1-1-2014

Essays on Dynamic Games of Incomplete Information

Ilwoo Hwang
University of Pennsylvania, ilhwang@sas.upenn.edu

Essays on Dynamic Games of Incomplete Information

Abstract
This dissertation consists of three essays that study the dynamic games with incomplete information. In the first chapter, I study a dynamic trading game where a seller and potential buyers start out symmetrically uninformed about the quality of a good, but the seller becomes informed about the quality, so that the asymmetric information between the agents increases over time. The introduction of a widening information gap results in several new phenomena. In particular, the interaction between screening and learning generates nonmonotonic price and trading patterns, contrary to the standard models in which asymmetric information is initially given. If the seller's effective learning speed is high, the equilibrium features "collapse-and-recovery" behavior: Both the equilibrium price and the probability of a trade drop at a threshold time and then increase later. The seller's payoff is nonmonotonic in his learning speed, as a slower learning speed can lead to higher payoff for the seller.

In the second chapter, I study a dynamic one-sided-offer bargaining model between a seller and a buyer under incomplete information. The seller knows the quality of his product while the buyer does not. During bargaining, the seller randomly receives an outside option, the value of which depends on the hidden quality. If the outside option is sufficiently important, there is an equilibrium in which the uninformed buyer fails to learn the quality and continues to make the same randomized offer throughout the bargaining process. As a result, the equilibrium behavior produces an outcome path that resembles the outcome of a bargaining deadlock and its resolution. The equilibrium with deadlock has inefficient outcomes such as a delay in reaching an agreement and a breakdown in negotiations. Bargaining inefficiencies do not vanish even with frequent offers, and they may exist when there is no static adverse selection problem.

In the third chapter, I address the following question: when does an incumbent party have an incentive to experiment with a risky reform policy in the presence of future elections? I study a continuous-time game between two political parties with heterogeneous preferences and a median voter. I show that while infrequent elections are surely bad for the median voter, too frequent elections can also make him strictly worse off. When the election frequency is low, a standard agency problem arises and the incumbent party experiments with its preferred reform policy even if its outlook is not promising. On the other hand, when the election frequency is too high, in equilibrium the incumbent stops experimentation too early because the imminent election increases the incumbent's potential loss of power if it undertakes risky reform. The degree of inefficiency is large enough that too frequent elections are worse for the median voter than a dictatorship.

Degree Type
Dissertation

Degree Name
Doctor of Philosophy (PhD)

Graduate Group
Economics

First Advisor
George J. Mailath

Keywords
Bargaining, Experimentation, Game Theory, Incomplete Information, Learning, Microeconomic Theory

Subject Categories
Economics | Economic Theory

ESSAYS ON DYNAMIC GAMES OF INCOMPLETE INFORMATION

Ilwoo Hwang

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2014

Supervisor of Dissertation

George J. Mailath, Professor of Economics

Graduate Group Chairperson

George J. Mailath, Professor of Economics

Dissertation Committee:

Andrew Postlewaite, Professor of Economics

Itay Goldstein, Professor of Economics

Yuichi Yamamoto, Assistant Professor of Economics

To my father

Acknowledgments

Firstly I would like to express my gratitude to my supervisor Professor George Mailath. George taught the first course I took in my PhD program, and introduced me to the world of microeconomic theory. Ever since I begin to conduct my research on dynamic game theory, his dedication, his insights, and his consistent support have greatly inspired and motivated me. Always having a sharp eye for the inconsistencies in my arguments and analyses, George taught me how to do rigorous research. Above all, he has set an example of intellectual honesty and professionalism which will accompany me throughout my own career.

I also greatly thank for the help from the members of my dissertation committee: Professor Andrew Postlewaite, Professor Itay Goldstein, and Professor Yuichi Yamamoto. Andy, my academic role model, always challenged me in numerous conversations with his sharp intuition and deep understanding of economics. Itay strengthened my self-confidence, and his insightful comments and discussions were indispensable. Yuichi helped me develop a more rigorous explanation about my result, and his arguments were always crystal clear and thoughtful.

Special thanks go to Professor Teddy Kim, who has shaped my understanding of dynamic bargaining theory and encouraged my idea which eventually became my job market paper. This dissertation was benefited from the continuous discussions with him.

Daily interactions and friendship with my colleagues were very important for my research and my life in Philadelphia. Countless discussions and joyful conversations over lunch, dinner, and coffee not only benefited my research ideas, but also filled me with energy for the challenges ahead. I am especially grateful to Dr Naoki Aizawa, Dr Jeasung Choi, Dr Francesc Dilme, Selman Erol, Zehao Hu, Dr Chong Huang, Dr Karam Kang, Dr You Suk Kim, Dr Fei Li, Dr Suryun Rhee, and Ilton Soares.

It is hard to describe my gratitude to my family for all their love and support. My mom Haesook is the greatest woman I ever known, and her love, optimism, and support for all my endeavors have been endless. My sister Eunhoo always cheered me up with a good dose of humour and encouragement, and she has showed me how to live with true passion. My dad Hieonsan has always been my role model as a man with integrity and wisdom. An academic himself, he showed me a lifelong dedication to his research and taught me how to live in harmony with knowledge. With all my love and respect, I dedicate this dissertation to him.

ABSTRACT

ESSAYS ON DYNAMIC GAMES OF INCOMPLETE INFORMATION

Ilwoo Hwang

George J. Mailath

This dissertation consists of three essays that study the dynamic games with incomplete information. In the first chapter, I study a dynamic trading game where a seller and potential buyers start out symmetrically uninformed about the quality of a good, but the seller becomes informed about the quality, so that the asymmetric information between the agents *increases* over time. The introduction of a widening information gap results in several new phenomena. In particular, the interaction between screening and learning generates nonmonotonic price and trading patterns, contrary to the standard models in which asymmetric information is initially given. If the seller's effective learning speed is high, the equilibrium features "collapse-and-recovery" behavior: Both the equilibrium price and the probability of a trade drop at a threshold time and then increase later. The seller's payoff is nonmonotonic in his learning speed, as a slower learning speed can lead to higher payoff for the seller.

In the second chapter, I study a dynamic one-sided-offer bargaining model between a seller and a buyer under incomplete information. The seller knows the quality of his product while the buyer does not. During bargaining, the seller randomly receives an outside option, the value of which depends on the hidden quality. If the outside option is sufficiently important, there is an equilibrium in which the uninformed buyer fails to learn the quality

and continues to make the same randomized offer throughout the bargaining process. As a result, the equilibrium behavior produces an outcome path that resembles the outcome of a bargaining deadlock and its resolution. The equilibrium with deadlock has inefficient outcomes such as a delay in reaching an agreement and a breakdown in negotiations. Bargaining inefficiencies do not vanish even with frequent offers, and they may exist when there is no static adverse selection problem.

In the third chapter, I address the following question: when does an incumbent party have an incentive to experiment with a risky reform policy in the presence of future elections? I study a continuous-time game between two political parties with heterogeneous preferences and a median voter. I show that while infrequent elections are surely bad for the median voter, too frequent elections can also make him strictly worse off. When the election frequency is low, a standard agency problem arises and the incumbent party experiments with its preferred reform policy even if its outlook is not promising. On the other hand, when the election frequency is too high, in equilibrium the incumbent stops experimentation too early because the imminent election increases the incumbent's potential loss of power if it undertakes risky reform. The degree of inefficiency is large enough that too frequent elections are worse for the median voter than a dictatorship.

Contents

List of Tables

List of Figures

Chapter I

Dynamic Trading with Increasing Asymmetric Information

1 Introduction

Akerlof's seminal 1970 paper on asymmetric information shows that its existence can lead to inefficient trade outcomes. In the literature following Akerlof's work, many researchers have investigated the dynamic impact of the adverse selection problem. Yet despite this focus, most existing models assume that the asymmetric information exists initially, in the sense that one side of transaction starts with superior information than the other. However, there are many economic environments in which neither agent is perfectly informed in the beginning and one side gradually obtains information, so that the information gap between the agents grows over time. This observation relates to the main innovation of this paper: I consider a dynamic trading situation where the degree of asymmetric information between agents *increases* over time, and analyze its effects on trading patterns and efficiency.

Increasing asymmetric information is a general phenomenon that arises in many environments. Consider, for instance, an entrepreneur who wants to sell his start-up firm. When the entrepreneur starts the company, he is not sure about the prospects of his firm or the technology that his firm creates, but over time, he learns about the firm's viability. Trading of a securitized asset (where asset holders are gradually informed about the quality of complex assets, such as collateralized mortgage obligations) and a market for "talent" (where a manager gains an informational advantage regarding the potential of his talent agents) are other environments with increasing asymmetric information. The common theme underlying these examples is the feature of "learning-by-holding." As people hold or use a good, they observe more signals and thereby gain an informational advantage. If an economic environment has the feature of learning-by-holding, the degree of the asymmetric information may increase over time.

To investigate the impact of increasing asymmetric information, I study a stylized model of a dynamic trading game between a single seller and a sequence of potential buyers. The seller holds an indivisible unit of a good, the quality of which is either high or low. The potential buyers randomly arrive to be matched with the seller. Upon arrival, the buyer observes how long the good has been up for sale (time-on-the-market) and makes a take-it-or-leave-it offer to the seller. In contrast to existing models, all agents are initially uninformed about the quality of the good and have a common prior belief. Over time, the seller exogenously learns the quality of the good by observing the arrival of a perfectly informative signal. The buyers remain uninformed about the quality of the good; they also do not know whether the seller is informed about it.

The introduction of increasing asymmetric information results in several new phenomena. In particular, the interaction between the seller's learning and the buyers' equilibrium behavior generates *nonmonotonic* price and trading patterns, contrary to the standard models in which asymmetric information is initially given. Equilibrium dynamics depend on the effective speed of learning of the seller, which is the ratio of the seller's speed of learning to the arrival rate of the buyers.

In this model, the buyers form two layers of beliefs, the evolution of which works as one of the main driving forces of nonmonotonic equilibrium dynamics. Since the buyers observe neither the quality nor the seller's learning, they form beliefs about the quality of the good *and* about the *seller's belief* about the quality of the good. This belief structure is different from the one in the existing models of dynamic adverse selection in which it is common knowledge that the seller is informed. Specifically, in this model the buyers form beliefs about the seller's status, which fall into one of the following three types: (1), the seller is informed that his good is of high quality; (2), he is informed that his good is of low quality (a "lemon"); or (3), that he is uninformed about the quality of the good.

In the early stage of the game, the buyers believe that the seller is highly likely to be uninformed and that the degree of asymmetric information is small. Therefore, if the buyer arrives early, he targets the uninformed seller by offering a middle-range price. Over time, the seller becomes more informed. If the seller finds that his good is of high quality, then he rejects the middle-range price in hopes of selling at a higher price. But the informed seller with a lemon accepts the middle-range price as waiting is more costly for him. As a result, if the buyer who arrives late targets an uninformed seller by a middle-range price offer, the probability of getting a low-quality good is higher.

If the effective learning speed of the seller is sufficiently high (a fast-learning case), the equilibrium features a "collapse-and-recovery" pattern. If the learning speed is high, the probability that the seller is uninformed rapidly decreases, so buyers become increasingly worried about the quality of the good when targeting an uninformed seller. Therefore, there is a threshold time after which it is no longer optimal for buyers to target an uninformed seller. Therefore, after the threshold time buyers target only the informed seller of a lemon. As a result, both the equilibrium price and the probability of a trade drop at the threshold time. On the other hand, an informed seller with a high-quality good rejects both a middle-range price and a low price, so the overall expected quality of the good increases over time. Therefore, there exists a second threshold time at which the expected quality is high enough that the buyers begin to offer a high price to target all types of sellers. The equilibrium trading price thus jumps at the second threshold time.

If the seller's effective speed of learning is low (a slow-learning case), then the probability that the seller is uninformed remains sufficiently high for a long period, and it is optimal for buyers to offer a middle-range price for that period. Thus the overall expected quality of the good increases over time, because the informed seller with a high-quality good does not trade. Therefore, similar to the fast-learning case, there exists a threshold time at which the buyers begin to offer a high price to target all types of sellers.

On the other hand, the equilibrium price before the threshold time may also be nonmonotonic, because of the seller's value of information. In the early stage of the game, buyers target an uninformed seller. This behavior generates a positive value of information for the seller, since the informed seller can adjust his offer acceptance behavior depending on the information received, and achieve a strictly higher payoff. So the uninformed seller, who expects to be informed later, factors the value of the *future* information into his current reservation price. I show that the change in the value of information may lead to a nonmonotonic reservation price for the seller, leading to a nonmonotonic equilibrium trading price.

After analyzing the equilibrium behavior, I conduct some comparative statics. I show that the threshold time decreases as the learning speed of the seller increases. If the learning speed is arbitrarily small, then the equilibrium of this model converges toward the equilibrium in the model with symmetrically uninformed agents. On the other hand, as the learning speed increases to infinity, the model converges toward the model with initial asymmetric information, and hence the collapse occurs almost immediately after the beginning of the game.

Lastly, I show that the seller's payoff is nonmonotonic with regard to his own learning speed. It is well known that in a situation with initial asymmetric information, the trade surplus is lower (because of the adverse selection problem) and the seller's payoff is higher (because of information rent) compared to an environment with symmetric information. In my model, while the trade surplus decreases as the learning speed increases, the seller may achieve a higher payoff in a case with increasing asymmetric information than in a case where he is initially informed. The higher the seller's learning speed is, the greater division of the surplus the seller obtains. However, if the learning speed is too high, inefficiency caused by asymmetric information becomes too large, leading to a smaller payoff for the seller.

1.1 Related Literature

This paper contributes to the rich literature of dynamic adverse selection. These papers investigate the dynamic impact of asymmetric information in various contexts, such as a dynamic bargaining game with interdependent values (Evans, 1989; Vincent, 1989; Deneckere and Liang, 2006; and Fuchs and Skrzypacz, 2010), a sequential search model (Hörner and Vieille, 2009; Zhu, 2012; Kaya and Kim, 2013; and Lauermann and Wolinsky, 2013), an equilibrium search framework (Moreno and Wooders, 2010; Kim, 2011; Camargo and Lester, 2011; and Guerrieri and Shimer, 2013), and a dynamic signaling model (Janssen and

Roy, 2002; Daley and Green, 2012; and Fuchs and Skrzypacz, 2013). All of these papers assume that asymmetric information is initially given, so that from the beginning one side of transaction is perfectly informed about the quality of the good. On the other hand, the present paper considers an environment where asymmetric information increases. Moreover, the richer equilibrium trading dynamics of this paper contribute to the applicability of the literature.

Daley and Green (2012) consider a dynamic setting in which stochastic information (news) about the value of a privately-informed seller's asset is gradually revealed to a market of buyers. So in their model, asymmetric information is initially given and exogenously dissolves over time. In contrast, the present paper considers a case in which agents are initially symmetrically uninformed, and then asymmetric information exogenously increases. Both papers show trading patterns that differ from those in the standard model, but the trading dynamics are different, as is the intuition behind the results.

Plantin (2009) and Bolton, Santos, and Scheinkman (2011) consider finite-horizon models in which the seller learns the quality of his asset. In their models, the learning of the seller occurs in a single period. On the other hand, the present paper models the learning process in a full dynamic setting, and finds various equilibrium trading dynamics and underlying belief evolutions. Moreover, the dynamic model in the paper make it possible to conduct comparative statics.

Choi (2013) studies a stationary dynamic equilibrium model of a resale market with adverse selection in which new owners are uninformed and slowly learn the quality of their acquisitions. He characterizes steady-state equilibria of the model and shows that trade efficiency increases as the learning speed of the seller increases. In this paper, I consider a nonstationary environment and analyze the dynamics of trading patterns.

The remainder of the paper is as follows. Section 2 describes the model and shows some preliminary observations. Section 3 presents equilibria under the slow- and fast-learning cases and describes the equilibrium dynamics with the underlying belief evolution. Section 4 presents comparative statics of some important equilibrium values as well as the trade surplus and its division. Section 5 discusses the implications of the results for the recent financial crisis and the role of assumptions of the model. Section 6 concludes. Some of the proofs are presented in the Appendix.

2 Model

Time $t \geq 0$ is continuous. There is a long-lived seller with a countably infinite number of potential buyers. The seller holds an indivisible unit of a good. Buyers arrive at random times which correspond to the jumping times of a Poisson process with constant rate λ. Upon arriving, the buyer observes only how long the the seller has stayed in the game, that is, the calendar time t. In particular, the buyer does not observe the history of past offers.[1] Then the buyer makes a take-it-or-leave-it offer p. If the seller accepts the offer, then the game ends. Otherwise, the buyer leaves and the seller waits for subsequent buyers.[2] The seller discounts future payoffs at a rate $r > 0$.

The quality θ of the good is determined by Nature and is either high (H) or low (L). At time zero, all agents of the game are uninformed, and they form a common prior belief

[1] So the model considers a case in which previous offers are kept hidden to future buyers. To read about the effect of the information available to potential buyers on trading dynamics and efficiency, see Noldeke and van Damme (1990); Swinkels (1999); Hörner and Vieille (2009); Kim (2011); Fuchs, Öry, and Skrzypacz (2012); and Kaya and Liu (2013).

[2] The assumptions on the arrival process and on the information of the buyers are similar to those of Kim (2011) and Kaya and Kim (2013).

q_0 that the quality of the good is high. Over time, the seller privately receives a series of perfectly informative signals which arrive according to a Poisson process of constant rate ρ. The processes of the arrival of signals and the arrival of the buyers are independent. Since each signal is perfectly informative, upon the first arrival of the signal the seller is perfectly informed about the quality of the good.[3]

The valuation of the good to the buyers is common to all of them and is denoted by v_θ, where $v_H > v_L$. The seller values the good at a discounted proportion of $\alpha < 1$. Therefore, the trading of a quality-θ good yields $(1-\alpha)v_\theta$ of trade surplus.[4]

An outcome of the game is a triple (θ, t, p), with the interpretation that the realized type is θ and that the trade occurs at time t with price p. The case $t = \infty$ (with $p = 0$) corresponds to the outcome in which the trade does not occur. The payoff of the buyer at time t is $v_\theta - p$ if the outcome is (θ, t, p), and zero otherwise. There are two ways to represent the seller's payoff. The first interpretation, which I adopt in the following analysis, assumes that each signal carries a dividend of size $x_\theta = \frac{r}{\rho} v_\theta$. The size of each dividend is precisely determined to ensure that the present expected value of the dividend from quality-θ good is v_θ. Then it is assumed that the seller values each dividend at a rate $\alpha < 1$.[5] Alternate interpretation is that the seller incurs a production cost αv_θ at the time of trade, so the payoff is realized after the trade occurs. It is immediate to verify that this interpretation yields the same incentives of the agents.

The paper analyzes the environment where there is a sufficiently high probability of a

[3]Models with different information processes are discussed in Section 5.

[4]The fact that the trade surplus increases in the quality of the good is not crucial in deriving the equilibrium of the model. Indeed, the result is robust under cases in which the trade surplus is independent or decreasing in the quality of the good, as long as the parameter values satisfy a relevant assumption (counterpart to Assumption 1).

[5]One interpretation is that the seller is more impatient than the buyers.

low-quality good (lemon). Consider a static bargaining game where the seller knows the quality of his good. In order to attract all types of sellers, the buyer must offer no less than αv_H, the minimum reservation price of the seller with the high-quality good. So the trade outcome is not efficient if offering such a price yields negative payoffs to the buyer, that is,

$$v(q_0) < \alpha v_H,$$

where $v(q_0) = q_0 v_H + (1 - q_0)v_L$ is the ex ante value of the good to the buyers. I call the above inequality the *static lemons condition*. Note that the condition holds if the prior q_0 is sufficiently small. In fact, define q^* such that $q^* v_H + (1 - q^*)v_L = \alpha v_H$. Then the static lemons condition can be equivalently written as

$$q_0 < q^*.$$

I am particularly interested in the case where the seller is sufficiently patient. Specifically, I make the following parametric assumption:

Assumption 1.

$$v(q_0) < \frac{r}{r+\lambda}\alpha v(q_0) + \frac{\lambda}{r+\lambda}\alpha v_H.$$

Assumption 1 ensures that the seller has non-trivial intertemporal incentives. It implies that the buyer's offer targeted to the uninformed seller (which is at most $v(q_0)$) is rejected if the uninformed seller expects that he will receive a non-screening offer (at least αv_H) at the next match. Note that static lemons condition is a necessary condition for Assumption 1. Given the static lemons condition, the assumption is satisfied when the value of r/λ

is sufficiently small. Although Assumption 1 is not a necessary condition for the basic economic mechanism I highlight in this paper, it contributes to the analytical tractability of the model.[6]

The information process implies that, at any time $t > 0$ the seller is one of the following three types: 1) one who has received a lump-sum payoff x_H, and so is informed that his good is of high quality; 2) one who is informed that his good is of low quality; and 3) one who has not received a payoff and so is uninformed about the good's quality. I will denote g (good type) for the informed seller with the high-quality good, b (bad type) for the informed seller with the low-quality good, and u (uninformed) for the uninformed seller.

Since the signal is perfectly informative, the good-type (bad-type) seller believes that the quality is high (low) with probability one. The uninformed seller's belief stays the same at the prior q_0. Because the arrival rate of the information is the same for all θ, not receiving any signal does not provide additional information.

The buyers' beliefs are represented by a function $\phi : \mathbb{R}_+ \to \Delta\{g, u, b\}$. Let $\phi_z(t) = \phi(t)(z)(z = g, u, b)$ be the belief of the buyer at time t that the seller is type z. Then it is straightforward that $\phi_u(0) = 1$, and that $\phi_g(t) + \phi_u(t) + \phi_b(t) = 1$ for any $t \geq 0$. Let $q(t)$ be the buyer's (unconditional) belief at time t that the quality of the good is high. Then $q(0) = q_0$, and $q(t)$ can be expressed as a function of $\phi_z(t)$:

$$q(t) = \phi_g(t) + \phi_u(t)q_0.$$

[6] If the static lemons condition is not satisfied, then there exists an equilibrium where the first buyer offers a trade-ending price to end the game. If the static lemons condition is satisfied, for a range of parameters that does not satisfy the assumption, there exists an equilibrium whose structure is similar to the one described in the paper. However, in this case it is difficult to get a clear equilibrium characterization result, such as a payoff equivalence result within the set of equilibria of the model.

The offer strategies of the buyers are represented as a mapping σ_B from $\mathbb{R}_+$ to a set of probability distributions over $\mathbb{R}$, where $\sigma_B(t)$ denotes a probability distribution of the buyer's offer at time t. I denote $\sigma_B(t) = p'$ when $\sigma_B(t)$ is a degenerate distribution at price p'. The acceptance strategy of the seller is represented by a function $\sigma_S : \{g, u, b\} \times \mathbb{R}_+ \times \mathbb{R}_+ \to [0, 1]$ where $\sigma_S(z, t, p)$ denotes the probability that a type-z seller accepts price p at time t.

I use the perfect Bayesian equilibrium concept throughout this paper.

Definition 1. A tuple $(\sigma_S, \sigma_B, \phi)$ is a *perfect Bayesian equilibrium (PBE)* if (1) given σ_S and ϕ, for any t, $\sigma_B(t)$ assigns a positive probability to a price p only if p maximizes the expected payoff of the buyer at time t, (2) given σ_S, for any z and t, $\sigma_S(z, t, p) > 0$ only if p is weakly greater than the type-z seller's continuation payoff at time t, and (3) given σ_S and σ_B, ϕ is derived through Bayesian updating.

2.1 Preliminary Observations

I begin by presenting lemmas that help in characterizing the equilibrium structure. The proofs of the lemmas are straightforward, so are omitted. The following lemma states that in any equilibrium of the model, there exists a reservation price function $R_z(t)$ for each type of the seller such that the type-z seller at t accepts $p > R_z(t)$ and rejects $p < R_z(t)$ with probability one.

Lemma 1. *(Reservation Price Strategy) In equilibrium, there exists a function $R_z : \mathbb{R}_+ \to \mathbb{R}$ for each $z = g, u, b$ such that $\sigma_S(z, t, p) = 1$ for any $p > R_z(t)$ and $\sigma_S(z, t, p) = 0$ for any $p < R_z(t)$.*

It is easy to show that $R_z(t)$ equals the type-z seller's continuation payoff if he rejects the buyer's offer at t. This is due to the information structure of the game whereby the current offer is not revealed to future buyers. Note that $R_z(t)$ is continuous in t because the probability that either the buyer or the lump-sum payoff arrives at a given time interval vanishes as the length of the interval shrinks to zero. Moreover, $R_g(t) > R_u(t) > R_b(t)$ for all t because of the heterogeneous expected value of lump-sum payoffs.

Given the seller's reservation price strategy, the buyer's equilibrium offer satisfies the following lemma:

Lemma 2. *In equilibrium, if the buyer's equilibrium offer is accepted with nonzero probability, then it is equal to $R_z(t)$ for some $z = g, u, b$.*

The intuition of the lemma is straightforward: If the offer is above the reservation price of some type of seller, then the buyer can lower his offer slightly and still trade with the same probability. Note that the above lemma does not rule out the case where the buyer's equilibrium offer is rejected with probability one at some t. In that case, the buyer's offer p must be a price between zero and $R_b(t)$.

The seller always has an option to hold the good, which gives lower bounds on the reservation price functions. They are given by

$$\begin{aligned} R_g(t) &\geq \alpha v_H, \\ R_u(t) &\geq \alpha v(q_0), \\ R_b(t) &\geq \alpha v_L. \end{aligned}$$

The following lemma places an upper bound on the buyer's equilibrium offer, and hence provides an upper bound on the reservation price of the good-type seller:

Lemma 3. *In equilibrium, the buyers never offer a price strictly more than αv_H. Therefore, $R_g(t) = \alpha v_H$ for any t.*

The intuition for this lemma is as follows. Suppose not, and let $\bar{p} > \alpha v_H$ be the supremum of the buyer's equilibrium offer. Then there exists $\bar{t}$ such that the buyer at time $\bar{t}$ offers a price arbitrarily close to $\bar{p}$. Then all types of sellers strictly prefer to accept the offer because the seller discount the future payoffs. Now consider a deviation of the buyer at time $\bar{t}$ to lower his offer by sufficiently small $\epsilon > 0$. Then all types of sellers would still accept the offer as long as the expected cost from discounting is greater than ϵ. But then offering such price is a profitable deviation of the buyer, leading to a contradiction.

Note that Lemma 3 implies that if the buyer offers αv_H, then the offer is accepted by all types of sellers, so the game ends with probability one. Therefore αv_H serves as the trade-ending offer in this model.

3 Equilibrium

In this section I construct an equilibrium of the model, and present a full characterization result of the equilibria for a range of parameters.

Because of the static lemons condition, offering the trade-ending price αv_H in the early stage yields a negative payoff to the buyer. Then one might expect that the buyer who arrives in the early stage submits a screening offer and targets either the uninformed seller or the bad-type seller. In this case, the expected quality of the good increases gradually over time.

On the other hand, the buyers' beliefs about the seller's type also evolve over time because of the seller's learning. The buyer who arrives in the early stage believes that the seller is likely to be uninformed. So the buyer targets the uninformed seller by offering a middle-range price, which equals to the reservation price of the uninformed seller. But the seller is getting informed over time, hence there is a growing probability that the seller is the bad type. The bad-type seller accepts the middle-range price offer, since it is strictly higher than his reservation price. In this case, the buyer becomes increasingly worried about the possibility of getting a lemon.

It turns out that the seller's speed of learning determines the rate of increase of the probability that the seller is bad type, which in turn affects the equilibrium behavior. Specifically, the equilibrium behavior is qualitatively different depending on the seller's effective speed of learning (ρ/λ).

In the following analysis, I first present the equilibrium when the effective speed of learning is low (the slow-learning case) with the characterization results. After that I turn to the case when the effective speed of learning is high (the fast-learning case).

3.1 Slow-learning Case

In this subsection I consider the case where the seller's effective speed of learning is low. I begin by defining a class of candidate equilibrium strategy profiles.

Definition 2. A strategy profile (σ_S, σ_B) is called a *two-phase strategy profile* if there exists $t^* > 0$ and $\hat{\sigma} \in [0, 1]$ such that the profile satisfies the following:

1. Phase I: for any $t < t^*$,

    ▷ $\sigma_B(t) = R_u(t)$;

- ▷ $\sigma_S(g, t, R_u(t)) = 0$; $\sigma_S(z, t, R_u(t)) = 1$ for $z = u, b$.

2. Phase II: for any $t \geq t^*$,

    - ▷ $\sigma_B(t)$ assigns a probability $\hat{\sigma}$ to $R_g(t) = \alpha v_H$ and a probability $1-\hat{\sigma}$ to $p_l \leq R_b(t)$;
    - ▷ $\sigma_S(z, t, \alpha v_H) = 1$ and $\sigma_S(z, t, p_l) = 0$ for $z = g, u, b$.

In the two-phase strategy profile, the agents' behavior is divided into two phases by a threshold time $t^* > 0$. In the first phase, the buyer targets the uninformed seller by offering a middle-range price which equals to the reservation price of the uninformed. The uninformed and the bad-type seller accept the offer for sure, while the good-type seller rejects the offer. In the second phase, the buyer randomizes between submitting the trade-ending offer $R_g(t) = \alpha v_H$ and the "losing offer" p_l. The losing offer is any price below or equal to $R_b(t)$ and all types of sellers reject it with probability one. Note that the buyer's randomization probability in the second phase is restricted to be constant over time.

A tuple $(\sigma_S, \sigma_B, \phi)$ is called a *two-phase equilibrium* if it is PBE and (σ_S, σ_B) is a two-phase strategy profile. An outcome of the game is called a two-phase equilibrium outcome as an equilibrium outcome induced by a two-phase equilibrium strategy profile. The following proposition (whose proof is presented in the Appendix) states that if the seller's effective learning speed is smaller than a threshold, then there exists a unique two-phase equilibrium outcome.

Proposition 1. *There exists $\underline{\eta} > 0$ such that for $0 < \rho/\lambda < \underline{\eta}$, there exists a unique two-phase equilibrium outcome.*

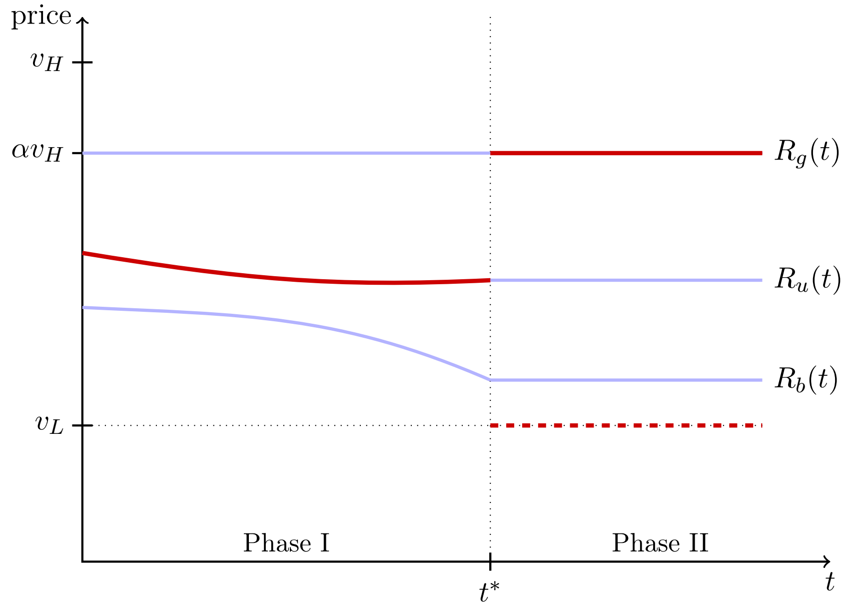
The uniqueness result in Proposition 1 depends on the stationary restriction imposed on the buyer's randomization probability in the second phase. Indeed, one can construct an equilibrium where the randomization probability of the buyers in the second phase follows non-stationary path. However, the threshold time t^* in any such non-stationary equilibrium is the same as one in the two-phase equilibrium, as well as the equilibrium behavior of the agents at any time before t^*. Moreover, the payoff of the buyer at any t and the payoff of each type of seller at any time $t \leq t^*$ is identical. I provide the intuition for the payoff equivalence after I describe the two-phase equilibrium.

The remainder of this subsection is organized as follows. First, I describe the price and belief evolution of the two-phase equilibrium and underlying incentives of the agents. I begin with the equilibrium behavior in the first phase then discuss the behavior in the second phase. Then I present an outline of the proof of the equilibrium construction. Finally, I discuss the multiplicity of the equilibria of the model and present a full characterization result.

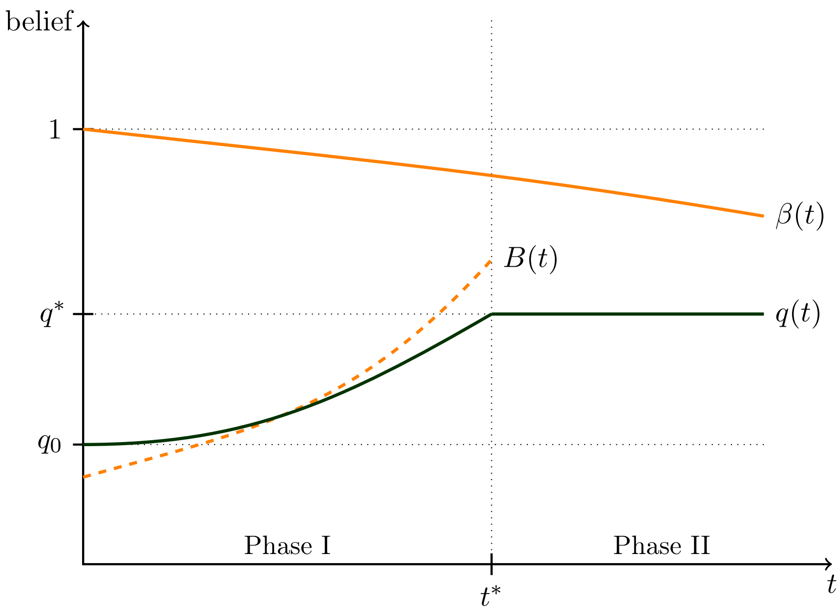
First Phase: Price Evolution The upper panel of Figure 1 shows the evolution of the reservation price and equilibrium offer in a two-phase equilibrium in which the price of the losing offer is v_L. The blue lines represent the reservation price of each type of the seller. Note that the reservation price of the good type seller is constant and equals to αv_H. The dark red line represents the equilibrium price offer.

