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# Essays on the Macroeconomics of Labor Markets

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# Essays on the Macroeconomics of Labor Markets

## **Abstract**

This dissertation consists of two essays studying macroeconomics questions about labor markets. The research in this document is separated into chapters that study distinct features of aggregate labor market outcomes.

The first essay documents the change in behavior of fertility rate at business cycle frequencies in the United States between the 1970s and 1990s and shows how the cyclical and secular properties of fertility can be used to distinguish among several proposed theories that account for the rise in labor force participation of married mothers. The model, in which households make fertility, female labor force participation and asset accumulation decisions, is estimated using data for the 1960s and 1970s. The model shows how fertility and women's labor participation decisions are related and replicates countercyclical fertility. The changes in the determinants of female labor supply are introduced into the model and the implications for female labor force participation and properties of fertility are analyzed.

The second essay (co-authored with Marcus Hagedorn and Iourii Manovskii) studies the relation between taxes and the unemployment rate using the Mortensen and Pissarides search and matching equilibrium theory of unemployment. The proposed quantitative model with ex-ante worker skill heterogeneity and two technology shocks is consistent with a strong response of labor market variables to cyclical fluctuations in productivity and a relatively weak response to changes in tax rates. The model also matches the properties of group-specific labor market variables. The key to achieve these results is endogenous response of aggregate and group-specific productivities.

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## **Third Advisor**

Jesús Fernandez-Villaverde

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Sergiy Stetsenko

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

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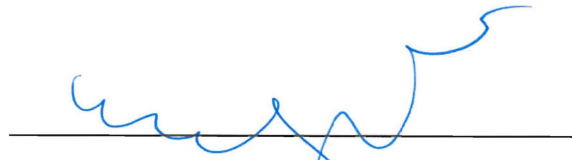
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Supervisor of Dissertation



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Graduate Group Chairperson

*For my parents: Alexander and Nina*

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## ABSTRACT

### ESSAYS ON THE MACROECONOMICS OF LABOR MARKETS

Sergiy Stetsenko

Iourii Manovskii

This dissertation consists of two essays studying macroeconomics questions about labor markets. The research in this document is separated into chapters that study distinct features of aggregate labor market outcomes.

The first essay documents the change in behavior of fertility rate at business cycle frequencies in the United States between the 1970s and 1990s and shows how the cyclical and secular properties of fertility can be used to distinguish among several proposed theories that account for the rise in labor force participation of married mothers. The model, in which households make fertility, female labor force participation and asset accumulation decisions, is estimated using data for the 1960s and 1970s. The model shows how fertility and women's labor participation decisions are related and replicates countercyclical fertility. The changes in the determinants of female labor supply are introduced into the model and the implications for female labor force participation and properties of fertility are analyzed.

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relatively weak response to changes in tax rates. The model also matches the properties of group-specific labor market variables. The key to achieve these results is endogenous response of aggregate and group-specific productivities.



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# Chapter 1

## Introduction

This dissertation consists of two independent essays on the macroeconomics of labor markets: “Female Labor Force Participation and Fertility” (Chapter 2) and “Taxation and Unemployment” (Chapter 3).

In the first essay I study the interaction between timing of fertility and female labor force participation decisions. There has been an increase in labor force participation of married women, especially women with young children, between the 1970s and 1990s, fact which has received substantial attention in the literature. Over the same period, cyclical and secular properties of fertility have changed significantly. In particular, I document that fertility is strongly countercyclical at business cycle frequencies in the 1960s and 1970s and procyclical thereafter. In addition, women have postponed childbearing substantially.

Using a life-cycle incomplete markets model with aggregate and idiosyncratic uncertainty, I show that cyclical properties and timing of fertility are related to labor force participation decisions of married women. The model calibrated to 1960s and 1970s generates countercyclical fertility. The intuition is the following: women with a high value of staying home prefer to have a child earlier, stay at home and save on child care costs; those at the margin between working and staying home prefer to have a child during a recession.

Women who are relatively more productive in the market prefer to have a child later and pay child care costs without leaving the workforce. They prefer to give birth during an expansion as a way to smooth consumption. The size of the latter group is relatively small in the 1960s and 1970s and the countercyclical effect dominates.

A number of explanations that have been proposed to account for the increase in female labor supply, in particular a decrease in the gender wage gap, an increase in women's returns to experience and a decrease in child care costs. All of these mechanisms either treat fertility as exogenous or do not model fertility at all. Nevertheless, fertility would have been affected by each of the proposed mechanisms had it been a choice. The objective of this work is to nest various mechanisms that explain the rise in married women's labor supply combined with fertility choice and to analyze their impact on properties of fertility.

I introduce these changes into the calibrated model and evaluate the implications for female labor force participation and properties of fertility. I find that each of them separately and all combined can explain some but not all features of the data. Taking into account the flattening of life-cycle earnings profile for males helps to account for the data but a significant discrepancy remains.

In the second essay we study the interaction between taxation and unemployment. A leading theory of equilibrium unemployment, the search and matching Mortensen and Pissarides model, has an important limitation for studying the effects of policies such as taxation. A one percentage point permanent decrease in productivity and a one percentage point permanent increase in labor income or sales taxes increase unemployment by the same amount. However, the data suggest that the elasticity with respect to productivity necessary to replicate business cycles is considerably larger than the elasticity with respect to taxes required to explain cross-country differences.

We propose a framework that can resolve this dilemma. The problem is that produc-

tivity is exogenous in the standard model and do not respond to changes in tax rates. We endogenize productivity allowing for ex-ante heterogeneity in skill (high and low) that interact on the production side of the economy.

We find that: 1) the model with ex-ante worker heterogeneity accounts well for the cyclical behavior of labor market variables in the aggregate and for each group, 2) the response of unemployment to changes in taxes is substantially reduced relative to the homogeneous model.



## Chapter 2

# Female Labor Force Participation and Fertility

### 2.1 Introduction

A fact that received substantial attention in the literature is that there has been an increase in labor force participation of married women, especially women with young children, between the 1970s and 1990s. For example, the employment rate has increased from 15% to 37% for married women with an infant and from 54% to 70% for married women without children under the age of 18 from 1970 to 1990.

A number of explanations have been proposed to account for the increase in the labor supply of married women between the 1970s and 1990s.<sup>1</sup> Jones, Manuelli, and McGrattan (2003) argue that the decline in the gender wage gap can explain the increase in the average hours of work for married women. Olivetti (2006) argues that the increase in women's returns to experience can account for the changes in women's hours of work. Attanasio, Low, and Sanchez-Marcos (2008) find that the reduction in the cost of children

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<sup>1</sup>A number of other studies focus on earlier period, for example, Greenwood, Seshardi, and Yorukoglu (2005) and Albanesi and Olivetti (2009) among others.

and in the gender wage gap combined can explain the increase in labor force participation of married mothers. All of these papers either treat fertility as exogenous or do not model fertility at all. Nevertheless, each of the proposed mechanisms would have had an impact on fertility had it been a choice. If women expect cyclical movements in their income and spend some time away from market work during pregnancy and upon birth, then parents prefer to time a birth when income is low. The procyclical response may result if households are liquidity constrained. Women who stay in the workforce when they have a child and outsource child care prefer to time a birth when income is high as a way to smooth consumption.

A strong impact of young children and time spent on child care on labor force participation of mothers is confirmed by the studies of Eckstein and Wolpin (1989) and Hotz and Miller (1988). Women's attachment to the labor force and the time at which they start childbearing are also connected.<sup>2</sup> Women who are relatively more productive at home are likely to have children early in the life-cycle. Women who are relatively more productive at work and outsource child care are likely to have children later, when household income is higher, if households face liquidity constraints.<sup>3</sup> Different mechanisms that lead to the increase in female labor supply may affect fertility behavior in different ways.

Indeed, I document that there have been significant changes in the cyclical properties of fertility simultaneous with the change in female labor supply. In particular, the fertility rate is strongly countercyclical in the 1960s and 1970s and becomes procyclical thereafter in the United States. More specifically, the correlation of the fertility rate with the business

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<sup>2</sup>The studies by Caucutt, Guner, and Knowles (2002), Conesa (2000) and Mullin and Wang (2002) find a strong link between the timing of fertility over the life-cycle and women's labor supply decisions.

<sup>3</sup>Goldin and Katz (2002) argue that the diffusion of the birth control pill in the 1960s had a significant influence on women's career decisions. The diffusion of the pill could lower the cost of professional education and lead to a delay in fertility. At the same time, Caucutt, Guner, and Knowles (2002) find that changes in the length of women's education can explain at most 30% of fertility delay.

cycle is negative for younger mothers (25 years old or less) in both periods. For older mothers (above 25 years old) it is slightly positive in the former period and strongly positive in the latter period, and this change accounts for the overall change. There has also been a delay in fertility over the same period as pointed out in the literature. For example, the average age of mothers at first birth has increased from 21.4 in 1970 to 24.2 in 1990.<sup>4</sup>

Scientific interest in the relationship between births and economic activity was significant at the end of 19th century and in the first half of the 20th century in the United States. Silver (1965) surveys the findings covering various time periods from 1870 until 1957. Pro-cyclical behavior of births has been established as one of the strongest empirical observations of that time.<sup>5</sup> Taking this finding into account, there have been two changes in the cyclical properties of the fertility rate: around 1960 and around 1980. The focus of this work is on the latter change, although the economic forces that shape cyclical properties of fertility are likely the same in the earlier period. In a related work, Jones and Schoonbroodt (2007) show that fertility is pro-cyclical in a stochastic version of the dynastic model between 1910 and 1970. They consider 10-year period fluctuations in productivity, not the business cycle frequency fluctuations.

The objective of this work is to nest various mechanisms explaining the rise in married women's labor supply combined with fertility choice and to analyze their impact on secular and cyclical properties of fertility. I also want to understand what features of the model are needed to account for the cyclical properties of the fertility rate. I consider a life-cycle overlapping generations model with aggregate and idiosyncratic uncertainty, female labor force participation, fertility and asset accumulation decisions. Women's productivity at

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<sup>4</sup>For more evidence of fertility delay since 1970s, see studies by Caucutt, Guner, and Knowles (2002), Hotz, Klerman, and Willis (1997) and Rindfuss, Morgan, and Offutt (1996).

<sup>5</sup>See, for example, Galbraith and Thomas (1941)

home is stochastic. A woman with a child saves on child care costs if she stays home. I allow for asset accumulation because the role of fertility timing as a tool for consumption smoothing may be exaggerated without assets. I calibrate the model parameters to match the facts about married women's employment and fertility in 1960s and 1970s. Cyclical behavior of fertility is not targeted.

The model calibrated to the first period produces countercyclical fertility driven by younger women as in the data. The intuition is the following: women with high value of staying home prefer to have a child earlier, stay at home and save on child care costs; those at the margin between working and staying home prefer to have a child during a recession. Women who are relatively more productive in the market prefer to have a child later and pay child care costs without leaving the workforce. They prefer to give birth during an expansion as a way to smooth consumption. The size of the latter group is relatively small in the 1960s and 1970s and the countercyclical effect dominates.

The results of the benchmark model show that cyclical properties of fertility and timing of fertility over the life-cycle are closely related to labor force participation decisions of married women. The proposed mechanisms that lead to the increase in women's labor supply may have different impacts on women with relatively high productivity in the market and women with relatively high productivity at home and thus, secular and cyclical properties of fertility. One of the goals of this work is to evaluate which mechanisms are consistent with observed properties of fertility.

I introduce changes in the determinants of female labor supply (a decrease in the gender wage gap, an increase in women's returns to experience, a decrease in child care costs) into the benchmark model and analyze the implications for female labor force participation and fertility. The decrease in the gender wage gap and the increase in the returns to experience for women separately can explain about a half of the increase in participation of mothers

with an infant and women without children. The decrease in child care costs can account for about a third of the increase in participation of mothers with an infant and does not affect participation of women without children. The decrease in the gender wage gap and the increase in the returns to experience lead to a delay in fertility, while the decrease in child care costs decreases the average age at first birth. Each alternative decreases the negative correlation of fertility with the business cycle for younger women and overall but does not change the correlation for older women. Combining all three alternatives together can account for the increase in participation of women without children and overshoots slightly the participation of mothers with an infant. It does not lead to a delay in fertility observed in the data and leads to overall procyclical fertility, but driven by younger women, not older as in the data. Summarizing the findings, each alternative separately can explain some but not all features of the data and the decrease in child care costs is the least successful candidate.

A change in the earnings of husbands can potentially have an impact on women's labor participation and fertility. As documented in Kambourov and Manovskii (2005), there has been a significant flattening of life-cycle earnings profiles for the successive cohorts of male workers entering the labor market starting from late 1960s. I find that this change can account for about 15% of the increase in participation of mothers with an infant and about a half of the increase in participation of women without children. It leads to a delay in fertility and stronger countercyclical fertility driven by younger women. Combined with other alternatives, it leads to a delay in fertility and dampens the strong procyclical fertility rate for younger women, though it is not enough to generate the change in the cyclical properties of fertility observed in the data. The effect on the cyclical behavior of fertility for older women is very small.

The rest of the work is organized as follows. Section 2 presents the empirical facts that

motivate the paper, in particular facts about married women’s participation and cyclical and secular behavior of fertility. In section 3, I develop a quantitative life-cycle overlapping generations model with discrete employment and fertility choices and aggregate and idiosyncratic uncertainty. In section 4, I describe how I calibrate the model parameters to match the facts about married women’s employment and fertility in 1960s and 1970s. Section 5 presents the results of the benchmark model. In Section 6, I conduct quantitative experiments for the changes in the determinants of women’s labor supply and discuss their results. Section 7 concludes the paper.

## **2.2 Facts**

In this section, I describe the facts about fertility and married women labor force participation. The data sources I use come from U.S. Bureau of Labor Statistics (BLS), National Center for Health Statistics (NCHS), United States Census of Population, Panel Study of Income Dynamics (PSID), Survey of Income and Program Participation (SIPP) and Current Population Survey (CPS). The variables and data sources are described in Appendix 3.7.1.

### **2.2.1 Employment**

In Table 2.1, I report employment rates for married women with their first child less than one year old and married women without children under age 18 in 1970 and 1990. The employment rate more than doubled from 15% to 37% for women with an infant and increased from 54% to 70% for women without children under age of 18 during that time.

There has been an increase in the proportion of educated women during the period of study. It is possible that changes in women’s labor force participation are driven by differences in education. Panel A of Table 2.2 shows the employment rate for married

Table 2.1: Employment Rate, 22-44 Year Old Married Women.

	1970	1990
Women with first child under age 1	0.145	0.372
Women with no children under age 18	0.544	0.701

Source: US Census of Population. Employment rate is the proportion of women who worked more than 30 hours during a reference week.

women with their first child less than one year old along with the proportion of women by educational attainment for 1970 and 1990. First, we can see that the employment rate of married women with their first child less than one year old is approximately the same for all four education categories in 1970 and higher for all groups in 1990, with the increase being larger for more educated women. To understand the role of changes in education, I carry out a simple counterfactual experiment. Suppose that education distribution had remained fixed as in 1970 and only participation behavior had changed. The employment rate would have increased from 0.145 to 0.352. Assuming that participation choices had stayed the same and education distribution had changed the employment rate would have changed from 0.145 to 0.140. These calculations show that the increase in labor participation of married women with an infant is not due to a composition effect.

Panel B of Table 2.2 shows the same statistics as in Panel A for married women without children under age 18. Unlike for women with a child, the employment rate is higher for more educated women comparing to their less educated counterparts in 1970. If education distribution had remained fixed as in 1970 and participation behavior had changed the employment rate would have increased from 0.544 to 0.640. If participation choices had stayed the same and education distribution had changed the employment rate would have

Table 2.2: Employment Rate, 22-44 Year Old Married Women by Education.

Education	1970		1990	
	Employment Rate	Proportion	Employment Rate	Proportion
Panel A: First Child Less Than 1 Year Old.				
Less Than High School	0.140	0.11	0.218	0.04
High School	0.160	0.49	0.351	0.27
Some College	0.136	0.23	0.385	0.36
College and Higher	0.128	0.17	0.394	0.33
Panel B: No Children Under Age 18.				
Less Than High School	0.415	0.25	0.438	0.08
High School	0.593	0.41	0.659	0.30
Some College	0.579	0.18	0.734	0.33
College and Higher	0.645	0.16	0.804	0.29

Source: US Census of Population. Note: Employment rate is the proportion of women who worked more than 30 hours during a reference week.

increased from 0.544 to 0.589. These results imply that while the composition effect plays a role, it cannot account for the rise in employment of married women without children entirely.

In Table 2.3, I report average hours worked per person, average hours worked conditional on employment, employment rate and full-time employment rate for married women in 1970 and 1990. We can see that there has been a large increase in average hours worked and only a marginal increase in hours worked conditional on employment between 1970 and 1990. At the same time, there has been a sharp rise in the employment rate and



Table 2.3: Employment Rate and Hours Worked, 22-44 Year Old Married Women.

	1970	1990
Average Hours	13.6	23.4
Average Hours, Employed	33.8	35.9
Employment Rate	0.42	0.68
Full-Time Employment Rate	0.29	0.51

Source: March CPS. Note: Employment rate is the proportion of women who are employed during a reference week. Full-time employment rate is the proportion of women who worked more than 30 hours during a reference week.

full-time employment rate. These statistics suggest the main change in women’s labor supply occurred along the extensive margin. Attanasio, Low, and Sanchez-Marcos (2008) reach the same conclusion using PSID data.

## 2.2.2 Fertility

### Cyclical Properties of Fertility

First, consider the behavior of fertility over the business cycle at aggregate level. I use labor productivity, defined as business output per worker, as a business cycle indicator. Table 2.4 shows the correlation between the business cycle frequency components of the fertility rate and productivity for two periods, 1961-1981 and 1982-2007. The difference between two periods is remarkable. The fertility rate is countercyclical in period one and procyclical in period two.<sup>6</sup>

Since I am interested in the fertility decision rather than birth itself, I use productivity lagged four quarters and also report the results for three and five quarter lags. Other com-

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<sup>6</sup>I use the band pass filter instead of the Hodrick-Prescott filter to detrend the series, because it allows for the isolation of business cycle frequencies and removes the high frequency noise from the fertility rate series. The results for series detrended using the Hodrick-Prescott filter are qualitatively similar.

Table 2.4: Correlation of Fertility Rate and Productivity: 1961Q1-2007Q4.

	Period I	Period II
	1961-1981	1982-2007
Productivity Lagged 3 Quarters	-0.45	0.48
Productivity Lagged 4 Quarters	-0.38	0.50
Productivity Lagged 5 Quarters	-0.29	0.44

Source: Fertility rate - National Center for Health Statistics. Productivity - BLS. Note: Fertility rate - number of births per 1000 women aged 15-44 years. The quarterly series is obtained by averaging the original seasonally adjusted monthly data. Productivity is business output per worker. Both variables are detrended using band pass filter with frequency parameters 6 and 32 for quarterly data.

monly used cyclical indicators, such as output and unemployment, are clearly endogenous with respect to fertility and participation decisions. The results based on these indicators are presented in Appendix 2.8.3 and confirm the findings reported here.

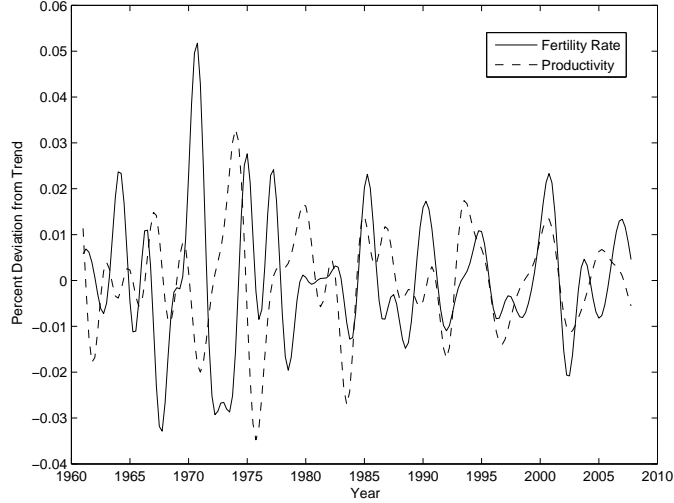
The choice of the break point, year 1981, is illustrated by Figure 2.1, which shows the detrended fertility rate and productivity series, with the latter lagged four quarters. We can see that the change in cyclical of fertility occurred around 1981.<sup>7</sup>

Figure 2.2 shows the change in cyclical of the fertility rate graphically. The solid line shows the correlation of the fertility rate with productivity lagged four quarters over the eighty quarters, with the last observation given by the value of the coordinate on the horizontal axis. Dotted lines show a 95% confidence interval.

Next, I use SIPP data to document the relationship between births and the business cycle at a more disaggregated level. I use the data from a 1984 survey for the first period and a 2001 survey for the second period. The details of the sample construction are described in Appendix 3.7.1. Using the data on the date of birth and link to mother for every individual, I construct fertility and marital histories for all women and estimate the

<sup>7</sup>The results presented in Table 2.4 are not sensitive to the choice of the break point.

Figure 2.1: Fertility Rate and Productivity Lagged 4 Quarters, Percent Deviation from Trend.



Source: Fertility rate - National Center for Health Statistics. Productivity - BLS. Note: Fertility rate - number of births per 1000 women aged 15-44 years. The quarterly series is obtained by averaging the original seasonally adjusted monthly data. Productivity is business output per worker. Both variables are detrended using band pass filter with frequency parameters 6 and 32 for quarterly data.

following linear probability model:

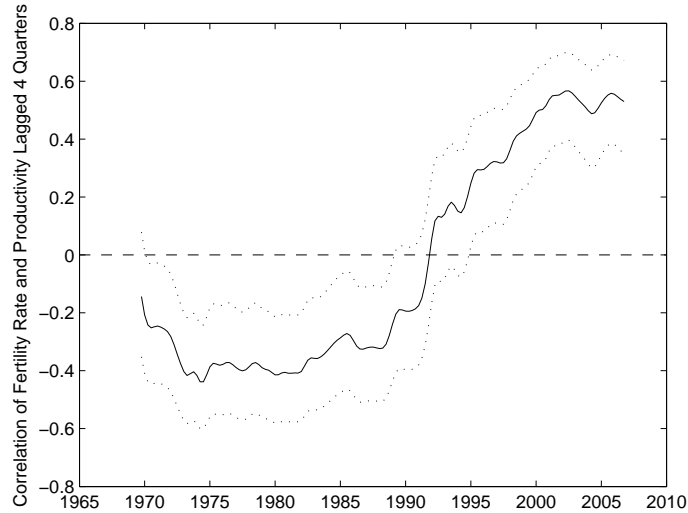
$$b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}, \quad (2.1)$$

where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ . The sample is restricted to married women. Table 2.5 shows the results.

I report the results restricting the age at birth to start from 15 to be consistent with statistics based on aggregate data described above and from 22 as will be relevant for the quantitative analysis below.<sup>8</sup> They are very similar to the results for the aggregate series,

<sup>8</sup>Data limitations do not allow me to use exactly the same periods as for aggregate series.

Figure 2.2: Correlation of Fertility Rate and Productivity Lagged 4 Quarters over the Previous 80 Quarters.



Source: Fertility rate - National Center for Health Statistics. Productivity - BLS. Note: Fertility rate - births per 1000 women aged 15-44 years. The quarterly series is obtained by averaging the original seasonally adjusted monthly data. Productivity is business output per worker. Both variables are detrended using band pass filter with frequency parameters 6 and 32 for quarterly data. For a given quarter, the correlation between the detrended series is computed over the previous 80 quarters and represented by a solid line. Dotted lines show 95% confidence intervals.

with the probability of birth being countercyclical in the 1960s and 1970s and procyclical thereafter. These results also suggest that the change in the cyclicity of fertility is driven by the change in behavior of older women. While  $\hat{\beta}_1$  is negative in both periods for younger women, it is slightly positive in the first period and strongly positive in the second period for older women. The results provided in Appendix 2.8.3 show that the findings of this subsection hold for women with different level of education and for the first births only.

### Secular Properties of Fertility

Table 2.6 shows that the average age of mothers at first birth has increased from 21.4 in 1970 to 24.2 in 1990.

Table 2.5: Probability of Birth over the Business Cycle, SIPP Data.

	Period I	Period II
	1966-1981	1984-2003
Age Group	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)
Age 22-44	-0.024 (0.025)	0.178 (0.027)
Age 22-25	-0.116 (0.049)	-0.077 (0.057)
Age 26-44	0.023 (0.028)	0.226 (0.030)
Age 15-44	-0.032 (0.021)	0.093 (0.024)
Age 15-25	-0.085 (0.030)	-0.160 (0.040)

Note: Estimates from the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ .

Table 2.6: Average Age of Mother at First Birth.

1970	1990
21.4	24.2

Source: Natinal Center for Health Statistics.

Table 2.7 illustrates this delay in fertility from a different angle. The share of first time mothers who are 25 years old or younger decreases from 78% in 1970 to 41% in 1990.

## 2.3 The Model

In this section, I describe the model I use to analyze the change in cyclical properties of fertility and the link between the changes in women's labor force participation and properties of fertility. I consider a stochastic life-cycle overlapping generations model with aggregate and idiosyncratic uncertainty.

Table 2.7: Share of First Time Mothers with an Infant by Age.

Age	1970	1990
15-21	0.42	0.16
22-25	0.36	0.25
26-30	0.17	0.36
31-44	0.05	0.23

Source: US Census of Population.

### 2.3.1 Environment

The economy is populated by overlapping generations of agents. The unit of analysis is a unitary household. Each household consists of a wife and her husband. In each period, a new generation of households of measure one enters the economy at age  $j_1$ . Households live  $T$  periods with certainty and leave the economy at age  $J$ .

