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# Essays in Macroeconomics

Mauricio Calani

University of Pennsylvania, [mcal@sas.upenn.edu](mailto:mcal@sas.upenn.edu)

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# Essays in Macroeconomics

## **Abstract**

This dissertation consists of two chapters on macroeconomic dynamics. The first chapter examines the consequences of technological diversification on long-run growth. I claim that the typical undiversified emerging economy would grow 0.23% faster if it were as technologically diversified as advanced economies. I obtain such estimate by building a growth model in which technological diversification facilitates inventor mobility across sectors, and through it, R\&D. Invention-relevant-knowledge is costly to accumulate, and more importantly, it is inalienable; inventors cannot disinvest cumulative knowledge capital when hit by a bad shock, and cannot sell it. Anticipation to this asymmetric effect leads inventors to refrain to accumulate too much capital. Technological diversification provides a way out. Inventors in a diversified economy can transport themselves to a better sector. A second result of the model is the quantification of the effect of volatility on growth. The model predicts that reducing standard deviation by 1% implies 0.08% faster growth. In the second chapter I extend the sovereign default model in the tradition of Eaton-Gersovitz (1981), to consider the consequences of strategic bailout from a lender country. Default is strategic as debt is not enforceable, and bailouts are strategic, as there is no obligation to extend them. The introduction of this implicit guarantee on sovereign debt has two opposing effects on its pricing. First, spreads are lower because the expected recovery rate is higher. However, if bailouts are a possibility after declaring default, and part of outstanding debt will dilute, then the value of exerting the default option is higher, and these events more frequent. This would raise spreads. I show that bond price schedule is decreasing in the haircut fraction of debt after re-structuring, is bounded from below, and is less sensitive to income fluctuations with the inclusion of a bailout probability. I also show that bailouts are more likely to happen when there are good realizations of income for the creditor economy. A final application can generate spreads close to zero for Italy even when income fluctuations and debt accumulation would predict otherwise. Shutting off the bailout implicit guarantee would have raised spreads from 0.03% to 1.8%.

## **Degree Type**

Dissertation

## **Degree Name**

Doctor of Philosophy (PhD)

## **Graduate Group**

Economics

## **First Advisor**

Guillermo L. Ordoñez

## **Keywords**

Innovation, Inventor mobility, Strategic Bailout, Technological diversification

## **Subject Categories**

Economics

ESSAYS IN MACROECONOMICS

Miguel Mauricio Calani Cadena

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2016

Supervisor of Dissertation

---

Guillermo Ordoñez

Associate Professor, Department of Economics

Graduate Group Chairperson

---

Jesús Fernández-Villaverde

Professor, Department of Economics

Dissertation Committee:

Ufuk Akcigit

Assistant Professor, Department of Economics, University of Chicago

Jesús Fernández-Villaverde

Professor, Department of Economics

ESSAYS IN MACROECONOMICS

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*To my family*

## ACKNOWLEDGEMENT

I owe my advisor, Guillermo Ordoñez my deepest gratitude for his patience and encouragement from the very first time I stepped into his office. I am also indebted to my committee members, Ufuk Akcigit, Jesús Fernández-Villaverde, and Guillermo Ordñez for their continuous support, and for teaching me how to think carefully, and rigorously, about macroeconomics.

Besides my committee members I am grateful to all the faculty of the Department of Economics, in particular to Dirk Krueger and Jeremy Greenwood, from whom I first learned to enjoy macroeconomics. The Penn Macro Club is an amazing group of people from whom I benefited immensely. My conversations with Murat Alp Celik, Gustavo Camilo, Juan Hernández, Olga Itenberg, Yena Park, Constanza Vergara, David Weiss and Daniel Wills, were much appreciated.

I also owe my friends Gloria, Francesco, Michael, Pau, Alberto, Ana María, Rodrigo, Rossa, and Andrea my gratitude for making my years in Philadelphia unforgettable. Yumi Koh's friendship was honest and unconditional, and helped me in the worst days of this adventure. This dissertation would not have been possible without the unconditional support and love of my parents, Sonia and Martín, and my siblings

Alfredo and Javier. I will always be immensely grateful to them for believing in me, even more than I did.

Finally, I cannot express well enough my gratitude to Jenny, the strongest, warmest and most challenging woman I ever met. Her love was stronger than our circumstances. She made a home and a shelter for the both of us, and gave me the strength to complete this endeavor. I will always treasure her love and sacrifices.

## ABSTRACT

### ESSAYS IN MACROECONOMICS

Miguel Mauricio Calani Cadena

Guillermo L. Ordoñez

This dissertation consists of two chapters on macroeconomic dynamics. The first chapter examines the consequences of technological diversification on long-run growth. I claim that the typical undiversified emerging economy would grow 0.23% faster if it were as technologically diversified as advanced economies. I obtain such estimate by building a growth model in which technological diversification facilitates inventor mobility across sectors, and through it, R&D. Invention-relevant-knowledge is costly to accumulate, and more importantly, it is inalienable; inventors cannot disinvest cumulative knowledge capital when hit by a bad shock, and cannot sell it. Anticipation to this asymmetric effect leads inventors to refrain to accumulate too much capital. Technological diversification provides a way out. Inventors in a diversified economy can transport themselves to a better sector. A second result of the model is the quantification of the effect of volatility on growth. The model predicts that reducing standard deviation by 1% implies 0.08% faster growth. In the second chapter I extend the sovereign default model in the tradition of Eaton and Gersovitz (1981) to consider the consequences of strategic bailout from a lender country. Default is strategic as debt is not enforceable, and bailouts are strategic, as



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## Chapter 1

# Technological Diversification, Volatility, and Innovation

### Abstract

Are there any consequences on long-run growth from having a more diversified economy? In this paper I claim, yes: the typical undiversified emerging economy would grow 0.23% faster if it was as technologically diversified as the typical advanced economy. I obtain such estimate by building a growth model in which technological diversification facilitates *inventor mobility across sectors*, and through it, has the ability to affect the level of research investment each innovator makes. Inventors care about diversification because invention-relevant-knowledge (which I label *knowledge-capital*), is costly to accumulate, and more importantly, it is *inalienable*. Inventors cannot disinvest previous research capital accumulation when hit by a bad shock, and cannot sell it. Anticipation to this asymmetric effect of positive and negative shocks leads inventors to refrain to accumulate too much capital. Technological diversification provides a way out. While unable to undo or sell knowledge capital, inventors in a diversified economy can transport it to another sector where such capital is valuable for producing ideas. A second result of the model is the quantification of the effect of volatility on long-run growth. In particular, the model implies that reducing standard deviation by 1% implies 0.08% higher annual growth for the average economy in terms of diversification.



## 1.1 Introduction

Inventors are at the center of the creation of new ideas, improving processes, productivity, and ultimately economic growth. To create new and useful ideas, these inventors need to learn skills, customize equipment, accumulate experience, and formal knowledge in their respective field. We will refer to “knowledge capital”<sup>1</sup> when thinking about this cumulative stock of know-how, which serves as input for the production of ideas. We will also refer to “research and development” spending, or simply R&D, to the flow of effort, time and physical resources spent in order to augment this knowledge capital<sup>2</sup>.

Knowledge capital is different from physical capital in two important aspects. First, knowledge capital can only be adjusted slowly. While physical capital can rise substantially with one periods abrupt investment, knowledge capital accumulation requires learning, understanding, and trial and error, all of which require time. That is, there are diminishing returns to R&D when attempting to augment the level of knowledge capital in a given period. Second, and much more important, knowledge capital is different from physical capital in that it is “inalienable”. It is difficult to sell or transfer it to another inventor.

The second characteristic becomes crucial for the analysis of knowledge-capital

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<sup>1</sup>This term was first used by Klette and Kortum (2004) pp.991, standing for “...all the skills, techniques, and know-how that [a firm] draws on as it attempts to innovate”.

<sup>2</sup>The way we are thinking about innovation can be illustrated by a well-known quote by Thomas Edison. It is said that after trying 1000 times to develop a successful light-bulb prototype, a reporter once asked him: “How did it feel to fail 999 times?”, to whom Edison replied: “I did not fail 999 times; the light bulb was an invention with 1000 steps”

accumulation because inalienability eliminates the option for the inventor to ex-post reduce the level of knowledge capital, if he deems his knowledge capital is above the optimal. This is particularly interesting if sectors are subject to shocks. If good shocks (those that raise the value of holding knowledge-capital in such sector) affect the inventor's sector, he benefits from them, and from accumulating more knowledge capital. On the other hand, if bad shocks affect their sector, the inventor would like to sell previous investment, but he cannot. Anticipating this possibility, a rational inventor would be cautious not to accumulate as much capital as one who can freely adjust their holdings of knowledge capital upwards as well as downwards.

A “technologically diversified” economy could provide a way out to the inventor with above-optimal holding of knowledge capital. And this is what this paper is mostly about. I refer to a “technologically diversified” economy, as one in which there are many production sectors where inventors could innovate<sup>3</sup>. In a diversified economy, knowledge-capital may be useful to innovate in more than one sector. Therefore, while inventors may not be able to sell and adjust downwards their holdings of knowledge capital, they may be able to move themselves to a different sector with better prospects. The decision to move to a different sector will depend on how technologically distant the origin and destination sectors are, that is, which is going to be strongly correlated to how diversified the economy is.

In this paper we build an endogenous growth model with inventor mobility

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<sup>3</sup>Note that we do not use the term “diversification” in the finance sense, where it implies reducing variance of returns or hedging risks in a deliberate way.

across sectors, in order to quantify the contribution of technological diversification to economic growth. In this model, any given inventor, in every period, decides on two things. First, she decides on how much to invest in R&D to augment her knowledge capital, and second, on which sector to attempt to innovate. For the second decision, she needs to know not only how profitable working in the destination sector is, but also how relevant her knowledge-capital is going to be in such sector. It may be very useful if the origin and destination sectors are somehow intellectually related, or it may be completely useless if the sectors are very different. Those sectors in which her knowledge capital is more useful are said to be “closer”, and those sectors in which her knowledge capital is irrelevant or useless are said to be “distant”. Technologically diversified economies are populated with many sectors, and as such it is more likely that there are sectors “close” to each other, facilitating inventor mobility across them. If inventors know that after a bad shock in the future, they will have the option to move to a different sector where their knowledge-capital will be useful, they will be more prone to invest more today. On the other hand, if they know that after a bad shock in the future they will have to stay in their sector with more-than-optimal knowledge-capital, they will cut R&D investment today, in anticipation. The former situation is the case of a well diversified economy, and the latter of a poorly diversified one. A calibrated version of this model predicts that – keeping the volatility of shocks constant – if the typical emerging economy (namely, Mexico) would be as diversified as a developed economy (namely, Korea),

then innovators in the former would on average be 7% more productive, which in the model would translate into growing an extra 0.23% every period. In addition to this last calculation we can also weight the cost of volatility on output growth. In particular, for the median economy in terms of diversification, shutting down volatility – which is on average 3% – implies 0.24% higher growth every period.

### 1.1.1 Evidence on inventor mobility across sectors

How does sector mobility of inventors look like in reality? Consider the story of Mr. Shigeru Morita, a Japanese inventor who in 1969 applied for a metallurgy patent called “*a process for the purification of lower olefin gases, which comprises removing and/or converting into easily removable materials harmful ingredients*”. To be capable of achieving such innovation one requires a very specific skill: *manipulation and degradation of poisonous gases*. After the oil crisis that hit Japan in 1973, the metallurgy sector became lethargic, and by year 2000 Mr. Morita was working in the pharmaceutical sector and applied for a patent which claims the “*invention of a DNA encoding protein to treat infectious airborne diseases such as asthma and chronic obstructive pulmonary disease*”. The medical trials involved in this patent required infecting mice with airborne diseases: a task suited for the skills developed by Mr. Morita. For him, moving to the pharmaceutical sector was not a lucky deviation from his initial sector. It was an informed decision when circumstances changed, and Morita was able to do this, because he could transport his expertise

to an existing sector. How would the inventor reshuffling be for a Venezuelan oil-engineer after the 2015 oil price bust?. More importantly, how invested would she be in R&D, knowing that she has little option to change sectors? The answer this paper provides is that she will be less likely to invest as much as Morita would have.

The former example on inventor mobility across sectors is far from an outlier in the data. In particular, let us analyze in a more systematic way two observations about the behavior of inventors, which we will use to build the model in Section 1.2.

**Stylized Fact 1** *Inventors do move sectors: There is a sizeable fraction of inventors who patent inventions in sectors they did not work in before*

**Stylized Fact 2** *Conditional on moving sectors, inventors in more diversified economies move shorter intellectual distances than those in low diversified economies*

Next, we elaborate on each of these two facts using information from patent applications and patent citations.

**Stylized Fact 1** *Inventors do move sectors.*

For this we need to consider events in which inventors switch sectors, and events in which they do not switch sectors in their next patent application. Let  $\mathcal{A}$  be the set of inventor  $A$ 's lifetime innovations, with cardinality  $\#\mathcal{A}$ . Consider the probability of

observing two consecutive successful innovations  $\{a^k, a^{k+1}\}$  in two different sectors  $m, n$  by the same inventor  $A$ ,  $P[a_n^{k+1}|a_m^k]$ .

$$P[a_n^{k+1}|a_m^k] = \frac{1}{|\#\mathcal{A} - 1|} \sum_{i \in \mathcal{A}} \mathbf{1}\{a^{i+1} \in n, a^i \in m\} \quad (1.1.1)$$

for  $n \neq m$ , and where  $\mathbf{1}\{\cdot\}$  is the indicator function. Note that equation (1.1.1) considers every two consecutive inventions by any inventor. If an inventor switches a few times only, and  $\#\mathcal{A} \rightarrow \infty$ , then this probability goes to zero. If this inventor switches sectors every time she applies to a new patent, (1.1.1) tends to one.

Alternatively we can “correct” equation (1.1.1) to not consider returning to a previous research sector. We could think that a lucky deviation is sometimes observed, but there is no reason to think that the inventor deliberately wanted to innovate in a different sector. Or, that an inventor works in two sectors at the same time and we observe innovations in two sectors, but it does not mean she switches all the time. For that, define  $\mathcal{S}_A^k$  the set of sectors that inventor  $A$  has visited up until innovation  $k$ . A robust measure of inventor mobility –which does not consider sector migration if the destination sector has previously been visited by the inventor– is defined by  $P[a_n^{k+1}|n \notin \mathcal{S}_A^k]$  with,

$$P[a_n^{k+1}|n \notin \mathcal{S}_A^k] = \frac{1}{|\#\mathcal{A} - 1|} \sum_{i \in \mathcal{A}} \mathbf{1}\{a^{i+1} \in n | n \notin \mathcal{S}_A^k\} \quad (1.1.2)$$

Note  $\mathcal{S}^{(\#\mathcal{A})}$  is the set of all the sectors an inventor has visited in their lifetime.

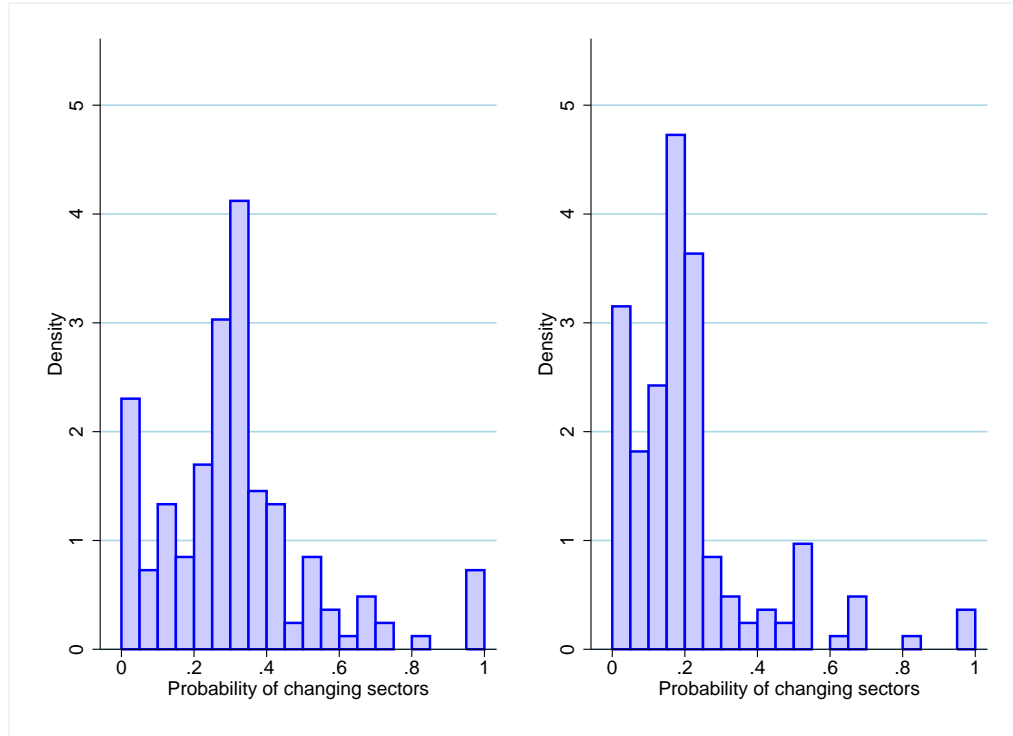


Figure 1.1: **Frequency of Inventor Mobility Across Sectors.** (a) Left hand side panel shows the frequency of sector mobility of two consecutive sectors using equation 1.1.1. (b) Right hand side panel corrects mobility by not considering sector movements to a returning sector, considering only events when the inventor moves to a new sector they have never been before. **Source:** NBER Patent Database and Li et al. (2014)

In figure (1.1) I plot the histogram of both measures of inventor mobility. The horizontal axis represents the probability of switching, and the vertical the frequency of observing such probability. The message is clear: we do observe that inventors visit new sectors and it is not unusual for them not to return to their original sector.

**Stylized Fact 2** *Conditional on moving sectors, inventors in more diversified economies move shorter intellectual distances than those in low diversified economies*

First, we need to define what we mean by “distance” between two technology classes. Intuitively we want to capture the fact that two sectors are similar enough that an inventor working in one of them, could easily use knowledge-capital from her current sector, to innovate on her destination sector. If knowledge capital is very relevant in the destination sector, then we say that it is “close”. Alternatively, if knowledge capital is not very useful in the destination sector, then we say that it is “distant” from her current sector.

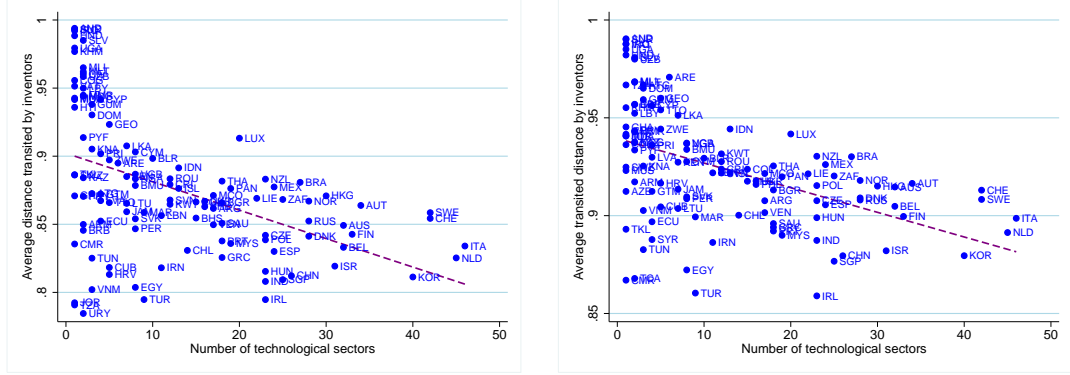
Empirically, technological classes can be represented by the first two digits of their International Patent Classification (IPC) code (IPC2)<sup>4</sup>. We use data from the NBER-USPTO Patent Database which contains information about application of patents, their technology class, and their inventors, and combine it with data on cross-citations from Li et al. (2014). According to our definition, a destination sector is close to another, if patents in the former frequently cite previous work on the latter. Citations provide valuable information about the relevance and usefulness of certain type of knowledge for producing innovation in any field. We exploit such information in our measure of distance.

We will consider two definitions of distance, and asymmetric and a symmetric one, in that order. First, consider an inventor who wants to move from sector “E” to sector “F”. If every patent in sector “F” cites a patent in “E”, an inventor with expertise in “E” should find easy to work in sector “F”. Accordingly, define

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<sup>4</sup>The analysis is robust to considering only the first digit of the IPC Code, or the first two. We follow Akcigit, Celik and Greenwood (2016) in considering the first two digits as an industry cluster.





(a) Asymmetric distance

(b) Symmetric distance

Figure 1.2: **Technological Distance Transited by Inventors and Technological Diversification.** Panels show in the vertical axes the (a) asymmetric and (b) symmetric intellectual distance country average, and the horizontal axis shows the number of sectors in each economy.. Source: NBER Patent Database and Li et al. (2014)

$\#cit(E \leftarrow F)$  as the number of patents in sector “F” that cite any patent in sector “E”. Also, let  $\#cit(F)$  be the number of patents in sector “F”. Then, I will define the asymmetric intellectual distance to go from “E” to “F” as,

$$d_A(E, F) \equiv 1 - \frac{\#cit(F \leftarrow E)}{\#cit(F)} \quad (1.1.3)$$

with  $0 < d_A(E, F) < 1$ . This measure is also intuitive. If all patents in F cite at least one patent in E, then this measure delivers  $d(E, F) = 0$ . If not one patent in F cites a previous invention in E, then this distance is  $d(E, F) = 1$ . The shorter the distance, the easier it should be for an inventor in sector E to move to F. In the vertical axis of panel (a) in figure (1.2) I compute the the average across inventors, of the transited intellectual distance by inventors who resided in each country, and who decided to move to a different sector. In the horizontal axis I use the number

of intellectual technology classes available in every country for a base year (in this case 1999). The choice of the base year is immaterial for the empirical fact, and shows that conditional on moving sectors, the transited distance by inventors is shorter when the economy is more diversified. Alternatively we can also say that short distance movements are only possible in highly diversified economies.

For robustness, let us also consider the symmetric measure of distance proposed by Akcigit, Celik and Greenwood (2016). In particular, they are –to my knowledge– the first ones to propose a notion of intellectual distance. Following their notation to define the distance between sectors  $X$  and  $Y$ , consider the number of patents (in any sector) which cite a patent in sector  $X$  or in sector  $Y$ ,  $\#(X \cup Y)$ . Also consider all patents that cite at least one patent in  $X$  and  $Y$  simultaneously, and name it  $\#(X \cap Y)$ . The definition of distance between two intellectual sectors  $X$  and  $Y$  the authors propose, is then given by

$$d(X, Y) \equiv 1 - \frac{\#(X \cap Y)}{\#(X \cup Y)} \quad (1.1.4)$$

with  $0 \leq d(X, Y) \leq 1$ . If every patent that cites  $X$  also cites  $Y$  then this distance is  $d(X, Y) = 0$ . If no patent that cites both classes but cites one of them, then this measure delivers  $d(X, Y) = 1$ . In panel (b) of figure (1.2) we use this measure to calculate the average transited distance of an inventor in a given country who moves from any sector  $X$  to  $Y$ . Again, in the horizontal axis we use number of available intellectual technology class sectors in a base year. The two measures of distance,

asymmetric and symmetric are computed differently and consider different sets of patents in each calculation but deliver the same message. Inventors who move to another sector, in highly diversified economies usually do not move long distances<sup>5</sup>.

### 1.1.2 Position and contribution to the literature

This paper is inserted in two strands of the literature. First, it contributes to the literature on firm heterogeneity and industry dynamics. The literature on firm dynamics is well established but is mostly centered around entry and exit decisions, and firm size, not on sector migration decisions. Starting from Hopenhayn (1992), who first provides a tractable model of firm entry and exit in a competitive industry, to the work of Grossman and Helpman (1991), Klette and Kortum (2004), and Acemoglu and Cao (2015), firms produce in an economy with a continuum of goods and decide on research expenditure considering entry and exit probabilities<sup>6</sup>. By assuming there is a continuum of goods or products there is no room for thinking about sector migration. Models outright assume diversification is total if every variety is an industry, or that diversification is null if every variety is a product within an industry<sup>7</sup>. This paper expands the literature by considering how the existence of

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<sup>5</sup>These measures show average inventor migration behavior by country with respect to the level of technological diversification. However any given inventor can move to distant sectors even when there are closer sectors. To get a glimpse of the distribution of technological distances irrespective of inventor behavior refer to Figure (A.1) which depicts the minimum distance transited in a country contrasted with the level of technological diversification. I will use this latter information in order to calibrate the parameter governing technological distance.