In the first phase, the reservation price of the uninformed seller $R_u(t)$, which is the equilibrium price, must satisfy the recursion

$$R_u(t) = rdt\alpha v(q_0) + (1 - rdt) \left[\rho dt(q_0\alpha v_H + (1 - q_0)R_b(t + dt)) + (1 - \rho dt)R_u(t + dt)\right].$$



(A) PRICE EVOLUTION



(B) BELIEF EVOLUTION

FIGURE 1
EQUILIBRIUM BEHAVIOR OF THE TWO-PHASE EQUILIBRIUM

Letting $dt \to 0$ and rearranging yield

$$R_u'(t) = r \underbrace{(R_u(t) - \alpha v(q_0))}_{\text{discounting}} - \rho \underbrace{B_I(t)}_{\text{learning}}, \tag{1}$$

where

$$B_I(t) \equiv q_0 \alpha v_H + (1 - q_0) R_b(t) - R_u(t).$$

The first term on the right-hand side of (1) captures the effect of discounting. Note that its effect on the equilibrium price $R_u(t)$ is nonnegative. In the first phase, the uninformed seller is indifferent between acceptance and rejection, and he discounts future payoffs. Therefore, absent other effects, the buyers who arrive in the future must offer a higher price to attract the uninformed. The term $\alpha v(q_0)$ in the first term captures the effect of the expected dividend until the next buyer arrives.

The second term, however, has a negative effect on the equilibrium price. It captures the effect of the uninformed seller's learning. I define $B_I(t)$ as the *value of information* for the uninformed seller, since it measures the difference in the payoff between the informed seller $(q_0 R_g(t) + (1 - q_0) R_b(t))$ and the uninformed seller $(R_u(t))$. Under the given profile, $B_I(t)$ is strictly positive in the first phase. The intuition is as follows. Consider the uninformed seller who becomes informed at time t. Then the seller chooses different behavior according to the information: If the information is good $(\theta = H)$, the seller rejects the offer $R_u(t)$ in the first period. If the information is bad $(\theta = L)$, he takes the offer $R_u(t)$, since it is strictly higher than his reservation price. This adjusted behavior gives the seller a strictly higher expected payoff when he is informed.

Equation (1) implies that the positive value of information has a negative effect on the slope of $R_u(t)$. Since the uninformed seller expects the possibility of future learning in the case of rejection, his current reservation price must take into account the value of information. Furthermore, when the seller is sufficiently patient (more precisely, if r/ρ is sufficiently small), the effect of learning on $R'_u(t)$ may be greater than the effect of discounting, so that $R_u(t)$ may decrease over time.

On the other hand, a similar recursive argument for the bad-type seller yields another differential equation for $R_b(t)$ and $R_u(t)$ in the first phase, which is

$$R'_b(t) = r \underbrace{(R_b(t) - \alpha v_L)}_{\text{discounting}} + \lambda \underbrace{(R_b(t) - R_u(t))}_{\text{buyer's offer}}. \tag{2}$$

Similar to (1), the first term on the right-hand side captures the effect of discounting. The second term represents the effect of the buyer's offer of $R_u(t)$, which the bad-type seller accepts for sure. Note that the second term is negative and is proportional to the arrival rate of the buyer. Therefore, similar to $R_u(t)$, $R_b(t)$ may decrease over time in the first phase. Equations (1) and (2) form a system of ordinary differential equations for $R_u(t)$ and $R_b(t)$ in the first phase.

First Phase: Belief Evolution How do the buyers' beliefs evolve over time? Recall that $q(t)$ represents the buyers' beliefs about the quality of the good. But in this paper, the buyers also form beliefs about the seller's belief about the quality. To capture the second-order beliefs of the buyers, define $\beta(t) = \frac{\phi_u(t)}{\phi_u(t)+\phi_b(t)}$ as the buyers' *confidence* at time t. Note that $\beta(t)$ is the probability of buying the uninformed seller's good when the buyer targets the uninformed seller. The buyer's confidence, together with beliefs about quality $q(t)$, plays an important role in determining the equilibrium price.

To understand the role of the buyers' confidence, note that the buyer at time t is better off when he offers $R_u(t)$ than when he offers $R_b(t)$ if and only if

$$\beta(t)(v(q_0) - R_u(t)) + (1 - \beta(t))(v_L - R_u(t)) > (1 - \beta(t))(v_L - R_b(t)),$$

which is equivalent to

$$\beta(t) > \frac{R_u(t) - R_b(t)}{v(q_0) - R_b(t)} \equiv B(t). \tag{3}$$

Therefore, the buyer targets the uninformed seller only if his confidence is higher than a threshold $B(t)$. Note that $B(t)$ is a function of reservation prices and hence is determined by the equilibrium price evolution.

The lower panel of Figure 1 describes the belief evolution in the two-phase equilibrium. In the first phase, the buyer's belief about quality $q(t)$ increases over time. The intuition is straightforward: Suppose the buyer submits a losing offer, so there is no trade. Then $q(t)$ does not change as the seller's learning process is a martingale. Then offering $R_u(t)$ increases $q(t)$, since all but the good-type seller accept the offer and leave. However, $q(t)$ is less than the threshold belief q^* throughout the first phase, which makes it suboptimal to make a trade-ending offer.

On the other hand, the buyer's confidence $\beta(t)$ is decreasing over time in the first phase. The buyer's offer $R_u(t)$ does not affect $\beta(t)$, since both the uninformed seller and the low-type seller leave the game at the same rate. But the seller's learning decreases the buyers' confidence, since there is a growing probability that the seller is informed.

However, if the seller's effective speed of learning is slow, the rate of decrease of the buyers' confidence is low. Therefore the buyers remain confident until the expected quality of the good becomes sufficiently high so that submitting the trade-ending offer does not yield negative payoff.

Second Phase The second phase begins as the belief about quality $q(t)$ reaches q^* for the first time. In the second phase, the buyer randomizes between a trade-ending offer $R_g(t) = \alpha v_H$ and a losing offer p_l. The losing offer p_l can be any price below or equal to the bad type's reservation price. Since the all types of sellers reject p_l, the trade occurs only at αv_H. In the upper panel of Figure 1, αv_H is represented as a solid line while the losing offer p_l is represented as a dashed line, illustrating that no trade occurs at p_l.[7]

Since the buyer in the second phase purchases a good from all types of sellers or does not buy the good at all, (conditional on the game continues) the buyer's beliefs about quality $q(t)$ is constant and equals q^* in the second phase. Therefore, offering αv_H yields zero payoff, so the buyer in the second phase is indifferent between submitting the trade-ending offer and the losing offer.

The buyers in the second phase randomize their offers in order to satisfy the uninformed seller's intertemporal incentives. Suppose that the buyer in the second phase offers αv_H with probability one. Then the uninformed seller in the first phase would reject the offer in favor of future high offers, leading to the breakdown of the equilibrium structure.

The reservation prices of the bad-type seller and the uninformed seller in the second phase are, respectively,

[7] In Figure 1, losing offer is equal to v_L, but the offer price can be any price less than or equal to $R_b(t)$.

$$R_b(t) = R_b^* = \underbrace{\frac{r}{r+\lambda\hat{\sigma}}\alpha v_L}_{\text{dividend}} + \underbrace{\frac{\lambda\hat{\sigma}}{r+\lambda\hat{\sigma}}\alpha v_H}_{\text{buyer's offer}}, \tag{4}$$

$$R_u(t) = R_u^* = q_0 \alpha v_H + (1-q_0) R_b^*, \tag{5}$$

where $\hat{\sigma}$ is the probability that the buyer offers the trade-ending offer. The bad-type seller's reservation price represented in (4) is a weighted average of the value of holding the asset (αv_L) and the trade-ending offer (αv_H). The reservation price of the uninformed seller (5) is a simple expectation of reservation prices of the good type and the bad type. This is because the value of the seller's information is zero in the second phase. Since the buyers target either all types of the seller or none, becoming informed does not change the seller's strategy, so the information does not provide any value.

The randomization probability $\hat{\sigma}$ is uniquely determined by the indifference condition of the buyer at the threshold time t^*: Targeting the uninformed seller at time t^* must yield zero payoff. The intuition is as follows. Suppose that targeting the uninformed seller at time t^* yields a positive payoff. Then since both $R_u(t)$ and the confidence $\beta(t)$ are continuous over time, there exists $\epsilon > 0$ such that targeting the uninformed at $t \in (t^*, t^* + \epsilon)$ yields a positive payoff, violating the optimality condition. Now suppose that targeting the uninformed yields a negative payoff at time t^*. Again the continuity of $R_u(t)$ and $\beta(t)$ implies that for sufficiently small $\epsilon' > 0$ targeting the uninformed at $t \in (t^* - \epsilon', t^*)$ is suboptimal, leading to a contradiction. Using the buyer's indifference condition at time t^*,

R_u^* is uniquely determined and is given by

$$R_u^* = \beta(t^*)v(q_0) + (1 - \beta(t^*))v_L. \tag{6}$$

One can then determine the value of R_b^* from (5). Finally, the randomization probability $\hat{\sigma}$ is determined by (4).

Is the randomizing behavior optimal for the buyers? First, recall that $q(t) = q^*$ implies that the buyer is indifferent between submitting the trade-ending offer and the losing offer. Second, given that the indifference condition (6) is satisfied, then targeting the uninformed seller at any $t > t^*$ yields a strictly negative payoff to the buyer. This is because while $R_u(t) = R_u^*$ is constant, the buyer's confidence $\beta(t)$ decreases because of the seller's learning. Finally, targeting the bad-type seller must yield a nonpositive payoff, so the probability of the trade-ending offer must satisfy

$$R_b^* \geq v_L. \tag{7}$$

Construction Given the above analysis, the two-phase equilibrium is constructed by the following steps:[8]

1. Determine t^* from the condition $t^* = \inf\{t : q(t^*) = q^*\}$.

2. Determine $\beta(t^*)$ from the evolution of the buyer's confidence.

3. Determine $\hat{\sigma}$ by conditions (4)-(6).

4. Check if $\hat{\sigma}$ satisfies (7).

[8] The formal proof of the construction result is given in the Appendix (Subsection A.3.1).

5. Determine $R_u(t)$ and $R_b(t)$ in the first phase, by differential equations (1) and (2) with the boundary conditions at $t = t^*$.

I show in the Appendix that step 4 is satisfied if the seller's learning speed is slow enough relative to the arrival rate of the buyers. Intuitively, a higher learning speed leads to a rapid decrease in the buyer's confidence, which in turn results in lower R_u^* (equation (6)). But if R_u^* is too low, then the correspondingly small $\hat{\sigma}$ may violate the incentive condition (7).

Characterization The two-phase equilibrium described above has a special characteristic: The randomization probability of the buyers in the second phase is constant over time. But there are other equilibria where the probability of the trade-ending price changes over time. In these equilibria, the corresponding $R_u(t)$ and $R_b(t)$ in the second phase are also non-stationary, but they must satisfy the incentive conditions

$$R_u(t) \geq \beta(t)v(q_0) + (1 - \beta(t))v_L,$$

$$R_b(t) \geq v_L,$$

for any $t \geq t^*$. The above incentive conditions imply that there is a continuum of equilibria in this model.

The above argument of equilibrium construction implies that any such non-stationary equilibrium share the main qualitative features with the two-phase equilibrium. As long as the buyers in the first phase target the uninformed seller, the evolution of the belief is identical, hence the value of t^* is the same. Then the indifference condition of the buyer at t^* (equation 6) implies that the boundary of $R_u(t)$ and $R_b(t)$ at t^* is the same, hence it must be that equilibrium behavior before t^* is identical. The only difference between any non-stationary equilibrium and the two-phase equilibrium is the randomization probability of the buyers and the reservation price of the uninformed and the bad-type seller in the second phase.

Moreover, the payoff of the buyer at any t and the payoff of the seller at any $t \leq t^*$ in any non-stationary equilibrium is same as those in the two-phase equilibrium. The discussion in the last paragraph clearly implies that the payoff of all agents in the first phase is the same. In the second phase, while the payoff of the uninformed and the bad-type seller is different, the payoff of the buyers (equals to zero) and the good-type seller (equals to αv_H) is identical across equilibria.

The following proposition (whose proof is presented in the Appendix) states that if the seller is sufficiently patient, there exists no equilibrium of the model other than the class of equilibria discussed above. Since all equilibria are payoff-equivalent, one can conduct the comparative statics in the slow-learning case using the two-phase equilibrium.

Proposition 2. *There exists $\bar{r} > 0$ such that for $r < \bar{r}$ and $0 < \rho/\lambda < \underline{\eta}$ (where $\underline{\eta} > 0$ is the bound from Proposition 1), the equilibrium of the model satisfies the following properties:*

- ▷ *in any equilibrium, behavior is divided into two phases, divided by the same threshold time t^*;*
- ▷ *the equilibrium behavior of every agent in the first phase is identical across all equilibria;*
- ▷ *the payoff of the buyer at each t and the ex ante payoff of the seller is the same across all equilibria.*

3.2 Fast-learning Case

The strategy profile in the previous subsection cannot be supported as an equilibrium when the seller's effective speed of learning (ρ/λ) is high. High learning speed leads to a rapid decrease in the buyer's confidence. Therefore there is a threshold time where the buyers find it suboptimal to target the uninformed seller while the expected quality of the good is still low.

In this case, the equilibrium consists of three phases, divided by two threshold times t_1^* and t_2^*. Similar to the slow-learning case, I define the following class of candidate equilibria:

Definition 3. A strategy profile (σ_S, σ_B) is called a *three-phase strategy profile* if there exist t_1^* and t_2^* $(0 < t_1^* < t_2^*)$ and $\hat{\sigma} \in [0, 1]$ such that the profile satisfies the following:

1. Phase I: for any $t < t_1^*$,

   ▷ $\sigma_B(t) = R_u(t)$;

   ▷ $\sigma_S(g, t, R_u(t)) = 0$; $\sigma_S(z, t, R_u(t)) = 1$ for $z = u, b$.

2. Phase II: for any $t \in [t_1^*, t_2^*)$,

   ▷ $\sigma_B(t) = R_b(t)$;

   ▷ $\sigma_S(z, t, R_u(t)) = 0$ for $z = g, u$; $\sigma_S(b, t, R_u(t)) = 1$.

3. Phase III: for any $t \geq t_2^*$,

   ▷ $\sigma_B(t)$ assigns a probability $\hat{\sigma}$ to $R_g(t) = \alpha v_H$ and a probability $1-\hat{\sigma}$ to $p_l \leq R_b(t)$;

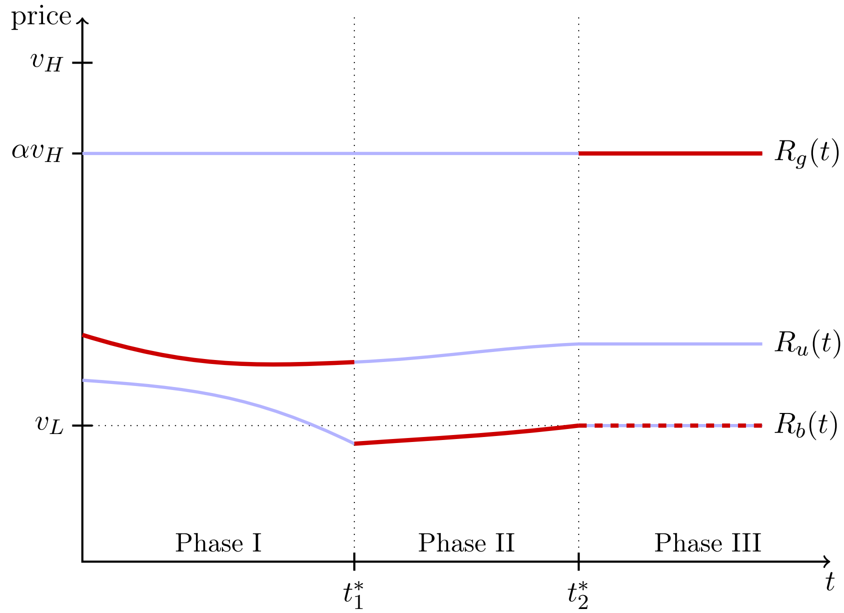
   ▷ $\sigma_S(z, t, \alpha v_H) = 1$ and $\sigma_S(z, t, p_l) = 0$ for $z = g, u, b$.

The agents' behavior is divided into three phases by two threshold times t_1^* and t_2^*. Same as the two-phase strategy profile, the buyer in the first phase targets the uninformed seller by offering the reservation price of the uninformed. The uninformed and the bad-type seller accept the offer for sure, while the good-type seller rejects the offer. At time t_1^*, the second phase begins where the buyer targets the bad-type seller by offering his reservation price, and only the bad-type seller accepts the offer. Behavior in the third and final phase is similar to that in the second phase of the two-phase strategy profile, where the buyer randomizes between submitting the trade-ending offer and the losing offer. Again, the stationary restriction is imposed on the randomization probability of the buyers.

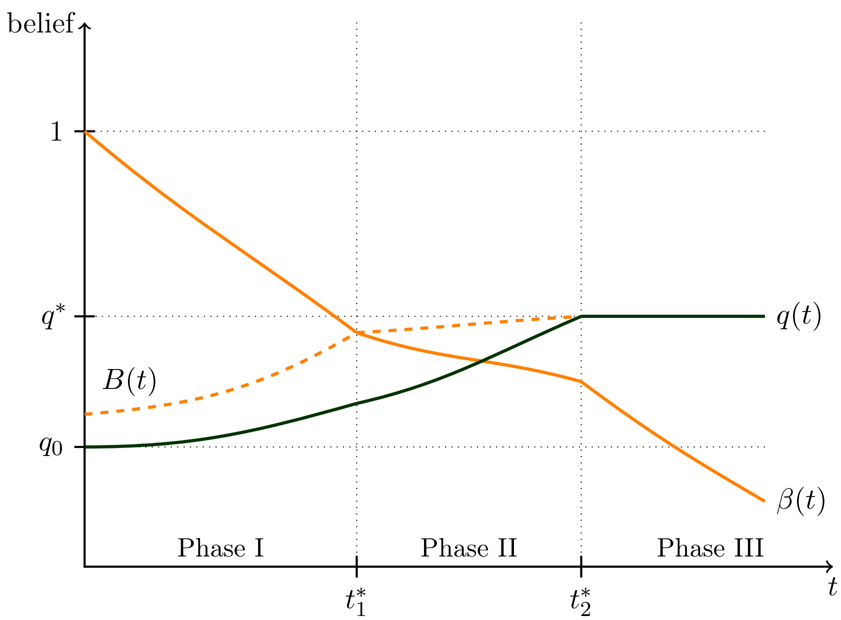
A tuple $(\sigma_S, \sigma_B, \phi)$ is called a *three-phase equilibrium* if it is PBE and (σ_S, σ_B) is a three-phase strategy profile. A three-phase equilibrium outcome is defined similar to one of the two-phase equilibrium. The following proposition (whose proof is presented in the Appendix) states that if the seller's effective learning speed is larger than a threshold, then there exists a unique three-phase equilibrium outcome.

Proposition 3. *There exists $\bar{\eta} > 0$ such that for $\rho/\lambda > \bar{\eta}$, there exists a unique three-phase equilibrium outcome.*

The upper panel of Figure 2 shows the evolution of the equilibrium price in the three-phase equilibrium. Same as Figure 1 the blue lines represent the reservation price of each type of the seller, and the dark red line represents the equilibrium price offer. In the first phase, the buyers target the uninformed seller by offering his reservation price. Similar to the two-phase equilibrium, if the seller's effective discount rate (r/ρ) is small, the equilibrium price decreases in the first phase because the seller takes into account the value of future



(A) PRICE EVOLUTION



(B) BELIEF EVOLUTION

FIGURE 2
EQUILIBRIUM BEHAVIOR OF THE THREE-PHASE EQUILIBRIUM

information.

However, the buyers' confidence rapidly decreases in the first phase because the seller's learning speed is high. The evolution of the buyers' beliefs described in the lower panel of Figure 2 shows how the equilibrium behavior is affected by the interaction between the buyers' beliefs about quality and the confidence. Contrary to the two-phase equilibrium in the slow-learning case, the buyers' confidence hits the threshold $B(t)$ *before* the belief about quality $q(t)$ reaches q^*.

So there is a threshold time t_1^* such that the buyers find it no longer optimal to target the uninformed seller, and submitting a trade-ending offer still yields a negative payoff. Therefore, the second phase begins at time t_1^* where the buyers only target the bad-type seller. Therefore, at time t_1^* the equilibrium trading price drops from the reservation price of the uninformed seller to that of the bad-type seller. Moreover, the probability of trade also drops because the uninformed seller begins to reject the buyer's offer.

In the second phase, trade only occurs with the bad-type seller at a price $R_b(t)$. Both $R_b(t)$ and $R_u(t)$ increase in the second phase. Since the bad-type seller receives an offer which is equal to his reservation price, getting an offer does not affect his reservation price. So contrary to $R_b(t)$ in the first phase (2), $R_b(t)$ in the second phase is affected only by the effect of the seller's discounting, and it satisfies the following differential equation:

$$R_b'(t) = r \underbrace{(R_b(t) - \alpha v_L)}_{\text{discount}} > 0. \tag{8}$$

On the other hand, the uninformed seller's reservation value satisfies $R_u(t) = q_0 \alpha v_H + (1 - q_0) R_b(t)$. Note that the value of information to the uninformed seller is zero in the second phase. While the seller also has zero value of information in the final phase (as I discussed in the previous subsection), the underlying intuition is different. Contrary to the final phase, the informed seller in the second phase behaves differently according to the quality of his good. But he does not gain higher payoff because the offer the bad-type seller accepts is precisely equal to his reservation value.

The buyers' confidence $\beta(t)$ in the second phase stays below the threshold $B(t)$ so that the buyers find it optimal to target the bad-type seller[9]. On the other hand, throughout the first and second phase the belief about quality $q(t)$ increases over time because the expected quality of the good that is traded is lower than the quality of the remaining good. Therefore there exists a second threshold time, t_2^*, where the belief about the quality $q(t)$ reaches q^*.

The third and final phase begins at t_2^*, and the equilibrium behavior is similar to the final phase of the two-phase equilibrium. The buyers randomize between a trade-ending offer, at which the trade occurs, and a losing offer. Therefore, the equilibrium price at which a trade occurs jumps at t_2^* from the bad type's reservation price to a trade-ending offer. Moreover, trade of the high-quality good resumes at t_2^* as all types of sellers trade.

Figure 3 describes the probability of trade in the three-phase equilibrium. The solid red (dashed blue) line depicts the distribution of the timing of a trade conditional on the good being low- (high-) quality. Note that the probability of trade of the high-quality good is zero in the second phase, because the trade occurs only with the bad-type seller.

In the three-phase equilibrium, the equilibrium behavior is uniquely determined given the threshold times t_1^* and t_2^*. There are two indifference conditions of the buyers which jointly determines two thresholds times: 1) indifference condition between targeting the

[9]In Section 5, I discuss the case of intermediate learning speed where fast screening behavior may lead to increase in the buyers' confidence more than the threshold.

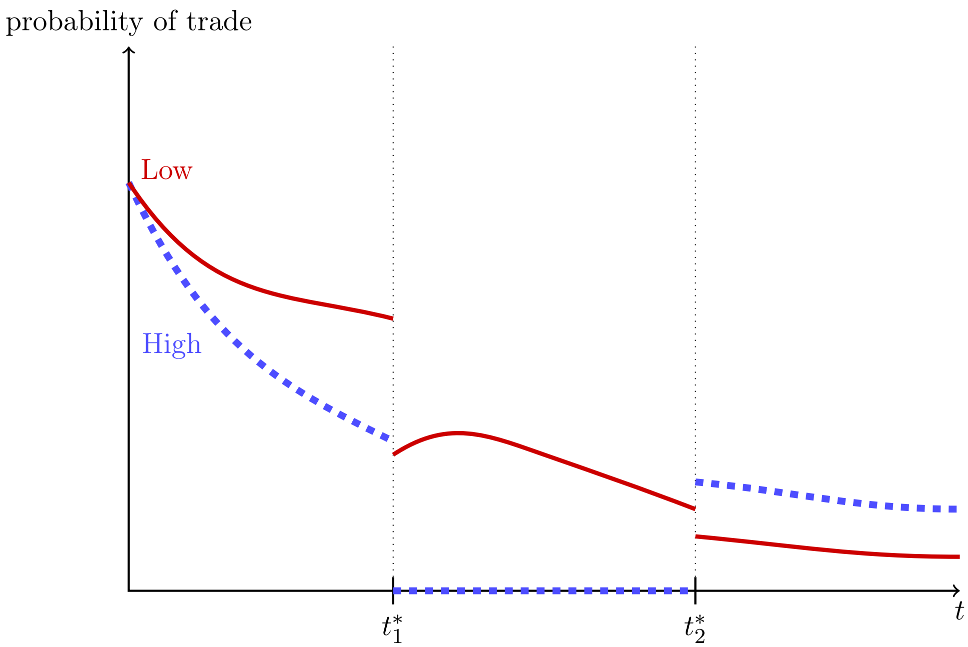


Figure 3
Probability of trade in the three-phase equilibrium

uninformed and the bad type at t_1^* ($\beta(t_1^*) = B(t_1^*)$), and 2) indifference condition between a trade-ending offer and a losing offer at t_2^* ($q(t_2^*) = q^*$). The following proposition states that if the effective learning speed of the seller is large enough, then there exists a unique pair of threshold times.

Similar to the slow-learning case, the model has multiplicity of equilibrium in the fast-learning case. The following proposition (whose proof is presented in the Appendix) states that every equilibrium of the model differs only in the randomization probability of the buyers in the final phase, and all equilibria are payoff-equivalent. Note that the characterization result in the fast-learning case does not need additional restriction on the seller's discount rate.

Proposition 4. *Suppose that $\rho/\lambda > \bar{\eta}$ (where $\bar{\eta} > 0$ is the bound from Proposition 3). Then* *the equilibrium of the model satisfies the following properties:*

- ▷ *in any equilibrium, behavior is divided into three phases, divided by the same threshold times* t_1^* *and* t_2^**;*
- ▷ *the equilibrium behavior of every agent in the first two phases is identical across all equilibria;*
- ▷ *the payoff of the buyer at each* t *and the ex ante payoff of the seller is the same across all equilibria.*

When the seller's effective learning speed is between $\underline{\eta}$ (the upper bound of the slow-learning case) and $\bar{\eta}$ (the lower bound of the fast-learning case), then there exists an equilibrium where the buyers use a mixed strategy even before the belief about quality $q(t)$ reaches q^*. In Section 5 I discuss the equilibria of the model in this case. The following proposition (whose proof is presented in the Appendix) shows that when the prior q_0 is not too small, there is no such range of parameter.

Proposition 5. *There exists* $\underline{q} < q^*$ *such that if* $q_0 \in (\underline{q}, q^*)$*, then* $\bar{\eta} = \underline{\eta}$*.*

In the following section, I present the results of comparative statics when $q_0 \in (\underline{q}, q^*)$.

4 Comparative Statics

In this section, I present several comparative statics results with respect to the seller's learning speed.

4.1 Threshold Time

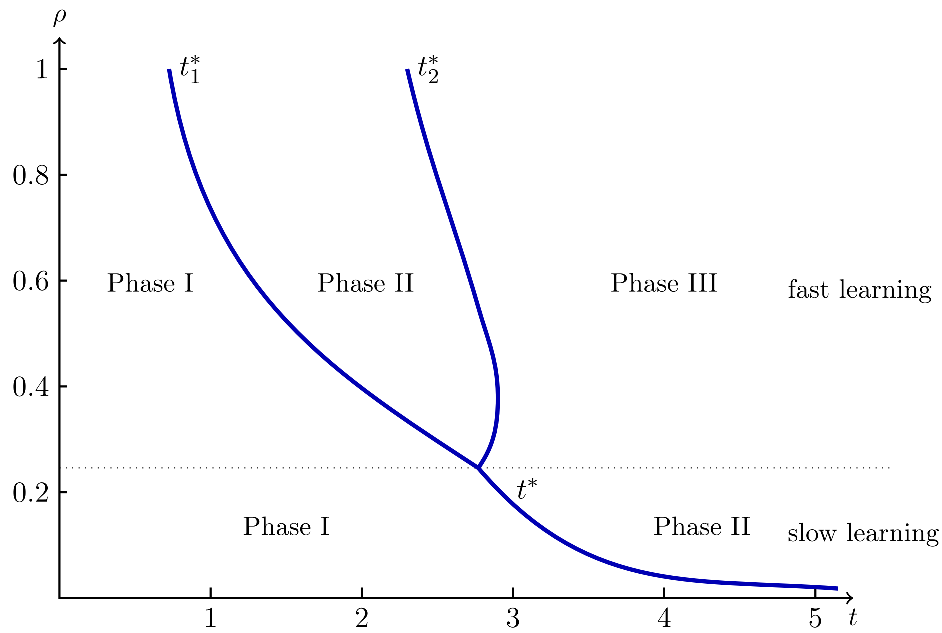
As shown in the previous section, the threshold times (t^* in the slow-learning case; t_1^* and t_2^* in the fast-learning case) are important equilibrium values that determine other equilibrium behavior. The following proposition (whose proof is presented in the Appendix) presents comparative statics results of the threshold times with respect to the learning speed of the seller:

Proposition 6.

- ▷ *In the two-phase equilibrium, t^* is decreasing in ρ;*
- ▷ *In the three-phase equilibrium, t_1^* is decreasing in ρ;*
- ▷ $\lim_{\rho \to 0} t^* = \infty$*;* $\lim_{\rho \to \infty} t^* = 0$.

Figure 4 depicts how the threshold times change with the seller's learning speed. In the slow-learning case, there is one threshold time t^* which decreases in ρ. Note that t^* diverges to infinity as ρ goes to zero. When ρ is arbitrarily close to zero, the environment is close to the one having symmetrically uninformed agents, so the trade occurs at the reservation price of the uninformed seller for an arbitrarily long horizon.

In the fast-learning case there are two threshold times t_1^* and t_2^*. Proposition 6 states that t_1^* decreases in ρ and converges to zero as ρ goes to infinity. The intuition is straightforward, since as ρ goes to infinity the environment converges to one that has initial asymmetric information, so the buyers target the bad type immediately after the beginning of the game. On the other hand, both t_2^* and $t_2^* - t_1^*$ are nonmonotonic under some parameter value.



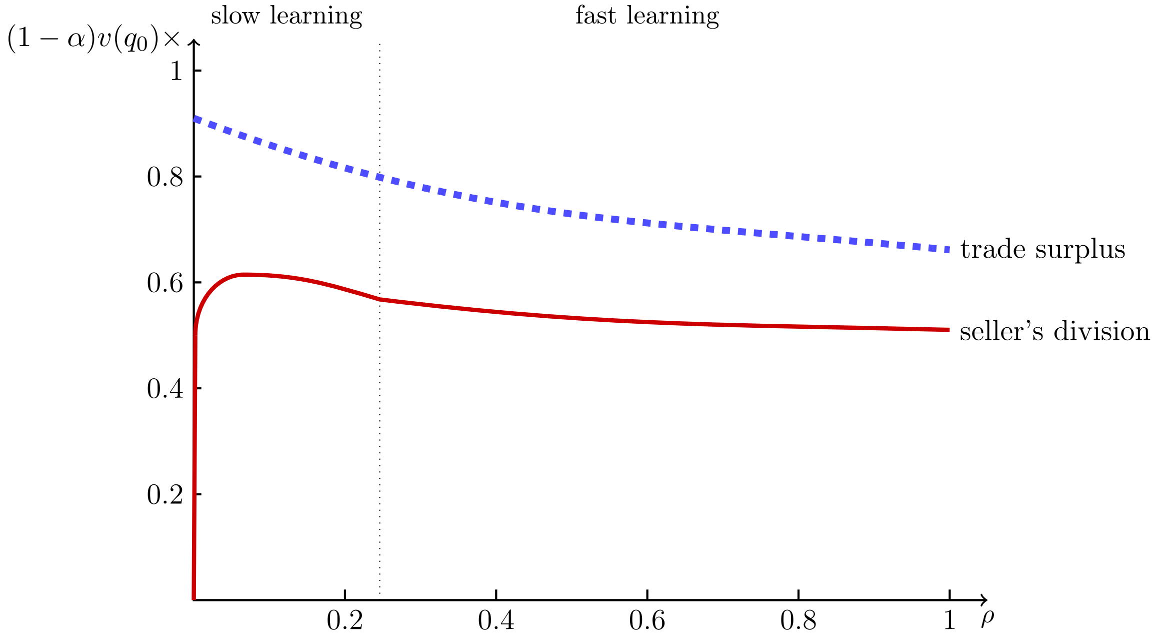
(Parameter values: $\lambda = 1, r = 0.1, v_H = 2, v_L = 1, \alpha = 0.8, q_0 = 0.3$)

FIGURE 4
THRESHOLD TIMES

4.2 Trade Surplus and Division of the Surplus

How do the trade surplus and the division of the surplus change as the learning speed changes? Standard models of adverse selection show that in the presence of initial asymmetric information, 1) the trade surplus is lower because the adverse selection problem leads to inefficient trade outcomes, and 2) the payoff of the informed agent is higher because he has a positive information rent. In this subsection I change the learning speed of the seller from zero (symmetrically uninformed agents) to infinity (initially informed seller) and simulate the value of the trade surplus and its division.

Let S_θ be the trade surplus when the quality of the good is θ. Let $f_\theta(t)$ be the probability



(Parameter values: $\lambda = 1, r = 0.1, v_H = 2, v_L = 1, \alpha = 0.8, q_0 = 0.3$)

FIGURE 5

TRADE SURPLUS AND THE SELLER'S DIVISION OF THE SURPLUS

distribution of trade of the quality-θ good at time t. Then we have

$$S_\theta = (1-\alpha)v_\theta \int_0^\infty e^{-rt} f_\theta(t)dt.$$

Then the ex ante trade surplus S is given by

$$S = q_0 S_H + (1-q_0)S_L.$$

The ex ante payoff of the seller is $R_u(0)$, because the seller is uninformed at $t = 0$ and his reservation price equals the continuation payoff. From the seller's ex ante payoff, his division of trade surplus is calculated.[10]

[10]Details of the calculation are in the Appendix (Subsection A.4).

The solid red line in Figure 5 is the trade surplus as a function of the seller's learning speed ρ. Note that the trade surplus is decreasing in the seller's learning speed. This result is related to one in Dang, Gorton, and Holmström (2012), who argue that trade is most efficient when the agents are symmetrically uninformed.[11]

On the other hand, the seller's ex ante payoff is nonmonotonic in the seller's speed of learning. The dashed blue line in Figure 5 is the seller's division of the surplus as a function of ρ. Note that the seller's surplus (hence his ex ante payoff) increases when ρ is small, but decreases when ρ is high. This is because there is a trade-off between the value of information and the adverse selection problem. If the degree of asymmetric information is small, then the seller's value of information increases in his learning speed. But if ρ is large, then the buyers' equilibrium behaviors takes into account the effect of seller's asymmetric information. Therefore, the inefficiency caused by severe adverse selection decreases the seller's payoff.