#### Preferences

The expected lifetime utility of a household is:

$$\mathbb{E} \sum_{j=j_1}^J \beta^{j-j_1} U(c_j, n_j, v_j; e_j), \quad (2.2)$$

where  $c_j$  - household's consumption,  $n_j \in \{0, 1\}$  - number of children in the household,  $v_j$  - value of wife staying home,  $\beta \in (0, 1)$  - discount factor and  $e_j$  - equivalence scale for consumption. Successive cohorts of households are different depending on realization of aggregate shock but I suppress the time index for convenience.

#### Stochastic Processes

Since this work studies the cyclical properties of fertility, an aggregate shock is an essential ingredient of the model. Each period, a household faces aggregate shock, which

is assumed to follow AR(1) process:

$$\log z' = \rho_z \log z + \epsilon_z, \epsilon_z \sim N(0, \sigma_z^2), \quad (2.3)$$

where  $\epsilon_z$  is standard normal random variable with standard deviation  $\sigma_z$ . Women's productivity at home is stochastic. A household entering the economy draws a value of home production for wife from a random distribution. Each period, wife's home production value is disturbed by idiosyncratic shock and evolves according to AR(1) process during the life-cycle:

$$\log v' = \rho_v \log v + \epsilon_v, \epsilon_v \sim N(0, \sigma_v^2). \quad (2.4)$$

The initial value,  $v_{j_1}$ , is drawn from the stationary distribution of  $v$ . The parameter  $\mu_v$  is used to locate the mean of the distribution of  $v$ :

$$v := v - \mu_v. \quad (2.5)$$

### Earnings

Husband and wife are endowed with one unit of time each. A husband plays a simple role in the model, he works and brings income to the household. I assume that husband always works since the participation rate is close to one for working age married men.<sup>9</sup> Husband's human capital,  $k_j^h$ , depends exogenously on his age  $j$ . Husband's earnings depend on the level of his human capital and aggregate state of the economy:

$$\log y_j^h = \log z + \log k_j^h. \quad (2.6)$$

Since a woman's labor supply decision is essential in this study, women are modeled in a more complicated way. As shown in Table 2.3, the major change in the labor supply of women occurred along the extensive margin. Based on this result, I assume that market

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<sup>9</sup>See, for example Blau (1998).

time is indivisible and wife can either work in the market or stay home. Earnings of age  $j$  wife depend on the aggregate state of the economy and level of her human capital  $k_j^w$ :

$$\log y_j^w = \log z + \log k_j^w. \quad (2.7)$$

Contrary to her husband, wife's human capital is determined endogenously depending on her employment history:

$$k_{j+1}^w = k_j^w + (\eta_0 + \eta_1 j)k_j^w I(E_j = 1), \quad (2.8)$$

where  $I(\cdot)$  - indicator function. The level of wife's human capital in the next period depends on the current level of human capital and the amount of human capital acquired on the job if she works in the current period. As in Attanasio, Low, and Sanchez-Marcos (2008) and Olivetti (2006), I assume that the increase in human capital associated with one more year of work depends on age and diminishes with age if  $\eta_1 < 0$ .

### **Budget Constraints**

Each period, household income consists of the income of the husband, the income of the wife if she is employed and assets brought from the previous period. The income is divided into consumption, assets carried into the next period and the cost of child care, which is paid if there is a child under age 18 in the household and wife works in the market:

$$y_j^h + y_j^w I(E_j = 1) + a = c + \frac{a'}{(1+r)} + p_c G(d) I(E_j = 1), \quad (2.9)$$

where  $G(d)$  - units of child care required for a child of age  $d$  and  $I(\cdot)$  - indicator function. Price per unit of child care is denoted by  $p_c$ . I assume that households can borrow up to a certain limit, so that:

$$a' \geq a_{min}. \quad (2.10)$$

Households enter the economy with zero assets,

$$a_{j_1} = 0, \quad (2.11)$$



and cannot leave the economy in debt:

$$a_{J+1} \geq 0. \quad (2.12)$$

### 2.3.2 Household Decision Problem

Consider the dynamic programming problem of age  $j$  household. Denote the household's value if wife works by  $W(x, d)$ , the household's value if wife stays home by  $H(x, d)$ . A household state is given by  $x := (z, j, k_w, v, a)$  and  $d$ , where  $d$  is the age of child.

The value function for a household without children ever born is given by:

$$V(x, 0) = \max_{a'} \{ \max \{ W(x, 0), H(x, 0) \} \}, \quad (2.13)$$

where

$$W(x, 0) = U(c, 0, 0) + \beta \max \{ EV(x', 0), E(p_j V(x', 1) + (1 - p_j) V(x', 0)) \}, \quad (2.14)$$

$$H(x, 0) = U(c, 0, v) + \beta \max \{ E(V(x', 0)), E(p_j V(x', 1) + (1 - p_j) V(x', 0)) \}, \quad (2.15)$$

and  $p_j$  is the probability of having a child next period conditional on a household's conception decision in the current period.

The interpretation is straightforward. A household without children makes fertility, participation and consumption decisions simultaneously. If the household makes a conception decision this period, a child appears next period with probability  $p_j$ . If wife does not work, the household enjoys the value of her home production.

The value function for a household with a child of age  $d \in [1, 17]$  is given by:

$$V(x, d) = \max_{a'} \{ \max \{ W(x, d), H(x, d) \} \}, \quad (2.16)$$

where

$$W(x, d) = U(c, 1, 0) + \beta EV(x', d + 1), \quad (2.17)$$

$$H(x, d) = U(c, 1, v) + \beta EV(x', d + 1). \quad (2.18)$$

A household with a child makes only participation and consumption decisions. If wife works then the household has to pay the cost of child care  $p_c G(d)$ , which depends on the age of the child  $d$ . If wife stays at home then the household enjoys the value of home production  $v$  and does not have to pay the cost of child care.

The value function for a household after the child leaves is given by:

$$V(x, 18) = \max_{a'} \{ \max \{ W(x, 18), H(x, 18) \} \}, \quad (2.19)$$

where

$$W(x, 18) = U(c, 0, 0) + \beta EV(x', 18), \quad (2.20)$$

$$H(x, 18) = U(c, 0, v) + \beta EV(x', 18). \quad (2.21)$$

This household solves the same problem as a household without children ever born but does not make a fertility decision. Parents do not derive utility from a child after the child leaves the household.

Denote household decision rules by  $a'(x, d)$  for asset choice,  $f(x, d)$  for conception decision and  $l(x, d)$  for wife's labor participation decision. A solution to the household problem is a set of decision rules,  $a'(x, d)$ ,  $f(x, d)$  and  $l(x, d)$  such that given interest rate  $r$ ,  $a'(x, 0)$ ,  $f(x, 0)$  and  $l(x, 0)$  solve equations (13)-(15) subject to the budget constraints (10)-(12) for the household without children ever had, and  $a'(x, d)$  and  $l(x, d)$  solve equations (16)-(18) for the household with a child under age 18 and equations (19)-(21) for the household after child leaves subject to the same budget constraints (10)-(12).

## 2.4 Quantitative Analysis

In this section, I describe how I choose functional forms and the parameters for the benchmark model.

### 2.4.1 Calibration

#### Functional Forms

Utility function is separable, that is:

$$U(c_j, n_j, v_j; e_j) = \log \frac{c_j}{e_j} + \gamma n_j + v_j \quad (2.22)$$

Following Hotz and Miller (1988), I specify the functional form for  $G$  as:

$$G(d) = \phi^{d-1} I(d \in [1, 17]). \quad (2.23)$$

Parameter  $\phi$  allows to account for the difference in need for the child care for children of different ages. The logarithm of husband's human capital is assumed to be a cubic polynomial in age:

$$\log k_j^h = a_0 + a_1 j + a_2 j^2 + a_3 j^3. \quad (2.24)$$

The probability of birth function  $p_j$  is parametrized using a cubic polynomial:

$$p_j = b_0 + b_1 j + b_2 j^2 + b_3 j^3. \quad (2.25)$$

I assume that women are not fertile after age 44.

#### Parameters Set A Priori

Some of the model parameters can be independently determined. Their values are described in Table 2.8. The model period is chosen to be one year. This is a reasonable amount of time between the decision to have a child and the birth. In addition, this choice

substantially reduces computing time. I assume that households enter the economy at age 21 and leave the economy at age 65. To determine  $e_j$ , I use McClements scale, which depends on the age and number of children.<sup>10</sup> The parameters for the earnings profile for males are taken from Kambourov and Manovskii (2005) for the cohort of males entering the labor market in 1968 at age 18. The earnings profile is normalized to 1 in the first period at age 21. The initial value for the wife's human capital process is chosen to match the ratio of female and male median earnings at the beginning of their career.<sup>11</sup> The interest rate is set equal to 4%.

Table 2.8: Parameter Values Chosen A Priori.

Description	Values
Model Period	1 year
Age	$J = 65, j_1 = 21, d \in \{0, 1, 2, \dots, 18\}$
Equivalence Scale	McClements scale, $e_j = 1$ for childless couple, increases with age of child
Human Capital, Husband	$a_0 = 9.3224, a_1 = 0.102,$ $a_2 = -0.00322, a_3 = 0.000029$ Normalized to 1 at $j_1$
Initial Human Capital, Wife	$k_{j_1}^w / k_{j_1}^h = 0.805$
Interest Rate	$r = 0.04$

### Calibrated Parameters

Table 2.9 shows the set of parameters that I calibrate along with the description of

<sup>10</sup>McClements scale assigns value 1 for a childless couple, 1.08 if a child is less than 2 years old and values increasing with age of child. See McClements (1977) for details.

<sup>11</sup>The value is computed using the CPS March 1971 data. The sample is restricted to include married men and women who worked full time 50 weeks or more during the previous year, 18-19 years old with high school degree and 21-22 years old with college degree.

calibration targets. There are 15 parameters that are calibrated to match the same number of the data statistics for 1970. It is clear that a change in each parameter leads to the changes in all statistics so the mapping between the parameters and targets is intended to show what parameters play a main role in determining respective statistics.

The calibrated model parameters are: utility of having a child, three coefficients for the birth probability equation<sup>12</sup>, persistence and standard deviation of aggregate shock, two parameters governing human capital accumulation for females, price of child care, units of child care function parameter, discount factor, borrowing limit and three parameters for value of home production (persistence, standard deviation and mean locator).

The parameters are calibrated to match the following selected statistics:

1. The fertility rate for 22-25 years old married women, computed using the US 1970 census of population data as the ratio of number of first time births and number of women between ages 22 and 25.

2. Shares of first time mothers for the following three age categories: 22-25, 26-30 and 31-35. These statistics are computed using the US 1970 census of population data and rescaled to account for the fact that households enter the economy at age 21. The original statistics are shown in Table 2.7 in the Facts section above.

3. Persistence and volatility of productivity, where productivity is output per worker BLS series. The annual productivity series is detrended using the Hodrick-Prescott filter with smoothing parameter 100.

4. The wage growth for two groups of women: younger than 34 years old and 34 years old or older. Following Attanasio, Low, and Sanchez-Marcos (2008), these two statistics are computed using PSID data for married women who have worked 90% of their lifetime

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<sup>12</sup>The fourth is set so that probability of birth next period equals to 0 for women of age 44 and older.

at each age. The wage growth is measured as a parameter  $\beta_1$  in the following regression:

$$\log y_j^w = \beta_0 + \beta_1 j + \epsilon_j \quad (2.26)$$

5. Wealth to income ratio. The choice of this statistic is not straightforward since the model considered in this work does not have many features that determine wealth accumulation. In particular, there is no retirement and no health shock. The ratio of household financial wealth (net worth excluding owners' equity in household real estate) to disposable personal income is 3.87 in 1970.<sup>13</sup> I assume that the model has to account for a third of that number and do a sensitivity analysis with respect to this choice.

6. Debt to income ratio. For the total debt to income ratio I use the ratio of consumer debt outstanding to disposable personal income.<sup>14</sup>

The following statistics are computed using the US 1970 census of population data.

7. Employment rate for women with six year old child.<sup>15</sup>
8. Employment rate for women younger than 34 with infant, their first child.
9. Employment rate for women aged 34 or more with infant, their first child.
10. Employment rate for women younger than 34 without children under age 18.
11. Employment rate for women aged 34 or more without children under age 18.

Thus, there are fifteen targets to pin down fifteen parameters.

## 2.5 Results from the Calibrated Model

### 2.5.1 Benchmark Calibration

To find the parameter values the model is solved numerically according to the computational algorithm described in Appendix 2.8.2. Table 2.10 shows the performance of the

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<sup>13</sup>See Table B.100 at <http://www.federalreserve.gov/releases/z1/Current/annuals/a1965-1974.pdf>.

<sup>14</sup>See Table B.100 at [http://www.federalreserve.gov/releases/g19/hist/cc\\_hist\\_r.html](http://www.federalreserve.gov/releases/g19/hist/cc_hist_r.html).

<sup>15</sup>Employment rate is the proportion of married women who worked more than 30 hours during a reference week.

Table 2.9: Calibrated Parameters.

Parameters	Description	Calibration Targets
$\gamma$	Utility from a Child	Fertility Rate, Age 22-25
$b_1$	Probability of Birth	Share of Births, Age 22-25
$b_2$	Probability of Birth	Share of Births, Age 26-30
$b_3$	Probability of Birth	Share of Births, Age 31-35
$\rho_z, \sigma_z$	Shock	Volatility and Persistence of Productivity
$\eta_0$	Women's HK	Wage Growth, Age 22-33
$\eta_1$	Women's HK	Wage Growth, Age 34-44
$p_c$	Child Care Cost	Employment Rate, Age 22-33, with Infant
$\phi$	Child Care Function Parameter	Employment Rate, Women with 6 y.o. Child
$\beta$	Discount Factor	Wealth Income Ratio
$a_{min}$	Borrowing Limit	Debt Income Ratio
$\sigma_v^2, \mu_v, \rho_v$	Value of staying home	Employment Rate, Age 34-44, with Infant Employment Rate, Age 22-33, no Children Employment Rate, Age 22-33, no Children

model in matching targets. We can see that the model matches the important features of the data. Calibrated parameter values are shown in Table 2.11 and they are quite reasonable. For example, child care function parameter,  $\phi$ , which determines how the need for child care depends on the age of the child, equals to 0.901. Hotz and Miller (1988) estimate the same parameter in their micro study and obtain the value 0.89. The price of the unit of child care relative to women's earnings is similar to that obtained by Attanasio, Low, and Sanchez-Marcos (2008). Borrowing limit approximately equals to a household's period income at age 25 if wife has always worked, which is not unreasonable. The implied probability of birth given a conception effort is about 0.4 at age 21 and decreases to zero at age 44. The probability of conception is somewhat lower relative to natural fertility for a modern sect practicing no birth control, which equals 0.55 for 20-24 years old women (See Clark (2007)) but similar to the estimates reported in Hotz and Miller (1988) and Rosenzweig and Schultz (1985), who find a monthly conception probability of around 2.5% on average during fertile years.<sup>16</sup>

### 2.5.2 Properties of the Model

The cyclical properties of the fertility rate, computed using the SIPP data and the simulated data from the calibrated model are shown in Table 2.12. We can see that the benchmark model produces countercyclical fertility, driven by younger women. To understand the results, let us consider a new cohort of households entering the economy. First, the utility from a child is high enough to guarantee that all agents want to have a child. The question is about timing and it depends on the value of home production and aggregate state of the economy. Recall that households draw a value of wife's staying home in the first period from a stochastic distribution. Women who have a high value prefer to stay home and have a child early since they derive utility from having a child

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<sup>16</sup>Annual probability of conception equals  $1 - (1 - 0.025)^{12} = 0.262$



Table 2.10: Benchmark Economy.

Statistic	Data	Model
Fertility Rate, Age 22-25	0.217	0.221
Employment Rate, with an Infant, Age 22-33	0.145	0.146
Employment Rate, with an Infant, Age 34-44	0.164	0.164
Employment Rate, no Children, Age 22-33	0.596	0.598
Employment Rate, no Children, Age 34-44	0.472	0.483
Share of Births, Age 22-25	0.621	0.601
Share of Births, Age 26-30	0.288	0.298
Share of Births, Age 31-35	0.060	0.059
Wage Growth, Age 22-33	0.026	0.026
Wage Growth, Age 34-44	0.013	0.013
Productivity, Standard Deviation	0.015	0.015
Productivity, Persistence	0.456	0.459
Wealth Income Ratio	1.290	1.295
Debt Income Ratio	0.178	0.174

Note: The table describes the performance of the model in matching the calibration targets.

Table 2.11: Calibrated Parameter Values.

Parameter	Definition	Value
$\gamma$	Utility from a Child	0.308
$b(1)$	Probability of Birth Function	-0.001
$b(2)$	Probability of Birth Function	0.008
$b(3)$	Probability of Birth Function	-0.312
$\rho_z$	Persistence of Aggregate Shock	0.896
$\sigma_z$	Standard Deviation of Aggregate Shock	0.016
$\eta_0$	Women's Human Capital Accumulation	0.056
$\eta_1$	Women's Human Capital Accumulation	0.001
$p_c$	Child Care Cost	0.573
$\phi$	Child Care Function Parameter	0.901
$\beta$	Discount Factor	0.971
$a_{min}$	Borrowing Limit	-2.128
$\sigma_v^2$	Value of Staying Home, Standard Deviation	0.297
$\mu_v$	Value of Staying Home, Mean Locator	0.446
$\rho_v$	Value of Staying Home, Persistence	0.476

Note: The table contains the calibrated parameter values in the benchmark calibration.

and do not pay child care costs if they stay home. Women at the margin between working and staying home choose to give birth during a recession when the opportunity cost of staying home is lower. Women who draw a low value of staying home face the following trade off. On the one hand, they want to have a child early because of discounting. On the other hand, the opportunity cost of staying home is high, so they choose to work in the market and pay the cost of child care. The desire to smooth consumption is a force to have a child later, when income is higher. These women prefer to have a child during an expansion as a way to smooth household consumption. This intuition is clear if we assume that the value of staying home is drawn randomly in the first period and stays constant over the life-cycle. One undesirable implication of this assumption is that the employment rate is close to zero for younger women with a young child and close to one for older women with a young child. However, the employment rate varies little by age for women with an infant as shown in Table 2.10. The introduction of persistent stochastic process for the value of staying home allows to obtain the employment rate for women of different ages with an infant as in the data and at the same time preserves the cyclical properties of fertility. Another property of the model is that fertility is more important as a tool to smooth consumption at an early age and assets are more important later in life. This explains the much stronger cyclical response of fertility for younger households compared to older households.

## 2.6 Experiments

In this section, I consider several changes in the determinants of female labor supply that occurred between the two periods and have been proposed in the literature to explain the increase in married women labor supply. I focus on the following candidates: 1) an increase in wage level for females (implying a decrease in the gender wage gap), 2) an increase in

Table 2.12: Data and Results from the Benchmark Model.

Statistic	Data	Model
$\hat{\beta}_1$ (Age 22-44)	-0.024	-0.037
(s.e.)	(0.025)	
$\hat{\beta}_1$ (Age 22-25)	-0.116	-0.130
(s.e.)	(0.049)	
$\hat{\beta}_1$ (Age 26-44)	0.023	0.002
(s.e.)	(0.028)	

Note: Estimates from the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ .

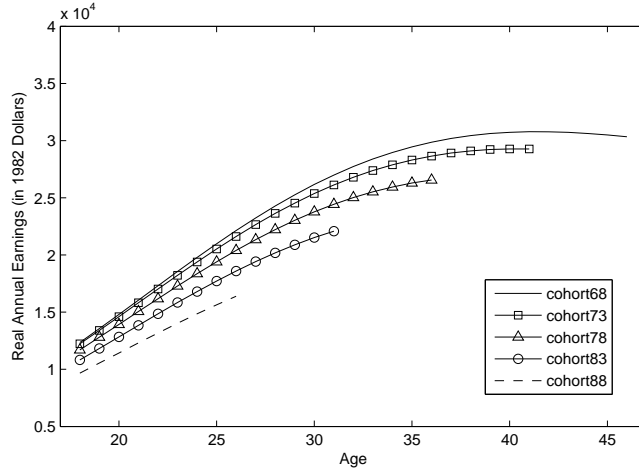
the returns to experience for females and 3) a decrease in child care cost. It has been argued that each of these changes is a major contributor to the changes in female labor supply.<sup>17</sup> The goal here is to evaluate the implication of each alternative for the cyclical and secular properties of fertility in the model with endogenous fertility.

I also consider one more potential candidate that may have contributed to the rise in female labor supply. As documented in Kambourov and Manovskii (2005) and shown in Figure 2.3, a significant flattening of life-cycle earnings profiles for successive cohorts of males occurred since the late 1960s. It is clear that this change in the earnings of their husbands may induce women to increase their labor supply. Based on the properties of the model described above, it may also lead to delay of fertility for women who are more productive in the market relative to home. I want to evaluate these effects quantitatively.

The nature of the experiments is the following. I introduce changes in the determinants

<sup>17</sup>See Jones, Manuelli, and McGrattan (2003), Olivetti (2006), Attanasio, Low, and Sanchez-Marcos (2008) for candidates 1), 2) and 3) respectively.

Figure 2.3: Life-Cycle Male Earnings Profile.



Source: Kambourov and Manovskii (2005)

of female labor supply that occurred between the two periods considered in this work, compute the new steady state using the benchmark model with appropriate changes in parameters and analyze the changes in female labor participation and properties of fertility between the two steady states. To carry out the experiments, I need to quantify the changes in the determinants of female labor supply and map them into changes in the parameters of the benchmark model. For the change in females' wage level experiment, I compute the ratio of female and male median earnings when they enter the workforce for the first time in 1990 using the CPS March 1991 data. Following the same way as in the benchmark case, the sample is restricted to include married men and women who worked full time 50 weeks or more during the previous year, 18-19 years old with high school degree and 21-22 years old with college degree. The ratio changes from 0.805 in 1970 to 0.907 in 1990, an increase by about 12.7%. For the flattening males' life-cycle earnings profile experiment, I use the parameters for the males earnings profile from Kambourov and Manovskii (2005)

for the cohort of males entering the labor market in 1988 at age 18. As in the benchmark case, the earnings profile is normalized to 1 in the first period at age 21. There is no direct measure of historic child care price but Attanasio, Low, and Sanchez-Marcos (2008) argue that a 15% decline is not unreasonable. I use this number and also 20% increase in the marginal returns to experience for females averaged over the life-cycle used in their work.<sup>18</sup> This value is of the same order of magnitude as the estimate reported in Olivetti (2006). Using the PSID data she finds a 25% increase in the elasticity of growth of hourly wages with respect to hours of work for women between 1970s and 1990s. Since there is empirical evidence of changes in all determinants of female labor supply considered in this work, I consider all changes simultaneously, then one by one, and study the implications for the cyclical and secular properties of fertility using the framework developed in this work.

### **Experiment I. Changing All Determinants Proposed in the Literature Combined**

In the first experiment, I introduce changes in all determinants proposed in the literature: 1) 12.7% increase in the initial females' wage level (implying a decrease in the gender wage gap), 2) 20% increase in the returns to experience for females and 3) 15% decrease in the cost of child care. Table 2.13 shows the results. Combining all three alternatives together can account for the increase in participation of women without children, overshoots the participation of mothers with an infant by about 23% for younger women and about 15% for older women and does not lead to a delay in fertility. The last result stems from the fact that the increase in the females' wage level and the returns to experience on the one hand, and the decrease in child care costs on the other hand, balance each other out as explained in the discussion of the Experiment II results below. Table 2.18 shows the

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<sup>18</sup>They consider increases by 10%, 20% and 40%.

Table 2.13: Experiment I, Changing All Determinants Proposed in the Literature Combined, Exhibit A.

Statistic	Data	Data	Model
	Period I	Period II	Period II
Fertility Rate, Age 22-25	0.217	0.154	0.227
Emp Rate w/ Infant, Age 22-33	0.145	0.368	0.451
Emp Rate w/ Infant, Age 34-44	0.164	0.405	0.461
Emp Rate, no Kids, Age 22-33	0.596	0.738	0.765
Emp Rate, no Kids, Age 34-44	0.472	0.664	0.662
Share of Births, Age 22-25	0.621	0.297	0.578
Share of Births, Age 26-30	0.288	0.438	0.321
Share of Births, Age 31-35	0.060	0.205	0.061

changes in the cyclicalness of fertility as a result of the changes in all three determinants combined. We can see that the fertility rate becomes procyclical as in the data but this result is driven by the change in behavior of younger women, not older as in the data.

### **Experiment II, Changing Each Determinant Separately**

To understand the role of each alternative, I carry out the set of experiments changing the determinants of female labor supply one at a time. The results are shown in Table 2.15. Column (W) shows the results of the increase in females' wage level. In particular, I change the ratio of female and male median earnings at the beginning of their career from 0.805 in 1970 to 0.907 in 1990. These statistics are computed using the CPS data as explained in the

Table 2.14: Experiment I, Changing All Determinants Proposed in the Literature Combined, Exhibit B.