<sup>6</sup>Among others, some papers who address directly firm heterogeneity and innovation include Klepper (1996), Aghion and Howitt (1996), but none gives a role to the abundance – or lack thereof – of sectors in the innovation process

<sup>7</sup>Again, when I refer to diversification I do not mean any hedging strategy, financial portfolio decision or risk reducing decision. In Acemoglu and Zilibotti (1998), the authors use the term

a finite and small set of industries affects the incentives to accumulate knowledge-capital. In our model there is a continuum of varieties *within each* industry and there is entry and exit within each industry, but there are a finite number of industries available for firm/inventor migration. There are a few exceptions to implicitly assuming that diversification is total. First, Akcigit, Hanley and Serrano-Velarde (2014) build a model to contrast basic and applied research by firms who can be present in several industries. They give a rationale as to why firms may be operating in several industries at the same time: basic research is more likely to be applied in several industries, and therefore, it is potentially more profitable to be present in all such industries [Evidence supporting their story can also be found in Garcia-Vega (2006), who use data from 15 EU countries at the firm level]. However, their focus is not on the abundance or scarcity of industries, nor on the implications of such cardinality on innovation incentives, but the choice of how many of them are visited by any firm. The second exception that gives a role to the number of industries is Acemoglu and Zilibotti (1998), who build a model of technology adoption, where capital scarcity determines that only a small number of technologies can be adopted at early stages of development, and as such, industry idiosyncratic risk is not well diversified. As an economy receives good shocks, it transits to adopting more technologies and reduces variance of output. However they do not address directly the implications of the number of sector in the commodity space on entrepreneur

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“diversification” to refer to the choice of activities/projects within a given commodity space, considering risk diversification.

decisions. This paper does.

Secondly, this paper also contributes, on the theory side, to the discussion of the link of growth and volatility, which has been mostly empirical. In an early influential paper, Ramey and Ramey (1995) presented empirical evidence, that there is negative and significant relation between volatility and growth in non-OECD economies <sup>8</sup>. For OECD economies this relationship seems to vanish, which motivates Aghion et al. (2005)(AABM). Under the assumption that the key difference between OECD and non-OECD economies is financial development, AABM build a model of financial friction in which higher volatility is jointly determined with lower growth for financially under-developed economies. Our model provides a causal link for the negative correlation of growth and volatility but the mechanism is rather different. In particular, we do not assume credit constraints or any financial friction. The main friction is real, that moving from one sector to another hinges on how technologically “distant” these sectors are. The easier it is to move sectors (diversified economies), the higher the knowledge-capital accumulation and the innovation rate.

Methodologically, this paper contributes to the literature on endogenous technical change by incorporating additional and alternative ways of thinking about the innovation arrival rate functional form. In particular, I argue that innovation is the result of a lengthy process of experience accumulation, *knowledge capital*. To

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<sup>8</sup>Of course, they are not alone in the literature. Among others that address this correlation are Acemoglu and Zilibotti (1998), Koren and Tenreyro (2007), Kormendi and Meguire (1985), and McConell and Perez-Quiros (2000) and Angeletos (2007)

the extend of our knowledge, all endogenous growth models assume that arrival rate of ideas is a function of current research effort alone. One notable exception is the notion of “knowledge-capital” used in Klette and Kortum (2004), who argue that R&D is more productive for firms who have a history of past innovation. The functional form of innovation arrival rate is a function of current R&D and the cardinality of previous innovations, which have not been yet superseded,  $n$ . Klette and Kortum (2004) is different from this paper in that the contribution of  $n$  to the innovation arrival rate is independent of the the sector or discipline in which these past innovations occurred<sup>9</sup>. In this paper inventors accumulate knowledge in a given sector.

On the empirical side, we contribute to the literature providing evidence on inventor mobility across sectors using micro data. We show how inventor productivity and mobility depends on the diversification of the economy they work in, on top of profitability and cost of research. This paper is not alone in this extension. Recent data availability on inventor identities has resulted in a fruitful research agenda . Perhaps one of the most interesting examples of it is recent work by Akcigit, Baslandze and Stantcheva (forthcoming), who consider a different type of inventor mobility, namely geographical mobility due to taxation motives.

**Outline:** Under the light of the empirical facts sketched previously, in the next section we build a multi-sector endogenous growth model. In this model inventors

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<sup>9</sup>Which in their setup is innocuous, as they have a continuum of varieties of products

accumulate knowledge-capital in a given sector, which they also choose. To decide whether to move sectors, they consider how valuable their previous knowledge capital is in the destination sector. Knowledge capital accumulation in this setup is an irreversible investment as it is inalienable. In Section 1.3 I analyze the main predictions of the model and in Section 1.4 I explore the validity of the main predictions of the model quantitatively and contrast it to patent/inventor data, cross country aggregate data. Section 1.5 concludes.

## 1.2 A Simple Model

The simplest possible model which incorporates the stylized facts in Section 1.1 consists of a consumer which supplies labor inelastically, and every period consumes a good  $c_t$ ; which is a composite different sectoral final goods. In each *industry*, the sectoral final good requires labor and a continuum of intermediate goods. Innovators work on enhancing the quality of these latter goods. In particular, each of them is produced only by the entrepreneur who holds the latest blueprint (vintage) and therefore, can do it most efficiently. She does so until superseded by another innovator. This is the engine of economic growth in this model, just as in the basic Schumpeterian model in Aghion and Howitt (1992)<sup>10</sup>. In order to innovate, inven-

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<sup>10</sup>While the simplest version of the model in this paper, presents the number of sectors in a given economy as fixed, there exists evidence that research effort has a positive and non-negligible probability of producing innovations in different sectors, and also creating new sectors. That is, innovation can endogenously make an economy more diverse [see Akcigit, Hanley and Serrano-Velarde (2014) for a model in which firms may have incentives to be in many sectors simultane-

tors accumulate knowledge capital specific to their sector, but they may move to a different sector. Let us then start with preferences and technology in the three layers of production. Later let us refer to the innovation problem and the equilibrium conditions.

### 1.2.1 Preferences

The representative household supplies inelastic labor which I normalize to unity, discounts the future according to a subjective discount factor  $\beta$ , and consumes a final good  $c_t$ , from which he derives instant utility  $u(c_t)$ , with  $u(\cdot)$  non-decreasing, strictly concave, and that satisfies  $u(0) = 0$ , and Inada conditions  $\lim_{c \rightarrow 0} u_c(\cdot) = \infty$  and  $\lim_{c \rightarrow \infty} u_c(\cdot) = 0$ . The sources of income for this household are her wage,  $w_t$ , interest payments on past savings in uncontingent assets ( $a_t$ ), and net profits from firms,  $\bar{\Pi}_t$ . In particular, the problem of the household is as follows. Taking as given wages, interest rates and profits,  $\{w_t, r_t, \bar{\Pi}_t\}_{t=0}^{\infty}$ , the representative household chooses  $\{c_t, a_{t+1}\}_{t=0}^{\infty}$ , to solve the following program,

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \quad (1.2.1)$$

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ously]. This feedback channel would only make the argument of this paper even more relevant. The two channels combined would result in a virtuous circle of diversification, or a poverty trap of non-diversification. In the basic version of the model I will abstract from this feedback effect and leave it as an extension. There are two reasons to do so. First, pedagogically, abstracting from the feedback channel highlights the main economic mechanism of the effect of diversity on innovation. Second, we can think of this model as one that explains innovation dynamics taking diversification of countries as initial condition, and which allows us to think about comparative statics, or, we can think that while the feedback channel is present, diversification of economies evolves slowly.



subject to

$$c_t + a_{t+1} \leq (1 + r_t)a_t + w_t L_t + \bar{\Pi}_t \quad (1.2.2)$$

where the instant utility function is constant relative risk aversion with parameter  $\gamma$ ,  $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$ . The first order condition of this problem is the usual Euler Equation which relates consumption in  $t$  and  $t + 1$ ,

$$\left( \frac{c_{t+1}}{c_t} \right)^\gamma = \beta(1 + r_{t+1}) \quad (1.2.3)$$

### 1.2.2 Technology

There are three layers of production. One final good, sectoral final goods and intermediate goods. The production technology in this model follows standard models of endogenous growth.

#### Final Good Producer

There is one final good that is ready for consumption. It is the aggregation of the sectoral final goods of every industry  $m$ ,  $m \in \mathcal{M}_t = \{1, 2, \dots, M_t\}$ , with  $M_t$  countable and finite. Let  $Y_t$  be the final good, and  $y_{m,t}$  the sectoral final good in sector  $m$ , such that,

$$Y_t = \left( \sum_{m \in \mathcal{M}_t} y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (1.2.4)$$

the final goods produced in each industry can either be perfect complements (if  $\epsilon$  approaches zero), perfect substitutes (when  $\epsilon$  approaches infinity) or be somewhere in between.

In each period  $t$  and sector  $m$ , final sectoral output  $y_{m,t}$  is produced using a continuum of varieties  $i \in [0, 1]$  of intermediate capital goods  $x_{i,m,t}$  using the same technology as in Grossman and Helpman (1991),

$$\log y_{m,t} = \int_0^1 \log x_{i,m,t} di \quad (1.2.5)$$

The problem of the (sectoral) final good producer is static as intermediate goods  $x_{i,m,t}$  are fully depleted in the production of the final good. In particular, given prices of intermediate goods  $\{p_{i,m,t}\}_{m \in \mathcal{M}, i \in [0,1], t=0}^{t=\infty}$ , the  $m$ -good producer chooses intermediate goods  $\{x_{i,m,t}\}_{i \in [0,1]} \geq 0$  to solve the following program every period  $t$ ,

$$\max_{\{x_{i,m,t}\}_{i \in [0,1]} \geq 0} p_{m,t} \exp \left( \int_0^1 \log x_{i,m,t} di \right) - \int_0^1 p_{i,m,t} x_{i,m,t} di \quad (1.2.6)$$

which readily yields the following first order condition (demand for intermediate goods),

$$x_{i,m,t}^D = \left( \frac{p_{m,t}}{p_{i,m,t}} \right) y_{m,t} \quad (1.2.7)$$

### Intermediate Good Producer

Each variety of intermediate good  $i \in [0, 1]$  (for sector  $m$ , in period  $t$ ) is produced with a linear technology on labor  $\ell_{i,m,t}$ ,

$$x_{i,m,t} = (\ell_{i,m,t} + \xi_{m,t})q_{i,m,t} \quad (1.2.8)$$

where  $\xi_{m,t}$  is a sector specific productivity shock. The production of the  $i$  variety grows proportionally to its quality  $q_{i,m,t}$ , which in turn grows with innovation according to a quality ladder of step size  $\sigma > 0$ .

$$q_{i,m,t+1} = \begin{cases} (1 + \sigma)q_{i,m,t} & \text{if there exists innovation} \\ q_{i,m,t} & \text{otherwise} \end{cases} \quad (1.2.9)$$

The labor augmenting shock  $\xi_{m,t}$  evolves exogenously according to

$$\xi_{m,t+1} = \rho_\xi \xi_{m,t} + \sigma_\xi \epsilon_{m,t+1} \quad (1.2.10)$$

with random shock  $\epsilon_{m,t} \sim N(0, 1)$ , independent from  $\epsilon_{n,t}$  for any  $n \neq m$ . Note that the unconditional mean of this process is zero which means that if  $\sigma_\xi = 0$  we are back to the usual deterministic production function. The incumbent producer charges a price consistent with Bertrand competition, the marginal cost of the closest follower.

This pricing deters the latter to use her vintage of quality  $\tilde{q}_{i,m,t}$ . Then,

$$p_{i,m,t} = \frac{w_t}{\tilde{q}_{i,m,t}} = \frac{(1 + \sigma)}{q_{i,m,t}} w_t \quad (1.2.11)$$

In every variety  $i$ , the owner of the latest blueprint takes home the monopolistic profit  $\pi_{i,m,t}$  every period:

$$\pi_{i,m,t} = \left[ \frac{w_t}{q_{i,m,t}} (1 + \sigma) x_{i,m,t} - w_t \frac{x_{i,m,t}}{q_{i,m,t}} + w_t \xi_{m,t} \right]$$

which can be expressed in terms of average sector qualities  $\{Q_{m,t}\}_m$ , sector shocks  $\{\xi_{m,t}\}$ , and average total quality  $Q_t$ ,

$$\pi_{i,m,t} = \left[ \left( \frac{\sigma}{1 + \sigma} \right) \left( \frac{Q_t}{Q_{m,t}} \right)^{1-\varepsilon} \left( 1 + \sum_m \xi_{m,t} \right) + \frac{\xi_{m,t}}{1 + \sigma} \right] Q_t \quad (1.2.12)$$

where  $\log Q_{m,t} = \exp(\int_0^1 \log q_{i,m,t} di)$ , and  $Q_t = (\sum_m Q_m^{\varepsilon-1})^{\frac{1}{1-\varepsilon}}$ . We can readily see that profits are sector-specific and not firm-specific. This is a consequence of assuming sector-specific aggregate shocks and not firm-specific idiosyncratic shocks. Appendix A2 elaborates on the algebra that leads to expression (1.2.12).

### 1.2.3 Research and Development

Up until this point, the ingredients of the model are standard. The problem of the inventor, however, is novel and deviates substantially from the previous literature in

three main aspects. First, we incorporate the fact that the arrival rate of new ideas is a function of cumulative research effort, in the form of knowledge-capital. Second, we explicitly consider the fact that knowledge-capital is inalienable, meaning that it cannot be sold if one inventor wished to reduce her holdings of knowledge capital<sup>11</sup>. And finally, we allow inventors to migrate to a different sector, taking with them a fraction of their cumulative knowledge capital.

A country is defined by a finite collection of industries (sectors)  $m \in \mathcal{M}_t$ , and the cardinality of this set will play a role on innovation even if it remains fixed<sup>12</sup>. In every industry, inventors accumulate sector specific knowledge-capital to innovate. We will assume that any inventor  $j$  can work on one sector at the time.

**Assumption 1.** *A given researcher  $j$  can only do research in one industry  $m \in \mathcal{M}_t$  at the time and will produce in only one variety in such a sector, at the same time.*

For any given entrepreneur  $j$  working in sector  $m$ , in period  $t$ , knowledge-capital law of motion is,

$$\omega_{m,t+1}(j) = (1 - \delta)\omega_{m,t}(j) + f\left(\frac{R_{m,t}(j)}{Q_{m,t+1}}\right) \quad (1.2.13)$$

where  $R_{m,t}(j)$  are the real resources invested in augmenting knowledge capital, namely R&D. Note R&D is proportional to sector quality, which captures the fact

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<sup>11</sup>While inalienable, it could be argued that an inventor could “rent” her knowledge capital in the form of employment for a larger firm. As long as the payment for this rent is proportional to the profits of the innovations she produces, this distinction does not change the analysis we are about to elaborate here.

<sup>12</sup>In this basic model we will abstract from  $M_t$  growing to highlight that the main mechanism in this paper is not related with expanding variety models as in Romer (1990)

that more complex goods require more resources in R&D in order to be improved upon. The reason to use average and not any particular quality  $q_{i,m,t}$  is that once an inventor is successful in sector  $m$ , she will become the incumbent of a *random variety in the sector*. This simplifies the problem drastically at no cost<sup>13</sup>. Function  $f(\cdot)$  transforms this normalized innovation effort into stock of knowledge, and is assumed to be increasing, concave and to satisfy Inada conditions;  $\lim_{x \rightarrow 0} f'(x) = \infty$  and  $\lim_{x \rightarrow \infty} f'(x) = 0$  and  $f(0) = 0$ . These assumptions on the functional form of  $f(\cdot)$  capture the fact that while investment in one period can increase cumulative know-how, large amounts of know-how require several periods. Finally, *inalienability* of knowledge capital implies that it cannot be less than the un-depreciated part of previous holdings.

Then, when researcher  $j$  has  $\omega_{m,t}(j)$  know-how in sector  $m$ , she can produce ideas or innovations in the same sector, and does so with probability:

$$\mu_{m,t}(j) = \mu(\omega_{m,t}(j)) \tag{1.2.14}$$

where  $\mu(\cdot)$  satisfies  $\mu_x(x) > 0$ ,  $\mu_{xx}(x) < 0$  and is bounded to  $\mu \in [0, 1]$ ,  $\lim_{x \rightarrow \infty} \mu(x) = 1$ ,  $\lim_{x \rightarrow 0} \mu(x) = 0$ <sup>14</sup>. In particular, I will use the following functional form for the

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<sup>13</sup>This assumption follows previous work by Klette and Kortum (2004), Lentz and Mortensen (2008), Akcigit, Hanley and Serrano-Velarde (2014), Acemoglu, Hanley and Kerr (forthcoming), and Acemoglu et al. (2013). Research is referred to as *directed* toward particular industries, but *indirected* within those industries.

<sup>14</sup>Note that the functional form chosen is actually the cumulative distribution function of a truncated ( $> 0$ ) exponential random variable. Only one parameter,  $\nu$ , governs this distribution. While we could use more flexible functional forms such as the Weibull or Shifted Gompertz distributions, that use two parameters, these are not always weakly concave and are usually convex for low levels of the argument.

probability of innovation given in equation (1.2.14),

$$\mu(\omega) = \{1 - \exp(-\nu \times \omega)\} \quad , \quad \omega \geq 0 \quad (1.2.15)$$

This is a model of sector mobility of inventors. If an inventor in sector  $m$  moves sectors, she transports her knowledge capital with her. However, in the destination sector  $m'$ , her knowledge of sector  $m$  may not be totally useful, and only a fraction  $1 - \phi_{m,m'}$  may be relevant to produce ideas in  $m'$ <sup>15</sup>. Diversified (non-diversified) economies display small (large) values of  $\phi_{m,m'}$ , therefore it is easier to move sectors in diversified economies, as the loss of previous capital accumulation is small. In particular, the law of motion of knowledge capital for the inventor  $j$  who moves from  $m$  to  $m'$  is,

$$\omega_{j,t+1}(j) = (1 - \delta)(1 - \phi_{m,m'})\omega_{m,t}(j) + f \left( \frac{R_{m',t}(j)}{Q_{m',t+1}} \right) \quad (1.2.16)$$

that is different from (1.2.13) only in that there is some loss of knowledge capital from the fact that the destination sector may be distant from the original  $m$  sector.

The timing of events (figure 1.3) is as follows. Any entrepreneur enters the period as an incumbent or an outsider, with knowledge capital  $\omega_{m,t}(j)$ . According to (1.2.15) she has a breakthrough and becomes an incumbent if she was an outsider. Nature also decides on the productivity shocks  $\{\xi_{m,t}\}_{m=1}^M$  which the amount

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<sup>15</sup>In the model  $\phi$  is the counterpart of the empirical notion of intellectual distance of equation (1.1.3).

of profits  $\pi_{m,t}$  incumbents take home. Then she decides on knowledge capital accumulation for next period. First she decides if she stays in sector  $m$  or moves, and later she decides on how much R&D to pursue in the destination sector.

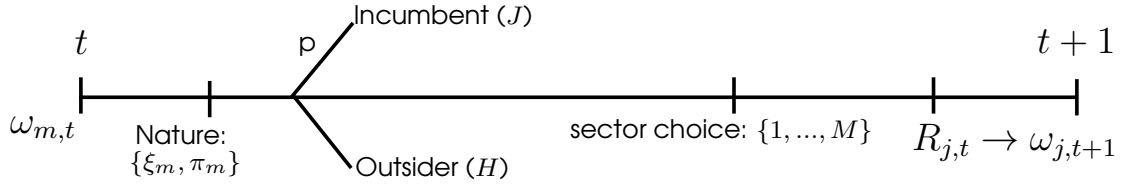


Figure 1.3: Timeline of Events in Innovation

Conditional on already choosing a sector for next period, the second decision ( $\omega_{t+1}$ ) is constrained by non negativity of R&D investment.

### Entrepreneurs across sectors

The collection of all entrepreneurs will be referred to as  $\mathcal{E}$ , and has mass  $E \propto M$ , which we will normalize to  $M + 1$ . In each sector there will be mass one of incumbents, one for each variety  $i$ . There will also be mass 1 of “outsiders” for the  $M$  sectors. In this model, some inventors will have positive holdings of knowledge-capital and yet not be producing in some period. Consider for instance an inventor who was an incumbent but was just superseded, losing her monopoly until she innovates again. She holds at least  $(1 - \delta)\omega(j) > 0$  for next period. She is an outsider. Even if she does not invest again in R&D she still has positive probability to become an incumbent next period. Denote  $J(\cdot)$  values for incumbents and  $H(\cdot)$ ,



values for outsiders.

We can think of the decisions of every entrepreneur as happening in two stages. First, they decide on which sector to work in (stay or move to  $m'$ ). Second, they decide how much research capital to bring to the next period in the destination sector. Once we solve the second problem it is easy to solve the first one. In particular, the first problem is reduced to comparing alternatives: moving or staying, conditional on doing what is best in each of these cases. The following problems, determine how much knowledge capital to bring to the next period in every possible case, and after nature has decided on the cost shocks and on the success/failure of innovation in the current period.

***Incumbent's Problem.*** This is the producer of good  $x_{i,m,t}$ . Let his identity be  $j$ . The aggregate state variables are the vector of qualities in every sector,  $Q = \{Q_m\}$  and shocks,  $Z = \{\xi_m\}$ . The individual state variable will be the current level of research capital  $\omega_m(j)$ . Let  $J^S(Q, Z, \omega(j)|m)$  be the value of being an incumbent in period  $t$  in sector  $m$ , and deciding to stay in sector  $m$  after learning the realization of nature in the current period. For this decision the only control variable is the stock of knowledge capital next period in  $m$ ,  $\omega'_m(j)$ . In particular, the problem for this entrepreneur is: Given, current knowledge capital, qualities and shocks for the current period  $\{Q, Z, \omega(j)\}$ , and given an interest rate  $r'$  and probability of replacement  $\lambda_m$ , the incumbent entrepreneur who decides to stay in the same sector

$m$ , chooses  $\omega'(j)$  to solve the program,

$$J^S(Q, Z, \omega_m) = \max_{\omega'_m(j)} \left\{ \begin{array}{l} \pi_m(Q_m, \xi_m) - Q'_m f^{-1}(\omega'_m(j) - (1 - \delta)\omega_m(j)) \\ + \frac{1}{1+r'} \int_{Z'} \left[ \left(1 - \lambda_m + \mu(\omega'_m(j))\lambda_m\right) J^0(Q', Z', \omega'_m(j)) \right. \\ \left. + \lambda_m \left(1 - \mu(\omega'_m(j))\right) H^0(Q', Z', \omega'_m(j)) \right] dF(Z'|Z) \end{array} \right\} \quad (1.2.17)$$

*subject to:*

$$\omega'_m(j) \geq (1 - \delta)\omega_m(j)$$

$$\lambda_m = \mathcal{L}^E(Q, Z; m)$$

$$\xi'_m = \rho_\xi \xi_m + \sigma_\xi \epsilon$$

$$\log Q'_m = \lambda_m \log(1 + \sigma) + \log Q_m$$

Equation (1.2.17) is the value of staying in sector  $m$  for the incumbent. At the beginning of the period, nature reveals the results of research and the cost shock in period  $t$ , the incumbent observes  $\{Q, Z\}$ , and is entitled to produce and obtain the flow profit  $\pi_{m,t}$ . Since she has decided to stay in sector  $m$  – and therefore her cumulative know-how does not erode – her research efforts are given by the second term in the first line of (1.2.17). The next lines stand for the continuation value. With probability  $\mu(\omega)$  the entrepreneur is successful and gets a random variety  $i$  within the same sector. With complementary probability she does not innovate, but is not superseded by anyone with probability  $1 - \lambda_m$ , and therefore remains an

incumbent (with value  $J^0$ ). Finally, with complementary probability the incumbent does not innovate, and is superseded (with value  $H^0$ ). The first constraint reflects the inalienability of knowledge-capital. The second constraint is the perceived law of motion of the probability of being replaced. The third and fourth constraints are the laws of motion of sector socks and quality.

The incumbent entrepreneur, however, may also choose to *abandon* sector  $m$  for sector  $m'$ . Then, given current knowledge capital, qualities and shocks for the current period  $\{Q, Z, \omega\}$ , and aggregate variables,  $\{r', \lambda_{m'}\}$ , the incumbent inventor  $j$ , who decides to move to sector  $m'$ , chooses  $\omega'_{m'}(j)$  to solve the program

$$J_{m'}^M(Q, Z, \omega_m) = \max_{\omega'_{m'}(j)} \left\{ \begin{array}{l} \pi(Q_m, \xi_m) - Q'_{m'} f^{-1}(\omega'_{m'}(j) - (1 - \delta)(1 - \phi_{m,m'})\omega_m(j)) \\ + \frac{1}{1+r} \int_{Z'} \left[ \mu(\omega'_{m'}(j)) J^0(Q', Z', \omega'_{m'}(j)) \right. \\ \left. + (1 - \mu(\omega'_{m'}(j))) H^0(Q', Z', \omega'_{m'}(j)) \right] dF(Z'|Z) \end{array} \right\} \quad (1.2.18)$$

*subject to*

$$\omega'_{m'}(j) \geq (1 - \delta)(1 - \phi_{m,m'})\omega_m(j)$$

and using analog perceived laws of motion for aggregate state variables as in (1.2.17). Conditional on moving from sector  $m$  to sector  $n$ , original know-how in sector  $m$  is destroyed not only because of depreciation, by a factor  $\delta$ , but because a fraction  $\phi_{mn}$  of the cumulative knowledge is not portable. When moving to  $n$  the entrepreneur

is automatically an outsider in the new sector. Next period, with probability  $\mu(\omega')$  she will become an incumbent to the sector and will acquire the value of being an incumbent with the latest innovation of quality in some variety. If she cannot innovate then she remains an outsider with positive know-how in sector  $n$ , whose value is given by the third line in equation (1.2.18).