5 Discussion

Implication for the Financial Crisis An important feature of the equilibrium in the fast-learning case is the impact of the buyers' second-order beliefs on the equilibrium dynamics. Before the first threshold time t_1^*, trade occurs at a middle-range price $R_u(t)$ and the trading patterns are relatively stable. However, the buyers' confidence $\beta(t)$ rapidly decreases, and eventually hits the threshold level at time t_1^*, leading to drops in both equilibrium price

[11] Levin (2001) shows in a static lemon market model that as the quality of seller information increases, trade may decrease *or* increase depending on the information structure. His result implies that the trade surplus in this model can be nonmonotonic in the seller's learning speed under a different learning process of the seller.

and the probability of a trade.

The results may help to understand what was observed at the beginning of the recent financial crisis. One of the main narratives of the crisis was the collapse of confidence in the market. For example, regarding the timing of the run on the sale and repurchase market (the "repo market") in August 2007, Gorton and Metrick (2012) argue the following:

> ...One large area of securitized banking, the securitization of subprime home mortgages, began to weaken in early 2007 and continued to decline throughout 2007 and 2008 ...The first systemic event occurs in August 2007 ...The reason that this shock occurred in August 2007, as opposed to any other month of 2007, is perhaps unknowable. We hypothesize that the market slowly became aware of the risks associated with the subprime market, which then led to doubts about repo collateral and bank solvency. At some point (August 2007 in this telling) *a critical mass of such fears* led to the first run on repo, with lenders no longer willing to provide short-term finance at historical spreads and haircuts. [Italics added]

Morris and Shin (2012) set up a static model of the adverse selection problem and show that a small amount of adverse selection can lead to the breakdown of "market confidence," defined as the approximate common knowledge of an upper bound on expected losses. In this paper, the dynamic structure of the model can illustrate the evolution of the beliefs and their effect on equilibrium behavior. Investigating the effect of the evolution of the higher-order beliefs in various trading institutions in financial markets is an interesting topic for future potential research.

Intermediate Speed of Learning If the seller's effective learning speed is between $\underline{\eta}$ (upper bound in the slow-learning case) and $\bar{\eta}$ (lower bound in the fast-learning case), then the buyers may not use a pure strategy even before the belief about quality $q(t)$ reaches q^*. Since $\rho/\lambda > \underline{\eta}$, there exists a threshold time where targeting the uninformed is no longer optimal. On the other hand, if $\rho/\lambda < \bar{\eta}$, targeting the bad type increases buyers' confidence so that the buyers' confidence becomes greater than the threshold $B(t)$, so it is suboptimal to target the bad type. In this case, the buyer in the second phase uses a mixed strategy, randomizing between targeting the uninformed and targeting the bad type. Constructing and characterizing the equilibrium in this parameter range is another area of future research.

Pure Good News and Pure Bad News Case One of the assumptions of the model is that the arrival rate of information is same regardless of the quality of the good. If the information arrival rate is quality-dependent, then not receiving a signal would also provide information about the item's quality. An environment with pure good news (bad news) is an example of a quality-dependent arrival rate, where the arrival rate of the information is zero for the low- (high-) quality good. Preliminary results show that for both cases, the equilibrium dynamics are similar to those of either the slow- or fast-learning cases examined in this paper.[12]

6 Conclusion

This paper has introduced a framework with which to study the trading patterns in an environment in which asymmetric information increases over time. In this framework, the interaction between the buyers' screening and the seller's learning generates nonmonotonic pricing and trading patterns, contrary to standard models in which asymmetric information

[12] A partial result for the equilibrium construction and characterization is available upon request.

is initially given. If the seller's effective learning speed is high, a rapid decrease of the buyers' confidence leads to drop in the equilibrium price and the probability of a trade. While the trade surplus decreases as the seller's learning speed increases, the seller's payoff is nonmonotonic in his learning speed, as a slower learning speed can lead to higher payoff for the seller.

The findings in this paper have implications for the process of designing optimal interventions for environments with increasing asymmetric information. The nonstationarity of the equilibrium trading pattern implies that the timing of an intervention would be crucial for its effectiveness. Suppose, for instance, that an asset market is hit by a shock which creates symmetric uncertainty about the value of an asset. It may then be the case that the government should not intervene immediately, because at the moment incomplete but symmetric information is not overly harmful to efficiency and only later becomes harmful as the asymmetric information grows worse. Investigating dynamic effects of an intervention and the design of optimal intervention in an environment with increasing asymmetric information are interesting topics for future research.

Chapter II

A Theory of Bargaining Deadlock

7 Introduction

There are many bargaining processes in which a bargainer may receive an outside offer during the process. Moreover, preferability of the outside offer often depends on the private information of the informed party. For example, consider an entrepreneur who negotiates to sell his company to an equity fund. The entrepreneur knows a company's fundamentals but is not able to verify them. During the bargaining process, a competitor might arrive and make an offer to buy the firm. The competitor is better informed about the fundamentals than the equity fund, so his offer is high if the fundamentals are good. For our purposes, the competitor's offer serves as an attractive outside option for the entrepreneur.[13]

In this example, when a bargainer is deciding whether or not to take the outside option, he must take into account the fact that choosing not to opt out may signal his private information. This paper analyzes the interplay of outside options and incomplete information in bargaining. Specifically, this paper analyzes the equilibrium effects of additional information provided by how bargainers respond to the outside option.

I study a model of an infinite-horizon bargaining game between a seller and a buyer. The seller privately knows the quality of his product. In each period, the buyer offers a price and the seller decides whether or not to accept the offer. After rejection, the seller's outside

[13]In corporate finance, buyers of businesses are generally classified into two different categories: financial buyers and strategic buyers. Financial buyers are mostly equity funds interested in the return they can achieve by buying a business. Strategic buyers are typically a competitor or a company in the same industry, and they look for companies that will create a synergy with their existing businesses.

option randomly arrives. The value of the outside option is increasing in the product quality. If the seller does not receive an outside option or he chooses not to opt out, bargaining continues into the next period.

There are two sources of information which the buyer uses to update his belief about quality: the seller's decision to accept/reject the buyer's offer (acceptance behavior) and his decision about whether to take the outside option (opting-out behavior). Suppose the buyer proposes an offer that is rejected. Then the buyer believes that the quality is more likely to be high, since the high-quality seller has a higher reservation value. This informational effect of the acceptance behavior is common in the standard models of incomplete-information bargaining (Fudenberg, Levine, and Tirole (1985); Deneckere and Liang (2006)). There is no outside option in their models, and so they only consider the effect of the acceptance behavior. As a result, the buyer's equilibrium belief moves only in one direction as the buyer becomes more confident that he is facing the high-quality seller.[14] This equilibrium dynamic of belief is known as the *skimming property* (Gul, Sonnenschein, and Wilson (1986)).

However, additional information is provided by the seller's opting-out behavior in this model and it has an opposite affect on belief updating. The buyer infers that the seller has not opted out by observing him still at the negotiation table. It might be that the seller has yet to receive an outside option, or he has received an outside option that he did not take. Since the value of the outside option is greater for the high-quality seller, he is more likely to opt out when the option arrives. Therefore, after observing that the seller has not opted out, the buyer adjusts his belief in the direction of low quality.

I show that when the outside option is sufficiently important, there is an equilibrium in

[14] Similarly, in the context of a durable goods monopoly, the uninformed seller becomes more confident that the remaining buyers have low valuation.

which the two countervailing forces in belief updating exactly offset one another. As a result, the buyer's belief does not change over time and he continues to make the same randomized offer throughout the bargaining process. Since the buyer does not make more generous offers in response to continued rejections, and the seller's behavior does not change, the equilibrium behavior produces an outcome path that resembles an outcome of a *bargaining deadlock*. For simplicity, I refer to such an equilibrium as a deadlock equilibrium.

In the deadlock equilibrium, there is a threshold belief such that once the buyer's posterior belief reaches that point, it does not change until the bargaining ends. If the buyer is more confident that he faces a high-quality seller than the threshold, he offers a sufficiently high price that bargaining ends. If the buyer is less confident, he makes an agreement only with the low-quality seller by offering no more than the value of his outside option. In this case, in each period the buyer adjusts his belief in the direction of high quality. This equilibrium behavior lasts for a finite number of periods until the posterior reaches the threshold point.

If the buyer's posterior is equal to the threshold, the buyer uses a mixed strategy between offering the bargaining-ending price and the low-quality seller's outside option value. The mixing probability is determined to satisfy the low-quality seller's indifference condition, and as the time between periods becomes vanishingly small, the buyer offers the low price with a probability close to one. In response to the buyer's low price offer, only the low-quality seller accepts it with a probability equal to the arrival probability of the outside option. The high-quality seller takes the outside option for sure if it arrives, while the low-quality seller does not. Since both types of sellers exit the game with the same probability, the posterior belief of the buyer remains the same in the next period, and the players continue to play in the same way.

If the buyer's prior expected quality is lower than the threshold belief, in equilibrium there is positive probability that an apparent bargaining deadlock arises: a sequence of the same low price offer is rejected by the seller, followed by a sudden resolution by either the buyer's high bargaining-ending offer, the low-quality seller's agreement on the low price, or the high-quality seller's opting out. Note that although the realization of the equilibrium outcome resembles a bargaining deadlock, the bargainers are not that uncompromising. Instead, the buyer and the low-quality seller are indifferent between a full compromise, and the bargaining ends with positive probability in each period. In this sense, the model provides an explanation of situations that look like bargaining deadlocks without the need to appeal to behavioral types.

I show that as the time between periods becomes vanishingly small, and if the buyer forms a prior belief such that the expected quality is lower than the threshold belief, then the bargaining reaches a deadlock phase almost immediately. Hence an outcome path of the equilibrium under frequent offers exhibits one of the following: either the bargaining ends immediately, or the aforementioned deadlock phase lasts for positive real time before a sudden resolution.

In the deadlock equilibrium, there are non-trivial bargaining inefficiencies. There is a bargaining delay in the deadlock equilibrium, and the expected length of delay is positive in the limit case of frequent offers. While the bargaining terminates (either by an agreement or an opt-out) incrementally over time, the failure of learning keeps the parties from reaching an agreement with certainty at any point in the bargaining process. Indeed, for any finite time, the bargaining continues beyond that point with positive probability. Moreover, the equilibrium exhibits the possibility of bargaining breakdown.

The inefficiencies found in the deadlock equilibrium have distinctive features compared to the ones in the standard model of incomplete-information bargaining. The standard model explains delay as a device by which the parties can credibly convey their genuine bargaining positions. Therefore, the adverse selection problem is alleviated over time as the uninformed party gradually learns private information. In this model, however, the adverse selection problem does not disappear because the buyer fails to learn the quality; hence the bargaining inefficiencies remain strong as long as the bargaining continues. Furthermore, a bargaining deadlock and real-time delay may exist even when there is no static adverse selection problem,[15] which contrasts with the result in the standard model (Deneckere and Liang (2006)).

In general, the model has multiple equilibria. There may exist an equilibrium where the informational effect of the acceptance behavior dominates that of the opting-out behavior, so that the equilibrium exhibits Coasian dynamics and so is approximately efficient when offers are frequent. But I show that under stronger parametric assumptions, the deadlock equilibrium is the only equilibrium that satisfies a natural monotonicity criterion that requires that the buyer's equilibrium offer be nondecreasing in the posterior belief of expected quality. Moreover, I show that under the same condition, all equilibria exhibit similar characteristics, specifically the partial failure of learning and the inefficiency in the bargaining outcome, so neither source of information dominates each other.

The paper contributes to a rich literature on dynamic bargaining with incomplete information. Standard models of incomplete-information bargaining do not model outside

[15] A static adverse selection problem arises when the average value of the product is lower than the highest possible reservation value of the seller (Akerlof (1970)).

options (See Gul, Sonnenschein, and Wilson (1986) and Ausubel and Deneckere (1989) for a durable goods monopoly; Deneckere and Liang (2006) for bargaining with interdependent values; Cho (1990) for two-sided private information; Abreu and Gul (2000) for reputational bargaining), or they model them as an exogenous breakdown (See Sobel and Takahashi (1983); Spier (1992); Fuchs and Skrzypacz (2012) for breakdown after a finite-horizon bargaining; Fuchs and Skrzypacz (2010) for stochastic breakdown). Since the players do not have an opting-out decision, information is revealed only through the offer/response behavior. In the present paper, information is revealed via both the acceptance and opting-out behaviors, which is the main driving force of the bargaining deadlock.

A few papers have an equilibrium structure similar to the one studied here, although the underlying mechanism is different. Evans (1989) and Hörner and Vieille (2009) (public offer case) consider bargaining with interdependent values and show that the bargaining may result in an impasse when the buyer is too impatient (or short-lived) relative to the seller. On the other hand, the present paper assumes a common discount factor, and a bargaining deadlock may exist even in the private value case. Abreu and Gul (2000) study a reputational bargaining game where each agent may be a behavioral type who demands a certain share of the pie and show that the equilibrium has a war of attrition structure exhibiting a deadlock. Even though each bargainer becomes less confident that the opponent is a normal type, they stick to imitating the behavioral type's behavior until a bargainer finally gives up. Compared to Abreu and Gul (2000), the present model does not assume behavioral types and a bargaining deadlock is associated with the uninformed buyer's failure of learning. Also it is known that introducing an outside option into their model may completely cancel out the deadlock and delay (explained in the next paragraph), while deadlock in this paper is a result of an interplay between the outside options and incomplete information.

There are papers in which some or all players can take an outside option that is available in every period. Compte and Jehiel (2002) (in the context of reputational bargaining) and **?** (in a durable goods monopoly) show that the introduction of an outside option may completely cancel out the impact of asymmetric information. In these papers, the players either agree with each other or opt out at the beginning of the game, so the equilibrium is efficient and information is revealed immediately. On the other hand, the stochastic arrival of outside options in this paper leads to non-trivial equilibrium dynamics.

Lee and Liu (2013) study a repeated bargaining game between a long-run player and a sequence of short-run players, where a stochastic disagreement outcome in each bargaining partially reveals private information of the long-run player. They focus on the incentive of the long-run player to build a reputation by choosing to gamble with the outside option, while the present paper analyzes the bargaining inefficiency caused by the informational effect of the outside options.[16,17]

The rest of the paper is organized as follows. Section 8 describes the model. Section 9 constructs the deadlock equilibrium and describes the equilibrium dynamics and the outcome path. In Section 10 I analyze the equilibrium behavior under the limit case of frequent offers and discuss real-time delay as well as other equilibrium characteristics. Section 11 finds sufficient conditions under which the deadlock equilibrium is the only equilibrium that satisfies a natural monotonicity criterion, and under which all equilibria have similar

[16] Compte and Jehiel (2004) raise an opposite question about bargaining dynamics and identify a source of gradualism in bargaining and contribution games.

[17] For other models that explain delay, Merlo and Wilson (1995) consider a complete information bargaining game where the bargaining surplus stochastically changes over time and derive an equilibrium delay. Yildiz (2004) considers a sequential bargaining model in which players are optimistic about their bargaining power and shows that there exists a uniquely predetermined settlement date as players learn over time.

characteristics. Section 12 discusses the role of assumptions and the robustness of the result under several extensions. Section 13 concludes. Some of the proofs are relegated to the Appendix.

8 Model

Consider an infinite-horizon, discrete-time bargaining game between a seller and a buyer. Periods are indexed by $n = 0, 1, 2, \ldots$. Let Δ be the length of the time interval between two successive periods, so period k occurs at time $k\Delta$.[18] Let $\delta = e^{-r\Delta}$ be a common discount factor, where $r > 0$ is a discount rate. Note that the discount factor becomes arbitrarily close to one as Δ converges to zero.

The seller holds an indivisible product that can be either high type (H) or low type (L). The type of the product is the seller's private information, and the buyer forms a prior belief $\pi_0 \in (0, 1)$ that $\theta = H$. The buyer's value of the type-θ product is $u_\theta > 0 (u_H \geq u_L)$. For simplicity, assume that the seller has zero production cost.[19]

Each period consists of an offer stage and an outside option stage. In the offer stage, the buyer offers a price p to the seller. Then the seller decides either to accept or reject the offer. If he accepts the offer, the game ends, and the seller and the buyer obtain payoffs p and $u_\theta - p$, respectively. In the case of rejection, the game continues to the outside option stage where the outside option arrives to the seller with probability $\xi = 1 - e^{-\lambda\Delta}$.[20] I assume that the arrival of the outside option is private information to the seller. If the seller opts

[18] This is a common modeling scheme in the literature on bargaining theory. The literature mainly considers the case where Δ is arbitrarily small, so that the commitment power of the uninformed player disappears.

[19] The robustness of the result to the case of a positive production cost is discussed in Section 12.

[20] Note that $\lambda > 0$ represents a Poisson arrival rate of the outside options.

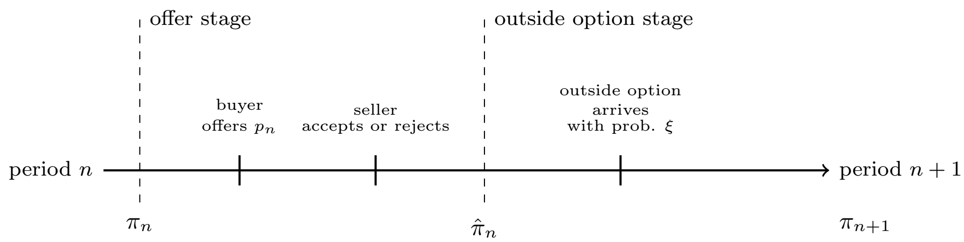


FIGURE 6
TIMELINE

out, the game ends, and the seller and the buyer obtain payoffs of v_θ and zero, respectively. Assume that $v_H > v_L > 0$ and that the buyer's value of the product is no less than the seller's value from the outside option ($u_\theta \geq v_\theta$). If either no outside option arrives or the option is rejected by the seller, the game continues into the next period. Figure 6 describes the timeline of the game.

Consider a seller's strategy in which he rejects any offer and opts out whenever the outside option arrives. Then the type-θ seller's expected payoff is

$$v_\theta^* \equiv \xi v_\theta + \delta(1-\xi)\xi v_\theta + \cdots = \frac{\xi}{1-\delta(1-\xi)} v_\theta.$$

Note that $v_\theta^* < v_\theta$, since the arrival of the outside option is delayed with positive probability.

It is clear that in any equilibrium of the game, the ex ante payoff of the type-θ seller must be no less than v_θ^*, and that the seller always rejects any offer below v_θ^*. Hereafter I call v_θ^* the *reservation price* of the type-θ seller. The following proposition says that in the case of complete information, v_θ^* is not only a lower bound but also the unique equilibrium payoff of the seller. The main intuition behind the proposition is similar to Diamond's paradox.

Proposition 7. *(Complete information) Suppose that the seller is type θ with probability* *one. Then there exists a unique subgame perfect equilibrium in which the buyer always offers* v_θ^*, *and the seller accepts any offer no less than* v_θ^*.

Proof. See the Appendix. $\square$

A public history $h^n \in H^n$ is a sequence of rejected offers $\{p_k\}_{k=0}^{n-1}$ from period 0 to $n-1$. In addition to that, the seller privately knows the availability of outside options in the past. Let $o_k \in \{Y, N\}$ denote the availability of an outside option for the seller in period k. Then the seller's private history $h_S^n \in H_S^n$ at the offer stage is $h_S^n = (h^n, \{o_k\}_{k=0}^{n-1})$. I also define a public *interim* history $\hat{h}^n = (h^n, p_n) \in \hat{H}^n$ and private interim history $\hat{h}_S^n = (h^n, p_n, \{o_k\}_{k=0}^{n-1}) \in \hat{H}_S^n$ at the outside option stage.

The buyer's strategy is his offer $p_n : H^n \to \Delta(\mathbb{R}_+)$ at the offer stage. The type-θ seller's strategy consists of the acceptance probability $\sigma_{\theta n} : H_S^n \times \mathbb{R}_+ \to [0,1]$ at the offer stage, and the opting-out probability $c_{\theta n} : \hat{H}_S^n \times \{Y\} \to [0,1]$ at the outside option stage. Finally, define $\pi_n = \Pr(\theta = H | h^n)$ and $\hat{\pi}_n = \Pr(\theta = H | \hat{h}^n)$ as a posterior belief and an interim belief of the buyer in period n, respectively.

We use the perfect Bayesian equilibrium (PBE) concept as defined in Fudenberg and Tirole (1991, Definition 8.2).[21] PBE implies that upon receiving an out-of-equilibrium offer, the continuation strategy of the seller is optimal.

As Δ goes to zero, the type-θ seller's reservation value converges to $\frac{\lambda}{r+\lambda} v_\theta$. Define $\eta = \frac{\lambda}{r+\lambda}$ as the seller's *effective discount rate*. Note that η can be any number between zero and one, depending on the ratio of the discount rate and the arrival rate of the outside option.

[21] Formally speaking, Fudenberg and Tirole defined perfect Bayesian equilibria for finite games of incomplete information. The suitable generalization of their definition to infinite games is straightforward and is omitted.

In this paper, I consider the case where outside options arrive frequently enough (relative to the discount rate), so that the outside options generate a sufficiently heterogeneous bargaining position of the seller according to his type. Specifically, I assume that the high type's reservation value is greater than the low type's payoff from the outside option.

Assumption. *(A1)*

$$\delta v_H^* > v_L + \frac{(1-\delta)(1-\xi)}{\xi} u_L.$$

Assumption 1 holds if (1) $\delta = e^{-r\Delta}$ is sufficiently large so that the interval between the periods is small enough, and (2) $\eta = \frac{\lambda}{r+\lambda}$ is sufficiently large so that the outside options arrive frequently enough that v_H^* is close to v_H. Note that Assumption 1 encompasses a case with private value ($u_H = u_L$).

The following lemma shows that in any (perfect Bayesian) equilibrium, the buyer's equilibrium offer is bounded above by the high type's reservation value. The intuition is similar to Proposition 7. This lemma and the following corollary helps in understanding the equilibrium structure of the game.

Lemma 4. *Suppose (A1) holds. Then in equilibrium, after any history h^n, the buyer never offers $p_n > v_H^*$.*

Proof. See the Appendix. □

Lemma 4 implies the following corollary:

Corollary 1. *Suppose (A1) holds. Then in equilibrium,*

(1) The high type accepts any $p \geq v_H^*$, rejects any $p < v_H^*$, and takes the outside option whenever the option arrives.

(2) The low type accepts any $p \geq \delta v_H^*$.

Note that the first part of Corollary 1 completely specifies the high type's equilibrium behavior after any history. So the equilibrium profile only needs to specify the behaviors of the low type and the buyer. Lemma 4 and Corollary 1 describe how the bargaining ends in any equilibrium. After any history, the buyer offers either $p_n = v_H^*$ or $p_n < v_H^*$. If he offers v_H^*, then both types of sellers accept it for sure, and the bargaining ends in period n with probability one. If $p_n < v_H^*$, then the high type rejects it for sure and takes the outside option if the option arrives. Therefore, the bargaining continues into the next period with positive probability, as the outside option does not arrive with probability one.

9 Deadlock Equilibrium

In this section I construct an equilibrium of interest. A heuristic argument for the equilibrium construction is provided here, while the complete description of the equilibrium (including behavior off the equilibrium path) is provided in the Appendix.

Definition. A perfect Bayesian equilibrium is called a *deadlock equilibrium* if the equilibrium behavior satisfies the following properties: there exists $\hat{p} < v_H^*$, $\pi^* \in (0,1)$ and $q \in (0,1)$ such that

1. If $\pi_n > \pi^*$,

    ▷ the buyer offers v_H^* for sure; bargaining ends immediately.

2. If $\pi_n = \pi^*$,

    ▷ the buyer offers either v_H^* or $\hat{p}$, or uses a mixed strategy between the two;

- ▷ if $p_{n-1} = \hat{p}$, he offers v_H^* or $\hat{p}$ with probability q and $1-q$, respectively;
- ▷ only the low type accepts $\hat{p}$ with probability ξ;
- ▷ only the high type opts out for sure;
- ▷ $\pi_{n+1} = \pi^*$.

3. If $\pi_n < \pi^*$,
   - ▷ the buyer offers some $p \leq \hat{p}$;
   - ▷ only the low type accepts p with positive probability;
   - ▷ $\pi_{n+1} \in (\pi_n, \pi^*]$.

In the deadlock equilibrium, there exists a cutoff belief π^* where the posterior, given that the bargaining continues, does not change once it reaches π^*. I call π^* a *deadlock belief* since the bargaining parties' behaviors do not change once the posterior reaches π^*; hence, the equilibrium behavior produces an outcome that resembles a bargaining deadlock.

The buyer's equilibrium offer sharply changes at the deadlock belief. If the posterior is greater than the deadlock belief, then the buyer offers v_H^* to end the bargaining process with both types of sellers. On the other hand, when the posterior is lower than π^* the buyer offers a much lower price and targets only the low type. Note that if the prior is less than π^*, the posterior is always less than or equal to π^* (unless bargaining ends) and the buyer never buys a high-type product.

I claim that the above profile is an equilibrium only if $\hat{p} = v_L$. Recall that by Corollary 1, if the buyer offers any price less than v_H^* the high type rejects the offer and opts out if the option is available, so he exits the game with probability ξ.

- ▷ Since the low-type seller accepts any $p \in [\delta v_H^*, v_H^*)$ with probability one (Corollary 1), $\hat{p}$ must be less than δv_H^*.

- ▷ Suppose that $\hat{p} \in (v_L, \delta v_H^*)$. Fix a history h^n with $\pi_n = \pi^*$. Let $\epsilon > 0$ be small that $\hat{p} - \epsilon > \max\{v_L, \delta \hat{p}\}$. Consider the buyer's deviation at h^n to offer $\hat{p} - \epsilon$.

  - I claim that in response to $\hat{p} - \epsilon$, the low type exits the game with probability ξ. If he exits with probability greater than ξ, the buyer's posterior becomes $\pi_{n+1} > \pi^*$. Hence the buyer offers v_H^* in period $n+1$. But then it is strictly optimal for the low type not to exit in period n, so his behavior is inconsistent with the belief. If he exits with probability less than ξ, then $\pi_{n+1} < \pi^*$, so the buyer offers $p_{n+1} \leq \hat{p}$ in period $n+1$. But then it is strictly optimal for the low type to accept p_n at period n.

  - Then the low type must accept $\hat{p} - \epsilon$ with probability ξ and not take the outside option because $\hat{p} - \epsilon > v_L$. Hence offering $\hat{p} - \epsilon$ is a profitable deviation for the buyer, contradiction.

- ▷ Suppose that $\hat{p} < v_L$. Then it is suboptimal for the low type not to opt out when the posterior is π^* and the buyer offers $\hat{p}$, because the outside option's value is more than the buyer's offer.

Given that $\hat{p} = v_L$, the value of π^* and q is uniquely determined by the indifference conditions of the players at the deadlock belief. At $\pi_n = \pi^*$ the buyer must be indifferent between offering v_H^* and v_L. If the buyer offers v_H^*, then both types of sellers accept it for sure and the buyer obtains

$$U_F^* \equiv (1 - \pi^*)(u_L - v_H^*) + \pi^*(u_H - v_H^*). \tag{9}$$

On the other hand, if $p_n = v_L$, the low type's response is $(\sigma_{Ln}, c_{Ln}) = (\xi, 0)$ and the buyer obtains

$$(1-\pi^*)\xi(u_L - v_L) + \delta(1-\xi)U_F^*. \tag{10}$$

Combining the above two formulas pins down the unique deadlock belief

$$\pi^* = \frac{(v_H^* - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - v_L)}{(u_H - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - v_L)}.$$

Now consider the seller's indifference condition. At the deadlock belief, the low type uses a mixed strategy between acceptance and rejection when the buyer offers v_L. So it must be the case that

$$v_L = \delta(qv_H^* + (1-q)v_L), \tag{11}$$

which uniquely determines q.

But then why is the above profile an equilibrium? Can the buyer induce a higher acceptance probability by offering a higher price? For any $p < v_H^*$, if the low type accepts p with probability greater than ξ, then in the next period, the posterior becomes greater than π^* and the buyer offers v_H^*. So as long as the price is less than δv_H^*, the acceptance probability must be no greater than ξ. Therefore, if the buyer wants to increase the acceptance probability, he needs to raise the price at least to δv_H^*.

What if the buyer offers δv_H^*? If the seller is the low type, he accepts the offer with probability one. However, if the seller is the high type, he rejects the offer and opts out if the option is available, and in that case, the buyer receives zero payoff. So if the outside

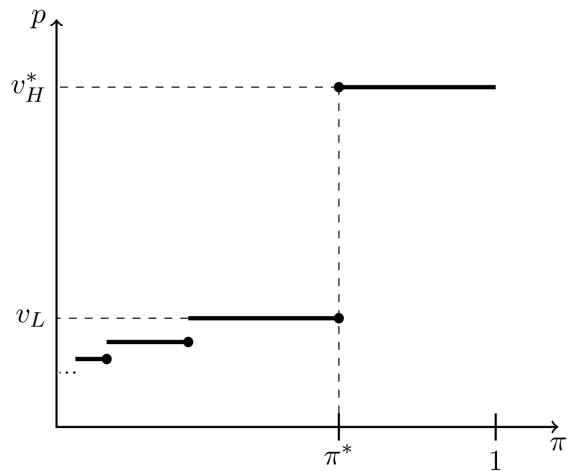


FIGURE 7
BUYER'S EQUILIBRIUM OFFER

option arrives with a high probability, the cost from the high type's opting out is greater than the benefit from trading with the low type. Assumption 1 necessitates such a high arrival rate of the outside option to guarantee the existence of the deadlock equilibrium. The following proposition summarizes the argument:

Proposition 8. *Suppose (A1) holds. Then the model generically has a unique deadlock equilibrium.*

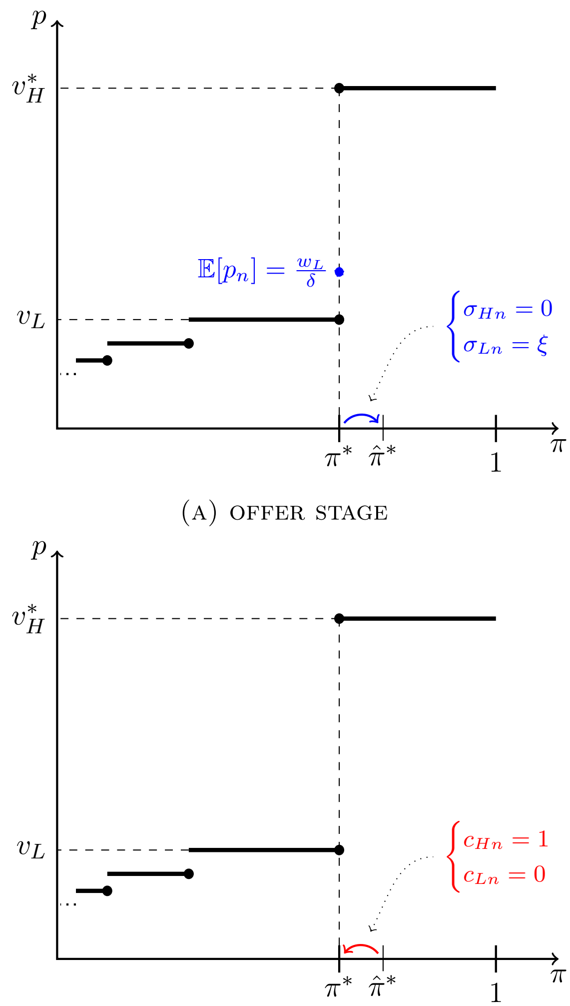
Proof. See the Appendix. □

Figure 7 describes the buyer's equilibrium offer of the buyer as a function of the posterior belief. If the buyer's belief is greater than the cutoff belief π^*, he offers v_H^* and both types of sellers accept the offer for sure; hence the game ends immediately. When the belief is less than π^*, his offer is no more than the low type's value of the outside option (v_L), and the offer is nondecreasing in the belief. Later in this subsection I describe more details of the equilibrium offer when the belief is less than π^*.

Equilibrium behavior at the cutoff belief π^* is depicted in Figure 8. At the offer stage (described in the left panel), the buyer offers either v_H^* or v_L. If the buyer had offered v_L in the previous period, then he plays a mixed strategy between offering v_H^* and v_L, which satisfies the low type's indifferent condition (11). If the buyer offers v_H^*, then both types of sellers accept it and hence the game ends. If the buyer offers v_L, then the high type rejects it, since it is lower than his reservation value. The low type accepts the offer with probability ξ. Therefore after the offer stage ends, the buyer's interim belief becomes $\hat{\pi}_n = \hat{\pi}^* \equiv \frac{\pi_n}{\pi_n+(1-\pi_n)(1-\xi)} > \pi^*$. At the outside option stage (right panel), only the high type exercises the outside option when it is available. Since the high type exits with probability ξ, the posterior belief π_{n+1} decreases back to π^*. From then on, the bargaining parties repeat the same behavior in each period: the buyer mixes between offering v_H^* and v_L; the low type accepts v_L with probability ξ while the high type rejects it; only the high type opts out. Note that the buyer's belief does not change unless bargaining ends, since the information from the seller's acceptance behavior and his opting-out behavior exactly offset one another.

What happens if the prior is lower than the deadlock belief? In the Appendix, I construct a sequence of prices $\{p_k^\dagger\}(p_0^\dagger = v_L, p_k^\dagger \in (v_L^*, v_L)$ for $k \geq 1)$ and a sequence of cutoff beliefs $\{\pi_k^\dagger\}(\pi_0^\dagger = \pi^*, \pi_k^\dagger \in (0, \pi^*)$ for $k \geq 1)$ that describe the equilibrium behavior when the belief is smaller than π^*. It is shown in the Appendix that both $\{p_k^\dagger\}$ and $\{\pi_k^\dagger\}$ are decreasing, and that for any prior $\pi_0 < \pi^*$, there exists $N \in \mathbb{N} \cup \{0\}$ such that $\pi_{N+1}^\dagger \leq \pi_0 < \pi_N^\dagger$. Here I consider the generic case that $\pi_{N+1}^\dagger < \pi_0$.

In the equilibrium, the buyer offers $p_N^\dagger$ in the first period. Then the low type accepts with positive probability such that the interim belief becomes $\pi_{N-1}^\dagger$. In the outside option stage, both types of sellers opt out if the outside option arrives, so the belief does not change at



(a) offer stage

(b) outside option stage

Figure 8
Equilibrium behavior at $\pi = \pi^*$

$\pi^\dagger_{N-1}$. In the second period, the buyer increases his offer to $p^\dagger_{N-1}$ which will induce another mixed acceptance by the low type, and both types opt out when possible, and the posterior becomes $\pi^\dagger_{N-2}$. This behavior continues until the posterior reaches $\pi^\dagger_0 = \pi^*$. Hence, it takes $\max\{N, 1\}$ periods for the posterior to reach the deadlock belief. Note that information about the seller's type comes only from his acceptance behavior, and the posterior strictly increases in each period. Figure 9 describes the dynamics of belief on the equilibrium path when $N = 2$.

