e_{ij} comes with a very small extra cost $c_{ij} = c_{ji}$ to both n_i and n_j. These additional costs $\{c_{ij}\}_{\{i,j\} \in [N]^2}$ are drawn jointly from a distribution $\mathcal{P}_\varepsilon$ with support $[0, \varepsilon]$ before the network is formed. All costs are common knowledge. Assume away integer problems: for $d^* = argmax_d V(d)$, k is divisible by d^*+1. There is second-order counterparty risk: $R(d^*) > d^* - 1$.

Proposition 26. *There exists $\varepsilon > 0$ such that for any $\mathcal{P}_\varepsilon$:*
In the absence of bailouts, any strongly stable network consists of disjoint cliques of order d^*+1.[57]
Each such network is d^*-stable.[58]

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[57] This is necessary, not sufficient.

[58] t-stable: There is no profitable deviation by coalitions with t or less number of members. See Erol and Vohra (2015) for more on this solution concept.

In the presence of bailouts, consider the following algorithmic construction. Start with, and keep taking the smallest remaining c_{ij} and form the link e_{ij}, if that does not make any firm exceed degree d^*. If it does, skip and move on to the next smallest c_{ij}. Stop when no more links can be added. The resulting network is almost-d^*-regular and strongly stable.

Proof. Let $\varepsilon < \frac{1}{k} min_{d \neq d', d, d' < k} |V(d) - V(d')|$. Then the lexicographic priority for firms is to get $V(d^*)$ modulo link costs. That needs having d^* counterparties with no second-order counterparty risk. Following the proof of Theorem 6, if there are no profitable deviations, network must consist disjoint cliques of order $d^* + 1$.

Take any such network. Suppose that a group of firms have a profitable deviation. In this deviation, they all must be getting $V(d^*)$ modulo some reduced link costs. Then each deviator has degree d^*, and deviators who are counterparties have the identical set of counterparties. Take a component of deviators after the deviation. They all must have the identical set of counterparties. If these deviators have a common non-deviator counterparty, that means they were all in the same clique before the deviation, and kept all links across each other. Then if they cut some links with non-deviators within their clique, their degree is now strictly less than d^*. If they kept their links with non-deviators, but cut links with some other deviators, that their degree is again strictly less than d^*. Otherwise, they are still in the same clique with all links intact, meaning they actually did not deviate. Therefore, these deviators do not have a non-deviator counterparty: they all cut their links with non-deviator members from their old clique. That means these deviators are from various old cliques, and formed a genuine new clique of order $d^* + 1$. But that needs $d^* + 1$ deviators. Hence the network is d^*-stable.

In the presence of bailouts, following the algorithm, if the resulting network had two firms with degree less than d^*, the algorithm would form a link between them. Consider the network formed. Suppose that there is a subset of firms that have a profitable deviation. Take the firm n among the deviators that received its d^*'th link first from the algorithm among other deviators. Now this n must have degree d^* after the deviation as well, otherwise it can't achieve $V(d^*)$ modulo link costs. Hence, it must be replacing some links with some other links. Since this is a profitable deviation, there exists a link e that n cut which has more cost than a link n added, say e'. n can't add links with non-deviators, so all added links are with deviators. But then, the link e' would have already been assigned to n before the link e because neither n nor the counterparty incident to e had reached degree d^* when algorithm was passing through e', which is before e. □

This result is does not guarantee existence of strongly stable networks in the absence of intervention, rather ensures that a d^*-stable network exists. Consider the following sub-case which guarantees existence. Consider any probability distribution $\mathcal{P}^*$ with support 2^N. Each subset M of N is drawn with probability $\mathcal{P}^*(M)$. Members of M become cost-type μ_1 and members of $N \backslash M$ become cost-type μ_2. $c_{ij} = 0$ if n_i and n_j have different cost-types. $c_{ij} = \varepsilon$ if n_i and n_j have the same cost-types.

Proposition 27. *There exists $\varepsilon > 0$ such that for any $\mathcal{P}^*$ the following holds. Let M be the realized subset of μ_1 cost-types and let $m = |M|$.*

In the absence of intervention:

- *The following is the unique strongly stable network: disjoint cliques of order $d^* + 1$ such that each clique includes $\lfloor \frac{m}{k}(d^* + 1) \rfloor$ or $\lceil \frac{m}{k}(d^* + 1) \rceil$ many μ_1 types.*

In the presence of intervention:

- *(Sufficiency) The following is strongly stable. If $m \leq k/2$, μ_1 types have no edges among each other. Each μ_1 type has d^* edges with μ_2 types, distributed over μ_2 types in a way that each μ_2 type has degree $\lfloor \frac{m}{k-m} d^* \rfloor$ or $\lceil \frac{m}{k-m} d^* \rceil$. Then μ_2 types have a network among each other such that each of them has $\lceil \frac{k-2m}{k-m} d^* \rceil$ or $\lfloor \frac{k-2m}{k-m} d^* \rfloor$ degree. If $m \geq k/2$, similarly for the other direction.*

- *(Necessity) Each strongly stable network must have the property that if $m \leq k/2$, each μ_1 type has d^* many μ_2 counterparties and no μ_1 counterparties. If $m \geq k/2$, similarly for the other direction.*

When there are no bailouts, firms tend to form cliques despite the small additional costs of links. This is because second-order counterparty losses exceed the additional costs of links. However, in the presence of intervention, second-order counterparty risk is eliminated hence the network formed is dictated by the costs c_{ij} modulo regularity of the network. Random realizations of costs induce an interconnected network with much higher ex-ante probability than the clique structure.

Summarizing remarks of the section:

Remark 9: If it is optimal to subsidize already solvent firms through bailing-out their defaulting counterparties, such as using debt guarantees, first-order counterparty risk gets strictly reduced by connecting with systemically important firms. In return, peripheral firms strictly prefer connecting with core firms rather than connecting with peripheral firms.

Remark 10: Committing to bailing out only systemically important firms strengthens the incentives for core-periphery structure, and reduces the incentives for interconnectedness among the periphery.

Remark 11: Government induced core-periphery is not necessarily an artifact of heterogeneity. Indeed, with homogenous firms, some firms downsize to become the periphery, in order to force the government to bailout the core even when core gets bad shocks. This way, the periphery reduces its first-order counterparty risk.

Remark 12: When there are limitations on bailouts proportional to the amount of transfer, such as a budget constraint or per-unit costs of transfers, government may be forced into selectively bailing out bad firms. In return, firms seek to benefit from the bailouts of bad firms executed for the sake of other other good firms. Firms can reduce their first-order counterparty risk by especially making the network more interconnected. This suggests increased network hazard. ‘Committing’ to less spending on bailouts may not always help in terms of systemic risk.

2.6 Other extensions

2.6.1 Productive bad firms and costly bailouts

• *'Homogeneity'* is maintained. *'Baseline government'* is relaxed to allow for $\lambda > 0$ and $W(C,d,B) - W(D,d,B) > 0$.

Two other aspects of bailouts that I have mentioned are the per-firm costs of bailouts and the possibility that bad firms may still have some merit in terms welfare, such as when the shocks are pure liquidity shocks. I lump these two cases into one subsection since they are technically similar.

When a firm which would otherwise default is bailed-out, the change in welfare directly from this firm is given by $\zeta(d_i,\theta_i) = -\lambda(d_i,\theta_i) + W(C,d_i,\theta_i) - W(D,d_i,\theta_i)$. In order to avoid complicating the analysis with the dependence on degree I assume that λ and W, thus ζ, do not depend on d_i. The analysis can be extended to the general degree dependent case using the techniques in Sections 2.5.1 and 2.5.2. Henceforth, let $\zeta_\theta \equiv \zeta(d,\theta)$, $\lambda_\theta \equiv \lambda(d,\theta)$, $\Delta W_\theta \equiv \zeta_\theta + \lambda_\theta$ for all θ.

In the benchmark case, since bad firms hurt welfare, $\zeta_B < 0$. If bad firms do not hurt welfare, and actually contribute enough relative to the cost of bailing them out, then bad firms would also be bailed-out for their own sake. Then, government then can make sure, by bailing-out all bad firms, that all firms continue, independent of the cost of bailing-out good firms. Let $d^{\star\star}(k) = argmax_{d<k} P(C,0,d,G)$.