Given equations (1.2.17) and (1.2.18), the researcher must decide if she wants to stay in her industry, or she wants to move. Furthermore, if she decided to move, she must decide where to. This option is summarized in the following program. Given (1.2.17) and (1.2.18), the value of having the choice is

$$J^0(Q, Z, \omega_m(j)) = \max \left\{ J^S(Q, Z, \omega_m(j)), \{J_{m'}^M(Q, Z, \omega_m(j))\}_{m' \neq m} \right\} \quad (1.2.19)$$

It is because of the fact that we need to compare all industries at the time in equation (1.2.19), that we need to keep track of the whole vector of aggregate qualities and cost shocks,  $\{Q, Z\}$  for each value function. Computationally,  $M_t$  being finite and small, makes this problem possible to solve. Else, a distribution of industries – which is in essence an infinite dimensional object–, would make the problem too hard to solve in practice.

**Outsider's Problem.** The problem of the outsider  $j$  entrepreneur in industry  $m$  ( $\omega_m(j) > 0$ ), who decides to stay in industry  $m$  is different from that of the

incumbent in that she does not enjoy the flow of profits. Let  $H^S(Q, Z, \omega_m(j))$  denote this value. Given, current research capital, qualities and shocks for the current period  $\{Q, Z, \omega_m(j)\}$ , interest rate  $r'$ , the outsider inventor who decides to stay in the same sector  $m$ , chooses  $\omega'_m(j)$  to solve the program,

$$H^S(Q, Z, \omega_m(j)) = \max_{\{\omega'_m(j)\}} \left\{ \begin{array}{l} -Q'_m f^{-1}(\omega'_m(j) - (1 - \delta)\omega_m(j)) \\ + \frac{1}{1+r'} \int_{Z'} \left[ \mu(\omega'_m(j)) J^0(Q', Z', \omega'_m(j)) \right. \\ \left. + (1 - \mu(\omega'_m(j))) H^0(Q', Z', \omega'_m(j)) \right] dF(Z'|Z) \end{array} \right\} \quad (1.2.20)$$

*subject to*

$$\omega'_m(j) \geq (1 - \delta)\omega_m(j)$$

There are no flow of profits, as the researcher is not an incumbent. The first line is the cost of engaging into research to expand the initial research capital  $\omega_m(j)$ . The second line is the value of becoming an incumbent and the third, the value of remaining an outsider. Again, the perceived laws of motion will be analogs to those in (1.2.17).

In a similar way, an entrepreneur  $j$  who is an outsider to sector  $m$  and holds research capital  $\omega_m(j)$  may choose to move to another sector that promises better prospects. If she decides to move, it will be effective from next period. Given, current knowledge-capital, qualities and shocks for the current period  $\{Q, Z, \omega_m(j)\}$ ,

and aggregate variable,  $r'$ , the outsider entrepreneur who decides to move to  $m'$ , chooses  $\omega'_{m'}(j)$  to solve the program,

$$H_{m'}^M(Q, Z, \omega_m(j)) = \max_{\omega'_{m'}(j)} \left\{ \begin{array}{l} -Q'_{m'} f^{-1}(\omega'_{m'}(j) - (1 - \delta)(1 - \phi_{m,m'})\omega_m(j)) \\ + \frac{1}{1+r'} \int_{Z'} \left[ \mu(\omega'(j)) J^0(Q', Z', \omega'_{m'}(j)) \right. \\ \left. + (1 - \mu(\omega'_{m'}(j))) H^0(Q', Z', \omega'_{m'}(j)) \right] dF(Z'|Z) \end{array} \right\} \quad (1.2.21)$$

*subject to*

$$\omega'_{m'}(j) \geq (1 - \delta)(1 - \phi_{m,m'})\omega_m(j)$$

Finally, once the outsider has the value of moving to each of the available sectors in the country, she will choose the one that gets her the largest value,

$$H^0(Q, Z, \omega_m(j)) = \max \left\{ H^S(Q, Z, \omega_m(j)), \{H_{m'}^M(Q, Z, \omega_m(j))\}_{m' \neq m} \right\} \quad (1.2.22)$$

Given equations (1.2.20), (1.2.21) and (1.2.22), the outsider researcher decides whether to persist in industry  $m$  or to move to a different sector  $m'$ .

## Industry Entry & Exit, and Aggregate Innovation

We are now in position to examine the dynamics of  $\{\lambda_m\}$ , the hazard rate that measures the probability of being replaced by a successful inventor in the same sector. While each entrepreneur uses a perceived law of motion  $\mathcal{L}^E(\cdot)$ , in the definition of the recursive competitive equilibrium this is actually an equilibrium object  $\mathcal{L}(Q, Z)$ .

Denote  $\mathcal{F}_t$  the  $M \times M$  matrix which collects the flows of entrepreneurs who move from one sector to another. Positive entry,  $\mathcal{F}_t(n, n')$  represents migration of entrepreneurs from sector  $n$  to sector  $n'$ . The probability of incumbent  $j$  in sector  $m$  of being replaced is given by<sup>16</sup>,

$$\lambda_m(Q, Z) \equiv 1 - \left[1 - \mu(\omega_m^{JS})\right] \left[1 - \mu(\omega_m^{HS})\right]^{F_m - \sum \mathcal{F}(m, \cdot)} \prod_{s \neq m} \left[1 - \mu(\omega_m^{HM(s)})\right]^{\mathcal{F}(s, m)} \quad (1.2.23)$$

which is simply one minus the probability not being replaced by any other inventor, outsider or incumbent. Note that the populations – given initial conditions – should satisfy certain restrictions. For instance, the sum of outflows should be equal to the sum of inflows, and the total mass of outsider entrepreneurs should add up to 1,  $\sum_n F'_n = 1$ <sup>17</sup>. Free mobility condition for every industry should hold.

---

<sup>16</sup>Note that the probability of not being replaced, means that neither incumbents, nor remaining or new outsiders innovate upon the current incumbent. Thus,  $prob[\text{not being replaced}]$  is equal to  $prob[\text{incumbents in } m \text{ fail to replace } j] \times prob[\text{outsiders who stay in } m \text{ fail to replace } j] \times prob[\text{new outsiders in } m \text{ fail to replace } j]$

<sup>17</sup>In the same way as in Akcigit, Hanley and Serrano-Velarde (2014), the population of entrants will control the relative importance of outside entry and therefore the creative destruction arising from new entrants.

This means that no entrepreneur should have incentive to migrate in equilibrium, because everyone who wanted to move already has. In order to solve for the population of entrepreneurs in each sector, it will be key to use a result of the equilibrium, that outsiders move before incumbents because they have more to gain of it. Actually, even when incumbents can move, all adjustment will be given by outsiders. Outsiders migration will change the value  $\lambda_m(Q, Z)$  and therefore the value of moving for incumbents, until they do not wish to anymore. We require that in every sector outsider entrepreneurs do not have the incentive to move. Thus the vector  $\{F'_m\}$  will change until the following ‘free mobility condition’ holds, for every  $m$

$$H^S(Q, Z, \omega_m) = H^0(Q, Z, \omega_m) \tag{1.2.24}$$

**Proposition 1.** *If an incumbent with knowledge capital  $\omega$  has the incentive to move from sector  $m$  to  $m'$ , then outsiders with the same level of knowledge capital  $\omega$ , in the same sector has greater incentive, and moves first.*

*Proof.* In Appendix A.2 □

Note that in every case (incumbent/outsider, staying/moving) all inventors behave identically within the same group. This is not an assumption. The absence of idiosyncratic shocks implies that a transition function for the distribution of  $\omega'$  is degenerate. Every entrepreneur in the same group with the same level of  $\omega$ , is identical, therefore their policy function and decisions will be identical.



## 1.3 Equilibrium

### 1.3.1 Mechanism and Intuition

Before computing the fully fledged equilibrium dynamics of the model, it is useful to distill the main mechanism. This is easier to do in a simplified version of the model in Section 1.2. For a moment let there only be one sector, let also the quality ladder be flat  $\sigma = 0$ , and the function  $f(\cdot)$  be the identity function. This means that the only two states to keep track of, are the sector shock and inventor  $j$ 's knowledge-capital,  $s(j) = (\xi, \omega(j))$ . Given  $\lambda$ , and  $r_d^{-1} = \frac{1}{1+r'}$ , the value of starting period  $t$  with knowledge capital  $\omega$ , after nature decides the entrepreneur is an incumbent with technology shock  $\xi$  is given by the solution of the following program,

$$J(s) = \max_{\omega'} \pi - q(\omega' - (1-\delta)\omega) + r_d^{-1} \mathbb{E} \left\{ \left( 1 - \lambda + \lambda \mu(\omega') \right) J(s') + \lambda \left( 1 - \mu(\omega') \right) H(s') \right\} \quad (1.3.1)$$

*subject to*

$$\omega' \geq (1 - \delta)\omega$$

$$\xi' = \rho_\xi \xi + \sigma_\xi \epsilon$$

By the same token, the value of being an outsider is the solution to the following program,

$$H(s) = \max_{\omega'} -q\left(\omega' - (1 - \delta)\omega\right) + r_d^{-1}\mathbb{E}\left\{\mu(\omega')J(s') + (1 - \mu(\omega'))H(s')\right\} \quad (1.3.2)$$

*subject to*

$$\omega' \geq (1 - \delta)\omega$$

$$\xi' = \rho_\xi \xi + \sigma_\xi \epsilon$$

It will be helpful for further proofs to note that being an incumbent is more valuable than being an outsider.

**Lemma 1.** *The value of being an incumbent will be higher than the value of being an outsider, i.e.  $J(\omega, \xi) > H(\omega, \xi)$ .*

*Proof.* In Appendix A.2 □

One of the main predictions of this model is that accumulation of knowledge capital is lower due to the fact that it is inalienable. Later, I will make the argument that diversification relaxes this constraint, thereby offsetting the following result, at least partially. Note that I will be using, to get a closed form result, the simplified model of equations (1.3.1) and (1.3.2).

**Proposition 2.** *The combination of inalienability of knowledge capital ( $\omega'$ ) and stochastic shocks, results in lower previous investment.*

*Proof.* In Appendix A.2

□

The intuition is the following. The first order condition implies that the marginal cost of investment  $q'$  equates the marginal benefit plus a wedge which captures the shadow price ( $\chi$ ) of constraint  $\omega'(j) \geq (1-\delta)\omega(j)$ . If the inventor increases  $\omega'(j)$  she can escape the current period constraint. However, this will directly impact future constraints. In particular, by raising holdings of next-period knowledge capital  $\omega'(j)$ , the constraint  $\omega''(j) \geq (1-\delta)\omega'(j)$  is more likely to bind. The anticipation of hitting the constraint in the future reduces the value of accumulating research capital today. If the inventor anticipates that bad shocks will hit her in the future, the value of the shadow prices of the constraints in the future are positive and therefore it is optimal not to accumulate too much capital today in the first place.

The analysis is similar to that of irreversible physical capital, which has been studied before notably by Pindyck (1988), focusing on capacity choice under irreversibility, and highlighting the sunk cost of forfeiting the value of the option to expand at a later date (a call option). This *option value* notion is carefully expanded in Abel et al. (1996), to correct the simple net present value discount rate, not only by the call option, but the put option of reselling at a lower price the invested marginal unit. On the macroeconomic implications of irreversibility, while Olson (1989) introduces irreversibility in the representative agent model, investment dynamics are not variable enough for it to have a bite. Bertola and Caballero (1994), on the other side argue that it is idiosyncratic (micro) irreversibilities which can

help explain aggregate dynamics in the presence of important sources of idiosyncratic irreversibility. That is, efforts have been directed towards understanding the effects of irreversibility on volatility. This paper takes a different direction, assessing the effect of irreversibility on the level, rather than volatility of: innovation, income and ultimately consumption.

**Proposition 3.** *Higher volatility of the industry idiosyncratic shock will lower the value of holding knowledge capital, and hence, lower innovation rates.*

*Proof.* In Appendix A.2 □

The intuition of the proof relies on the fact that the inventor  $j$  considers, in her decision about  $\omega'(j)$ , the impact on the next period inalienability constraint. We have already proved that the existence of such constraints creates a wedge between the marginal cost and benefit of accumulating knowledge capital and that she reduces knowledge capital  $\omega'$  in order to avoid hitting the constraint in future periods (Proposition 2). But also, *higher volatility* of profits means that such constraint is more likely to be hit, which further reduces the optimal decision of next period research capital. The formal proof investigates a mean-preserving expansion of the variance and the behavior of the expectation of the Lagrange multiplier. Volatility lowers the value of research without assuming further frictions like credit constraints as in Aghion et al. (2005). This is the second important message in this paper; volatility will affect innovation but will affect it less the less severe the inalienability constraint is. This will happen when there is a way to exit the industry

by moving to a sister industry, namely when the economy grows in diversity.

Without prejudice of the previous assertion, volatility has a second round effect. Good shocks – to the degree that there is some persistence in the process – are associated with marginal positive accumulation of knowledge capital, but they are also associated with industry entry. These intensive, and extensive margins translate into higher hazard rate  $\lambda_m$ ; the probability of replacement of a current incumbent, which partially wipes out the original positive shock observable to an individual incumbent. When the opposite happens, however, the reverse is not true. While inventors can exit the sector, they cannot undo their previous capital investment. There is room for adjustment at the extensive margin, but not at the intensive margin. Those who stay in the sector have to endure the bad shock. They are affected more by the negative shocks than by the positive shocks; *symmetric* increases in the variance of the shock result in *asymmetric* responses due to the irreversibility constraint.

Now we can proceed, in the rest of this section, to define the competitive equilibrium and proceed to the quantitative analysis in the next one.

### 1.3.2 Competitive Equilibrium

Having introduced the main components of the model we can proceed to examine the definition of equilibrium and examine the existence and characterization of an expected balanced growth path (EBGP) for this economy.

**Definition 1 (Equilibrium).** *A competitive equilibrium for this economy with innovation and sector mobility of inventors, is aggregate allocations and prices  $\{c_t, a_t, Y_t, r_t w_t\}_{t=0}^\infty$ , sectoral allocations, prices and profits  $\{\{y_{m,t}, p_{m,t}, \pi_{m,t}\}_{m=1}^M\}_{t=0}^\infty$ , intermediate goods allocations and prices  $\{\{\{\ell_{i,m,t}, x_{i,m,t}, p_{i,m,t}^x\}_{i \in [0,1]}\}_{m=1}^M\}_{t=0}^\infty$ , distribution of investors population  $\{\{F_{m,t}\}_{m=1}^M\}_{t=0}^\infty$ , value functions for incumbents and outsiders  $\{J^S, H^S, \{J_n^M, H_n^M\}_{n \neq m}\}_{m=1}^M$ , aggregate qualities and perceived hazard rates  $\{\{Q_{m,t}, \lambda_{m,t}\}_{m=1}^M\}_{t=0}^\infty$ , such that:*

1. Given  $\{w_t, r_t\}_{t=0}^\infty, \{c_t, a_{t+1}\}_{t=0}^\infty$  solve 1.2.1 subject to 1.2.2
2. Given  $\{p_{i,m,t}^x\}_{i \in [0,1]}$ , the final good producer of sectoral output  $m$  demands each variety of  $\{x_{i,m,t}\}_{i \in [0,1]}$  to solve program 1.2.6
3. Given  $\{w_t, \xi_{m,t}, q_{i,m,t}\}$  the producer of  $x_{i,m,t}^S$  sets price  $p_{i,m,t}^x$  according 1.2.11 (Bertrand monopolistic competition) and earns profits 1.2.12
4. Given  $\{\lambda_m\}$  and  $\{Q, Z\}$ , then values  $\{J^S, H^S, \{J_n^M, H_n^M\}_{n \neq m}\}_{m=1}^M$  and associated policy functions solve problems 1.2.17, 1.2.18, 1.2.19, 1.2.20, 1.2.21 and 1.2.22.
5. Markets clear,  $\forall i, m, t$

$$x_{i,m,t}^D = x_{i,m,t}^S \tag{1.3.3}$$

$$L = \sum_m \int_0^1 \ell_{i,m,t} di \tag{1.3.4}$$

$$a_{t+1} = 0 \tag{1.3.5}$$

6. Finally, we require consistency of perceived and actual laws of motion for sector quality, shocks and hazard rates,

$$\xi'_m = \rho_\xi \xi_m + \sigma_\xi \epsilon \quad (1.3.6)$$

$$\log Q'_m = \lambda_m \log(1 + \sigma) + \log Q_m \quad (1.3.7)$$

$$\mathcal{L}_m^E(Q, Z) = \mathcal{L}_m(Q, Z) \quad (1.3.8)$$

where  $\mathcal{L}(Q, Z)$  is pinned down by the free mobility condition 1.2.23

7. Share of entrepreneurs are given by 1.2.24

**Definition 2 (Expected Balanced Growth Path).** *The economy is in expected balanced growth path (EBGP) in period  $\tau$ , if it is in a trajectory such that,  $\forall t > \tau$ , aggregate variables  $\{c_t, w_t, Y_t\}_{t>\tau}$  grow at a constant **average** common rate, while prices, population shares and hazard rates are stationary around a constant.*

This economy lacks a balance growth path. The fundamental reason is that it lacks a continuum of sectors which average out, and on which dimension we can invoke a law of large numbers. Instead, we have finite sectors and shocks are sector specific. For any variable which would be a constant in the BGP,  $x_t = x$ , the expected balanced growth path definition revolves around  $\bar{x} = \lim_{b \rightarrow \infty} \frac{1}{b} \sum_{\tau=t}^{t+b} x_\tau = x$ <sup>18</sup>. The system of equations which represents the economy in EBGP is presented in detail in Appendix C for the interested reader.

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<sup>18</sup>It can be shown that most of the markovian stability notions are extended to this case [Stokey, Lucas and Prescott (1989), ch. 11 & 12]

However, some insights are worth being mentioned. First, note that this economy resembles a collection of multisector Aghion-Howitt economies, in which the innovation rate  $\{\lambda_m\}_m$  is different, and determined in each sector by the distribution of knowledge capital, in particular by equation (1.2.23). While this innovation rate is not constant, the average of a finite sequence  $\{\lambda_m^j\}_{j=1}^b$  of size  $b$  is constant, i.e.  $\bar{\lambda}_m = \frac{1}{b} \sum_{\tau=t}^{t+b} \lambda_{m,\tau}$ . Second, in the long run we require that all sectors  $m$  grow at the same average rate. This means  $\bar{\lambda}_m = \bar{\lambda}_n$ , for  $n \neq m$ . If not, the relative size of one sector with respect to the whole economy will diverge and become negligible.

The system of equations characterizing equilibrium in Appendix C are all expressed in stationary notation. The growth rate of the economy is equal to the growth rate of  $Q$ , where we know  $Q = (\sum_m Q_m^{\varepsilon-1})^{\frac{1}{\varepsilon-1}}$ . At the same time, the growth rate of  $Q_m$  can be calculated by noting that  $\log Q'_m = \int_0^1 \log q'_{i,m,t} di$ , which implies that the growth rate  $g_{m,t}$  of every sector in each period of time is equal to

$$(1 + g_{m,t}) \simeq (1 + \sigma)^{\lambda_{m,t}} \tag{1.3.9}$$

Since the average innovation rate is  $\bar{\lambda}_m$ , then the average of a sequence of sectoral growth also converges to a constant  $\bar{g}_m$ , such that  $\bar{g} = \bar{g}_m$ . Growth depends positively on the quality ladder step size, but more importantly on what determines  $\lambda$ . Further, we know that  $\lambda$  is the aggregation of innovation effort by researchers whose accumulation of knowledge-capital depends positively on the diversification of the economy. That is, diversification is embedded in (1.3.9). This completes the



description of the equilibrium of this economy and the main predictions of the basic model. Section 1.4 investigates further how valid these predictions are in the data, and the ability of the model to replicate the major stylized facts.

## 1.4 Quantitative Analysis

Two main predictions distill from this model. First, diverse economies provide more incentives to innovation and therefore are more innovative at the inventor level. Second, volatility hinders this innovation, but it does so more, in less diverse economies. In this section I will first use data from patent and inventor applications to test the predictions that stem from the model. In the second part I calibrate the model, show that it is a good laboratory, and contrast the data generated by the model with the empirical findings found in the first subsection below, which I do not explicitly target in the calibration process. This allows me to quantify the contribution of the channel stressed in this paper vis a vis other potential explanations for the coefficients found in the empirical part of this section<sup>19</sup>.

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<sup>19</sup>In particular there are two other potential explanations, “availability externalities” and “knowledge spillovers”. which are explained in detail and in the context of the same basic model in Appendix A.5. Notably, “availability externalities” hypothesis stresses that more diverse economies make the production process easier by having all inputs at hand. Alternatively the “knowledge spillovers” hypothesis stresses the fact that an innovation in one sector creates innovations in other sectors, and that having many sectors increases this spillover effect. Not any one of these channels creates a link between volatility and innovation, as the mechanism presented in this paper does.

### 1.4.1 Regression Based Analysis

#### Data Sources

The main sources of information are the National Bureau of Economic Research & United States Patent and Trademark Office (NBER-USPTO) Utility Patents Grant Data. The USPTO grants a property right on those inventors applying for patent protection. A patent gives the holder a temporal right to sell or use their idea. It is up to the patent holder to enforce her own rights if she is granted a patent. The main types of patents are (i) Utility patents, (i) Design patents, and (i) Plant patents. Utility patents are those we refer to when thinking about economic growth. These are issued for the invention of a “*new and useful process, machine, manufacture, or composition of matter...*”, and cover around 90% of the patent documents issued by the USPTO. It is useful for the analysis of this paper that each of these applications contains the International Patent Classification (IPC) code which pins each invention down to a technology class. For the analysis below I work with the first two digits in the IPC code to denote a technology class.

The second main source of information is the “Disambiguated Inventor Data Set” (DID) by Li et al. (2014), which is based on the NBER-USPTO project, and contains data on around 3.1 million inventors for the period 1975-2010. It contains data on cross citations of patents. If patent A cites patent B, then we observe a forward citation from A to B, and backward citation from B to A.

## Evidence from micro data: inventors' data

In this sub-section I test the main predictions of the model on inventor level data. This data contains the history of more than two million inventors, and almost seven million patents that can be traced back to authors [Table 1.1]). The average number of inventions by researcher is 3.3, but eliminating those inventors who appear only once in the dataset we end up with 1.1 million inventors who on average invent 6 patents in their careers. Separating productivity of inventors between countries according to their diversification a difference of at least one more invention per career in the diversified economies.

Table 1.1: **Innovation productivity and Diversification:** Inventors are more productive in diverse economies. Serial inventor sample is the truncation of inventors to those with more than one patent. High(Low) diversification is defined as countries in percentiles 75 to 95 (05 to 25), in the 1980 measure of the ECI index by Hausmann et al. (2013)

	(1)	(2)	(3)	(4)
	<i>Number</i>	<i>Share (%)</i>	<i>High Div</i>	<i>Low Div.</i>
Total Inventors	2107420	100		
Once in a lifetime inventors	1139231	54.06	54.64	64.41
Inventions / inventor	3.3	100	3.21	2.11
Inventions / serial-inventor	6.0		5.89	4.13

**Empirical fact 1.** *Inventors in more diverse economies are more innovative.*

This fact is the counterpart of Proposition 2. A first pass at the data [Table 1.1] shows that inventors who reside in the 75-95 percentile of countries according to product diversification [ECI measure by Hausmann et al. (2013)], innovate on av-

average 3.21 patents during their lifetime in contrast to 2.11 patents in the 5-25 percentiles group of inventors. If we exclude those inventors that are only a once-in-a-lifetime inventors, and focus on those that have two (2) or more patents, inventors in diversified economies innovate 5.89 patents on average compared to 4.13 patents in the second group.

For a more detailed analysis of productivity let us consider Table 1.2, which regresses the (log) number of inventions of an inventor in her lifetime (measure of inventor productivity) with the technological diversification and volatility of the economy where she resides at the time of inventions, and other controls which include scale (total log employment), initial per capita income and education. Volatility is calculated in two different ways. First, we use the standard deviation of the PPP GDP growth rate of the 1975-2011 sample for each country, and secondly, we follow Neumeyer and Perri (2005) and compute the time variant measure of deviation from trend.

The sample size is about 2 million inventors. We measure diversification using the (log) of the number of technology classes that are visited by inventors in each economy. These range widely for different countries, from 1 to 137 with a average (median) of 18 (7) sectors. The estimations indicate that the effect of higher diversification (1%) implies inventors being from 13% to 23% more productive in terms of the number of patents they produce in a lifetime.