Statistic	Data	Data	Model
	Period I	Period II	Period II
$\hat{\beta}_1$ (Age 22-44)	-0.024	0.178	0.178
(s.e.)	(0.025)	(0.027)	
$\hat{\beta}_1$ (Age 22-25)	-0.116	-0.077	0.730
(s.e.)	(0.049)	(0.057)	
$\hat{\beta}_1$ (Age 26-44)	0.023	0.266	0.065
(s.e.)	(0.028)	(0.030)	

Note: Estimates from the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ .



beginning of this section. The implied female-male earnings ratio increases from 0.597 to 0.697. These numbers are very close to the numbers estimated in Blau (1998), who reports an increase from 0.562 in 1969 to 0.692 in 1989 for full-time workers between ages 25 and 64. Column (E) shows the results of the increase in the returns to experience for females. More specifically, I increase  $\eta_0$  so that the implied marginal returns to experience averaged over the life-cycle increase by 20%.<sup>19</sup> Column (C) shows the results of the decrease in the cost of child care,  $p_c$ , by 15%. Finally, Column (M) shows the results of the flattening males' life-cycle earnings profile as estimated in Kambourov and Manovskii (2005). We can see that the change in each determinant leads to the rise in the employment rate for women but none of them can account for the increase entirely. The results of the increase in females' wage level and the returns to experience are similar, they produce the increase in employment rate of younger and older women with an infant and without children under age of 18. They also deliver a delay in fertility. The only difference is that, not surprisingly, the increase in the returns to experience leads to a higher employment rate for older women comparing to the increase in the wage level case. In the former case, women accumulate more human capital at an early age and participate more when they become older. Attanasio, Low, and Sanchez-Marcos (2008) find similar results for the gender wage gap experiment but they find that the increase in the returns to experience has a small effect on women's labor supply. They contribute this limited impact of the returns to experience to the presence of uncertainty in their model as households work, save more early in life and do not respond fully to intertemporal incentives. After careful examination, the results of the decrease in child care costs experiment are similar to those reported in Attanasio, Low, and Sanchez-Marcos (2008) in terms of female labor force participation. Participation of women without children remains unchanged, participation

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<sup>19</sup>Note that the marginal returns to experience depend on age.

of women with young children increases by about the same percent, 26% for women under age 29 and 36% for women above 29 in their work and 26% for younger women and 48% for older women in this paper. The differences are that they consider changes between the cohorts of women born in 1944-1948 and 1954-1958, consider women with children under age of three and account for the change in participation rates from 0.42 to 0.53 for younger women with children under age three and from 0.53 to 0.72 for older women with children under age three. So the impact of the decrease in child care costs is very similar in two models but it is not enough to account for the changes in the women's labor participation statistics used in this work. The decrease in the child care costs counterfactually predicts that women begin childbearing earlier. The last experiment, flattening of males' life-cycle earnings profile, delivers the increase in employment rate for all categories of women but, as in previous cases, not enough to account for the changes in the data. It also produces a delay in fertility.

Before discussing the intuition behind the results of the experiments let us consider their impact on the cyclical properties of the fertility rate. Table 2.16 shows the results. Each alternative except flattening of males' life-cycle earnings profile decreases the negative correlation of fertility with the business cycle for all women and younger women but does not change the correlation for older women.

To understand the economics behind the results of each experiment it is useful to recall the mechanism behind the results of the benchmark model. Women who draw a high value of staying home prefer to have a child as soon as they enter the economy and those at the margin between working and staying home prefer to do it during a recession. Women who draw a low value of staying home prefer to work in the market, have a child later when household's income is high enough and pay child care costs. They time their fertility to good times to smooth household's consumption.

Table 2.15: Experiment II, Changing Each Determinant Separately, Exhibit A.

Statistic	Data I	Data II	(W)	(E)	(C)	(M)
Fertility Rate, 22-25	0.217	0.154	0.177	0.183	0.278	0.125
ER w/ Infant, 22-33	0.145	0.368	0.242	0.246	0.215	0.170
ER w/ Infant, 34-44	0.164	0.405	0.249	0.310	0.207	0.219
ER, no Kids, 22-33	0.596	0.738	0.675	0.682	0.593	0.678
ER, no Kids, 34-44	0.472	0.664	0.548	0.595	0.479	0.553
Share of Births, 22-25	0.621	0.297	0.509	0.510	0.686	0.407
Share of Births, 26-30	0.288	0.438	0.367	0.362	0.239	0.416
Share of Births, 31-35	0.060	0.205	0.075	0.077	0.045	0.107

Note: ER - Employment Rate. Columns Data I and Data II show the statistics from the data for Period I and Period II respectively. Columns (W), (E), (C) and (M) show the statistics computed using the model simulated series obtained increasing wage level for females, increasing the returns to experience for females, decreasing child care costs and flattening life-cycle earnings profile for males respectively.

Table 2.16: Experiment II, Changing Each Determinant Separately, Exhibit B.

Statistic	Data I	Data II	(W)	(E)	(C)	(M)
$\hat{\beta}_1$ (Age 22-44)	-0.024	0.178	0.021	0.059	0.037	-0.143
(s.e.)	(0.025)	(0.027)				
$\hat{\beta}_1$ (Age 22-25)	-0.116	-0.077	0.050	0.009	0.060	-0.754
(s.e.)	(0.049)	(0.057)				
$\hat{\beta}_1$ (Age 26-44)	0.023	0.266	-0.001	0.048	-0.013	-0.033
(s.e.)	(0.028)	(0.030)				

Note: Estimates from the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ . Columns Data I and Data II show the statistics from the data for Period I and Period II respectively. Columns (W), (E), (C) and (M) show the statistics computed using the model simulated series obtained increasing the wage level for females, increasing the returns to experience for females, decreasing child care costs and flattening life-cycle earnings profile for males respectively.

To understand how the experiments work we need to consider how those two groups of households are affected. The intuition behind the increase in females' wage level and returns to experience experiments is similar. Higher current or expected future wage level decreases the threshold value of staying home and induces more women to work in the market compared to the benchmark case. These women delay their fertility and the fertility rate declines for younger women. Women with low value of staying home prefer to have a child earlier because the household's income is higher. The effect for the former group dominates and there is a delay in fertility as a result. Since the market wage increases while the value of staying home and child care costs remain unchanged, the employment rate is higher for younger and older women with and without children. The correlation of fertility with the business cycle increases for younger women and overall, because of the change in behavior of women with low value of staying home. The number of women with high value of staying home who prefer to have a child during a recession stays about the same while the number of women who prefer to work in the market and pay child care costs when they give a birth increases.<sup>20</sup> At the same time, women with low value of staying home prefer to have a child earlier compared to the benchmark case because household income is higher.

The decrease in the child care costs case is different from the experiments described above because women with high value home production stay home when they have a child, do not pay child care costs and therefore, they are not affected. Women with low value of staying home have a child earlier compared to the benchmark case and childbearing shifts to earlier ages as a result, the opposite to what we see in the data. The employment rate of women without children is not affected while it increases for women with a child as in

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<sup>20</sup>The change in the number of women in the former case depends on the change in the mass of agents around the threshold value of home production and it is relatively small because the threshold is located around the median.

the wage level and returns to experience cases. As women with low value of staying home start bearing a child earlier and the employment rate increases for women with a child, the correlation of fertility with the business cycle increases for all and younger households.

The flattening of males' life-cycle earnings profile operates in the following way. As their husbands' income decreases more women work in all categories compared to the benchmark case. The impact on women with high value of staying home is the same as in the wage level and returns to experience experiments. The threshold value of staying home decreases, more women work and delay their fertility. The impact on women with low value of staying home is unique for this experiment. The decline in household income leads to a delay in fertility for these women since they wait longer till the household income is high enough. Both groups of women delay fertility, that is why the delay in fertility is the most pronounced among all experiments. The delay in fertility by the group with low value of staying home is the reason that fertility becomes stronger countercyclical and that this is driven by younger women. As women with low value of staying home, whose fertility response to aggregate shock is procyclical, delay the birth of their child and fertility becomes less important as a tool to smooth consumption, the countercyclical response of younger mothers becomes more pronounced and dominates the overall response.

### **Experiment III. Changing All Four Determinants Combined**

In experiment III, I introduce the changes in all four determinants: 1) 12.7% increase in wage level for females (implying a decrease in the gender wage gap), 2) 20% increase in the returns to experience for females, 3) 15% decrease in child care costs and 4) flattening males' life-cycle earnings profile. Column (L) in Table 2.17 shows the results from experiment I (changes 1), 2) and 3) combined) and Column (L+M) shows the results obtained changing all four determinants. All changes combined lead to the higher employment rates for all categories of women in the second period comparing to the data. The employment

Table 2.17: Experiment III, Changing All Four Determinants Combined, Exhibit A.

Statistic	Data I	Data II	(L)	(L+M)
Fertility Rate, Age 22-25	0.217	0.154	0.227	0.169
Emp Rate w/ Infant, Age 22-33	0.145	0.368	0.451	0.495
Emp Rate w/ Infant, Age 34-44	0.164	0.405	0.461	0.546
Emp Rate, no Kids, Age 22-33	0.596	0.738	0.765	0.790
Emp Rate, no Kids, Age 34-44	0.472	0.664	0.662	0.730
Share of Births, Age 22-25	0.621	0.297	0.578	0.446
Share of Births, Age 26-30	0.288	0.438	0.321	0.415
Share of Births, Age 31-35	0.060	0.205	0.061	0.084

Note: Columns Data I and Data II show the statistics from the data for Period I and Period II respectively. Columns (L) and (L+M) show the statistics computed using the model simulated series obtained changing all three determinants proposed in the literature and all four determinants respectively.

rate increases by 35% higher for women with an infant and by about 7 – 10% higher for women without children comparing to the data. Adding flattening of the life-cycle earning profile generates a delay in fertility as observed in the data. The fertility rate declines for the youngest households and women have their first child later comparing to the benchmark case.

Table 2.18 shows the changes in the cyclical properties of fertility. Combined with other alternatives, the flattening of the life-cycle earnings profile for males dampens the strong procyclical fertility for younger women, though it is not enough to generate the change in the cyclical properties of fertility observed in the data.

Table 2.18: Experiment III, Changing All Four Determinants Combined, Exhibit B.

Statistic	Data I	Data II	(L)	(L+M)
$\hat{\beta}_1$ (Age 22-44)	-0.024	0.178	0.179	0.118
(s.e.)	(0.025)	(0.027)		
$\hat{\beta}_1$ (Age 22-25)	-0.116	-0.077	0.730	0.614
(s.e.)	(0.049)	(0.057)		
$\hat{\beta}_1$ (Age 26-44)	0.023	0.266	0.065	0.084
(s.e.)	(0.028)	(0.030)		

Note: Estimates from the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t-4$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t-4$ . Columns Data I and Data II show the statistics from the data for Period I and Period II respectively. Columns (L) and (L+M) show the statistics computed using the model simulated series obtained changing all three determinants proposed in the literature and all four determinants respectively.



## 2.7 Conclusion

I document and analyze the change in the cyclical behavior of the fertility rate at business cycle frequencies. I find that fertility is countercyclical in the 1960s and 1970s and procyclical thereafter. Countercyclical fertility is shaped by the behavior of younger women in the first period and the change in the second period is driven by the change in the behavior of older women. I find that a standard model with incomplete markets can generate countercyclical fertility in the 1960s and 1970s. The model implies that properties of fertility are related to labor force participation decisions of married women. The following candidates have been suggested in the literature to explain the rise in married women's labor supply between the 1970s and 1990s:

1. A decrease in the gender wage gap.
2. An increase in the returns to experience for females.
3. A decrease in child care costs.

These changes have implications for the properties of fertility. The decrease in the gender wage gap and the increase in the returns to experience lead to fertility delay while the decrease in child care costs shifts childbearing to earlier ages contrary to the data. Each alternative decreases the negative correlation of fertility with the business cycle for younger women and overall but does not change the correlation for older women. Combining all three alternatives together does not change the age of women at first birth and leads to overall procyclical fertility as observed in the data, but driven by younger women, not older.

The flattening of life-cycle earnings profile for males leads to a delay in fertility and stronger countercyclical fertility, driven by younger women. Combining it with other candidates dampens the strong procyclical fertility for younger women, though it is not

enough to generate the change in the cyclical properties of fertility observed in the data.

The key message of this work is that female labor force participation and timing of fertility are determined by the same economic forces, and implications for fertility can be used to distinguish among theories of the rise in labor force participation of married women. The question that remains open is what drives the strong procyclical fertility for older women in the second period.

## 2.8 Appendix

### 2.8.1 Data

**Output.** Output is business output series constructed by BLS.<sup>21</sup>

**Productivity.** Productivity is business output per worker constructed by BLS.

**Unemployment Rate.** Unemployment rate is civilian unemployment rate computed using the Current Population Surveys (CPS) data.

**Employment Rates, Birth Shares.** Women's employment rates and birth shares for first time mothers in 1970 and 1990 are computed using the US census of population data available at <http://usa.ipums.org/usa/>.

**Gender Wage Gap.** Gender wage gap for individuals entering the labor market is computed as explained in the paper using the Current Population Survey March Supplement data available at <http://cps.ipums.org/cps/>.

**Wage Growth.** Females wage growth rates are computed as explained in the paper using the Panel Study of Income Dynamics (PSID) data available at <http://simba.isr.umich.edu/>.

**Fertility Rate.** The seasonally adjusted monthly fertility rate is taken from National Vital Statistics Reports for the years 1998-2007 (available at <http://www.cdc.gov/nchs/products/nvsr.htm>), Monthly Vital Statistics Reports for the years 1970-1997 (available at <http://www.cdc.gov/nchs/products/mvsr.htm#vol12s>) and U.S. U.S. Census (1939-2002) Vital Statistics for the years 1951-1969. Monthly series is averaged into quarterly series.

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<sup>21</sup>BLS data are available at <http://data.bls.gov/cgi-bin/dsrv?pr>.

**SIPP data.** SIPP 1984 and 2001 Panels are formed from nationally representative samples of individuals of 15 years of age and older of the civilian noninstitutionalized population. Information is collected about sampled individuals and their household members. 1984 Panel began interviews in October 1983 with sample members in 19,878 households. The interviews were conducted once every four months over a 32-month period. 2001 Panel began interviews in February 2001 with sample members in 36,700 households. The interviews were conducted once every four months over a 36-month period. The Panel was divided into four rotation groups. Each rotation group was interviewed in a separate month. An interview wave is a set of interviews covering all four rotation groups during four months. Respondents were asked questions about the previous four months during each interview. A core set of questions was repeated at each wave of interviewing. Some sets of questions, labeled ‘Topical Modules’, were assigned to particular interviewing waves. These modules were designed to obtain the detailed information about a variety of topics including marital and fertility history. Marital history contains information about the first, the second and the last marriages for individuals ever married. Fertility history contains information about the first and the last child for women who had children. In particular, the 1984 Panel includes a month and a year of the beginning of each marriage, divorce and separation and a month and a year of birth of the first and the last child. The 2001 Panel includes only a year of all aforementioned events.

To construct the fertility histories for women in 1984 and 2001 Panels, I identify all individuals of age 18 and below at time of an interview and locate their mothers using the person number of parent variable, *PNPT*<sup>22</sup>, from 1984 Panel and the person number of mother variable, *EPNMUM*, from 2001 Panel. I use variable *ETYPMOM* to consider only biological children of women in 2001 Panel. Panel 1984 does not have this information.

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<sup>22</sup>This variable identifies mother if she lives in the same household.

Since the majority of mothers are biological mothers for children who live with mothers in their households, the results will not be affected most likely<sup>23</sup>. Since a month and a year of birth are available for all individuals in the data; for a given women, I obtain the dates of births of her children who live in the same household. I link core files and topical modules files<sup>24</sup> and construct women's marital histories using the topical module data. The resulting 1984 sample contains about 10,000 women. The total number of births is about 400 on average every year during the 1960s and 1970s. The number of first births is about 160 on average every year during the same time. The resulting 2001 sample contains about 21,000 women. The total number of births is about 900 on average every year during 1980s and 1990s. The number of first births is about 400 on average every year during the same period.

## 2.8.2 Numerical Solution and Algorithm

Since agents face a finite horizon, the numerical solution of the model is obtained recursively starting from the terminal period. Given the household's state vector and the value function for the next period, the current value function and decision rules are solved for. A state vector,  $x := (z, j, k_w^j, v^j, a, 0)$ , consists of six variables: aggregate shock, age, wife's human capital, wife's value of staying home, asset stock and age of child. Given a state vector, a household that never had children ( $d = 0$ ) makes a labor participation decision for the wife, a fertility decision and an asset accumulation decision according to Bellman equations (13)-(15) and subject to the budget constraints (10)-(12). A household with a child ( $d \in [1, 17]$ ) makes a labor participation decision for a wife and asset accumulation decision according to Bellman equations (16)-(18) and subject to the same budget

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<sup>23</sup>For example, the likelihood that a child lives with his or her biological mother given that this child has a mother in a household is above 97% based on 2001 Panel data.

<sup>24</sup>See <http://www.census.gov/sipp/linking.html> for details about using and linking files.

constraints (10)-(12). A household after leaving of the child ( $d > 17$ ), makes a labor participation decision for a wife and asset accumulation decision according to Bellman equations (19)-(21) and subject to the same budget constraints (10)-(12).

The combination of the discrete and continuous choices implies that the value functions are not necessarily concave or differentiable. The problem arises because of participation and fertility decisions in future periods. As asset level increases, consumption can decrease because of the changes in the future labor force status or presence of child. Therefore, I discretize continuous state variables and solve for an approximate solution of the household problem.

There are four continuous state variables: the aggregate shock, wife's human capital, the value of staying home and the asset stock. The state space of the problem is the subset of  $\mathbb{R}^6$  space:  $(\mathbb{R}^+ \times \{j_1, \dots, J\} \times \mathbb{R}^+ \times \mathbb{R} \times [a_{min}, \infty] \times \{0, \dots, 18\})$ . Continuous stochastic processes for aggregate shock,  $z$ , and value of staying home,  $v$ , are approximated by discrete processes with 7 and 15 states respectively using Tauchen (1986) algorithm. Given the initial value of wife's human capital, the maximum value is computed assuming she never stays home during her life and a nonlinear grid with 30 points is employed with points concentrated near the initial value. The upper bound for asset stock of 22 is chosen so that it never binds and a nonlinear grid with 40 points is used with points concentrated near the borrowing limit and zero. As a result, the discretized state space has the size  $(7 \times 45 \times 30 \times 15 \times 40 \times 19)$ . To reduce the approximation error, I solve for optimal asset decision rule,  $a'$ , in two steps. In the first step, given a current state, I find an optimal  $a'$  among the grid points, in the second step, I use a golden search method to find an optimal  $a'$  around the point obtained in the first step and do a sensitivity analysis with respect to this procedure. I use a weighted linear approximation of expected continuation value to obtain its value at a point outside of the set of grid points for the asset stock and wife's

human capital state variables.

I employ the simulated method of moments (SMM) to find the parameter values that produce the target statistics. The following algorithm is used to find a solution of household problem. First, guess values are assigned to the calibrated parameters summarized in Table 2.9. Using these parameters as well as parameters set a priori, optimal decisions rules for asset holding, labor participation and fertility are obtained employing finite dynamic program. In the next step, I simulate the aggregate shock history for 4,000 periods. Every period, a value of staying home is drawn from a stationary distribution for 5,000 households entering the economy and simulated for the rest of the households<sup>25</sup>. Using the simulated values and optimal decision rules, the target statistics and the value of the SMM objective function are calculated for the model economy. The procedure that I use to minimize the objective function is Downhill Simplex. Since this is a local optimization procedure I use different initial parameter values and Simulated Annealing global routine to make sure that the optimal parameter values represent a unique solution of the optimization problem.

### 2.8.3 Sensitivity Analysis

#### **Alternative Business Cycle Indicators and 1st Order Fertility Rate**

Table 2.19 shows the correlation of the fertility rate with several business cycle indicators, in particular productivity, output and unemployment rate. We can see that all in all the results are not sensitive to the choice of the business cycle indicator. It is not clear a priori what indicator is more appropriate to measure the cyclicity of fertility rate since it is not known what information households use to form expectations about the state of the economy. The results are reasonable in terms of the lag structure since productivity leads output by about two quarters and unemployment rate is sluggish. I use the band pass filter

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<sup>25</sup>There are 5,000 households of each age from 21 to 65 in any given period in the economy.

(Baxter and King (1999)) rather than Hodrick-Prescott filter to isolate frequencies that are relevant for business cycle analysis, because the former removes the high frequency fluctuations from the fertility rate series.

Since a household can have only one child in the model, in Table 2.20 I report the correlation of the 1st order fertility rate with the business cycle indicators. The results are virtually unchanged compared the overall fertility rate case.

At the micro level, Table 2.21 shows the results for the first births using the SIPP data. We can see that the results are very similar to the results for all births shown in Table 2.5. Countercyclical fertility in the first period is driven by younger women while procyclical fertility in the second period is driven by older women.

Since the proportion of educated women has increased substantially during the period of study (See Panels A and B in Table 2.2), it is possible that women with different educational achievements behave in a different way and the changes in the cyclical properties of fertility are driven by the composition effect. Table 2.22 shows that this is not the case since women with different level of education experienced similar changes as all women. Table 2.23 shows the results for the first births by education. Again, the results are qualitatively the same as for all births.

### **Benchmark Model Assumptions**

Here, I discuss the sensitivity of the results to the choice of two target statistics: wealth to income ratio and debt to income ratio. Discount factor,  $\beta$ , and borrowing limit,  $a_{min}$ , are the most important parameters determining wealth to income and debt to income ratios in the model. Instead of changing the target statistics and recalibrating the model, I change  $\beta$  and  $a_{min}$  and analyze the impact on the results of the benchmark model. One more important issue to consider is the assumption about the price of the child care unit,  $p_c$ . I assume that it is constant but it may be argued that  $p_c$  may change over the business



cycle since child care expenditures are used to pay wages of those who provide child care services and if wages are procyclical than so should be the price of child care. There is no direct evidence about the behavior of the price of child care over the business cycle so I do a sensitivity analysis assuming that elasticity of  $p_c$  with respect to wage equals one.

Tables 2.24 shows the sensitivity of the benchmark model results to the changes in  $\beta$ ,  $a_{min}$  and child care price elasticity. Column (B) shows the results of the benchmark model. Columns (1) and (2) show the results of setting  $\beta = 0.965$  and  $\beta = 0.975$  respectively. We can see that households have a child earlier and the fertility rate becomes less countercyclical as a result of the decrease in  $\beta$ . This happens because women with low value of staying home want to have a child earlier. The increase in  $\beta$  has the opposite effect. The effect on employment rate is very small. Columns (3), (4) and (5) show the results of setting  $a_{min} = -2.5$ ,  $a_{min} = -1.5$  and  $a_{min} = 0.0$  respectively. Employment and fertility rates are not affected significantly. As expected, increasing the borrowing limit leads to stronger countercyclical fertility rate as households can smooth consumption better and decreasing the borrowing limit leads to the opposite effect. It is clear that the fertility rate will be strongly countercyclical in the case of the “natural” borrowing constraint<sup>26</sup>. In case of no borrowing ( $a_{min} = 0.0$ ), the fertility rate becomes procyclical. Column (6) shows the results setting elasticity of child care price with respect to wage equal to one. The only significant change is that the fertility rate becomes stronger countercyclical.

This analysis shows that the results are not sensitive to small changes in  $\beta$  and  $a_{min}$ , which means that they are not sensitive to small changes in wealth to income and debt to income ratios. Introduction of the procyclical child care price leads to a stronger countercyclical fertility rate. In this case, setting the borrowing limit to zero or decreasing

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<sup>26</sup>The “natural” borrowing constraint arises if the utility function satisfies the Inada condition and households never choose an asset position such that they may end up with zero consumption in some future state with positive probability.

the intertemporal elasticity of substitution and recalibrating the parameters brings the results of the benchmark model and experiments back. Changing  $\sigma$ , I essentially target the cyclical of fertility rate. Once I get the cyclical of fertility rate as in the benchmark model, the results of the experiments still hold.

Table 2.19: Correlation of Fertility Rate and Business Cycle Indicators.

Lag	Productivity		Output		Unemployment	
	Period I	Period II	Period I	Period II	Period I	Period II
Current	-0.429*	0.145	-0.619*	0.399*	0.607*	-0.492*
1 Quarter	-0.489*	0.259*	-0.576*	0.485*	0.458*	-0.559*
2 Quarters	-0.492*	0.382*	-0.487*	0.526*	0.274*	-0.539*
3 Quarters	-0.449*	0.475*	-0.378*	0.502*	0.093	-0.438*
4 Quarters	-0.375*	0.500*	-0.273*	0.401*	-0.049	-0.281*
5 Quarters	-0.294*	0.436*	-0.189**	0.231*	-0.139	-0.097
6 Quarters	-0.226*	0.294*	-0.129	0.009	-0.180	0.095

Source: Fertility rate - National Center for Health Statistics. Output and productivity - BLS.

Note: Fertility rate is the number of births per 1000 women between the ages of 15 to 44. Output is business output, Productivity is business output per worker. Unemployment rate is civilian unemployment rate. All variables are detrended using band pass filter with frequency parameters 6 and 32 for quarterly data. Single '\*' and double '\*\*' indicate that the coefficient is statistically significant with 5% and 10% level of significance.

Table 2.20: Correlation of 1st Order Fertility Rate and Business Cycle Indicators.

Lag	Productivity		Output		Unemployment	
	Period I	Period II	Period I	Period II	Period I	Period II
Current	-0.509*	-0.091	-0.584*	0.334*	0.530*	-0.575*
1 Quarter	-0.531*	0.082	-0.476*	0.477*	0.372*	-0.674*
2 Quarters	-0.503*	0.284*	-0.330*	0.579*	0.077	-0.674*
3 Quarters	-0.427*	0.461*	-0.178	0.607*	-0.138	-0.578*
4 Quarters	-0.314*	0.558*	-0.044	0.543*	-0.288*	-0.414*
5 Quarters	-0.183	0.545*	0.056	0.386*	-0.359*	-0.213*
6 Quarters	-0.060	0.428	0.119	0.163	-0.366*	-0.002

Source: Fertility rate - National Center for Health Statistics. Output and productivity - BLS.

Note: 1st order fertility rate is the number of first time births per 1000 women between the ages of 15 to 44. Output is business output, Productivity is business output per person. Unemployment rate is BLS civilian unemployment rate. All variables are detrended using band pass filter with frequency parameters 6 and 32 for quarterly data. Single '\*' and double '\*\*' indicate that the coefficient is statistically significant with 5% and 10% level of significance respectively.

Table 2.21: Probability of Birth over the Business Cycle, 1st Order Births, SIPP data.

	Period I	Period II
	1966-1981	1984-2003
Age Group	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)
Age 22-44	-0.013 (0.017)	0.129 (0.017)
Age 22-25	-0.034 (0.035)	0.028 (0.041)
Age 26-44	0.019 (0.016)	0.130 (0.019)
Age 15-44	-0.019 (0.015)	0.094 (0.017)
Age 15-25	-0.039 (0.024)	-0.058 (0.032)

Note: Estimates the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a first birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  the percentage deviation of productivity from trend in period  $t - 4$ .