Proposition 28. *If $\zeta_B > 0$, optimal policy makes sure all firms continue. Pareto strongly stable networks consist of $d^{\star\star}(k)$-regular networks (modulo divisibility).*

Proof. Take any optimal policy. If it induced an action profile in which some firms default, then this policy could be improved by bailing out all bad firms that are not bailed-out. So the optimal policy, either by bailing-out good or bad firms, makes sure all firms continue.

Then a firms payoff is given by $P(C, 0, d, G)$. The rest is similar to previous cases. □

If ζ_B is not positive, then bad firms are not bailed-out for their own sake. The other extreme is $\zeta_B << 0$. That is, it is never optimal to bailout any bad firms. Now, the analysis is somewhat very close to the baseline case. The only difference is that now ζ_G now determines how welfare is affected by the costs of bailouts.

Proposition 29. *If $\zeta_B << 0$ and $\zeta_G > 0$, the optimal policy and the network formed is identical to the baseline case.*

Proof. Since ζ_B is very small, it is never optimal to bailout any bad firms. Hence any optimal policy induces all bad firms to default. Also, if there was any good firm left to default, by bailing it out, welfare would be directly improved by $\zeta_G > 0$, and possibly more through spillovers. Then the induced incentives for firms are also identical. □

The treatment of bail-out costs paves the way to a more detailed welfare analysis. If λ_G is close to zero, so that ζ_G is very close the welfare gain, welfare is improved. If, on the other hand, λ_G is very close to the welfare gain, so that ζ_G is very close to zero, then systemic risk is reflected onto the welfare as follows. Now for each bailout, welfare gain is ex-post almost 0 from the reduction in first-order indirect defaults. Then ex-ante, each excess bailout that the interconnectedness generates on top of what would be needed in a clique structure is an ex-ante welfare loss. The welfare loss in the absence of bailouts was proportional to the total number of indirect defaults, but now in the presence of bailouts, the welfare loss is proportional to the number of bailouts. Recall that anticipation of intervention increases spillovers: the expected number of bailouts under intervention is larger than the expected number of indirect defaults under no-intervention. Therefore, welfare decreases. For each selection of the regular network formed in the presence of intervention, there is a middle threshold value of the cost of bailouts so that the welfare decreases below the threshold and increases above the threshold. The threshold depends on the extent of contagion in each particular interconnected network that might be formed in the presence of intervention. Furthermore, note that the analysis here can directly be applied to 'Minimal heterogeneity' as well, if degree of H types do not increase welfare. Hence similar goes for core-periphery result. That is, if bailouts are sufficiently costly, but not too costly that bailouts are ex-post suboptimal, then systemic risk and volatility is reflected onto the welfare. If higher degree of H firms increases welfare, the analysis relies on the comparison between bailout costs of H types and welfare gains higher degree of H types.

The non-trivial tradeoff between bailing-out good or bad firms is the middle case of the comparison between ζ_B and ζ_G. Section 2.5.3 studies this case with an example. I leave the general treatment to future work.

2.6.2 Incompletely informed government

• *'Homogeneity'* and *'Baseline government'* are maintained.

Aside of the how second and first-order counterparty risk are affected by transfers, there is possibly another reason for why moral hazard might arise. If the government does not have precise information on the financial standing of firms, it may be unable to use ex-post minimal transfers, leading to some overcompensation. That in return generates incentives to exploit the expectation of overcompensation by becoming vulnerable to counterparty risk, on purpose.

For technical tractability, assume away screening problem. Let

$$P(D, -, d, B) - P(C, 0, d, B) > P(D, -, d, G) - P(C, d, d, G) > 0$$

for all d. This implies that a bad firm never finds it optimal to continue and mimic a good firm in order to benefit from transfers intended for indirect defaults. Clearly, this is a very strong assumption. I leave deeper treatment of incompletely informed government to future work.

Let $d^\star(k) := \arg\max_{d<k} V(d) + \alpha \times [P(D,-,d,G) - P(C,d,d,G)]$.

Proposition 30. *The optimal policy is to bailout all firms at an amount*

$$P(D,-,d,G) - P(C,d,d,G).$$

Only good firms utilize the bailout and continue. Unique (Pareto) strongly stable network structure is $(d^\star(k)+1)$-cliques or $d^\star(k)$-regular networks, depending on the resilience. Any almost-$d^\star(k)$-regular network is strongly stable. Any strongly stable network is approximately-$d^\star(k)$-regular.

Proof. The optimal action profile to induce is that all good firms continue and all bad firms default, if possible at all. By offering all firms $P(D,-,d,G) - P(C,d,d,G)$, this action profile would indeed be induced due to 'No need for screening'. Then the question is to induce this action profile using minimal transfers. Since, under any optimal policy, all bad firms default, and it is possible that a good firm n_i is surrounded by d_i many bad firms, and no amount less than $P(D,-,d,G) - P(C,d,d,G)$ can guarantee a good firm to continue.

Given the optimal policy, the expected payoff of a firm with degree d is given by $V(d)$ plus the expected transfer $\alpha T_i(C)$, which makes $d^\star(k) := \arg\max_{d<k} V(d) + \alpha \times [P(D,-,d,G) - P(C,d,d,G)]$. The rest follows the proof of previous theorems. □

Lemma 5. *Suppose that $P(D,-,d,G) - P(C,d,d,G)$ is increasing in d. Then $d^\star(k) \geq d^*(k)$.*

Proof. Increasing differences. □

2.6.3 Incompletely informed firms

•*'Baseline government'* is maintained.

Now I introduce incomplete information on the firm side into the model. Each firm observes the shocks to itself, its counterparties, and possibly some more firms. Formally, n_i observes the shocks to a subset $\mathcal{I}_i(E, \vec{\theta})$ of firms which includes n_i and all counterparties of n_i: $\{n_i\} \cup N_i \subset \mathcal{I}_i(E, \vec{\theta}) \subset N$.

Cooperating equilibrium is defined for complete information in the model for simplicity. Vives (1990) also shows that any Bayesian game with supermodular ex-post payoff functions has a maximal pure strategy Bayesian-Nash equilibrium. Thus cooperating equilibrium is identically defined.

Most theorems hold almost identically. The idea is that, under no-bailouts, if firms form cliques, their shocks are common knowledge across clique members. This allows them to enjoy $V(d, \gamma)$ for a $(d+1)$-clique, still. However, still under incomplete information, a firm with degree d cannot get more than $V(d, \gamma)$ in any other configuration of the network. Hence similar proofs apply. On the other hand, when there are government bailouts, a firm need not know anything more than his counterparties shocks, because government makes sure that all good firms play C and all bad firms play D. Then each firm still enjoys $V(d, \gamma)$ and still does not care about the network topology, so similar proofs apply again. Therefore the interconnection and core-periphery intuition is robust to incompletely informed firms, as long as every firm can observe its counterparties shocks.

2.6.4 Generalized cooperating equilibrium and transfers

• *'Baseline government'* is maintained.

The transfer function $\{T_i\}_{i \in N}$ can depend on the action profile in general. But then, the induced game in stage three may not be supermodular so that the cooperating equilibrium may not be well defined. Here I state the appropriate definition of cooperating equilibrium for games that are not supermodular, and show that the optimal policy indeed has the property that transfers to a firm depends only on its own action.

Consider any function $\{U_i^*\}_{i\in N} : \{C,D\}^N \times \{G,B\}^N \times [N]^2 \times \Gamma^N \to \mathbb{R}$. For fixed $(\vec{\theta}, E, \vec{\gamma})$, $\{U_i^*\}_{i\in N}$ induces a binary game. Let the set of Nash equilibria be denoted $\mathcal{N}(\vec{\theta}, E, \vec{\gamma})$. Since the game is possibly not supermodular, the set of Nash equilibria may not be a complete lattice, yet it is still a partial order.[59] Take *any* selection, $\sigma(\vec{\theta}, E, \vec{\gamma}) \in \mathcal{N}(\vec{\theta}, E, \vec{\gamma})$ such that there is not element of $\mathcal{N}(\vec{\theta}, E, \vec{\gamma})$ that ranks higher in the order. Call this selection the σ-generalized cooperating equilibrium.