Table 1.2: **Systematic Evidence on Innovation Productivity and Diversification:** Dependent variable is log of number of inventions per inventor. Observations are at inventor level. Volatility measures are the standard deviation yearly PPP GDP (7y window) and the Neumeyer and Perri (2005) measure of country volatility. Standard errors in parentheses, \*  $prob < 0.01$ , \*\*  $prob < 0.05$ , \* $prob < 0.1$

	(1)	(2)	(3)	(4)	(5)	(6)
Diversification (log)	0.14*** (0.00)	0.13*** (0.002)	0.15*** (0.005)	0.07*** (0.005)	0.23*** (0.007)	0.04*** (0.006)
Volatility measure 1 (x100)	-1.53*** (0.09)		-1.64*** (0.09)		-1.69*** (0.13)	
Volatility 1 * Diversification	0.39*** (0.02)		0.41*** (0.02)		0.36*** (0.03)	
Volatility measure 2 (x100)		-5.32*** (0.15)		-5.90*** (0.18)		-7.92*** (0.24)
Volatility 2 * Diversification		1.54*** (0.03)		1.71*** (0.04)		2.15*** (0.05)
Employment (log)			-0.01*** (0.00)	0.02*** (0.00)	-0.02*** (0.00)	0.02*** (0.00)
Per capita income (log, 1980)			-0.01*** (0.00)	0.04*** (0.00)	-0.03*** (0.00)	0.04*** (0.00)
Human Capital Index					-0.07*** (0.00)	0.00* (0.00)
Constant	-0.00 (0.01)	-0.01 (0.01)	0.06*** (0.02)	-0.23*** (0.02)	0.18*** (0.022)	-0.13*** (0.02)
<i>Observations (000)</i>	2,082	2,089	2,081	2,083	2,081	2,083
<i>R-squared</i>	0.007	0.009	0.007	0.009	0.007	0.009

**Empirical fact 2:** *Volatility reduces innovation, but does so more in less diverse economies.*

This fact verifies Proposition 3 and is at the core of this paper. It provides a rationale as why we could causally observe the well documented negative correlation between growth and volatility in Ramey and Ramey (1995) and Aghion et al. (2005). One basis point of standard deviation (in a 7 year rolling windows) amounts to 1.5% less patent applications per inventor. On our second measure of volatility this effect goes from 5% to 8% less innovations. This amounts to roughly 3% to

10% less innovation in a typical emerging market economy, compared to the typical OECD economy, depending on the chosen volatility variable.

Table 1.3: **Evidence on Inventor Mobility Across Sectors:** Binary choice model (logit), 1 = inventor changes sector, 0 = does not change sector. Observations are at inventor/patent level. (1) Volatility measures are the standard deviation of yearly PPP GDP, (2) and the Neumeyer and Perri (2005) measure of country volatility. (3) Barro and Lee (2010) index of human capital. Standard errors in parentheses, \*\*\**prob* < 0.01, \*\**prob* < 0.05, \**prob* < 0.1.

	(1)	(2)	(3)	(4)
	<i>All</i>	<i>All</i>	<i>All</i>	<i>Exclude Superstars</i>
<i>Diversification</i>	0.15** (0.01)	0.14** (0.01)	0.19** (0.01)	0.13** (0.01)
<i>Volatility (1)</i>		0.77** (0.05)		0.69** (0.05)
<i>Volatility (2)</i>			-0.53** (0.05)	
<i>(log) # inventions</i>	0.01** (0.00)	0.01** (0.00)	0.01** (0.00)	-0.00** (0.00)
<i>(log) employment</i>	-0.07** (0.00)	-0.05** (0.00)	-0.08** (0.00)	-0.03** (0.00)
<i>(log) GDPpc 1990</i>	0.02** (0.01)	0.09** (0.01)	-0.02* (0.01)	0.12** (0.01)
<i>Human Capital (3)</i>	-0.06** (0.00)	-0.07** (0.00)	-0.07** (0.00)	-0.09** (0.00)
<i>Trade Openness</i>	-0.00** (0.00)	-0.00** (0.00)	-0.00** (0.00)	-0.00** (0.00)
<i>Constant</i>	-0.92** (0.06)	-1.71** (0.08)	-0.57** (0.06)	-1.99** (0.08)
Observations (thousands)	4767	4638	4765	4312
<i>Pearson p-value</i>	0.00	0.00	0.00	0.00
<i>Hosmer-Lemeshow p-value</i>	0.00	0.00	0.00	0.00

**Empirical fact 3:** *There is a higher probability that an inventor switches sectors in a more diverse economies.*

Diverse economies make it easier for inventors to move. We have already established

that they accumulate more knowledge capital based on the option of changing sectors. But do they? In table 1.3 we present evidence that inventors are more likely to change sectors when they work in a diverse economy. In particular, we model the probability of changing sectors – defined as IPC2 technology classes – as a binary decision and estimate a logit model that considers the diversification, two measures of volatility and controls for inventor productivity, scale, convergence, human capital and openness to trade. Statistically inventors in more diverse economies do move more often than those in non-diverse economies.

In Table 1.3, the first column shows the results of including only the (log) of the number of technological sectors available for any inventor in her economy, the second and third columns include different measures of volatility, and for robustness in the last column we exclude “superstar” inventors, who are so productive that they could have a large influence in the results. The conclusions hold. More diversified economies have inventors who change sectors more often. For illustration purposes, let us use column 2 to calculate the marginal probabilities of changing sectors in two different economies in terms of diversification. Take the typical emerging market economy, Mexico and the typical advanced economy, Korea. Mexico has 27 technological sectors and Korea has 49 (the US has 137 and Germany 79). According to table 1.3 an inventor in Mexico has a marginal probability of changing sectors, on each of her inventions, of 0.338 and an inventor in Korea of 0.368. The median inventor in the sample submits 7 patents and the average one submits 18. This

implies that the median (average) inventor changes technology classes 2.31 (5.94) times in Mexico, but she would change 2.52 (6.48) times in Korea. This concludes our assessment of data in order to contrast the implications of our model.

### 1.4.2 Solution and Calibration

In this subsection we describe the solution method, describe the calibration of the model and later check the reach of the model to generate moments different from the calibration targets. Later we use the calibrated model to quantify the effect of diversification and volatility on innovation and ultimately on economic growth.

#### On the solution method

One of the key difficulties for solving this model is that the number of state variables grows geometrically with the number of sectors in the economy ( $M$ ). In particular, note that an entrepreneur in every sector needs to know the realizations of quality  $\{Q_m\}_{m=1}^M$ , and stochastic shock  $\{\xi_m\}_{m=1}^M$ , in order to compare the value of staying in their sector to the value of moving to different sector, which need not be an immediate neighbor. The number of state variables for each dynamic problem is  $2 \times M + 1$ . On top of that, there is little reason to think that a discrete decision can be accurately approximated with local solution methods – namely, using perturbation methods–. It is more appropriate to use a global method. In particular, we use the collocation method with Chebyshev polynomials [For a detailed explanation see Aruoba, Fernandez-Villaverde and Rubio-Ramirez (2006)]. This approach, requires



solving the model exactly for a finite number of nodes (the collocation points) for each state variable. With more than one variable, the tensor product of these nodes rapidly grows in dimension and turns the problem into unsolvable. To deal with this problem we implement the Smolyak (sparse-grid) algorithm first proposed in Krueger and Kubler (2004) and later applied and explained pedagogically in Malin, Krueger and Kubler (2011). In the latter, the authors show that their method can easily accommodate solving a multi-country real business cycle model with at least 20 countries making this method a natural solution to my problem. The algorithm I implement is,

### **Solution Algorithm**

1. Define  $M$  sectors, distributed in a unitary circle and  $\phi_M$  the symmetric distance between them.
2. Define the Smolyak collocation points  $c_j$  for  $\{Q, Z, \omega\}$ , following Malin, Krueger and Kubler (2011). In particular  $q = 5, d = 3, \& \mu = 2$  (fineness), in their nomenclature. The functions to be approximated by Chebyshev polynomials are 1.2.19 and 1.2.22.
3. Guess, for each  $\{Q, Z\}$  the associated hazard rate  $\lambda_m$ . Note that collecting these probabilities in  $\Lambda$  results in a  $(q)^{2M}$  entries tensor. Label it  $\Lambda_0$ , and  $\lambda_m = \Lambda_0(s)$  with  $s \equiv \{Q(c_j), Z(c_j)\}$  the vector of collocation points.

4. Fix sector  $m$  and  $\{r, \lambda_m\}$ . Using the polynomial approximations of (1.2.19) and (1.2.22) functions solve (1.2.17), (1.2.18,  $\forall n \neq m$ ), (1.2.20) and (1.2.21,  $\forall n \neq m$ ), using time iteration and any optimizer<sup>20</sup>.
5. With proper value functions for staying and moving, for incumbents and outsiders, check if the no-motion condition (1.2.24) holds. If not, adjust population flow  $\mathcal{F}_{(m,n)}$  and  $\{F_j\}$  accordingly by small number  $\epsilon_j, \forall j$  such that  $\sum_j F_j = 1$  holds.
6. Adjust probability (1.2.23) accordingly, store in  $\Lambda_1$  and compute distance  $d_\lambda = |\Lambda_1 - \Lambda_0|_\infty$ . If  $d_\lambda < \text{tol}$ , with  $\text{tol} = 1e - 4$ , then stop, else go to (3).

## Calibration

The basic model uses ten parameters, which we can divide into three sub-groups:  $\Theta_1 = \{\beta, \gamma, \varepsilon, \rho_\varepsilon, \delta\}$ ,  $\Theta_2 = \{\sigma_\varepsilon^2, \phi\}$  and  $\Theta_3 = \{\nu, \kappa_1, \kappa_2\}$ . The first group I will pin down using what we know from the literature. The second group we do not really need to pin down, as we require variance in these parameters in order to develop the comparative static exercises (more on this later). Finally, I will pin down parameters in  $\Theta_3$  in the basic model matching average entrepreneur productivity during average lifetime –which will pin down productivity parameter,  $\nu$ –, and the speed of completion of total innovations in such life-time to calibrate the form of the cost function of research and development ( $\kappa_1, \kappa_2$ ). We summarize this information

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<sup>20</sup>For Fortran code, in particular, I use the MIDACO initiative (Mixed Integer Distributed Ant Colony Optimization) available at [www.midaco-solver.com](http://www.midaco-solver.com)

in table (1.4).

Table 1.4: **Parameter Calibration:** For the internally calibrated parameters the data on the “speed of completion” of life-time patents considers a career of 8 years (sample average), from the first invention. Simulated data constructs a panel of entrepreneurs with 15 years, and is also censored to start with one innovation and eight periods after. This procedure makes the two panels of data comparable.

<b>(a) Externally calibrated</b>					
Description	Parameter	Value	Target Moments	Data	Model
Relative Risk Aversion	$\gamma$	1.00	RBC literature	-	-
Discount Factor	$\beta$	0.98	2% interest rate	-	-
Elasticity Substitution	$\varepsilon$	2.00	Mild Substitute	-	-
Research Cap. Dep.	$\delta$	0.12	Nadiri and Prusha (1996)	-	-
Step quality ladder	$\sigma$	.061	4% growth rate	-	-
Shock persistence	$\rho_\varepsilon$	0.41	TFP from PWT	-	-
<b>(b) Internally calibrated</b>					
Description	Parameter	Value	Target Moments	Data	Model
Innovation intensity	$\nu$	3.7	Av. # inventions	4.51	4.57
			Compleat. %(y1)	0.247	0.219
			Compleat. %(y2)	0.356	0.328
R&D spending efficiency into research capital	$\kappa_1$	1.8	Compleat. %(y3)	0.464	0.425
			$\kappa_2$	2.1	Compleat. %(y4)
	Compleat. %(y5)	0.677			0.665
	Compleat. %(y6)	0.780			0.777
	Compleat. %(y7)	0.884			0.888
	Compleat. %(y8)	1.000	1.000		

We will assess the performance of the model generating an artificial panel of inventors and running the same regression as in table (1.2). Entrepreneurs have an eight year career on average, thus the artificial panel generates 15 periods and considers the eight periods of every inventor after her first invention. This is done to mimic the data, where the first time we observe an inventor is with her first patent application. We consider different degrees of volatility and diversification for the regressions to check if the model can generate data-like patterns in the innovation process. Note that we are not trying to target the regression outcome, but

rather just checking if the model can accommodate realistic outcomes. In order to complete this exercise, one last piece of data-model match has to be considered. In the data we observe the number of available technology classes available for inventors, and use it as a regressor. In the model it is intellectual distances  $\phi_{m,m'}$  which largely determines the effect of diversification on inventor mobility. There is a way to reconcile these two variables, as shown in in stylized fact 1. Transited distance is strongly correlated with the number of available technological sectors. In particular, consider a starker correlation, between the number of sectors in each economy and the minimum distance transited by its inventors. In the Appendix, figure A.1 shows how related these two series are. Then, we take the distance from the model which generates data on inventors and project it through this linear correlation and obtain the same regressor used in table 1.2. The estimated coefficients are now comparable. We are particularly interested in  $\{\beta_1, \beta_2, \beta_3\}$ , the coefficients related to diversification, volatility and their interaction. The results of the regression on the baseline calibration use variance from the regressors in the supports for  $\phi \in [.01, .3]$  and  $\sigma_\varepsilon \in [.01, .05]$ . The estimation method is ordinary least squares, and no fixed effect or other control is added, since this is simulated data. The model is able to generate realistic features that resemble data regressions. In particular, from the data we know that, all else equal, one percent of increase in the number of technology classes implies 14% average more innovations in a lifetime. Our benchmark calibration can generate 8.1% more innovations. Higher volatility operates in the

same way that in the data as well. One extra point of standard deviation implies 0.015% less innovations on average, whereas the model can generate 0.023% less innovation when volatility raises by 1%. Finally, the coefficient on the interaction of the (log) number of sectors and volatility implies a positive coefficient of 0.039 in the data. The simulated data can accommodate 0.017. The signs are correct, even when the exact numbers of the regression are not the same. However we consider this good performance of the model and validation of the mechanism. Note that the mechanism of this paper is not the only potential way in which diversification can affect innovation. In particular, Appendix A.5 elaborates on two alternative mechanisms which can potentially generate the positive partial correlation  $\beta_1$ , but fail to generate the other two correlations.

Table 1.5: **Regression on Simulated Panel Data from the Model.** Dependent variable is (log) number of innovations at inventor data. Each observation is one inventor. Simulated data considers  $N_E = 500$  entrepreneurs who live  $T = 20$  periods. (1) Units of variable are log of number of innovation, (2) Units of variable are standard deviation expressed in percentage.

Coefficient	<i>Diversification</i> (1) $\beta_1$	<i>Volatility</i> (2) $\beta_2$	<i>Interaction</i> $\beta_3$
Data (Table 2, column 1)	0.140 (.000)	-0.015 (.000)	0.004 (.000)
Simulation	0.081 (0.002)	-0.023 (0.011)	0.018 (0.001)

In order to answer the question on what is the cost of volatility for this economy, we need to solve the model twice, considering the different volatility parameters (one positive, one zero). This, exercise, however, is precisely what is behind what we did

to elaborate table (1.5); solving the model under different parameter families and then pooling the results in a linear regression. Under this lens, the cost of volatility is precisely the foregone innovation. The cost of 1% of standard deviation is 2.3% less innovation at the inventor level. This would mean that for the median country in terms of diversification and volatility, reducing by one hundred basis points the standard deviation of shocks means (i.e. from 2% to 1%) implies output grows 0.08% faster on average. Now consider the effect of higher diversification. In particular, consider again our example of the typical emerging and advanced economy. Namely, Mexico and Korea. Mexico has 27 technological sectors compared to 49 in Korea. The numbers in table 1.5 imply that Mexico would grow an extra 0.23% (one fourth of 1%) more every year had it enjoyed the technological diversification of Korea. While these numbers may seem small, they are expressed in yearly growth rates, and in that perspective are not small at all. In particular, the order of magnitude is similar to the effect of augmenting one year of schooling in the growth model of Bils and Klenow (2000). This completes our measurement of the effect of volatility and diversification on innovation and growth.

## 1.5 Conclusions

Inventors are at the center innovation; the creation of new ideas and and improvement of how we do things. This innovation process requires them to learn skills, customize equipment, and accumulate formal knowledge and above all, accumulate

experience, in their respective field. In this paper we have labeled this cumulative effort as “knowledge capital”. Knowledge capital has two characteristics. First, it can only be adjusted slowly, and second, it cannot easily be sold or transferred to another party. That is, it is *inalienable*. This characteristic becomes crucial for the analysis of knowledge-capital accumulation because inalienability eliminates the option for the inventor to ex-post reduce the level of knowledge capital, if he deems her holdings are above optimal. This is particularly interesting if sectors are subject to shocks. If good shocks (those that raise the value of holding knowledge-capital in such sector) affect the inventor’s sector, he benefits from them, and from accumulating more knowledge capital. On the other hand, if bad shocks affect their sector, the inventor would like to sell previous investment, but she cannot. Anticipating this possibility, a rational inventor would be cautious not to accumulate as much capital as one who can freely adjust their holdings of knowledge capital upwards as well as downwards.

A “technologically diversified” economy (one in which there are many production sectors where inventors could innovate) provides a way out to the inventor with above-optimal holding of knowledge capital. In a diversified economy, knowledge-capital may be useful to innovate in more than one sector. Therefore, while inventors may not be able to sell and adjust downwards their holdings of knowledge capital, they may be able to *move* themselves to a different sector with better prospects. The decision to move to a different sector will depend on how technologically distant the

origin and destination sectors are, which on average is shorter the more diversified the economy is.

In this paper we build an endogenous growth model with inventor mobility across sectors, in order to quantify the contribution of technological diversification to innovation. In this model, any given inventor, in every period, decides on two things, which sector to work in and how much to invest and accumulate in knowledge capital. To determine if she moves she considers how “close”, the destination sector is, which will be determined by how technologically diversified her economy is. If inventors know that after a bad shock in the future, they will have the option to move to a different sector where their knowledge-capital will be useful, they will be more prone to invest more today. On the other hand, if they know that after a bad shock in the future they will have to stay in their sector with more-than-optimal knowledge-capital, they will cut R&D investment today, in anticipation. The former situation is the case of a well diversified economy, and the latter of a poorly diversified one. A calibrated version of this model to mimic some features of inventor patenting dynamics, predicts that – keeping the volatility of shocks constant – if the typical emerging economy (namely, Mexico) would be as diversified as the typical developed economy (namely, Korea), then innovators in the former would on average be 7% more productive, which in the model would translate into growing an extra 0.23% every period. Alternatively the model allows us to calculate the forgone economic growth caused by the stochastic realizations of profit streams



to inventions. In particular, shutting down volatility completely – for a country with 3% standard deviation of output growth – implies 0.22% higher growth every period. While 1/4 of a percentage point seems not too important, its impact can grow large in long periods of time.

## Chapter 2

# Saving thy Neighbor: A Model of Cross Country Bailouts

### **Abstract**

We extend the sovereign default model in the tradition of Eaton and Gersovitz (1981) to consider the consequences of strategic bailout from a third country. In this type of models, default is strategic as debt is not enforceable. In this model in particular, bailout is strategic, as there is no obligation to extend it. We show that this policy has the potential to lower spreads through the implicit guarantee it implies, but also creates moral hazard which could raise default probabilities, and hence, spreads. We also show that higher income realizations for the creditor economy make bailouts more likely to be extended. Finally, we use our model to assess the spread dynamics of the periphery countries in the 1999-2008 period and find that the model can reconcile aspect that the standard model would fail to.

## 2.1 Introduction

We extend the canonical Eaton and Gersovitz (1981) / Arellano (2008) model of sovereign strategic default to understand the consequences of strategic bailout from another country who has investment exposure to defaultable debt. Default is strategic because debt is not enforceable. Bailout is strategic because there is no obligation of the creditor economy to carry it out. Every period both countries decide on whether it is convenient for themselves to carry out any given policy. The form of the bailout we consider in this model aims to mimic reality. Bailout is understood as rolling-over of outstanding external debt with a haircut. Bailouts are not a certain event, but the mere possibility of it changes the incentives of the debtor country, in the form of moral hazard. This is the key contribution of this paper, that differs from previous work analyzing implicit debt guarantees.

In the model we consider there are three agents. The debtor (periphery) country, the creditor (core) country, and international investors. International investors are deep pocketed and risk neutral. Agents in both countries are risk averse and of unit measure. In both countries the government is benevolent and uses transfers to maximize welfare of its inhabitants using world financial markets. The creditor country can borrow and later decide if it is in its best interest to default. If the country defaults, then it is excluded from world financial markets indefinitely, and its output reduced while this exclusion lasts. By introducing the possibility of a bailout, defaulting incentives change. First, debt is priced accordingly, and the me-

chanical effect is to lower spreads because the expected pay off increases. However, the inclusion of a bailout probability raises the value of defaulting because there is a chance that the country will start next period, not excluded from financial markets, and with lower debt due to the haircut embedded in the rescue package. Which effect dominates on the consequences for spreads, is a quantitative question, and in particular of the bailout probability. The creditor country also acts strategically in the best interest of its inhabitants. When faced to a non-default episode it can choose how much to invest in the periphery debt and in a risk free instrument. However, after the periphery country decides to default on its obligations, the core economy can either assist the former in the form of a coordinated bailout, or can do nothing and allow it to default. A bailout would be beneficial because it saves a part of the outstanding debt, but it comes to a cost<sup>21</sup>. There is not just one creditor who can roll over debt. The cost can be interpreted as coming from coordination of other creditors, real resources disbursed as a signaling mechanism on the confidence of the bailout operation, or more pragmatically, the initial equity for a permanent bailout institution<sup>22</sup>. On the hand, not bailing out the periphery economy has the

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<sup>21</sup>In this model, the bailout benefits the creditor because it partially saves the previous investment in periphery debt. In reality there are many reasons why a core economy would be interested in not allowing the periphery to fail. In particular, if the smaller economy is left out of financial markets it is concomitant that trade runs into a bottleneck [see Mendoza and Yue (2012), and trade is a bilateral gain. Also, there might be banking exposure from core economy banks to subsidiaries in the periphery. In this paper we abstract from financial linkages, and deterioration of trade. The main cost of allowing the debtor to fail is the extinction of the obligation in which the core country invested

<sup>22</sup>It could be argued that a permanent bailout institution will no longer act strategically if it has a financial stability mandate. However the initial disbursement of equity for it, is a strategic decision. We refer to the European Financial Stability Facility, which has €440 billion to borrow, of which 27% was financed by German equity contribution

advantage of not incurring in any cost, but amounts seeing the invested money in the periphery country vanish overnight, with all its consequences.

We investigate the main lessons of this setup and how they compare to the canonical model. In particular, we illustrate how two forces are at play in the pricing of periphery debt. The implicit guarantee extended by the core government to anyone holding periphery debt and the moral hazard this very same guarantee creates. All in all, the main lessons in Arellano (2008)'s work hold. And we can also assess the effect of incomplete debt write-off after the extension of a bailout. We show that incomplete haircut can have disciplinary effects and make default more costly, and therefore a less attractive option. We also analyze the interaction of the perceived bailout probability and the exogenous exit from financial exclusion. Finally on the side of the lender economy, we show that the strategic decision of extending a bailout is more likely to occur with higher than lower income realizations. That is, when the lender economy is going through a bad period, marginal utility of forgone consumption, that any cost the bailout entails, is higher, and therefore the perceived cost of extending a bailout is higher as well.

In this paper, we rule out “selective default” of external debt. That is, the periphery economy cannot selectively decide what creditor not to pay based on its nationality. During default episodes of Russia, Ecuador, Pakistan, Ukraine, and more notably, Argentina their own national banking systems suffered tremendous losses [See Gennaioli, Martin and Rossi (2014) and IMF (2002)]. Defaulting gov-

ernments cannot perfectly discriminate creditors. By the same token, bailouts are not selective either. The core government is not able to exercise perfect discrimination and simply bailout its own banking system. Selective bailout would require to target perfectly the bondholdings of national banks and investors at the time of default, but this is extremely difficult as in practice these bonds are actively traded within the second in secondary markets [see Broner, Martin and Ventura (2010)]. Thus the available option is to bailout the borrower altogether or not at all. Note that the non-discriminatory bailout feature of financial assistance creates a positive externality of the bailout on the foreign investor who purchased periphery bonds, but does not chip in when it comes to extend a bailout program to a defaulting economy.

We use the model to investigate the unfolding of the European Debt Crisis (EDC), but more importantly the decade before it, which exhibited remarkable low and stable spreads on periphery debt. From the standard model of sovereign debt we know that it is reasonable to expect the pricing of debt is a reflection of the ability of particular country to honor its commitments. Lower income and higher debt burden should be reflected in greater vulnerability to default and hence, in higher financing spreads. This, apparently, did not happen for a decade, and somehow the mechanism was triggered back in 2008. We offer a mechanism that can reconcile this pitfall. In particular we use the bailout argument to defend this spread dynamics, which the model is able to accommodate.

## **Contribution to the literature**

This paper builds on a large literature which examines the motivation of a country to pay its debt when there is no enforceability. Quantitative recent work on strategic default includes Arellano (2008), Aguiar and Gopinath (2007), Yue (2010) and Mendoza and Yue (2012) and Hatchondo, Martinez and Saprizza (2010) among many others. However the literature on strategic bailouts is far thinner. We contribute to the literature with a model that considers jointly these two strategic behaviors.

Bailouts take different interpretations in the literature. For Farhi and Tirole (2012), a bailout is an action of monetary policy to reduce interest rates in order to provide liquidity (when necessary) to save risky projects. In Farhi and Tirole's model bailouts are directed transfers from liquidity owners to firms in liquidity distress. In our model bailouts are not a transfers, but roll-over of debt. There is also a branch of the literature that considers bailouts mainly from the point of view of institutional agents, like the IMF, and concentrate in particular on the potential moral hazard effect this mechanism would create. See Corsetti, Guimaraes and Roubini (2006), Eichengreen (2003), and Jeanne and Zetterlmeyer (2001). Our view in this paper differs from this international architecture perspective. We think about bailouts in a strategic way, by a country that does not have a mandate for financial stability but extends a bailout because it benefits its own inhabitants.