Table 2.22: Probability of Birth over the Business Cycle, by Education, SIPP data.

Age Group	Low Skilled		High Skilled	
	Period I	Period II	Period I	Period II
	1966-1981	1984-2003	1966-1981	1984-2003
	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)
Age 22-44	-0.004 (0.031)	0.232 (0.043)	-0.059 (0.041)	0.145 (0.034)
Age 22-25	-0.127 (0.065)	0.025 (0.101)	-0.101 (0.073)	-0.112 (0.067)
Age 26-44	0.055 (0.034)	0.223 (0.046)	-0.034 (0.050)	0.226 (0.040)
Age 15-44	-0.024 (0.027)	0.163 (0.040)	-0.047 (0.033)	0.050 (0.030)
Age 15-25	-0.105 (0.041)	-0.082 (0.068)	-0.055 (0.043)	-0.167 (0.048)

Note: Estimates the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ . Low skilled category includes women with high school degree or lower education attainment at time of interview. High skilled category includes women with some college or higher education attainment.

Table 2.23: Probability of Birth over the Business Cycle, 1st Order Births, by Education, SIPP data.

	Low Skilled		High Skilled	
	Period I	Period II	Period I	Period II
	1966-1981	1984-2003	1966-1981	1984-2003
Age Group	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)	$\hat{\beta}_1$ (s.e.)
Age 22-44	0.019 (0.019)	0.113 (0.026)	-0.029 (0.029)	0.126 (0.023)
Age 22-25	-0.026 (0.044)	0.079 (0.066)	-0.046 (0.059)	-0.004 (0.052)
Age 26-44	0.041 (0.018)	0.104 (0.026)	-0.020 (0.031)	0.143 (0.026)
Age 15-44	-0.001 (0.018)	0.116 (0.026)	-0.021 (0.024)	0.080 (0.022)
Age 15-25	-0.048 (0.031)	-0.012 (0.051)	-0.023 (0.035)	-0.073 (0.040)

Note: Estimates the linear probability model  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a first birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t-4$ . Low skilled category includes women with high school degree or lower education attainment at time of interview. High skilled category includes women with some college or higher education attainment.

Table 2.24: Benchmark Economy: Sensitivity Analysis.

Statistic	(B)	(1)	(2)	(3)	(4)	(5)	(6)
Fertility Rate, Age 22-25	0.221	0.278	0.151	0.232	0.211	0.197	0.235
ER w/ Infant, Age 22-33	0.146	0.144	0.159	0.153	0.162	0.166	0.160
ER, w/ Infant, Age 34-44	0.164	0.156	0.176	0.163	0.164	0.163	0.163
ER, no Children, Age 22-33	0.598	0.564	0.654	0.588	0.615	0.636	0.599
ER, no Children, Age 34-44	0.483	0.460	0.501	0.484	0.483	0.484	0.483
Share of Births, Age 22-25	0.601	0.676	0.418	0.619	0.589	0.504	0.613
Share of Births, Age 26-30	0.298	0.242	0.414	0.283	0.305	0.369	0.288
Share of Births, Age 31-35	0.059	0.048	0.099	0.058	0.062	0.075	0.059
Wage Growth, Age 22-33	0.026	0.026	0.026	0.026	0.026	0.026	0.026
Wage Growth, Age 34-44	0.013	0.013	0.013	0.013	0.013	0.013	0.013
Productivity, Std Deviation	0.015	0.015	0.015	0.015	0.015	0.015	0.015
Productivity, Persistence	0.459	0.447	0.469	0.464	0.471	0.474	0.455
$\hat{\beta}_1$ (Age 22-44)	-0.037	0.041	-0.196	-0.052	-0.021	0.185	-0.057
$\hat{\beta}_1$ (Age 22-25)	-0.130	0.074	-0.914	-0.276	-0.040	1.015	-0.464
$\hat{\beta}_1$ (Age 26-44)	0.002	0.015	-0.063	-0.022	-0.009	0.051	0.011
Debt Income Ratio	0.174	0.278	0.051	0.221	0.133	0.000	0.177
Wealth Income Ratio	1.295	0.672	2.247	1.125	1.322	1.602	1.284

Note: ER - Employment rate,  $\hat{\beta}_1$  is obtained estimating the following linear probability model:  $b_{it} = \beta_0 + \beta_1 d_{t-4} + \epsilon_{it}$ , where  $b_{it} = 1$  if woman  $i$  gives a first birth in period  $t$  and  $b_{it} = 0$  otherwise,  $d_{t-4}$  is the percentage deviation of productivity from trend in period  $t - 4$ . Column (B) shows the results of the benchmark model ( $\beta = 0.97$ ,  $a_{min} = -2$ ), column (1):  $\beta = 0.965$ , column (2):  $\beta = 0.975$ , column (3):  $a_{min} = -2.5$ , column (4):  $a_{min} = -1.5$ , column (5):  $a_{min} = 0$ , column (6): procyclical child care price.

## Chapter 3

# Taxation and Unemployment

### 3.1 Introduction

The facts describing the secular evolution of unemployment and taxes in the U.S. and continental Europe are well known. In the 1960s unemployment rates were quite similar in the U.S. and in the continental European countries. While the unemployment rate in the U.S. has remained at almost the same level until now, the rates in many European countries have increased starting in the late 1970s and stayed considerably higher than in the U.S. since then. At the same time the tax wedge, measured as the sum of labor and sales taxes, has increased in those European countries relative to the U.S.

A natural framework to understand the relationship between taxes and unemployment is the leading theory of equilibrium unemployment, the Mortensen and Pissarides (MP) search and matching model. However, this simple framework has an important limitation for studying the effects of policies, such as taxation. Productivity is assumed to be exogenous so that it does not respond to changes in tax rates. This seems restrictive both from an empirical and a theoretical perspective. Empirically, Prescott (2004), among others, documents that the increase in tax rates was accompanied by an increase in aggregate productivity (most notably in France and Germany) relative to productivity in the U.S.



Furthermore, we document that the skill premium, the relative productivity of college and high school graduates, is strongly negatively related to the tax wedge.

Theory also suggests that large differences in policy do not leave productivity and technology unaffected. The literature on induced technical change, pioneered by Acemoglu (2002, 2007) predicts a non-neutral shift in productivity in response to the change in relative abundance of productive inputs. If, for example, unemployed low-skilled labor becomes more abundant due to a change in the tax policy, technologies that are biased toward low-skilled labor and thus increase its productivity are more likely to be developed in the long run. In the theory developed in Krusell, Ohanian, Ríos-Rull, and Violante (2000), changes in productivity are due to a technology which features capital-skill complementarity. The adjustment of the stocks of capital as well as of high- and low-skilled labor in response to a change in policy lead to an endogenous change in productivity. This theory is a natural candidate to conduct a quantitative analysis with because changes in productivity can be accounted for by changes in observed factor quantities, most notably the stock of capital equipment. Thus, building on the standard MP setup we allow for ex-ante heterogeneity in skills (high-skilled and low-skilled workers) that interact on the production side of the economy as in Krusell, Ohanian, Ríos-Rull, and Violante (2000).

The endogenous response of productivity in our model has several important implications. First, in the standard MP model, a one percentage point permanent decrease in productivity and a one percentage point permanent increase in sales taxes increase unemployment by the same amount. The finding that these two responses are very close is not a coincidence but a feature of many models driven by productivity, including the MP model. However, the data suggest that the elasticity with respect to productivity necessary to replicate business cycles is considerably larger than the elasticity with respect to taxes required to explain cross-country differences (Costain and Reiter (2008), Mortensen

and Nagypal (2005), Hornstein, Krusell, and Violante (2005b)).<sup>1</sup> Our framework can resolve this dilemma. The endogenous response of productivity mitigates the policy response substantially without sacrificing the business cycle properties.

Second, Hornstein, Krusell, and Violante (2005a) have pointed out that the MP model has the counterfactual implication that the rise in unemployment in response to, e.g., skill-biased technical change is concentrated among the low-skilled workers<sup>2</sup>, whereas Nickell and Bell (1996) and Gottschalk and Smeeding (1997), among others, conclude that data from many European countries support the conclusion that unemployment rose proportionately across the entire skill spectrum. We show that the change in productivities in our model induced by an increase in the tax wedge shifts the rise in unemployment toward high skilled workers.

We calibrate the model following the strategy of Hagedorn and Manovskii (2008) and find that the two-skill version of the MP model is consistent with the cyclical volatility of the aggregate and group-specific labor market variables in the data. The model generates a high unemployment volatility among low-skilled workers because their productivity in the market is estimated to be relatively close to their productivity at home. The model also matches a high volatility of unemployment among high skilled workers despite the fact that their estimated value of non-market activity is substantially lower than their market productivity.<sup>3</sup>

To understand the cyclical behavior of labor market variables for different groups

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<sup>1</sup>For example, the MP model calibrated in Hagedorn and Manovskii (2008) generates the observed amount of volatility of unemployment and vacancies but generates very large policy effects.

<sup>2</sup>See, for example, Mortensen and Pissarides (1999) and Albrecht and Vroman (2002) for alternative models that share this prediction.

<sup>3</sup>This is consistent with the common prior articulated in e.g., Mortensen and Pissarides (1999) who argue that it is a “plausible assumption that the economic value of non-employment (other than UI benefits) does not increase proportionately with skill.” Moreover, they argue that the same is true of the UI benefits which are closer to the productivity level of less skilled workers.

of workers it is essential to identify the cyclical behavior of their productivities.<sup>4</sup> The aggregate production function estimated by Krusell, Ohanian, Ríos-Rull, and Violante (2000) provides the way to do so. This production function accounts exceptionally well for the trends in wages of skilled and unskilled workers over the last several decades. It thus appears to be a natural candidate to provide an accurate and parsimonious way to also measure the business-cycle properties of the marginal productivities of the two labor inputs it considers: high-skilled and low-skilled workers. Measuring the evolution of worker productivity using this production function, we find that the (endogenously determined) marginal product of high-skilled workers is considerably more volatile over the business cycle than the marginal product of low-skilled workers. One important reason for this finding is that Krusell, Ohanian, Ríos-Rull, and Violante (2000) estimate that high-skilled workers and capital equipment are complements in the production process. Since investment-specific shocks are an important contributor to business cycle fluctuations (Fisher (2006)), they amplify the volatility of productivity of high-skilled workers. This explains why the cyclical volatility of unemployment is high for high-skilled workers despite them having a relatively low value of non-market activities.

The paper is organized as follows. A discrete time stochastic version of the Pissarides (1985, 2000) search and matching model with two skill groups and capital-skill complementarity is laid out in Section 3.2. In Section 3.3 we develop our calibration strategy. In Section 3.4 we describe the quantitative behavior of the model over the business cycle, both in the aggregate and for both groups of workers. We find that the model matches the cyclical volatility of labor market variables very well. A comparison with the results from

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<sup>4</sup>We cannot use wages to infer the cyclical behavior of productivity because wages are not equal to the marginal product of labor in a search model. In most parameterizations of the MP model, including the one in this paper, the level of wages is very close to average productivity. The cyclical properties of wages, however, are different from the cyclical properties of productivity.

the homogeneous worker model (with exogenous productivity) implies that the model with worker heterogeneity generates higher volatility of aggregate labor market statistics and is closer to the data than the homogeneous worker model.

Having verified that the model is a good quantitative laboratory, we conduct the analysis of policy effects in Section 3.5. The analysis is subdivided in two parts. First, we analyze the effects of policies theoretically to better understand how the model works and what features of the model are important for dampening the effects of policies. One important result is that introducing curvature in the production side of the MP model is not sufficient *per se* to dampen the effects of policies. It is only if the production function includes heterogeneous and imperfectly substitutable labor inputs that the effects of policies will be dampened relative to the effects of cyclical movements in productivity. Next, we use the calibrated model to evaluate the effects of policies quantitatively. We find that the effects of policies are dampened substantially compared to the homogeneous agent version of the model, and are in line with the effects of policies implied by the data. Moreover, consistent with the U.S. and European experiences, higher taxes increase the productivity of low skilled workers and (slightly) decreases the productivity of high skilled workers, so that aggregate productivity increases and the skill premium decreases. The relative change in productivities also shifts the rise in unemployment toward high skilled workers. Since the productivity of low skilled workers increases in equilibrium, firms incentives to post vacancies for low skilled workers increases and thus lowers their unemployment rate whereas for high skilled workers the opposite holds. Section 3.6 concludes.

## 3.2 The Model

We consider a stochastic discrete time version of the Pissarides (1985, 2000) search and matching model with aggregate uncertainty and workers of two types  $T \in \{L, H\}$ , referring

to low- and high-skilled workers, respectively.

### 3.2.1 Workers and Firms

There are measures  $N^T$  of infinitely lived workers of each type and a continuum of infinitely lived firms. Workers maximize their expected lifetime utility:

$$\mathbb{E} \sum_{t=0}^{\infty} \delta^t y_t^T, \quad (3.1)$$

where  $y_t^T$  represents income in period  $t$  and  $\delta \in (0, 1)$  is workers' and firms' common discount factor.

There is a competitive final goods sector that combines 4 inputs to produce the final good - low-skilled labor  $l_t$ , high-skilled labor  $h_t$ , capital structures  $k_{st}$  and capital equipment  $k_{et}$  - through the following production function (Krusell, Ohanian, Ríos-Rull, and Violante (2000)):

$$y_t = F(l_t, h_t, k_{st}, k_{et}) = A_t k_{st}^\alpha \left[ \mu l_t^\sigma + (1 - \mu) (\lambda k_{et}^\rho + (1 - \lambda) h_t^\rho)^{\frac{\sigma}{\rho}} \right]^{\frac{1-\alpha}{\sigma}}, \quad (3.2)$$

where  $A_t$  is a neutral technology shock.

The resource constraint is

$$F(t) = C_t + i_{st} + \frac{i_{et}}{q_t}, \quad (3.3)$$

where  $i_{st}$  is investment in capital structures,  $i_{et}$  is investment in capital equipment,  $C_t$  is consumption, and where the technology parameter  $q_t$  determines the amount of equipment that can be produced by one unit of final output. In a perfectly competitive market,  $q_t$  is also the relative price between consumption and equipment, a feature we exploit to measure  $q$  in the calibration (as in Greenwood, Hercowitz, and Krusell (1997) and Krusell, Ohanian, Ríos-Rull, and Violante (2000)). The two stocks of capital evolve according to

the following dynamic equations:

$$k_{s,t+1} = (1 - d_s)k_{st} + i_{st} \quad (3.4)$$

$$k_{e,t+1} = (1 - d_e)k_{et} + i_{et}, \quad (3.5)$$

where  $d_e$  and  $d_s$  are the depreciation rates of capital equipment and capital structures respectively.

Both  $A_t$  and  $q_t$  are assumed to follow AR(1) processes,

$$A_t = \bar{A} + \kappa_A A_{t-1} + \epsilon_{A,t}, \quad (3.6)$$

$$q_t = \bar{q} + \kappa_q q_{t-1} + \epsilon_{q,t}. \quad (3.7)$$

The two shocks,  $\epsilon_{A,t}$  and  $\epsilon_{q,t}$  are independent normal variables with respective standard deviations  $\eta_A$  and  $\eta_q$ .

Each firm operating in the intermediate goods sector is either matched with an unskilled worker, matched with a skilled worker or posts a vacancy. If matched, it receives, from the competitive final sector,  $p_{lt} = F_l(t)$  or  $p_{ht} = F_h(t)$ . There is free entry of firms. Firms attract unemployed workers by posting a vacancy at the flow cost  $c^T$ . Once matched, workers and firms separate exogenously with probability  $s^T$  per period. Employed workers are paid a wage  $w_t^T$ , and firms make accounting profits of  $p_t^T - w_t^T$  per worker each period in which they operate. Unemployed workers get flow utility  $z^T$  from leisure/non-market activity.

### 3.2.2 Matching

Let  $u_t^T$  denote the number of unemployed people and  $n_t^T = N^T - u_t^T$  the number of employed people from group  $T$  ( $n^L = l$  and  $n^H = h$ ). Let  $v_t^T$  be the number of vacancies posted in period  $t$ . We refer to  $\theta_t^T = v_t^T / u_t^T$  as the market tightness at time  $t$  for type  $T$ . The aggregate market tightness is defined as  $\theta_t = (v_t^H + v_t^L) / (u_t^H + u_t^L)$ .

The number of new matches (starting to produce output at  $t + 1$ ) is given by a constant returns to scale matching function  $m^T(u_t^T, v_t^T) \leq \min(u_t^T, v_t^T)$ . Employment evolves according to the following law of motion:

$$n_{t+1}^T = (1 - s^T)n_t^T + m^T(u_t^T, v_t^T). \quad (3.8)$$

The probability that an unemployed worker will be matched with a vacancy next period equals  $f^T(\theta_t^T) = m^T(u_t^T, v_t^T)/u_t^T = m^T(1, \theta_t^T)$ . The probability that a vacancy will be filled next period equals  $\phi^T(\theta_t^T) = m^T(u_t^T, v_t^T)/v_t^T = m^T(1/\theta_t^T, 1) = f^T(\theta_t^T)/\theta_t^T$ .

### 3.2.3 Equilibrium

Denote the firm's value of a job (a filled vacancy) by  $J^T$ , the firm's value of an unfilled vacancy by  $V^T$ , the worker's value of having a job by  $W^T$ , and the worker's value of being unemployed by  $U^T$ . Bellman equations (3.9)-(3.12) describe an equilibrium of the model where  $J^T$ ,  $W^T$ ,  $U^T$  and  $V^T$  depend on the current shocks to productivity  $A_t$  and  $q_t$  and the stock of low-skilled  $l_t$  and the stock of high-skilled  $h_t$ . Let  $x_t = (A_t, q_t, l_t, h_t)$  be today's state vector and  $x_{t+1} = (A_{t+1}, q_{t+1}, l_{t+1}, h_{t+1})$  be next period's state vector. The two capital stocks  $k_e$  and  $k_s$  do not have to be included in the state vector, since risk neutrality implies that they are already functions of  $x$ .<sup>5</sup>

$$J_{x_t}^T = p_{x_t}^T - w_{x_t}^T + \delta(1 - s^T)E_{x_t}J_{x_{t+1}}^T \quad (3.9)$$

$$V_{x_t}^T = -c^T + \delta\phi^T(\theta_{x_t}^T)E_{x_t}J_{x_{t+1}}^T \quad (3.10)$$

$$U_{x_t}^T = z_t^T + \delta\{f^T(\theta_{x_t}^T)E_{x_t}W_{x_{t+1}}^T + (1 - f^T(\theta_{x_t}^T))E_{x_t}U_{x_{t+1}}^T\} \quad (3.11)$$

$$W_{x_t}^T = w_{x_t}^T + \delta\{(1 - s^T)E_{x_t}W_{x_{t+1}}^T + s^T E_{x_t}U_{x_{t+1}}^T\}. \quad (3.12)$$

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<sup>5</sup>The two first-order conditions for capital equipment and capital structures describe period  $t$  capital stocks as functions of  $x_t$  only because risk neutrality implies that the real interest rate is constant. Without risk neutrality this simplification would not be possible, since the period  $t$  interest rate would depend on consumption in period  $t$  and  $t + 1$ .

The interpretation is straightforward. Operating firms earn profits  $p_{x_t}^T - w_{x_t}^T$  and the matches are exogenously destroyed with probability  $s^T$ . A vacancy costs  $c^T$  and is matched with a worker (becomes productive in period  $t + 1$ ) with probability  $\phi^T(\theta_{x_t}^T)$ . An unemployed worker derives utility  $z^T$  and finds a job next period with probability  $f^T(\theta_{x_t}^T)$ . An employed worker earns wage  $w_{x_t}^T$  but may lose her job with probability  $s^T$  and become unemployed next period. Nash bargaining implies that a worker and a firm split the surplus  $S_{x_t}^T = J_{x_t}^T + W_{x_t}^T - U_{x_t}^T$  such that

$$J_{x_t}^T = (1 - \beta^T)S_{x_t}^T, \quad (3.13)$$

$$W_{x_t}^T - U_{x_t}^T = \beta^T S_{x_t}^T. \quad (3.14)$$

Free entry implies that the value of posting a vacancy is zero:  $V_{x_t}^T = 0$  for all  $x_t$  and, therefore,

$$\begin{aligned} c^T &= \delta \phi^T(\theta_{x_t}^T) E_{x_t} J_{x_{t+1}}^T \\ &= \delta \phi^T(\theta_{x_t}^T) (1 - \beta^T) E_{x_t} S_{x_{t+1}}^T. \end{aligned} \quad (3.15)$$

The Bellman equation for the surplus is:

$$S_{x_t}^T = p_{x_t}^T - (z^T + \delta f^T(\theta_{x_t}^T) \beta^T E_{x_t} S_{x_{t+1}}^T) + \delta (1 - s^T) E_{x_t} S_{x_{t+1}}^T. \quad (3.16)$$

To compute expectations, one has to know how the state variables evolve. The two productivity processes evolve according to the VAR(1) described above. The value of marginal productivity  $p^T$  next period is endogenous and depends on how many workers are working today, how many vacancies are posted and how much capital is invested.

The market for capital equipment and structures is perfectly competitive and, each period, firms can rent capital to maximize profits. Households own the capital stock and



invest to maximize their utility, which leads to the two first-order conditions for capital:<sup>6</sup>

$$E_t F_{k_s}(t+1) + (1 - d_s) = \frac{1}{\delta}, \quad (3.17)$$

$$q_t E_t F_{k_e}(t+1) + (1 - d_e) E_t \frac{q_t}{q_{t+1}} = \frac{1}{\delta}. \quad (3.18)$$

Note, that the decision on  $k_{e,t+1}$  is taken in period  $t$ , but that the relative price of investment goods next period,  $q_{t+1}$ , matters for this decision as well.

We now derive the expressions for equilibrium wages and profits, which, except for being dependent on the type, take the usual form.<sup>7</sup> Because firms can buy and sell capital in a competitive market, the wage bargain is not affected as in Pissarides (2000). Using equation (3.13), it follows from the free-entry condition (3.15) and the flow equation (3.9) for  $J^T$  that:

$$(1 - \beta^T) S_{x_t}^T = p_{x_t}^T - w_{x_t}^T + (1 - s^T) c^T / \phi^T(\theta_{x_t}^T). \quad (3.19)$$

Free entry and (3.16) imply that

$$S_{x_t}^T = p_{x_t}^T - z^T + (1 - s^T - f^T(\theta_{x_t}^T)\beta) \frac{c^T}{\phi^T(\theta_{x_t}^T)(1 - \beta^T)}. \quad (3.20)$$

Thus, we have that

$$(1 - \beta^T) S_{x_t}^T = (1 - \beta^T)(p_{x_t}^T - z^T) + c^T \frac{1 - s^T - f^T(\theta_{x_t}^T)\beta^T}{\phi^T(\theta_{x_t}^T)}. \quad (3.21)$$

Rearranging (3.19) and substituting using (3.21), we find that wages are given by

$$\begin{aligned} w_{x_t}^T &= p_{x_t}^T - (1 - \beta^T) S_{x_t}^T + (1 - s^T) c^T / \phi^T(\theta_{x_t}^T) \\ &= \beta^T p_{x_t}^T + (1 - \beta^T) z^T + c^T \beta^T \theta_{x_t}^T, \end{aligned} \quad (3.22)$$

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<sup>6</sup>To see the second equation note that the  $k_{e,t+1}$  is chosen to maximize  $\dots - \frac{k_{e,t+1}}{q_t} + \delta E_t(r_{t+1} k_{e,t+1} + \frac{(1-d_e)k_{e,t+1}}{q_{t+1}}) + \dots$ , where  $r = F_{k_e}$  is the interest rate in the rental market.

<sup>7</sup>It is well known that only the present value of wages and not the specific sequence of wages matters. We adopt here the standard assumption of Nash bargaining to pin down this sequence. Hagedorn and Manovskii (2009) provide evidence that this assumption is more consistent with the data than the alternative based on contracts through which firms insure workers against aggregate shocks.

and accounting profits are given by

$$\Pi_{x_t}^T = p_{x_t}^T - w_{x_t}^T = (1 - \beta^T)(p_{x_t}^T - z^T) - c^T \beta^T \theta_{x_t}^T. \quad (3.23)$$

### 3.3 Calibration

In this section we calibrate the model to match U.S. labor market facts. We define the variables consistently with Krusell, Ohanian, Ríos-Rull, and Violante (2000) and conduct a measurement that ensures the comparability of our results to the large body of existing work on the cyclical behavior of unemployment and vacancies. In particular, we measure capital structures and equipment, output and employment in the non-farm business sector. As in Krusell, Ohanian, Ríos-Rull, and Violante (2000), the sample is restricted to individuals between 16 and 70 years old. The unskilled category includes individuals who have a high school diploma or less. The skilled category includes college-educated workers. Labor market data for the two subgroups comes from the monthly Current Population Surveys (CPS) from January 1976 to December 2006 and the CPS Outgoing Rotation Groups (ORG) covering the period January 1979 to December 2006. To aggregate individual observations we use CPS sample weights. On average over the sample period there are 2.6379 unskilled workers for each skilled worker. Whenever we are interested in cyclical properties of a variable observed at quarterly frequency, we use the HP-filter (Prescott (1986)) with a smoothing parameter of 1600. The data and variable construction procedures we use are detailed in Appendix 3.7.1.

**Basics.** We choose the model period to be one week (one-twelfth of a quarter), which is lower than the frequency of the employment data we use, but necessary to deal with time aggregation. The data used to compute some of the targets have monthly, quarterly or annual frequency, and we aggregate the model appropriately when matching those targets.

We set  $\delta = 0.99^{1/12}$ .