Suppose that the firms play σ-generalized cooperating equilibrium, for some arbitrary selection σ. Quiet trivially, for any $(\vec{\theta}, E, \vec{\gamma})$, the profile in which all good firms firms play C and all bad firms play D must be induced. Otherwise, government could use the optimal transfer scheme of the baseline case, which would make sure the game is supermodular, so that any selection σ had to uniquely choose this profile. If, for any $(\vec{\theta}, E, \vec{\gamma})$, this action profile is being induced, following the proof of Proposition 21, the transfers must be at least as much as T^* in order to induce this profile. Then minimality requires that T^* is still the unique optimal policy. T_i^*, conditional on $(\vec{\theta}, E, \vec{\gamma})$, depends only on the action of n_i.

Summarizing remarks of the section:

Remark 13: If the effective cost of bailouts are large, welfare gains diminish and systemic risk reflects onto welfare.

Remark 14: The lack of precise information on government's side is another source of moral hazard. It leads to overcompensation and overconnection. Interconnectedness remains.

Remark 15: The lack of precise information on firm's side do not have dramatic effects in my model. As long as firms can observe the financial situation of their immediate counterparties, there is no change since no firm wants to have a second-order counter-

[59]For mixed strategies, the order is analogously defined using the probabilities on higher actions.

party even under complete information. As for the case of intervention, second-order counterparty is eliminated by bailouts, so firms don't need the corresponding information.

2.7 Discussions

2.7.1 Other interpretations of the model

Here I discuss how the model can be interpreted in other ways. These are not formal microfoundations, rather illustrations of the main forces.

Origination and diversification of risk and investment Each firm has one proprietary investment which has to be managed by the firm and can't be sold,[60] and a non-proprietary project.[61] The firm can sell parts of the non-proprietary project to each of its counterparties, in ε portions each.[62] Each firm initially has illiquid assets of worth $A(\gamma)$ and liabilities at amount $X(\gamma)$.

All returns from all investments realize after stage three, in "stage four". Liabilities also must be paid in stage four. Each non-proprietary project returns $r \sim F$, where F has support $(\underline{r}, \bar{r})$. Each proprietary project returns $R(G,\gamma) = \bar{R}(\gamma)$ with probability α and $R(B,\gamma) = 0$ with probability $1-\alpha$. The information on the proprietary projects is revealed in stage two.

If a firm defaults in stage three, it liquidates its illiquid assets and its proprietary project at ratio $\phi_1 < 1$, so it gets $\phi_1(A + R(\theta,\gamma))$. The liquidation value of its shares of non-proprietary projects are 0. The non-proprietary project that it originated fails and

[60] Think of this project as having high cost of monitoring and operating. If sold, originating firm does not exert the effort to monitor the project, so that the project fails. In return, no firm wants to buy shares of this asset.

[61] These have smaller costs of monitoring so other firms are willing to buy this asset.

[62] This quantities can be thought of as the scale of the project.

returns 0 to other firms that holds shares of it. Therefore,

$$P(D,d,f,\theta,\gamma) = \phi_1(A(\gamma) + R(\theta,\gamma)) - X(\gamma).$$

Firms that continue in stage three move onto stage four. After the returns from non-proprietary projects are realized, if a firm's liquid assets can pay for its liabilities, it is solvent. Thus, for $\psi = (1 - d\varepsilon)r_i + \varepsilon \sum_{j \in N_i, a_j = C} r_j$, if $\psi + R(\theta,\gamma) - X(\gamma) \geq 0$, firms payoff is $A(\gamma) + \psi + R(\theta,\gamma) - X(\gamma)$. Otherwise, it goes bankrupt, and has to liquidate its illiquid assets at rate $\phi_2 < \phi_1$. Accordingly, its realized payoff is given by $\phi_2 A(\gamma) + \psi + R(\theta,\gamma) - X(\gamma)$. All in all, its payoff is $A(\gamma) + \psi + R(\theta,\gamma) - X(\gamma) - (1 - \phi_2) \times A(\gamma) \times \mathbb{1}_{\psi < X(\gamma) - R(\theta,\gamma)}$. Note that in stage three, ψ is a random variable which depends on d and f. Denote the distribution of the random variable $\psi(d,f)$. Then the expected payoff in stage three is

$$P(C,d,f,\theta,\gamma) = A(\gamma) + R(\theta,\gamma) - X(\gamma) + \mathbb{E}[\tilde{\psi} - (1 - \phi_2) \times A(\gamma) \times \mathbb{1}_{\tilde{\psi} < X(\gamma) - R(\theta,\gamma)}]$$

where $\tilde{\psi} \sim \psi(d,f)$. Notice that by buying assets originated by other firms, firms diversify against the risk of losing the full value of their illiquid assets A.

Notice that $P(C,d,f,\theta,\gamma)$ is strictly decreasing in f and $P(D,d,f,\theta,\gamma)$ is constant in f. Hence Assumption 6 is satisfied. As for Assumption 7, assume that $\bar{r} < X(\gamma) - A(\gamma)$ for all γ. That is, a firm with a bad shock is certain to go bankrupt if it continues. Hence, by continuing, it would get at most $\phi_2 A(\gamma) - X(\gamma) + \bar{r}$. Assume that $\bar{r} < (\phi_1 - \phi_2)A(\gamma)$ for all γ. Thus, a firm with a bad shock strictly prefers to default. Therefore Assumption 7 is also satisfied.

Diversification against liquidity risk This interpretation follows the lines of Holmstrom and Tirole (1997) and Allen and Gale (2000). Firms are financial institutions. Each firm has one risky project. There is another risk that a project may require additional investments before its maturity. The liquidity shock is negatively correlated: at most one bank receives this shock. Links are credit lines across firms in order to insure each other against the liquidity risk.

There is a cost x of forming each link with other firms, such as search costs or management costs. Each credit line has a limit 1 unit of credit. After credit lines are formed in stage one, information about the productivity of projects arrive in stage two. Each project returns $R(G, \gamma) = \bar{R}(\gamma)$ with probability α and $R(B, \gamma) \approx 0$ with probability $1 - \alpha$, which is due the end of stage four. In stage three, each firm chooses to continue managing its project or default. The cost of effort for continuing to manage the project is y. If effort is not exerted, project fails and returns 0. In stage four, at most one project is chosen to randomly requires an additional $\rho \sim F$ unit of investment, where F has support $(0, \infty)$. The probability of being selected is $\varepsilon < 1/k$. Each firm that continued has access to 1 unit of deposits, which can be lent out via the credit lines. If a project that belongs to a firm that has chosen to continue requires additional investment, then the firm can use its credit lines. If it can't find ρ units of additional funds, it does not borrow any and defaults. Project fails. Otherwise, it provides the additional funds and project returns additional ρ on top of $R(\theta, \gamma)$ which the firm uses the pay back its lenders. Thus, in stage four, a firm that continued which received the liquidity shock has payoff given by $-dx - y$ if $d - f + 1 < \rho$ and $R(\theta, \gamma) - dx - y$ if $d - f + 1 \geq \rho$. Firms that did not receive the liquidity shock has $R(\theta, \gamma) - dx - y$ payoff. Therefore, in stage three, a firm that continued has expected payoff

$$P(C, d, f, \theta, \gamma) = R(\theta, \gamma) - dx - y - \varepsilon(1 - F(d - f + 1))R(\theta, \gamma).$$

A firm that defaults in stage three gets $P(D, d, f, \theta, \gamma) = -dx$.

$P(C, d, f, \theta, \gamma)$ is strictly decreasing in f and $P(D, d, f, \theta, \gamma)$ is constant in f. Moreover, $P(D, d, -, B, \gamma) = -dx > -dx - y > P(C, d, f, \theta, \gamma)$. Therefore both Assumptions 6 and 7 are satisfied.