More closely related to the concept of bailout of this model are the definitions used by Aguiar and Gopinath (2006) who model bailouts as a grant to creditors in the case of default from an unmodelled third party. The form of this grant is that creditors are reimbursed the amount of debt up to some limit, a guaranteed *debt ceiling*. Any unpaid debt beyond the debt ceiling is loss to the creditor if the country defaults. Then, a bailout implies that up to the debt ceiling the debt carries risk free interest rate, and that default probability is used only to price any debt beyond this ceiling. A second alternative to understand bailouts in the context of sovereign debt is given by Roch and Uhlig (2012) who consider a minimal level of debt that must be guaranteed in order to obtain a “good equilibrium”. This minimum level imposes an assisted price schedule which feeds back into the actual pricing of debt in the form of a *price floor*. Any amount of debt can be sold at least at the floor price. The bailout policies in any of these two papers changes the pricing equation, but does not directly enter in the default decision of the periphery country. Our model addresses this issue directly. More importantly, we can address the changing nature of bailout incentives through the business cycle of the creditor economy.

## **Outline**

The next section (2.2) elaborates on the model. Section 2.3 examines the main properties of the model and characterizes the notion of equilibrium. Next, we elaborate an application of the model under the context of the European Debt



Crisis, and the decade before in Section 2.4.

## 2.2 A Simple Model

This is a model of three agents. A periphery economy, a core economy and the rest of the world in the form of deep pocketed investors. Both countries are small enough not to affect some prices of the international capital markets, but act strategically with respect to one another. Let there be a borrower (*periphery*) country  $\mathcal{P}$ , who decides strategically on reneging from its outstanding external debt, and there be a creditor economy (*core*) country  $\mathcal{C}$ , with exposure to this periphery debt. The government of the creditor country,  $\mathcal{C}$ , may intervene, in order to protect its investment, after a default episode by the periphery economy. The form of this “financial assistance” is in the form of roll-over periphery debt with a “haircut”. There are many sensible reasons to think  $\mathcal{C}$  would want to assist  $\mathcal{P}$ . A prominent one is that its financial system has exposure to the periphery’s sovereign debt, and in the absence of assistance, default means interruption of the chain of payments and financial fragility in the domestic credit flows. That is, there is something to lose from letting the  $\mathcal{P}$  country fall. While we can argue that the government of  $\mathcal{C}$  may assist their exposed banks only, in reality it is hard for governments to exercise perfect discrimination because bonds are actively traded in secondary markets within the second, and such a policy would turn impracticable [See for example Gennaioli, Martin and Rossi (2014), and Broner, Martin and Ventura (2010)]. In

this paper, then, I abstract from banks and therefore there is no choice between bailing out national banks directly or the periphery country. The absence of perfect discrimination is an assumption that results in that the government has to choose between assisting the periphery economy or doing nothing at all.

From the perspective of the borrowing country I will follow closely the canonical model by Arellano (2008) incorporating the possibility of international assistance by the core economy in the pricing of periphery sovereign debt. The inclusion of international assistance changes the expected recovery value of defaulted debt, and acts as an implicit guarantee, but also changes the recurrence of default episodes. The final effect on the pricing of periphery debt is then a quantitative question. All this analysis is done on one period, uncontingent, debt.

### **2.2.1 Borrower Country**

Consider a small open economy ( $\mathcal{P}$ ) populated by a representative household that receives a stochastic stream of income  $y$ . The government can issue debt  $b < 0$  in the international capital markets, and sell it to risk neutral investors, but also to the core economy. This debt is not enforceable and the government can default on it at any time. If the country decides to default it is temporarily excluded from the financial markets, and incurs in a direct output cost<sup>23</sup>. But even if the country decides to default, it may receive financial assistance in the form of preferential roll

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<sup>23</sup>In this paper, as in Arellano (2008) we model a reduced from output cost. There are micro-founded ways to argue for this output cost and its convex nature, as in Mendoza and Yue (2012)

over of outstanding debt. If the core economy bails out the small economy, the latter resumes its participation in international markets.

The government in the borrowing country is benevolent, and fully incorporates the utility function of its inhabitants. It uses the international financial market to smooth their consumption via transfers. Households are identical and risk averse. Their preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.2.1}$$

where  $0 < \beta < 1$  is the discount factor,  $c_t$  is the period consumption and the instantaneous utility function  $u(\cdot) : [0, \infty) \rightarrow \mathbb{R}$ , is increasing and strictly concave. Also, this is an endowment economy with a Markov transition function  $f(y'|y)$  in a compact support  $\mathcal{Y}$ . As mentioned earlier, this government can issue one period bonds, that are uncontingent to the extend that they are not defaulted on. The price schedule for selling these bonds is given by  $q(b', y)$  that depends on both, the size of the outstanding debt  $b'$  and income shock  $y$  because the default probability depends on them too. Positive values of  $b$  imply the government owns a contract that entitles it to be paid  $b$  dollars. We use negative values of  $b$  for debt. Then in any period that there is periphery debt,  $\mathcal{P}$  issues a contract to receive  $-q(b', y)b' > 0$  and pay back  $-b' > 0$  next period. The resource constraint is then given by,

$$c = y + b - q(b', y)b' \tag{2.2.2}$$

Should the country decide to default, financial exclusion implies that consumption equals income, and income is lower than output as in,

$$c = y^{\text{def}} = h(y) \leq y$$

with  $h(y)$  increasing and cuasi-concave. However, there is a probability  $g$  that it receives international assistance and takes it out from financial exclusion. This is one of the main deviation from the canonical model. Later we will elaborate on the determinants of  $g$ , but for the periphery economy, this is a given; it cannot affect it directly.

### Recursive formulation

Let the state variables for  $\mathcal{P}$  be given by  $s = \{b, y\}$ , the bonds outstanding and the income shock. Conditional on deciding to honor their obligations,  $\mathcal{P}$  chooses next period debt  $b'$  to solve the following problem.

$$v^c(b, y) = \max_{b'} \left\{ u(y + b - q(b', y)b') + \beta \int_{y'} v^0(b', y') dF(y'|y) \right\} \quad (2.2.3)$$

Note  $\mathcal{P}$  pays  $b$  in full and rolls borrows again  $-q(b', y)b'$ . The continuation value  $v^0$  summarizes the value of starting next period with both alternatives available, defaulting or honoring the contract  $\{q(b', y), b'\}$ .

Alternatively,  $\mathcal{P}$  may decide to default on outstanding debt  $b$ . If it does, output

falls in the current period to  $h(y) < y$ . There is a probability  $g$  that it is assisted by the core economy and that debt is reduced by a fraction  $\alpha$ . If there is no international assistance, with probability  $\theta$ ,  $\mathcal{P}$  returns to good financial standing exogenously. This is meant to capture the fact that after a finite number of periods the default episode is let go by international investors in terms of reputation. The value of going to default is then given by,

$$v^d(b, y) = u(y^{\text{def}}) + \beta \int_{y'} \left\{ g \left[ v^0((1 - \alpha)b, y') \right] + (1 - g) \left[ \theta v^0(0, y') + (1 - \theta) v^d(0, y') \right] \right\} dF(y'|y) \quad (2.2.4)$$

There is no access to international markets, so consumption is equal to disposable income. Note that in the case of defaulting on current debt there is no choice of next period debt. On the one hand if there is financial assistance, next period debt will automatically be lower than the original  $b$  defaulted debt by a fraction  $\alpha$ , but will not be zero. The haircut  $\alpha$  parameter is assumed to be constant in this model<sup>24</sup>. On the other hand if there is not financial assistance there is total debt write off and therefore  $b' = 0$ .

Now we are in position to write the value of starting a period with total debt  $b$  and income  $y$ , before the decision of defaulting or honoring the contract has happened. In particular, considering the solution for problems 2.2.3 and 2.2.4 this

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<sup>24</sup>The haircut parameter is the focus of the bargaining problem proposed in the model by Yue (2010). Here we abstract from this issue.

value is,

$$v^0(b, y) = \max \left\{ v^c(b, y), v^d(b, y) \right\} \quad (2.2.5)$$

Let us follow Arellano (2008) in defining a repayment set of the form

$$A(b) = \{y \in \mathcal{Y} : v^c(b, y) \geq v^d(b, y)\}$$

and a default set  $D(B)$

$$D(b) = \{y \in \mathcal{Y} : v^d(b, y) > v^c(b, y)\}$$

Using this notation we can define the probability of country  $\mathcal{P}$  defaulting on debt  $b'$  in  $t + 1$ ,  $\delta(b', y)$ .

$$\delta(b', y) = \int_{D(b')} f(y'|y) dy' \quad (2.2.6)$$

Sovereign debt pricing is given by the no profit condition of the international investors,  $\phi$ .

$$\phi = q(b', y)b' - \frac{(1 - \delta(b', y))b'}{1 + r} - \frac{g\delta(b', y)(1 - \alpha)b'}{1 + r}$$

which implies that the pricing schedule is

$$q(b', y) = \frac{1}{1 + r} \left( 1 - \delta(b', y) + g(1 - \alpha)\delta(b', y) \right) \quad (2.2.7)$$

which incorporates the fact that default is a possibility but conditional on it, there is a chance that there will be international assistance which will result in every creditor getting back a fraction  $(1 - \alpha)$  of their original asset  $b'$ . The price of bonds then are increasing in  $g$  –which will result in lower spreads–, but this abstracts from the fact that this new setup might result in higher default incentives, that go in the opposite direction.

The model sketched so far is based on the canonical model of sovereign default but is different in a number of important ways besides the inclusion of  $g$ . In particular, if the bailout probability would just imply that the defaulter country transits from exclusion to inclusion to the financial markets with an exogenous probability  $g$ , this amounts to increasing exogenously  $\theta$ . This is not the case as re-entering international financial markets implies only partial debt write off, which can substantially change the incentives to default. On section **2.3** we will elaborate on how these interactions.

### **2.2.2 Creditor Country**

Whether  $\mathcal{P}$  receives financial assistance from  $\mathcal{C}$ , or not, hinges on the costs and benefits for  $\mathcal{C}$  of engaging in a bailout operation. The upside for  $\mathcal{C}$  is that investment in the periphery economy is only partially lost, to the extent that the agreed haircut is not complete. Also, by impeding that the borrowing economy is left out of world capital markets, trade linkages –which are inextricably related to capital linkages–

are preserved. The downside for  $\mathcal{C}$ , is that in addition to forfeit accrued capital gains, it has to incur in coordination costs with other debt holders, and has to be willing to not to receive its capital in the current period but in the future, with a probability lower than one. Now let us turn to describing the setup of the core economy  $\mathcal{C}$ .

The core economy also has the possibility of issuing its own, riskless debt in international markets. We are assuming that the reputation the  $\mathcal{C}$  economy enjoys is such that any debt it issues can be sold at rate  $r$ . This is also an endowment economy with income  $AY_t$ . Where  $Y_t$  evolves according to an AR(1) process and where  $A$  is a scale parameter. In particular

$$\log Y_t = \rho_Y \log Y_{t-1} + \varepsilon_t^Y + \nu_t$$

where  $\varepsilon_t^Y$  is the idiosyncratic core country shock, not correlated with  $\varepsilon_t^y$  – the idiosyncratic periphery country shock–, and variance  $\eta_Y^2$ . Finally,  $\nu_t$  is a common shock with variance  $\sigma_\nu^2$ . This implies that unconditional variances of incomes are then given by  $\text{Var}(\log y) = \frac{\sigma_\nu^2 + \eta_y^2}{1 - \rho_y^2}$  and  $\text{Var}(\log z) = \frac{\sigma_\nu^2 + \eta_Y^2}{1 - \rho_Y^2}$ .

The time line of decisions is the following. Before investing in risky sovereign debt the core country does not know if the periphery country will effectively default or not. In particular, the core economy can only assess such events with probability  $\delta(s)$ . Once in the period, the periphery country that has already engaged in  $b_t$  debt in  $t - 1$ , learns  $y_t$  and decides strategically to default or not. Conditional on the



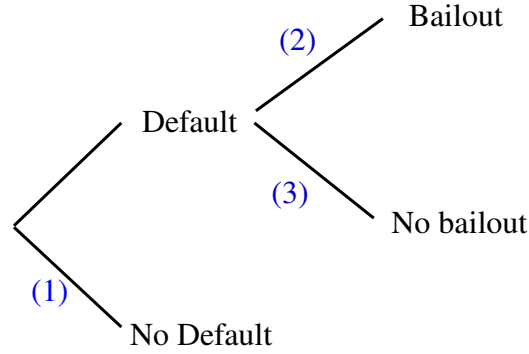


Figure 2.1: **Sequence of Events.** Possible scenarios that  $\mathcal{C}$  has to consider.  $\mathcal{P}$  moves first. If  $\mathcal{P}$  does not default then  $\mathcal{C}$  can only choose asset holdings (1). If  $\mathcal{P}$  defaults then  $\mathcal{C}$  decides on bailing out (2) or not (3)

sovereign default episode, the core country, decides on whether or not to engage in a bailout plan. The outcome of such a decision hinges on the cost benefit analysis for its household [choose (2) or (3) in figure 2.1].

The government in this economy is benevolent and uses international markets to maximize intertemporal welfare of its households. There are basically two instruments the household can use to transfer consumption from one period to another. First, it can access the international markets to issue (positive or negative) risk free debt ( $D_t$ ) which pays  $1 + r_t > 1$  units in return at maturity in  $t + 1$  for a unit investment in  $t$ . The second alternative is investing in periphery debt  $B_t$  which is subject to the possibility of default and therefore riskier than  $D_t$ . While the core economy cannot influence prices of risky sovereign debt, it can assess the probability of default for next period because it observes the state variables  $s_t$  and knows the problem solved by the periphery economy. The household in the  $\mathcal{C}$  economy chooses on bailing out  $\mathcal{P}$ , and on allocations for consumption and assets to solve:

$$\max_{C_t, D_{t+1}, B_{t+1}} E_0 \sum_{t=0}^{+\infty} \beta^t u(c_t) \quad (2.2.8)$$

subject to the relevant budget constraints, each of which depends on the actions of both countries. Then, the three potential scenarios are

- (1)  $\mathcal{P}$  does not default. There is no room for government intervention. This is by assumption but also would be an equilibrium outcome as there is nothing to gain from rolling over debt and conceding a haircut if the borrower country is willing to pay in full. The budget constraint is given by

$$C_t = AY_t + D_t - B_t - (1+r)^{-1}D_{t+1} + -B_t + q(b_t, y_t)B_{t+1}$$

- (2)  $\mathcal{P}$  defaults and there is a bailout plan with haircut  $\alpha$  and coordination cost  $\lambda^{25}$ . For this, all creditors who own periphery debt must chip in an extra exogenous cost every time a bailout operation is in place. This reduced form cost is meant to capture not only any administrative cost of bailing out the country but also the signaling each creditor must do, in order to credibly state its commitment to saving the troubled economy.

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<sup>25</sup>The core country is not the sole creditor of the periphery economy, and thus it cannot simply agree not to receive the promised revenue and completely solve  $\mathcal{P}$ 's debt problem. In particular, it is one of several creditors and thus must convince them to bailout the periphery country. This resembles the creation European Financial Stability Facility (EFSF)

$$C_t = AY_t + D_t - (1 + r_t)^{-1}D_{t+1} - \lambda - B_t(1 - \alpha) + q((1 - \alpha)b_t, y_t)(1 - \alpha)b_t$$

- (3) Finally, if the core economy decides not to assist the troubled economy, it does not incur in the  $\lambda$  cost but since default has already been declared it will not see its original investment  $b_t$ . Also, the periphery economy is in no position to issue debt during the defaulting period, and thus the core economy cannot use this investment instrument

$$C_t = AY_t + D_t - (1 + r_t)^{-1}D_{t+1}$$

### Recursive Formulation

This problem can be rewritten in recursive form which makes it easier to be computationally solved. Again the state variables are  $s = (b, y)$ ,  $S = (B, D, Y)$ . I will therefore denote  $V^0(s)$  to be the value function of the core government at the beginning of the period. Also, I will use  $V^B(S)$  and  $V^{NB}(S)$  to refer to the value functions after taking the decision to bailout or not bailout  $\mathcal{P}$  respectively. Finally, let me refer to  $V^{ND}(s)$  as the value function for the economy if there is no sovereign default to begin with.

If there is no default then the problem for  $\mathcal{C}$  is simple. Given interest rate, and non-default decision by  $\mathcal{P}$ , and bond schedule  $(r, q)$ ,  $\mathcal{C}$  chooses asset positions

$(B', D')$  to solve

$$V^{ND}(S) = \max_{B', D'} \left\{ u \left( AY + D - (1+r)^{-1}D' - B + q(s)B' \right) + \beta \int_{Y'} V^0(S') dF(Y'|Y) \right\} \quad (2.2.9)$$

Once the  $\mathcal{P}$  defaults, then the core economy has to decide if it is in its best interest to assist  $\mathcal{P}$ . For that, the former compares the value of bailout,  $V^B(s)$ , and the value of not doing so,  $V^{NB}(s)$ . When a bailout plan is unfolded,  $\mathcal{C}$  coordinates creditors such that a fraction  $(1 - \alpha)$  of outstanding debt is rolled over. In the current period there are no debt repayments, on the promise that next period the initial obligations are  $(1 - \alpha)b$ .

$$V^B(S) = \max_{D'} \left\{ u \left( AY + D - (1+r)^{-1}D' - \lambda \right) + \beta \int_{Y'} V^0(S') dF(Y'|Y) \right\} \quad (2.2.10)$$

subject to

$$B' = B(1 - \alpha)$$

Alternatively, the core government may decide not to bailout the  $\mathcal{P}$ , and allow it to enter default. Debt is completely repudiated, the current investment in the periphery vanishes, and so does the possibility to invest in risky assets for the next period. However, as of next period there is a chance ( $\theta$ ) that the periphery economy will exit the default state exogenously. Then the value of not bailing out

the periphery economy is the solution to (2.2.11). Given interest rate and exogenous probability of financial inclusion

$$V^{NB}(S) = \max_{D'} \left\{ u(AY + D - (1+r)^{-1}D') + \beta \int_{Y'} [\theta V^0(S') + (1-\theta)V^{NB}(S')] dF(Y'|Y) \right\} \quad (2.2.11)$$

with

$$B' = 0$$

Finally, consider the main value function of the core economy when it starts the period. The value of being a creditor with  $(B, D, Y)$  who faces  $\delta$  default probability in  $t$  is given by,

$$V^0(S) = (1 - \delta)V^{ND}(S) + \delta \max \{ V^{NB}(S), V^B(S) \} \quad (2.2.12)$$

Define the set of income realizations such that the value of bailing out  $\mathcal{P}$  is higher than the value of letting it enter default.

$$F(B, D, Y) = \left\{ Y \in \mathcal{Y} : V^B(B, D, Y) > V^{NB}(B, D, Y) \right\}, \quad (2.2.13)$$

Therefore, conditional on a credit event the probability of assisting financially  $\mathcal{P}$  for country  $\mathcal{C}$  with assets  $B', D'$  due next period is given by,

$$g(B', D', Y) = \int_{F(B', D', Y)} f(Y'|Y)DY' \quad (2.2.14)$$

This probability is relevant for the periphery country as it changes the value of default vis a vis the value of honoring its debt, and is the main feedback from the core economy to the periphery country. While it may lower spreads by reducing the expected loss in times of default to foreign investors of periphery debt, it also may induce moral hazard by making default relatively less costly. The final outcome will, of course, be a quantitative issue but closer inspection leads to some more general results which I will develop in the next section <sup>26</sup>.

## 2.3 Equilibrium

### 2.3.1 The benchmark case

It is interesting to understand the basic effects a bailout implicit-guarantee has on the incentives to default and the pricing of sovereign debt. In particular, to what extent the main conclusions of the canonical model hold? Let us briefly

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<sup>26</sup>An earlier version of this paper consisted of a core economy populated by banks and households. While a banking sector is crucial to determine the costs and benefits of assisting another country in times of default because of financial exposure we can abstract from it in a first approximation for two reasons. First, it is still true that the government in the core economy can invest in the periphery country without bank intermediation, although we will miss the overexposure/overleverage feature of the European financial system in the midst of the European debt crisis. Second, even if we include a financial system which is overleveraged we still would need to model the fundamental financial friction that would impair economic functioning if banks were to fail. That is, we would need to model the interaction of a productive sector and banking sector which clearly is out of the scope of this paper and is a well known research agenda in itself. On top of that, the numerical exercises get too complicated because of the curse of dimensionality given the high number of state variables (6 in this case)

accommodate these lessons to the new environment. All proofs will be elaborated in Appendix **B.1** and use two main simplifying assumptions, (i) that income shocks are *iid*, and (except for Proposition 8) that the exogenous probability of re-entry in world financial markets is zero,  $\theta = 0$ .

### **Periphery ( $\mathcal{P}$ ) Country**

First, assume there is total debt write off. That is  $\alpha = 1$ . Later we relax this assumption to examine the bite of decreasing the haircut on defaulted debt.

**Proposition 4.** *For all  $b^1 \leq b^2$ , if default is optimal for  $b^2$ , in some states  $y$ , then default will be optimal for  $b^1$  for the same states  $y$ , that is,  $D(b^2) \subseteq D(b^1)$*

*Proof.* See Appendix B.1 □

Total debt write-off makes the value of default be independent of  $b$ , and then the main result follows from the fact that the value of honoring the contract is increasing in the level of assets the country enters the period with. That is, as debt increases the value of the contract decreases as well. If default was chosen with a level of assets, decreasing this level only reduces the value of the contract and thus does not change the default decision.

**Proposition 5.** *There exists a well defined threshold  $\bar{g}(b')$  below which the following holds. If for some  $b$ , the default set is nonempty  $D(b) \neq \emptyset$ , then there are no contracts available  $\{q(b', y), b'\}$  such that the economy can experience capital inflows,  $b - q(b', y)b' > 0$*

*Proof.* See Appendix B.1 □

If default is the optimal choice, then it must be true that there are no contracts which allow  $\mathcal{P}$  to grow their debt. If there are any available, the optimal strategy is to consume more today and default tomorrow on higher debt. This is the case when bailout probability is low. Alternatively, if there is a good chance that the country will be saved, and will start with zero debt and not excluded from world financial markets, then it pays consume more today and default the next period. But this is true for every possible realization of the income shock, which drastically changes the pricing of debt. Nobody is willing to lend if they know there is no chance of recovering the debt. Then it must be true that if default is the optimal strategy, then only capital outflows are available for the borrowing country.

**Proposition 6.** *For bailout probability low enough,  $g < \bar{g}(\cdot)$ , default incentives are stronger the lower the endowment. That is,  $\forall y_1 \leq y_2$  if  $y_2 \in D(B)$  then so does  $y_1 \in D(B)$ .*

*Proof.* See Appendix B.1 □

This result is important because what we observe in the data is that countries usually tend to default when income is low rather than high. This result is a result of the concavity of the utility function. When output is lower, then it must be the case that it is more expensive in terms of marginal utility to honor the contract instead of repaying it. Results so far are only confirmation of what we know about



the canonical model of sovereign debt, and are presented here to show they are not reversed under the implicit bailout guarantee. Next we start analyzing the direct implications on the incentives to default of two key characteristics of the environment, namely, the probability to being bailed out, and the haircut percentage. The most direct effect of a bailout guarantee is to lower spreads as is evident from 2.2.7. However this guarantee creates moral hazard. On the expectation of being assisted, the default option becomes more valuable. This second effect is not considered in the cited work of Aguiar and Gopinath (2007) and Roch and Uhlig (2012), since both of them treat the bailout as not internalized by the  $\mathcal{P}$  economy. That is one of the main contributions of this paper.

Now, let us relax the assumption of  $\alpha = 1$ . So far we have assumed that if  $\mathcal{P}$  chooses to default, and is later saved by a third country, then debt is totally written off. However, episodes of total debt write-off are not the rule, but rather the exception, if at all existent. What is more plausible to happen is that after a bailout, the debtor country is benefited from only a partial haircut [ $\alpha \in (0, 1)$ ] of total outstanding debt which reduces the debt burden to  $\tilde{b}(t+1) = (1-\alpha)b(t) \geq b(t)$ .

**Proposition 7.** *(Default sets are shrinking in post-bailout balance of debt). For all  $\alpha_0 < \alpha_1$ , if default is optimal for  $b$  under parameter  $\alpha_0$ , for some states  $y$ , then default will be optimal for  $(b, y)$  under  $\alpha_1$ . That is  $D(b|\alpha_0) \subseteq D(b|\alpha_1)$ .*

*Proof.* See Appendix B.1 □

The intuition of this proposition is simple. Given a probability of bail out in-

creasing the haircut of debt after it only gives a better financial position to the defaulting country. Haircut then, turns out to be a key element shaping the incentives to default on sovereign debt. The formal proof of this proposition is also in Appendix **B.1**. An indirect effect is that the haircut will shape the effect of increasing the bailout probability on the incentives to default. For instance take the extreme case of no haircut ( $\alpha = 0$ ), then increasing  $g$  only raises the probability that the country has to honor their commitments anyway, even after it decided to default in the first place. This lowers the value of default. Alternatively if the haircut is large or close to one, then increasing the probability of bailout amounts to being saved with no repercussions, which raises the value of default. Finally, it is interesting to examine the interaction of the exogenous probability of re-entry into the financial markets,  $\theta$ , along with the bailout probability,  $g$ .