So if the prior is less than the deadlock belief, the equilibrium behavior produces an outcome path with the following characteristics:

- ▷ Bargaining starts with a *pre-deadlock phase*. In this phase the buyer plays a pure offer strategy, and his offer is increasing over time so that the low type is indifferent between acceptance and rejection. Only the low type accepts the offer with positive probability, and both types of sellers opt out if possible. So an observed outcome in this phase has the following characteristics: the buyer offers a price less than v_L; the buyer's offer increases over time; bargaining might end with either acceptance of the buyer's offer (by the low type) or opting out (by both types).

- ▷ A *deadlock phase* begins once the buyer offers v_L. In this phase, the buyer continues to make the same randomized offer throughout the bargaining process. Only the low type accepts v_L with positive probability, and only the high type opts out if possible. Therefore, an outcome path features a sequence of the same offer of v_L being rejected repeatedly before bargaining ends.

- ▷ Bargaining ends with a sudden resolving behavior that is either 1) the buyer's bargaining-

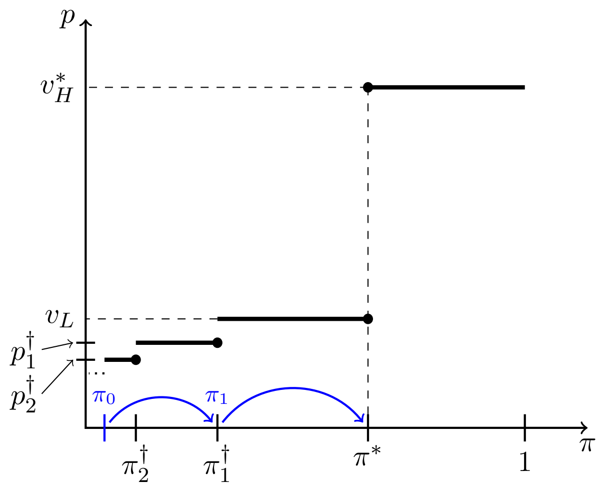


FIGURE 9
EQUILIBRIUM BEHAVIOR WHEN $\pi_0 < \pi^*$

ending offer (v_H^*), 2) the low type's acceptance of v_L, or 3) the high type's opting out. Note that bargaining ends (either by an acceptance or an opt-out) in a finite number of periods with probability one.

10 Frequent Offers

Consider the limit case of frequent offers by letting the time between periods (denoted by Δ) converge to zero. Recall that $\eta = \frac{\lambda}{r+\lambda}$ is the effective discount factor.

Proposition 9. *Suppose $\eta v_H > v_L + \frac{1-\eta}{\eta} u_L$. Then,*

1. *The deadlock equilibrium exists for sufficiently small Δ.*

2. *In the deadlock equilibrium, as Δ converges to zero, the buyer's equilibrium offer when $\pi < \pi^*$ converges to v_L; the length of the pre-deadlock phase (measured in real time) shrinks to zero; the expected length of the deadlock phase does not shrink to zero.*

Proof.

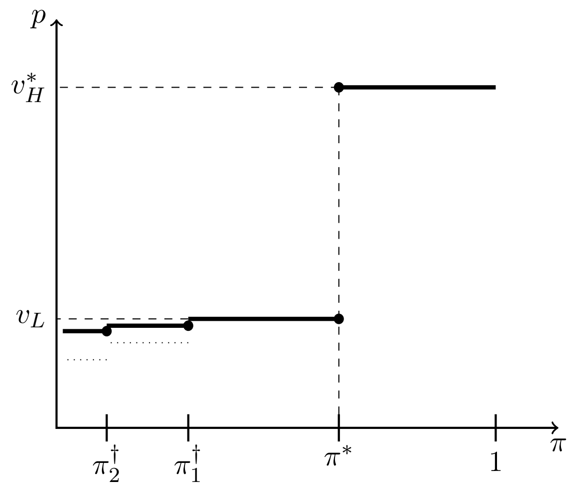


Figure 10
Offer function when Δ is small

1. (A1) is satisfied if $\eta v_H > v_L + \frac{1-\eta}{\eta} u_L$ and Δ is sufficiently small.

2. The proof is based on the construction of the deadlock equilibrium and is relegated to the Appendix.

□

As mentioned before, the deadlock equilibrium exists when the outside options are sufficiently important. In the limit case of frequent offers, this condition is represented by the effective discount factor being sufficiently high.

In the pre-deadlock phase, the equilibrium exhibits Coasian dynamics at a price v_L. Since the discount factor goes to one as Δ converges to zero, the difference between the buyer's successive offers vanishes as the buyer makes the low-type seller indifferent between acceptance and rejection. Moreover, the same force behind the Coase conjecture results in the pre-deadlock phase shrinking to zero. Figure 10 describes the limit equilibrium offer by the buyer when Δ is close to zero.

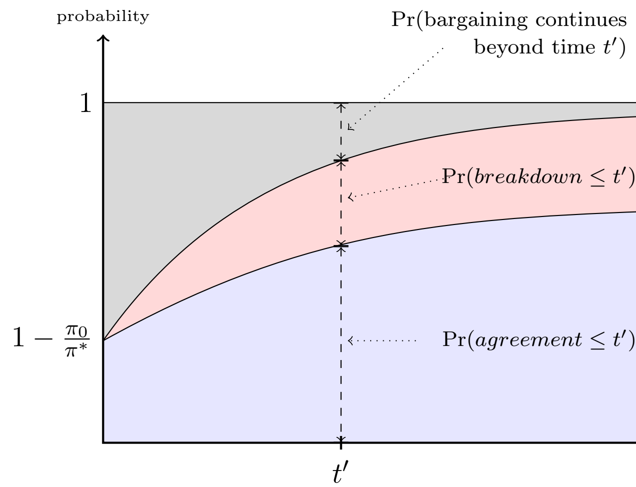


FIGURE 11
LIMIT DISTRIBUTION OF THE EQUILIBRIUM OUTCOME

However, the deadlock phase does not shrink in the limit case of frequent offers. More specifically, each resolution behavior of the deadlock phase (the buyer's bargaining-ending offer, the low type's acceptance of the low offer, and the high type's opt-out) converges to a Poisson arrival process. The indifference condition (11) implies that, as Δ converges to zero the probability of the buyer offering v_H^* converges to zero at the same rate. As a result, the buyer's equilibrium offer path (in real time) converges to the base offer of v_L with the endogenous Poisson arrival of v_H^*.[22] The low type's acceptance of offer v_L and the high type's opt-out occurs with probability $\xi = 1 - e^{-\lambda\Delta}$; hence, they converge to Poisson processes with parameter λ. Note that the Poisson arrivals of resolution behaviors are independent of each other.

[22] To see this, note that

$$\begin{aligned} q &= \frac{v_L/\delta - v_L}{v_H^* - v_L} \\ &= \frac{v_L}{v_H^* - v_L}(e^{r\Delta} - 1) = \frac{v_L r}{v_H^* - v_L}\Delta + o(\Delta), \end{aligned}$$

so as $\Delta \to 0$, the arrival of the buyer's offer $p = v_H^*$ converges to a Poisson process of rate $\frac{v_L r}{v_H^* - v_L}$.

Figure 11 summarizes the discussion above by depicting the limit distribution of the equilibrium outcome as $\Delta \to 0$. At any real time t', the height in the blue (red) area indicates the probability that the agreement (breakdown) happens anytime before t'. The height in the grey area is the probability that the bargaining continues beyond time t'. Note that for any finite t, bargaining will continue beyond time t with positive probability.

10.1 Real-Time Delay and Breakdown

The outcome of the deadlock equilibrium exhibits various bargaining inefficiencies. Several key values, such as the expected length of delay and the probability of a breakdown, are derived in closed form.

The equilibrium behavior described in Section 9 implies that in the deadlock equilibrium, the bargaining is delayed with positive probability before it ends either in an agreement or in a breakdown. The following corollary states that the expected length of delay in real time is positive even when the time between periods becomes arbitrarily small. So in the deadlock equilibrium, inefficiency does not disappear when offers are frequent. Let T_d be the (unconditional) expected length of delay, and let $\hat{T}_d$ be the expected length of delay conditional on deadlock.

Corollary 2. *In the deadlock equilibrium, the expected length of delay is positive if the prior is less than π^*. Moreover, as Δ converges to zero,*

$$\begin{aligned}\hat{T}_d &\to \frac{Z}{Z+\mu} \cdot \frac{1}{\lambda}, \\ T_d &\to \frac{\pi_0}{\pi^*}\hat{T}_d,\end{aligned}$$

where $Z = \frac{v_H^* - v_L}{v_L}$ *and* $\mu = \frac{r}{\lambda}$.

Proof. See the Appendix. □

Recall that Assumption 1 encompasses both the private and correlated value case, so that the real-time delay associated with the deadlock equilibrium can be found in both cases.

Another source of inefficiency in the deadlock equilibrium is the possibility of a breakdown resulting from the high type's opt-out. Let P_b be the ex ante probability of a breakdown, and $\hat{P}_b$ be the breakdown probability conditional on deadlock.

Corollary 3. *In the deadlock equilibrium, as* Δ *converges to zero,*

$$\begin{aligned} \hat{P}_b &\rightarrow \pi^* \frac{Z}{Z+\mu}, \\ P_b &\rightarrow \pi_0 \frac{Z}{Z+\mu}. \end{aligned}$$

Proof. See the Appendix. □

10.2 High Arrival Rate of the Outside Option

The assumption in Proposition 9 implies that when the effective discount rate $\eta = \frac{\lambda}{r+\lambda}$ is arbitrarily close to one, the deadlock equilibrium exists in the limit of frequent offers. On the other hand, Corollary 2 implies that the expected length of the delay converges to zero as λ becomes arbitrarily high. So it is of interest to analyze the equilibrium behavior under sufficiently high λ.

Recall that at the deadlock belief, the low type accepts v_L with probability ξ. If ξ is close to one, almost every low-type seller accepts v_L and the interim belief after the offer stage becomes close to one. Then the high type exits the game with probability ξ and the posterior becomes π^*. Therefore, even though the equilibrium structure is preserved, the bargaining ends with a probability close to one.

Similar intuition can be applied to the limit case of frequent offers. Recall that as Δ goes to zero, each type of resolution behavior in the deadlock phase converges to a Poisson arrival process. As λ becomes arbitrarily high, the arrival rates of resolution behaviors also become arbitrarily high, and the (expected) length of the deadlock phase shrinks to zero.

What is the limit of π^* when λ becomes arbitrarily high? Fixing the discount rate r, as λ goes to infinity, the indifference condition of the buyer at π^* (from (9) and (10)) becomes

$$(1-\pi^*)(u_L - v_L) = \pi^*(u_H - v_H) + (1-\pi^*)(u_L - v_H).$$

Consider a static bargaining game where the buyer makes a take-it-or-leave-it offer to the seller and the seller has an outside option of v_θ. Then the left-hand side (right-hand side) of the above equation is the payoff to the buyer when he offers $v_L(v_H)$ to target low-type seller (both types of sellers). In other words, the buyer's optimal offer under arbitrarily high λ converges to one of static bargaining.

Interestingly, the limit distribution of the equilibrium outcome under high λ converges to the monopoly pricing equilibrium in **?**, with the role of seller and buyer reversed. They consider a model of a seller-offer bargaining game where the buyer has private information about his valuation of the seller's good, and they assume that the buyer has an outside option available at any period. They show that there is a unique sequential equilibrium where the seller always offers an optimal monopoly price, and the buyer either accepts the offer or opts out immediately. So there is no bargaining delay in the equilibrium. If I switch the role of the seller and the buyer in **?**, their equilibrium coincides to the limit distribution of the deadlock equilibrium with $\Delta \to 0$ and $\lambda \to \infty$.

11 Uniqueness

In general, there are multiple equilibria of this model. In particular, there may exist an equilibrium where the buyer uses an offer strategy similar to the 'Coasian' pricing (Fudenberg, Levine, and Tirole (1985); Gul, Sonnenschein, and Wilson (1986)). In this equilibrium, as the time between the periods becomes vanishingly small, the buyer's offer converges to v_H^* and the expected delay converges to zero, so the equilibrium outcome is approximately efficient. In the equilibrium with Coasian dynamics, although there are two sources of information, the information revealed by the seller's acceptance behavior dominates the information revealed by his opting-out behavior.[23]

Then the question is whether the deadlock equilibrium is one equilibrium of the model where two sources of information happen to offset one another. In this section, I show that under a stronger parametric assumption, the offsetting effect can be found in all PBE of the model. First, I present the parametric assumption stronger than (A1).

Assumption. *(A2)* $\frac{\xi}{1-\delta(1-\xi)}\delta v_H^* > u_L$.

A necessary condition for (A2) is $v_H^* > u_L$. Since $u_H \geq v_H$, the private value case ($u_H = u_L$) does not satisfy (A2). More important, $v_H^* > u_L$ is a necessary condition for the existence of the static adverse selection problem. Suppose there is a static market where

[23] More discussion about Coasian equilibrium is in Section 12.

the buyer's value is u_θ and the seller's reservation value is v_θ^*. Then adverse selection in the trade exists if and only if $\mathrm{E}[u_\theta] < v_\theta^*$. Therefore, if $v_H^* > u_L$, the adverse selection problem arises for sufficiently low π_0.

Since (A2) implies (A1), (A2) guarantees the existence of the deadlock equilibrium. The following proposition shows that under (A2), the deadlock equilibrium is the only PBE satisfying a monotonicity property. The property, called *nondecreasing offers*, requires that when the buyer's expected quality is higher, he tends to offer a higher price to the seller.

Definition. A strategy profile satisfies *nondecreasing offers* if for any history $h^n, h'^{n'}$ with $\pi_n < \pi_{n'}$, if the buyer offers p at h^n and p' at $h'^{n'}$, then $p \leq p'$.

Proposition 10. *Suppose (A2) holds. Then the deadlock equilibrium is the unique perfect Bayesian equilibrium that satisfies nondecreasing offers.*

Proof. See the Appendix. □

The following proposition states that under (A2), in every equilibrium neither source of information dominates the other, so the equilibrium has characteristics similar to those of the deadlock equilibrium.

Proposition 11. *Suppose (A2) holds. Then in every perfect Bayesian equilibrium of the game, if prior is low enough,*

(1) the posterior belief π_n never exceeds the deadlock belief π^* (defined in Section 9) conditional on the bargaining continues, and

(2) for any finite n, bargaining continues beyond period n with positive probability.

Proof. See the Appendix. □

12 Discussions

Random Arrival of Outside Options The random arrival of outside options assumed in this paper is a simple way of modeling a stochastic payoff of opt-out behavior. In principle, the bargaining parties can break the negotiation process at any point in time. However, the value of opting out typically changes over time. First, the value of the best available outside option may change over time. As in the example given in the introduction, a satisfactory outside offer often does not exist. Second, the cost of opting out may also change over time. Several external factors, such as the bargaining party's decision-making procedure and time-varying external environment, can affect the cost of taking the outside option. [24]

Theoretically, the random arrival of outside options provides an alternative perspective on bargaining dynamics. Standard models of bargaining with an outside option assume that the option is available in every period to some or all of the bargaining parties. In bargaining with complete information, the outside option is either completely ineffective (when the value is low) or crucially effective (when the value is high) in determining an equilibrium behavior. In bargaining with incomplete information, an outside option may almost completely cancel out the impact of incomplete information, and the equilibrium features immediate termination of bargaining when the bargaining party either agrees or opts out depending on his private type.[25] In this paper the outside option is not available with positive probability. As a result, the bargaining continues into the next period with positive probability unless the buyer offers v_H^*; hence, the equilibrium shows non-trivial dynamics.

[24]In Section 13, I discuss a possible extension of the random value of the outside option.

[25]See Compte and Jehiel (2002) for the effect of outside options on reputational bargaining, and **?** for the effect on a dynamic durable goods monopoly.

Production Cost and Heterogeneous Arrival Rate The result of this paper extends to the case where the seller has a positive cost of production. Suppose the type-θ seller has a production cost of $c_\theta > 0$. Recall that the seller's payoff is v_θ when he takes an outside option. Then the type-θ seller never accepts an offer if

$$p - c_\theta < \frac{\xi}{1-\delta(1-\xi)} v_\theta = v_\theta^*,$$

or $p < c_\theta + v_\theta^*$. To guarantee the existence of the deadlock equilibrium, a modified version of (A1) needs to be imposed:

Assumption. *(A1') $\delta(c_H + v_H^*) > v_L + \frac{(1-\delta)(1-\xi)}{\xi} u_L + (1 - \frac{1-\delta}{\xi}) c_L$.*

Note that if $c_H = c_L = 0$, (A1) and (A1') are equivalent.

Proposition 12. *Suppose (A1') holds. Then there exists a deadlock equilibrium of the model.*

Proof. See the Appendix. □

Note that (A1') encompasses a case where the value of the outside option is the same for both types ($v_H = v_L$). As long as the model's parameters induce the high type to have a stronger incentive to opt out, the deadlock equilibrium exists.

Similar intuition can be applied to check the robustness of the deadlock equilibrium when the arrival rate of the outside option is different across types. For example, the deadlock equilibrium may exist in the model where the outside option arrives only to the high-type seller.

Existence of Coasian Equilibrium When (A2) is not satisfied, the model has an equilibrium where Coasian dynamics lead to an approximately efficient outcome. In the Coasian equilibrium, the buyer plays a pure strategy at any history on the equilibrium path. The buyer gradually increases his offer over time. On the equilibrium path, the high type rejects the buyer's offer in all but the final period, and the low type uses a mixed acceptance strategy. If the initial offer is high enough, only the high type opts out in every period before the game ends. If the initial offer is low, then both types take the outside option until the offer exceeds some cutoff where it becomes suboptimal for the low type to opt out. As the time between periods becomes arbitrarily small, the initial offer converges to v_H^* and the equilibrium yields an approximately immediate trade.[26]

If the parameters satisfy (A2), then because of the static adverse selection problem, playing a Coasian strategy yields a negative payoff to the buyer. So the Coasian strategy profile does not hold as an equilibrium, and the deadlock equilibrium becomes a unique equilibrium under the monotonicity condition.

Permanent Outside Option Consider the case where the outside option does not disappear once it arrives. In the model of the permanent outside option, there exists a deadlock equilibrium that has the same equilibrium outcome as the one in the original model. The key reason is that both types of sellers have no incentive to keep the outside option. For the high type, since the buyer's maximum offer is strictly smaller than the payoff from the outside option ($v_H^* < v_H$), he opts out once the option is available. The low type cannot signal that he has an outside option, since he always uses a mixed strategy on the equilibrium path.

[26] A detailed description of the Coasian equilibrium in the presence of a stochastic outside option is available upon request. Hwang and Li (2013) construct a Coasian equilibrium in a model similar to the present paper, where the roles of the seller and the buyer are reversed.

So he cannot use the outside option as a threat in future periods. Since the buyer's offer is increasing over time, the low type has no incentive to keep the outside option.

Change in Timeline How robust is the deadlock equilibrium under different timelines of the game? Consider the case where the seller receives an outside option before the buyer offers a price. First, suppose that the seller's opting-out decision comes before the buyer offers a price, so the offer stage comes later than the outside option stage. Then a simple calculation shows that the equilibrium structure is unchanged. It is not surprising since the effect of switching the two stages only accounts for the discount factor.

What happens if the outside option arrives before the buyer makes an offer? In this case, there may exist multiple equilibria even if there is complete information about the quality. Under some range of parameters, the buyer's offer and the seller's opting-out decision have a self-fulfilling effect on each other. If the arrival rate of the outside option is low, there exists an equilibrium where the buyer makes an offer that is accepted only by the seller without the outside option, and the seller with the outside option rejects the offer and opts out. If the arrival rate of the outside option is high, there exists another type of equilibrium where the buyer makes an offer high enough so that the seller accepts it. And for the intermediate arrival rate, both equilibria may exist.

Two-Sided Incomplete Information It is generally known that in the model of a two-sided incomplete information bargaining game, severe multiplicity arises. The attempt to narrow down the equilibrium set results in either implausibility of the criterion or the non-existence of the equilibrium under certain parameters. I expect that in the model with the stochastic outside option, a similar multiplicity would arise.

Continuum of Types If the model assumes a continuum of seller's types, the main difficulty in the analysis is tracking the belief. Since the outside option does not arrive with probability one, the belief after an outside option stage has the same support as the one before the stage, but the belief about the high quality would decrease. Therefore, the posterior belief is not a truncation of the prior and therefore cannot be simplified to a state variable. So the equilibrium profile must describe the bargainer's behavior for any possible posterior belief.[27]

I conjecture that as in the two-types case, there are two countervailing forces in belief updating: the lower types tend to accept the buyer's offer and the higher types tend to opt out. However, it is unclear whether these countervailing forces would lead to a bargaining deadlock or to another equilibrium dynamic.

13 Concluding Remarks

One interesting extension is to assume a random value of the outside options instead of random availability. Consider the model where the type-θ seller receives an outside option in each period, and the value of the outside option is randomly drawn from distributions F_θ. Assume that F_H is first-order stochastic dominant over F_L. I conjecture that under some conditions on the distributions, there exists a deadlock equilibrium. In this case, neither type of seller plays a mixed strategy towards the outside option. Moreover, the low type would also opt out if he received a good enough outside option. But similar to the benchmark

[27] Fuchs, Öry, and Skrzypacz (2012) analyzed an equilibrium where the posterior belief is an addition of multiple truncated beliefs. In their paper, such beliefs are formed when the future buyer cannot observe past offers, so the price history does not affect future buyers' beliefs and hence it does not affect their strategies. In this paper, there is a single buyer and he observes history of past offers. So an out-of-equilibrium offer affects the future belief of the buyer, which makes the analysis difficult.

model, not taking the outside option conveys a bad signal about the quality of the product. It would be interesting to investigate whether and how a bargaining deadlock occurs in this extension.

Chapter III

Experimentation with Repeated Elections

14 Introduction

The peaceful transition of power marks a well-functioning democratic system. When an incumbent party decides whether to undertake a reform, the possibility of losing its current power affects its policy decision. In particular, if the consequences of the reform are unknown until after the reform has been implemented, then the incumbent has to consider the effects of the reform's outcomes on future elections and the action of the opposing party, which may have different preferences over policy alternatives.

For example, suppose that an incumbent party decides to implement health care reform. The diverse effects of the reform cannot be fully anticipated. Suppose also that the incumbent prefers to extend health care, while the opposing party prefers the opposite. Then the incumbent takes into account the fact that it might lose its control and the opposing party would reverse the policy. In this case, the presence of a change in power affects the incumbent's incentive to experiment with policy. Other examples of reforms whose outcome is uncertain and for which political parties have heterogeneous preferences include hawkish and dovish approaches to foreign policy, legalization of drugs, and social insurance.

In this paper, I study the incumbent party's incentives to experiment in the presence of elections, when the political parties have heterogeneous preferences over the outcomes. I address the following questions: How do repeated elections affect incentives to experiment? What is the equilibrium level of experimentation, and how does it depend on the frequency of elections? What is the socially efficient level, and is it achievable?

To address these questions, I analyze a continuous-time three-player game with two political parties and a voter. The policy experimentation process is modeled by a three-armed bandit model in which a *safe* policy yields a constant payoff, and two *risky* policies yield outcomes whose distribution, or type, is unknown. The safe policy is interpreted as a status-quo policy, while the risky policies are reform policies in different directions. At each instant, the incumbent party chooses one of the three alternatives. The parties and the voter learn the types of risky policies only through experimentation. Each risky policy is either productive or unproductive, and it is commonly known that exactly one alternative is productive while the other is unproductive. This means that one of the two mutually exclusive reform policies will turn out to be good if explored long enough. Moreover, any news shock fully reveals that the risky policy under experimentation is productive (and the other is unproductive), so all uncertainty is resolved.

Each party is biased toward a different risky alternative. This means that each reform policy has an ideological characteristic that is in accordance with one party's value. Each party gets the greatest value from its preferred risky alternative if it is productive. Furthermore, the party does not value the opposite risky alternative regardless of its types, so even if it is known that its preferred risky policy is unproductive, it prefers to choose the safe one. This payoff structure captures the loss of enthusiasm or support among the party's partisans when the opposing risky alternative is implemented.

I model elections as a Poisson arrival process. This stationarity assumption enhances the tractability of the model, while it is still enough to analyze the paper's main question: the incumbent's incentive to experiment. At each election, the voter chooses the party that will have power until the next election. The voter has unbiased preferences and prefers any productive risky alternative to the safe one. There exists a political agency problem in the sense that the voter cannot control the incumbent party while there is no election, and it is the incumbent party that chooses the policy alternatives. For the voter, the only way to control the party is to replace it with the other at the next election.

I restrict players to stationary Markov strategies with the common posterior belief as the state variable. I characterize a Markov perfect equilibrium (MPE) in which the voter elects the party whose preferred risky alternative offers a more promising belief about its productivity type. I show that there exists a unique equilibrium for a broad range of parameters. Then I analyze the efficiency of policy experimentation from the voter's perspective and conduct comparative statics with respect to the parameters such as election frequency, speed of learning and the value of a productive risky alternative.

The equilibrium shows an interesting implication in regard to the optimal frequency of elections. A common intuition is that the voter would be better off under more frequent elections, since he would have more control over the political parties. However, I show that while infrequent elections are surely bad for the voter, too frequent elections can also make him strictly worse off.

It is not surprising that if elections are infrequent, then the equilibrium exhibits inefficiency caused by political agency. If the next election is far away from now, then the incumbent party will not worry much about the future loss of power, and so its behavior is similar to that of a dictator. So the incumbent keeps experimenting with its preferred risky alternative even when the belief about its productivity type is pessimistic. The voter knows that it is better to experiment with the opposite risky policy, but he cannot control the incumbent's behavior. In this case, inefficiency decreases as election frequency increases.

A more surprising result is that there exists a different type of inefficiency under high frequency of elections. If the election frequency is greater than a certain threshold, in equilibrium the incumbent ceases to experiment at a certain point of belief. Instead, the incumbent party chooses the safe alternative and the learning stops. If elections occur too frequently, then each party would have to give up its power right after it generates the negative information. Therefore, the value of experimentation to the incumbent becomes small enough to avoid risky policy. This shows that there is another source of inefficiency from political agency: potential loss of power prevents the incumbent from conducting risky policy. I show that the degree of this inefficiency is so large that the voter is worse off under too frequent elections than under a dictatorship.

The above argument implies that there exists an optimal election frequency under which inefficiency is minimized. The frequency of elections must be high enough so that the inefficiency from a standard political agency problem is small. On the other hand, it must not be too high; otherwise it triggers a cessation of experimentation. I show that there exists a unique frequency of elections where the voter's expected payoff is maximized. Moreover, I show that the optimal frequency of elections is increasing in the value of the productive risky alternative. When the risky policy has a high value, the incumbent has enough incentive to explore the risky policy even under frequent elections.

The paper contributes to a developing literature on experimentation with multiple agents. Bolton and Harris (1999) and Keller, Rady, and Cripps (2005) study a two-armed bandit problem in which different agents may choose different arms. Klein and Rady (2011) consider a similar case but assume that the expected payoffs of risky arms are negatively correlated across players, and this information structure is applied in the present paper. Bonatti and Hörner (2011) consider the case in which each agent's action is unobservable and find that the moral hazard problem leads not only to a reduction in effort but also to procrastination. In all of these papers, an informational free-riding problem leads to underinvestment in the acquisition of information. On the other hand, in the present paper the driving force for ceasing experimentation is the presence of a potential loss of control, which is crucially related to the political agency problem.[28]

The paper also contributes to the literature on political agency (Ferejohn (1986); Banks and Sundaram (1998); Besley (2004); Maskin and Tirole (2004)). These papers consider a potential moral hazard problem of elected politicians, so their results imply that it is always better for the voter to have more frequent elections. In contrast, the present paper shows that if we consider the uncertainty of policy implementation, there exists another type of political agency problem that occurs when election frequency is too high.

The paper is also related to the literature of alternating political power. Dixit and Gul (2000) use a repeated game argument to show that the presence of alternating power enables two parties to make political compromises. Aragones, Palfrey, and Postlewaite (2007) develop a model of repeated elections and analyze conditions under which candidates' reputations may affect voters' beliefs over what policy will be implemented by the winning candidate of an election.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3

[28] Some papers apply a model of strategic learning in the context of political economy. Strulovici (2010) considers the case in which a number of agents collectively decide which of two alternatives to choose according to some voting rule. He finds that the control-sharing effect leads to an inefficiently low level of experimentation in equilibrium. Callander (2011) considers a two-period model to show that alternating political power can benefit voters when the policy outcome is unknown. Gul and Pesendorfer (2012) consider a model of a political campaign in which two parties of opposing interests provide costly information to voters.

derives the Hamilton-Jacobi-Bellman equation for a party's best response. Section 4 characterizes the Markov perfect equilibria of the non-cooperative game and derives the optimal election frequency. Section 5 argues how the voter's payoff improves in the incumbency advantage equilibrium and discusses several extensions of the model. Section 6 concludes. Some of the proofs are relegated to the Appendix.

15 Model

Time $t \in [0, \infty)$ is continuous. There are two political parties ($i = 1, 2$) and a median voter (m). Both parties and the median voter are forward-looking and they have a common discount rate $r > 0$. At each instant, one of two political parties is determined to be an active party and chooses a policy $X_t \in \{S, R_1, R_2\}$. The other party, a passive party, cannot affect the active party's decision.

The first policy S is a *safe* policy and generates a deterministic flow payoff. There are two *risky* policies R_i ($i = 1, 2$), which can be either a *productive* type or an *unproductive* type. The types of risky policies are unknown at the beginning. We will further assume a perfect negative correlation between two risky policies: it is common knowledge that exactly one risky policy is productive, while the other one is unproductive.

Payoffs for each political party are as follows. If the active party chooses to play S, then it yields a flow payoff of $s > 0$ to both parties. If the active party plays R_i and if it is unproductive, it generates zero payoff. If R_i is productive, then it pays a lump-sum payoff h at random times *only to party* i. Party j gets zero payoff from R_i regardless of its type. These heterogeneous preferences can be interpreted such that the party j is biased toward the risky action R_j, so that party j does not value the outcomes from R_i. Assume that the lump-sum arrival times correspond to jumping times of a Poisson process with intensity $\lambda > 0$. Then $g = h\lambda$ is the expected payoff to party i per unit of time, conditional on R_i being productive. We assume $g > s > 0$, so party i strictly prefers R_i to S and S to R_j if R_i is productive, and strictly prefers S, if it is unproductive, to R_i and R_j.

While each political party is biased toward the outcomes of one risky policy, the median voter is unbiased toward both risky policies and hence prefers *any* productive risky policy. That is, he gets the expected payoff of g from any productive risky policy and the flow payoff of s from a safe policy.

I model elections as Poisson arrivals. At random time, which corresponds to jumping times of a Poisson process with arrival rate $\xi > 0$, the median voter chooses one of the two parties to be the active party. Once a party is chosen, then it is guaranteed to have control over the action choices until the next election. The election process and the lump-sum payoff process of a productive risky policy are independent. The types of the risky policies stay the same at every regime change, that is, Nature conducts a random draw only once at the beginning of the game. Finally, we assume no private information: both parties can observe the active party's choice of action and the resulting outcome.

Let $\{\sigma_{i,t}\}_{t\geq 0}(i = 1, 2)$ and $\{\sigma_{M,t}\}_{t\geq 0}$ be the actions of the parties and the median voter, where $\sigma_{i,t} \in \{S, R_1, R_2\}$ and $\sigma_{M.t} \in [0, 1]$ (probability of choosing party 1) is measurable with respect to the information available at time t. Let $\{\iota_t\}_{t\geq 0}$ ($\iota_t \in \{1, 2\}$) be a stochastic process of the active party, which is determined by Poisson arrivals of the elections and $\{\sigma_{M,t}\}_{t\geq 0}$. Then a policy decision rule is a stochastic process $\{X_t\}_{t\geq 0}$, where $X_t = \sigma_{\iota_t,t}$ is an action taken by the active party at time t.

Let p_t be the common posterior belief at time t that R_1 is productive. Let $\{\mathcal{F}_t\}_{t\geq 0}$ be a filtration generated by $\{X_t\}_{t\geq 0}$ and the corresponding outcome process, then the stochastic process $\{p_t\}_{t\geq 0}$ is adapted to $\{\mathcal{F}_t\}_{t\geq 0}$, and p_t evolves according to Bayes' rule. The posterior belief jumps up to one once there is a breakthrough on R_1, and jumps down to zero if a breakthrough on R_2 is observed. In either case, learning is complete and p_t stays the same. If there has been no breakthrough until t, then p_t obeys the following differential equation:

$$\dot{p}_t = p_t(1-p_t)\lambda(\mathbf{1}_{X_t=R_2} - \mathbf{1}_{X_t=R_1}).$$

Note that $\dot{p}_t < 0 (> 0)$ when the active party plays $R_1 (R_2)$ and no breakthrough is discovered.

Party 1's total discounted expected payoff, expressed in per-period units, can be written as

$$\mathrm{E}_0\left[\int_0^\infty re^{-rt}[\mathbf{1}_{X_t=R_1}\cdot p_t g + \mathbf{1}_{X_t=S}\cdot s]dt\right]$$

where the expectation is taken over the stochastic processes $\{X_t\}_{t\geq 0}$ and $\{p_t\}_{t\geq 0}$. Similarly, the payoff for party 2 is

$$\mathrm{E}_0\left[\int_0^\infty re^{-rt}[\mathbf{1}_{X_t=R_2}\cdot (1-p_t)g + \mathbf{1}_{X_t=S}\cdot s]dt\right],$$

and

$$\mathrm{E}_0\left[\int_0^\infty re^{-rt}[\mathbf{1}_{X_t=R_1}\cdot p_t g + \mathbf{1}_{X_t=R_2}\cdot (1-p_t)g + \mathbf{1}_{X_t=S}\cdot s]dt\right]$$

for the median voter.

A sequential equilibrium is called a Markov perfect equilibrium (MPE) if the agents of the game play stationary Markov strategies with the common posterior belief $p_t \in [0,1]$ as a state variable. Party i's strategy is then given by a function $\sigma_i : [0,1] \to \{S, R_1, R_2\}$, and the voter's strategy is given by $\sigma_M : [0,1] \to [0,1]$.

Let Σ^* be the set of strategy profiles where the median voter chooses the candidate whose preferred reform policy has a more optimistic belief. In other words, the median voter's strategy is $\sigma_M(p) = 1$ whenever $p > 1/2$ and $\sigma_M(p) = 0$ whenever $p < 1/2$. In this paper, I focus on Markov perfect equilibria in Σ^*.

Observe that in any equilibrium in Σ^*, each party never plays the reform policy that the opposite party prefers, that is, party i always chooses either S or R_i. This is because for party i, playing R_j gives the same amount of information as R_i while generating a zero flow payoff. In the rest of the paper, I will define $k_i(p) \in \{0,1\}$ as the probability that party i chooses its preferred reform policy.