Intermediation and allocation of funds This interpretation follows the lines of Moore

(2011).[63] Firms under this interpretation are banks. Each bank has one proprietary project. Projects return small dividends over time, and finally a lumpsum return at the end of the project. Each project occasionally receives an opportunity to scale up the investment. There is a flow of depositors to each bank, who ask for dividends to keep their deposits in the bank. Banks can channel these deposits to other banks that have an opportunity to scale up their investments. The network serves the purpose intermediating and allocating funds.

Formally, there are $\bar{t}$ mini-periods of length $1/\bar{t}$ between stage one and two. Consider one bank and its proprietary project. The projects starts off at scale s_0. A project at scale s_t in mini-period t pays εs_t dividends in that mini-period. At stage two, there are no more opportunities left to rescale the investment and the scale is fixed at $s_{\bar{t}}$. The final gross rate of return $R(\theta)$ is determined via the shock θ. It takes value $R(G) = \bar{R}$ with probability α and $R(B) = 0$ with probability $1 - \alpha$. Thus, in stage three, the project returns $R(\theta)s_{\bar{t}}$.

Depositors ask for ε unit of dividends for 1 unit of dividends every mini-period. Therefore, all mini-period deposits are channeled to depositors. Moreover, bank promises r^H gross rate of return per 1 unit of deposits at stage three due the final gross returns from projects.

The dynamics of mini-periods are as follows. At each mini-period, exactly one bank randomly receives an opportunity to scale up its investment, whereas each bank receives one new depositor with $x = Xk/\bar{t}$ units of deposits. If the bank can promise εx dividends each mini-period and $r^H x$ in stage three, depositors deposit their money. Otherwise, they leave.

Banks can issue interbank bonds to borrow from other banks. The incompleteness in the market is that a bank can borrow only from banks that it has links with. Interbank bonds promise ε units of dividends per mini-period and r^I units of return in stage three

[63] Farboodi (2015) and others also have models that examine interbank lending between trading partners. Afonso et al. (2013) document existence of these partnerships.

per 1 unit borrowed.

In stage two, banks can default or not. Deposits are fully insured by the government, hence it is optimal for depositors to keep their assets in the bank in order to enjoy dividends, if the bank can promise dividends each mini-period.

All in all, in each mini-period, all dividends flow back to depositors. Through the interbank bonds, banks build exposures to each other. A bank that continues business in stage two receives a return $R(\theta)s_{\bar{t}}$, pays back r^I rate of return to its lender banks, receives r^I rate of return from its borrower banks that has not defaulted, pays back r^H rate of return to its depositors.

Take $\bar{t} \to \infty$. Each bank has the rescaling opportunity $1/k$ portion of the mini-periods, at each of which it borrowed $Xk/\bar{t}$ from each of its counterparties. Hence, in total, the interbank bonds it has issued binds it to pay $r^I X$ to each of its counterparties. Each bank has borrowed from depositors each time any of its counterparties or itself had the opportunity to rescale, hence, it owes the depositors $r^I X(d+1)$. As a result, it scaled its project up by $X(d+1)$; X directly from its own depositors, and Xd with funds from its counterparties.

A bank that defaults walks away from all obligations, pays nothing and gets 0 payoff. Therefore, a bank that continues business receives a payoff

$$P(C,d,f,\theta) = R(\theta) \times (s_0 + X(d+1)) + (d-f)r^I X - dr^I X - r^H(s_0 + X(d+1))$$

$$= (R(\theta) - r^H)(s_0 + X(d+1)) - fr^I X.$$

Notice that $P(C,d,f,\theta)$ is strictly decreasing in f and $P(D,d,f,\theta) = 0$ is constant in f. Moreover, $P(C,d,f,B,\gamma) = -r^H(s_0 + X(d+1)) - fr^I X < 0 = P(D,d,f,\theta,\gamma)$. Therefore, both Assumptions 6 and 7 are satisfied.

Joint projects and syndication of loans This interpretation follows the lines of Erol and Vohra (2014). Firms are financial institutions take undertake joint and safe projects. Projects are initiated and funded by deposits in stage one, with an interest rate $r > 1$ promised due the realization of returns from projects. Each project requires x units of investment from each from each counterparty. Projects also have to be managed by costly effort which has to be exerted at the end of stage two. Cost of effort for each firm is $y(\theta, \gamma) < X$ per project undertaken. Each project returns X if the project succeeds. Projects succeed if and only if both counterparties exert effort. Project requires expertise of each counterparty, hence project fails if any counterparty fails.

If a firms continues business, its payoff is

$$P(C, d, f, \theta, \gamma) = (d - f)(X - y(\theta, \gamma)) - dxr.$$

If a firm defaults, all of its projects fail, and walks away with an outside option

$$P(D, d, f, \theta, \gamma) = 0.$$

Notice that $P(C, d, f, \theta, \gamma)$ is strictly decreasing in f and $P(D, d, f, \theta, \gamma)$ is constant in f. Assume that $y(B, \gamma) > X - rx$ so that $P(C, d, f, B, \gamma) \leq d\,(X - y(\theta, \gamma) - xr) < 0 = P(D, d, -, \theta, \gamma)$. That is, Assumptions 6 and 7 are satisfied.

2.7.2 Future work

Aside of government intervention, ex-ante regulation is another important avenue in the context of network formation.

On top of the bailout question, there is also the *bail-in* question. Transfer could be negative, meaning that government can force some firms into chipping in for bailouts of others. This would stress the insurance aspect of intervention as well. In this case, firms need to take into account their expected losses due to transfers to troubled institutions.

In the simple form of incompletely informed government, I assumed that government has no information. However, government can indeed acquire information. This might have time costs or monetary costs. *Costly information acquisition* on government's side, and how that information would be shared with firms as a *signaling* tool can be another aspect of government's main considerations.

The main emphasis in this paper is the network formation. A deeper treatment of the mechanism design part is an important avenue for future research.

2.8 Conclusion

This paper provides a framework to study contagion with endogenous networks, and the adverse effects of government intervention with the market. It can be seen as a first attempt at mechanism design with endogenous network externalities, and can be applied to various other settings. The model is tractable and has wide room for generalizations and extensions.

In the benchmark case of no intervention, second-order counterparty risk generates a market discipline and leads to the formation of isolated and dense small clusters. Intervention, in form of bailouts, however, eliminates second-order counterparty risk as a byproduct. Hence the presence of bailouts dissolves the clusters into an interconnected network. Firms do not overconnect or underconnect, they just interconnect. Moreover, firms do not make riskier individual investment choices. Interconnectedness makes the system more susceptible to various measures of systemic risk. Moreover, when second-order counterparty risk vanishes, some firms become systemically important. This way, the network becomes a core-periphery network. Core serves as both a buffer against and an amplifier of contagion, making very good and very good outcomes more likely, generating volatility. It is notable that core-periphery structure can emerge as a consequence of intervention even for homogenous firms. When there are restrictions on government with regards to how bailouts can be executed, the incentives for forming interconnected networks and forming core-periphery networks strengthen. These results suggest that the presence of bailouts might indeed be contributing to the observed interconnectedness of the financial system and the core-periphery structure of financial networks.

Several questions are immediate. Can prevention in form of capital and liquidity requirements particular to systemic-importance undo the network moral hazard (designer at both stages) ? If government can use 'bail-in's forcing good firms to chip in to the bailouts, what kinds of effects would emerge on the network (negative transfers) ? In case of incomplete information, what is the best way to acquire information for government (costly information acquisition) ? I leave these to future research.

Chapter 3

Network Reactions to Banking Regulations

This chapter is co-authored with Guillermo L. Ordoñez.

Abstract

The optimal level of liquidity requirements for a single bank should balance a delicate trade-off. Tighter requirements reduce the chances of liquidity shortages and bank runs. Looser requirements foster the allocation of funds towards productive investments. With multiple banks, however, characterizing the optimal level of liquidity requirements is not straightforward. Banks form lending partnerships in the overnight interbank lending market to satisfy these requirements. We show that the interbank network suddenly becomes highly interconnected around a critical level of liquidity requirements, with discontinuous changes in systemic risk and welfare. Further, we show that the effects of discontinuity can be partly undone by using capital requirements in conjunction with liquidity requirements.