**Proposition 8.** *(Default sets are shrinking on  $g$  for large  $\theta$  and expanding on  $g$  for small  $\theta$ ): For  $g_L \leq g_H < \bar{g}$ , (i) if default is optimal for  $b$  in some states  $y$ , with  $g_H$ , then default will be optimal in the same states with  $g_L$  when  $\theta$  is high, (ii) if default is optimal for  $b$  in some states  $y$ , with  $g_L$ , then default will be optimal in the same states with  $g_H$  when  $\theta$  is low.*

*Proof.* See Appendix B.1 □

The key insight of Proposition **8** is that the exogenous re-entry into financial markets competes with the bailout probability. There are two opposing cases. First, if it is likely that the defaulting country will enter financial markets anyway (high

$\theta$ ) higher bailout probability means that at least in this case some of the debt will be repaid. If there is exogenous probability of re-entering financial markets, all debt is repudiated. This lowers the value of default and shrinks the set. The second case is that it is very unlikely to enter the financial markets exogenously with a clean start of no debt (low  $\theta$ ). Then, the bailout probability raises the value of default. In the alternative case, default is an absorbent state. Next we elaborate on how the bailout decision and probability are formed looking at the problem that the core economy has to solve.

### **Core Country**

The benefit for the core economy of saving its neighbor is that only a portion of the initial investment is lost rather than the whole  $b_t$  investment. In a more general model including trade and production technology, it would also prevent that the concomitant recession in the periphery economy translates in lower local output. The cost of assisting the defaulting economy comes in several ways. First, the lower return on the initial investment (by a fraction  $\alpha$ ). But more importantly there is a coordination cost with other creditors that is a real cost in itself, but many times comes with signaling disbursements, lines of credit or simply the creation of a special facility for the troubled economy. All these are captured by the parameter  $\lambda$ .

The result that will be critical for this paper is that when output in the core

economy is lower, then the incentives to assist the periphery economy are lower. The model of the core economy is even less tractable than the periphery economy and for the next proposition to hold let me restrict to the benchmark case, and also assume that periphery bonds are the only investment instrument available.

**Proposition 9.** (*Bailout sets are expanding in income*). For all  $Y_1 < Y_2$ , if  $Y_2 \in F(B)$ , then  $Y_1 \in F(B)$

*Proof.* See Appendix **B.1**

□

Proposition 9 establishes a key result. When the core country is doing well in terms of output there is a higher chance that it will bail  $\mathcal{P}$  out. In particular, the application in the following section uses this result, and Proposition 6 to establish that when a negative output shock hit, core and periphery economy, the bailout probability collapsed and the default probability grew, but also there was a reinforcement effect from the former to the latter which would help explain the surge or spreads in the periphery country.

## 2.4 Implementation and numerical application

The computational challenge of the periphery country is that it features a bond price schedule,  $q(s)$  which is highly nonlinear. This makes perturbation methods unlikely to be useful [See Hatchondo, Martinez and Saprizza (2010)]. The literature typically uses discretization methods and value function iteration (VFI). VFI is a robust way of solving these models but the literature has found that for an accurate

solution in such a non-linear model, one needs to use a very fine grid. Else the margin of error is too large to extract credible conclusions out of a simulation I solve the periphery economy, approximating equations (2.2.5) and (2.2.4) with collocation of orthogonal Chebyshev polynomials. The problem of the core economy is closer to a standard optimal consumption problem and does not feature high nonlinearities. For this economy I use standard VFI. Also, the solutions for the core and periphery economies have to be consistent between each other. In particular,  $\mathcal{P}$  takes  $g$  as given, and its behavior shapes  $q$ . Simultaneously,  $\mathcal{C}$  takes  $q$  from the pricing equation of international investors to shape  $g$ . All these must be consistent among them. Next I sketch the algorithm used in the numerical application.

### Solution algorithm

1. Fix all states  $\{B, Y, b, y\}$ , with  $S = \{B, Y\}$  and  $s = \{b, y\}$  for the core and periphery economy.
2. Guess  $g^{(0)} = g_0(B, Y)$ , and fix the number of collocation points for  $b, y$ :  $N_b, N_y$
3. For variables  $y, b$  compute the collocation points in  $[b_{min}, b_{max}]$  and  $[y_{min}, y_{max}]$
4. Solve the periphery economy problem: Guess the value of  $\theta_{ij}^k = \theta_{ij}^{k,(0)}$  for  $k = \{d, c\}$ ,  $i \in \{1, \dots, N_y\}$  and  $j \in \{1, \dots, N_b\}$  and compute the Chebyshev approximation  $v_\theta^d(y, b) \simeq \sum_i \sum_j \theta_{ij}^d \psi(b_i) \psi(y_j)$  and  $v_\theta^c(s) \simeq \sum_i \sum_j \theta_{ij}^c \psi(v_i) \psi(y_j)$  and plug into 2.2.4 and 2.2.3

5. Compute the residuals of the function  $\mathcal{H}(\theta_{ij}^d, \theta_{ij}^c) = [v^d - v_{\theta}^d, v^c - v_{\theta}^c]'$  which is a vector of  $2 \times N_y \times N_B$ . If the residuals are equal to zero ( $< \text{tol}$ ) then stop, otherwise adjust  $\theta^d, \theta^c$  using a Newton Raphson algorithm and go back to step (3)
6. For  $s = \{b, y\}$  in the periphery economy, use  $v_{\theta}^k(b, y)$ ,  $k = c, d$  to determine if there is default, and if there is not, compute price  $q(b, y)$  from 2.2.7
7. Given  $q(b, y)$  solve the core economy's problem: Use value function iteration to solve 2.2.9, 2.2.10, 2.2.11, and 2.2.12
8. Calculate matrix  $g_1(B, Y)$ , and  $g^{(1)} = g_1(S)$  from 2.2.14. Compute  $\text{dis} = |g^{(1)} - g^{(0)}|$
9. If  $\text{dis} < \text{tol}$ , stop and save  $g(S) = g^{(1)}$ . Else feed  $g^{(0)} = g^{(1)}$  to the periphery problem in (2).

Note that the bailout probability is not a number but a function of  $S$ , in the same way that the bond price schedule is a function of  $s$ .

### **Functional forms and parameter calibration**

A crucial functional form which will determine incentives for the periphery economy is the instantaneous cost of default given by function  $h(y)$ . Ideally one would like to account for the transmission mechanism which makes default to be costly for production as in Mendoza and Yue (2012). In their paper, a key implication of their general equilibrium model with production, is that shocks to TFP are amplified by a

default decision and default costs are increasing and convex in TFP. This convexity is key to sustain higher ratios of debt to GDP than the previous literature. Based on this motivation I use a reduced form functional form for the function  $h(y)$  for the calibration.

$$y^{\text{def}} = \begin{cases} y_t - \gamma_0 & \text{if } y_t < \bar{y} \\ \bar{y} + (y_t - \bar{y})\gamma_1 & \text{if } y_t \geq \bar{y} \end{cases} \quad (2.4.1)$$

The parameter values used for this numerical exercise are presented in table (2.1). For the income realizations we take the AR(1) detrended log process for both Italy and Germany, before the European Union. The rest of the parameters do not need further explanation, except for the exogenous probability of re-entry. Europe is radically different from Argentina, and that is reflected in the choice of  $\theta$ . Fuentes and Saravia (2010) document that international investors do not outright punish defaulters. Only those who suffered losses in the credit episode do. Being Europe more connected than Argentina, it makes sense that this parameter should be closer to one than to zero.

### **Comparative statics**

With the solution of the model we can examine in detail partial equilibrium results on both sides, the periphery and the core economy. Consider first the periphery economy. The periphery economy takes the bailout probability as given. For a moment consider this probability fixed around  $g = 0.2$ . Then the solution of the

Table 2.1: Model Benchmark Calibration.

(\*) Calculation using International Financial Statistics data and log regression.

Calibration	Values	Target Statistics
Risk - free interest rate	$r = 1.31\%$	Germany 5 year bond yearly yield of 5.4%
Risk aversion	$\sigma = 2$	Arellano (2008)
Income $\mathcal{P}$ (*)	$\rho = 0.83$ $\eta = 0.048$	Italy GDP (1993q1-1999q2)
Income $\mathcal{C}$ (*)	$\rho = 0.38$ $\eta = 0.067$	Germany GDP (1993q1-1999q2)
Probability of re entry	$\theta = 0.282$	Fuentes and Saravia (2010)
Discount factor $\mathcal{P}$	$\beta_p = 0.975$	1.5% default yearly probability
Discount factor $\mathcal{C}$	$\beta_c = 0.98$	
Bailout cost /GDP	$\lambda = 0.02$	
Output cost	$\gamma_0 = 0.01$ $\gamma_1 = 0.969$	2.51 % debt quarterly service to GDP Trade balance standard deviation = 0.02

model reduces to computing the dynamics of the periphery economy under the benchmark calibration. In figure 2.2, panel (a) shows two different price schedules  $q(b', y_L)$  and  $q(b', y_H)$ . When income is higher it is understood that the default probability is lower and therefore the discount on next period debt is lower. On panel (b) we show the effect of two different perceived bailout probabilities ( $g = 0.1$  and  $g = 0.5$ ) on the same bond schedules for the same income realization (namely,  $y = 1$ ). There are two main effects on the pricing of bonds. First, there is a lower bound on the recovery rate of defaulted debt and, this in turn bounds periphery debt price from below. In the extreme case where bailouts are a sure event, the canonical model would predict bonds are worth zero, but the introduction of a bailout guarantee would predict  $q(b', y) = \frac{1-\alpha}{1+r}$ . Second, and by the same token, the bond price schedule is less steep which implies that this modified model can sustain higher levels of debt before entering default.



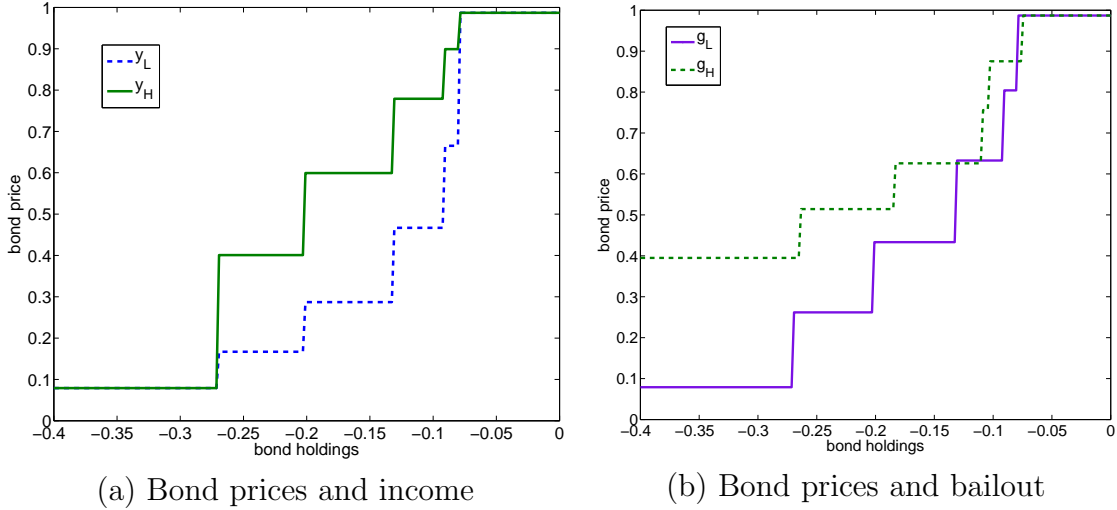


Figure 2.2: **Bond Price Schedules Under Different Income Realizations and Perceived Bailout Probabilities.** Vertical axis shows price  $q(b', y)$ , and the horizontal axis shows the bond holdings in  $t + 1$  (negative, as they are expressed as assets). Panel (a) compares income differences in  $t$  with 5 percent deviations upwards and downwards from steady state. Panel (b) show bond price schedules under perceived bailout probabilities of 10 and 50 percent

In particular consider the sensitivity of bond price schedules to income under different bailout regimes. This is the case of panel (a) in figure 2.3. Dashed lines consider the price schedule under low bailout probability, and the straight lines consider a high bailout probability. The second case is characterized not only by having a clear lower bound, and as a consequence less step change with debt issuance, but also the sensitivity to different income schedules is reduced.

Another key parameter in the model, and in particular for proposition 4, is the haircut parameter. That is, the fraction of debt that is diluted after a default episode, and not paid even after financial assistance,  $\alpha$ . The fraction  $1 - \alpha$  that must be paid after default, changes incentives. The higher the fraction that must nevertheless be repaid in a bailout scenario lowers the value of entering default in

the first place, making it a less likely event, raising bond prices. This can be seen in figure 2.3, panel (b).

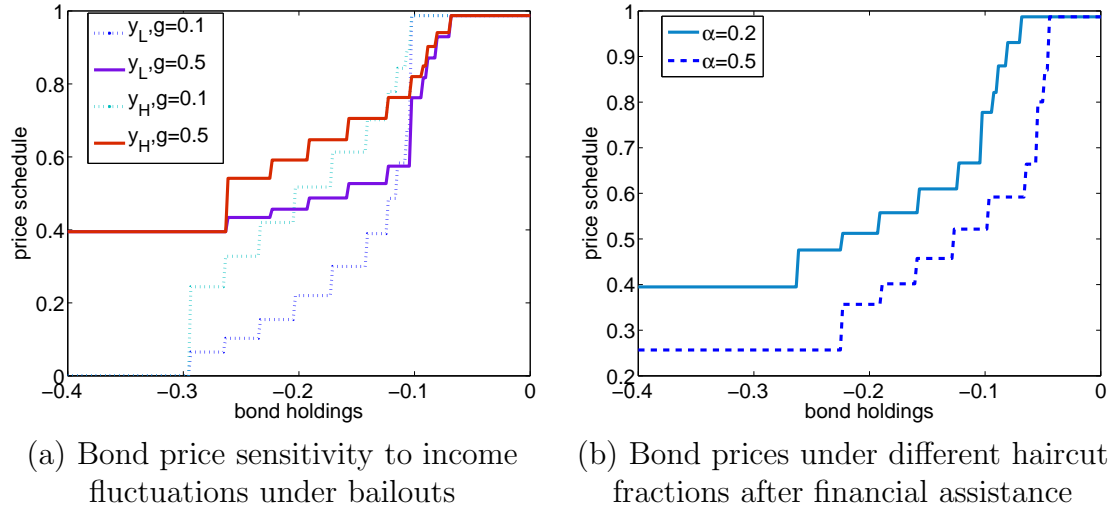
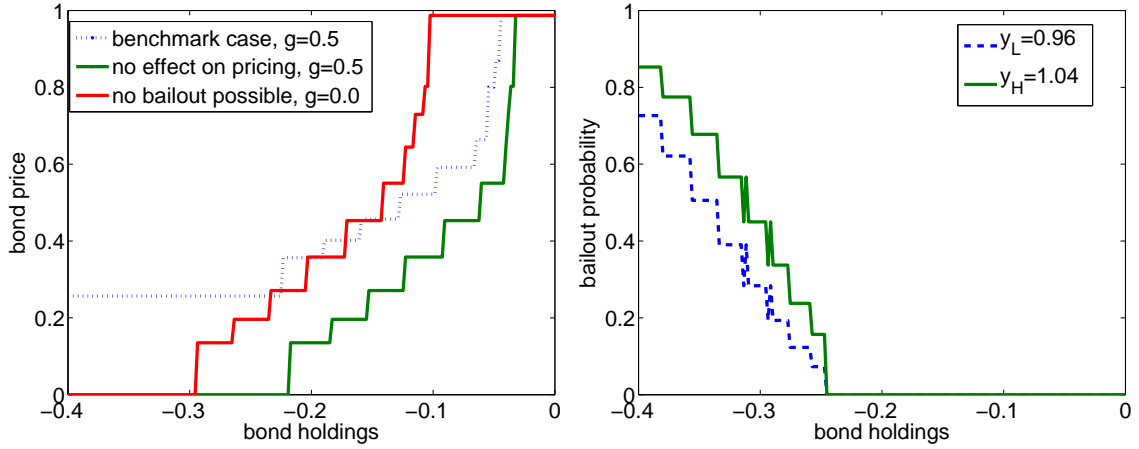


Figure 2.3: **Bond price schedules:** Vertical axis shows price  $q(b', y)$ , and the horizontal axis shows the bond holdings in  $t + 1$ . Panel (a) compares income differences in  $t$  with 5 percent deviations upwards and downwards from steady state under two different perceived bailout probabilities. Panel (b) show bond price schedules under bailout probability  $g = 0.2$ , and haircut fractions of 0.2 and 0.5

There are two forces moving bond prices in different directions. On one hand we have that according to equation 2.2.7, with  $g > 0$  bond prices are higher. On the other hand, the introduction of such a possibility changes incentives to default, making it more likely. The answer to the question of which effect dominates is not straightforward, and will depend on the particular parameter values one assumes. Panel (a) in figure 2.4 attempts at showing the decomposition of this effect. Consider the case of a high enough bailout probability,  $g = 0.5$ . Then the benchmark solution for bond prices is sketched by the dashed blue line. Let us shut down the first effect on pricing. That is, solve the model under the assumption that only incentives are



(a) Bond prices and the effect of the introduction of a bailout probability      (b) Bailout probabilities for different output realizations in the core economy

Figure 2.4: **Bond price schedules.** Figures show in the vertical axis price  $q(b', y)$ , and the horizontal axis shows the bond holdings in  $t + 1$ . Panel (a) shows the pricing of debt incorporating a potential bailout, shutting down the pricing channel and shutting down the bailout altogether. Panel (b) shows the bailout probability for realizations of income 4% below and above steady state, for the Core economy and under different exposures to periphery debt.

distorted for the borrower country, but the pricing of international financial markets do not include the same beliefs. We obtain the green solid line that is lower than the benchmark case for every value of debt. Now, eliminate the bailout probability for the solution of the model altogether and we end up with the red solid line. The difference between the green and red lines captures the moral hazard effect on bond prices.

Finally let us examine the behavior of the model for the core economy. In particular, consider equation 2.2.14 and two different realizations of income for the core economy. According to Proposition 9 we know that lower output is associated with lower probability of extending a bailout. This is sketched in panel (b) of figure

2.4. Bailout probability is higher the higher the output, and also, the higher the exposure of the core economy to periphery debt. When the core economy has a lot of resources invested in the periphery economy, paying the fixed cost  $\lambda$  is a good idea. On the other hand, if there is little exposure the core economy will certainly not extend financial assistance. The figure displays a flat area for low debt exposures. How large this area is hinges on the marginal dis-utility of paying  $\lambda$  cost.

In the next subsection we develop an application of the preceding model. In particular, we consider the 2000-2007 data on sovereign spreads and give a rationale as to why these were anchored around zero for such a long period of time. Our argument can also accommodate the decoupling of spreads when a macro shock hit both the core and periphery economies.

### **2.4.1 The European Debt Crisis**

The chronology of events for the EDC is roughly as follows. Before the introduction of the Euro in 2001, spreads were heterogeneous. Periphery debt, Spain and Italy, for instance were priced significantly differently from debt of the European core. In 1997 the monetary union started. The Euro entered as the official unit of account, before actually being a medium of exchange currency. Not all current members of the current EU were members by that time though. Countries were required to fulfill a set of fiscal discipline targets and not all of them did (what was later known as the Maastricht Treaty). All in all, periphery countries did put effort

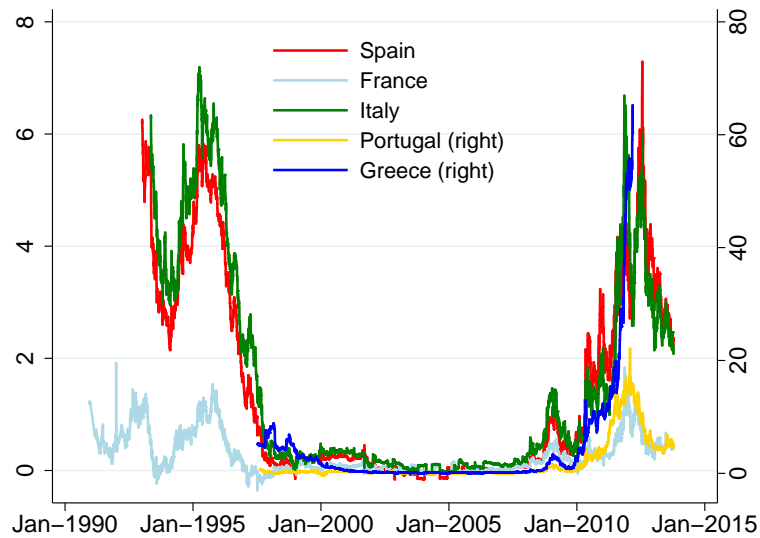


Figure 2.5: **Sovereign Spreads - Europe Periphery Countries** Five year sovereign bond spreads with respect to German bonds. Expressed in percentage. Greece and Portugal are on the right axis. Source: Bloomberg.

into cleaning up their fiscal balances in the final years of the nineties, and then sovereign spread differentials started to decline. By the time the Euro was formally introduced in 2001, spread differentials plummeted to zero, but by then, all fiscal discipline efforts had been abandoned in practice. Somehow, entering the European Union insulated periphery spreads against their weakening fundamentals, and the cheap financing allowed them to borrow freely. For a decade the level and variance of sovereign spreads for Ireland, Greece, Italy and Spain remained close to zero, as is shown in figure 2.5 that plots the time line of five year sovereign spreads with respect to the same maturity German debt.

In late 2007 the world observed the first signs of what would later become the

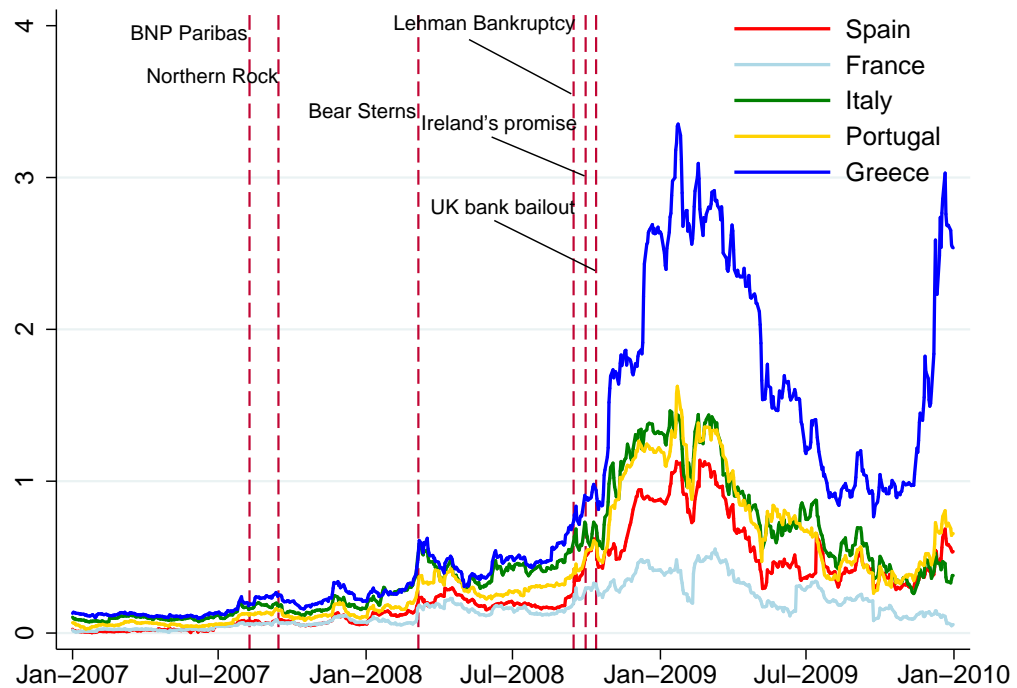


Figure 2.6: **Chronology of Events and the Decoupling of Sovereign Spreads.** Five year sovereign bond spreads with respect to German bonds. Expressed in percentage. The figure signals event dates which in chronological order are: (1) 9 August 2007, BNP Paribas freeze three of their funds. (2) Run on Northern Rock. (3) JP Morgan buys Bear Stearns. (4) Lehman Brothers files for bankruptcy. (5) Ireland goes into recession and promises to underwrite entire banking system. (6) UK government bails out banking system. Source: Bloomberg.