16 Hamilton-Jacobi-Bellman equation

16.1 Single party's problem

Before I analyze the model, consider a benchmark case where there is no election, that is, $\xi = 0$. In this case, only one party is active for all $t \in [0,\infty)$ and it faces the single decision-maker problem described in Keller, Rady, and Cripps (2005). Suppose party 1 is always active, and let $V_1(p)$ be its value as a function of posterior belief p. Then it solves the following Hamilton-Jacobi-Bellman equation:

$$V_1(p) = s + \max_{k_1 \in \{0,1\}} k_1 \left\{ -s + pg + \frac{p\lambda}{r}\{g - V_1(p) - (1-p)V_1'(p)\} \right\}. \tag{12}$$

The first part of the maximand corresponds to action S, and the second corresponds to R_1. The effect of R_1 on the value of party 1 can be decomposed into three elements: (i) an expected flow payoff pg, (ii) a jump in value function when party 1 discovers a breakthrough on R_1, captured by $\frac{p\lambda}{r}(g - V_1(p))$, and (iii) a decrease in value function when no breakthrough is observed, captured by $-\frac{\lambda}{r}p(1-p)V_1'(p)$. The difference between expected flow payoffs from S and R_1, which is

$$c_1(p) \equiv s - pg,$$

is called the *opportunity cost* of experimentation for party 1. The sum of the second and third elements,

$$b_1(p) \equiv \frac{p\lambda}{r}\{g - V_1(p) - (1-p)V_1'(p)\},$$

is called the *value of experimentation* of party 1. Then party 1 experiments if and only if $b_1(p) > c_1(p)$. The value of information $b_1(p)$ is nonnegative in the single party problem. However, I show that if $\xi > 0$, it can be negative for a range of p under some equilibria.

If party 1 were myopic, i.e., merely maximizing current flow payoffs, then it plays the risky action if and only if $c_1(p)$ is negative. So party 1 plays the cutoff strategy with cutoff $p^m = \frac{s}{g}$. If it were forward-looking, then it values the information from a risky action to use it for future decisions. In this case, the optimal decision rule is to play a risky action if and only if $b_1(p) > c_1(p)$, so it uses the optimal single party cutoff

$$p^0 = \frac{\mu s}{\mu g + (g - s)} < p^m, \tag{13}$$

where $\mu = \frac{r}{\lambda}$.

16.2 Hamilton-Jacobi-Bellman equation

Now I introduce elections by assuming $\xi > 0$. Let $V_1(p)(W_1(p))$ be the value function of party 1 when it is active (passive). Then for any open interval of beliefs where the actions of party 2 and the median voter are constant, party 1's payoff function is differentiable and solves the following set of Hamilton-Jacobi-Bellman (HJB) equations:[29]

$$\begin{aligned} V_1(p) &= s - (1-\sigma_M(p)) \cdot \frac{\xi}{r}(W_1(p) - V_1(p)) + \max_{k_1 \in \{0,1\}} k_1 \left\{ -s + pg + \frac{p\lambda}{r}\{g - V_1(p) - (1-p)V_1'(p)\} \right\}, \\ W_1(p) &= \begin{cases} \sigma_M(p) \cdot \frac{\xi}{r}(V_1(p) - W_1(p)) + s & \text{if } \sigma_2(p) = S, \\ \sigma_M(p) \cdot \frac{\xi}{r}(V_1(p) - W_1(p)) + \frac{(1-p)\lambda}{r}\{W_1(0) - W_1(p) + pW_1'(p)\} & \text{if } \sigma_2(p) = R_2. \end{cases} \end{aligned} \tag{14}$$

Similar to the single decision-maker problem, party 1, when active, faces the trade-off between the opportunity cost and the value of experimentation. However, there exists an election after which party 1 can lose its power and become a passive player if the median voter chooses party 2. The first term of the equation for $V_1(p)$ represents such a possible

[29] Similarly, for any open interval of beliefs where party 1's action is constant, $V_2(p)$ and $W_2(p)$ are differentiable and they solve

$$\begin{aligned} V_2(p) &= \mathbf{1}_{\{\sigma_m(p)=1\}} \cdot \frac{\xi}{r}(W_2(p) - V_2(p)) + \max\left\{ s \ , \ (1-p)g + \frac{(1-p)\lambda}{r}\{V_2(0) - V_2(p) + pV_2'(p)\} \right\}, \\ W_2(p) &= \begin{cases} \mathbf{1}_{\{\sigma_m(p)=2\}} \cdot \frac{\xi}{r}(V_2(p) - W_2(p)) + s & \text{if } \sigma_1(p) = S, \\ \mathbf{1}_{\{\sigma_m(p)=2\}} \cdot \frac{\xi}{r}(V_2(p) - W_2(p)) + \frac{p\lambda}{r}\{W_2(1) - W_2(p) - (1-p)W_2'(p)\} & \text{if } \sigma_1(p) = R_1. \end{cases} \end{aligned}$$

regime change. Note that party 1's problem is essentially the same as that of the single decision-maker for the range of beliefs where the median voter chooses party 1. Since it cannot affect the action choice when it is passive, there is no maximization problem in the formula of $W_1(p)$, and $W_1(p)$ depends on the opponents' strategy. The first term of $W_1(p)$ disappears when the median voter chooses party 2.

For the median voter, let $Z_m(p,i)$ be the value function of the median voter when there is no election and when party i is active, and $V_m(p)$ be the value function at the time of election. Then the HJB equations are given by

$$
\begin{aligned}
Z_M(p,i) \quad &= \quad \frac{\xi}{r}(V_M(p) - Z_M(p,i)) \\
&\quad +\mathbf{1}_{\{\sigma_{\mathbf{i}}(\mathbf{p})=\mathbf{S}\}} \cdot \mathbf{s} \\
&\quad +1_{\{\sigma_i(p)=R_1\}} \cdot \{pg + \frac{p\lambda}{r}\left\{Z_M(1,i) - Z_M(p,i) - (1-p)Z'_M(p,i)\right\}\} \\
&\quad +\mathbf{1}_{\{\sigma_i(p)=R_2\}} \cdot \{(1-p)g + \frac{(1-p)\lambda}{r}\left\{Z_M(0,i) - Z_M(p,i) + pZ'_M(p,i)\right\}\},
\end{aligned}
$$

$$
V_M(p) \quad = \quad \max_{\sigma_M \in [0,1]} Z_M(p,2) + \sigma_M[Z_M(p,1) - Z_M(p,2)].
$$

Note that there is no maximization problem in the expression for $Z_M(p,i)$. This is because the median voter does not take any action between elections, so only the active party's policy choice affects the value of $Z_M(p,i)$. When the election comes, the median voter chooses the party that gives him higher $Z_M(p,i)$.

When the uncertainty is resolved, i.e., when $p = 0$ or 1, there exists a dominant strategy for each player. Since there is no uncertainty at those beliefs, both parties choose the myopically optimal action. Therefore, $(\sigma_1(0), \sigma_2(0)) = (S, R_2)$, and $(\sigma_1(1), \sigma_2(1)) = (R_1, S)$. Moreover, the median voter chooses the party that is biased toward the productive risky arm, so $\sigma_M(0) = 0$ and $\sigma_M(1) = 1$. Using this, the values of V_1 and W_1 in the certainty case are calculated and given by

$$V_1(1) = g, \qquad V_1(0) = (1-\chi)s$$

$$W_1(1) = \chi g + (1-\chi)s, \qquad W_1(0) = 0.$$

where $\chi = \frac{\xi}{\xi + r}$.

17 Equilibrium

In this section, I fully characterize the Markov perfect equilibria in Σ^*. There are three classes of MPEs for which the equilibrium outcome is qualitatively different. I show that each class of MPE appears in a different range of parameters.

17.1 Low stake

The following theorem (whose proof is in the appendix) states that in the case of a small stake (low value of g/s), the there exists a unique MPE which generates an efficient outcome from the voter's perspective. Recall that p^0 is an optimal cutoff of the single decision-maker problem.

Proposition 13. *If $\frac{g}{s} < \alpha_0 \equiv \frac{1+2\mu}{1+\mu}$, then a strategy profile in Σ^* is an MPE if and only if* $\sigma_1^{-1}(R_1) = (p^0, 1]$ and $\sigma_2^{-1}(R_2) = [0, 1 - p^0)$.

From (13) it is easy to see that $p^0 > \frac{1}{2}$ if and only if $\frac{g}{s} < \alpha_0$. Therefore, in the above equilibrium, once the belief falls in the range $[1-p^0, p^0]$, both parties choose the safe policy and the reform policy is never explored.

In this equilibrium, each party's equilibrium strategy is the same as the that of the single decision-maker's problem in Section 16.1. This is because given the median voter's strategy, there is essentially no strategic interaction between the two parties. Note that the incumbent is in the "safe region" if its preferred reform policy has a more optimistic belief, because the voter would reelect the incumbent if there was an election at that instant. Then the incumbent in its safe region chooses the policy as if it is a single decision-maker. Here, each party stops experimentation in the safe region; hence, its optimal strategy does not depend on the opponent party's strategy. The similar intuition explains the fact that the upper bound α_0 on the size of the stake does not depend on the election frequency (ξ).

Figure 12 describes party 1's payoff functions $V_1(p)$ (dark red line) and $W_1(p)$ (bright blue line) in the equilibrium in the low-stake case. Note that $W_1(p)$ is less than or equal to $V_1(p)$ for any belief point, and the difference between the two functions captures the cost of losing control. Both $V_1(p)$ and $W_1(p)$ are equal to s in the middle range of the belief space, since both parties choose the safe policy.

In the low-stake case, the unique equilibrium outcome is optimal for the median voter, and there is no inefficiency from the political agency problem.

17.2 High stake case: Infrequent elections

Now consider the high-stake case where $\frac{g}{s} \geq \alpha_0$. In this case, the optimal cutoff p^0 of the single decision-maker problem becomes strictly less than 1/2. So playing p^0 induces a

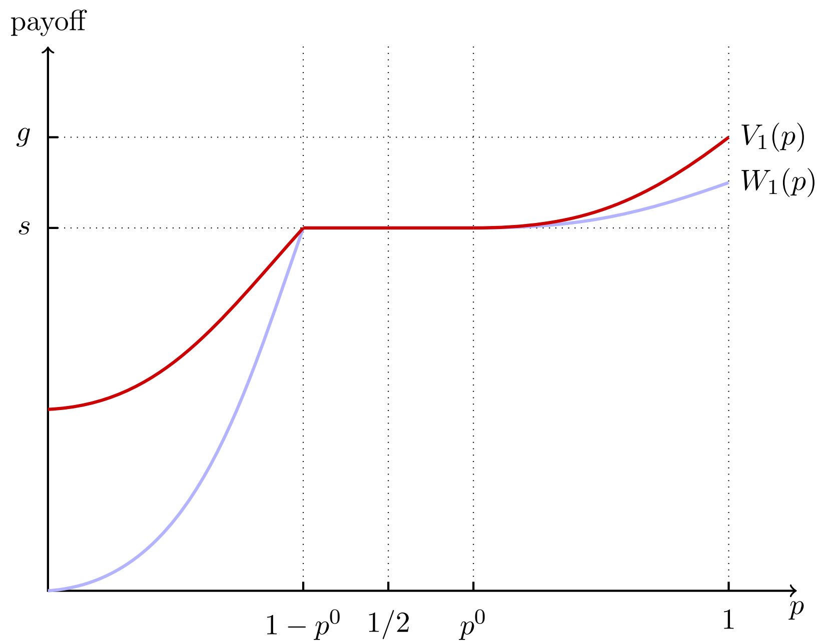


FIGURE 12
PARTY 1'S PAYOFF FUNCTION IN THE MPE OF THE LOW-STAKE CASE
(parameter values: $g/s = 1.2, r = 0.05, \lambda = 0.1, \xi = 0.05$)

nontrivial strategic interaction between the parties, so the equilibrium strategy profile would differ from that of the single decision-maker's problem. There are two types of equilibria in the high-stake case: one emerges in the case where the election frequency is low and the other emerges in the frequent elections case. Both types of equilibria show inefficient outcomes (from the voter's perspective), but the underlying forces for the inefficiency is different in each type of equilibrium.

Proposition 14. *There exists $\alpha_3(\chi)$ such that if $\frac{g}{s} > \alpha_3$, there exists a unique $p^* < \frac{1}{2}$ such that a strategy profile in Σ^* is an MPE if and only if* $\sigma_1^{-1}(R_1) = (p^*, 1]$ and $\sigma_2^{-1}(R_2) = [0, 1-p^*)$. *Moreover, $\alpha_3(\chi)$ is increasing in χ.*

In this equilibrium, each party chooses the reform policy even when the belief is unfavorable to its preferred reform policy. Hence failure to find a breakthrough eventually leads

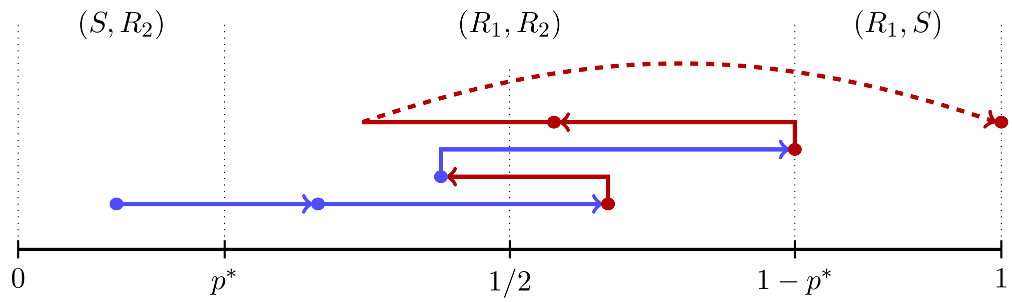
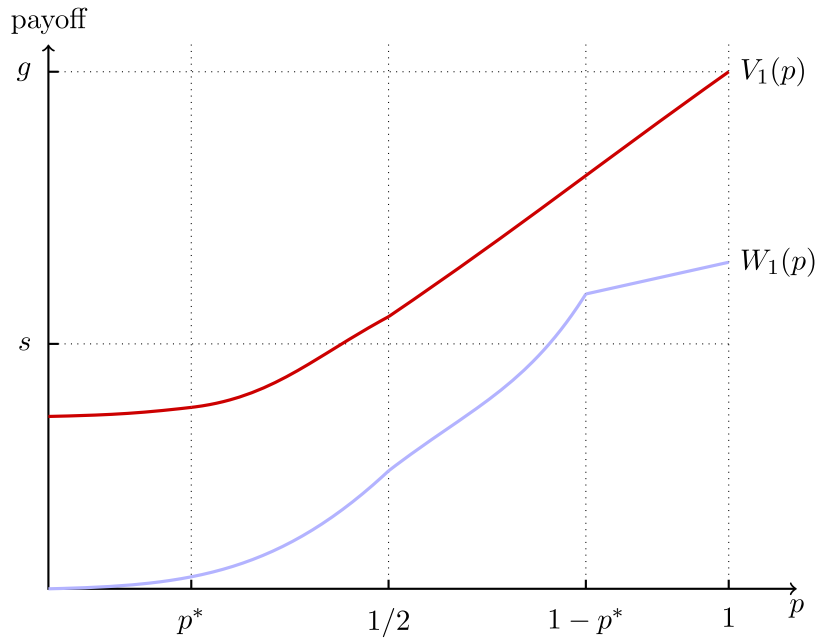


Figure 13
An equilibrium outcome path in the infrequent election case

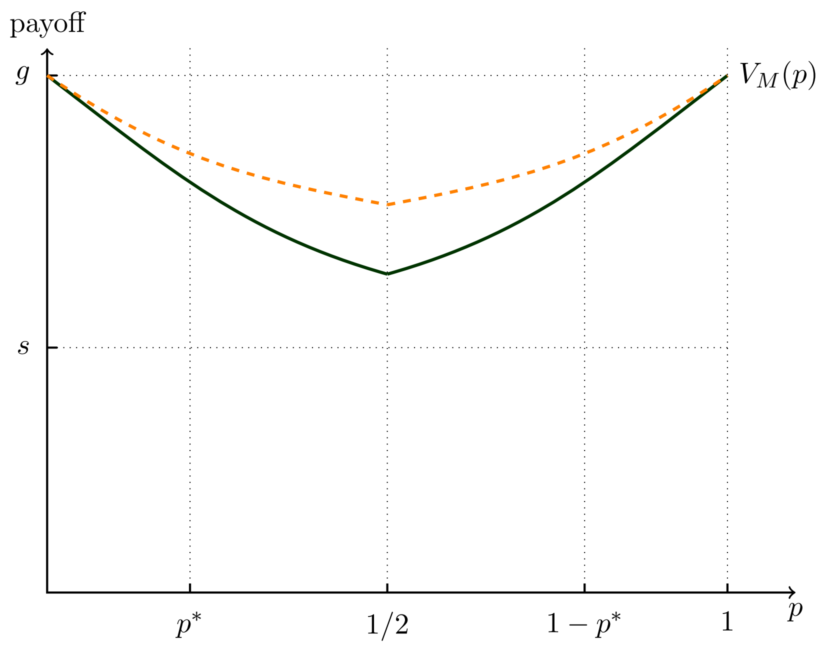
to political turnover. Since the voter always elects the party that would conduct one of the reform policies, (with probability one) there is a breakthrough in the reform policy in finite time; hence, the uncertainty is resolved.

Figure 13 describes an equilibrium outcome path of the infrequent election case in terms of belief dynamics. The dark red line (bright blue line) represents the belief dynamic when party 1 (party 2) is a ruling party, and a circle represents an election. In this case, the prior belief p_0 is less than a half, so initially the median voter elects party 2, which chooses its preferred reform policy (R_2). Once the belief goes above a half, the voter chooses party 1 at the election. There will be successive political turnovers until one of the parties receives a breakthrough from its preferred reform policy.

Figure 14 describes party 1's payoff functions (upper panel) and the median voter's payoff function (lower panel) in the equilibrium in Theorem 14. In the lower panel, the green line represents the median voter's expected payoff function $V_M(p)$. The dashed yellow line is the median voter's payoff under no political agency problem, that is, the payoff when he could choose the policy by himself. It turns out that for any $p \in (0, 1)$, the voter's expected payoff is strictly less than the payoff with no agency problem. In the equilibrium of the



(A) PARTY 1'S PAYOFF FUNCTION



(B) MEDIAN VOTER'S PAYOFF FUNCTION

FIGURE 14
EQUILIBRIUM BEHAVIOR OF THE MPE IN THE INFREQUENT ELECTION CASE
(parameter values: $g/s = 2.2, r = 0.05, \lambda = 0.1, \xi = 0.02$)

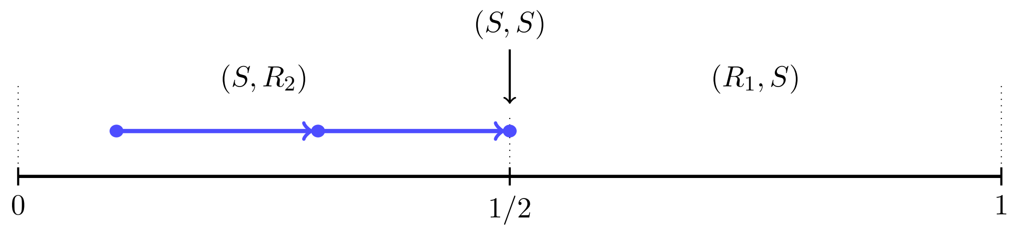


FIGURE 15
AN EQUILIBRIUM OUTCOME PATH IN THE FREQUENT ELECTION CASE

infrequent election case, the inefficiency comes from the suboptimal choice of a risky policy by the ruling party.

17.3 High stake case: Frequent elections

If the election is frequent, there exists another type of inefficiency. In the equilibrium of the frequent election case, the incumbent stops experimentation too early because the imminent election increases the incumbent's potential loss of power if it undertakes risky reform. The degree of inefficiency is large enough that too frequent elections are worse for the median voter than a dictatorship.

In the frequent election case, there exist three types of equilibria that appear in the different range of parameters. However, all types of equilibria share the common feature that the expected length of experimentation is shorter than the efficient level, and uncertainty is not resolved with positive probability.

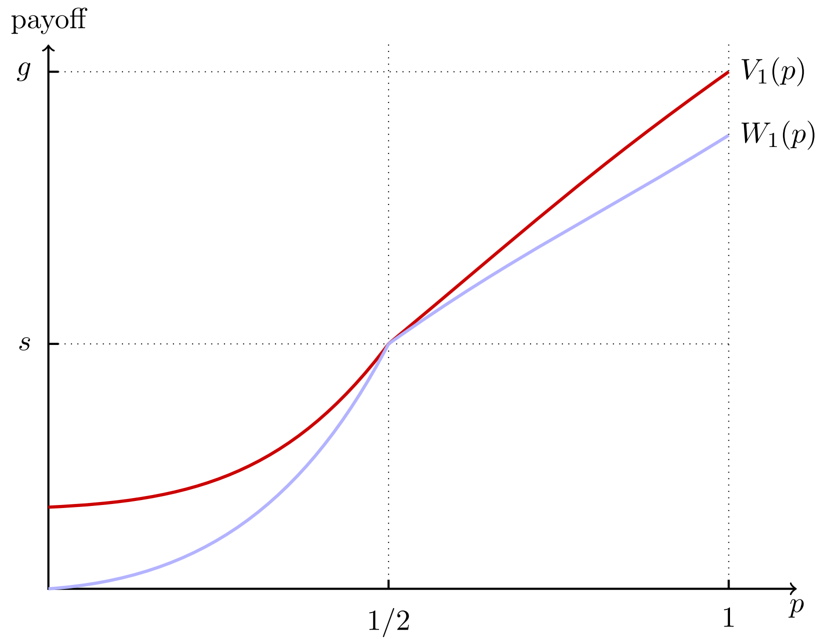
Proposition 15. *There exists $\alpha_1(\chi)$ such that for $\frac{g}{s} \in [\alpha_{0,}\alpha_1]$, a strategy profile in Σ^* is an MPE if and only if $\sigma_1^{-1}(R_1) = (\frac{1}{2}, 1]$ and $\sigma_2^{-1}(R_2) = [0, \frac{1}{2})$. Furthermore, $\alpha_1(\chi)$ is increasing in χ.*

Figure 15 describes a possible belief path in the equilibrium of Theorem 15. In this equilibrium, each incumbent initially chooses its preferred reform policy. But if a breakthrough has not been discovered until the belief reaches $1/2$, then the incumbent stops experimentation and switches to the safe policy. Since the opponent party would also choose the safe policy at $p = 1/2$, replacing the incumbent does not help in terms of more reform policy. This implies that for any $p \in (0, 1)$, the uncertainty is not resolved with positive probability.

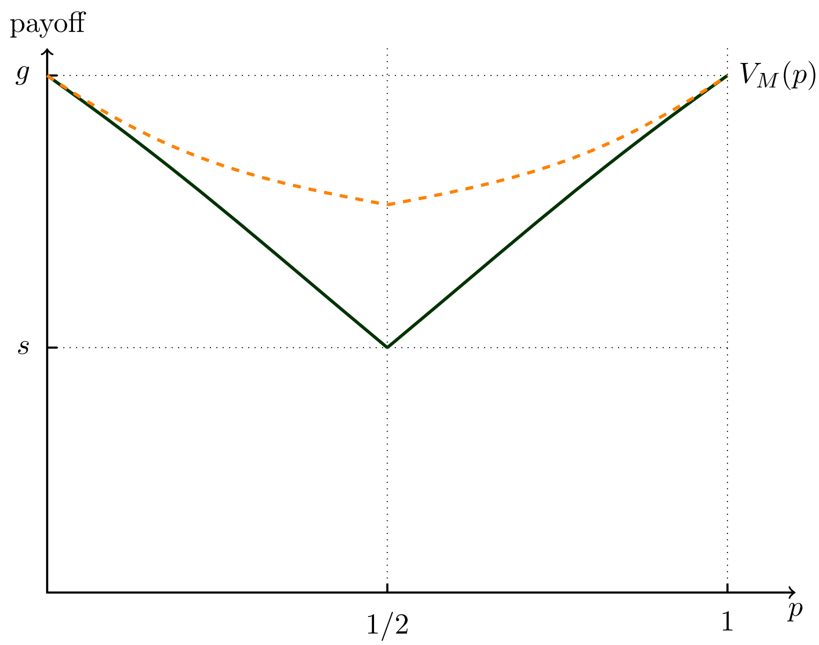
Figure 16 describes party 1's payoff function (upper panel) and the median voter's payoff function (lower panel) in the equilibrium in Theorem 15. Note that in the upper panel, $V_1(p)$ and $W_1(p)$ have the same value at $p = 1/2$ as both parties play a safe policy at that point. In the lower panel, the median voter's expected payoff function $V_M(p)$ (green line) hits the value s at the belief $1/2$ as there is no experimentation at that point. Similar to Figure 14, the degree of inefficiency is captured as the distance between the dashed yellow line and the green line. In the equilibrium of the frequent election case, the inefficiency comes from underinvestment in the reform policy. Note that all parameter values used for Figures 14 and 16 are the same except the election frequency, and that the voter's expected payoff function is lower in the frequent election case (Figure 16). In the next subsection, I will discuss more about the relationship with the election frequency and the voter's welfare.

There are other types of equilibria in the frequent election case depending on the parameter values. But all share the same feature that the experimentation stops at $p = 1/2$.

Proposition 16. *There exists $\alpha_2(\chi)$ such that for $\frac{g}{s} \in (\alpha_1,\alpha_2]$, and there exists a unique pair $p_a, p_b (p_a < p_b \leq \frac{1}{2})$ such that a strategy profile in Σ^* is anMPE if and only if $\sigma_1^{-1}(R_1) = (p_a, p_b] \cup (\frac{1}{2}, 1]$, $\sigma_2^{-1}(R_2) = [0, \frac{1}{2}) \cup [1 - p_b, 1 - p_a)$. Moreover, $\alpha_2(\chi) < \alpha_0(1 + \chi)$ and $\alpha_2(\chi)$ is increasing in χ.*



(A) party 1's payoff function



(B) median voter's payoff function

Figure 16
Equilibrium behavior of the MPE in the frequent election case
(parameter values: $g/s = 2.2, r = 0.05, \lambda = 0.1, \xi = 0.2$)

In this type of equilibrium, the belief dynamic is the same as that of the equilibrium in Theorem 3 (except that it can be different until the first election). Moreover, the median voter's value function $V_M(p)$ is the same.

Finally, there exists an asymmetric equilibrium of the model:

Proposition 17. *For $\frac{g}{s} \in (\alpha_2, \alpha_3)$, there exists an asymmetric equilibrium where*

$$
\begin{aligned}
k_1^{-1}(1) =& (\hat{p}_1, \frac{1}{2}) \cup (\frac{1}{2}, 1], \\
k_2^{-1}(1) =& [0, p_2^*), \\
\sigma_M(p,i) =& \begin{cases} 1 & \text{if } p > \frac{1}{2} \text{ or } \{p = \frac{1}{2}\} \cap \{i = 1\}, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}
$$

17.4 Median voter's welfare and optimal election frequency

The equilibrium analysis in the previous subsection suggests that there is an optimal frequency of elections (from the voter's perspective) that trades off the two types of inefficiencies.

Proposition 18. *Suppose $g/s > \alpha_0$. Then for any $p \in (0,1)$,*

1. *if the parameter values are in the 'infrequent elections' range, $V_M(p;\xi)$ is increasing in the election frequency (ξ);*

2. *$V_M(p)$ in the MPE of the frequent election case is smaller than the one in the MPE of the infrequent election case;*

3. *$V_M(p)$ in the MPE of the frequent election case is smaller than the one in the dictatorship case ($\xi = 0$).*

If the parameter values are in the infrequent elections range, the degree of inefficiency decreases as election frequency increases. However, too frequent elections would result in the worst outcome from the perspective of the median voter, as the parties stop experimentation in the equilibrium of the frequent election case. In fact, the median voter's expected payoff in the frequent election case is lower than the one where there is no election, as the length of experimentation is shorter.

18 Discussions

18.1 Incumbency advantage

For the frequent election case, I conjecture that that efficiency can be restored by giving an advantage to the incumbent in the election.

Conjecture 1. *Suppose $\frac{g}{s} \in [\alpha_0, \alpha_1]$. Then there exists $\epsilon_1, \epsilon_2 > 0$ such that the following strategy profile is an MPE:*

1. On the equilibrium path, party i plays cutoff strategy with cutoff $\frac{1}{2} + \epsilon_1 \cdot (-1)^i$, and the median voter plays

$$\sigma_M(p,i) = \begin{cases} 1 & \text{if } i = 1, p \geq \frac{1}{2} - \epsilon_1 \text{ or } i = 2, p > \frac{1}{2} + \epsilon_1 \\ 0 & \text{otherwise.} \end{cases}$$

2. If the median voter deviates, then the agents play a Markovian profile with $\sigma_1^{-1}(R_1) = (\frac{1}{2}, 1]$ and $\sigma_2^{-1}(R_2) = [0, \frac{1}{2})$.

3. If the party i deviates, then the agents play agents play a Markovian profile with

$\sigma_1^{-1}(R_1) = (\frac{1}{2} + \epsilon_2 \cdot (-1)^{i-1}, 1]$, $\sigma_2^{-1}(R_2) = [0, \frac{1}{2} + \epsilon_2 \cdot (-1)^{i-1})$, and $\sigma_M^{-1}(1) = (\frac{1}{2} + \epsilon_2 \cdot (-1)^{i-1}, 1]$.

The above equilibrium is non-Markovian where the voter chooses the incumbent at the neighbor of $p = 1/2$. By giving advantage to the incumbent, the voter can induce each party to experiment and induce endogenous political turnover. I conjecture that the above profile is still an equilibrium even if the election frequency is arbitrarily high, so the median voter can approximately achieve first-best.

In the incumbency advantage equilibrium, the voter is more generous to the incumbent in the sense that he may reelect the incumbent even when its preferred risky alternative is less promising than the opposite one. Knowing that, the incumbent experiments aggressively with its preferred risky policy even under frequent elections. Therefore, the incumbency advantage strategy can introduce frequent switches of power without causing the cessation of the experimentation with the risky alternatives, which is optimal from the voter's perspective. This result provides a normative argument for the incumbency advantage and contributes to previous positive arguments about the incumbency advantage.[30]

18.2 Time in power and electability

The equilibrium outcome in the infrequent election case in Figure 13 suggests that there is a correlation between the time in power and the electability of the incumbent. More formally, let T_0 be the length of time in which the current ruling party has been in power, and let T_{-k} be the length of time in which the kth previous ruling party had been in power. Furthermore, let $\tilde{\pi}$ be the probability that the current ruling party will be reelected. Then

[30] For the positive argument of incumbency advantage, see Samuelson (1987) and Ashworth and de Mesquita (2008).

I conjecture that under the equilibrium in the infrequent election case, if there has been no breakthrough, $\tilde{\pi}$ is decreasing (increasing) in T_{-k} for k even (odd).

19 Conclusion

In this paper, I study a continuous-time game between two political parties with heterogeneous preferences and a median voter. At each election, the voter chooses a party to which he gives power until the next election. Then the incumbent chooses a policy from among a safe alternative with known payoffs or two risky ones with initially unknown expected payoffs. I show that while infrequent elections are surely bad for the median voter, too frequent elections can also make him strictly worse off. When the election frequency is low, a standard agency problem arises and the incumbent party experiments with its preferred reform policy even if its outlook is not promising. On the other hand, when the election frequency is too high, in equilibrium the incumbent stops experimentation too early because the imminent election increases the incumbent's potential loss of power if it undertakes risky reform. The degree of inefficiency is large enough that too frequent elections are worse for the median voter than a dictatorship. There is an optimal frequency of elections (from the voter's perspective) that trades off the two types of inefficiencies.

Chapter IV

Appendix to Chapter 1

A Preliminaries

In this section I provide basic results which help to prove the results of the paper. First, I state differential equations which describe the dynamics of the buyers' beliefs and the reservation prices of the seller. Then I provide a detailed construction method for the equilibria described in Section 3. Proofs for the propositions of the paper are given in Section B.

A.1 Belief Dynamics

Let $m_z(t)(z = g, u, b)$ be the probability that the seller is type z *and* he is still available at time t. Then the buyers' beliefs about the seller's type $\phi_z(t)$ can be written as $\phi_z(t) = \frac{m_z(t)}{m_g(t)+m_u(t)+m_b(t)}$. Similarly, the beliefs about the quality $q(t)$ and the confidence $\beta(t)$ can be written as functions of $m_z(t)$, which are given by

$$q(t) = \frac{m_g(t) + m_u(t)q_0}{m_g(t) + m_u(t) + m_b(t)},$$
$$\beta(t) = \frac{m_u(t)}{m_u(t) + m_b(t)}.$$

Later it is shown that the evolution of $m_z(t)$ is given by a simple form of differential equations, which makes the equilibrium analysis easier.

By Lemma 2 the equilibrium offer of the buyer at time t is either $R_z(t)(z = g, u, b)$ or a losing offer. Suppose the buyer at time t offers $R_z(t)$ with probability $\sigma_{Bz}(t)$, and submits a losing offer with probability $\sigma_{B\chi}(t) = 1 - (\sigma_{Bg}(t) + \sigma_{Bu}(t) + \sigma_{Bb}(t))$. Then each $m_z(t)$ satisfies

$$
\begin{aligned}
m_g(t+dt) &= (m_g(t) + \rho q_o m_u(t)dt)(1 - \lambda\sigma_{Bg}(t)dt),\\
m_u(t+dt) &= m_u(t)(1-\rho dt)(1 - \lambda(\sigma_{Bg}(t) + \sigma_{Bu}(t))dt),\\
m_b(t+dt) &= (m_b(t) + \rho(1-q_o)m_u(t)dt)(1 - \lambda(\sigma_{Bg}(t) + \sigma_{Bu}(t) + \sigma_{Bb}(t))dt).
\end{aligned}
$$

Letting $dt \to 0$ and arranging yield

$$m_g'(t) = \rho q_0 m_u(t) - \lambda\sigma_{Bg}(t)m_g(t), \tag{15}$$

$$m_u'(t) = -(\rho + \lambda(\sigma_{Bg}(t) + \sigma_{Bu}(t)))m_u(t), \tag{16}$$

$$m_b'(t) = (1-q_0)\rho m_u(t) - \lambda(\sigma_{Bg}(t) + \sigma_{Bu}(t) + \sigma_{Bb}(t))m_b(t). \tag{17}$$

Solving (15)-(17), combined with boundary conditions $m_u(0) = 1$ and $m_g(0) = m_b(0) = 0$, gives the value of $m_z(t)$ at each t. Moreover, the evolution of the confidence $\beta(t)$ is given by

$$
\begin{aligned}
\beta'(t) &= \frac{m_b(t)m_u'(t) - m_u(t)m_b'(t)}{(m_u(t)+m_b(t))^2}\\
&= \beta(t)\cdot\left[-\rho(1-q_0\beta(t)) + \lambda\sigma_{Bb}(t)(1-\beta(t))\right].
\end{aligned} \tag{18}
$$

A.2 Price Dynamics

Suppose the buyer offers $R_z(t)$ with probability σ_z, and offers p_l with complementary probability. Then $R_u(t)$ and $R_b(t)$ satisfy the following recursions:

$$R_u(t) = rdt\alpha v(q_0) + (1-rdt)[\rho dt(q_0\alpha v_H + (1-q_0)R_b(t+dt)) +$$

$$(1-\rho dt)(\lambda\sigma_{Bg}(t)dt\alpha v_H + (1-\lambda\sigma_{Bg}(t)dt)R_u(t+dt)],$$

$$R_b(t) = rdt\alpha v(q_0) + (1-rdt)[\lambda\sigma_{Bg}(t)dt\alpha v_H + \lambda\sigma_{Bu}(t)dtR_u(t+dt) +$$

$$(1-\lambda(\sigma_{Bg}(t)+\sigma_{Bu}(t))dt)R_b(t+dt)].$$

Letting $dt \to 0$ and rearranging yield

$$R'_u(t) = r(R_u(t) - \alpha v(q_0)) - \rho B_I(t) - \lambda\sigma_{Bg}(t)(\alpha v_H - R_u(t)), \tag{19}$$

$$R'_b(t) = r(R_b(t) - \alpha v_L) - \lambda\sigma_{Bg}(t)(\alpha v_H - R_b(t)) - \lambda\sigma_{Bu}(t)(R_u(t) - R_b(t)), \tag{20}$$

where $B_I(t) = q_0\alpha v_H + (1-q_0)R_b(t) - R_u(t)$ is the seller's value of information. Solving (19) and (20) jointly with the boundary conditions yields the reservation price functions of each type.