3.1 Introduction

Banks provide a key intermediation function in the economy. They extend loans to individuals with productive investment opportunities using funds obtained from savings of individuals without access to those opportunities. If a bank's investments fail depositors may loose their savings – a solvency failure. While solvency failures are not a source of inefficiency, their possibility may trigger bank runs and liquidity failures – when depositors fear insolvency, they may run on the bank to withdraw their funds, forcing the bank to liquidate assets and fail, even when the bank would have been solvent in the absence of a run.

To discourage bank runs, the government usually provides deposit insurance. In principle, in the absence of insolvency the deposit insurance is not to be used. The provision of deposit insurance, however, subsidizes the cost of funds for the banks and induce them to over-extend loans, up to a point of inefficient investments and excessive risk taking, increasing the likelihood of solvency failures. This implies that deposit insurance ends up being used excessively, generating a social cost in terms of distortionary taxation.

To counteract this effect, the government also tends to impose liquidity requirements, which put a direct upper bound on the amount of loans that can be extended by a bank per U.S. dollar kept physically in possession of the bank. Too tight liquidity requirements would reduce excessive risk-taking but may end up choking the efficiency of banks in channeling funds to productive investment opportunities. When these two forces change smoothly in the level of liquidity requirements for an individual bank, it is possible to obtain the optimal regulation level for liquidity.

This logic is very sound when considering a single bank but may fail when considering multiple banks that interact with each other. Banks react to liquidity requirement by making low interest loans to each other in the overnight interbank lending market and covering liquidity shortages. After extending loans for a lucrative investment opportunity, a bank may fall short of the liquidity needed to fulfill the imposed regulatory requirement and refer to other banks for short term loans in order to satisfy it. This implies that the level of liquidity requirements have a first order impact on the vibrancy and interconnectedness of the interbank market. How does the interbank network react to changes in liquidity requirements? How do liquidity requirements should be determined in a setting with interacting banks? How does the endogenous formation of interbank networks affect the welfare effects of liquidity requirements?

In this paper we show that the topology and the level of interconnectedness of the interbank network may react discontinuously to liquidity requirements. This reaction features a phase transition: beyond a tipping point of liquidity requirements, the network becomes disproportionately more interconnected, with systemic risk changing discontinuously in response to this abrupt change in the network architecture. While the marginal benefit of an additional counterparty increases as liquidity requirements loosens, its marginal cost is constant since it is determined purely by the probability the counterpart defaults. Once the benefit is larger than the cost, a bank would discontinuously prefer to increase its counterparties, interbank lending network becomes very dense and the aggregate level of interbank activity jumps radically.

This result has important welfare implications. We show that the highest welfare is achieved exactly at the tipping point, as around the tipping point, welfare also changes discontinuously. One can then argue that the liquidity requirement should simply be set at the tipping point. We claim, however, that the optimal distance between the regulatory liquidity requirements and the tipping point should balance the loss in welfare from being farther away from the tipping point and the gains from reducing the probability of the system moving beyond the tipping point (a crisis), which depends on the structure of shocks that affect the location of the tipping point. In this sense, financial crises may be the result of small changes in fundamentals and the optimal level of regulation may optimally allow for the possibility of financial crises to happen.

We finalize the paper by arguing that if capital requirements are used in conjunction with liquidity requirements, they can restrict the interconnectedness of the network when the system enters into a financial crisis zone with high systemic risk. The optimal level of capital requirements should balance the costs of raising capital during normal times and the gain from buffering the increase in systemic risk during crises. There will still be a discontinuous change during crises, but limited by capital requirements.

Allen (2014) makes a survey of the recent literature on liquidity regulation, concluding that "much more research is required in this area. With capital regulation there is a huge literature but little agreement on the optimal level of requirements. With liquidity regulation, we do not even know what to argue about." Some of this literature discusses theoretically what should be the optimal level of a bank's liquidity to prevent bank runs (see Cooper and Ross (1998) and Ennis and Keister (2006)). They do not study regulation as banks can hold enough reserves to avoid runs. Vives (2014) and Diamond and Kashyap (2015) introduce the need for regulation in the presence of private information about the bank's solvency, analyzing the optimal combination of capital and liquidity requirements. Farhi et al. (2009) study the role of liquidity requirements to prevent free riding that occurs when the liquidity of one bank reduces the likelihood of a run in another bank, Calomiris et al. (2014) in limiting risk-shifting, and Santos and Suarez (2015) in giving time for depositors to distinguish between liquidity and solvency problems. Finally Ordonez (2016) study the unforeseen effects of banking regulations on spurring shadow banks.

Even though there is also a recent literature on endogenous networks in financial markets, such as Erol and Vohra (2016), there is not systematic study to understand the effects of liquidity and capital requirements on shaping the network of interconnections between banks, and then their effects on systemic risk and welfare.

3.2 Model

3.2.1 Environment

There are k **banks.** Denote by $N = \{n_1, ..., n_k\}$ the set of banks. Each bank n_i has equity $q_i > 0$ and deposits $p_i > 0$. There are three stages in the benchmark model.

In **stage one**, government sets policy parameters $\phi \geq 1$ and $\psi > 0$. ϕ is called liquidity requirement and ψ is called capital requirement.

In **stage two**, the network formation stage, banks form **links** by mutual consent, that serve as mutual borrowing and lending partnerships. A formed link between agents n_i and n_j is denoted $e_{ij} = e_{ji}$, and the resulting set of links is denoted $E \subset [N]^2$. (N, E) is the realized **network**. Denote $N_i = \{j : e_{ij} \in E\}$ the set of **partners** of n_i and $d_i = |N_i|$ its **degree**.

After links are formed, each bank n_i extends l_i amount of loans to its outside borrowers. l_i is restricted by regulation ϕ and ψ, and the liquidity provided by depositors and secondary market trading. In reduced form, $l_i \leq max\{q_i \times \psi, (p_i + \zeta d_i) \times \phi\}$. The idea is that capital requirements prohibit the bank from making more than ψq_i loans and reserve requirements prohibit the bank from making more than ϕp_i loans. However, by the liquidity provided by secondary market trading, banks are able find more liquidity on demand, which serve to relax the reserve requirement to an extent, captured by ζd_i.

In **stage three**, each bank n_i receives a shock $\theta_i \in \{G, B\}$, G being the good shock and B being the bad shock. Shocks capture the productivity of borrowers of bank n_i. Productivity of each borrower is subject to both aggregate and idiosyncratic risk. Accordingly, shocks are correlated and determined as follows. With probability $1 - \sigma$, all shocks are bad capturing the aggregate risk. With probability σ, each shock is i.i.d., good with probability α and bad with probability $1 - \alpha$.

A borrower with a good shock returns Rl_i to bank n_i. A borrower with a bad shock returns rl_i, where $r < 1 < R$. Let $r = 0$. Banks need to pay depositors p_i back. Depositors are protected by deposit insurance and don't seek interest. Moreover, banks need to fulfill their interbank obligations arising from secondary market trading. Each bank pays each of its counterparties η. Banks cannot net out their loans ex-ante since it requires costly effort to evaluate complicated contracts with different types of assets and maturities, and to evaluate how these contracts would net out ex-ante. Ex-post, if two counterparties are solvent, they can pay each other and net out their loans η.

Aside of interbank claims, banks with bad shocks have at most $rl_i + \eta d_i$ liquid assets from their counterparties, yet are liable for $p_i + \eta d_i$. Therefore, they are illiquid and have to liquidate their indivisible illiquid equity at a loss, and receive γq_i. For simplicity, $\gamma = 0$. Since $rl_i + \gamma q_i < p_i$, bank is insolvent, but the banker is protected by limited liability. Thusly the bank has payoff 0. For simplicity, lump illiquidity and insolvency all under the name default, and denote $a_i = D$.

After shocks, banks continue (C) or default (D). Defaulting banks receive 0 payoff. Bank n_i with a good shock $\theta_i = G$ that continues receives $(R-1)l_i - \eta \times d_i^D$ where d_i^D denotes the number of defaulting counterparties of n_i and $l_i = max\{q_i\psi, (p_i + \eta d_i)\phi\}$. Notice that the game after the network formation and shock realization is supermodular in the action profile a.