Great Financial Crisis and Recession of 2008-2010. Figure 2.6 shows the evolution of sovereign spreads with milestones in the unfolding of the financial crisis with particular importance for European countries. European spreads started to feel the crisis in March 2008, when investment bank JP Morgan bought Bear Stearns. This event took spreads away from zero, but the real decoupling started later, after there was evidence that there was some European exposure to the developments in the United States. Three events can be singled out before spreads decoupled notoriously. First the Lehman Brothers file for bankruptcy, followed by the Ireland's government promise to underwrite its entire banking system, and finally, the bank run on UK Royal Bank of Scotland, Lloyds TSB, and HBOS, when the British government had to step in in a bank bailout. In 2009 Greece is forced to disclose that their budget deficit is higher than previously reported, and by 2010 the European Debt Crisis was completely unfolded. The events following the surge in interest rates and how they evolved into a fully fledged debt crisis are out of the scope of this paper, and admit explanations such as those in Lorenzoni and Werning (2014) or Cole, Neuhann and Ordoñez (2016). This application is concerned with the decade before the surge in interest rates and the early stages of the debt crisis. We argue that the bailout mechanism sketched in section **2.2** has the ability to account for this dynamics. In particular we propose that this event is informative of the mechanism because, unlike the only other monetary union – the United States– the first signs of a crisis caused spreads to decouple. On the other hand, US states were in the middle of the

problem and their funding spreads did not spike at all. There are many differences between any US state and a periphery economy in Europe, but one of the most noteworthy for debt markets is that the EU “no-bailout clause” had never been tested, and it was hard to defend the idea that it was fully credible. On the other hand, the federal US government refused to bailout troubled states already by 1840, and hence the no bailout clause has been fully credible.

If one takes the standard model of endogenous default from Arellano (2008) and calibrate it for the Italian economy at the moment of entering the European Union, we cannot rationalize the cheap financing these economies were getting. In particular, the fundamentals of the Italian economy speak of a spread that should have been in the neighborhood of 180 basis points above the spread we actually observed. Our case rests on how suddenly spreads plummeted to zero while a standard model would predict spreads that looked similar to the 1990s and in the neighborhood of 2.0%. The introduction of the implicit guarantee enabled periphery countries to access cheap financing, in spite of their deteriorating conditions. When it was clear that the financial crisis spread over Europe, the bailout guarantee was less likely to be honored, as the incentives of the core changed with the state of the economy, and that alone removed the insulation of periphery bonds to its fundamentals.



## 2.4.2 Numerical application

Direction of results are readily intuitive and can be derived from the propositions in the previous section. This application provides two lessons, first that the stand alone Arellano (2008) model would imply a different path of periphery spreads in the 2000's, and second, that the inclusion of a bailout mechanism can reconcile this with the 2008 initial surge in interest rates. For this, I will use Italy and Germany as the periphery and the core country, respectively. Italian stylized facts are presented in Table (2.2). One can see that the spread of Italian debt over German Bunds was highest in the period 1993-1999, and as figure (2.5) depicts, plummets abruptly to an average spread of 0.15 percent in the period 2000-2007, a period which coincides with the introduction of the Euro.

Table 2.2: Italian statistics on five year government debt by periods. Source: Bloomberg tickers GDBR5, GBTPGR5 are used for yields. B stands for debt, DS stands for debt service. Source: Bloomberg.

	Gov. Bond 5y yield	Spread	B/GDP	Debt Serv.	CA/GDP var	CA/ GDP
1993-2013	5.18	1.51	113.59	1.47	0.0004	-0.01
1993-1999	8.51	3.25	118.19	2.51	0.0004	0.01
1999-2007	4.01	0.15	107.42	1.08	0.0001	-0.01
2007-2013	3.80	1.94	117.88	1.12	0.0002	-0.02

Table 2.3: Model Performance

Statistic	Spreads (%)	Debt service to GDP (%)	CA/GDP std	Default probability
Data	3.25	2.51	0.02	0.02
Simulation	2.28	2.17	0.03	0.06

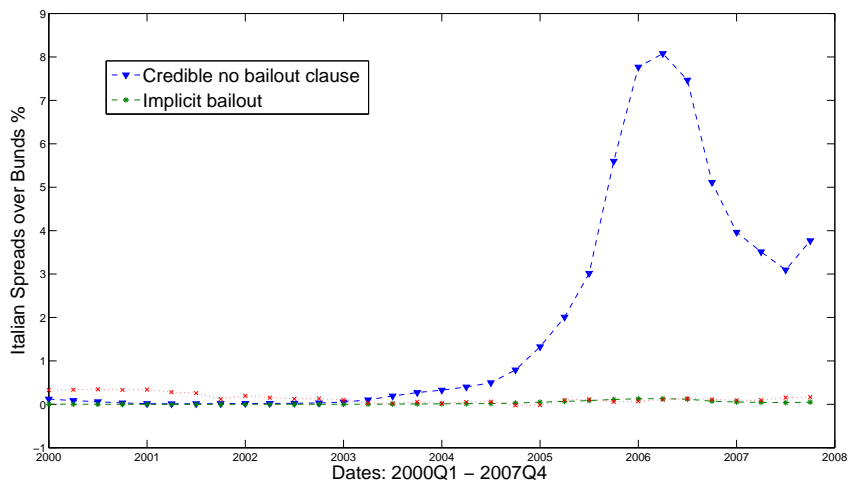


Figure 2.7: **Simulation counterfactual:** Spreads for 2000-2007 in a calibrated model to 1993-1999 for Italy in the absence of implicit guarantees

Table 2.3 reports the performance of the model. In particular, the parameter values of the calibration can achieve 2.28% spread for the 90's period, when the actual spread on Italian debt was 3.25%. That is, the model slightly under-performs on this dimension. Let us now assess the counterfactual spread dynamics for Italy, under the baseline calibration. Now we want to extend the model into the 2000s, in order to compare it to actual spreads. To do this, we save the estimated forecast errors from OLS output regressions to the model, take the endowment process as given and compute the default probabilities and spreads. Figure 2.7 depicts in blue a completely different story. Italian spreads should have been above 5% by 2006, but instead we actually observed a mean 0.15% spread in the 2000-2007 period. The mean of spreads in that period, in the absence of a bailout guarantee would have been instead 1.806%. Using the same calibration, we now investigate if the

model can insulate interest rates. In figure 2.7, we see that the bailout guarantee can easily do that.

After the collapse of Lehman Bro. in September 2008, the world economy cut projections for future income for both the periphery economy and the core economy. This event had two main consequences. First, spreads for periphery economies was higher only because their own income was lower. Second, this model predicts that if income of the core economy is lower it is less likely for the latter to save its neighbor in case of default which amplifies the initial spike in periphery sovereign spreads.

## 2.5 Conclusions

We consider the introduction of strategic bailouts from a creditor country (core) to a defaulting borrower country (periphery), in the context of the sovereign default model in the tradition of Eaton and Gersovitz (1981). In the same way that default is strategic because debt is not enforceable, bailout extensions from a creditor country are strategic as there is no obligation to provide such alleviation. Thus, this is a theory of bailout extension from a creditor country, and not from an international financial institution like the IMF, whose mandate would be global financial stability. The creditor economy wants to extend a bailout because there is money at play. Its inhabitants have invested resources in periphery debt, and would lose their savings if default is to materialize. However bailouts are costly. Either because there is a cost in coordinating all creditors, or simply because creating a fund like

the European Financial Stability Mechanism requires initial equity.

The introduction of this implicit guarantee on sovereign debt has two opposing effects on its pricing. First, spreads are lower simply because the expected recovery rate is higher. However incentives to default are also changed. In particular, if there is a chance that the defaulting economy will receive financial assistance after declaring default and part of its outstanding debt will dilute, then the value of exerting the default option is higher, and these events more frequent. This would raise spreads. We show that bond price schedule is decreasing in the haircut fraction of debt after re-structuring, is bounded from below, and is less sensitive to income fluctuations with the inclusion of a bailout probability. We also show that bailouts are more likely to happen when there are good realizations of income for the creditor economy. A final application of our model and a benchmark calibration can generate spreads close to zero for Italy even when income fluctuations and debt accumulation would predict otherwise. Shutting off the bailout implicit guarantee would have raised spreads from 0.03% to 1.8%.

# Appendices

# Appendix Chapter 1

## A.1 Derivations

### A.1.1 Preferences

Given prices  $r_t, w_t$ , and profits  $\bar{\Pi}_t$  the problem in (1.2.1) and (1.2.2) results in

$$\beta(1 + r_{t+1}) = \left( \frac{c_{t+1}}{c_t} \right)^\gamma$$

In EBCP, we know that  $c_t$  grows at the average constant economy rate  $1 + g$  and therefore we can pin down the interest rate along the balance trajectory:  $r_{t+1} = \frac{(1+g)^\gamma}{\beta} - 1$ .

### A.1.2 Technology

From the main text we know that there are three levels of production; final good,  $M_t$  sectoral final goods, and a continuum of intermediate goods for each sector. The final good firm produces  $Y_t$  which is ready to be consumed and, is therefore in the same units as  $c_t$ . Its price is normalized to one,  $p_t = 1$ . I will first characterize

the demand for each of these market layers, then interact them with the production technologies and finally impose market clearing in each market, and the labor market.

The final good firm operates in perfect competition, aggregates the sectoral final goods in a basket, and sells it competitively to the consumer. In particular, given the price of final goods in each sector  $m$ ,  $\{p_{m,t}\}_{m=1}^M$  the final good producer chooses  $\{y_{m,t}\}_{m=1}^M$  to solve,

$$\max_{\{y_m\}_{\geq 0}} \left( \sum_m y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \sum_m p_{m,t} y_{m,t}$$

which yields the demand for each sectoral final good  $\{y_{m,t}^D\}_{m=1}^M$

$$y_{m,t}^D = Y_t \left( \frac{1}{p_{m,t}} \right)^{\varepsilon}$$

We can learn more about if we calculate total cost of assembling the final good  $\sum_m y_{m,t} p_{m,t}$ . Plugging in the demand for  $y_{m,t}^D$  we obtain  $\sum_m y_{m,t} p_{m,t} = Y_t$ , which will be handy when we calculate equilibrium wages,  $w_t$ .

Next, let us characterize the demand for intermediate goods  $x_{i,m,t}^D$  from the producer of good  $y_{m,t}$ . Every  $t$ , for each  $m$ , given sequences of prices  $p_{m,t}$ , and sequence  $\{p_{i,m,t}\}$  the sectoral final good producer chooses  $\{x_{i,m,t}\}_{i \in [0,1]}$  to solve,

$$\max_{\{x_{i,m,t}\}_{i \in [0,1]} \geq 0} p_{m,t} \exp \left( \int_0^1 \log x_{i,m,t} di \right) - \int_0^1 p_{i,m,t} x_{i,m,t} di$$

from which we can solve an expression for demand of intermediate goods,  $x_{i,m,t}^D$ ,

$$x_{i,m,t}^D = \left( \frac{p_{m,t}}{p_{i,m,t}} \right) y_{m,t}$$

Finally, labor demand will be pinned down by the equilibrium production of variety  $x_{i,m,t}$ ,

$$\ell_{i,m,t}^D = \frac{x_{i,m,t}}{q_{i,m,t}} - \xi_{m,t}$$

Let us start to introduce supply functions and market clearing conditions. First, in the sectoral final good market Bertrand pricing will pin down the level of production. The closest follower owner will not compete if the marginal revenue is lower than  $w_t/\tilde{q}_{i,m,t}$  where  $\tilde{q}_{i,m,t}$  is the quality of her vintage. Then pricing is given by equation (1.2.11). Plugging this result in  $x_{i,m,t}^D$  yields equilibrium production of intermediate good

$$x_{i,m,t} = \left( \frac{p_{m,t} y_{m,t}}{(1+\sigma)w_t} \right) q_{i,m,t}$$

.

Note that we can plug in the previous result on the labor demand function  $\ell_{i,m,t}^D$ , which results in  $\ell_{i,m,t}^D = \frac{p_{m,t} y_{m,t}}{(1+\sigma)w_t} - \xi_{m,t}$ . This result is the same across varieties, which allows us to aggregate across varieties and across sectors to get,

$$\sum_m \int_0^1 \ell_{i,m,t} di = \sum_m \frac{p_{m,t} y_{m,t}}{(1+\sigma)w_t} - \sum_m \xi_{m,t}$$



Using market clearing for the labor market, we can then obtain an expression for wages

$$\begin{aligned} w_t &= \frac{\sum_m p_{m,t} y_{m,t}}{(1 + \sigma)(1 + \sum_m \xi_{m,t})} \\ &= \frac{Y_t}{(1 + \sigma)(1 + \sum_m \xi_{m,t})} \end{aligned}$$

We have by now equilibrium expressions for  $x_{i,m,t}$  and  $w_t$ . Next we can calculate equilibrium  $y_{m,t}$ . For this, define average sectoral quality by  $\log Q_{m,t} = \int_0^1 \log q_{i,m,t} di$ . Use equation (1.2.5) and the solution for  $x_{i,m,t}$  to obtain

$$p_{m,t} = \frac{(1 + \sigma)w_t}{Q_{m,t}}$$

which can be further plugged in the demand for  $y_{m,t}$  to obtain

$$y_{m,t} = Y_t \left( \frac{Q_{m,t}(1 + \sum_m \xi_{m,t})}{Y} \right)^\varepsilon$$

Finally, we can obtain two expressions for sectoral final output and final output that are functions of sector quality and shocks only,

$$Y_t = \left[ 1 + \sum_m \xi_{m,t} \right] Q_t \tag{A.1.1}$$

$$y_{m,t} = \left[ 1 + \sum_m \xi_{m,t} \right] Q_t^{1-\varepsilon} Q_{m,t}^\varepsilon \tag{A.1.2}$$

where the economy-wide average quality is defined by  $Q_t = (\sum_m Q_{m,t}^{\varepsilon-1})^{\frac{1}{\varepsilon-1}}$ . This allows us to write our profit function, and obtain equation (1.2.12), which is also a function only of sectoral quality and shocks.

## A.2 Proofs

**Proposition 1.** *If an incumbent with knowledge capital  $\omega$  has the incentive to move from sector  $m$  to  $n$ , then an outsider with the same level of knowledge capital  $\omega$ , in the same sector has greater incentive, and moves first.*

*Proof.* Define  $A$  to be the set of all knowledge capital values,  $\{\omega_m\}$  for which it is convenient to move from  $m$  to sector  $m'$ , given that the entrepreneur is an incumbent. That is,

$$A = \{\omega_m \in \mathcal{W} : J_{m'}^M(Q, Z, \omega_m) > J^S(Q, Z, \omega_m)\}$$

Define, as well, the set of all values of knowledge capital for which it is convenient to move to sector  $m'$  from sector  $m$ , if the entrepreneur is an outsider rather than an incumbent,

$$\tilde{A} = \{\omega_m \in \mathcal{W} : H_{m'}^M(Q, Z, \omega_m) > H^S(Q, Z, \omega_m)\}$$

It is sufficient to prove that  $A \subseteq \tilde{A}$ . Let notation be,  $\omega_1^* = \arg \max\{J_{m'}^M\}$ ,  $\omega_2^* = \arg \max\{J^S\}$ ,  $\omega_3^* = \arg \max\{H_{m'}^M\}$ ,  $\omega_4^* = \arg \max\{H^S\}$ . Note that  $\omega_1^* = \omega_3^*$ . By

definition, we have that the value of moving is larger than the value of staying,

$$\begin{aligned} \mu(\omega_1^*)\mathbb{E}J_{m'}^0(\omega_1^*) + (1 - \mu(\omega_1^*))\mathbb{E}H_{m'}^0(\omega_1^*) - Q'_{m'}f^{-1}(\omega_1^* - (1 - \delta)(1 - \phi_{m,m'})\omega_m) &> \\ (1 - \lambda + \mu(\omega_2^*)\lambda)\mathbb{E}J_m^0(\omega_2^*) + (1 - \mu(\omega_2^*))\lambda\mathbb{E}H_m^0(\omega_2^*) - Q'_mf^{-1}(\omega_2^* - (1 - \delta)\omega) & \end{aligned}$$

and we need to show that

$$\begin{aligned} \mu(\omega_3^*)\mathbb{E}J_{m'}^0(\omega_3^*) + (1 - \mu(\omega_3^*))\mathbb{E}H_{m'}^0(\omega_3^*) - Q'_mf^{-1}(\omega_3^* - (1 - \delta)(1 - \phi)\omega) &> \\ \mu(\omega_4^*)\mathbb{E}J_{m'}^0(\omega_4^*) + (1 - \mu(\omega_4^*))\mathbb{E}H_{m'}^0(\omega_4^*) - Q'_mf^{-1}(\omega_4^* - (1 - \delta)\omega) & \end{aligned}$$

Noting the properties of a maximum, and  $\lambda > 0$  it is straightforward to see that

$$\begin{aligned} (1 - \lambda + \mu(\omega_2^*)\lambda)\mathbb{E}J_m^0(\omega_2^*) + (1 - \mu(\omega_2^*))\lambda\mathbb{E}H_m^0(\omega_2^*) - Q'_mf^{-1}(\omega_2^* - (1 - \delta)\omega) &\geq \\ (1 - \lambda + \mu(\omega_4^*)\lambda)\mathbb{E}J_m^0(\omega_4^*) + (1 - \mu(\omega_4^*))\lambda\mathbb{E}H_m^0(\omega_4^*) - Q'_mf^{-1}(\omega_4^* - (1 - \delta)\omega) &> \\ \mu(\omega_4^*)\mathbb{E}J_{m'}^0(\omega_4^*) + (1 - \mu(\omega_4^*))\mathbb{E}H_{m'}^0(\omega_4^*) - Q'_mf^{-1}(\omega_4^* - (1 - \delta)\omega) &= \\ &H_m^S(Q, Z, \omega_m) \end{aligned}$$

Then,  $\omega \in A$  implies  $\omega \in \tilde{A}$ . That is  $A \subseteq \tilde{A}$ . Outsiders have a greater incentive to move first.  $\square$

**Lemma 1.** *The value of being an incumbent will be higher than the value of being an outsider, i.e.  $J(\omega) > H(\omega)$ .*

*Proof.* Without loss of generality consider  $\omega = \bar{\omega}$ , for an arbitrary  $\bar{\omega}$ . Also, for equations (1.3.1) and (1.3.2) consider  $\omega^* = \operatorname{argmax} J(s)$  and  $\hat{\omega} = \operatorname{argmax} H(s)$ . In the simplest case  $s = \omega$ , then,

$$\begin{aligned}
J(\omega) &= -q[\omega^* - (1 - \delta)\omega] + \mu(\omega) [\pi + r_d^{-1}J(\omega^*)] \\
&\quad + (1 - \mu(\omega)) [\lambda r_d^{-1}H(\omega^*) + (1 - \lambda)(\pi + r_d^{-1}J(\omega^*))] \\
&\geq -q[\hat{\omega} - (1 - \delta)\omega] + \mu(\omega) [\pi + r_d^{-1}J(\hat{\omega})] \\
&\quad + (1 - \mu(\omega)) [\lambda r_d^{-1}H(\hat{\omega}) + (1 - \lambda)(\pi + r_d^{-1}J(\hat{\omega}))] \\
&= H(\omega) + (1 - \mu(\omega))(1 - \lambda) [\pi + r_d^{-1}J(\hat{\omega}) + r_d^{-1}H(\hat{\omega})] \\
&> H(\omega) + \frac{(1 - \mu(\omega))(1 - \lambda)}{1 + r} [r_d^{-1}J(\hat{\omega}) + r_d^{-1}H(\hat{\omega})] \tag{A.2.1}
\end{aligned}$$

Then, we know that it must be the case that,

$$J(\omega) - H(\omega) > \frac{(1 - \mu(\omega))(1 - \lambda)}{1 + r} [J(\hat{\omega}) - H(\hat{\omega})]$$

Consider the first case in which  $J(\hat{\omega}) - H(\hat{\omega}) > 0$ , which then results in  $J(\omega) - H(\omega) > 0$  as required. In the alternative case we could assume that  $J(\hat{\omega}) - H(\hat{\omega}) < 0$ .

Then, it is also useful to consider the sequence conformed by the optimal policy function result for  $H(\cdot)$ :  $\{\omega_i\}_{i=1}^N = \{\omega_1 = \omega, \omega_2 = \hat{\omega}, \omega_3 = \omega'(\omega_2), \omega_4 = \omega'(\omega_3), \dots, \omega_N\}$ .

Then we know from (A.2.1) that

$$\begin{aligned}
J(\omega_1) - H(\omega_1) &> \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} [J(\omega_2) - H(\omega_2)] \\
J(\hat{W}) - H(\hat{W}) &> \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} [J(\omega_3) - H(\omega_3)]
\end{aligned}$$

In particular, for  $A_i < \infty$  (which we know holds),

$$\begin{aligned}
J(\omega_2) - H(\omega_2) &= \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} [J(\omega_3) - H(\omega_3)] + A_2 \\
&= \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} \\
&\quad \times \left[ \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} [J(\omega_4) - H(\omega_4)] + A_3 \right] + A_2 \\
&= \left( \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} \right)^{N-2} [J(\omega_N) - H(\omega_N)] \\
&\quad + \sum_{i=2}^{N-1} \left( \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} \right)^{i-2} A_i \\
&> \left( \frac{(1 - \mu(W))(1 - \lambda)}{1 + r} \right)^{N-2} [J(\omega_N) - H(\omega_N)]
\end{aligned}$$

The right hand side goes to zero as  $N$  goes to infinity, as  $\frac{(1 - \mu(W))(1 - \lambda)}{1 + r} \in (0, 1)$ .

Then we get that  $J(\omega_2) > H(\omega_2)$ , which contradicts the premise of the second case, therefore we can conclude that  $J(\omega_1) = J(\omega) > H(\omega) = H(\omega_1)$   $\square$

**Proposition 2.** *The combination of inalienability of knowledge capital ( $\omega'$ ) and stochastic shocks, results in lower previous investment.*

*Proof.* Consider the case in which an outsider yields zero value. This is without loss

of generality since the main role of outsiders will be to shape general equilibrium research intensity in every sector. This proposition, however, is about the individual behavior of every entrepreneur who faces irreversibility constraints every period. More importantly, they create their own future inalienability constraints by building too much capital in the present. Let the Lagrange multiplier for equation (1.3.1) be  $\chi$ . This multiplier represents the shadow value of relaxing the irreversibility constraint today. Namely, lowering the value of  $(1 - \delta)\omega$ . High values of  $\omega'$ , beyond this lower bound result in  $\chi = 0$ . At the same time, the same high values of  $\omega'$  result in positive values of  $\chi'$ . By choosing high  $\omega'$  for next period, the future constraint  $\omega'' \geq (1 - \delta)\omega'$  becomes more stringent.

The first order condition on this simplified problem along with the associated envelope condition is then,

$$q - \chi = r_d^{-1} \mathbb{E} \left[ \lambda \mu'(\omega') (J(\xi', \omega')) + (1 - \lambda + \lambda \mu(\omega')) \underbrace{J'(\xi', \omega')}_{(1-\delta)(q' - \chi')} \right] \quad (\text{A.2.2})$$

with  $r_d^{-1} = \frac{1}{1+r}$ , and complementary slackness conditions,  $\chi \omega' = 0$ ,  $\chi > 0$  and  $\omega' > 0$ . It must be the case that, compared to the unrestricted benchmark case, research capital accumulation is lower. That means that when future constraints activate, the choice of  $\omega'$  is lower. By the implicit function theorem we have that

$$\frac{\partial \omega'}{\partial \chi'} = \frac{r_d^{-1}(1 - \lambda + \mu(\omega')\lambda)(1 - \delta)}{r_d^{-1}\mathbb{E}[\lambda\mu''(\omega')J(\omega', \xi') + 2\lambda\mu'(\omega')(1 - \delta)(q - \chi')]}$$

and we are only left to prove that second term in the denominator is non positive since the first one is negative by the concavity of  $\mu(\cdot)$ . Note that if  $\chi' > 0$ , which is the case under analysis here, then it must be the case that relaxing the one period ahead constraint impacts the objective function positively. This means that reducing  $\omega'$  (relaxing the one period ahead constraint), raises the value of the objective function, or that  $J'(\omega, \xi') < 0$ , which we know to be equal to the one period ahead envelope condition  $(1 - \delta)(q - \chi') < 0$ . Else,  $\chi' = 0$ , which completes this proof.

If we want to avoid assuming continuity of the policy function  $\omega'$ , then we can proceed with the following reasoning. Consider again the first order condition

$$q - \chi = r_d^{-1}\mathbb{E}[\lambda\mu'(\omega')J(\omega', \xi') + (1 - \lambda + \mu(\omega')\lambda)(1 - \delta)(q - \chi')]$$

since the control variable is known before hand, it can be taken out the expectation.