Recall that $B(t) = \frac{R_u(t)-R_b(t)}{v(q_0)-R_b(t)}$ is the function used to determine the optimality of the buyer between targeting the uninformed and the bad type (equation 3). Then the evolution of $B(t)$ is given by

$$\begin{aligned} B'(t) &= \frac{v(q_0)(R'_u(t) - R'_b(t)) - R_b(t)R'_u(t) + R_u(t)R'_b(t)}{(v(q_0) - R_b(t))^2}, \\ &= B(t) \cdot \left[\rho F_\rho(t) + rF_r(t) + \lambda\sigma_{Bu}(t)(1 - B(t))\right], \end{aligned} \tag{21}$$

where $F_\rho(t) = \frac{R_u(t)-q_0\alpha v_H-(1-q_0)R_b(t)}{R_u(t)-R_b(t)} \leq 0$ and $F_r(t) = \frac{B(t)(v(q_0)-\alpha v_L)-\alpha q_0(v_H-v_L)}{R_u(t)-R_b(t)}$.

A.3 Equilibrium Construction

In this subsection, I provide a complete description of the equilibrium profile in Section 3. In the Section B I prove the existence of the equilibrium as well as characterization result.

A.3.1 Slow-learning Case

The equilibrium behavior in the second phase is analyzed in the main text (Subsection 3.1).

Belief dynamics in the first phase is as follows. Since the buyers target the uninformed with probability one for any time between zero and t, each $m_z(t)$ is given by

$$
\begin{aligned}
m_g(t) &= \frac{\rho}{\lambda+\rho} q_0 (1 - e^{-(\lambda+\rho)t}) \\
m_u(t) &= e^{-(\lambda+\rho)t} \\
m_b(t) &= (1-q_0) e^{-\lambda t} (1 - e^{-\rho t}),
\end{aligned}
$$

therefore $q(\hat{t})$ and $\beta(\hat{t})$ are

$$
q(t) = \frac{\frac{\rho}{\lambda+\rho} q_0 + \frac{\lambda}{\lambda+\rho} q_0 e^{-(\lambda+\rho)t}}{\frac{\rho}{\lambda+\rho} q_0 + \frac{\lambda}{\lambda+\rho} q_0 e^{-(\lambda+\rho)t} + (1-q_0) e^{-\lambda t}}, \tag{22}
$$

$$
\beta(t) = \frac{e^{-\rho t}}{q_0 e^{-\rho t} + (1-q_0)}. \tag{23}
$$

It remains to analyze the price dynamics in the first phase. This can be done by solving (1) and (2) jointly, which yields

$$
\begin{pmatrix} R_u(t) \\ R_b(t) \end{pmatrix} = C_1 \begin{pmatrix} D_1 \\ 2\lambda \end{pmatrix} e^{\gamma_1 t} + C_2 \begin{pmatrix} D_2 \\ 2\lambda \end{pmatrix} e^{\gamma_2 t} + \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}, \tag{24}
$$

where C_1, C_2 are integration constants, $X = (\lambda+\rho)^2 - 4\lambda\rho q_0$, $\gamma_1 = \frac{2r+\lambda+\rho+\sqrt{X}}{2}$, $\gamma_2 = \gamma_1 - \sqrt{X}$, and

$$\begin{pmatrix} Z_1 \\ \\ Z_2 \end{pmatrix} = \frac{1}{r(r+\lambda+\rho)+\rho\lambda q_0} \begin{pmatrix} r(r+\lambda+\rho)\alpha v(q_0) + \rho\lambda q_0 \alpha v_H \\ \\ r(r+\lambda+\rho)\alpha v(q_0) + \rho\lambda q_0 \alpha v_H - r(r+\rho) q_0 \alpha (v_H - v_L) \end{pmatrix}.$$

Note that $\gamma_1 > \gamma_2 > 0$, and $D_1 = \lambda - \rho - \sqrt{X} < 0, D_2 = \lambda - \rho + \sqrt{X} > 0$.

The equilibrium is constructed by the following steps:

1. From the condition $q(t^*) = q^*$, the threshold time t^* is uniquely determined from (22), which is

$$\frac{\rho}{\rho+\lambda} e^{\lambda t^*} + \frac{\lambda}{\rho+\lambda} e^{-\rho t^*} = C, \tag{25}$$

   where $C = \frac{q^*}{1-q^*} \cdot \frac{1-q_0}{q_0} > 1$.

2. Calculate $\beta(t^*)$ from equation (23) and calculate $R_u^* = \beta(t^*) v(q_0) + (1 - \beta(t^*)) v_L$.

3. Determine unique value of $\hat{\sigma}$ from equations (4) and (5),

$$\hat{\sigma} = \frac{r}{\lambda} \cdot \frac{R_b^* - \alpha v_L}{\alpha v_H - v_L}.$$

4. Determine $R_u(t)$ and $R_b(t)$ in the first phase, by putting boundary conditions

$$R_u(t^*) = R_u^*, \;\; R_b(t^*) = R_b^*,$$

into (24) to get integration constants, which are given by

$$4\lambda\sqrt{X}\begin{pmatrix} e^{\gamma_1 t^*}C_1 \\ \\ e^{\gamma_2 t^*}C_2 \end{pmatrix} = \begin{pmatrix} -2\lambda \\ \\ 2\lambda \end{pmatrix}(R_u^* - Z_1) + \begin{pmatrix} D_2 \\ \\ -D_1 \end{pmatrix}(R_b^* - Z_2).$$

A.3.2 Fast-Learning Case

Belief evolution in the first phase is same as the slow-learning case, which is summarized by equations (22) and (23). In the second phase, each $m_z(t)$ satisfies

$$\begin{aligned} m_g'(t) &= \rho q_0 m_u(t), \\ m_u'(t) &= -\rho m_u(t), \\ m_b'(t) &= -\lambda m_b(t) + \rho(1-q_0)m_u(t). \end{aligned}$$

Solving with the boundary condition at t_1^*, we have

$$\begin{aligned} m_g(t) &= q_0\left[\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho}e^{-(\lambda+\rho)t_1^*} - e^{-(\lambda t_1^*+\rho t)}\right], \\ m_u(t) &= e^{-(\lambda t_1^*+\rho t)}, \\ m_b(t) &= (1-q_0)\left[(1-\frac{\lambda}{\lambda-\rho}e^{-\rho t_1^*})e^{-\lambda t} + \frac{\rho}{\lambda-\rho}e^{-(\lambda t_1^*+\rho t)}\right], \end{aligned}$$

hence

$$q(t) = \frac{q_0\left(\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho}e^{-(\lambda+\rho)t_1^*}\right)}{q_0\left(\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho}e^{-(\lambda+\rho)t_1^*}\right) + (1-q_0)\left(e^{-\lambda t} + \frac{\lambda}{\lambda-\rho}(e^{-(\lambda t_1^*+\rho t)} - e^{-(\lambda t+\rho t_1^*)})\right)}, \quad (26)$$

and

$$\beta(t) = \frac{e^{-(\lambda t_1^* + \rho t)}}{e^{-(\lambda t_1^* + \rho t)} + (1-q_0)\left((1 - \frac{\lambda}{\lambda-\rho}e^{-\rho t_1^*})e^{-\lambda t} + \frac{\rho}{\lambda-\rho}e^{-(\lambda t_1^* + \rho t)}\right)}. \tag{27}$$

Price dynamics are as follows. In the third (and final) phase, the reservation price is determined by (4) and (5), same as the slow-learning case. In the second phase, reservation prices satisfy

$$R_u'(t) = r(R_u(t) - \alpha v(q_0)) + \rho(R_u(t) - q_0\alpha v_H - (1-q_0)R_b(t)),$$

$$R_b'(t) = r(R_b(t) - \alpha v_L).$$

Solving with the boundary conditions $R_u(t_2^*) = R_u^*, R_b(t_2^*) = R_b^*$ yields

$$R_b(t) = \alpha v_L + (R_b^* - \alpha v_L)e^{r(t-t_2^*)}, \tag{28}$$

$$R_u(t) = q_0\alpha v_H + (1-q_0)R_b(t). \tag{29}$$

Note that the reservation value of the uninformed is the expectation of those of the good type and the bad type, as the buyer's equilibrium offer $R_b(t)$ gives the seller zero value of information. Last, in the first phase, the reservation values satisfy the same differential equations in the slow-learning case, hence their functional forms are given by (24), but the boundary conditions are different ($R_b(t_1^*)$ and $R_u(t_1^*)$ from the above equations (28) and (29)).

Then the equilibrium is constructed by the following steps:

1. The condition $q(t_2^*) = q^*$, yields

$$\frac{\frac{\rho}{\lambda+\rho} + \frac{\lambda}{\lambda+\rho} e^{-(\lambda+\rho)t_1^*}}{e^{-\lambda t_2^*} + \frac{\lambda}{\lambda-\rho} e^{-(\lambda+\rho)t_1^*} (e^{-\rho(t_2^*-t_1^*)} - e^{-\lambda(t_2^*-t_1^*)})} = C, \tag{30}$$

where $C = \frac{q^*}{1-q^*} \cdot \frac{1-q_0}{q_0} > 1$.

2. From the equations (23), (28) and (29), the buyer's indifference condition at t_1^*, $\beta(t_1^*) = B(t_1^*)$, is given by

$$e^{-r(t_2^*-t_1^*)} = \frac{(1-\alpha)\left\{\frac{q_0}{1-q_0} v_H + v_L\right\} + \alpha(v_H - v_L) q_0 (1 - e^{\rho t_1^*})}{(1-\alpha) v_L (1 + q_0(1 - e^{\rho t_1^*}))}. \tag{31}$$

Equations (30) and (31) jointly give the unique values of t_1^* and t_2^*.

3. From the optimality condition $R_b^* = v_L$, R_u^* and $\hat{\sigma}$ are given by

$$R_u^* = q_0 \alpha v_H + (1 - q_0) v_L.$$
$$\hat{\sigma} = \frac{r}{\lambda} \cdot \frac{(1-\alpha) v_L}{\alpha v_H - v_L}.$$

4. Determine $R_u(t)$ and $R_b(t)$ in the second phase, from (28)-(29) and the boundary conditions at t_2^*. They are given by

$$R_b(t) = \alpha v_L + (1-\alpha) v_L e^{r(t-t_2^*)}, \tag{32}$$

$$R_u(t) = q_0 \alpha v_H + (1 - q_0) R_b(t). \tag{33}$$

5. Determine $R_u(t)$ and $R_b(t)$ in the first phase, from (24) and the boundary conditions

at t_1^*.

A.4 Calculation of the Trade Surplus

Recall that $f_\theta(t)$ is the probability distribution of trade of the quality-θ good over time. Let $F_\theta(t)$ be the cdf of $f_\theta(t)$. Then

$$\begin{aligned}
F_H(t) &= 1 - \frac{m_g(t) + q_0 m_u(t)}{q_0}, \\
F_L(t) &= 1 - \frac{(1-q_0)m_u(t) + m_b(t)}{1-q_0},
\end{aligned}$$

hence

$$\begin{aligned}
f_H(t) &= -\frac{m_g'(t) + q_0 m_u'(t)}{q_0}, \\
f_L(t) &= -\frac{(1-q_0)m_u'(t) + m_b'(t)}{1-q_0}.
\end{aligned}$$

Recall that S_θ is the trade surplus when the quality of the good is θ. Since $S_\theta = (1-\alpha)v_\theta \int_0^\infty e^{-rt} f_\theta(t)dt$, the following can be shown using the results in the previous subsection:

▷ In the equilibrium under the slow-learning case,

$$\begin{aligned}
\frac{S_H}{(1-\alpha)v_H} &= \frac{\lambda}{r+\rho+\lambda}(1 - e^{-(r+\rho+\lambda)t^*}) + \frac{\lambda\sigma}{r+\lambda\sigma} e^{-rt^*} \left(\frac{\rho + \lambda e^{-(\rho+\lambda)t^*}}{\rho+\lambda} \right), \\
\frac{S_L}{(1-\alpha)v_L} &= \frac{\lambda}{r+\lambda}(1 - e^{-(r+\lambda)t^*}) + \frac{\lambda\sigma}{r+\lambda\sigma} e^{-(r+\lambda)t^*}.
\end{aligned}$$

▷ In the equilibrium under the fast-learning case,

$$
\begin{aligned}
\frac{S_H}{(1-\alpha)v_H} &= \frac{\lambda}{r+\rho+\lambda}(1-e^{-(r+\rho+\lambda)t_1^*}) + \frac{\lambda\sigma}{r+\lambda\sigma}e^{-rt_2^*}\left(\frac{\rho+\lambda e^{-(\rho+\lambda)t_1^*}}{\rho+\lambda}\right), \\
\frac{S_L}{(1-\alpha)v_L} &= \frac{\lambda}{r+\lambda}(1-e^{-(r+\lambda)t_1^*}) \\
&\quad + \lambda\left(1-\frac{\lambda}{\lambda-\rho}e^{-\rho t_1^*}\right)\frac{1}{r+\lambda}\left(e^{-(r+\lambda)t_1^*}-e^{-(r+\lambda)t_2^*}\right) \\
&\quad + \lambda\frac{\rho}{\lambda-\rho}e^{-\lambda t_1^*}\frac{1}{r+\rho}\left(e^{-(r+\rho)t_1^*}-e^{-(r+\rho)t_2^*}\right) \\
&\quad + \frac{\lambda\sigma}{r+\lambda\sigma}e^{-rt_2^*}\left[e^{-\lambda t_2^*}+\frac{\lambda}{\lambda-\rho}\left(e^{-(\lambda t_1^*+\rho t_2^*)}-e^{-(\lambda t_2^*+\rho t_1^*)}\right)\right].
\end{aligned}
$$

B Proofs

B.1 Proof of Propositions 1-4

Here I prove the optimality of the two-phase and three-phase equilibria, and provide the characterization result for both slow- and fast-learning case. I start with the characterization of the final phase, which is common for both cases. Then I analyze characterization for the cases with slow and fast learning, respectively.

B.1.1 Equilibrium Behavior in the Final Phase

Recall that $\beta(t) = \frac{\phi_u(t)}{\phi_u(t)+\phi_b(t)}$ is the buyers' confidence at time t. The following lemma summarizes the results derived in this subsection:

Lemma 5. *In equilibrium, there exists $t^* < \infty$ and $\sigma : \{t : t \geq t^*\} \to [0,1]$ such that the equilibrium behavior after t^* is the following:*

▷ *the buyer at time t submits the trade-ending offer αv_H with probability $\sigma(t)$ and submits* *a losing offer $p_\ell \le R_b(t)$ with probability $1-\sigma(t)$.*

▷ *the seller's reservation value:*

$$R_b(t) = \alpha v_L + \alpha(v_H - v_L)\int_t^\infty e^{-r(\tilde{t}-t)} d(1 - e^{-\int_t^{\tilde{t}} \lambda\sigma(s)ds}),$$

$$R_u(t) = q_0 \alpha v_H + (1-q_0) R_b(t);$$

▷ *the belief $q(t) = q^*$ for $t \ge t^*$;*

▷ *the buyer's offer $\sigma(t)$ must satisfy*

$$R_b(t) \ge v_L, \tag{34}$$

$$R_u(t) \ge \beta(t) v(q_0) + (1-\beta(t)) v_L, \tag{35}$$

for any $t \ge t^*$; at least one of the above conditions binds at $t = t^*$.

Fix an equilibrium. Let $t^* = \inf\{t : q(t) \ge q^*\}$ be the time when the buyer's unconditional belief reaches q^* for the first time.

Step 1 t^* is finite.

Proof. Suppose not, i.e., $q(t) < q^*$ for any t. Then the buyers never offer αv_H as it yields a negative payoff. Then the similar argument as Lemma 3 shows that the buyers never offer more than $\alpha v(q_0)$, and hence $R_u(t) = \alpha v(q_0)$ for all t.

There must exist finite $\bar{t}$ such that for all $t > \bar{t}$, offering $R_u(t)$ gives negative payoff (if not, there must be a lots of agreement with type-B seller, hence $q(t) > q^*$ for t large). Then after $\bar{t}$, trade is occurred only with type-B seller, so $R_b(t) = \alpha v_L$ for any $t > \bar{t}$. Since this is profitable for the buyer, the trade will occur and eventually $q(t) > q^*$, contradiction. $\square$

Step 2 For any $t \geq t^*$, trade occurs only at $R_g^* = \alpha v_H$. Therefore, $q(t) = q^*$ for any $t \geq t^*$.

Proof. Suppose not, then there exists t_1 and t_2 such that $t^* \leq t_1 < t_2 < \infty$ and the trade happens at $p \neq \alpha v_H$ only if $t \in [t_1, t_2]$.[31]

Then for any $t > t_1$, the buyers' belief $q(t)$ is greater than q^*, so offering αv_H yields positive payoff. Hence the buyer after t_1 never submits a losing offer. That implies the buyer after t_2 offers αv_H for sure. Then $R_u(t)$ and $R_b(t)$ as t approaches to t_2 are given by

$$\lim_{t \to t_2} R_u(t) \to \frac{r}{\lambda + r}\alpha v(q_0) + \frac{\lambda}{\lambda + r}\alpha v_H,$$
$$\lim_{t \to t_2} R_b(t) \to \frac{r}{\lambda + r}\alpha v_L + \frac{\lambda}{\lambda + r}\alpha v_H.$$

However, submitting either offer is suboptimal for the buyer, since under Assumption 1,

$$v_L - \left(\frac{r}{\lambda + r}\alpha v_L + \frac{\lambda}{\lambda + r}\alpha v_H\right) < 0,$$
$$\phi_u(t)\left[v(q_0) - \left(\frac{r}{\lambda + r}\alpha v(q_0) + \frac{\lambda}{\lambda + r}\alpha v_H\right)\right] + \phi_b(t)\left[v_L - \left(\frac{r}{\lambda + r}\alpha v(q_0) + \frac{\lambda}{\lambda + r}\alpha v_H\right)\right] < 0.$$

So if t is arbitrarily close to t_2, trade must happen only at αv_H, which contradicts to the definition of t_2. $\square$

[31] It must be the case that there exists finite t_2: suppose not. Then $q(t)$ converges to one as t goes to infinity, since no buyer submits a losing offer after t_1. Furthermore, since the speed of learning $\rho > 0$ is positive, the probability of the good type $\phi_g(t)$ converges to one as $t \to \infty$. However, if $\phi_g(t)$ is sufficiently close to one, expected payoff from targeting the uninformed or the bad type is arbitrarily small because there exist lower bounds for $R_u(t)$ and $R_b(t)$. Therefore, there exists $\hat{t} < \infty$ such that it is strictly optimal to offer $R_g(t) = \alpha v_H$ for $t > \hat{t}$, contradiction.

Step 3 $R_u(t)$ and $R_b(t)$ satisfy (34) and (35) for any $t \geq t^*$; at least one of the conditions binds at $t = t^*$.

Proof. By step 2, the buyers arrive at $t \geq t^*$ receives zero payoff. If either (34) or (35) is violated, then the buyer has a profitable deviation to target the low type or the uninformed, respectively.

Suppose that both (34) and (35) are strict at $t = t^*$. Then since $R_z(t)$ and $\phi_z(t)$ are continuous in t, there exists $\epsilon > 0$ such that offering $R_b(t)$ or $R_u(t)$ yields negative payoff to the buyer for all $t \in (t^* - \epsilon, t^*)$. But it contradicts to the definition of t^*. □

B.1.2 Equilibrium Before the Final Phase: Preliminary Observations

By the definition of t^*, offering $p = \alpha v_H$ at any $t < t^*$ yields negative payoff to the buyer, hence it is suboptimal. So the buyer either offers $R_u(t)$ to target the uninformed or offers $R_b(t)$ to target the bad type. Recall that the buyer receives more payoff by offering $R_u(t)$ than $R_b(t)$ if and only if

$$\beta(t) > B(t),$$

where $B(t) = \frac{R_u(t) - R_b(t)}{v(q_0) - R_b(t)}$.

The result in Step 3 implies that there are three cases at $t = t^*$:

1. (35) is binding, but (34) is not: if this is the case, then $R_u(t^*) = \beta(t^*)v(q_0) + (1 - \beta(t^*))v_L$ and $R_b(t^*) > v_L$. Moreover, $R_u(t^*) = q_0\alpha v_H + (1 - q_0)R_b(t^*)$ since the value

of information is zero in the final phase. Hence

$$\beta(t^*) = \frac{R_u(t^*) - v_L}{v(q_0) - v_L}$$
$$= \frac{q_0 \alpha v_H + (1-q_0)R_b(t^*) - v_L}{v(q_0) - v_L} > q^*.$$

Similar calculation shows that $B(t^*) > q^*$. On the other hand, $B(t^*) < \beta(t^*)$ since targeting the bad type is worse than targeting the uninformed. As a result, $\beta(t^*) > B(t^*) > q^*$.

2. both (34) and (35) are binding: then $R_u(t^*) = \beta(t^*)v(q_0) + (1-\beta(t^*))v_L$ and $R_b(t^*) = v_L$. Similar calculation shows that $\beta(t^*) = B(t^*) = q^*$.

3. (34) is binding, but (35) is not: in this case, we have $\beta(t^*) < B(t^*) = q^*$.

B.1.3 Slow-learning Case: Proof of Propositions 1 and 2

Lemma 6. *Fix an equilibrium. Suppose at time t^*, (35) is binding, but (34) is not. Then the buyers at any $t < t^*$ offer $R_u(t)$ for sure.*

Proof. First I show that the buyers at any $t < t^*$ do not target the bad type, that is $\sigma_{Bb}(t) = 0$ for any $t < t^*$. Suppose to the contrary that $\sigma_{Bb}(t) > 0$ for some $t < t^*$. Let $t^\dagger = \sup\{t < t^* : \sigma_{Bb}(t) > 0\}$. Then $t^\dagger < t^*$ because (34) does not bind at t^*. Since $R_b(t^\dagger) \leq v_L$ and $R_u(t) \leq q_0\alpha v_H + (1-q_0)R_b(t)$ for all t,

$$B(t^\dagger) = \frac{R_u(t) - R_b(t)}{v(q_0) - R_b(t)} \leq \frac{q_0(\alpha v_H - R_b(t))}{v(q_0) - R_b(t)} \leq q^*,$$

so it must be the case that $\beta(t^\dagger) \leq q^*$. On the other hand, $\beta(t^*) > q^*$ since (35) binds at t^*

while (34) does not bind. Moreover, since $\sigma_{Bb}(t) = 0$ for $t \in (t^\dagger, t^*]$, (18) implies that $\beta(t)$ is decreasing for $t \in (t^\dagger, t^*]$, leading to a contradiction.

Now it remains to show that any buyer at $t < t^*$ has no incentive to submit a losing offer. Define $\tilde{p}(t) = \beta(t)v(q_0) + (1-\beta(t))v_L$ be a expected value of traded good to the buyer then he targets the uninformed seller . Then submitting a losing offer is no worse than targeting the uninformed at time t if and only if $R_u(t) \geq \tilde{p}(t)$. I claim that $R_u(t) < \tilde{p}(t)$ for any $t < t^*$. From the previous argument,

$$R_u(t^*) = \tilde{p}(t^*) \geq q_0\alpha v_H + (1-q_0)v_L = q^* v(q_0) + (1-q^*)v_L.$$

Since $\beta'(t) = -\rho\beta(t)(1-q_0\beta(t))$ and $\beta(t^*) \geq q^*$ from the above equation,

$$\begin{aligned}\tilde{p}'(t) &= -\rho\beta(t)(1-q_0\beta(t)) \cdot q_0(v_H - v_L) \\ &\leq -\rho q_0(v_H - v_L)\min\{1-q_0, q^*(1-q_0q^*)\},\end{aligned}$$

and $\tilde{p}(t) > q^*v(q_0) + (1-q^*)v_L$. On the other hand, since $R_u'(t) = -\rho(q_0\alpha v_H + (1-q_0)R_b(t) - R_u(t))$, So either $R_u(t) \leq q^*v(q_0) + (1-q^*)v_L$ or

$$\begin{aligned}R_u'(t) &\geq -\rho q_0(\alpha v_H - R_u(t)) \\ &> -\rho q_0(1-q_0)(\alpha v_H - v_L) = -\rho q_0(v_H - v_L)q^*(1-q_0).\end{aligned}$$

Therefore, whenever $R_u(t) \geq q^*v(q_0) + (1-q^*)v_L$ it must be that $R_u'(t) > \tilde{p}'(t)$, leading to the desired result. $\square$

Let $z(t) = e^{-\rho t}$ and $z^* = z(t^*)$. Let $\kappa = \frac{\lambda}{\rho} = \frac{1}{\eta}$ be the inverse of the seller's effective learning speed. Consider a strategy profile in which the buyers target the uninformed for any $t < t^*$. Then the condition (25) can be rewritten as

$$z^*\kappa + (z^*)^{-\kappa} = C(1+\kappa), \tag{36}$$

where $C = \frac{q^*}{1-q^*}\frac{1-q_0}{q_0} > 1$. By the implicit function theorem, $\frac{\partial z^*}{\partial \kappa} > 0$.[32] Moreover, it can be shown that $\lim_{\kappa\to 0} z^* = 0$ and $\lim_{\kappa\to\infty} z^* = 1$.

On the other hand, equation (23) can be rewritten as

$$\beta(t) = \frac{z(t)}{q_0 z(t) + (1-q_0)}, \tag{37}$$

so $\beta(t^*) = \frac{z^*}{q_0 z^* + (1-q_0)} \equiv \beta^*$. Since β^* is increasing in z^*, there exists $\bar{\kappa}$ such that $\kappa \geq \bar{\kappa}$ if and only if $\beta(t^*) \geq q^*$.

Lemma 7. *(1) $\kappa \geq \bar{\kappa}$ if and only if there exists an equilibrium of the game where the buyers at $t < t^*$ target the uninformed for sure, and hence t^* is uniquely determined by equation (36).*

(2) There exists $\bar{r} > 0$ such that if $\kappa \geq \bar{\kappa}$ and $r < \bar{r}$, then in any equilibrium of the game,

[32] Let $F(z^*,\kappa) = z^*\kappa + (z^*)^{-\kappa} - C(1+\kappa)$. Then

$$\begin{aligned}\frac{\partial z^*}{\partial \kappa} &= -\frac{\frac{\partial F}{\partial \kappa}}{\frac{\partial F}{\partial z^*}}\\ &= \frac{z^* + (z^*)^{-\kappa}\log\frac{1}{z^*} - C}{\kappa((z^*)^{-\kappa-1} - 1)}\\ &= \frac{z*\kappa + (z^*)^{-\kappa}\log(z^*)^{-\kappa} - C\kappa}{\kappa^2((z^*)^{-\kappa-1} - 1)}\\ &= \frac{-(z^*)^{-\kappa}(1 - \log(z^*)^{-\kappa}) + C}{\kappa^2((z^*)^{-\kappa-1} - 1)}\end{aligned}$$

Since $x(1-\log x) < 1$ if $x \in (0,1)$, $\frac{\partial z^*}{\partial \kappa} > 0$.

buyers at $t < t^*$ target the uninformed for sure, and hence t^* is determined by equation (36).

Proof. (1) Consider a strategy profile in which the buyers at any $t < t^*$ target the uninformed seller. Then (35) must binds at t^*, that is,

$$R_u(t^*) = \beta(t^*)v(q_0) + (1 - \beta(t^*))v_L.$$

On the other hand, by Lemma 5, $R_u(t^*) = q_0\alpha v_H + (1-q_0)R_b(t^*)$. Then a simple calculation shows that $R_b(t^*) \geq v_L$ if and only if $\beta(t^*) \geq q^*$. Therefore, the incentive constraint for the bad type (34) is satisfied if and only if $\kappa \geq \bar{\kappa}$.

(2) Suppose to the contrary that there exists an equilibrium where $\sigma_{Bu}(t) < 1$ for some $t < t^*$. Then Step 3 in Subsection B.1.1 and Lemma 6 imply that (34) must bind at $t = t^*$, and hence $B(t^*) = q^*$. Moreover, proof of Lemma 6 implies that $\sigma_{Bb}(t) > 0$ for $t < t^*$. Let $\hat{t} = \inf\{t < t^* : \sigma_{Bb}(t) > 0\}$. Then since $R_b(\hat{t}) \leq v_L$ and $R_u(t) \leq q_0\alpha v_H + (1 - q_0)R_b(t)$ for all t,

$$B(\hat{t}) = \frac{R_u(\hat{t}) - R_b(\hat{t})}{v(q_0) - R_b(\hat{t})} \leq \frac{q_0(\alpha v_H - R_b(\hat{t}))}{v(q_0) - R_b(\hat{t})} \leq q^*,$$

and hence $\beta(\hat{t}) \leq B(\hat{t}) \leq q^*$. Furthermore, $\beta(\hat{t}) = B(\hat{t})$ by the following argument: Suppose to the contrary that $\beta(\hat{t}) < B(\hat{t})$. Then there exists $\epsilon > 0$ such that $\sigma_{B\chi}(t) = 1$ for any $t \in [\hat{t} - \epsilon, \hat{t})$. However, then from (20) $R_b(t)$ is strictly increasing $t \in [\hat{t} - \epsilon, \hat{t})$, so the buyer at $\hat{t} - \epsilon$ has a profitable deviation to offer $R_b(t)$, contradiction.

Therefore, it must be that there exists a time before $\hat{t}$ where the buyer submits a losing offer with positive probability, that is, $\sigma_{B\chi}(t) > 0$ for some $t < \hat{t}$ (if not, $q(\hat{t}) > q^*$ because $\kappa \geq \bar{\kappa}$, so it contradicts to the definition of t^*). Let $\tilde{t} = \sup\{t \leq \hat{t} : \sigma_{B\chi}(t) > 0\}$. Then the buyer at time $\tilde{t}$ must be indifferent between submitting a losing offer and targeting the uninformed seller, that is, $R_u(\tilde{t}) = \tilde{p}(\tilde{t}) = \beta(t)v(q_0)+(1-\beta(t))v_L$[33]. Moreover, the definition of $\tilde{t}$ implies that $\tilde{p}'(t) \geq R_u'(t)$. From the equation (18),

$$\begin{aligned}\tilde{p}'(\tilde{t}) &= q_o(v_H - v_L)\beta'(\tilde{t}) \\ &= -\rho q_0(v_H - v_L)\beta(\tilde{t})(1 - q_0\beta(\tilde{t})).\end{aligned}$$

On the other hand, using equation (19) and the condition $R_u(\tilde{t}) = \tilde{p}(\tilde{t})$, lower bound on $R_u'(\tilde{t})$ is given by

$$\begin{aligned}R_u'(\tilde{t}) &> -\rho(q_0\alpha v_H + (1-q_0)R_b(\tilde{t}) - R_u(\tilde{t})) \\ &> -\rho q_0(\alpha v_H - R_u(\tilde{t})) \\ &= -\rho q_0(\alpha v_H - \tilde{p}(\tilde{t})) \\ &= -\rho q_0(v_H - v_L)(q^* - \beta(\tilde{t})q_0).\end{aligned}$$

Simple calculation gives that $\tilde{p}'(\tilde{t}) \geq R_u'(\tilde{t})$ only if

$$\beta(\tilde{t}) \leq \beta^\dagger \equiv \frac{1 + q_0 - \sqrt{(1+q_0)^2 - 4q^*q_0}}{2q_0} \in (0, q^*).$$

Note that $\beta^\dagger$ is independent of the seller's discount rate r. Since $\beta(t)$ is decreasing for $t \in [0, \hat{t})$, it follows that $B(\hat{t}) = \beta(\hat{t}) \leq \beta^\dagger$. However, the price dynamics described in

[33]Suppose the contrary that $R_u(\tilde{t}) < \tilde{p}(\tilde{t})$. Then at $\tilde{t}$ the buyer must be indifferent between submitting a losing offer and targeting the bad type, that is, $R_b(\tilde{t}) = v_L$. Moreover, it must be that $\sigma_{B\chi}(t) = 1$ at $t \in (\tilde{t} - \epsilon, \tilde{t})$ for sufficiently small $\epsilon > 0$. But then the price dynamics described in Subsection A.2 implies that $R_b(t) < R_b(\tilde{t}) = v_L$ for $t \in (\tilde{t} - \epsilon, \tilde{t})$, so the buyers at $t \in (\tilde{t} - \epsilon, \tilde{t})$ are better off by targeting the bad type, a contradiction.

Subsection A.2 implies that there exists $\bar{r} > 0$ such that if $r < \bar{r}$, then the value of $\hat{t} - t^*$ must be sufficiently large to satisfy $B(\hat{t}) \leq \beta^\dagger$, so that the condition $q(t^*) = q^*$ is violated, leading to a contradiction. □

B.1.4 Fast-learning Case: Proof of Propositions 3 and 4

Now consider the case in which $\kappa \leq \bar{\kappa}$.