3.2.2 Solution concepts

In **stage three**, banks are playing a supermodular game over the already determined network and states. The solution concept is the **cooperating equilibrium**: it is the Nash equilibrium in which the largest set of agents, with respect to set inclusion, play D among all Nash equilibria. Due to supermodularity, this equilibrium notion is well-defined. Supermodularity, by Tarski's Theorem, implies that the set of Nash equilibria is a complete lattice, and the cooperating equilibrium is the highest element of the lattice.

An alternative definition of the cooperating equilibrium can be given via a **rationalizable contagion** argument, which has a natural interpretation in financial contagion setup. Banks who receive bad shocks are insolvent and find it strictly dominant to play default. After some banks become insolvent in this way, some solvent banks who are tightly connected to insolvent banks will become illiquid, and find D iteratively strictly dominant, and so on. The iterated dominance argument resembles the financial contagion black-boxed into a single period. Along the iteration, at the point that remaining banks can rationalize C, the contagion stops. The resulting profile is the rationalizable strategy profile in which anyone who can rationalize C do play C. Supermodularity ensures that this profile is also a Nash equilibrium.

In **stage one**, agents evaluate network with the expectation of their payoffs in the cooperating equilibrium in stage three. Agents form a strongly stable network which is defined as follows. Consider a candidate interbank network (N, E) and a subset N' of agents. A **feasible deviation** by N' from E is one in which **1)** N' can add any missing links or cut any existing links that stays within N', **and 2)** N' can cut any of the links between N' and N/N'. A **profitable deviation** by N' from E is a feasible deviation in which the resulting network yields strictly higher expected payoff to every member of N'. An interbank network (N, E) is **strongly stable** if there are no subsets of N with a profitable deviation from E.

This network formation solution concept can be microfounded by a proposal game. Each bank pays $c > 0$ to make a proposal to another bank. Mutual proposals turn into links, and cost c is refunded to both. One-sided proposals do not turn into links, and c is lost. After being formed, links represent secondary market trades that serve to provide liquidity to banks. The strong Nash equilibria of this game corresponds to strongly stable networks.

3.3 Regulation

3.3.1 Reserve requirements

Let's ignore the capital requirements for now: $\psi = \infty$. Also let $\eta = 1$ for now. $l_i = (p_i + d_i)\phi$, where $\phi \in [1, \phi^*]$. Government, at the beginning of stage one, can set $\phi \in [1, \phi^*]$.[1] From this point on, all the relevant notation is indexed with ϕ. Define $\tilde{\phi} := \frac{(1-\alpha)\eta}{R-1}$, called the **tipping point** for ϕ.

Assumption 9. $\phi^* > \tilde{\phi}$.

This assumption is very mild and only says that life is not horrible for banks. By having one more interbank link, a bank, conditional on not being insolvent, is incurring an expected cost $(1-\alpha)\eta$ due to counterparty risk. On the other hand, this extra link allows the bank, at the feasible limit, to make ϕ^* more loan so that it makes $\phi^* r$ profit conditional on not being insolvent. Therefore, the assumption states that the expected gain from having one more interbank link, at the upper limit of loans a bank would be willing to make, is positive. Otherwise, the entire exercise technically still works through without any change, but it is less meaningful.

Lemma 6. *Suppose that* $\phi \neq \tilde{\phi}$.[2] *Then* $d^*_\phi < \infty$ *if and only if* $\phi < \tilde{\phi}$.

Lemma 7. *If* $\phi > \tilde{\phi}$ *then there exists* n^{**}_ϕ *such that* $V(d, \phi)$ *is strictly increasing for* $d > n^{**}_\phi$.

Theorem 10. *If* $\phi < \tilde{\phi}$, *the strongly stable network includes* $\lfloor \frac{n}{d^*_\phi + 1} \rfloor$ *disjoint cliques of order* $d^*_\phi + 1$.[3] *If* $\phi > \tilde{\phi}$ *then there exists* n^*_ϕ *such that, for* $n > n^*_\phi$ *the strongly stable network is the complete graph* K_n.

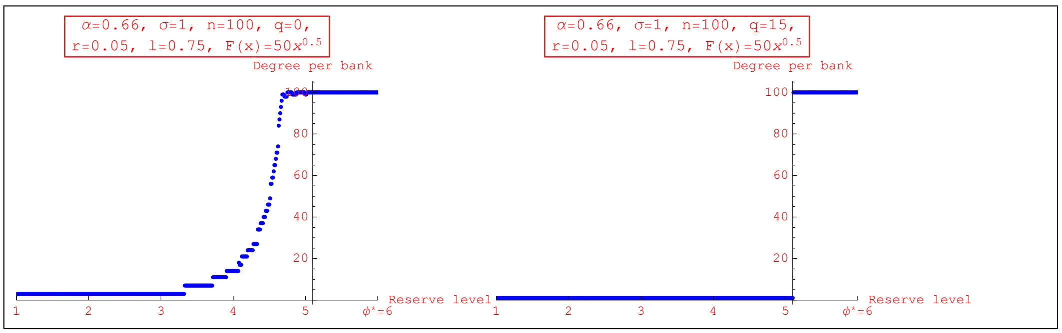
---

[1] This first tool of prevention here can be thought of as setting the reserve requirements $1/\phi$ and $\phi = \phi^*$ refers to no reserve requirement. Then it is natural to think of l then as being linear in reserve ratios ϕ. In this case, ϕ^* would be voluntary reserve ratio that banks indeed hold in order to protect themselves from bankruns, or the liquid they need for their daily operations.

[2] d^*_ϕ is the size of clusters formed under regulation ϕ. (See Erol and Vohra (2016) and Erol (2015) for details.)

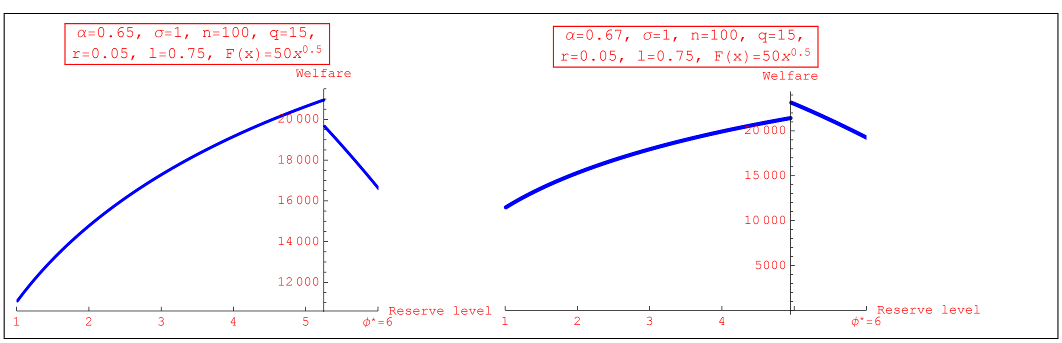
[3] If there is a residual, one or more additional smaller disjoint cliques whose orders can be found via Theorem 1 in Erol (2015).