**Job-Finding and Separation Rates.** Using the CPS, we estimate, using the Shimer (2005b) two-state model described in Appendix 3.7.1, the average monthly job-finding rate to be 0.3618 for skilled workers and 0.4185 for unskilled workers. The total separation rate (into unemployment, non-employment and job-to-job), not adjusted for time aggregation, for high-skilled equals 0.042 and for low-skilled 0.064 (Fallick and Fleischman (2004)). The separation rate into unemployment, also not adjusted for time aggregation, equals 0.0097 for the skilled and 0.0378 for the unskilled. We make this distinction between the rates of total separation and separation into unemployment, since what matters for firms' decisions is the expected duration of an employment spell, and this duration depends on the total rate of separation. We use this separation rate when modeling firms' decisions. To describe the average level and the evolution of unemployment for the two groups (Equation 3.8) we use the separation rate into unemployment only.

At a weekly frequency these estimates imply job-finding rates of  $f^H = 0.1062$  and  $f^L = 0.1268$ , total job separation rates of  $s^H = 0.0105$  and  $s^L = 0.016$ , rates of separation into unemployment  $s_U^H = 0.0029$  and  $s_U^L = 0.0117$ , and steady state unemployment rates of  $u^H = s_U^H / (s_U^H + f^H) = 0.0262$  and  $u^L = s_U^L / (s_U^L + f^L) = 0.0846$ .<sup>8</sup> These steady state unemployment rates are very similar to the average unemployment rate in the data of

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<sup>8</sup>We now illustrate this adjustment procedure in the case of skilled workers. The probability of not finding a job within a month is  $1 - 0.3603 = 0.6382$ . The probability of not finding a job within a week then equals  $0.6382^{1/4} = 0.8938$  and the probability of finding a job equals  $1 - 0.8938 = 0.1062$ . The probability of observing someone not having a job who had a job one month ago equals (counting paths in a probability tree):  $s\{(1-f)(fs+(1-f)^2)+f(s(1-f)+(1-s)s)\}+(1-s)\{s(fs+(1-f)^2)+(1-s)(s(1-f)+(1-s)s)\} = 0.0097$ . Solving for  $s$ , we obtain  $s = 0.0029$ .

The *total* separation rate does not have to be adjusted for time aggregation, since it does not matter whether a worker switches employers once or multiple times between observation points. All we need to know is that the previous employment relationship ended.

0.0263 for skilled workers and 0.0838 for unskilled ones.<sup>9</sup>

Petronglo and Pissarides (2001) survey the empirical evidence and conclude that the value of 0.5 for the elasticity of the aggregate job-finding rate with respect to aggregate labor market tightness is appropriate. By skill group, the elasticity of the job-finding probability with respect to overall labor market tightness is higher for high-skilled workers by a factor of 1.3345.

**Production Function Parameters.** We use the elasticity parameters of the production function  $\alpha = 0.117$ ,  $\sigma = 0.401$ , and  $\rho = -0.495$  and weekly depreciation rates of structures and equipment  $d_s = 0.001068$  and  $d_e = 0.002778$  estimated by Krusell, Ohanian, Ríos-Rull, and Violante (2000). Given the values of these parameters and the average employment levels of high- and low-skilled workers, we normalize the average stock of capital structures,  $k_s = 399.7251$ , capital equipment,  $k_e = 389.8385$ , and aggregate productivity  $A = 0.4197$ , and find the distribution parameters  $\lambda = 0.9341$  and  $\mu = .7445$  as solutions to a system of five equations. The system includes the first-order conditions (3.17) and (3.18) for structures and equipment, the normalization that the marginal product of low-skilled labor is equal to 1, the condition that the labor share is 2/3 of output, and the condition that the ratio of the marginal products of skilled and unskilled workers is equal to 1.9846, on average.<sup>10</sup>

The productivity of the two labor inputs is affected by the volatility of capital structures and equipment over the business cycle. In the data, the standard deviation of HP-filtered

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<sup>9</sup>Those workers who get separated from firms but do not become unemployed can be thought of as being hired by a large firm or by the government. This hiring presumably involves no search frictions due to the sheer size of these employers. These large firms hire at a constant rate  $s^T - s_U^T$  and workers get separated at rate  $s_U^T$  into unemployment.

<sup>10</sup>The last target is consistent with the competitive model but may not hold exactly in the model with search frictions. This theoretical inconsistency has a negligible impact on our findings because, in our calibration, the average wage is close to the marginal product.

log capital structures is 0.0028 and the standard deviation of HP-filtered log quality-adjusted capital equipment is 0.0100 (see Appendix 3.7.1). To ensure that the model matches the cyclical volatility of the capital series, we allow the depreciation rates for capital structures and equipment to depend on aggregate productivity. In particular, we introduce and calibrate a parameter  $d_e^*$  and specify the depreciation of capital equipment at time  $t$  to equal  $d_e * (k_{e,t}/\bar{k}_e)^{d_e^*}$ . Thus, if equilibrium capital equipment stock  $k_{e,t}$  in period  $t$  is equal to the average capital equipment  $\bar{k}_e$ , the depreciation rate is given by  $d_e$ . If capital in some periods deviates from its steady state value, the depreciation rate deviates in the same direction. The strength of the response of the depreciation rate is governed by parameter  $d_e^*$ . The depreciation rate for capital structures is defined symmetrically with parameters  $d_s$  and  $d_s^*$ .<sup>11</sup>

**Neutral and Capital Equipment-Specific Technologies.** We use the estimated production function parameters and compute the quarterly series for  $A_t$  and  $q_t$ . We set  $q_t$  equal to the NIPA price of consumption goods (non-durables and services),  $p_t^c$ , divided by the price of equipment investment goods,  $p_t^e$ . We use the  $p_t^e$  series constructed by Schorfheide, Rios-Rull, Fuentes-Albero, Santaaulalia-Llopis, and Kryshko (2007). (They extend the annual series of Cummins and Violante (2002) to 2006 and convert the annual series to quarterly frequency similar to Fisher (2006)). We use the resulting price series to construct the quality-adjusted stock of capital equipment using the perpetual inventory

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<sup>11</sup>The only reason for variable depreciation rates is to generate the right volatilities of the two capital stocks. Alternatively we could treat the capital stocks as exogenous and just calibrate the two processes. This approach would leave our quantitative analysis unchanged. However, we could not conduct any policy experiments, a main objective of this paper, since the capital stock responds to changes in taxation in our environment but would not if capital is exogenous. Departing from linear utility would also reduce the volatility of capital and capital would be an endogenous variable. However, the data imply an asymmetric adjustment of the volatilities of the two capital stocks whereas a departure from linear utility would presumably lead to a symmetric reduction.

method.

We log and linearly de-trend the  $A_t$  and  $q_t$  series and use the resulting series to estimate the VAR in (3.6) and (3.7). To calibrate this process in the model, we consider quarterly averages of weekly productivity. We find that at weekly frequency we must set  $\kappa_A = 0.9936$ ,  $\kappa_q = 0.9988$ ,  $\epsilon_A = 0.0035$ , and  $\epsilon_q = 0.0019$  to match the process in the data. We also normalize the average  $\bar{q} = 1$  and the average productivity of an unskilled worker equal to one, which requires setting  $\bar{A} = .4197$ .

**Labor Market Tightness.** Hagedorn and Manovskii (2008) estimate an average value of labor market tightness of 0.634. This value lies between the estimates of 0.539 obtained by Hall (2005) and 0.72 obtained by Pissarides (2007).

In Hagedorn and Manovskii (2008) we used data on the time and costs involved in recruiting workers from the 1982 Employment Opportunity Pilot Project survey and the 1992 Small Business Administration survey reported in Barron, Berger, and Black (1997). These authors also estimate the vacancy duration equation  $D = c_0 + c_1X$  using the same datasets, where  $D$  is the log of the duration time and  $X$  is the set of controls including the log number of years of education, and report that the education coefficient is statistically significant in both datasets and equal to 0.886 and 2.432, respectively. The average years of education in our sample for high-skilled and low-skilled workers are equal to  $ed^H = 16.54$  and  $ed^L = 10.83$ , respectively. This implies that vacancies for high skilled workers last  $d^{rel} = (ed^H/ed^L)^{c_1} = 2.128$  times longer, where the actual number represents the average across the two data sets. The ratio of the market tightnesses across groups is then given by  $\theta^{rel} = f^H/f^L * d^{rel} = 1.78$ . Finally, using the data on the numbers of skilled and unskilled unemployed workers, the aggregate  $\theta$ , and the relative  $\theta^{rel}$  of high skilled workers we obtain that  $\theta^L = 0.5858$  and  $\theta^H = 1.0442$ .

**Matching Functions.** We choose the Cobb-Douglas functional form of the matching functions of skilled and unskilled workers:

$$m(u^T, v^T) = \chi^T (u^T)^{\gamma^T} (v^T)^{1-\gamma^T}. \quad (3.24)$$

The two parameters,  $\chi^T$ ,  $\gamma^T$ , that characterize the matching function differ for the two types. This allows us to match a different job-finding probability and a different elasticity of the job-finding probability with respect to labor market tightness.

**The Cyclicity of Wages.** Over the 1979:1-2006:4 period we find that a 1-percentage-point increase in labor productivity is associated with a 0.674-percentage-point increase in average real wages. Wages are measured as the non-farm business labor share constructed by the BLS times labor productivity defined as seasonally adjusted real non-farm business output constructed by the BLS from the NIPA divided by seasonally adjusted non-farm business employment from the monthly Current Population Survey. Both time series are in logs and HP-detrended. We also use CPS data to estimate the wage elasticity with respect to average output per person for each group separately. We find that wages for high-skilled workers are more cyclical than wages of low-skilled. The ratio of the two elasticities equals 1.771. The estimates reported in Castro and Coen-Pirani (2008) imply a very similar ratio of wage elasticities. To obtain the corresponding estimates in the model, we first aggregate the weekly model-generated data to replicate the quarterly frequency of the data. We then log and HP-filter the time series and estimate regressions identical to those estimated in the data.<sup>12</sup>

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<sup>12</sup>Keane and Prasad (1991) and Prasad (1996) report, using NLSY from 1966-1981, similar magnitudes for the cyclicity of wages of skilled and unskilled workers. We replicated their analysis using the NLSY 1979 data (over the 1979:1-2006:4 period that corresponds to the coverage of the CPS data we use). See Hagedorn and Manovskii (2009) for a detailed description of NLSY 1979 and the variable construction procedures. We found a ratio of the wage elasticities for high and low skilled workers that is remarkably close to the number we computed based on the CPS data. These findings suggest that the properties of

**The Costs of Posting Vacancies.** In Hagedorn and Manovskii (2008) we found that the expected labor costs of posting vacancies equals 50.23% of average weekly labor productivity. The flow capital costs of posting vacancies equals 47.4% of average weekly labor productivity, which equals 1.2707, so that the capital costs equal 0.6023. The analysis from Hagedorn and Manovskii (2008) for these average numbers applies here as well. However, the presence of capital-skill complementarity and two types of capital implies that the numbers for the two groups are different.

For labor costs it is simple. We find that the skill premium in the data equals 1.9846. The expected costs of a vacancy in the model equals  $\frac{c_W^T}{\phi^T}$ , where  $c_W^T$  is the flow cost and  $\phi^T$  is the probability of filling a vacancy. The numbers we report above imply that  $\phi^H = 0.1017$  and  $\phi^L = 0.2165$ . Solving  $\frac{c_W^L}{\phi^L} = 0.5023$  and  $\frac{c_W^H}{\phi^H} = 1.9846 \cdot 0.5023$ , we find  $c_W^H = 0.1014$  and  $c_W^L = 0.1087$ .

The specification of the production function in Krusell, Ohanian, Ríos-Rull, and Violante (2000) features capital-skill complementarity, so that more capital is bought when a high-skilled worker is hired than when a low-skilled worker is hired. The relative sizes of capital equipment and capital structures needed can be computed from the first-order conditions (3.17) and (3.18). For skilled workers, the implicit function theorem provides us with two functions  $k_s(h)$  and  $k_e(h)$  solving the two first-order conditions, keeping the number of unskilled workers fixed. Analogously for unskilled workers, we get two functions  $k_s(l)$  and  $k_e(l)$ . The relative capital needs for capital equipment then equals  $\frac{\partial k_e(h)}{\frac{\partial h}{\partial k_e(l)}}$  and the relative capital needs for capital structures equals  $\frac{\partial k_s(h)}{\frac{\partial h}{\partial k_s(l)}}$ . Evaluating these expressions at the steady state gives  $\frac{\partial k_e(h)}{\frac{\partial h}{\partial k_e(l)}} = 8.3384$  and  $\frac{\partial k_s(h)}{\frac{\partial h}{\partial k_s(l)}} = 1.9846$ .

We can now compute the flow capital costs for high-skilled  $c_e^H$  (for equipment) and wages shifted in the early 1980s. This is also consistent with the finding of Parker and Vissing-Jorgensen (2009), who report that the relative volatility of labor income of high earners (likely correlated with being more educated) increased sharply around the early 1980s.



$c_s^H$  (for structures) and for low skilled:  $c_e^L$  (for equipment) and  $c_s^L$  (for structures). The different capital needs imply that  $c_e^H = 8.3384c_e^L$  and  $c_s^H = 1.9846c_s^L$ .

The average flow cost for equipment equals  $c_e^H \frac{v^H}{v^H+v^L} + c_e^L \frac{v^L}{v^H+v^L}$  and that for structures equals  $c_s^H \frac{v^H}{v^H+v^L} + c_s^L \frac{v^L}{v^H+v^L}$ . Since the capital income share for structures equals 0.117 and that for equipment equals  $(1/3 - 0.117)$  we solve

$$c_e^H \frac{v^H}{v^H+v^L} + c_e^L \frac{v^L}{v^H+v^L} = \frac{1/3 - 0.117}{2/3} 0.6023 \quad (3.25)$$

$$c_s^H \frac{v^H}{v^H+v^L} + c_s^L \frac{v^L}{v^H+v^L} = \frac{0.117}{1/3} 0.6023 \quad (3.26)$$

We find  $c_e^H = 1.4359$ ,  $c_s^H = 0.3585$ ,  $c_e^L = 0.1722$  and  $c_s^L = 0.1806$ . Thus, overall, the flow costs of posting a vacancy for high-skilled workers equals  $c^H = 1.4359 + 0.3585 + 0.1014 = 1.8958$  and for low-skilled workers it equals  $c^L = 0.1722 + 0.1806 + 0.1087 = 0.4615$ .

**Remaining Parameters.** Ten parameters remain to be determined: the values of non-market activity,  $z^H$ ,  $z^L$ , worker's bargaining weights,  $\beta^H$ ,  $\beta^L$ , the matching function parameters,  $\chi^H$ ,  $\chi^L$ ,  $\gamma^H$ ,  $\gamma^L$ , and depreciation factors for capital structures and equipment,  $d_s^*$ ,  $d_e^*$ . We choose the values for these parameters to match the data on the average value for labor market tightness for skilled and unskilled workers, the elasticity of wages with respect to aggregate productivity, the relative elasticity of wages with respect to aggregate productivity of skilled and unskilled workers, the average values for the job-finding rates of skilled and unskilled workers, the elasticity of the aggregate job-finding rate with respect to aggregate labor market tightness, the relative elasticity of the job-finding rate with respect to aggregate labor market tightness of skilled and unskilled workers, and the standard deviations of capital structures and equipment. Thus, there are ten targets, all described in the previous paragraphs, to pin down ten parameters.

To find the values of these parameters we solve the model numerically according to the computational algorithm described in Appendix 3.7.2. The performance of the model in

matching calibration targets is described in Table 3.1. We are able to match the targets almost exactly. Calibrated parameter values can be found in Table 3.2. To understand these results, it is useful to recall how the two key parameters - the bargaining power and the value of non-market activity - are determined in the homogeneous worker case (Hagedorn and Manovskii (2008)). The bargaining power is chosen to match the elasticity of wages, since a higher bargaining power of workers makes wages more responsive to changes in productivity. The level of non-market activity is then chosen to match the average level of wages. The average level of wages, holding fixed other parameters such as the separation rate and the interest rate, depends one-for-one on expected hiring costs  $c/\phi$ , since a higher level of expected costs requires higher profits and thus lower average wages. The same logic applies here. Since expected vacancy posting costs  $c/\phi$  are about four times higher for high-skilled workers than for low-skilled workers (relative to productivity),  $z^H$  is substantially lower than  $z^L$  (relative to productivity). The bargaining power is again chosen to match the elasticity of wages with one modification. We match the elasticity of wages with respect to average productivity and not with respect to marginal productivity, since the latter is not directly observable. For the targeted elasticity it holds that  $\epsilon_{w^T,p} = \epsilon_{w^T,p^T} \cdot \epsilon_{p^T,p}$ , which makes a difference, since  $\epsilon_{p^T,p}$  does not equal one ( $\epsilon_{x,y}$  denotes the elasticity of  $x$  with respect to  $y$ ). We find that  $\epsilon_{p^H,p} = 1.316$  and  $\epsilon_{p^L,p} = 0.935$ , since changes in capital equipment mainly affect  $p^H$ . In equilibrium, the effect due to a higher volatility of productivity for high-skilled workers outweighs the effect due to a higher productivity elasticity of their wages, implying a lower bargaining power for them compared to low-skilled workers. A similar reasoning applies to measuring the elasticity of the matching function. It is identified by the elasticity of the job-finding rate with respect to the aggregate market tightness  $\theta$  because  $\theta^T$  is not observable. It holds that

Table 3.1: Matching the Calibration Targets.

Target	Value	
	Data	Model
1. Elasticity of wages wrt agg. productivity, $\epsilon_{w,p}$	0.674	0.671
2. Relative elasticity of wages wrt agg. productivity, $\epsilon_{w^H,p}/\epsilon_{w^L,p}$	1.770	1.774
3. Skilled job-finding rate, $f^H$	0.106	0.105
4. Unskilled job-finding rate, $f^L$	0.127	0.126
5. Skilled average market tightness, $\theta^H$	1.044	1.047
6. Unskilled average market tightness, $\theta^L$	0.586	0.585
7. Elasticity of agg. job-finding wrt agg. market tightness, $\epsilon_{f,\theta}$	0.500	0.513
8. Relative elas. of job-finding wrt agg. mrkt tightness, $\epsilon_{f^H,\theta}/\epsilon_{f^L,\theta}$	1.335	1.332
9. Standard deviation of capital structures	0.003	0.003
10. Standard deviation of capital equipment	0.010	0.010

Note: The table describes the performance of the model in matching the calibration targets.

$\epsilon_{f^T,\theta} = \epsilon_{f^T,\theta^T} \cdot \epsilon_{\theta^T,\theta}$ . We find that  $\epsilon_{\theta^H,\theta} = 0.837$  and  $\epsilon_{\theta^L,\theta} = 1.056$ .<sup>13</sup> The choice of the remaining parameters is simple. The matching function efficiency parameter  $\chi^T$  determines the job finding rate and the depreciation factors are chosen to match the standard deviations of capital structures and equipment.

### 3.4 Business-Cycle Properties of the Model

The statistics of interest, computed from quarterly U.S. data from 1979:1-2006:4 and the results from the calibrated model are presented in Table 3.3.

**Aggregate Results.** A comparison between the corresponding statistics reveals that

<sup>13</sup>Note that  $f^T(\theta^T) = \chi^T(\theta^T)^{1-\gamma^T}$  and  $\frac{(1-\gamma^H)\epsilon_{\theta^H,\theta}}{(1-\gamma^L)\epsilon_{\theta^L,\theta}} = \frac{0.801 \cdot 0.837}{0.471 \cdot 1.056} = 1.348$ , very close to the target for  $\epsilon_{f^H,\theta}/\epsilon_{f^L,\theta}$ . The small difference arises since we compute our targets on model generated data.

Table 3.2: Calibrated Parameter Values.

Parameter	Definition	Value
$z^H$	skilled value of non-market activity (share of their productivity)	0.813
$z^L$	unskilled value of non-market activity (share of their productivity)	0.929
$\beta^H$	skilled workers' bargaining power	0.069
$\beta^L$	unskilled workers' bargaining power	0.112
$\gamma^H$	skilled matching function elasticity	0.199
$\gamma^L$	unskilled matching function elasticity	0.529
$\chi^H$	skilled matching function efficiency	0.102
$\chi^L$	unskilled matching function efficiency	0.164
$d_s^*$	depreciation factor of capital structures	11.200
$d_e^*$	depreciation factor of capital equipment	1.399

Note: The table contains the calibrated parameter values in the benchmark calibration.

the model matches the key business-cycle facts quite well. In particular, the volatility of aggregate labor market tightness, unemployment, and vacancies is quite close to that in the data. Moreover, the model generates a strong negative correlation between unemployment and vacancies, i.e., the Beveridge curve.

**Results by Skill Group.** In the data the unemployment rate is 2.6% for skilled workers and 8.4% for unskilled ones. Both of these rates are highly volatile, with respective standard deviations of the HP-filtered logged unemployment rate of 0.111 and 0.085. Thus, while low-skilled workers account for most of the fluctuations in unemployment, the unemployment rate of skilled workers is even more volatile in percentage terms.<sup>14</sup>

The model does an excellent job in matching these observations. It generates unem-

<sup>14</sup>Interestingly, Castro and Coen-Pirani (2008) show that over the time period that we study even *employment* of skilled workers is more volatile than that of low-skilled workers.

Table 3.3: Data and Results from the Calibrated Model.

Statistic	Value	
	Data	Model
1. St. dev. of agg. productivity, $p$	0.013	0.013
2. Autocorr. of agg. productivity, $p$	0.765	0.765
3. St. dev. of agg. unemployment, $u$	0.090	0.086
4. St. dev. of agg. vacancies, $v$	0.116	0.110
5. St. dev. of agg. market tightness, $\theta$	0.202	0.196
6. Corr. of agg. unemployment and vacancies	-0.910	-0.777
1. St. dev. of skilled productivity, $p^H$	—	0.018
2. Autocorr. of skilled productivity, $p^H$	—	0.782
3. St. dev. of skilled unemployment, $u^H$	0.111	0.114
4. St. dev. of skilled vacancies, $v^H$	—	0.078
5. St. dev. of skilled market tightness, $\theta^H$	—	0.162
1. St. dev. of unskilled productivity, $p^L$	—	0.013
2. Autocorr. of unskilled productivity, $p^L$	—	0.763
3. St. dev. of unskilled unemployment, $u^L$	0.085	0.083
4. St. dev. of unskilled vacancies, $v^L$	—	0.133
5. St. dev. of unskilled market tightness, $\theta^L$	—	0.206

Note: Seasonally adjusted aggregate unemployment,  $u$ , is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). Seasonally adjusted skill-group unemployment,  $u^H$  and  $u^L$ , is constructed by the authors from the monthly Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index,  $v$ , is constructed by the Conference Board.  $u$ ,  $u^H$ ,  $u^L$ , and  $v$  are quarterly averages of monthly series. Average labor productivity  $p$  is seasonally adjusted quarterly real non-farm business output constructed by the BLS from the NIPA divided by non-farm business employment from the monthly Current Population Survey. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

ployment rates of 2.6% for skilled workers and 8.4% for unskilled ones, with respective standard deviations of the HP-filtered logged unemployment rate of 0.114 and 0.083. To understand these results we compute these statistics twice for two economies populated by homogeneous agents only. The first economy is populated by low skilled workers only and we thus use the parameters for unskilled workers from our heterogeneous agent economy. The second economy is populated by high skilled workers only and we thus use the parameters for skilled workers from our heterogeneous agent economy. We find that for the skilled worker economy, market tightness is 9.2 times more volatile than their productivity. This high value, despite a low value of  $z^H = 0.813$ , is mainly due to two differences between an economy consisting only of skilled workers and the representative agent economy (the homogeneous agent economy calibrated to the same aggregate statistics as in this paper). First, the productivity process for high-skilled workers is more persistent than for the representative agent. Second, the matching function elasticity for skilled workers,  $\gamma^H$ , equals 0.199 whereas this elasticity equals 0.5 in the representative agent case. Equation (3.27) in Footnote 15 explains why such a difference in the matching function elasticities results in a different productivity elasticity of market tightness. The high ratio of the volatility of market tightness to the volatility of productivity then translates into a high volatility of market tightness since the productivity process for high-skilled workers is also more volatile than for the representative agent.

For unskilled workers, the standard deviation of HP-filtered log market tightness  $\theta^L$  is 0.206, which is 15.8 times higher than the volatility of their productivity. This higher volatility for low-skilled workers is due to a higher value of  $z^L = 0.929$  (relative to their productivity). In the representative agent model of Hagedorn and Manovskii (2008), a value of  $z = 0.94$  would be required to generate a volatility of market tightness of 0.206. A value of  $z = 0.929$  would generate a volatility of only 0.177 in that model. The difference

is due to a separation rate of low-skilled workers that is higher than the one used in the representative agent economy in Hagedorn and Manovskii (2008).<sup>15</sup>

The matching function translates the volatility of market tightness into volatile unemployment. For each group, the steady state elasticity of unemployment with respect to productivity can be expressed as

$$\epsilon_{u^T,p^T} = \epsilon_{u^T,f^T} \cdot \epsilon_{f^T,\theta^T} \cdot \epsilon_{\theta^T,p^T} = -\left(1 - \frac{u^T}{N^T}\right)(1 - \gamma^T)\epsilon_{\theta^T,p^T} \quad (3.28)$$

Our finding that  $1 - \gamma^H = 1 - 0.199$  is substantially larger than  $1 - \gamma^L = 1 - 0.529$  explains why high-skilled unemployment is more volatile than low-skilled unemployment, although the opposite ordering between groups holds for market tightness.