Lemma 8. *In equilibrium, there exists $t < t^*$ such that $\sigma_{Bb}(t) > 0$.*

Proof. Suppose not; that is, $\sigma_{Bb}(t) = 0$ for any $t < t^*$. Then from (18), $\beta(t)$ is given by

$$\beta(t) = \frac{z(t)}{q_0 z(t) + (1 - q_0)}.$$

Then by the definition of $\bar{\kappa}$, $\beta(t^*) < q^*$. Since $R_u(t^*) = \beta(t^*)v(q_0) + (1 - \beta(t^*))v_L = q_0 \alpha v_H + (1 - q_0)R_b(t^*)$, it must be that $R_b(t^*) < v_L$, which contradicts to Step 3. □

Let $\hat{t} < t^*$ be the first time in which the buyer offers $R_b(\hat{t})$ with positive probability, that is, $\hat{t} = \inf\{t < t^* : \sigma_{Bb}(t) > 0\}$. Then the proof of Lemma 7 implies that $\beta(\hat{t}) = B(\hat{t}) \leq q^*$, and hence that $\hat{t} > 0$.

Lemma 9. *There exists $\underline{\kappa} > 0$ such that if $\kappa < \underline{\kappa}$, for any fixed $x > 0$, if a strategy profile with $t^* - \hat{t} = x$ is an equilibrium, then*

(1) the buyers at $t \in (\hat{t}, t^*)$ offer $R_b(t)$ with probability one and the buyers at $t \in [0, \hat{t})$ offer $R_u(t)$ with probability one;

(3) the value of $\hat{t}$ (hence t^*) is uniquely determined.

Proof. (1) Let $\tilde{q} = q_0 \frac{\alpha v_H - \alpha v_L}{v(q_0) - \alpha v_L} \in (q_0, q^*)$. I claim that if $\kappa < \frac{1-\tilde{q}q_0}{1-\tilde{q}}$, the buyers at $t \in (\hat{t}, t^*)$ offer $R_b(t)$ with probability one. Suppose to the contrary that $\sigma_{Bb}(t) < 1$ for some $t \in (\hat{t}, t^*)$. Let $\tilde{t} = \sup\{t : \sigma_{Bb}(t) < 1\}$. To derive contradiction, it is sufficient to show that $\beta'(\tilde{t}-) < B'(\tilde{t}-)$.

From (18),

$$\beta'(\tilde{t}-) \leq -\rho\beta(\tilde{t}) \cdot \left[(1 - q_0\beta(\tilde{t})) - \kappa(1 - \beta(\tilde{t}))\right].$$

Since the value of information at $\tilde{t}$ is zero (that is, $R_u(\tilde{t}) = q_0\alpha v_H + (1 - q_0)R_b(\tilde{t})$), and $R_b(\tilde{t}) \geq \alpha v_L$, hence $B(\tilde{t}) \geq \tilde{q}$. Since $\kappa < \frac{1-\tilde{q}q_0}{1-\tilde{q}}$, it is easy to verify that $\beta'(\tilde{t}-) < 0$. On the other hand, from (21),

$$B'(\tilde{t}-) \geq B(\tilde{t}) \cdot \left[\rho F_\rho(\tilde{t}) + rF_r(\tilde{t})\right].$$

Since $R_u(\tilde{t}) = q_0\alpha v_H + (1 - q_0)R_b(\tilde{t})$, $F_\rho(\tilde{t}) = 0$ and $F_r(\tilde{t}) \geq 0$. Therefore, $B'(\tilde{t}-) \geq 0 > \beta'(\tilde{t}-)$, leading to the contradiction.

On the other hand, Since the buyers offer $R_b(t)$ with probability one for all $t \in (\hat{t}, t^*)$, from Subsection A.3.2 the reservation prices of the uninformed and the bad type at $\hat{t}$ are given by (since $t^* - \hat{t} = x$)

$$R_b(\hat{t}) = \alpha v_L + (v_L - \alpha v_L)e^{-rx},$$

$$R_u(\hat{t}) = q_0\alpha v_H + (1 - q_0)R_b(\hat{t}).$$

From equations (24) with the above boundary conditions at time $\hat{t}$, it is easy to show that there exists $\kappa^\dagger > 0$ such that for any $\kappa < \kappa^\dagger$, $R_u(t) < \tilde{p}(t)$ for any $t < \hat{t}$. Then defining $\underline{\kappa} = \min\{\frac{1-\tilde{q}q_0}{1-\tilde{q}}, \kappa^\dagger\}$ leads to the desired result.

(2) Since the buyer at time $\hat{t}$ is indifferent between targeting the uninformed and targeting the bad type, it must be that $B(\hat{t}) = \beta(\hat{t})$. The value of $R_u(\hat{t})$ and $R_b(\hat{t})$ calculated above imply that $B(\hat{t}) = \beta(\hat{t}) = q_0 \frac{\alpha v_H - (\alpha v_L + (v_L - \alpha v_L)e^{-rx})}{v(q_0) - (\alpha v_L + (v_L - \alpha v_L)e^{-rx})}$. Then from the belief evolution equation (23), the value of $\hat{t}$ is uniquely determined. Note that $B(\hat{t}) = \beta(\hat{t})$ is decreasing in the value of x, so $\hat{t}$ is increasing in x. $\square$

Lemma 10. *Suppose $\kappa < \underline{\kappa}$ where $\underline{\kappa}$ is determined in Lemma 9. Then there exists unique x such that the strategy profile in the previous lemma with $t^* - \hat{t} = x$ is an equilibrium.*

Proof. Let $\tilde{q}(t_1, t_\Delta)$ be the value of $q(t_1 + t_\Delta)$ under the strategy profile in which the buyers at any $t \in [0, t_1)$ offer $R_u(t)$ for sure and the buyers at any $t \in [t_1, t_1 + t_\Delta)$ offer $R_b(t)$ for sure. Then it is sufficient to show that $\tilde{q}(t_1, t_\Delta)$ is strictly increasing in t_1 and t_Δ. From equation (26),

$$\begin{aligned}
\frac{\tilde{q}(t_1, t_\Delta)}{1 - \tilde{q}(t_1, t_\Delta)} &= \frac{q_0}{1 - q_0} \cdot \frac{\frac{\rho}{\lambda+\rho} e^{(\lambda+\rho)t_1} + \frac{\lambda}{\lambda+\rho}}{e^{\rho t_1 - \lambda t_\Delta} + \frac{\lambda}{\lambda-\rho}(e^{-\rho t_\Delta} - e^{-\lambda t_\Delta})} \\
&= \frac{q_0}{1 - q_0} \cdot \frac{e^{(\lambda+\rho)t_1} + \frac{\lambda}{\rho}}{e^{\rho t_1} + \frac{\lambda}{\lambda-\rho}(e^{(\lambda-\rho)t_\Delta} - 1)} \cdot \frac{\frac{\rho}{\lambda+\rho}}{e^{-\lambda t_\Delta}}.
\end{aligned}$$

Since

$$\frac{\partial \left(\frac{e^{(\lambda+\rho)t_1} + \frac{\lambda}{\rho}}{e^{\rho t_1} + \frac{\lambda}{\lambda-\rho}(e^{(\lambda-\rho)t_\Delta} - 1)} \right)}{\partial t_1} = \frac{\lambda \left\{ (e^{(\lambda+\rho)t_1} - 1) + (\lambda+\rho) e^{(\lambda+\rho)t_1} \frac{e^{(\lambda-\rho)t_\Delta} - 1}{\lambda - \rho} \right\}}{\left\{ e^{\rho t_1} + \frac{\lambda}{\lambda-\rho}(e^{(\lambda-\rho)t_\Delta} - 1) \right\}^2} > 0,$$

and

$$\frac{\partial \left(\left\{ e^{\rho t_1} + \frac{\lambda}{\lambda-\rho}(e^{(\lambda-\rho)t_\Delta} - 1) \right\} e^{-\lambda t_\Delta} \right)}{\partial t_1} = -\frac{\lambda \rho}{\lambda - \rho}(e^{-\rho t_\Delta} - e^{-\lambda t_\Delta}) < 0,$$

$\frac{\partial \tilde{q}(t_1, t_\Delta)}{\partial t_1} > 0$ and $\frac{\partial \tilde{q}(t_1, t_\Delta)}{\partial t_\Delta} > 0$. $\square$

B.2 Proof of Proposition 5

It is sufficient to show that if q_0 is close to q^*, $z^* = e^{-\rho t^*}$ calculated when $\kappa = \frac{1-\tilde{q}q_0}{1-\tilde{q}}$ satisfies $\frac{z^*}{q_0 z^* + (1-q_0)} > q^*$, or $z^* > \frac{q^* - q^* q_0}{1-q^* q_0}$. It can be shown from equation (36) that z^* is increasing in q_0, and that z^* converges to one as q_0 converges to q^*. Moreover, $\frac{q^*-q^*q_0}{1-q^*q_0}$ is decreasing in q_0, and converges to $\frac{q^*}{1+q^*} < 1$ as q_0 converges to q^*, completing the proof.

B.3 Proof of Proposition 6

Suppose $\kappa > \bar{\kappa}$. Then by equation (22), t^* is determined by

$$\rho e^{\lambda t^*} + \lambda e^{-\rho t^*} = C(\rho + \lambda), \tag{38}$$

where $C = \frac{q^*}{1-q^*} \cdot \frac{1-q_0}{q_0} > 1$. Let $Y(\rho, t^*) = \rho e^{\lambda t^*} + \lambda e^{-\rho t^*} - C(\rho + \lambda)$, then

$$\begin{aligned} \frac{\partial t^*}{\partial \rho} &= -\frac{\frac{\partial Y}{\partial \rho}}{\frac{\partial Y}{\partial t^*}} \\ &= \frac{1}{\lambda \rho} \cdot \frac{C + t^* \lambda e^{-\rho t^*} - e^{-\lambda t^*}}{e^{\lambda t^*} - e^{-\rho t^*}}. \end{aligned}$$

Since $e^{\lambda t^*} - e^{-\rho t^*} > 0$, it remains to show that $\Phi \equiv C + t^* \lambda e^{-\rho t^*} - e^{-\lambda t^*} < 0$. Let $w^* = \rho t^*$. Then by (25),

$$\begin{aligned} \Phi &= C + \kappa w^* e^{-w^*} - C(1+\kappa) + \kappa e^{-w^*} \\ &= \kappa(e^{-w^*}(1+w^*) - C). \end{aligned}$$

Since $(1-a)e^{-a} < 1 < C$ for any $a > 0$, we have $\Phi < 0$.

Now suppose that $\kappa < \bar{\kappa}$. Let $t_\Delta^* = t_2^* - t_1^*$, and define

$$g_1(t_1^*, t_\Delta^*; \rho) \equiv \frac{\frac{\rho}{\lambda+\rho} e^{(\lambda+\rho)t_1^*} + \frac{\lambda}{\lambda+\rho}}{e^{\rho t_1^* - \lambda t_\Delta^*} + \frac{\lambda}{\lambda-\rho}(e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*})} - C,$$

$$g_2(t_1^*, t_\Delta^*; \rho) \equiv e^{-r t_\Delta^*} - \frac{(1-\alpha)\left\{\frac{q_0}{1-q_0} v_H + v_L\right\} + \alpha q_0 (v_H - v_L)(1 - e^{\rho t_1^*})}{(1-\alpha) v_L (1 + q_0 - q_0 e^{\rho t_1^*})},$$

Then by the implicit function theorem,

$$\frac{\partial t_1^*}{\partial \rho} = \frac{-A_{22} B_1 + A_{12} B_2}{A_{11} A_{22} - A_{12} A_{21}},$$

$$\frac{\partial t_\Delta^*}{\partial \rho} = \frac{A_{21} B_1 - A_{11} B_2}{A_{11} A_{22} - A_{12} A_{21}},$$

where

$$A_{11} \equiv \frac{\partial g_1(t_1^*, t_\Delta^*; \rho)}{\partial t_1^*} = \frac{1}{\Lambda^2} \cdot e^{-\lambda t_\Delta^*} \frac{\lambda \rho}{\lambda + \rho} \left\{ (e^{(\lambda+\rho)t_1^*} - 1) + (\lambda+\rho) e^{(\lambda+\rho)t_1^*} \frac{e^{(\lambda-\rho)t_\Delta^*} - 1}{\lambda - \rho} \right\} > 0,$$

$$A_{12} \equiv \frac{\partial g_1(t_1^*, t_\Delta^*; \rho)}{\partial t_\Delta^*} = \frac{1}{\Lambda^2} \cdot \left\{ \frac{\rho}{\lambda+\rho} e^{(\lambda+\rho)t_1^*} + \frac{\lambda}{\lambda+\rho} \right\} \frac{\lambda \rho (e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*})}{\lambda - \rho} > 0,$$

$$\begin{aligned} B_1 \equiv \frac{\partial g_1(t_1^*, t_\Delta^*; \rho)}{\partial \rho} &= \frac{1}{\Lambda^2} \cdot \left\{ e^{\rho t_1^* - \lambda t_\Delta^*} + \frac{\lambda (e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*})}{\lambda - \rho} \right\} \cdot \frac{(\lambda+\rho) t_1^* \rho e^{(\lambda+\rho)t_1^*} + \lambda (e^{(\lambda+\rho)t_1^*} - 1)}{(\lambda+\rho)^2} \\ &- \frac{1}{\Lambda^2} \cdot \left\{ t_1^* e^{\rho t_1^* - \lambda t_\Delta^*} + \frac{\lambda(-(\lambda-\rho) t_\Delta^* e^{-\rho t_\Delta^*} + \lambda (e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*}))}{(\lambda-\rho)^2} \right\} \cdot \frac{\rho e^{(\lambda+\rho)t_1^*} + \lambda}{\lambda + \rho} > 0, \end{aligned}$$

$$A_{21} \equiv \frac{\partial g_2(t_1^*, t_\Delta^*; \rho)}{\partial t_1^*} = \frac{\rho q_0 (\alpha v_H - v(q_0)) e^{\rho t_1^*}}{(1-q_0)(1-\alpha) v_L (1 + q_0 - q_0 e^{\rho t_1^*})^2} > 0,$$

$$A_{22} \equiv \frac{\partial g_2(t_1^*, t_\Delta^*; \rho)}{\partial t_\Delta^*} = -r e^{-r t_\Delta^*} < 0,$$

$$B_2 \equiv \frac{\partial g_2(t_1^*, t_\Delta^*; \rho)}{\partial \rho} = A_{21} \cdot \frac{t_1^*}{\rho} > 0,$$

and $\Lambda = e^{\rho t_1^* - \lambda t_\Delta^*} + \frac{\lambda}{\lambda-\rho}(e^{-\rho t_\Delta^*} - e^{-\lambda t_\Delta^*})$. Then it is easy to check that $\frac{\partial t_1^*}{\partial \rho} < 0$.

Chapter V

Appendix to Chapter 2

C Proofs

C.1 Proof of Proposition 7

It is sufficient to show that in any equilibrium, after any history the buyer never offers a price p greater than v_θ^*. First, observe that the buyer never makes an offer above u_θ, since his equilibrium payoff must be nonnegative. Given that, the seller's expected payoff after the rejection is no more than

$$z_1 \equiv \max\{\delta u_\theta, \xi v_\theta + \delta(1-\xi)u_\theta\} < u_\theta.$$

Note that the first (second) term in the bracket denotes the seller's maximum expected payoff when it is optimal for him to reject (accept) an outside option. So the seller accepts any offer $p > z_1$ after any history; hence, such offer is suboptimal for the buyer, since he can always make a lower offer $p - \epsilon > z_1$ and buy the product.

Proceeding with the same argument, given that the buyer's offer is bounded by z_m, the seller always accepts any offer above

$$z_{m+1} \equiv \max\{\delta z_m, \xi v_\theta + \delta(1-\xi)z_m\} < z_m,$$

so any offer greater than z_{m+1} is suboptimal for the buyer. Since $\{z_m\}$ is decreasing and converges to v_θ^*, for any $\epsilon > 0$ the buyer's offer $v_\theta^* + \epsilon$ is accepted by the seller, and hence is suboptimal.

C.2 Proof of Lemma 4

It is clear that $p \leq u_H$ after any history. Now I claim that if the buyer never offers more than $z_m > v_H^*$ in the equilibrium, both types surely accept any offer greater than

$$z_{m+1} = \max\{\delta z_m, \xi v_H + \delta(1-\xi) z_m\} < z_m.$$

The low type accepts p_n for sure, since his maximum payoff after the rejection is no more than $\max\{\delta z_m, \xi v_L + \delta(1-\xi) z_m\}$, which is less than z_{m+1}. Since $\{z_m\}$ is decreasing and converges to v_H^*, making an offer $v_H^* + \epsilon$ for any $\epsilon > 0$ is suboptimal for the buyer.

C.3 Proof of Proposition 8

Equilibrium Behavior at $\pi < \pi^*$

In this subsection, I construct sequences of prices $\{p_k^\dagger\}$ and cutoff beliefs $\{\pi_k^\dagger\}$ that describe the equilibrium behavior when the posterior is less than the deadlock belief π^*. In the deadlock equilibrium, at $\pi < \pi^*$ the buyer offers a price less than or equal to v_L and the low type uses a mixed acceptance strategy, and the bargaining reaches the deadlock phase in a finite number of periods.

Let $p_0^\dagger = v_L$, and let $p_k^\dagger$ be the equilibrium price when there are k periods until the bargaining reaches the deadlock phase. The low type is indifferent between accepting $p_k^\dagger$ and waiting k periods to accept $p_0^\dagger = v_L$. Since $p_k^\dagger$ must be strictly lower than v_L, opting out (when the option is available) is strictly optimal for the low-type seller in these k periods. Then the low type's indifference condition, which gives a recursive equation for $\{p_k^\dagger\}$, is given by

$$p_k^\dagger = \xi v_L + \delta(1-\xi)p_{k-1}^\dagger. \tag{39}$$

For the construction of $\{\pi_k^\dagger\}$, I define notions that make the analysis easier. Let $\beta(\pi, \pi')$ be the low-type seller's acceptance probability, which changes the posterior belief from π to π', given that both types of sellers opt out. That is, $\beta(\pi, \pi')$ satisfies

$$\frac{\pi'}{1-\pi'} = \frac{\pi}{1-\pi} \cdot \frac{1}{1-\beta(\pi, \pi')},$$

so $\beta(\pi, \pi') = 1 - \frac{\pi}{1-\pi} \cdot \frac{1-\pi'}{\pi'}$. On the other hand, let $\tilde{\beta}(\pi, \pi')$ be the low type's acceptance probability, which changes the posterior belief from π to π', given that only the high type takes the outside option. So $\tilde{\beta}(\pi, \pi')$ satisfies

$$\frac{\pi'}{1-\pi'} = \frac{\pi}{1-\pi} \cdot \frac{1-\xi}{1-\tilde{\beta}(\pi, \pi')},$$

so $\tilde{\beta}(\pi, \pi') = 1 - \frac{\pi}{1-\pi} \cdot \frac{1-\pi'}{\pi'}(1-\xi)$.

Let $\pi_0^\dagger = \pi^*$, and let $\pi_k^\dagger$ be the maximum belief where the buyer offers $p_k^\dagger$. That is, the buyer offers $p_k^\dagger$ if $\pi \in (\pi_{k+1}^\dagger, \pi_k^\dagger]$. Then when the belief is $\pi_1^\dagger$, the buyer is indifferent between offering $p_0^\dagger = v_L$ and $p_1^\dagger$. Either price leads to the posterior belief equal to $\pi_0^\dagger = \pi^*$, but the low type's acceptance probability is different. If the buyer offers $p_1^\dagger$, both types of sellers opt out, and the low type accepts with probability $\beta(\pi_1^\dagger, \pi^*)$. On the other hand, if the buyer offers $p_0^\dagger = v_L$, only the high type opts out, and the low type accepts with probability

$\tilde{\beta}(\pi_1^\dagger, \pi^*)$.

Therefore, the payoff to the buyer when he offers $p_1^\dagger$ is

$$\left(1 - \frac{\pi_1^\dagger}{\pi^*}\right)(u_L - p_1^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\delta(1-\xi)U_F^*,$$

where $U_F^* \equiv (1-\pi^*)(u_L - v_H^*) + \pi^*(u_H - v_H^*)$ is defined in (9), and the payoff when he offers $p_0^\dagger = v_L$ is

$$\left(1 - \frac{\pi_1^\dagger}{\pi^*}\right)(u_L - p_0^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\xi(1-\pi^*)(u_L - p_0^\dagger) + \frac{\pi_1^\dagger}{\pi^*}\delta(1-\xi)U_F^*.$$

The indifference condition gives

$$(1 - \frac{\pi_1^\dagger}{\pi^*})(p_0^\dagger - p_1^\dagger) = \frac{\pi_1^\dagger}{\pi^*}\xi(1-\pi^*)(u_L - p_0^\dagger).$$

Note that the left-hand side of the equation above is the benefit of screening the low type at a lower price, while the right-hand side is the cost of the low type's opting-out. Hence $\pi_1^\dagger$ is given by

$$\frac{\pi_1^\dagger}{\pi^*} = \frac{(1-\delta)(1-\xi)v_L}{(1-\delta)(1-\xi)v_L + \xi(1-\pi^*)(u_L - v_L)}.$$

For $k > 1$, when the posterior is $\pi_k^\dagger$ the buyer is indifferent between offering $p_k^\dagger$, which the low type accepts with probability $\beta(\pi_k^\dagger, \pi_{k-1}^\dagger)$, and offering $p_{k-1}^\dagger$, which the low type accepts with probability $\beta(\pi_k^\dagger, \pi_{k-2}^\dagger)$. Let $W(\pi)$ be the buyer's expected payoff in the equilibrium when the posterior is π. Then

$$W(\pi_k^\dagger) = (1-\gamma_k)(u_L - p_k^\dagger) + \delta\gamma_k(1-\xi)W(\pi_{k-1}^\dagger) \tag{40}$$

$$= (1-\gamma_k)(u_L - p_{k-1}^\dagger) + \gamma_k W(\pi_{k-1}^\dagger), \tag{41}$$

where $\gamma_k = \frac{\pi_k^\dagger}{\pi_{k-1}^\dagger}$. Note that (40) and (41) are the buyer's expected payoff when he offers $p_k^\dagger$ and $p_{k-1}^\dagger$, respectively. The indifference condition gives

$$\gamma_k W(\pi_{k-1}^\dagger) = (1-\gamma_k)(p_{k-1}^\dagger - v_L^*) \tag{42}$$

Then plugging (42) into (41) gives

$$W(\pi_k^\dagger) = (1-\gamma_k)(u_L - v_L^*).$$

Finally, plugging the equation above into (42) leads to

$$\frac{1}{\gamma_k} = 1 + (1-\gamma_{k-1})\frac{u_L - v_L^*}{p_{k-1}^\dagger - v_L^*}. \tag{43}$$

Note that since $\lim_{k\to\infty} p_k^\dagger = v_L$, γ_k converges to zero as k goes to infinity. Therefore, for any $\pi_0 \in (0, \pi^*)$, there exists $N \in \mathbb{N}$ such that $\pi_N^\dagger \le \pi_0 < \pi_{N-1}^\dagger$. Here I consider the generic case that $\pi_{N+1}^\dagger < \pi_0$.

Strategy Profile

▷ Buyer:

$$p_n(h^n) = p_n(\pi_n, p_{n-1}) = \begin{cases} v_H^* & \text{if } \pi_n > \pi^*, \\ q(p_{n-1}) \circ v_H^* + (1 - q(p_{n-1})) \circ v_L & \text{if } \pi_n = \pi^*, \\ p_{k-1}^\dagger & \text{if } \pi_n \in [\pi_k^\dagger, \pi_{k-1}^\dagger), \end{cases}$$

where $\pi_0^\dagger = \pi^*$ and $q(p_{n-1}) = \max\{\frac{p_{n-1}/\delta - v_L}{v_H^* - v_L}, 0\}$.

▷ The low type:

$$\sigma_{Ln}(h_S^n) = \sigma_{Ln}(\pi_n, p_n) = \begin{cases} 1 & \text{if } p_n \geq \delta v_H^*, \\ \max\{0, \tilde{\beta}(\pi_n, \pi^*)\} & \text{if } p_n \in [v_L, \delta v_H^*), \\ \max\{0, \beta(\pi_n, \pi_k^\dagger)\} & \text{if } p_n \in [p_k^\dagger, p_{k-1}^\dagger), \\ 0 & \text{if } p_n < v_L^*, \end{cases}$$

$$c_{Ln}(\hat{h}_S^n) = c_{Ln}(\hat{\pi}_n) = \begin{cases} 1 & \text{if } \hat{\pi}_n < \hat{\pi}^*, \\ \max\{0, \tilde{\beta}(\hat{\pi}_n, \pi^*)/\xi\} & \text{if } \hat{\pi}_n \geq \hat{\pi}^*, \end{cases}$$

where $\hat{\pi}^* = \frac{\pi^*}{\pi^* + (1-\pi^*)(1-\xi)}$.

Optimality of Profile

The following lemma states that if (A1) holds, then offering any price $p \in [\delta v_H^*, v_H^*)$ at the deadlock belief is suboptimal for the buyer.

Lemma 11. *Suppose (A1) holds. Then at $\pi = \pi^*$, the buyer is better off by offering v_H^* than by offering a price $p \in [\delta v_H^*, v_H^*)$.*

Proof. Let $U_F(\pi) = (1-\pi)(u_L - v_H^*) + \pi(u_H - v_H^*)$ be the payoff to the buyer when he offers v_H^* to finish the bargaining at a posterior belief π. Then $U_F^* = U_F(\pi^*)$, where U_F^* is defined in (9).

Corollary 1 implies that offering a price $p \in (\delta v_H^*, v_H^*)$ is dominated by offering δv_H^*. If the buyer offers δv_H^*, the low type accepts it for sure. The high type opts out when the option is available, then the buyer offers v_H^* in the next period and bargaining ends. In this case the buyer's expected payoff is

$$(1-\pi)(u_L - \delta v_H^*) + \pi\delta(1-\xi)(u_H - v_H^*).$$

If the buyer instead offers v_H^* and finishes the bargaining immediately, he obtains $U_F(\pi)$. Then the difference is

$$\underbrace{(1-\pi)(1-\delta)v_H^*}_{\text{benefit of screening}} - \underbrace{\pi\xi(u_H - v_H^*)}_{\text{cost of breakdown}} - \underbrace{\pi(1-\xi)(1-\delta)(u_H - v_H^*)}_{\text{cost of delay}}.$$

Therefore offering v_H^* yields greater payoff if and only if

$$\pi > \tilde{\pi} \equiv \frac{(1-\delta)v_H^*}{(1-\delta(1-\xi))(u_H - v_H^*) + (1-\delta)v_H^*}.$$

A simple calculation shows that $\pi^* > \tilde{\pi}$ if and only if (A1) holds. □

The optimality of each action is as follows:

1. c_{Ln} :

   (a) If $\hat{\pi}_n < \pi^*$, then for any value of $c_{Ln} \in [0,1]$, $\pi_{n+1} < \pi^*$ so $p_{n+1} \leq v_L$. Therefore the continuation payoff is no greater than v_L, so taking an outside option is optimal for the low type.

   (b) If $\hat{\pi}_n \in [\pi^*, \hat{\pi}^*]$, the only consistent strategy is to use a mixed strategy to induce $\pi_{n+1} = \pi^*$.

   (c) If $\hat{\pi}_n > \hat{\pi}^*$, then for any value of $c_{Ln} \in [0,1]$, $\pi_{n+1} > \pi^*$ so the buyer offers v_H^* in the next period. Therefore, the low type strictly prefers not to take an outside option.

2. σ_{Ln} :

   (a) By Corollary 1, the low type accepts any $p_n \geq \delta v_H^*$ for sure.

   (b) $p_n \in [v_L, \delta v_H^*)$: If $\pi_n \leq \hat{\pi}^*$, accepting the offer with probability $\sigma_{Ln} = \tilde{\beta}(\pi_n, \pi^*)$ that, combined with $c_{Ln} = \max\{0, \tilde{\beta}(\hat{\pi}_n, \pi^*)/\xi\}$, induces $\pi_{n+1} = \pi^*$ is the only consistent strategy of the low-type seller. if $\pi > \hat{\pi}^*$, the low type is strictly better off by rejecting p_n, not taking outside option and accepting $p_{n+1} = v_H^*$.

   (c) $p_n < v_L$: The construction of the sequences $\{(\pi_k^\dagger, p_k^\dagger)\}_{k=0}^\infty$ implies that the low type is indifferent between acceptance and rejection by following the above strategy profile.

3. p_n : Lemma 11 and the construction of the sequences $\{(\pi_k^\dagger, p_k^\dagger)\}_{k=0}^\infty$ imply that offer strategy p_n is the best response to the seller's strategy $(\sigma_{\theta t}, c_{\theta t})$.

C.4 Proof of Proposition 9

Let $\{\hat{p}_k^\dagger\}$ and $\{\hat{\pi}_k^\dagger\}$ be the limit of sequences $\{p_k^\dagger\}$ and $\{\pi_k^\dagger\}$ when $\Delta \to 0$. Then the recursive equation (39) implies that $\hat{p}_k^\dagger = v_L$ for any k. Therefore, the recursive equation (43) for $\gamma_k = \frac{\pi_k^\dagger}{\pi_{k-1}^\dagger}$ becomes

$$\gamma_k = \frac{1}{1 + (1 - \gamma_{k-1})\frac{u_L - \eta v_L}{v_L - \eta v_L}},$$

where $\eta = \frac{\lambda}{r+\lambda}$. Since a function $g(x) = \frac{1}{1+(1-x)\frac{u_L - \eta v_L}{v_L - \eta v_L}}$ is convex and has fixed points of one and $\frac{v_L - \eta v_L}{u_L - \eta v_L} < 1$, γ_k converges to $\frac{v_L - \eta v_L}{u_L - \eta v_L}$. Therefore, for any prior $\pi_0 \in (0, \pi^*)$ there exists a finite K such that $\hat{\pi}_K^\dagger \leq \pi_0$. Therefore, as Δ goes to zero, the equilibrium offer at π_0 converges to v_L, and the real-time length of the pre-deadlock phase, $K\Delta$, shrinks to zero.

In the deadlock phase, in each period the bargaining ends by 1) agreement at $p = v_H^*$ with probability q, 2) agreement at $p = v_L$ with probability $(1-q)(1-\pi^*)\xi$, and 3) opting-out with probability $(1-q)\pi^*\xi$. Therefore, the resolution period of the deadlock phase is a geometric distribution with parameter $q + (1-q)\xi$. As $\Delta \to 0$, the limit distribution becomes Poisson arrival process with a finite arrival rate, so the deadlock phase does not shrink.

C.5 Proof of Corollaries 2 and 3

The probability of agreement at $t = 0$ is $(1-\pi_0)\beta(\pi_0, \pi^*) = 1 - \frac{\pi_0}{\pi^*}$. The proof of Proposition 9 implies that $\hat{T}_d = \frac{\Delta}{q+(1-q)\xi}$, and letting $\Delta \to 0$ provides the desired result.

The probability of a breakdown conditional on the bargaining reaching the deadlock phase is

$$\frac{(1-q)\pi^*\xi}{q + (1-q)\xi},$$

so letting $\Delta \to 0$ provides the desired result..

C.6 Proof of Proposition 11

Suboptimality of Two-Period Screening

Lemma 12. *Suppose (A2) holds. Then in equilibrium, $p_n \in [\delta v_H^*, v_H^*)$ is never offered after any history.*

Proof. Recall from Lemma 11 that offering v_H^* yields a greater payoff than offering a sequence of prices $\delta v_H^*, v_H^*$ if and only if

$$\pi > \tilde{\pi} \equiv \frac{(1-\delta)v_H^*}{(1-\delta(1-\xi))(u_H - v_H^*) + (1-\delta)v_H^*}.$$

On the other hand, from the inequality

$$(1-\pi)(u_L - \delta v_H^*) + \pi\delta(1-\xi)(u_H - v_H^*) < 0,$$

the offer sequence $\delta v_H^*, v_H^*$ yields a negative payoff if and only if

$$\pi < \underline{\pi} \equiv \frac{\delta v_H^* - u_L}{\delta(1-\xi)(u_H - v_H^*) + (\delta v_H^* - u_L)}.$$

Suppose (A2) holds. Then a simple calculation shows that (A2) implies $\underline{\pi} > \tilde{\pi}$. Then for any $\pi \in [0,1]$, $p_n \in [\delta v_H^*, v_H^*)$ is not offered in equilibrium, since either $p = v_H^*$ or $p = 0$ is a profitable deviation. $\square$

Upper Bound on the Equilibrium Posterior

Lemma 13. *In any equilibrium, there exists* $\bar{\pi} \in (0,1)$ *such that if* $\pi_n > \bar{\pi}$ *after any history, the buyer offers* $p_n = v_H^*$.

Proof. The maximum payoff of the buyer by screening the low type is

$$(1-\pi)\cdot(u_L - v_L^*) + \pi(1-\xi)\cdot\delta(u_H - v_H^*). \tag{44}$$

If instead the buyer offers v_H^*, then his payoff is $U_F(\pi)$. Therefore, if

$$\pi > \frac{v_H^* - v_L^*}{(1-\delta(1-\xi))(u_H - v_H^*) + v_H^* - v_L^*}$$

then the buyer strictly prefers to offer v_H^* regardless of the history. □

Lemma 14. *If* $\pi_n \leq \bar{\pi}$ *and* $p_n < v_H^*$, $\pi_{n+1} \leq \bar{\pi}$.

Proof. Suppose not; that is, there exists a history h^n where $\pi_n \leq \bar{\pi}$, $p_n < v_H^*$, and $\sigma_{Ln} + (1-\sigma_{Ln})\xi c_{Ln} > \tilde{\beta}(\pi_n, \bar{\pi})$. By Lemma 12, $p_n < \delta v_H^*$. Moreover, by Lemma 13, $p_{n+1} = v_H^*$. Then it is optimal for the low type to reject both p_n and an outside option and wait for the next period offer, which leads to a contradiction. □

Lemma 15. *(1) If* $\pi_n \leq \bar{\pi}$ *and* $p_n \in (v_L, \delta v_H^*)$, *then* $c_{Ln} = 0$.

(2) If $\pi_n = \bar{\pi}$ *and* $p_n < v_L$, *then* $\sigma_{Ln} = 0$.

Proof. (1) Suppose not; that is, there exists a history h^n where $\pi_n \leq \bar{\pi}$, $p_n \in (v_L, \delta v_H^*)$, and $c_{Ln} > 0$. Then opting-out must be at least as good as waiting, so $u_L(h^{n+1}) \leq v_L/\delta$. Then it is strictly optimal to accept p_n, contradicting Lemma 14.

(2) Suppose that there exists a history h^n where $\pi_n = \bar{\pi}$ and $p_n < v_L$, and $\sigma_{Ln} > 0$. Then by Lemma 14 $c_{Ln} < 1$, which implies $u_L(h^{n+1}) \geq v_L/\delta$. But then accepting p_n is suboptimal. Contradiction. □

Lemma 16. $\bar{\pi} \leq \pi^*$.

Proof. Suppose the contrary that there exists an equilibrium with $\bar{\pi} > \pi^*$. Then it suffices to show that for all history with a belief smaller than but sufficiently close to $\bar{\pi}$, offering v_H^* is optimal for the buyer.