Proof. Consider ϕ such that $\frac{\phi(R-1)}{\eta} + \alpha < 1$. Since $d^*_\phi = argmax_{d \in \mathbb{N}_0} V(d,\phi)$, along the sequence that defines sizes of the cliques, $d^*(n_t,\phi) = d^*_\phi$ as long as $n_t > d^*_\phi$. Thus, the first $\lfloor \frac{n}{d^*_\phi+1} \rfloor$ first order differences of elements of the sequence is $d^*_\phi + 1$. When $\frac{\phi R}{\eta} + \alpha > 1$, $V(d,\phi)$ is increasing for $d > n^{**}_\phi$. Also, $lim_{m\to\infty} d^*(m,\phi) = d^*_\phi = \infty$ meaning that V increases unboundedly. Thus there exists $n^*_\phi \geq n^{**}_\phi$ so that $d^*(k,\phi) = k-1$ for all $k > n^*_\phi$. Therefore, the order sequence of the strongly stable network has one element k. Banks form the largest clique they can: K_k. □



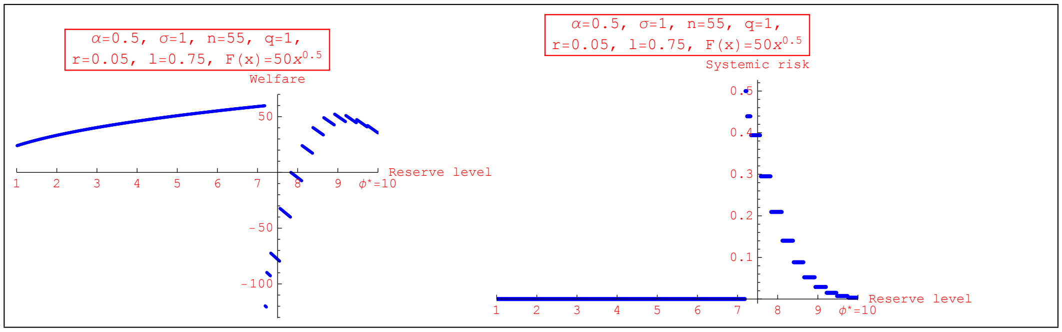
Remark 1. *This result stresses how the financial structure structure can be extremely sensitive to policy, in particular reserve requirements. The change in the network is discontinuous in the policy. This is suggestive to be careful about reserve requirements when they are the binding policy tools.*

Remark 2. *This is a phase transition property concerning the topology, even when networks are formed strategically without any apriori restrictions. This is the first such result to the best of our knowledge. This contributes to -robust-yet-fragile and tipping-point-property arguments in reaction to policy- by addressing the exogeneity issue in the network structure.*



Remark 3. *It is not clear in which direction welfare jumps around the tipping point. Around*

the critical level of , welfare can jump up or down. Therefore, neither relaxing, nor tightening the policy gives a robust outcome.



Remark 4. *The lack of robustness has another serious component regarding systemic risk. If banks are able to indulge in shadow banking type activities to relax the reserve requirement for themselves, they can effectively increase their ϕ . Then just beyond the tipping point, welfare and systemic risk can be affected dramatically by such unregulated activities. It might, then, be better to just relax the requirements all at once.*

3.3.2 Capital requirements

Capital requirements can be captured via a parameter $\psi \in \mathcal{G} \subset (1, \infty]$ as $l_i \leq q\psi$.[4,5] Hence government, by prenvention, chooses ψ to limit how much loan banks can extend as a function of their own resources, so that any bank n_i now invests $l_i = min\{q_i\psi, (p_i + \eta d_i)\phi\}$. The timing of the prevention is at the very beginning of stage one: (ψ, ϕ) is announced before the interbank network is formed.

For a fixed ϕ, l_i is not linear in d anymore due to the capital requirements. This partly breaks down the phase transition, and effectively eliminating the main source of instability in the network structure. Banks are not willing to form k links and be exposed to excessive counterparty risk since they cannot benefit from the upside of links due to binding capital requirements.

[4] $\mathcal{G}$ is a countable grid that spans $(1, \infty]$. This is solely due to divisibility issues arising from discreteness of the model. We make it explicit what the appropriate $\mathcal{G}$ is when it is needed.

[5] $\psi = \infty$ means there is no capital requirement.

3.3.3 Capital and Reserve requirements

Conjecture 13. *Reserve requirements increase systemic risk when capital requirements already bind.*

V (as a function of L of d) is supermodular in (L, ϕ) on an increasing subsequence of d that diverges. Also V in d is singlepeaked on this domain. Hence, when ψ binds, ϕ increases interconnectedness in order to induce the desired loan by banks. That increases the risk of default.

Appendix A

Appendix to Chapter 1

Proof of Proposition 6

Proof. (Sketch) By Cauchy-Schwarz inequality, the total expected payoff is less than

$$\sum_{i \in N} d_i \alpha^{2d_i},$$

which is less than $Nk^{**}\alpha^{2k^{**}}$. This bound is achieved if and only if the network is k^{**}-regular. □

Proof of Proposition 7

Proof. (Sketch)

1. A realized network is Nash if and only if the degree of all nodes is less than or equal to k^*.
2. Take a stable network. Let $\underline{d}$ be the smallest degree of any node in the network. If two nodes have degrees d_1 and d_2 such that $k^* + \underline{d} \geq d_1 + d_2 + 2$, then they are adjacent.
3. Any node with degree $d_i \leq k^* - 2$ must be adjacent to each node with degree

$\underline{d}$. Thus, there can be at most $\underline{d}$ nodes with degrees less than or equal to $k^* - 2$. ($\underline{d} \leq k^* - 1$).

4. Take a stable network. Take two nodes v', v'' with degrees at most d, which are adjacent to a third node with degree at least $d + 1$. Then v' and v'' are adjacent.
5. Nodes with degree at most $k^* - 1$ who have neighbors of degree k^* form a clique. Then, there can be at most $max\{m(k^* - 1 - m)\} \leq \left(\frac{k^*-1}{2}\right)^2$ nodes with degree k^* that have neighbors with degrees smaller than k^*.
6. If two nodes v', v'' have degrees k^*, and all their neighbors also have degree k^*, then v' and v'' are adjacent. Such nodes form a clique, so that there can be at most $k^* + 1$ many such nodes.
7. Bringing all the pieces together: there can be at most $k^* - 1 + \left(\frac{k^*-1}{2}\right)^2 + k^* + 1 = \left(\frac{k^*+1}{2}\right)^2$ many nodes with degree different from $k^* - 1$.

□

Proof of Proposition 8

Proof. (sketch)

1. All nodes with degree less than or equal to $k^* - 2$ form a clique.
2. There are at most $\left(\frac{k^*-1}{2}\right)^2$ nodes which have degree at most $k^* - 2$ or have neighbors with degree less than equal to $k^* - 2$. The remainder have degree at least $k^* - 1$ and all neighbors with degree $k^* - 1$ or k^*. Hence all these others have payoff at most $\frac{1}{\alpha}\left(k^* \alpha^{k^*}\right) \times \alpha^{k^*-1}$.
3. k^{**} is very close to $k^*/2$. Hence there are at least k^{**} people who would like to deviate and form an isolated clique. The core is empty.

□

Proof of Proposition 9

Proof. (sketch)

1. As nodes are already in cliques, no two non-adjacent nodes have a common neighbor. Hence, no node is willing to delete an edge to gain at most one other edge
2. No two nodes from disjoint cliques are willing to connect due to their already high degree.

□

Proof of Proposition 13

Proof. (sketch)

1. Any group has to be a clique.
2. All nodes of a C-group has to be adjacent to C.

□

Proof of Proposition 14

Proof. (sketch)

1. Let the C-groups be indexed by $t = 1, 2, ..., c$ and P_t be the probability that C plays B conditional on '-t has no bad edges'.
2. Among all C-groups, at most one can have $P_t < 1$.
3. Among all C-groups, at most one can have nodes less than d^*, all the rest have exactly d^* nodes.
4. If among all C-groups at most one has order less than d^*, all the rest have exactly d^* nodes, then $P_t = 1$ for all but at most one if and only if $c \leq s^*$.
5. If $c \leq s^*$, and all but one C-group has order d^*, then the remainder C -group is also of order d^*.
6. If all C-groups are of order d^*, and $c \leq s^*$, then $c = s^*$.
7. Among all NC-groups, at most one can have nodes less than d^*, all the rest have exactly d^* nodes.

□

Appendix B

Appendix to Chapter 2

All proofs of Chapter 2 are included within the chapter.

Appendix C

Appendix to Chapter 3

Lemma 1.