The aggregate statistics targeted in this paper differ from those in Hagedorn and Manovskii (2008). We now calibrate the linear model with homogeneous workers in Hagedorn and Manovskii (2008) to match these same aggregate statistics. In particular, we target a wage elasticity of 0.67 (instead of 0.45 in Hagedorn and Manovskii (2008)) and also make the distinction between the total separation rate and the separation rate into unemployment. We find a standard deviation of market tightness of 0.11 and a standard deviation of unemployment of 0.049, which represent only about one-half of the corresponding numbers in the data. As we have shown above, only after we allow for heterogeneity, the model is able to replicate the observed volatilities. Two simple observations explain this finding. First, the volatility of unemployment is an increasing and convex function of  $z$  (see equation (3.27)). Second, the calibrated value of  $z$  in the homogeneous worker model lies between the two values  $z^H$  and  $z^L$  and is close to their weighted average. As a

<sup>15</sup> In Hagedorn and Manovskii (2008) we derive, in the model without aggregate uncertainty, the elasticity of labor market tightness with respect to aggregate productivity to be:

$$\epsilon_{\theta,p} = \frac{p}{p-z} \frac{\beta f(\theta) + (1-\delta(1-s))/\delta}{\beta f(\theta) + (1-\eta)(1-\delta(1-s))/\delta}, \quad (3.27)$$

where  $\eta$  is the elasticity of  $f(\theta)$  with respect to  $\theta$ . This equation may be used to quite accurately evaluate the impact of various parameter values, such as the separation rate, on the volatility of market tightness.

consequence, low-skilled unemployment and thus also overall unemployment are substantially more volatile than unemployment in the homogeneous worker model. To summarize, we find that the extended MP model calibrated using the strategy proposed in Hagedorn and Manovskii (2008) is consistent with labor market volatilities in the aggregate, once we allow for heterogeneity, and in subgroups.

**Robustness.** The only target in our benchmark calibration that is not standard is the elasticity of wages with respect to aggregate productivity. Recall that we define productivity as non-farm business output divided by employment from the monthly Current Population Survey. Shimer (2005a) used the same measure of output but divided it by employment measured in the Current Employment Statistics. The estimated elasticity of wages with respect to aggregate productivity is affected by this choice. Our measure of productivity implies an elasticity of 0.67, while Shimer’s measure implies an elasticity of only 0.5.<sup>16</sup> We now recalibrate the model to match the same calibration targets but target a low wage elasticity of 0.5.

The performance of the model in matching the calibration targets with a low wage elasticity, the calibrated parameter values, and the results are described in Appendix Tables 3.7, 3.8, and 3.9, respectively. The changes in the calibrated values of the bargaining power  $\beta$  and the value of non-market activity  $z$  are as expected. A lower value for the targeted wage elasticity for both groups leads to lower values for the bargaining power of both types,  $\beta^H$  and  $\beta^L$ . Since the expected costs of posting vacancies remain unchanged, per period profits and thus average wages do not change either. To generate the same level of wages with a lower bargaining power requires then a higher value of non-market

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<sup>16</sup>The differences between the cyclical properties of these series are documented in Hagedorn and Manovskii (2007). There we argue why a productivity measure based on CPS employment might be preferred.



activity for both types,  $z^H$  and  $z^L$ . Higher values of non-market activity result in more volatile labor market variables in the aggregate and for each worker type as compared to the benchmark calibration.

An additional benefit of this experiment is that it (coincidentally) targets virtually the same aggregate statistics computed over the 1951-2004 period as in Shimer (2005a) and Hagedorn and Manovskii (2008). For comparison, we reproduce these statistics in Column (1) of Appendix Table 3.9 and the results from the calibration of the linear MP model with homogeneous workers in Column (3) (targeting the same aggregate statistics as in the model with heterogeneity). A comparison of the results based on the model with worker heterogeneity with the results from the linear model implies that the model with worker heterogeneity again generates a higher volatility of aggregate labor market statistics and is closer to the data than the homogeneous worker model.

A new feature of our calibration is that we make a distinction between the total separation rate and the separation rate into unemployment. We now recalibrate the model to match the same calibration targets but without making this distinction. The performance of the model in matching the calibration targets, calibrated parameter values, and results are described in Appendix Tables 3.10, 3.11, and 3.12, respectively. A lower total separation rate increases expected profits from a filled vacancy. Since vacancy posting costs are unchanged, a higher value of non-market activity  $z$  is required to keep profits unchanged. A higher value of  $z$  leads to more volatility in market tightness and in wages. Thus a lower value of the bargaining weight is chosen to match a wage elasticity of 0.67. Again the model with heterogeneity is closer to the data, since the linear model with homogeneous workers generates too little volatility.

Finally, we have assumed throughout the paper that the two shocks,  $\epsilon_{A,t}$  and  $\epsilon_{q,t}$  are independent. Estimating their correlation in the data, we obtain a correlation of 0.2644.

We have recalibrated the model with this correlation and found that our results are not affected. Introducing this correlation makes capital slightly more volatile because the price of capital equipment is lower when TFP is higher. However, the depreciation factors adjust to match capital volatility in the data, and all other statistics remain unchanged.

### 3.5 Policy Experiments

In this section we investigate the effects of changes in tax policies under two scenarios. First, when productivity is exogenous and second, when productivity is endogenous because workers are heterogeneous and interact through the production side of the economy. Specifically, we consider how unemployment responds to changes in the labor income tax rate, in the sales tax and in the capital income tax rate in the two scenarios. The effects of these policy changes are easy to compute since they are equivalent to changing the value of non-market activity, or equivalently changing labor productivity. Hagedorn and Manovskii (2008) show that the equilibrium with a labor tax rate  $\tau_w$  is equivalent to the equilibrium without a labor tax but where  $z$  is replaced by  $\frac{z}{1-\tau_w}$ . An equilibrium with a sales tax of  $\tau_s$  is equivalent to an equilibrium without a sales tax but where productivity  $p$  is replaced by  $p(1 - \tau_s)$ . Finally, imposing a capital income tax rate  $\tau_k$  changes optimal capital accumulation in a steady state (with the normalization  $q = 1$ ) according to the two equations:

$$(F_{k_s}(t+1) - d_s)(1 - \tau_k) = \frac{1}{\delta} - 1, \quad (3.29)$$

$$(F_{k_e}(t+1) - d_e)(1 - \tau_k) = \frac{1}{\delta} - 1. \quad (3.30)$$

The direct impact of taxing capital income is to lower investment, which then leads to a drop in labor productivity.

Another policy change, an increase in unemployment insurance, is theoretically equivalent to a change in sales taxes. However, as shown in Faig and Zhang (2008) and Zhang

(2008), such a policy change should take into account that entitlement to unemployment insurance benefits must be earned with employment. In this case, an increase in unemployment insurance generosity has a small effect on unemployment, much smaller than a change in tax rates.<sup>17</sup> We therefore concentrate on the evaluation of the effects of tax policies.

In the next section, we analyze how productivity and unemployment respond to policy changes theoretically before assessing its quantitative performance. We study the effects of changes in  $z$  (corresponding to a change in labor taxes), but all of our results fully apply to changes in consumption taxes and capital income tax rates since only the difference between  $p$  and  $z$ ,  $p - z$  matters.

### 3.5.1 Theoretical Analysis

In this section, we show that a change in  $z$  changes not only employment but also productivity, which can mitigate or amplify the changes in employment. If, for example, an increase in  $z$  increases productivity, the drop in employment is smaller than it would be with a constant level of productivity. To show this we consider a simplified (relative to (3.2)) constant return to scale (CRS) production technology

$$y_t = G(l, h, k), \tag{3.31}$$

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<sup>17</sup>They show that an increase in unemployment benefits by itself lowers the unemployment rate. The overall effect has however to take into account that higher unemployment benefits have to be financed with higher taxes which lead to a higher unemployment rate.

Another aspect is that changes in unemployment insurance lead to important substitution effects. For instance, Gruber and Cullen (2000) find that for each dollar of a husband's unemployment insurance received, wives earn 73 cents less. Moreover, a higher replacement rate crowds out private (precautionary) savings (Gruber and Engen (2001)). Taking into account the latter two effects will presumably further dampen the effect of changes in unemployment insurance.

where  $k$  is capital and  $l$  and  $h$  are two different labor types, and  $G$  is increasing and concave in each argument.<sup>18</sup>

A drop in  $l$  (due to an increase in  $z^l$ ) increases the productivity  $G_l$  of low-skilled workers if the levels of  $h$  and  $k$  are unchanged. But  $h$  and  $k$  adjust as well, and this adjustment can overturn this conclusion, depending on the properties of  $G$ . The following sections investigate these properties.

### Equilibrium Conditions for Capital, Employment and Market Tightness

Given the production function  $G$ , we now consider how the productivities  $G_l, G_h$  and  $G_k$ , labor inputs  $l$  and  $h$ , capital  $k$  and the policy parameter  $z$  are related.

Capital solves the first-order condition ( $d$  is the depreciation rate)

$$G_k = \frac{1}{\delta} - (1 - d), \quad (3.32)$$

which defines capital implicitly as a function of  $l$  and  $h$ :  $k(l, h)$ .

For the two labor inputs, we can derive in the case of no aggregate uncertainty (see Hagedorn and Manovskii (2008)) the following relationship between market tightness and productivity for each group (we suppress the dependence on type  $T$ ).

$$\frac{1 - \delta(1 - s)}{\delta q(\theta)} + \beta\theta = \frac{p - z}{c}(1 - \beta). \quad (3.33)$$

The steady state conditions for employment  $l$  and  $h$  are

$$l = \frac{f^L(\theta^L)}{s^L + f^L(\theta^L)} \quad \text{and} \quad h = \frac{f^H(\theta^H)}{s^H + f^H(\theta^H)}. \quad (3.34)$$

The last two equations together imply two functions that relate the level of employment

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<sup>18</sup>The technology in (3.2) takes this form for  $\alpha = 0$ . Since (3.2) combines capital structures and  $G$  through a Cobb-Douglas aggregator, assuming (3.2) would not change the conclusions of this section. The Cobb-Douglas specification implies that capital structures change one-for-one with  $G$ .

to  $p$  and  $z$ :

$$l = L(p^l, z^l), \quad (3.35)$$

$$h = H(p^h, z^h). \quad (3.36)$$

Denote the marginal productivity of group  $l$ :

$$p^l = G_l(l, h, k), \quad (3.37)$$

and the marginal productivity of group  $h$ :

$$p^h = G_h(l, h, k). \quad (3.38)$$

Taking into account that capital  $k$  is a function of  $l$  and  $h$ ,  $k(l, h)$ , allows us to express productivities as functions of  $l$  and  $h$  only

$$p^l = G_l(l, h, k(l, h)) = \pi^l(l, h), \quad (3.39)$$

$$p^h = G_h(l, h, k(l, h)) = \pi^h(l, h). \quad (3.40)$$

Plugging the expression for  $L(p^l, z^l)$  and  $H(p^h, z^h)$  into the functions  $\pi$ , results in two functions  $A$  and  $B$ :

$$p^l = A(p^l, z^l, p^h, z^h) = \pi^l(L(p^l, z^l), H(p^h, z^h)), \quad (3.41)$$

$$p^h = B(p^l, z^l, p^h, z^h) = \pi^h(L(p^l, z^l), H(p^h, z^h)), \quad (3.42)$$

which jointly describe the two productivity levels  $(p^l, p^h)$  as a fixed point, depending on the two parameters  $(z^l, z^h)$ . We now want to investigate how changing  $(z^l, z^h)$  affects the fixed point  $(p^l, p^h)$ .

### **Productivity changes**

To characterize how productivity  $(p^l, p^h)$  depends on  $(z^l, z^h)$  requires knowing how the functions  $A$  and  $B$  depend on productivities  $(p^l, p^h)$ . The next proposition accomplishes this.

**Proposition 1**

$$\epsilon_{A,p^l} = \epsilon_{\pi^l,l} \epsilon_{L,p^l} = \left\{ -\epsilon_{G_l,h} + \frac{\epsilon_{G_k,h} \cdot \epsilon_{G_l,k}}{\epsilon_{G_k,k}} \right\} \epsilon_{L,p^l}, \quad (3.43)$$

$$\epsilon_{A,p^h} = \epsilon_{\pi^l,h} \epsilon_{H,p^l} = \left\{ \epsilon_{G_l,h} - \frac{\epsilon_{G_k,h} \cdot \epsilon_{G_l,k}}{\epsilon_{G_k,k}} \right\} \epsilon_{H,p^l}, \quad (3.44)$$

$$\epsilon_{B,p^l} = \epsilon_{\pi^h,l} \epsilon_{L,p^h} = \left\{ \epsilon_{G_h,l} - \frac{\epsilon_{G_k,l} \cdot \epsilon_{G_h,k}}{\epsilon_{G_k,k}} \right\} \epsilon_{L,p^h}, \quad (3.45)$$

$$\epsilon_{B,p^h} = \epsilon_{\pi^h,h} \epsilon_{H,p^h} = \left\{ -\epsilon_{G_h,l} + \frac{\epsilon_{G_k,l} \cdot \epsilon_{G_h,k}}{\epsilon_{G_k,k}} \right\} \epsilon_{H,p^h}, \quad (3.46)$$

where  $\epsilon_{x,y}$  is the elasticity of  $x$  w.r.t.  $y$ .

We can consider two special cases in which productivity is invariant when policy is changed. The first case arises if the two types of workers are perfect substitutes, so that the production part of the model is equivalent to a model with homogeneous workers. In this case the invariance of productivity is not very surprising. Any drop in labor leads to a drop in capital, such that the capital-labor ratio remains unchanged. Since labor productivity is a function of the capital-labor ratio, it does not change either.

The assumption that the two labor inputs are perfect substitutes implies that  $G_{ll} = G_{hh} = G_{lh}$  and that  $G_{kl} = G_{kh}$  and it implies the following proposition:

**Proposition 2 (Special Case: L and H are Perfect Substitutes)** *If the two labor inputs  $l$  and  $h$  are perfect substitutes, then the labor productivities do not change with changes in labor inputs:  $\epsilon_{\pi^l,l} = \epsilon_{\pi^h,h} = \epsilon_{\pi^l,h} = \epsilon_{\pi^h,l} = 0$ .*

A similar logic applies when one of the two labor inputs is unrelated to the other labor input and capital, that is, either  $G_{lh} = 0$  and  $G_{kl} = 0$  or  $G_{lh} = 0$  and  $G_{kh} = 0$ . In each of these two cases, the economy consists of two unrelated economies, each of which has only one type of worker. Since “both economies” have a CRS production function with a representative agent, the previous proposition applies.

**Proposition 3 (Special Case: L and H are Unrelated Inputs)** *If either  $G_{lh} = 0$  and  $G_{kl} = 0$  or  $G_{lh} = 0$  and  $G_{kh} = 0$ , then productivity remains unchanged:  $\epsilon_{\pi^l, l} = \epsilon_{\pi^h, h} = \epsilon_{\pi^l, h} = \epsilon_{\pi^h, l} = 0$ .*

The production function we use in this paper does not fall into one of the two special cases. Instead it implies the following assumption:

**Assumption 1**  $G_{lh} \geq 0$  and  $G_{kl}G_{kh} \geq 0$ , where at least one inequality is strict.

With this assumption, we can show that productivity indeed changes when the policy parameter  $z$  is changed and we know the sign of this change. The key step is to show that labor productivity changes if the amount of labor input is changed. The reason why these changes are not zero is that the above logic does not fully apply anymore. With a representative agent, a fully flexible capital stock adjusts to keep the capital-labor ratio and thus labor productivity constant. If, instead, capital was fixed or not fully flexible, labor productivity would increase in response to a decrease in labor. With two types of labor a similar effect obtains. Capital cannot fully adjust to keep the two capital-labor ratios constant. Instead, there is only partial adjustment, as would be the case with a representative agent if the capital stock is a fixed factor. As a consequence, labor productivity is not constant. The next proposition states this and also establishes how the functions  $A$  and  $B$  respond to changes in  $p^l$  and  $p^h$ .

**Proposition 4** *If assumption 1 holds, then*

$$\epsilon_{\pi^l, l}, \epsilon_{\pi^h, h}, A_{p^l}, B_{p^h} < 0, \quad (3.47)$$

$$\epsilon_{\pi^l, h}, \epsilon_{\pi^h, l}, A_{p^h}, B_{p^l} > 0, \quad (3.48)$$

Once the signs of the derivatives of the functions  $A$  and  $B$  are known, the last step is easy:

**Proposition 5**

$$\frac{\partial p^l}{\partial z^l} = \frac{-A_{p^l}}{1 - A_{p^l} - B_{p^h}} > 0, \quad (3.49)$$

$$\frac{\partial p^h}{\partial z^l} = \frac{-B_{p^l}}{1 - A_{p^l} - B_{p^h}} < 0, \quad (3.50)$$

$$\frac{\partial p^l}{\partial z^h} = \frac{-A_{p^h}}{1 - A_{p^l} - B_{p^h}} < 0, \quad (3.51)$$

$$\frac{\partial p^h}{\partial z^h} = \frac{-B_{p^h}}{1 - A_{p^l} - B_{p^h}} > 0. \quad (3.52)$$

It also holds that  $\frac{\partial p^l}{\partial z^l} < 1$  and  $\frac{\partial p^h}{\partial z^h} < 1$ , so that  $\frac{\partial(p^l - z^l)}{\partial z^l} < 0$  and  $\frac{\partial(p^h - z^h)}{\partial z^h} < 0$ .

**What does this mean for employment changes?**

Once the change in productivity is known, it is sufficient to look at equations (3.35) and (3.36) to figure out the change in employment. For example, if  $p - z$  increases, employment increases, and if  $p - z$  decreases, employment decreases.

More generally, the change in total employment  $l + h$  in response to a change in  $z^l$  is:

$$\begin{aligned} \epsilon_{l+h,z^l} &= \left( (L_{p^l} \frac{\partial p^l}{\partial z^l} + L_{z^l}) + H_{p^h} \frac{\partial p^h}{\partial z^l} \right) \frac{z^l}{l+h} \\ &= (\epsilon_{L,p^l} \epsilon_{p^l,z^l} + \epsilon_{L,z^l}) \frac{l}{l+h} + \epsilon_{H,p^h} \epsilon_{p^h,z^l} \frac{h}{l+h}, \end{aligned} \quad (3.53)$$

which means that the total employment change is a weighted sum of the change in  $l$  and in  $h$ . Similarly, the change in total employment in response to a change in  $z^h$  is:

$$\begin{aligned} \epsilon_{l+h,z^h} &= \left( (H_{p^h} \frac{\partial p^h}{\partial z^h} + H_{z^h}) + L_{p^l} \frac{\partial p^l}{\partial z^h} \right) \frac{z^h}{l+h} \\ &= (\epsilon_{H,p^h} \epsilon_{p^h,z^h} + \epsilon_{H,z^h}) \frac{h}{l+h} + \epsilon_{L,p^l} \epsilon_{p^l,z^h} \frac{l}{l+h}. \end{aligned} \quad (3.54)$$

The total change, if  $z^l$  and  $z^h$  go up by 1% equals

$$\epsilon_{l+h,z^l} + \epsilon_{l+h,z^h}. \quad (3.55)$$



This expression equals

$$\begin{aligned}
\epsilon_{l+h,z^l} + \epsilon_{l+h,z^h} = & \tag{3.56} \\
& \frac{l}{l+h}(\epsilon_{L,p^l}(\epsilon_{p^l,z^h} + \epsilon_{p^l,z^l}) + \epsilon_{L,z^l}) + \frac{h}{l+h}(\epsilon_{H,p^h}(\epsilon_{p^h,z^l} + \epsilon_{p^h,z^h}) + \epsilon_{H,z^h}) \\
& \frac{l}{l+h}\left(\frac{\epsilon_{L,p^l}}{\vartheta}(\epsilon_{\pi^l,h}\epsilon_{H,z^h} + \epsilon_{\pi^l,l}\epsilon_{L,z^l}) + \epsilon_{L,z^l}\right) + \frac{h}{l+h}\left(\frac{\epsilon_{H,p^h}}{\vartheta}(\epsilon_{\pi^h,l}\epsilon_{L,z^l} + \epsilon_{\pi^h,h}\epsilon_{H,z^h}) + \epsilon_{H,z^h}\right) \\
& \frac{l}{l+h}(\epsilon_{L,p^l}\left(\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h}) + \epsilon_{L,z^l}\right) + \frac{h}{l+h}(\epsilon_{H,p^h}\left(\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{H,z^h} - \epsilon_{L,z^l}) + \epsilon_{H,z^h}\right)),
\end{aligned}$$

where  $\vartheta = 1 - A_{p^l} - B_{p^h}$ , which is positive under Assumption 1 (as established in Proposition 4). This expression is quite insightful. The change in  $l$ -productivity  $p^l$  due to a change in  $z$  equals  $\frac{1}{\vartheta}(\epsilon_{\pi^l,h}\epsilon_{H,z^h} + \epsilon_{\pi^l,l}\epsilon_{L,z^l})$  and similarly the change of the  $h$ -productivity  $p^h$  equals  $\frac{1}{\vartheta}(\epsilon_{\pi^h,l}\epsilon_{L,z^l} + \epsilon_{\pi^h,h}\epsilon_{H,z^h})$ . If these changes are zero, this means productivity is constant, and the change in employment would equal

$$\frac{l}{l+h}\epsilon_{L,z^l} + \frac{h}{l+h}\epsilon_{H,z^h}, \tag{3.57}$$

which is a weighted sum of the changes in  $l$  and in  $h$ . This *composition effect* strictly dampens the change in employment (and thus unemployment) relative to the group effects, whenever one group is more responsive to policy than the other group, for example, if  $|\epsilon_{L,z^l}| > |\epsilon_{H,z^h}|$ .

If, however, productivity responds to changes in  $z$ , the response of group employment changes. If productivity increases in response to an increase in  $z$ , the employment effect is mitigated ( $p - z$  decreases by less); if productivity decreases in response to an increase in  $z$ , the employment effect is amplified ( $p - z$  decreases by more).

Whether productivity increases or decreases for group  $l$  and group  $h$  is described by the signs of  $\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})$  and of  $\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{H,z^h} - \epsilon_{L,z^l})$ . Multiplying these expressions with  $\epsilon_{L,p^l}$  and  $\epsilon_{H,p^h}$ , respectively, translates the productivity changes into employment changes (higher productivity leads to higher employment).

One implication of the above expression is that the change in employment is equal to that with constant productivity if  $\epsilon_{L,z^l} - \epsilon_{H,z^h} = 0$  (both types of labor respond in the same way to changes in unemployment insurance), namely,

**Proposition 6** *If  $\epsilon_{L,z^l} - \epsilon_{H,z^h} = 0$ , then productivity does not change and the change in total employment equals*

$$\epsilon_{l+h,z^l} + \epsilon_{l+h,z^h} = \frac{l}{l+h}\epsilon_{L,z^l} + \frac{h}{l+h}\epsilon_{H,z^h}, \quad (3.58)$$

because in this case productivity would not move (endogenously).

Furthermore, it follows that if one group has a stronger labor demand elasticity, for example, group  $L$  ( $\epsilon_{L,z^l} - \epsilon_{H,z^h} < 0$ ), then the productivity of this group increases and the drop in employment is mitigated, whereas the productivity of the other group decreases (since  $\epsilon_{L,p^l} > 0$ ,  $\epsilon_{p^l,L} < 0$ ,  $\epsilon_{H,p^h} > 0$ ,  $\epsilon_{p^h,H} < 0$ ).

**Proposition 7** *If  $\epsilon_{L,z^l} - \epsilon_{H,z^h} < 0$ , then  $p^l$  increases and  $p^h$  decreases. As a consequence the employment response of group  $l$  is mitigated (relative to constant productivity) and the employment response of group  $h$  is amplified (relative to constant productivity).*

The overall effect on employment due to the change in productivity would be (since

$$\begin{aligned} \epsilon_{p^h,H} &= \frac{lp^l}{hp^h}\epsilon_{p^l,L}) \\ &\frac{l}{l+h}(\epsilon_{L,p^l}(\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})) + \frac{h}{l+h}(\epsilon_{H,p^h}(\frac{\epsilon_{\pi^h,h}}{\vartheta}(\epsilon_{H,z^h} - \epsilon_{L,z^l}))) \quad (3.59) \\ &= \frac{l}{l+h}\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})(\epsilon_{L,p^l} - \frac{p^l}{p^h}\epsilon_{H,p^h}), \end{aligned}$$

which is positive if  $\frac{p^l}{p^h}$  is not substantially larger than one (if group  $h$  are high-skilled workers with lower relative  $z$  and higher productivity, this conclusion obviously holds).

**Proposition 8** *The overall effect on employment due to the change in productivity equals*

$$\frac{l}{l+h}\frac{\epsilon_{\pi^l,l}}{\vartheta}(\epsilon_{L,z^l} - \epsilon_{H,z^h})(\epsilon_{L,p^l} - \frac{p^l}{p^h}\epsilon_{H,p^h}), \quad (3.60)$$

which is positive if  $\frac{p^l}{p^h}$  is not substantially larger than one.

## Comparative statics

Consider the impact of different parameter values on the overall effect on employment in equation (3.60).

**Proposition 9** *Consider the employment effect due to productivity changes:*

- **Skill premium:** *A decrease in  $\frac{p^l}{p^h}$  increases the effect if  $\epsilon_{L,z^l} - \epsilon_{H,z^h} > 0$ .*
- **Preferences:** *An increase in  $\epsilon_{L,z^l} - \epsilon_{H,z^h} (> 0)$  (for example if  $z^l - z^h$  increases) increases the effect.*
- **Production:** *Any change in the production function that lowers  $\epsilon_{\pi^l,l} < 0$  increases the effect. This would happen if one of the positive values  $G_{lh}, G_{lk}, G_{hk}$  increases.*

### 3.5.2 Quantitative Evaluation

In this section we investigate quantitatively the effect of labor, sales and capital income tax rates on unemployment and productivity. In each experiment we keep all the parameter values the same as in our benchmark calibration except for increasing the value of non-market activity  $z$  in the case of a labor tax or decreasing labor productivity  $p$  in case of a sales or capital income tax. An increase in the labor income tax rate by one percentage point amounts to increasing  $(z^L, z^H)$  to  $(\frac{z^H}{1-0.01}, \frac{z^L}{1-0.01})$ . A one percentage point increase in the sales tax rate leads to a decrease of labor productivity from  $(p^L, p^H)$  to  $((1 - 0.01)p^L, (1 - 0.01)p^H)$ . Finally a one percentage point increase in the capital income tax rate lowers the return on capital  $r$  to  $(1 - 0.01)r$ , as described above.