Define

$$\tilde{U}(\bar{\pi}) = (1-\bar{\pi})\xi(u_L - v_L) + \delta(1-\xi)U_F(\bar{\pi}),$$

and let $\hat{U}(h^n)$ be the supremum of the buyer's expected payoff at h^n, given that the buyer offers $p < v_H^*$ at h^n. I claim that for any history h^n with a belief $\bar{\pi}$, $\hat{U}(h^n) < \tilde{U}(\bar{\pi})$. Suppose the bargaining ends after k periods. Then by Lemma 14, the probability of an agreement between the low-type seller before bargaining ends is no more than $\frac{1-\xi^k}{1-\xi}$. Since the low type never accepts any offer less than v_L,[34] making an agreement at v_L with the least delay yields the highest possible payoff to the buyer. Therefore the buyer's payoff is bounded by

$$\tilde{U}_k(\bar{\pi}) = (1-\bar{\pi})\frac{\xi(1-\delta^k(1-\xi)^k)}{1-\delta(1-\xi)}(u_L - v_L) + \delta^k(1-\xi)^k U_F(\bar{\pi}).$$

Since $\bar{\pi} > \pi^*$, $(1-\delta(1-\xi))U_F(\bar{\pi}) > (1-\bar{\pi})\xi(u_L - v_L)$, so $k = 1$ is optimal.

Now consider histories with beliefs less than $\bar{\pi}$. Then the continuity of the previous argument implies that for any $\beta > 0$, there exists $\epsilon > 0$ such that for any history h^n with

[34] Suppose not; that is, there exists a history $\hat{h}^m$ where the buyer offers a price less than v_L and the low type accepts it with positive probability. Then, the low type must take the outside option for sure (if it is available) at every history between h^n and $\hat{h}^m$. Then at $\hat{h}^m$ the posterior is $\bar{\pi}$, which contradicts Lemma 15.

a belief $\pi(h^n) \in (\bar{\pi} - \epsilon, \bar{\pi})$, if the buyer offers $p < v_H^*$ with positive probability at h^n, then $U(h^n) < \tilde{U}(\bar{\pi}) + \beta$.

Equations (9) and (10) imply that $\bar{\pi} > \pi^*$ if and only if $\tilde{U}(\bar{\pi}) < U_F(\bar{\pi})$. Then since $U_F(\pi)$ are continuous, for sufficiently small $\beta > 0$, there exists $\epsilon > 0$ such that for any history h^n with a belief $\pi(h^n) \in (\bar{\pi} - \epsilon, \bar{\pi})$, if the buyer offers $p < v_H^*$ with positive probability at h^n, $U(h^n) < \tilde{U}(\bar{\pi}) + \beta < U_F(\bar{\pi} - \epsilon)$. So the buyer's optimal offer is v_H^* for any history with $\pi \in (\bar{\pi} - \epsilon, \bar{\pi})$, which contradicts the definition of $\bar{\pi}$. □

C.7 Proof of Proposition 10

Fix a perfect Bayesian equilibrium that satisfies nondecreasing offers.

Step 1 For any history h^n, $\pi_n \geq \pi_{n-1}$.

Proof. Suppose not; that is, there exists a history h^n such that $\pi_n < \pi_{n-1}$. Then in order to make the low type indifferent, the buyer's offer at h^n satisfies $\mathbb{E}[p_n] = p_{n-1}/\delta$. Therefore, the seller offers $p_n > p_{n-1}$ with positive probability, which violates the nondecreasing offers. □

In the proof of Proposition 11, I show that there exists $\bar{\pi} \leq \pi^*$ that bounds the posterior belief along the bargaining process. Then Step 1 implies that if $\pi_n = \bar{\pi}$ at some history h^n, then $\pi_{n+1} = \bar{\pi}$ after any $p_n < \delta v_H^*$, which implies that $\sigma_{Ln}(\bar{\pi}, x) = \xi$ for any $p \in (v_L, \delta v_H^*)$.

Step 2 At $\pi = \bar{\pi}$, the buyer's equilibrium offer is either v_L or v_H^*.

Proof. It is clear that offering $p < v_L$ is suboptimal for the buyer. Moreover, Lemma 12 says that any offer $p \in [\delta v_H^*, v_H^*)$ is suboptimal. Then it is sufficient to show that if any $p \in (\tilde{p}, v_H^*)$ is not offered, the same goes for any $p \in (\max\{\delta\tilde{p}, v_L\}, \tilde{p})$. Suppose at some history h^n, the buyer offers $p_n \in (\max\{\delta\tilde{p}, v_L\}, \tilde{p})$. Then in the next period, to make the low type indifferent, the buyer must use mixed offer between v_H^* and some (possibly multiple) $p \le \tilde{p}$. Therefore the buyer's expected payoff at history h^n is

$$(1-\bar{\pi})\xi(1-p_n) + \delta(1-\xi)(1-v_H^*).$$

Now consider the deviation of the buyer to offer $p' = p_n - \epsilon$, where ϵ is small enough such that $p' > \max\{\delta\tilde{p}, v_L\}$. Then the buyer can make an agreement with the low type at a lower offer with the same probability and still use a mixed offer in the next period. So offering p' is a profitable deviation, which proves that any $p \in (v_L, v_H^*)$ cannot be offered in equilibrium. □

Step 3 $\bar{\pi} = \pi^*$.

Proof. Suppose $\bar{\pi} < \pi^*$. First, I claim that if $\pi_n = \bar{\pi}$, the buyer's offer must be v_H^*. Suppose v_L is offered in some history h^n. Then in the next period the buyer must use a mixed strategy between v_H^* and some $p \le v_L$. Therefore, the buyer's expected payoff at history h^n is

$$(1-\bar{\pi})\xi(1-v_L) + \delta(1-\xi)(1-v_H^*),$$

which is greater than $1 - v_H^*$ since $\bar{\pi} < \pi^*$. So offering v_L at h^{n+1} is strictly better than v_H^*, contradictory to the fact that the buyer uses a mixed strategy.

Since the equilibrium satisfies nondecreasing offers, it must be that for all history h^n with $\pi_n < \bar{\pi}$, the buyer never offers v_H^*. Let $\bar{p}$ be a supremum of the buyer's offer at history h^n with $\pi_n < \bar{\pi}$. Then by Lemma 12, $\bar{p} \le \delta v_H^*$. Fix ϵ sufficiently small that $\bar{p} - \epsilon > \delta\bar{p}$.

Then there exists a history h^n with $\pi_n < \bar{\pi}$ where the buyer offers $p > \bar{p} - \epsilon$ with positive probability. Suppose that $\pi_{n+1} \geq \bar{\pi}$; then $p_{n+1} = v_H^*$ and the low type is strictly better off by rejecting p_n. If $\pi_{n+1} < \bar{\pi}$, then accepting p_n is a strict best response of the low type, violating consistency. $\square$

Step 4 Behavior at $\pi \leq \pi^*$ is determined uniquely.

Proof. By step 1, the equilibrium belief is nondecreasing. Therefore, the backward induction method in the proof of Proposition 8 yields unique equilibrium behavior. $\square$

C.8 Proof of Proposition 12

The buyer's indifference condition at the deadlock belief π^* is given by

$$(1 - \delta(1 - \xi))\{(1 - \pi^*)u_L + \pi^* u_H - (c_H + v_H^*)\} = (1 - \pi^*)\xi(u_L - (c_L + v_L)),$$

so

$$\pi^* = \frac{(c_H + v_H^* - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - (c_L + v_L))}{(u_H - u_L) + \frac{\xi}{1-\delta(1-\xi)}(u_L - (c_L + v_L))}.$$

The buyer can conduct a two-period screening by offering $(1 - \delta)c_L + \delta(c_H - v_H^*)$ to the low type then offering $c_H + v_H^*$ to the remaining high type. In this case, his payoff is

$$(1 - \pi)[u_L - (1 - \delta)c_L + \delta(c_H - v_H^*)] + \pi\delta(1 - \xi)[u_H - (c_H + v_H^*)].$$

Hence the two-period screening yields a higher payoff than offering $c_H + v_H^*$ if

$$\pi < \tilde{\pi} \equiv \frac{(1-\delta)[(c_H + v_H^*) - c_L]}{(1-\delta)[(c_H + v_H^*) - c_L] + (1-\delta(1-\xi))[u_H - (c_H + v_H^*)]}.$$

It can be shown that (A1') is satisfied if and only if $\pi^* > \tilde{\pi}$.

Chapter VI

Appendix to Chapter 3

D Preliminaries

D.1 Hamilton-Jacobi-Bellman equations

Given a Markov strategy of party 2 and the median voter (σ_2, σ_M), standard arguments imply that on any open interval where σ_2 and σ_M are constant, party 1's payoff functions (V_1, W_1) from playing a best response are once continuously differentiable and solve the system of Bellman equations

$$V_1(p) = s - (1 - \sigma_M(p)) \cdot \tau_1(p; V_1, W_1) + \max_{k_1 \in [0,1]} k_1(b_1(p; V_1) - c_1(p)), \tag{45}$$

$$W_1(p) = s + \sigma_M(p) \cdot \tau_1(p; V_1, W_1) + k_2(p)(\beta_1(p; W_1) - s), \tag{46}$$

where

- ▷ $\tau_1(p; V_1, W_1) = \frac{r}{\xi}(V_1(p) - W_1(p))$: value change from loss of control
- ▷ $b_1(p; V_1) = \frac{p\lambda}{r}\{V_1(1) - V_1(p) - (1-p)V_1'(p)\}$: value of party 1's experimentation
- ▷ $\beta_1(p; W_1) = \frac{(1-p)\lambda}{r}\{W_1(0) - W_1(p) + pW_1'(p)\}$: value of opponent's experimentation
- ▷ $c_1(p) = s - pg$: opportunity cost of experimentation

Party 2's value functions $V_2(p)$ and $W_2(p)$ satisfy a similar Bellman equation, with $1 - p$

replacing p.

Before we analyze the equilibrium, let us remember the important notations and define several new ones:

- ▷ $\mu = {}^{r}/_{\lambda}$: inverse of the effective success rate of the reform policy
- ▷ $\chi = \frac{\xi}{\xi + r}$: relative frequency of elections
- ▷ $\hat{\mu} = \frac{\mu}{1-\chi}$.
- ▷ $\Omega(p) = \frac{1-p}{p}$: (inverse) odds ratio
- ▷ $f(p) \equiv (1-p)\Omega(p)^{\mu} = (1-p)^{\mu+1}p^{-\mu}$.
- ▷ $\hat{f}(p) \equiv (1-p)\Omega(p)^{\hat{\mu}}$.

D.2 Explicit solution to HJB equations

For a range of beliefs where the Markov profile (k_1, k_2, σ_M) is constant, we solve (45) and (46) to get the explicit solutions of $V_1(p)$ and $W_1(p)$, with integration constants. Table 1 shows the explicit solutions for each set of values (k_1, k_2, σ_M). Integration constants C_k are determined by boundary conditions, such as value-matching conditions and smooth-pasting conditions.

E Equilibrium Characterization: Proof of Propositions 14-17

First, I will prove lemmas, which makes the analysis simpler. The first lemma states that in the belief range where there is no chance of losing power, each party will play as if it were a single decision-maker.

		$\sigma_M(p) = 1$	$\sigma_M(p) = 0$
$(0,0)$	$V_1(p)$	s	s
	$W_1(p)$	s	s
$(0,1)$	$V_1(p)$	s	$(1-\chi)s + \chi C_5 f(1-p)$
	$W_1(p)$	$\frac{\hat{\mu}\chi s}{\hat{\mu}+1} + \frac{\chi s}{\hat{\mu}+1}p + C_1 \hat{f}(1-p)$	$C_5 f(1-p)$
$(1,0)$	$V_1(p)$	$gp + C_2 f(p)$	$\chi s + \frac{g-\chi s}{\hat{\mu}+1}p + C_4 \hat{f}(p)$
	$W_1(p)$	$(1-\chi)s + \chi(gp + C_2 f(p))$	s
$(1,1)$	$V_1(p)$	$gp + C_2 f(p)$	$\frac{\mu+1}{\hat{\mu}+1}gp + \frac{\hat{\mu}\chi}{\hat{\mu}+\mu+1}C_5 f(1-p) + C_6 \hat{f}(p)$
	$W_1(p)$	$\chi gp + \frac{\hat{\mu}\chi}{\hat{\mu}+\mu+1}C_2 f(p) + C_3 \hat{f}(1-p)$	$C_5 f(1-p)$

Table 1
Explicit solutions to HJB equations

Lemma 17. *In any MPE in Σ^*, the action of party 1 (resp. party 2) at $p > \frac{1}{2}$ (resp. $p < \frac{1}{2}$) is the same as that of the optimal decision rule of the single decision-maker problem.*

Proof. First, consider party 1. Since $\sigma_M^*(p) = 1$ for $p > \frac{1}{2}$, party 1's HJB equations in this range of beliefs given σ_M^* are equal to those of the single decision-maker problem (12). Therefore, the corresponding value functions and the boundary conditions are also the same, which leads to the same optimal decision rule in the equilibrium. A similar argument can be applied to party 2. □

The next lemma shows that each party has the dominant strategy when the belief is close to certainty.

Lemma 18. *In any MPE in Σ^*, 1) party 1 (resp. party 2) chooses R_1 (resp. R_2) at a belief close to one (resp. zero), and 2) chooses S at a belief close to zero (resp. one).*

Proof. The first part is straightforward from Lemma 17. Suppose to the contrary that there exists an MPE where party 1 chooses R_1 for any $p > 0$. Then by the first part of the proof, there exists $\underline{p} \in (0, \frac{1}{2})$ such that $k_2(p) = 1$. Since $\sigma_M(p) = 0$ for $p < \underline{p}$, we can derive the functional form of value function $V_1(p)$ for $p \in (0, \underline{p})$. Then a simple calculation shows that

$\lim_{p\to 0} V(p) = 0$ (it is sufficient to show that $C_6 = 0$), but then party 1 can deviate to S to get the payoff of $(1-\chi)s$. Contradiction. $\square$

Lemma 18 implies that in any MPE, there exists at least one cutoff point $p_1 \in (0,1)$ on the belief space where party 1 switches action from R_1 to S, that is, there exists $\epsilon > 0$ such that $k_1(p) = \begin{cases} 1 & \text{if } p \in (p_1, p_1+\epsilon) \\ \\ 0 & \text{if } p = p_1 \end{cases}$. Let p_1^* be the greatest belief point of such p_1's, and let p_2^* be the smallest such cutoff of p_2. Then Lemma 17 implies that for $\frac{g}{s} < \alpha_0$, $p_1^* = 1 - p_2^* = p^0 > \frac{1}{2}$. This proves Theorem 1.

Now consider the case where $\frac{g}{s} \geq \alpha_0$. Then by Lemma 17, $p_1^* \leq \frac{1}{2}$ and $p_2^* \geq \frac{1}{2}$ in any MPE in Σ^*. Therefore, we have four cases to consider:

1. $p_1^* = p_2^* = \frac{1}{2}$,

2. $p_1^* < \frac{1}{2}$ and $p_2^* > \frac{1}{2}$,

3. $p_1^* = \frac{1}{2}$ and $p_2^* > \frac{1}{2}$,

4. $p_1^* < \frac{1}{2}$ and $p_2^* = \frac{1}{2}$.

In the following subsections, we characterize the equilibria in each case.

E.1 Case 1: Propositions 15 and 16

Suppose $p_1^* = p_2^* = \frac{1}{2}$. Then by definition of p_i^*, $k_1(p) = 1$ for $p > \frac{1}{2}$, $k_2(p) = 1$ for $p < \frac{1}{2}$, and $k_1(\frac{1}{2}) = k_2(\frac{1}{2}) = 0$. Since both parties play the safe action at $p = \frac{1}{2}$ when in control, $V_i(\frac{1}{2}) = W_i(\frac{1}{2}) = s$ for all $i = 1, 2$. These boundary conditions give us the value of the integration constants $C_2 = 2s - g$ and $C_5 = 2s$; hence, the payoff functions are given by (for party 1)

$$V_1(p) = gp + (2s - g)f(p), \quad p \geq \frac{1}{2},$$
$$W_1(p) = 2sf(1-p), \quad p \leq \frac{1}{2}.$$

Note that $V_1(p)$ for $p < \frac{1}{2}$ does not depend on party 2's behavior for $p > \frac{1}{2}$. The intuition is that given that the prior is less than $\frac{1}{2}$, the posterior never reaches $p \in (\frac{1}{2}, 1)$. Therefore, the question is essentially getting the best response of party 1 to $k_2(p) = 1$ and $\sigma_M(p) = 0$. By symmetry, if $k_1(p)$ is a best response of party 1 for $p < \frac{1}{2}$, then $k_2(p) = k_1(1-p)$ is a best response of party 2 for $p > \frac{1}{2}$, which constitutes an equilibrium. Therefore, it is sufficient to analyze party 1's best response.

We consider the following two subcases:

Subcase 1 Suppose $p_1^* = \frac{1}{2}$ is party 1's only cutoff point. Then $\sigma_1(p) = S$ for all $p < \frac{1}{2}$, and by Table 1,

$$V(p) = (1-\chi)s + 2\chi sf(1-p).$$

This is party 1's best response if and only if $b(p; V_1) \leq c(p)$ for any $p \in (0, \frac{1}{2})$, or

$$(1+\mu)2\chi sf(1-p) \geq -\mu s + (-(1-\chi)s + (1+\mu)g)p, \tag{47}$$

for any $p \in (0, \frac{1}{2})$, which gives the upper bound $\alpha_1(\mu, \chi)$ for the value of $\frac{g}{s}$[35].

[35]More precisely,

$$\alpha_1(\mu, \chi) \equiv \inf_{p \in (0, \frac{1}{2})} \frac{\mu + (1-\chi)p - 2\chi(1+\mu)f(1-p)}{(1+\mu)p}.$$

Subcase 2 Suppose that there exist additional cutoff points other than $p_1^* = \frac{1}{2}$. Let $\hat{p}_1$ be the smallest such cutoff point. Then since the value of (σ_2, σ_M) is constant in the neighbor of $\hat{p}_1$, by Table 1, $V(p)$ solves the value-matching condition at $p = \hat{p}_1$

$$(1-\chi)s + 2\chi s f(1-\hat{p}_1) = \frac{\mu+1}{\hat{\mu}+1} g\hat{p}_1 + \frac{2\hat{\mu}\chi s}{\hat{\mu}+\mu+1} f(1-\hat{p}_1) + \tilde{C}_6 \hat{f}(\hat{p}_1), \tag{48}$$

and the smooth-pasting condition at $p = \hat{p}_1$

$$2\chi s f'(1-\hat{p}_1) = \frac{\mu+1}{\hat{\mu}+1} g + \frac{2\hat{\mu}\chi s}{\hat{\mu}+\mu+1} f'(1-\hat{p}_1) - \tilde{C}_6 \hat{f}'(\hat{p}_1),$$

where $\tilde{C}_6$ is an integration constant. Combining the above two equations, we have

$$(1+\mu)2\chi s f(1-\hat{p}_1) = -\mu s + (-(1-\chi)s + (1+\mu)g)\hat{p}_1, \tag{49}$$

Notice that both sides of the above equation are identical to both sides of inequality (47) with $p = \hat{p}_1$. Therefore, the solution of equation (47) exists if and only if $\frac{g}{s} \geq \alpha_1(\mu, \chi)$.

Note that there exist at most two solutions of equation (49), since the left-hand side of (49) is convex in p, while the right-hand side is linear in p. It turns out that the smaller solution must be $\hat{p}_1$. To see this, suppose the contrary: that $\hat{p}_1$ is the greater solution of (49), and let $p^\dagger$ be the smaller solution. Then for any $p \in (p^\dagger, \hat{p}_1)$, it must be

$$(1+\mu)2\chi s f(1-p) < -\mu s + (-(1-\chi)s + (1+\mu)g)p.$$

Therefore, $b(p; V_1) > c(p)$ for $p \in (p^\dagger, \hat{p}_1)$, which contradicts to the definition of $\hat{p}_1$.

Plugging in $\hat{p}_1$ to (48), we have the integration constant

$$\tilde{C}_6(\hat{p}_1) = \frac{1}{f(\hat{p}_1)} \left\{ (1-\chi)s - \frac{\mu+1}{\hat{\mu}+1} g\hat{p}_1 + 2\chi s \frac{\mu+1}{\hat{\mu}+\mu+1} f(1-\hat{p}_1) \right\},$$

then a function

$$\tilde{V}_1(p) \equiv \frac{\mu+1}{\hat{\mu}+1} gp + \frac{2\hat{\mu}\chi s}{\hat{\mu}+\mu+1} f(1-p) + \tilde{C}_6 \hat{f}(p),$$

is party 1's payoff function at $p' < 1/2$ if party 1 plays R_1 for $p \in [\hat{p}_1, p')$. If party 1 plays a safe action at p', it receives the payoff of $(1-\chi)s + 2\chi s f(1-p)$. Therefore playing R_1 is its best response at $p' < 1/2$ if and only if $\tilde{V}_1(p) > (1-\chi)s + 2\chi s f(1-p)$ for all $p \in [\hat{p}_1, p')$.

Define $\alpha_2(\mu, \chi)$ be the value such that $\frac{g}{s} < \alpha_2(\mu, \chi)$ if and only if $\tilde{V}_1(\frac{1}{2}) = \frac{\mu+1}{\hat{\mu}+1} \cdot \frac{g}{2} + \frac{\hat{\mu}\chi}{\hat{\mu}+\mu+1} s + \frac{\tilde{C}_6}{2} \leq s$. Then a simple calculation proves the following lemma:

Lemma 19. 1) If $\frac{g}{s} \in (\alpha_1, \alpha_2)$, then there exists a unique $\tilde{p}_1 \in (\hat{p}_1, \frac{1}{2})$ such that

$$\tilde{V}_1(p) > (1-\chi)s + 2\chi s f(1-p) \text{if } p \in (\hat{p}_1, \tilde{p}_1),$$
$$\tilde{V}_1(p) < (1-\chi)s + 2\chi s f(1-p) \text{if } p \in (\tilde{p}_1, \frac{1}{2}).$$

2) For $\frac{g}{s} > \alpha_2$, $\tilde{V}_1(p) > (1-\chi)s + 2\chi s f(1-p)$ for all $p \in (\hat{p}_1, \frac{1}{2})$.

Using this, we prove the main result:

Proposition 19. *For $\frac{g}{s} \in (\alpha_1, \alpha_2)$, there is a unique MPE of the game where $k_1^{-1}(o) = [0, \hat{p}_1] \cup (\tilde{p}_1, \frac{1}{2}]$ and $k_2^{-1}(o) = [\frac{1}{2}, 1-\tilde{p}_1] \cup [1-\hat{p}_1, 1]$.*

Proof. It suffices to prove that the above $k_1(p)$ for $p \in (0, \frac{1}{2}]$ is the unique best response to

$k_2(p) = 1$ for $p \in (0, \frac{1}{2}]$. There must exist some $\tilde{p}$ such that $k_1(p)$ switches from 0 to 1. We claim that $\tilde{p} = \tilde{p}_1$. If $\tilde{p} < \tilde{p}_1$, then $V_1(\tilde{p}-) > V_1(\tilde{p}+)$, so party 1 has a profitable deviation to play R_1. If $\tilde{p} > \tilde{p}_1$, then for $p \in (\tilde{p}, \tilde{p}_1)$ playing S is profitable deviation for party 1. (we show by comparing $b(p)$ and $c(p)$.) $\square$

E.2 Case 2: Proposition 14

Suppose $p_1^* < \frac{1}{2}$ and $p_2^* > \frac{1}{2}$. First we show that for any $p_2^* > \frac{1}{2}$, party 1's best response cutoff p_1^* is uniquely determined. The intuition is as follows. Observe that if the prior belief were less than p_2^*, the posterior never falls into $p \in (p_2^*, 1)$. Therefore, the party 1's optimal response does not depend on party 2's action for $p \in (p_2^*, 1)$. Using the fact that party 2 plays the safe action for all $p \leq p_2^*$, party 1's best response is determined.

Fix any $p_2^* > \frac{1}{2}$. Then the following five boundary conditions determine the unique p_1^*:

1. value-matching condition of V_1 at $p = p_1^*$:

$$(1-\chi)s + \chi C_5 \cdot f(1-p_1^*) = Bgp^* + A\chi C_5 \cdot f(1-p_1^*) + C_6 \cdot \hat{f}(p_1^*),$$

2. smooth-pasting condition of V_1 at $p = p_1^*$:

$$\chi C_5 \cdot f'(1-p_1^*) = Bg + A\chi C_5 \cdot f'(1-p_1^*) + C_6 \cdot \hat{f}'(p_1^*),$$

3. value-matching condition of V_1 at $p = \frac{1}{2}$:

$$Bg + A\chi C_5 + C_6 = g + C_2,$$

4. value-matching condition of W_1 at $p = \frac{1}{2}$:

$$C_5 = \chi g + A\chi C_2 + C_3,$$

5. value-matching condition of W_1 at $p = p_2^*$:

$$\chi g(p_2^*) + A\chi C_2 \cdot f(p_2^*) + C_3 \cdot \hat{f}(1 - p_2^*) = (1-\chi)s + \chi g(p_2^*) + \chi C_2 \cdot f(p_2^*).$$

The above boundary conditions jointly determine the unique p_1^*. Therefore, there exists a unique pair of (p_1^*, p_2^*) such that each cutoff belief is the best response to the other cutoff. Furthermore, $p_2^* = 1 - p_1^*$. In the best response cutoff (p_1^*, p_2^*), there is no other cutoff belief of the party. That is, party 1 chooses the safe policy for any $p \in (0, p_1^*)$ and vice versa. There exists $\alpha_3(\chi, \mu)$ such that the profile with (p_1^*, p_2^*) is an MPE if and only if $\frac{g}{s} > \alpha_3$.

E.3 Cases 3 and 4: Proposition 17

Finally, consider the case where $p_1^* = \frac{1}{2}$ and $p_2^* > \frac{1}{2}$. Suppose that the median voter's election rule at $p = \frac{1}{2}$ is to elect the ncumbent. Combining with $p_1^* = \frac{1}{2}$, party 1's payoff at $p = \frac{1}{2}$ with power is $V_1(\frac{1}{2}) = s$.

First analyze party 2's best response. Since $W_2(\frac{1}{2}) = s$ and W_2 is continuous at $\frac{1}{2}$, we have

$$W_2(p) = 2sf(p),$$

for all $p \geq \frac{1}{2}$. Then as in Case 2, there exists a cutoff $p_2^* > \frac{1}{2}$ only if it satisfies

$$(1+\mu)2\chi s f(p_2^*) = -\mu s + (-(1-\chi)s + (1+\mu)g)(1-p_2^*).$$

Using this p_2^*, we can derive $V_2(p)$ (again similar to Case 2) for $p \in [\frac{1}{2}, p_2^*)$. Since $p_2^* > \frac{1}{2}$, it must be the case that $V_2(\frac{1}{2}) > s$, which is equivalent to $\frac{g}{s} > \alpha_2$.

Now let us consider party 1's best response. Having fixed p_2^*, the value matching condition of $W_1(p)$ at $p = p_2^*$ and $p^* = \frac{1}{2}$ gives a complete specification of $W_1(p)$. Using this, we can compute another cutoff $\hat{p}_1$, which finishes the construction of the asymmetric equilibrium in Theorem 5.

Bibliography

Abreu, D., and F. Gul (2000): "Bargaining and Reputation," *Econometrica*, 68, 85–118.

Akerlof, G. A. (1970): "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84(3), 488–500.

Aragones, E., T. Palfrey, and A. Postlewaite (2007): "Political Reputations and Campaign Promises," *Journal of the European Economic Association*, 5(4), 846–884.

Ashworth, S., and E. B. de Mesquita (2008): "Electoral Selection, Strategic Challenger Entry, and the Incumbency Advantage," *The Journal of Politics*, 70(04), 1006.

Ausubel, L. M., and R. J. Deneckere (1989): "Reputation in Bargaining and Durable Goods Monopoly," *Econometrica*, 57(3), 511–31.

Banks, J. S., and R. K. Sundaram (1998): "Optimal Retention in Agency Problems," *Journal of Economic Theory*, 82(2), 293–323.

Besley, T. (2004): "Joseph Schumpeter Lecture: Paying Politicians: Theory and Evidence," *Journal of the European Economic Association*, 2(2-3), 193–215.

Bolton, P., and C. Harris (1999): "Strategic Experimentation," *Econometrica*, 67(2), 349–374.

Bolton, P., T. Santos, and J. A. Scheinkman (2011): "Outside and Inside Liquidity," *Quarterly Journal of Economics*, 126(1), 259–321.

Bonatti, A., and J. Hörner (2011): "Collaborating," *American Economic Review*, 101(2), 632–663.

Callander, S. (2011): “Why Does Political Power Alternate?,” *Working Paper*.

Camargo, B., and B. Lester (2011): “Trading Dynamics in Decentralized Markets with Adverse Selection,” *Working Paper, FRB Philadelphia and Sao Paulo School of Economics*.

Cho, I.-K. (1990): “Uncertainty and Delay in Bargaining,” *Review of Economic Studies*, 57(4), 575–95.

Choi, M. (2013): “Asset Prices and Information Transmission,” *Working Paper, University of Iowa*.

Compte, O., and P. Jehiel (2002): “On the Role of Outside Options in Bargaining with Obstinate Parties,” *Econometrica*, 70(4), 1477–1517.

——— (2004): “Gradualism in Bargaining and Contribution Games,” *Review of Economic Studies*, 71(4), 975–1000.

Daley, B., and B. Green (2012): “Waiting for News in the Market for Lemons,” *Econometrica*, 80(4), 1433–1504.

Dang, T. V., G. B. Gorton, and B. Holmström (2012): “Ignorance, debt and financial crises,” *Working Paper, Yale University and MIT.*

Deneckere, R. J., and M.-Y. Liang (2006): “Bargaining with Interdependent Values,” *Econometrica*, 74(5), 1309–1364.

Dixit, A. K., and F. Gul (2000): “The Dynamics of Political Compromise,” *Journal of Political Economy*, 108(3), 531–568.

Evans, R. (1989): "Sequential Bargaining with Correlated Values," *Review of Economic Studies*, 56(4), 499–510.

Ferejohn, J. A. (1986): "Incumbent Performance and Electoral Control," *Public Choice*, 50(1/3), 5–25.

Fuchs, W., A. Öry, and A. Skrzypacz (2012): "Transparency and Distressed Sales under Asymmetric Information," *Working Paper, UC Berkeley and Stanford University.*

Fuchs, W., and A. Skrzypacz (2010): "Bargaining with Arrival of New Traders," *American Economic Review*, 100(3), 802–836.

——— (2012): "Bargaining with Deadlines and Private Information," *RCER Working Papers.*

——— (2013): "Costs and Benefits of Dynamic Trading in a Lemons Market," *Working Paper, UC Berkeley and Stanford University.*

Fudenberg, D., D. K. Levine, and J. Tirole (1985): "Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information," in *Bargaining with Incomplete Information.*

Fudenberg, D., and J. Tirole (1991): *Game Theory.* MIT Press.

Gorton, G. B., and A. Metrick (2012): "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, 104(3), 425–451.

Guerrieri, V., and R. Shimer (2013): "Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality," *American Economic Review.*

Gul, F., and W. Pesendorfer (2012): "The War of Information," *Review of Economic Studies*, 79(2), 707–734.

Gul, F., H. F. Sonnenschein, and R. Wilson (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39(1), 155–190.

Hörner, J., and N. Vieille (2009): "Public vs. Private Offers in the Market for Lemons," *Econometrica*, 77(1), 29–69.

Hwang, I., and F. Li (2013): "Coasian Bargaining with an Arriving Outside Option," *PIER Working Paper.*

Janssen, M. C. W., and S. Roy (2002): "Dynamic Trading in a Durable Good Market with Asymmetric Information," *International Economic Review*, 43(1), 257–282.

Kaya, A., and K. Kim (2013): "Trading Dynamics in the Market for Lemons," *Working Paper, University of Iowa.*

Kaya, A., and Q. Liu (2013): "Transparency and Price Formation," *Working Paper, University of Iowa and Columbia University.*

Keller, R. G., S. Rady, and M. W. Cripps (2005): "Strategic Experimentation with Exponential Bandits," *Econometrica*, 73(1), 39–68.

Kim, K. (2011): "Information about Sellers' Past Behavior in the Market for Lemons," *Working Paper, University of Iowa.*

Klein, N., and S. Rady (2011): "Negatively Correlated Bandits," *Review of Economic Studies*, 78(2), 693–732.

Lauermann, S., and A. Wolinsky (2013): "Search with Adverse Selection," *Working Paper, University of Michigan and Northwestern University.*

Lee, J., and Q. Liu (2013): "Gambling Reputation: Repeated Bargaining with Outside Options," *Econometrica*, 81(4), 1601–1672.

Levin, J. (2001): "Information and the Market for Lemons," *RAND Journal of Economics*, 32(4), 657–666.

Maskin, E., and J. Tirole (2004): "The Politician and the Judge: Accountability in Government," *American Economic Review*, 94(4), 1034–1054.

Merlo, A., and C. A. Wilson (1995): "A Stochastic Model of Sequential Bargaining with Complete Information," *Econometrica*, 63(2), 371–99.

Moreno, D., and J. Wooders (2010): "Decentralized Trade Mitigates the Lemons Problem," *International Economic Review*, 51(2), 383–399.

Morris, S., and H. S. Shin (2012): "Contagious Adverse Selection," *American Economic Journal: Macroeconomics*, 3(4), 1–21.

Noldeke, G., and E. van Damme (1990): "Signalling in a Dynamic Labour Market," *Review of Economic Studies*, 57(1), 1–23.

Plantin, G. (2009): "Learning by Holding and Liquidity," *Review of Economic Studies*, 76(1), 395–412.

Samuelson, L. (1987): "A Test of the Revealed-Preference Phenomenon in Congressional Elections," *Public Choice*, 54(2), 141–169.

Sobel, J., and I. Takahashi (1983): "A Multistage Model of Bargaining," *Review of Economic Studies*, 50(3), 411–426.

Spier, K. E. (1992): "The Dynamics of Pretrial Negotiation," *Review of Economic Studies*, 59(1), 93–108.

Strulovici, B. (2010): "Learning While Voting: Determinants of Collective Experimentation," *Econometrica*, 78(3), 933–971.

Swinkels, J. M. (1999): "Education Signalling with Preemptive Offers," *Review of Economic Studies*, 66(4), 949–70.

Vincent, D. R. (1989): "Bargaining with Common Values," *Journal of Economic Theory*, 48(1), 47–62.

Yildiz, M. (2004): "Waiting to Persuade," *Quarterly Journal of Economics*, 119(1), 223–248.

Zhu, H. (2012): "Finding a Good Price in Opaque Over-the-Counter Markets," *Review of Financial Studies*, 25(4), 1255–1285.