Proof. Let $f_0(k', n', p') = \binom{n'}{t}(1-p')^{n'-t}(p')^t$ and $\mathbb{F}_0(k', n', p') = \sum_{t=0}^{k'} \binom{n'}{t}(1-p')^{n'-t}(p')^t$ denote the pdf and cdf of the binomial distribution. Rewrite V as follows:

$$\frac{V(d,\phi)}{\alpha l} = \sum_{t=0}^{\lfloor R(d,\phi) \rfloor} \left\{ \binom{d}{t} \alpha^{d-t} (1-\alpha)^t \left(R(d,\phi) - t\right) \right\} ...$$

$$= R(d,\phi) \times \mathbb{F}_0\left(\lfloor R(d,\phi) \rfloor, d, 1-\alpha\right) - \sum_{t=0}^{\lfloor R(d,\phi) \rfloor} \left\{ \binom{d}{t} \alpha^{d-t} (1-\alpha)^t t \right\} ...$$

$$= R(d,\phi) \times \mathbb{F}_0\left(\lfloor R(d,\phi) \rfloor, d, 1-\alpha\right) - d(1-\alpha) \times \sum_{t=1}^{\lfloor R(d,\phi) \rfloor} \left\{ \binom{d-1}{t-1} \alpha^{d-t} (1-\alpha)^{t-1} \right\} ...$$

$$= R(d,\phi) \times \mathbb{F}_0\left(\lfloor R(d,\phi) \rfloor, d, 1-\alpha\right) - d(1-\alpha) \times \mathbb{F}_0\left(\lfloor R(d,\phi) \rfloor - 1, d-1, 1-\alpha\right).$$

Note that $\mathbb{F}_0(k', n', p') = \mathbb{F}_0(k'-1, n'-1, p') + (1-p') \times f_0(k', n'-1, p')$. Hence:

$$\frac{V(d,\phi)}{\alpha l} = [R(d,\phi) - d(1-\alpha)] \times \mathbb{F}_0 \left(\lfloor R(d,\phi) \rfloor - 1, d-1, 1-\alpha \right) + ...$$

$$R(d,\phi) \times f_0 \left(\lfloor R(d,\phi) \rfloor, d-1, 1-\alpha \right).$$

$R(d,\phi) = \left(q + r(q+d)\phi + ds\right)/l = d \times \frac{\phi r + s}{l} + \frac{q(1+\phi r)}{l}$, so that:

$$\frac{V(d,\phi)}{\alpha l} = d \times \left[\left(\frac{\phi r + s}{l} + \alpha - 1 \right) + \frac{q\,(1+\phi r)}{dl} \right] \times \mathbb{F}_0 \left(\lfloor R(d,\phi) \rfloor - 1, d-1, 1-\alpha \right) ...$$

$$+ R(d,\phi) \times f_0 \left(\lfloor R(d,\phi) \rfloor, d-1, 1-\alpha \right).$$

When $\frac{\phi r + s}{l} + \alpha - 1 > 0$, $\mathbb{F}_0 \left(\lfloor R(d,\phi) \rfloor - 1, d-1, 1-\alpha \right) \to 1$ as $d \to \infty$. Hence, $V(d,\phi) \to \infty$, so that $d^*_\phi = \infty$. When $\frac{\phi r + s}{l} + \alpha - 1 < 0$, there exists d^{**} such that for $d > d^{**}$ implies that $\left(\frac{\phi r + s}{l} + \alpha - 1 \right) + \frac{q(1+\phi r)}{dl} < 0$. So the $limsup$ of the first summand is non-positive. The second summand $R(d,\phi) \times f_0 \left(\lfloor R(d,\phi) \rfloor, d-1, 1-\alpha \right) \to 0$ as $d \to \infty$. $V(d,\phi)$ on the other hand is non-negative. Hence, $limsup_{d\to\infty} V(d,\phi) = 0$. Therefore, there exists a finite d^*_ϕ. □

Lemma 2:

Proof. Take any $\varepsilon > 0$.

$$\mathbb{F}_0 \left(\lfloor R(d,\phi) \rfloor - 1, d-1, 1-\alpha \right) > 1 - \varepsilon \iff \mathbb{F}_0 \left(d - \lfloor R(d,\phi) \rfloor, d-1, \alpha \right) < \varepsilon.$$

Recall the Chernoff Bound: $\mathbb{P}\left(X < (1-\delta)\mu \right) < \exp\left(-\frac{\delta^2 \mu}{2} \right)$, where μ is the mean of the random variable $X = \sum X_i$ for independent random variables X_i and $\delta \in (0,1)$ is a constant. So,

$$\mathbb{F}_0 \left(d - \lfloor R(d,\phi) \rfloor, d-1, \alpha \right) < \exp\left(- \frac{\left[(d-1)\alpha - (d - R(d,\phi) + 1) \right]^2}{2(d-1)\alpha} \right) ...$$

$$= \exp\left(-\frac{\left[\left(\frac{\phi r+s}{l}+\alpha-1\right)(d-1)+\left(2+\frac{q(1+\phi r)}{l}\right)\right]^2}{2(d-1)\alpha}\right).$$

Some algebra shows that this is less than ε if

$$d > \frac{\ln(\frac{1}{\varepsilon})}{\frac{1}{\alpha}\left(\frac{\phi r+s}{l}+\alpha-1\right)^2} - \left(\frac{2l+q(1+\phi r)}{\phi r+s+l(\alpha-1)}-1\right) =: d'_\varepsilon \in O\left(\ln\left(\frac{1}{\varepsilon}\right)\right)$$

around 0. Following identical steps, some similar $d''_\varepsilon \in O\left(\ln\left(\frac{1}{\varepsilon}\right)\right)$ guarantees

$$1 \geq \mathbb{F}_0\left(\lfloor R(d,\phi)\rfloor, d-1, 1-\alpha\right) > 1-\varepsilon.$$

Then $d > d''_\varepsilon$ implies

$$1 \geq \mathbb{F}_0\left(\lfloor R(d,\phi)\rfloor - 1, d-1, 1-\alpha\right) > 1-\varepsilon.$$

Then $d > d_\varepsilon := max\{d'_\varepsilon, d''_\varepsilon\} \in O\left(\ln\left(\frac{1}{\varepsilon}\right)\right)$ (around 0) gives $f_0\left(\lfloor R(d,\phi)\rfloor, d-1, 1-\alpha\right) < \varepsilon$.

Now $d > d_\varepsilon$ implies that

$$1 \geq \mathbb{F}_0\left(\lfloor R(d,\phi)\rfloor - 1, d-1, 1-\alpha\right) > 1-\varepsilon$$

and

$$0 \leq f_0\left(\lfloor R(d,\phi)\rfloor, d-1, 1-\alpha\right) < \varepsilon.$$

If, d also satisfies

$$d < d^+_\varepsilon = \frac{1}{\varepsilon} \times \frac{\phi r+s+l(\alpha-1)}{2\phi r+s+l(\alpha-1)} - \frac{2q\left(1+\phi r\right)+l(\alpha-1)}{2\phi r+s+l(\alpha-1)} \in O\left(\frac{1}{\varepsilon}\right)$$

then

$$\left[(d-1)\times\left(\frac{\phi r+s}{l}+\alpha-1\right)+\frac{q\left(1+\phi r\right)}{l}\right] + R(d,\phi)\times\varepsilon <$$

$$\left[d \times \left(\frac{\phi r + s}{l} + \alpha - 1\right) + \frac{q\,(1+\phi r)}{l}\right](1-\varepsilon)...$$

$$\implies [R(d-1,\phi) - (d-1) \times (1-\alpha)] + R(d-1,\phi) \times \varepsilon <$$

$$[R(d,\phi) - d \times (1-\alpha)] \times (1-\varepsilon) + R(d,\phi) \times 0$$

which implies that $V(d-1,\phi) < V(d,\phi)$. All in all, for all d such that $d_\varepsilon < d < d_\varepsilon^+$, one has $V(d-1,\phi) < V(d,\phi)$. Since $d_\varepsilon \in O\left(\ln\left(\frac{1}{\varepsilon}\right)\right) \in o\left(\frac{1}{\varepsilon}\right)$, and $d_\varepsilon^+ \in O\left(\frac{1}{\varepsilon}\right)$ around 0, as ε goes to 0 the union of regions $[d_\varepsilon, d_\varepsilon^+]$ cover a ray $[n_\phi^*, \infty)$ for some $n_\phi^* > 0$. Then V is increasing for $d \geq n_\phi^*$. □

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