In all these experiments we assume that the value of non-market activity is invariant with respect to tax policy.<sup>19</sup> This is obviously a strong assumption. For example, unemployed workers also have to pay a sales tax and thus suffer from a tax increase to the extent

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<sup>19</sup>Note, that as we mentioned above the increase in labor tax is equivalent to an increase in  $z$ . This does not contradict the invariance assumption because there are no additional effects of the tax change on  $z$ .

that  $z$  measures not only the value of leisure but also receiving unemployment benefits or being self-employed. For example, if a third of the value of  $z$  reflects unemployment insurance, then the value of  $z$  should be decreased by a third of a percentage point dampening the policy effects by about a third. A similar argument applies to a labor income tax rate if unemployment benefits are taxed as in, e.g., the U.S., or are determined as a fraction of after-tax wages, as in, e.g., Germany. If the latter case, an increase in labor tax rates lowers the net wage and thus unemployment benefits for a fixed replacement ratio. Finally, we did not model a direct link between the level of productivity and  $z$ . It is likely, however, that such a relationship exists. Hall and Milgrom (2008) introduce curvature into preferences in the MP model. With some assumption on preference parameters (they assume preferences inconsistent with balanced growth) they derive  $z$  as a function of consumption levels of employed and unemployed workers (which would be affected by, e.g., permanent changes in productivity). The RBC model with balanced growth preferences may provide some guidance to the direction and magnitude of the impact of a change in  $p$  on  $z$ . For example, a change in capital income tax rate in that model has no effect on employment (Prescott (2004)). This would correspond to  $z$  decreasing by the same amount as  $p$  in response to an increase in  $\tau_k$ . Therefore the numbers found here should be considered upper bounds on policy effects. However, this reasoning does not affect our comparison of policy effects in models with endogenous and exogenous productivity since we compute percentage differences.

The results of performing these experiments are presented in Column 1 of Table 3.4. For labor income tax rates we find that the overall unemployment rate increases by 6.7% (from 7.0% to 7.5%), the low skill unemployment rate increases by 6.6% (from 8.6% to 9.2%), and the high skill unemployment rate increases by 7.5% (from 2.7% to 2.9%). For the sales tax the findings are very similar (as they would also be in an RBC model with

Walrasian labor markets). The overall unemployment rate increases by 7.3%, the low skill unemployment rate increases by 7.1%, and the high skill unemployment rate increases by 9.1%. Finally, for the capital income tax we find that the overall unemployment rate increases by 0.9%, the low skill unemployment rate increases by 0.8%, and the high skill unemployment rate increases by 1.9%.

Section 3.5.1 implies that the change in unemployment can be decomposed into the effect due to productivity changes and a composition effect. Column 2 of Table 3.4 illustrates that with a constant level of productivity, the response of low-skilled unemployment to the same increase in labor tax rates would be to increase by 8.5% and high-skilled unemployment would increase by 7.2% leading to the overall increase in unemployment with unchanged productivity of 8.4%. The endogenous change in productivity reduces the strength of these effects. As reported in Table 3.5, productivity of low-skilled workers increases by 0.194% and productivity of high skilled decreases by 0.028%. This accounts for the much smaller increases in unemployment in the model with endogenous productivity, with the extent of the difference reported in Column 3 of Table 3.4. Tables 3.4 and 3.5 show that the effects of changes in sales taxes are very similar to the effects of changing the labor tax. In both cases the endogenous change in productivity dampens the effects of changes in tax rates by about 25%. The effects of changes in capital income tax rates are dampened even stronger, by about 38%. Since capital income tax affects labor productivity only indirectly, the unemployment rate responds by less than in the case of labor or sale taxes.

These policy effects are much lower than those implied by the standard MP model with homogeneous workers. A meaningful comparison of the size of policy effects between the two models requires that they both generate the same amount of volatility in market tightness. Otherwise, one model could generate small policy effects just because it does

Table 3.4: Semi-Elasticities of Unemployment with Respect to Changes in Tax Rates.

	Endogenous Prod. (1)	Exogenous Prod. (2)	Difference (3)
<i>Labor Income Tax</i>			
Overall Unemployment	6.705	8.409	25.416
Low Skilled Unemployment	6.611	8.547	29.291
High Skilled Unemployment	7.504	7.238	-3.545
<i>Sales Tax</i>			
Overall Unemployment	7.343	9.161	24.749
Low Skilled Unemployment	7.139	9.186	28.668
High Skilled Unemployment	9.077	8.951	-1.389
<i>Capital Income Tax</i>			
Overall Unemployment	0.907	1.249	37.689
Low Skilled Unemployment	0.793	1.274	60.830
High Skilled Unemployment	1.887	1.032	-45.312

Note: Entries are semi-elasticities with exogenous and endogenous productivity: percentage changes of overall unemployment, high skilled unemployment and low skilled unemployment in response to a one percentage point increase in the labor income tax rate, the sales tax rate and the capital income tax rate, respectively. The column “Difference” reports the percentage difference between Columns 1 and 2.

Table 3.5: Percentage Change of Productivity in Response to Changes in Tax Rates.

	Endogenous Prod.	Exogenous Prod.
	(1)	(2)
<i>Labor Income Tax</i>		
Overall Productivity	0.179	0.014
Low Skilled Productivity	0.194	0.000
High Skilled Productivity	-0.028	0.000
<i>Sales Tax</i>		
Overall Productivity	0.191	0.010
Low Skilled Productivity	0.204	0.000
High Skilled Productivity	-0.013	0.000
<i>Capital Income Tax</i>		
Overall Productivity	-0.127	0.000
Low Skilled Productivity	0.096	0.000
High Skilled Productivity	-0.226	0.000

Note: percentage changes of overall productivity (output per worker), high skilled productivity and low skilled productivity (both marginal productivities) in response to a one percentage point increase in the labor income tax rate, the sales tax rate and the capital income tax rate, respectively. Productivity is before subtracting sales taxes.

not generate much volatility (an arbitrarily low value of  $z$  would ensure this). To generate a volatility of 0.296 in the linear model requires that  $z = 0.928$  (all other parameters except for vacancy posting costs are chosen to match the same aggregate statistics as in our benchmark calibration). For this value of  $z$  we find a semi-elasticity of the overall unemployment rate of 9.5% for sales taxation, 8.6% for labor taxation and 1.2% for capital taxation.

The results are even stronger if one considers the low wage elasticity calibration. In that case we have to set  $z = 0.942$  in the standard model to generate a volatility of 0.246, the volatility generated by our model with heterogeneous agents (and a low wage elasticity) as described in Appendix Table 3.12. This implies a semi-elasticity of 13.3% for sales taxation, a semi-elasticity of 12.6% for labor taxation and a semi-elasticity of 2.0 for capital taxation, whereas our model with heterogeneity implies semi-elasticities of 9.3%, 8.7% and 1.2%, respectively.

### **3.5.3 Empirical Evidence on the Effects of Policy Changes**

In the previous sections we have established several results on the effects of changes in labor taxation, sales taxation and capital income taxation on unemployment and productivity. We found that the semi-elasticity of unemployment with respect to labor and sales taxes are of about equal size, 7%, whereas capital income taxes have only very small effects. We also showed that an increase in both labor and sales taxes leads to a decrease in the skill premium since the productivity of low skilled workers increases whereas the productivity of high skill workers decreases. Furthermore, because of these endogenous productivity responses, the percentage change in the unemployment rate is higher for high skilled than for low skilled workers. Due to this neutralizing effect, we do not expect to find that increases in unemployment are concentrated among low skilled workers. In this section we use cross-country evidence to verify whether these model predictions are consistent with



the data. In particular, we ask how much of the differences in unemployment rates between countries can be accounted for by differences in tax policy. Table 3.6 uses data on tax rates and unemployment rates for the OECD countries to provide some evidence on the empirical effects of taxation.<sup>20</sup> We regress the log of the unemployment rate on various tax measure. Thus, the numbers in the table represent the semi-elasticity of unemployment with respect to the respective tax variable. Column (1) establishes that, as expected, capital income taxes have virtually no effect on unemployment whereas the effects of labor and sales taxes are substantial and of similar magnitude. We can thus define a tax wedge as the sum of the labor tax rate and the sales tax rate and we do not expect the results to change. Column (2), which shows the result from a regression of unemployment on this tax wedge and capital income taxation, confirms this. A one percentage point increase in the tax wedge increases unemployment by 8.436 percent. Summing labor and sales taxation seems also appropriate from a fiscal perspective since different governments may choose different combinations of sales and labor taxation to generate the same tax revenue. It is then conceivable that by pure chance high unemployment countries choose, say, labor taxation and low unemployment countries choose sales taxation. A similar argument could apply to capital income taxation and unemployment insurance although the revenue from capital income taxes is much lower than that raised from labor taxes and the expenditures on unemployment insurance are only a small fraction of government expenditures. However, summing all these policy rates seems problematic since they have substantially different

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<sup>20</sup>We use data on effective labor, capital, and sales taxes for a number of the OECD countries over 1965-1996 period provided by Enrique Mendoza on his webpage <http://econserver.umd.edu/~mendoza/pp/taxdata.pdf> and [.../newtaxdata.pdf](http://econserver.umd.edu/~mendoza/pp/newtaxdata.pdf). The data were constructed using the method described in Mendoza, Razin, and Tesar (1994). Data on the unemployment rates for these countries was provided by Jim Costain on his webpage <http://www.econ.upf.edu/~costain/rbcmatch/webpage/bcui.html>. See Costain and Reiter (2008) for the detailed description of the data. Since unemployment data used in Costain and Reiter (2008) refers to five-year averages, we average the tax data similarly in the years when it is available.

Table 3.6: Evidence on the Effects of Taxes on Unemployment

Tax Measure	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Labor tax	8.465				9.171			
Sales tax	7.889				8.314			
Capital tax	0.507	0.518			-0.193	-0.203		
Tax Wedge 1		8.436				9.199		
Tax Wedge 2			3.746				3.129	
Tax Wedge 3				1.806				2.001
Other controls	No	No	No	No	Yes	Yes	Yes	Yes

Note: The dependent variable is the log of unemployment rate. The numbers represent the semi-elasticity of unemployment with respect to the respective tax variable. All regressions include country fixed effects. Following Nickell and Layard (1999) and Costain and Reiter (2008), Columns 5 through 8 include additional controls: indices of benefit duration, employment protection, union density, and bargaining coordination, and the percent of households who are owner-occupiers. Tax Wedge 1 = labor tax rate + sales tax, Tax Wedge 2 = labor tax rate + sales tax + capital income tax rate, Tax Wedge 3 = labor tax rate + sales tax + capital income tax rate + replacement rate.

effects on unemployment. Columns (3) and (4) show that indeed the effects of these wedges are diluted. Finally, Columns (5) - (8) redo the experiments from Columns (1) - (4) but add the additional controls used by Costain and Reiter (2008) and Nickell and Layard (1999) that may also affect unemployment. These controls include indices of benefit duration, employment protection, union density, and bargaining coordination, and the percent of households who are owner-occupiers. Adding these controls does not significantly affect our results. We therefore conclude that a one percentage point increase in labor or consumption tax rates increases unemployment by about 8 percent.<sup>21</sup>

We now provide evidence for how the differences in tax policies across countries affect

<sup>21</sup>An increase in the unemployment rate by 8 percent from 5.7% (sample mean) to 6.16% corresponds to a decrease of 100 \* employment by 0.456 (population has measure one). Gordon (2007) finds similar numbers in his survey of the literature. He reports  $-0.47$  for the response of hours per capita to tax changes and about  $-0.4$  for the response of employment per capita.

Figure 3.1: Skill Premium and Tax Wedge, Men

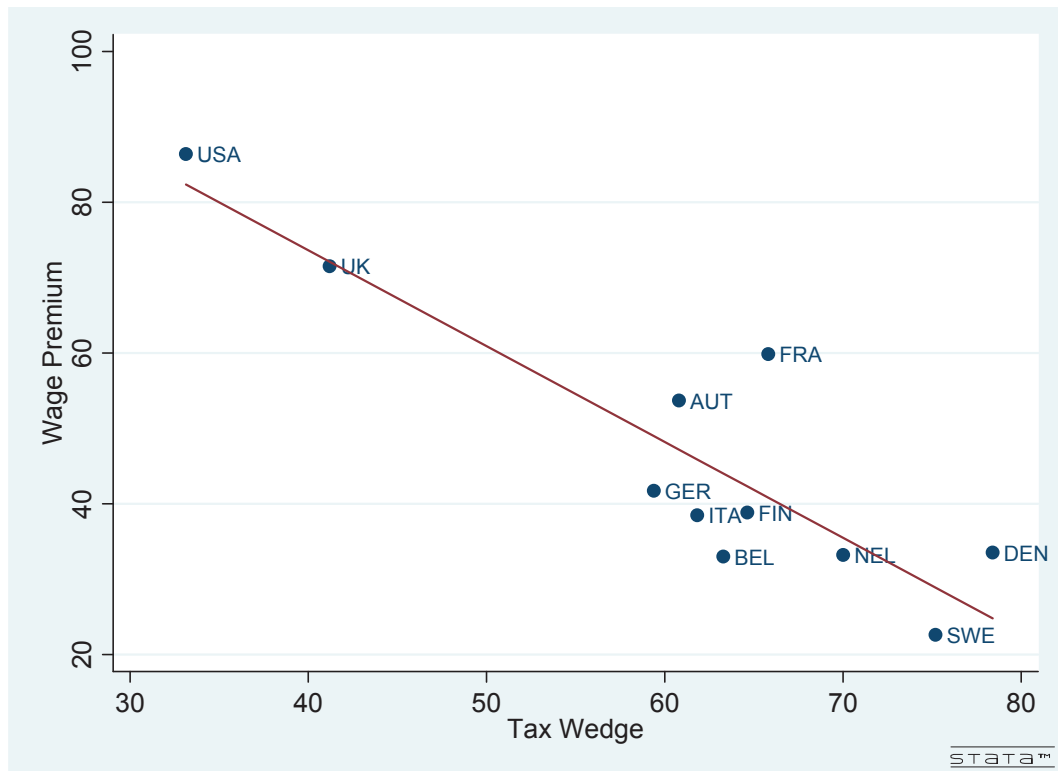


Tax wedge (sum of labor and sales taxes) in percentage points, Skill premium is the percentage difference in wages between high and low skilled workers

the skill premium and the relative unemployment rates for different skill groups. We use data from a number of the OECD countries from 1996-2000 on the skill premium for both men and women and for the unemployment rates for college and high school workers.<sup>22</sup>

<sup>22</sup>The data on skill premium come from Strauss and de la Maisonnieve (2007). They use households surveys of the OECD countries in the 1990s to measure the college premium constructed using comparable definitions of wages and schooling groups across countries. Data on unemployment rates by level of education come from Eurostat Table “Unemployment rates of the population aged 25-64 by level of education.” Because we have to use different data sources for constructing the variable, they do not always overlap in their coverage. We have data on skill premia, unemployment rates by skill and tax wedges for Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Sweden, United Kingdom and United

Figure 3.2: Skill Premium and Tax Wedge, Women



Tax wedge (sum of labor and sales taxes) in percentage points, Skill premium is the percentage difference in wages between high and low skilled workers

We compute the average of these premia and unemployment rates for each country and we also average the tax wedge (sum of labor and sales taxes) for these countries between 1990-1999.<sup>23</sup> Figure ?? confirms the predictions of our theory. We find a significant negative effect of the tax wedge on the skill premium for both men (significant slope  $-1.007$ ) and women (significant slope  $-1.271$ ) and virtually no effect on the ratio of low skill to high States. For Norway and Switzerland we have only data on tax wedges and unemployment.

<sup>23</sup>We average the tax wedge over a longer time period to maximize the number of countries in our sample. Restricting ourselves to the period 1995-1999 would shrink our sample to G-7 countries only. Our findings are insensitive to this choice, however.

Figure 3.3: Relative Unemployment and Tax Wedge



Tax wedge (sum of labor and sales taxes) in percentage points, relative unemployment rate is  $\log(\text{low skill unemployment rate over high skill unemployment rate})$

skill unemployment (insignificant slope  $-0.002$ ), echoing the view expressed in Nickell and Bell (1996) and Gottschalk and Smeeding (1997).

### 3.6 Conclusion

We extended the basic Mortensen-Pissarides search and matching model along two dimensions. First, we allowed for ex-ante heterogeneity between workers, low and high skilled. Second, we allowed two technology shocks, neutral and investment-specific, to be the driving forces of the economy. Specifically, we integrated the framework of Krusell, Ohanian,

Ríos-Rull, and Violante (2000) - a production function with capital-skill complementarity and two skill-groups - into a business-cycle search and matching model. We calibrated the model using the approach in Hagedorn and Manovskii (2008) and found that the model accounts well for the cyclical behavior of labor market variables in the aggregate and for each demographic group.

Our calibration implies that the flow value of non-market activity of high-skilled workers is considerably lower than the corresponding value for a representative worker in the model with homogeneous workers. For low-skilled workers the flow value of non-market activity is slightly higher than the value for a representative worker. Nevertheless, in the model, as in the data, the unemployment rates for these two groups of workers are highly and roughly equally volatile over the business cycle. The fact that the unemployment rate of low-skilled workers is highly volatile is not surprising given the results in Hagedorn and Manovskii (2008). The accounting profits that firms make on these workers are small and thus respond strongly in percentage terms to fluctuations in the marginal product of these workers. The fact that the unemployment rate of highly skilled workers is also highly volatile, despite the fact that the accounting profits firms make on them are relatively large, is due to the higher volatility and the higher persistence of their marginal product relative to the representative worker case.

We find that the response of unemployment to changes in taxes is substantially lower in the model with worker heterogeneity than in the model with homogeneous workers if both models generate the same volatility of market tightness. We show that this difference in policy effects is due to an endogenous response of productivity. Consider, for example, an increase in labor taxes. Because the flow utility of unemployment for high-skilled workers is relatively low, a change in taxes does not substantially affect the decisions of firms to post vacancies in a hope of hiring these workers. Thus, they serve as a fixed fac-

tor in the aggregate production. Because capital equipment is complementary with these workers and since the stock of high-skilled workers is little changed, the stock of capital equipment is little changed as well, even in the long run. In turn, if the productivity of low-skilled workers remained unchanged, a change in policy that squeezes the profits that firms make on them would induce firms to post fewer vacancies and the employment of low-skilled workers would fall. However, as their employment falls, their productivity increases because capital equipment and high-skilled workers remain in place. This increase in productivity of low-skilled workers acts to restore the profits that firms make on these workers and counteracts the effect of the change in the policy. Thus, the endogenous response of productivity significantly dampens the effect of a change in taxes on unemployment. Note that these effects are driven by the presence of worker heterogeneity and not by the curvature in the production *per se*. With a one-sector Cobb-Douglas production function, capital would adjust after a change in policy to keep the capital-labor ratio and thus productivity constant.

We have shown that the semielasticity of unemployment with respect to changes in the tax wedge implied by the model is quantitatively consistent with the data. Moreover, the model matches the evidence that countries with higher tax rates have higher aggregate productivity, lower skill premia, and higher unemployment rates among both high- and low-skilled workers. This evidence provides support for the key mechanism in the model based on worker heterogeneity.

Endogeneity of productivity in our model would also dampen the effects of changes in the generosity of unemployment insurance on unemployment. As mentioned above, we did not consider these policy experiments in this paper since theory suggests that these policy effects are small anyway. Faig and Zhang (2008) and Zhang (2008) show that this is the case once it is taken into account that entitlement to unemployment insurance benefits

must be earned with employment. Ljungqvist and Sargent (1998, 2008) argue that the rise in European unemployment can be accounted for by the increase in the depreciation rate of human capital upon job displacement interacted with more generous unemployment insurance in Europe. They do not consider the ability of their model to match business cycle facts. We think it would be fruitful to take into account the endogenous response of productivity to policy changes in their framework as well. It is clearly an important research agenda to sort out the joint effects of tax and benefit policies on unemployment outcomes.

While we study the effects of worker heterogeneity in the MP model, a number of papers have recently investigated the quantitative implications of heterogeneity of productivities across jobs (e.g., Hornstein, Krusell, and Violante (2007), Michelacci and Lopez-Salido (2007), Pissarides and Vallanti (2007)). Most closely related to our analysis is the contribution by Hornstein, Krusell, and Violante (2007) who study the effects of labor market policies, including tax wedges, in determining the effect of the faster capital embodied capital change on unemployment. They focus on steady states and do not investigate the response of unemployment to cyclical fluctuations in productivity. Their analysis abstracts from the endogenous response of productivity to changes in tax policies and thus likely features similar elasticities of unemployment with respect to cyclical fluctuations in productivity and taxes. Introducing worker heterogeneity along the lines of our paper into their model would likely help match the differences in these responses in the data. This appears an interesting extension to pursue.

Our analysis in this paper can be described as a comparison of two stationary economies (featuring aggregate shocks to productivity and price of capital equipment that do not have a trend) characterized by different tax rates. Thus, we have abstracted from secular changes in productivity and in the price of capital equipment. Relatedly, we did not study



the secular increase in the college premium observed in the U.S. in the 1980s and 1990s. A number of papers, including Acemoglu (1999), Albrecht and Vroman (2002), Shi (2002), Wong (2003), among others, have explored this issue in the MP model that includes worker heterogeneity but the productivity changes are exogenous. Krusell, Ohanian, Ríos-Rull, and Violante (2000) study the effect of a decline in the price of capital equipment on the college premium in a frictionless model. It seems to be an interesting extension to evaluate the effects of a decline in the price of capital equipment in our model and to compare the response of wage inequality, in particular, across countries with different levels of the tax wedge.

Finally, while our focus in this paper is on unemployment, Prescott (2004), Rogerson (2008), Rogerson and Wallenius (2009), among others, have studied the effects of tax policies on total hours worked using versions of the real business cycle model. The RBC model features the same trade-off as the basic MP model. As is shown in Prescott (2004), the standard RBC model with labor supply elasticity equal to 3 matches the cross-country differences in hours worked in response to differences in taxes. However, as pointed out in Hansen (1985), with this labor supply elasticity the RBC model generates only about one half of the observed volatility of hours worked over the business cycle. A higher labor supply elasticity is required to match the cyclical movements in hours. However, a higher elasticity would imply counterfactually strong policy effects. Incorporating worker heterogeneity into the RBC model along the lines proposed in this paper will help break the close linkage between the response of hours to changes in productivity and the response to changes in tax rates. Just as the version of the MP model that we proposed, a version of the RBC model with such a mechanism can feature a strong propagation of productivity shocks and simultaneously weaker policy effects.

## 3.7 Appendix

### 3.7.1 Data and Variable Construction Procedures

#### Aggregate Data

**Output.** Output is BLS non-farm business output.<sup>24</sup>

**Employment.** Aggregate employment is computed using monthly Current Population Surveys (MCPS) from January 1976 to December 2006. MCPS data are available at [http://www.nber.org/data/cps\\_basic.html](http://www.nber.org/data/cps_basic.html). To make this measure of employment consistent with the aggregate measure of output we exclude government, private households and unpaid family workers. We keep government agriculture workers because the CPS did not distinguish between private and government workers in agriculture before July 1985. Since there are only a few government agriculture workers in each sample after June 1985, they do not affect the results. The only inconsistency is that BLS business output does not include the output of non-profit institutions but our measure of employment includes employees of those institutions (because we cannot identify these people in the data). The resulting monthly employment series is seasonally adjusted using the ratio to moving average method and averaged into quarterly series.

**Productivity.** Aggregate productivity is defined as a ratio of output and employment.

**Wages.** Aggregate wage series is constructed as BLS labor share in non-farm business sector times productivity.

**Capital Structures.** We construct quarterly quality-adjusted stock of capital structures using the perpetual inventory method,  $k_{s,t+1} = (1 - d_{st})k_{st} + i_{st}$ . Annual series for depreci-

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<sup>24</sup>BLS data used are available at <http://data.bls.gov/cgi-bin/dsrv?pr>.

ation of capital structures,  $d_{st}$ , for the period from 1947 to 2000 comes from Cummins and Violante (2002). To compute the quarterly series we assume constant depreciation during a year. For the years 2001 through 2006 we assume that  $d_{st}$  is constant and equal to its value in the year 2000. Quality-adjusted investment in structures,  $i_{st}$ , is constructed using private fixed investment in structures (BEA-NIPA Table 5.3.5) deflated by the price index of non-durables consumption and services,  $PCONS_t$ .<sup>25</sup>  $PCONS_t$  is calculated using a Tornqvist procedure. If we have  $N$  goods, the change in the price index is

$$\Delta PCONS_t = \sum_{i=1}^N \log \left( \frac{p_t^i}{p_{t-1}^i} \right) \frac{s_t^i + s_{t-1}^i}{2},$$

and the price index is calculated then recursively

$$PCONS_t = PCONS_{t-1} \exp(\Delta PCONS_t),$$

where the initial value for the price index is set equal to 1. The price index for good  $i$ ,  $p_t^i$ ,  $i =$  non-durables consumption, services, is taken from BEA-NIPA Table 2.3.4 and the nominal share for good  $i$ ,  $s_t^i$ , is calculated using BEA-NIPA Table 2.3.5. The initial value (year 1947) for the stock of capital structures comes from BEA-FAT Table 2.1. The obtained series is then truncated for the years before 1976.

**Capital Equipment.** Quarterly quality-adjusted stock of capital equipment is also constructed using the perpetual inventory method,  $k_{e,t+1} = (1 - d_{et})k_{et} + i_{et}$ . Annual series for depreciation of capital equipment,  $d_{et}$ , is also taken from Cummins and Violante (2002), assuming that  $d_{et}$  is constant during a year and equal to its value in the year 2000 during the period from 2001 to 2006. We construct the series for nominal investment in equipment as the sum of private fixed investment in equipment (BEA-NIPA Table 5.3.5), changes in inventories (BEA-NIPA Table 1.1.5) and consumer durables (BEA-NIPA Table

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<sup>25</sup>As a robustness check we computed the price index of non-durables consumption and services excluding energy and housing and did not get any significant changes in the results.

1.1.5) and deflate it by the price index for equipment investment,  $PEQ_t$ , to get  $i_{et}$ . We use  $PEQ_t$  series constructed by Schorfheide, Rios-Rull, Fuentes-Albero, Santaaulalia-Llopis, and Kryshko (2007). It is constructed using the annual price index of equipment investment computed by Cummins and Violante (2002) and imputing the quarterly movements of the official NIPA price index of equipment investment.<sup>26</sup> The initial value (year = 1947) for the stock of capital equipment comes from BEA-FAT Table 2.1. The obtained series is then truncated for the years before 1976.<sup>27</sup>

### **Skill-Group Employment and Wages**

The sources of employment and wage data by skill group are monthly Current Population Surveys (MCPS) from January 1976 to December 2006 and CPS Outgoing Rotation Groups (ORG) covering the period January 1979 to December 2006. MCPS data are available at <http://www.nber.org/data/cps-basic.html> and CPS ORG data are available at <http://www.ceprdata.org/cps/org-index.php>. To compute the employment series by skill group we use the same procedure as for aggregate employment.

To compute wage series for skilled and unskilled categories we use data constructed by Schmitt (2003) from CPS ORG. Following the approach adopted in Krusell, Ohanian, Ríos-Rull, and Violante (2000) we divide our workers into 198 groups based on their demographic characteristics. There are 11 five-year age groups, 3 race groups (white, black and others), 2 gender groups and 3 education groups (less than high school diploma, high school diploma and college degree and more). Each group,  $g$ , is defined by age, race, gender and education. The set of groups is denoted by  $G$ . The measure of the group

<sup>26</sup>See Schorfheide, Rios-Rull, Fuentes-Albero, Santaaulalia-Llopis, and Kryshko (2007) for details.

<sup>27</sup>As a robustness check we computed the series for the stocks of capital structures and equipment for the period from 1976 to 2006 using 1976 stock as an initial value. There were no important changes in the results.

hourly wage is defined as

$$w_{gt} = \frac{\sum_{i \in g} w_{it} h_{it} \mu_{it}}{\sum_{i \in g} \mu_{it}},$$

where  $t = 01.1979, \dots, 12.2006$ ,  $\mu_{it}$  - individual's  $i$  earnings weight,  $h_{it}$  - individual's  $i$  usual weekly hours,  $w_{it}$  - the measure of individual  $i$  hourly wage constructed by Schmitt (2003) from CPS ORG. This measure uses a log-normal imputation to adjust for top-coding, trims data below US\$1 and US\$100 per hour (in constant 2002 dollars), includes overtime, tips and commissions for hourly paid workers and imputes usual weekly hours for those who report "hours vary" starting from 1994.

The measure of wages for skilled and unskilled workers in period  $t$  is constructed as follows

$$W_t^j = \sum_{g \in G_t^j} w_{gt} \bar{\mu}_g^j,$$

where  $j \in \{u, s\}$  indicates unskilled and skilled type, respectively,  $\bar{\mu}_g^j = \frac{\sum_{t=1}^T \mu_{gt}^j}{T}$  - temporal average proportion of group  $g$  workers in  $G^j$ ,  $T$  - number of time periods,  $\mu_{gt}^j = \frac{\sum_{i \in g} \mu_{i,t}}{\sum_{i \in G_t^j} \mu_{i,t}}$ . The resulting monthly series are deflated using monthly CPI-U, seasonally adjusted using the ratio to moving average method and averaged into quarterly series.

### Technology Shocks

The series of investment-specific technology change is calculated as

$$q_t = \frac{PCONS_t}{PEQ_t}.$$

To measure neutral technology shocks we use the production function parameters calibrated in Section 3.3. The monthly skill-group employment series constructed above are seasonally adjusted using the ratio to moving average method and averaged into annual series denoted by  $L_t$  and  $H_t$ , respectively. Low-skilled labor  $l_t$  and high-skilled labor  $h_t$

are normalized as follows

$$l_t = 2.6379 * 0.9162 \frac{L_t}{\sum_{t=1}^T L_t/T}, t = 1976, \dots, 2006$$

and

$$h_t = 0.9737 \frac{H_t}{\sum_{t=1}^T H_t/T}, t = 1976, \dots, 2006,$$

where 2.6379 is the measure of low-skilled workers,<sup>28</sup> and 0.9162 and 0.9737 are employment rates for low-skilled and high-skilled workers, respectively.

The series of neutral technology change is calculated as

$$A_t = \frac{Output}{k_{st}^\alpha \left[ \mu l_t^\sigma + (1 - \mu) (\lambda k_{et}^\rho + (1 - \lambda) h_t^\rho)^\frac{\sigma}{\rho} \right]^\frac{1-\alpha}{\sigma}}.$$

### Job-Finding and Job Separation Probabilities

To calculate job-finding and job separation probabilities we employ Shimer (2005b) two state approach. Assuming constant labor force,

$$u_{t+1} = u_t(1 - f_t) + u_{t+1}^s,$$

where  $u_{t+1}$  the number of unemployed individuals in month  $t$ ,  $u_{t+1}^s$  the number of individuals unemployed for less than one month in month  $t$ , and  $f_t \equiv \frac{m(u,v)}{u}$  is a probability that an unemployed individual finds a job. The measure of job separation probability is<sup>29</sup>

$$s_t = \frac{u_{t+1} - (1 - f_t)u_t}{e_t}.$$

We use basic monthly CPS data for the number of unemployed individuals and number of people unemployed for less than 4 weeks to construct  $f_t$  and  $s_f$  for skilled and unskilled categories.

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<sup>28</sup>This number is calculated as the average of  $\left\{ \frac{L_t}{H_t} \right\}_{t=01.1976, \dots, 12.2006}$ . The measure of high-skilled workers is normalized to 1.

<sup>29</sup>Note that this formula does not take time aggregation into account, since in our model inputs are measured at weekly frequencies.

Until 1994, all unemployed workers were asked about the duration of unemployment. Starting from 1994, the BLS adds the intervening time for unemployed individuals who have been asked about the duration of unemployment in the previous month. To account for this change in methodology we follow the procedure in Shimer (2005b) and multiply all computed series for short-term unemployment by 1.1 after 1994. The resulting monthly series are seasonally adjusted using the ratio to moving average method.

### 3.7.2 Computation

We use the free-entry condition (3.15) and flow equation for the surplus (3.16) to derive the following difference equations in  $\theta^T$ :

$$\frac{c_{x_t}^T}{\delta \phi^T(\theta_{x_t}^T)} = E_{x_t} \left\{ (1 - \beta^T)(p_{x_{t+1}}^T - z^T) - c_{x_t}^T \beta^T \theta_{x_t}^T + \frac{(1 - s^T)c_{x_{t+1}}^T}{\phi^T(\theta_{x_{t+1}}^T)} \right\}. \quad (3.61)$$

We solve this system of difference equations to find  $\theta^T$  as a function of  $x$ . Next, we simulate the model to generate artificial time series for neutral and investment-specific shocks, stocks of capital structures and equipment, unemployment, vacancies, and wages for each of the two worker types and the aggregate economy. To do so, we start with an initial value for unemployment of the two groups of workers, as well as neutral and investment-specific productivity shocks. Using the law of motion for employment, we compute next period's employment level  $n_{t+1}^l = l_{t+1}$  and  $n_{t+1}^h = h_{t+1}$ . Using these numbers compute capital  $k_{e,t+1}$  and  $k_{s,t+1}$  from the corresponding first-order conditions. Next, we draw a new pair of shocks to productivity and the price of capital equipment according to the stochastic process describing their evolution. We then know  $\theta^T$  and, thus, the job-finding rate and the new unemployment rate. Iterating this procedure generates the time series of interest.

### 3.7.3 Proofs

#### Implicit differentiation

We show how productivity changes in response to changes in  $z$ , where  $p^l$  and  $p^h$  are the fixed point of

$$A(p^l, z^l, p^h, z^h) - p^l = 0, \quad (3.62)$$

$$B(p^l, z^l, p^h, z^h) - p^h = 0. \quad (3.63)$$

It holds that

$$\begin{pmatrix} A_{p^l} - 1 & A_{p^h} \\ B_{p^l} & B_{p^h} - 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial p^l}{\partial z^l} \\ \frac{\partial p^h}{\partial z^l} \end{pmatrix} = \begin{pmatrix} -A_{z^l} \\ -B_{z^l} \end{pmatrix}.$$

This implies that

$$\begin{pmatrix} \frac{\partial p^l}{\partial z^l} \\ \frac{\partial p^h}{\partial z^l} \end{pmatrix} = -1/DD \begin{pmatrix} B_{p^h} - 1 & -A_{p^h} \\ -B_{p^l} & A_{p^l} - 1 \end{pmatrix} \cdot \begin{pmatrix} A_{z^l} \\ B_{z^l} \end{pmatrix},$$

where  $DD := (1 - A_{p^l})(1 - B_{p^h}) - A_{p^h}B_{p^l}$ . For the derivatives it holds (because equation (3.33) depends only on  $p - z$ ) that

$$A_{p^l} = -A_{z^l}, \quad (3.64)$$

$$A_{p^h} = -A_{z^h}, \quad (3.65)$$

$$B_{p^l} = -B_{z^l}, \quad (3.66)$$

$$B_{p^h} = -B_{z^h}. \quad (3.67)$$

This means that

$$\frac{\partial p^l}{\partial z^l} = \frac{-A_{p^l}(1 - B_{p^h}) - A_{p^h}B_{p^l}}{(1 - A_{p^l})(1 - B_{p^h}) - A_{p^h}B_{p^l}}, \quad (3.68)$$

$$\frac{\partial p^h}{\partial z^l} = \frac{-B_{p^l}(1 - A_{p^l}) - A_{p^l}B_{p^h}}{(1 - A_{p^l})(1 - B_{p^h}) - A_{p^h}B_{p^l}}. \quad (3.69)$$



To simplify this expression, we have to compute  $\frac{\partial \pi^l}{\partial l}$ ,  $\frac{\partial \pi^l}{\partial h}$ ,  $\frac{\partial \pi^h}{\partial l}$  and  $\frac{\partial \pi^h}{\partial h}$ .

First compute  $\frac{\partial \pi^l}{\partial l}$ :

$$\begin{aligned}
\frac{\partial \pi^l}{\partial l} &= G_{lk}k_l + G_{ll} \\
&= G_{lk} \frac{-G_{lk}}{G_{kk}} + G_{ll} \\
&= G_{lk} \frac{k}{l} + G_{lk} \frac{G_{kh}h}{G_{kk}l} + G_{ll} \\
&= \frac{h}{l} \left( -G_{lh} + G_{kh} \frac{G_{kl}}{G_{kk}} \right),
\end{aligned} \tag{3.70}$$

where the first equality follows from implicit differentiation of (3.32) and the second and third equalities are a consequence of constant returns to scale (which implies that  $G_k$  and  $G_l$  are homogeneous of degree zero):

$$G_{kk}k + G_{kh}h + G_{kl}l = 0, \tag{3.71}$$

$$G_{lk}k + G_{lh}h + G_{ll}l = 0. \tag{3.72}$$

Now compute  $\frac{\partial \pi^l}{\partial h}$ :

$$\begin{aligned}
\frac{\partial \pi^l}{\partial h} &= -G_{lk} \frac{G_{kh}}{G_{kk}} + G_{lh} \\
&= -\frac{l}{h} \frac{\partial \pi^l}{\partial l}.
\end{aligned} \tag{3.73}$$

Making use of similar arguments, it also holds that

$$\begin{aligned}
\frac{\partial \pi^h}{\partial h} &= G_{hk}k_h + G_{hh} \\
&= G_{hk} \frac{-G_{hk}}{G_{kk}} + G_{hh} \\
&= G_{hk} \frac{k}{h} + G_{hk} \frac{G_{kl}l}{G_{kk}h} + G_{hh} \\
&= \frac{l}{h} \left( -G_{lh} + G_{kh} \frac{G_{kl}}{G_{kk}} \right),
\end{aligned} \tag{3.74}$$

and

$$\begin{aligned}
\frac{\partial \pi^h}{\partial l} &= -G_{hk} \frac{G_{kl}}{G_{kk}} + G_{lh} \\
&= -\frac{h}{l} \frac{\partial \pi^h}{\partial h},
\end{aligned} \tag{3.75}$$

and

$$\frac{\partial \pi^l}{\partial l} = \frac{h^2}{l^2} \frac{\partial \pi^h}{\partial h}. \quad (3.76)$$

We can now simplify  $\frac{\partial \pi^l}{\partial z^l}$  and  $\frac{\partial \pi^h}{\partial z^l}$ :

$$\begin{aligned} A_{p^l} B_{p^h} - A_{p^h} B_{p^l} &= \left( \frac{\partial \pi^l}{\partial l} L_{p^l} \right) \left( \frac{\partial \pi^h}{\partial h} H_{p^h} \right) - \left( \frac{\partial \pi^l}{\partial h} H_{p^h} \right) \left( \frac{\partial \pi^h}{\partial l} L_{p^l} \right) \\ &= L_{p^l} H_{p^h} \left( \frac{\partial \pi^l}{\partial l} \frac{\partial \pi^h}{\partial h} - \left( \frac{-l}{h} \frac{\partial \pi^l}{\partial l} - \frac{h}{l} \frac{\partial \pi^h}{\partial h} \right) \right) = 0, \end{aligned} \quad (3.77)$$

and, thus,

$$\frac{\partial p^l}{\partial z^l} = \frac{-A_{p^l}}{1 - A_{p^l} - B_{p^h}}, \quad (3.78)$$

$$\frac{\partial p^h}{\partial z^l} = \frac{-B_{p^l}}{1 - A_{p^l} - B_{p^h}}. \quad (3.79)$$

By the same arguments it follows that

$$\frac{\partial p^l}{\partial z^h} = \frac{-A_{p^h}}{1 - A_{p^l} - B_{p^h}}, \quad (3.80)$$

$$\frac{\partial p^h}{\partial z^h} = \frac{-B_{p^h}}{1 - A_{p^l} - B_{p^h}}. \quad (3.81)$$

### Proof of Proposition 1

Using the above expressions for  $\frac{\partial \pi^l}{\partial l}$ ,  $\frac{\partial \pi^l}{\partial h}$ ,  $\frac{\partial \pi^h}{\partial l}$  and  $\frac{\partial \pi^h}{\partial h}$ , we find that

$$\begin{aligned} \epsilon_{\pi^l, l} &= \frac{h}{p^l} \left( -G_{lh} + G_{kh} \frac{G_{kl}}{G_{kk}} \right) \\ &= -\epsilon_{G_l, h} + \frac{\epsilon_{G_k, h} \cdot \epsilon_{G_l, k}}{\epsilon_{G_k, k}} \end{aligned} \quad (3.82)$$

$$\begin{aligned} \epsilon_{\pi^l, h} &= \frac{h}{p^l} \left( -G_{lh} + G_{kh} \frac{G_{kl}}{G_{kk}} \right) \\ &= -\epsilon_{\pi^l, l} \end{aligned} \quad (3.83)$$

$$\begin{aligned} \epsilon_{\pi^h, h} &= \frac{l}{p^h} \left( -G_{lh} + G_{kh} \frac{G_{kl}}{G_{kk}} \right) \\ &= -\epsilon_{G_h, l} + \frac{\epsilon_{G_k, l} \cdot \epsilon_{G_h, k}}{\epsilon_{G_k, k}} \\ &= \frac{lp^l}{hp^h} \epsilon_{\pi^l, l} \end{aligned} \quad (3.84)$$

$$\begin{aligned}
\epsilon_{\pi^h,l} &= -G_{hk} \frac{G_{kl}}{G_{kk}} + G_{lh} \\
&= -\epsilon_{\pi^h,h}
\end{aligned} \tag{3.85}$$

From the definitions of the functions  $A, B, \pi^l$  and  $\pi^h$  it follows that

$$\epsilon_{A,p^l} = \epsilon_{\pi^l,l} \epsilon_{L,p^l}, \tag{3.86}$$

$$\epsilon_{A,p^h} = \epsilon_{\pi^l,h} \epsilon_{H,p^l}, \tag{3.87}$$

$$\epsilon_{B,p^l} = \epsilon_{\pi^h,l} \epsilon_{L,p^h}, \tag{3.88}$$

$$\epsilon_{B,p^h} = \epsilon_{\pi^h,h} \epsilon_{H,p^h}, \tag{3.89}$$

which proves the proposition.

## Proof of Proposition 2

CRS and perfect substitutes imply that

$$G_{kk}k + G_{kh}h + G_{kl}l = G_{kk}k + G_{kl}(h+l) = 0, \tag{3.90}$$

$$G_{lk}k + G_{lh}h + G_{kk}l = G_{lk}k + G_{ll}(h+l) = 0. \tag{3.91}$$

The first equation implies that

$$(h+l) = -\frac{G_{kl}}{G_{kk}}k. \tag{3.92}$$

Plugging this into the second equation implies that

$$G_{lk}k + G_{ll} - \frac{G_{kl}}{G_{kk}}k = 0. \tag{3.93}$$

This implies that

$$\begin{aligned}
\epsilon_{\pi^l,l} &= \frac{h}{p^l} \left( -G_{lh} + G_{kh} \frac{G_{kl}}{G_{kk}} \right) \\
&= \frac{h}{p^l} \left( -G_{ll} + G_{kl} \frac{G_{kl}}{G_{kk}} \right) = 0.
\end{aligned} \tag{3.94}$$

Noting that all of the four elasticities are just a multiple of each other concludes the proof.

**Proof of Proposition 3**

Follows directly from inspection of  $\frac{\partial \pi^l}{\partial l}$ .

**Proof of Proposition 4**

Follows directly from Assumption 1 and Proposition 1.

**Proof of Proposition 5**

How the derivatives of  $p$  with respect to  $z$  are related to the derivatives of  $A$  and  $B$  was shown above. The sign of these derivatives then follows immediately from Proposition 3.

**Proof of Propositions 6, 7, 8 and 9**

The derivation of these results is discussed in the main text, which is based on Equation (3.56).

### 3.7.4 Appendix Tables

Table 3.7: Matching the Calibration Targets with Low Wage Elasticity.

Target	Value	
	Data	Model
1. Elasticity of wages wrt agg. productivity, $\epsilon_{w,p}$	0.500	0.498
2. Relative elasticity of wages wrt agg. productivity, $\epsilon_{w^H,p}/\epsilon_{w^L,p}$	1.770	1.775
3. Skilled job-finding rate, $f^H$	0.106	0.105
4. Unskilled job-finding rate, $f^L$	0.127	0.126
5. Skilled average market tightness, $\theta^H$	1.044	1.039
6. Unskilled average market tightness, $\theta^L$	0.586	0.584
7. Elasticity of agg. job-finding wrt agg. market tightness, $\epsilon_{f,\theta}$	0.500	0.497
8. Relative elas. of job-finding wrt agg. mrkt tightness, $\epsilon_{f^H,\theta}/\epsilon_{f^L,\theta}$	1.335	1.335
9. Standard deviation of capital structures	0.003	0.003
10. Standard deviation of capital equipment	0.010	0.010

Note: The table describes the model's performance in matching the calibration targets, including low wage elasticity.

Table 3.8: Calibrated Parameter Values with Low Wage Elasticity.

Parameter	Definition	Value
$z^H$	skilled value of non-market activity (share of their productivity)	0.848
$z^L$	unskilled value of non-market activity (share of their productivity)	0.945
$\beta^H$	skilled workers' bargaining power	0.043
$\beta^L$	unskilled workers' bargaining power	0.072
$\gamma^H$	skilled matching function elasticity	0.238
$\gamma^L$	unskilled matching function elasticity	0.544
$\chi^H$	skilled matching function efficiency	0.104
$\chi^L$	unskilled matching function efficiency	0.165
$d_s^*$	depreciation factor of capital structures	11.800
$d_e^*$	depreciation factor of capital equipment	1.460

Note: The table contains the calibrated parameter values in the low wage elasticity calibration.

Table 3.9: Results from the Calibrated Model with Low Wage Elasticity.

Statistic	Value		
	Data, 1951-2004 (1)	Model (2)	LM (3)
1. St. dev. of agg. productivity, $p$	0.013	0.013	0.013
2. Autocorr. of agg. productivity, $p$	0.765	0.765	0.765
3. St. dev. of agg. unemployment, $u$	0.125	0.104	0.071
4. St. dev. of agg. vacancies, $v$	0.139	0.142	0.101
5. St. dev. of agg. market tightness, $\theta$	0.259	0.246	0.163
6. Corr. of agg. unemployment and vacancies	-0.919	-0.782	-0.780
1. St. dev. of skilled productivity, $p^H$	—	0.018	—
2. Autocorr. of skilled productivity, $p^H$	—	0.779	—
3. St. dev. of skilled unemployment, $u^H$	—	0.138	—
4. St. dev. of skilled vacancies, $v^H$	—	0.103	—
5. St. dev. of skilled market tightness, $\theta^H$	—	0.207	—
1. St. dev. of unskilled productivity, $p^L$	—	0.013	—
2. Autocorr. of unskilled productivity, $p^L$	—	0.754	—
3. St. dev. of unskilled unemployment, $u^L$	—	0.100	—
4. St. dev. of unskilled vacancies, $v^L$	—	0.170	—
5. St. dev. of unskilled market tightness, $\theta^L$	—	0.258	—

Note: Column (1) contains aggregate statistics computed over the 1951:1 to 2004:4 period as in Shimer (2005a). Hornstein, Krusell, and Violante (2005b) report virtually identical numbers. In Column (1) seasonally adjusted unemployment,  $u$ , is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index,  $v$ , is constructed by the Conference Board. Both  $u$  and  $v$  are quarterly averages of monthly series. Average labor productivity  $p$  is seasonally adjusted real average output per person in the non-farm business sector, constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. Column (2) contains the results from the model calibrated with low wage elasticity. Column (3) reproduces the results from the linear model with homogeneous workers for the same aggregate calibration targets. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

Table 3.10: Matching the Calibration Targets with  $s = s_U$ .

Target	Value	
	Data	Model
1. Elasticity of wages wrt agg. productivity, $\epsilon_{w,p}$	0.670	0.669
2. Relative elasticity of wages wrt agg. productivity, $\epsilon_{w^H,p}/\epsilon_{w^L,p}$	1.770	1.801
3. Skilled job-finding rate, $f^H$	0.106	0.105
4. Unskilled job-finding rate, $f^L$	0.127	0.127
5. Skilled average market tightness, $\theta^H$	1.044	1.059
6. Unskilled average market tightness, $\theta^L$	0.586	0.604
7. Elasticity of agg. job-finding wrt agg. market tightness, $\epsilon_{f,\theta}$	0.500	0.501
8. Relative elas. of job-finding wrt agg. mrkt tightness, $\epsilon_{f^H,\theta}/\epsilon_{f^L,\theta}$	1.335	1.346
9. Standard deviation of capital structures	0.003	0.003
10. Standard deviation of capital equipment	0.010	0.010

Note: The table describes the model's performance in matching the calibration targets without distinguishing between the total separation rate and the separation rate into unemployment.



Table 3.11: Calibrated Parameter Values with  $s = s_U$ .

Parameter	Definition	Value
$z^H$	skilled value of non-market activity (share of their productivity)	0.897
$z^L$	unskilled value of non-market activity (share of their productivity)	0.943
$\beta^H$	skilled workers' bargaining power	0.064
$\beta^L$	unskilled workers' bargaining power	0.098
$\gamma^H$	skilled matching function elasticity	0.230
$\gamma^L$	unskilled matching function elasticity	0.540
$\chi^H$	skilled matching function efficiency	0.102
$\chi^L$	unskilled matching function efficiency	0.164
$d_s^*$	depreciation factor of capital structures	11.500
$d_e^*$	depreciation factor of capital equipment	1.420

Note: The table contains the calibrated parameter values in the calibration without distinguishing between the total separation rate and the separation rate into unemployment.

Table 3.12: Results from the Calibrated Model with  $s = s_U$ .

Statistic	Value		
	Data	Model	LM
	(1)	(2)	(3)
1. St. dev. of agg. productivity, $p$	0.013	0.013	0.013
2. Autocorr. of agg. productivity, $p$	0.765	0.765	0.765
3. St. dev. of agg. unemployment, $u$	0.090	0.096	0.061
4. St. dev. of agg. vacancies, $v$	0.116	0.130	0.086
5. St. dev. of agg. market tightness, $\theta$	0.202	0.227	0.139
6. Corr. of agg. unemployment and vacancies	-0.910	-0.780	-0.780
1. St. dev. of skilled productivity, $p^H$	—	0.018	—
2. Autocorr. of skilled productivity, $p^H$	—	0.778	—
3. St. dev. of skilled unemployment, $u^H$	0.111	0.129	—
4. St. dev. of skilled vacancies, $v^H$	—	0.096	—
5. St. dev. of skilled market tightness, $\theta^H$	—	0.192	—
1. St. dev. of unskilled productivity, $p^L$	—	0.013	—
2. Autocorr. of unskilled productivity, $p^L$	—	0.758	—
3. St. dev. of unskilled unemployment, $u^L$	0.085	0.093	—
4. St. dev. of unskilled vacancies, $v^L$	—	0.156	—
5. St. dev. of unskilled market tightness, $\theta^L$	—	0.238	—

Note: Column (1) reproduces Column (1) of Table 3.3. See notes to that table for details. Column (2) contains the results from the model calibrated without distinguishing between the total separation rate and the separation rate into unemployment. Column (3) shows the results from the linear model with homogeneous workers for the same aggregate calibration targets. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

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