The condition is then transformed into

$$\begin{aligned}
q - \chi &= r_d^{-1} \lambda \mu'(\omega') \mathbb{E}[J(\omega', \xi')] + (1 - \lambda + \mu(\omega') \lambda)(1 - \delta) \mathbb{E}(q - \chi') \\
&= r_d^{-1} \lambda \mu'(\omega') \mathbb{E}[J(\omega', \xi')] + (1 - \lambda + \mu(\omega') \lambda)(1 - \delta) \times \min\{q, \mathbb{E}(q - \chi')\} \\
&= r_d^{-1} \lambda \mu'(\omega') \mathbb{E}[J(\omega', \xi')] + (1 - \lambda + \mu(\omega') \lambda)(1 - \delta) \times \min\{q, \\
&\quad r_d^{-1} \lambda \mu'(\omega'') \mathbb{E}[J(\omega'', \xi'')] + (1 - \lambda + \mu(\omega'') \lambda)(1 - \delta) \times \min\{q, \mathbb{E}(q - \chi'')\}\}
\end{aligned}$$

That is, if in the future, any inalienability constraints binds, it will lower the net benefit of holding research capital today. This completes the proof. □

**Proposition 3.** *Higher volatility of the industry idiosyncratic shock will lower the value of holding knowledge capital, and hence, lower innovation rates.*

*Proof.* Consider, without loss of generality, the simpler version of the original model [equation (1.3.1)]. Again, the first order condition is given by,

$$q - \chi = r_d^{-1} \mathbb{E}[\lambda \mu'(\omega') J(\omega', \xi') + (1 - \lambda + \mu(\omega') \lambda)(1 - \delta)(q - \chi')]$$

We require to show that increasing volatility will make the wedge between marginal cost and marginal benefit wider. For that it is enough to note that the expectation of the Lagrange multiplier next period,  $\mathbb{E}[\chi']$  is higher, the higher  $\sigma_\xi^2$ . Note that the distribution of  $\xi'$  induces a distribution for the policy function  $\omega'_u$  in



the unrestricted case. Refer to it as  $f(W)$  (with cumulative distribution function  $F(W)$ ). The inalienability constraint bounds optimal values from below at some  $c^* = (1 - \delta)\omega$ . At the same time the inalienability constraint Lagrange multiplier is defined by

$$\chi' = \begin{cases} > 0 & \text{if } \omega'_u < c^* \\ = 0 & \text{if } \omega'_u \geq c^* \end{cases}$$

Also, notice that a mean-preserving expansion of the variance will result in reshuffling probability mass from mid-point events in  $f(W)$  to tail events. Refer to a low variance distribution as  $f_L(W)$  and a high variance distribution as  $f_H(W)$ . We know then that if this is a mean preserving expansion of variance, there exists a point  $\bar{c}$  in in which the cumulative distributions cross. Consider the case in which  $c^* \leq \bar{c}$  first. We know that to the right of  $c^*$  the shadow price is zero. Therefore since there is first order stochastic dominance of  $F_H(W)$  over  $F_L(W)$  on all non-zero values, the expectation of this multiplier is higher. Now consider the case where  $c^* > \bar{c}$ . Under symmetric distributions the cumulative probability mass for the support  $[c^*, \bar{c}]$  is perfectly offset by the mass in  $[2\bar{c} - c^*, c^*]$ , and the same argument as in the first case is true for the support below  $2\bar{c} - c^*$ . This completes the proof.  $\square$

### A.3 Stationary representation

The stationary system for the baseline model is<sup>27</sup>,

$$\begin{aligned}
r_{t+1} &= (1 + g_t)^\gamma / \beta - 1 \\
\frac{c_t}{Y_t} &= 1 \\
\frac{\pi_{m,t}}{Y_t} &= \frac{\sigma}{(1 + \sigma)^{1+\varepsilon}} \left[ \frac{w_t}{Q_{m,t}} \right]^{-\varepsilon} + \left( \frac{w_t}{Y_t} \right) \xi_{m,t} \\
\frac{w_t}{Y_t} &= \frac{1}{1 + \sigma} \left[ \frac{\sum \left( \frac{Q_m}{Y_t} \right)^{\varepsilon-1}}{1 + \sum \xi_m} \right]^{1/\varepsilon} \\
\frac{J^S(\omega_m)}{Y_t} &= \max_{\{\omega'_m\}} \left\{ \frac{\pi_t}{Y_t} - \frac{Q'_m}{Y_t} f^{-1}(\omega'_m - (1 - \delta)\omega) + (r_d Y_{t+1})^{-1} (1 + g_t) \times \right. \\
&\quad \left. + \mathbb{E} \left[ (1 - \lambda_m + \mu_m \lambda_m) J^0(\omega'_m) + (1 - \mu_m) \tilde{\lambda} H^0(\omega'_m) \right] \right\} \\
\frac{J^M_{m'}(\omega_m)}{Y_t} &= \max_{\{\omega'_{n'}\}} \left\{ \frac{\pi_t}{Y_t} - \frac{Q'_n}{Y_t} f^{-1}(\omega'_{m'} - (1 - \delta)(1 - \phi)\omega) + \right. \\
&\quad \left. + (r_d Y_{t+1})^{-1} (1 + g_t) \mathbb{E} \left[ \mu_{m'} J^0(\omega'_{m'}) + (1 - \mu_{m'}) H^0(\omega'_{m'}) \right] \right\} \\
\frac{H^S(\omega_m)}{Y_t} &= \max_{\{\omega'_m\}} \left\{ -\frac{Q'_m}{Y_t} f^{-1}(\omega'_m - (1 - \delta)\omega) \right. \\
&\quad \left. + (r_d Y_{t+1})^{-1} (1 + g_t) \mathbb{E} \left[ \mu_m J^0(\omega'_m) + (1 - \mu_m) H^0(\omega'_m) \right] \right\} \\
\frac{H^M_{m'}(\omega_m)}{Y_t} &= \max_{\{\omega'_{n'}\}} \left\{ -\frac{Q'_n}{Y_t} f^{-1}(\omega'_{m'} - (1 - \delta)(1 - \phi)\omega) \right. \\
&\quad \left. + (r_d Y_{t+1})^{-1} (1 + g_t) \mathbb{E} \left[ \mu_{m'} J^0(\omega'_{m'}) + (1 - \mu_{m'}) H^0(\omega'_{m'}) \right] \right\}
\end{aligned}$$

---

<sup>27</sup>For presentation simplicity note I am omitting aggregate state variables in the value functions and arguments of profits/probabilities which are functions rather than numbers. Also,  $\mu_j = \mu(\omega'_j)$ . This appendix is meant only to show how to twist the competitive equilibrium definition into the stationary trajectory which is used to solve the problem numerically.

$$\begin{aligned}
\frac{J^0(\omega_m)}{Y_t} &= \max \left\{ \frac{J^S(\omega_m)}{Y_t}, \left\{ \frac{J_{m'}^M(\omega_m)}{Y_t} \right\}_{m' \neq m} \right\} \\
\frac{H^0(\omega_m)}{Y_t} &= \max \left\{ \frac{H^S(\omega_m)}{Y_t}, \left\{ \frac{H_{m'}^M(\omega_m)}{Y_t} \right\}_{m' \neq m} \right\}
\end{aligned}
\tag{A.3.1}$$

## A.4 Supporting Tables and Figures

### A.4.1 Traction at the macro level

The argument in this paper is consistent with both micro and macro evidence, it is robust to specification and variable definition. Our argument is complementary to the mechanism in Aghion et al. (2005), and both channels may well be operative simultaneously. To that extent let us introduce diversification in the main specification presented in Aghion et al. (2005). For this refer to Table A.1.

### A.4.2 Matching diversification and cross-citation distances

Our assessment of the model performance relies in our model generating plausible numbers for the coefficients in table 1.2. In order to go from distances  $\phi$  to the regressor we observe in the data, the (log) number of technological sectors, we need a way to link these two variables. In our analysis we rely on the linear regression of  $d_A(m, m')$  on  $\log(M)$ , which is depicted in figure A.1.

Table A.1: **Alternative Cross Country Panel Data Evidence using Data from European Patent Office - PATSTAT:** Dependent variable is log of innovations per worker. Per worker granted patents, 1975-2011 from European Patent Office (PATSTAT data). Countries which are not included are those who did not submit patenting information. First three specifications are based on country-year observations, while specification 4 is based on cross country observations. Restricted to Utility Models. Standard errors in parenthesis,  $***prob < .01$ ,  $**prob < .05$ ,  $*prob < .1$

	(1)	(2)	(3)	(4)	(5)
<i>Diversification</i>	1.840*** (0.074)	1.784*** (0.080)	0.827*** (0.161)	0.575*** (0.158)	2.167** (1.074)
<i>Volatility</i>	-0.237*** (0.039)	-0.251*** (0.041)	-0.482*** (0.052)	-0.484*** (0.051)	-0.774** (0.316)
<i>Diversification * Volatility</i>			0.339*** (0.050)	0.365*** (0.048)	0.052 (0.328)
<i>Domestic Credit /GDP</i>		0.002 (0.001)	0.003** (0.001)	0.009*** (0.001)	-0.006 (0.009)
<i>Initial Income</i>	-0.156*** (0.046)	-0.176*** (0.047)	-0.169*** (0.047)	-0.213*** (0.045)	-0.354* (0.195)
<i>Constant</i>	4.993*** (0.542)	5.187*** (0.555)	5.777*** (0.552)	6.184*** (0.704)	8.663*** (2.644)
<i>Year dummies</i>	No	No	No	Yes	NA
<i>Observations</i>	1,316	1,272	1,272	1,272	54
<i>R-squared</i>	0.400	0.401	0.422	0.478	0.551

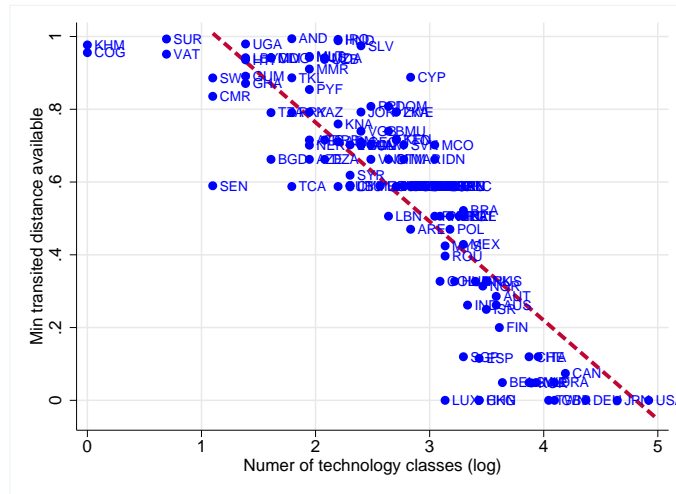


Figure A.1: **Matching diversification and cross-citation data:** Technological distance transited by entrepreneurs (min) vs Diversification of the Economy. Author's calculation on NBER-USPTO, and Li et al. (2014)

## A.5 Extensions to the baseline model

The model presented in Section 1.2 considers the interaction between irreversibility of investment decisions due to inalienability of knowledge capital, volatility, and different degrees of diversification, and provides predictions about inventor mobility across sectors and the implications this mechanism has on the innovation intensity. Alternatively, we can think of two (2) other potential mechanisms through which diversification has an effect on innovation, and ultimately on growth. In the next subsection I lay down natural extensions of the model that embed these channels and examine their bite relative to what we observe in the data and in terms of predictions of the model.

### A.5.1 Availability Externalities

I refer to “availability channel”, to the idea that an entrepreneur would benefit from a more diverse environment because, sometimes, the innovation process requires products or services which are outside the core competencies of the inventor. It is easier to find such services in more diverse economies. These requirements are not identifiable ex-ante by the entrepreneur. By facilitating availability at low cost, diversified environments reduce the expected cost of finding what is needed. In the absence of *availability*, the innovation endeavor runs into a bottle neck, is paused, and innovation cannot happen.

In particular, assume that an entrepreneur enters the period with  $\omega$  knowledge

capital and with the probability  $\mu(\omega)$  they come up with a breakthrough which would convert them into a new incumbent. But in order to accomplish full implementation of their idea they require help from a different sector in the economy with constant probability  $\bar{\theta}$ . Conditional on needing help, with probability  $\mathfrak{b}(M)$ ,  $\mathfrak{b}'(M) > 0$  the inventor “finds” help and carry on the process. This probability is increasing in the level of diversification of the economy, capturing the fact that it is easier to find help in a diversified economy. For simplicity the cost of this help is normalized to zero<sup>28</sup>. Then, the innovation does not run into a bottleneck with probability

$$\hat{\theta} \equiv 1 - \bar{\theta} + \bar{\theta}\mathfrak{b}(M)$$

The introduction of this type of externalities, changes the problem of the inventor presented in section **1.2** only slightly. In particular, probability  $\hat{\theta}$  will enter the problem by shifting the probabilities of being an incumbent next period for any value of  $\omega'$  the entrepreneur decides to accumulate. Then, given  $\{\hat{\theta}, \lambda_m, r'\}$ , the problem of the incumbent entrepreneur in sector  $m$ , who decided to stay in their sector is the analog of equation (1.2.17),

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<sup>28</sup>Alternatively, we can assume that the cost of finding help is lower in diversified economies and high in un-diversified economies and  $\mathfrak{b}(M) = 1$  every time. This modeling strategy however, requires deciding on the search intensity (another control variable), and makes the problem less tractable but not essentially different

$$J^S(Q, Z, \omega_m) = \max_{\omega'} \left\{ \begin{array}{l} \pi(Q_m, \xi_m) - Q'_m f^{-1}(\omega'_m - (1 - \delta)\omega_m) \\ + r_d^{-1} \mathbb{E} \left[ \left( 1 - \lambda_m + \hat{\theta}\mu(\omega')\lambda_m \right) J^0(Q', Z', \omega'_m) \right] \\ + \lambda_m (1 - \hat{\theta}\mu(\omega')) H^0(Q', Z', \omega'_m) \right] \end{array} \right\} \quad (\text{A.5.1})$$

subject to

$$\omega' \geq (1 - \delta)\omega$$

$$\lambda_m = \mathcal{L}^E(Q, Z)$$

$$\xi'_m = \rho_\xi \xi_m + \sigma_\xi \epsilon$$

$$\log Q'_m = \lambda_m \log(1 + \sigma) + \log Q_m$$

The problem of the incumbent that wishes to move from sector  $m$  to sector  $n$ , and the problem of outsiders follows accordingly using the same logic. Then, let us analyze the implications for the basic model. The analog to the simplification of section 1.2 is

$$J(\omega, \xi) = \max_{\omega'} \left\{ \pi(\xi) - q'[\omega' - (1 - \delta)\omega] + r_d^{-1} \mathbb{E} \left[ (1 - \lambda + \lambda \hat{\theta}\mu(\omega')) J(\omega', \xi') + \lambda (1 - \hat{\theta}\mu(\omega')) H(\omega', \xi') \right] \right\}$$

First order conditions of this problem, together with the usual envelope conditions imply the following equilibrium condition

$$-q' + r_d^{-1} \mathbb{E} \{ \hat{\theta} \lambda \mu'(\omega') J(\omega', \xi') - \mu'(\omega') \hat{\theta} \lambda H(\omega', \xi') \} + r_d^{-1} (1 - \delta) q' = 0$$

In the extreme case in which all innovations continue – i.e. there are no bottlenecks –,  $\hat{\theta} = 1$ . When  $\hat{\theta} < 1$  the distortion still has bite on the result. Since technological diversification raises the value of  $\hat{\theta}$ , by the implicit function theorem we can observe that

$$\frac{\partial \omega'}{\partial \hat{\theta}} = -\frac{\mu'(\omega')}{\hat{\theta} \mu''(\omega')}$$

which is equal to  $\frac{\partial \omega'}{\partial \hat{\theta}} = \frac{1}{\theta \nu} > 0$  with the particular functional forms adopted here. That is, diversification, by raising the probability of continuation raises the equilibrium holding of knowledge capital, which results in higher innovation rates. Note however, that volatility has no effect at all in this condition, beyond the mechanism sketched in Section 1.3. That is, it does not influence innovation positively or negatively under this hypothesis in the absence of the inalienability constraint.

### A.5.2 Knowledge Spillovers

The literature informally refers to these effect as *Jacobs externalities*<sup>29</sup>. Under these spillovers view, whenever there is an invention in one industry, there is a random event that this idea generates a subsequent one in a different industry.

This probability is increasing in the number of industries in the same economy.

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<sup>29</sup>Named after urbanist Jane Jacobs. Jacobs pushed the idea that one industry can affect the outcome of another related industry. In particular, she observed the Detroit region production in the 19th and 20th century. Detroit started exporting flour in 1820 by ships and an industry for repairing –and later producing– ships emerged. Engineers working on steam engines came up with gasoline engines which were not suited for boats but for automobiles. Steel industry combined with this new idea were the precursors of a later hub of automobile industry. For a time, both industries coexisted and benefited from each other. “Olds produced boat engines, and Dodge repaired them” [Carlino (2001), Jacobs (1961)]. More generally, evidence is available that innovation is positively related to the concentration of urban areas as this hypothesis would predict [Carlino (2001)]



This is in contrast to (the so called) Marshall-Arrow-Romer spillovers, which are about positive effects on firms within the same industry. The literature has used this intuition in order to rationalize the reason as to why a firm would go into research in very different technology classes; they would benefit from higher cross-fertilization of ideas between technology classes [Granstrand (1998), Suzuki and Kodama (2004)], and would therefore, invest a higher proportion of their sales in R&D [Garcia-Vega (2006)].

It would also be the reason why *basic* (more fundamental) research is done in highly diversified firms, as this would better solve the appropriability problem by expanding the span of activities and exploiting cross-fertilization of ideas [see Akcigit, Hanley and Serrano-Velarde (2014)].

Formally, we can think of these spillovers in the following way. An entrepreneur on industry  $m$  can improve on his technology by investing in R&D and innovating. When she is successful, another sector may also find her discovery useful<sup>30</sup>. The event that sector  $n$  finds an  $m$  sector discovery useful happens (conditional on innovating) with probability  $\iota$ . Therefore the probability of generating a spillover on any variety  $j \in [0, 1]$  in sector  $n \neq m$  is given by

$$\mathbf{a}_m = \lambda_m \iota$$

---

<sup>30</sup>Think of the automobile industry in Detroit, that benefited from research done by the transportation sector. Or consider the laser technology, whose first application was medical (wound treatment) but was later used in many other applications as different as semiconductor production.

Let us also define the probability of any variety in sector  $m$  of receiving a spillover from any other sector in the economy by  $\psi_m$

$$\psi_m = 1 - \prod_{j \neq m} (1 - \alpha_j) \quad (\text{A.5.2})$$

Note that in the symmetric case  $\psi_m = 1 - (1 - \alpha)^{M-1}$  is increasing in the number of sectors in the economy. This is no surprise. The more sectors there are in the economy, the more likely it is that an invention in one of them will benefit sector  $m$ . Now, the problem of the entrepreneur who is an incumbent in sector  $m$  [analog to equation (1.2.17)] is: Given  $\{\psi_m, \tilde{\lambda}, r'\}$ , the entrepreneur who decides to stay in their sector decides on  $\omega'$  to solve,

$$\begin{aligned} J(\omega, \xi) = \max_{\omega'} & \pi(\xi_m) - q'(\omega'_m - (1 - \delta)\omega_m) \\ & + r_d^{-1} \mathbb{E} \left[ (1 - \tilde{\lambda}(1 - \mu(\omega'))(1 - \psi_m)) J(\omega', \xi') \right. \\ & \left. + \tilde{\lambda}(1 - \mu(\omega'))(1 - \psi_m) H(\omega', \xi') \right] \end{aligned} \quad (\text{A.5.3})$$

Notably, probability  $\psi_m$  can act in both ways on innovation. At the aggregate level we know that every innovation generates spillovers and therefore the aggregate level of innovation is mechanically higher for a given level of research capital. At the individual level, however, the opposite is true. The spillover raises the probability that any entrepreneur will be an incumbent. The probability of receiving a spillover is disconnected from the amount of research capital, making low effort entrepreneurs

equally likely to become incumbents compared to those with high levels of research capital. In particular, the first order condition on the simplified model amounts to

$$\mu(\omega')(1 - \psi_m)\tilde{\lambda}_m(J(\omega', \xi') - H(\omega', \xi')) = (r + \delta)q'$$

where  $\tilde{\lambda}$  is the equilibrium replacement probability and by implicit function theorem we have that  $\frac{\partial \omega'}{\partial \psi_m} = -\frac{1}{\nu(1-\psi)} < 0$ . The effect of  $\psi_m$  is clear at the individual level; it discourages research capital accumulation as it raises the probability of being an incumbent without accumulating research capital. The net total effect is then, is ambiguous. On the one hand, each invention generates free inventions in other sectors, and on the other this effect discourages investment in research capital at the entrepreneur level.

All in all, in the same way as in the “availability channel”, the level of innovation is affected by the level of technological diversification, but not by volatility, if we do not consider the “inalienability channel”.

## Appendix Chapter 2

### B.1 Proofs of Propositions

**Proposition 1:** *For all  $B^1 \leq B^2$ , if default is optimal for  $B^2$ , in some states  $y$ , then default will be optimal for  $B^1$  for the same states  $y$ , that is,  $D(B^2) \subseteq D(B^1)$*

Note the following proof follows closely Arellano (2008), and uses two simplifying assumptions. First, there is total debt write off, and second shocks are *iid*. Also, for simplicity and more general results assume  $h(y) = y$ , and  $\theta = 0$ . Consider  $y \in D(b^2)$ . Then by definition we know that the following holds

$$v^d(y, b^2) > v^c(y, b^2)$$

This in turn means

$$u(y) + \beta E (g v^0(0, y') + (1 - g) v^d(0, y')) > \max_{b'} u(y + b^2 - q(b', y) b') + \beta E v^0(b', y')$$

Define  $b_j^* = \arg \max_{b'} u(y + b^j - q(b', y) b') + \beta E v^0(b', y')$ , for  $j = 1, 2$ , and note we

can bound this expression from below.

$$\begin{aligned}
u(y + b^2 - q(b_2^*, y)b_2^*) + \beta Ev^0(b_2^*, y') &\geq u(y + b^2 - q(b_1^*, y)b_1^*) + \beta Ev^0(b_1^*, y') \\
&> u(y + b^1 - q(b_1^*, y)b_1^*) + \beta Ev^0(b_1^*, y') \\
&= v^c(b^1, y)
\end{aligned}$$

By transitivity we can then argue that

$$v^d(y, b^1) > v^c(y, b^1)$$

which completes this proof. □

**Proposition 2:** *There exists a well defined  $\bar{g}$  such that for  $g < \bar{g}$  the following holds: If for some  $b$ , the default set is nonempty  $D(b) \neq \emptyset$ , then there are no contracts available  $\{q(b', y), b'\}$  such that the economy can experience capital inflows,  $b - q(b', y)b' > 0$*

This is a proof by contradiction. Assume there are contracts  $\{q(b', y), b'\}$  such that these allow capital inflows,  $b - q(b', y)b' > 0$ . Still the government maximizes the value of the contract  $v^c(b, y)$  with respect to the stock of debt for next period, choosing  $\hat{b}$  such that  $b - q(\hat{B}, y)\hat{B} < 0$ , and then finds it optimal to default. Then

it must be true that

$$v^d(b, y) > v^c(b, y)$$

$$u(y) + \beta E [g v^0(0, y') + (1 - g) v^d(0, y')] > u(y + b - q(\hat{b}, y) \hat{b}) + \beta E v^0(\hat{b}, y')$$

(i) First consider the case when  $g = 0$ . Then it must be true that

$$u(y) + \beta E v^d(0, y') > u(y + b - q(\hat{b}, y) \hat{b}) + \beta E v^0(\hat{b}, y')$$

Now suppose instead of choosing  $\hat{b}$  we consider  $b'$  that implies capital inflows.

Let us examine this inequality by parts, the period utility and continuation values. We can readily see  $u(y) < u(y + b - q(b', y) b')$  because utility is increasing. Also we know that

$$\begin{aligned} v^0(b', y') &= \max\{v^d(b', y'), v^c(b', y')\} \\ &= \max\{v^d(0, y'), v^c(b', y')\} \\ &\geq v^d(0, y') \end{aligned}$$

Therefore if default was optimal, and that  $b'$  was chosen to maximize the value of the contract, then it must be that  $\nexists \{q(b'), b'\}$  such that  $b - 1(b', y) b' > 0$

(ii) Now consider the alternative extreme case that  $g = 1$ . We know that if default

is the optimal strategy for some  $y$ , then it must be that

$$u(y) + \beta v^0(0, y') > u(y + b - q(\hat{b}, y)\hat{b}) + \beta E v^0(\hat{b}, y')$$

Consider any  $b'$  such that  $b + q(b', y)b' > 0$ . Then, since  $v^0(0, y') \geq v^0(b', y')$  there is room for this inequality to hold as long as the capital inflow is not too large.

Then, relying on the continuity  $v^d$  on  $g$ , which is evident, we can find  $\bar{g}$  such that for any  $g < \bar{g}$  the proposition holds. In particular, let us rearrange our original claim  $v^d(b, y) > v^c(b, y)$  in the following way

$$\begin{aligned} \beta E[gv^0(0, y') + (1 - g)v^d(0, y') - v^0(b', y')] &< u(y + b - q(b', y)b') - u(y) \\ \beta E[g[v^0(0, y') - v^d(0, y')] + v^d(0, y') - v^0(b', y')] &< u(y + b - q(b', y)b') - u(y) \end{aligned}$$

and solving for  $g$  that solves this with equality we have

$$\bar{g}(b') = \frac{1}{\beta} \left[ \frac{u(y + b - q(b', y)b') + E v^0(b', y) - u(y) - \beta E v^d(0, y')}{E[v^0(0, y') - v^d(0, y')]} \right]$$

Where we can inspect that  $\bar{g}$  is increasing in  $b'$  if  $b' < \hat{b}$  and decreasing in  $b'$  in the opposite case. □

**Proposition 3:** *For bailout probability low enough,  $g < \bar{g}$ , default incentives are*

stronger the lower the endowment. That is,  $\forall y_1 \leq y_2$  if  $y_2 \in D(B)$  then so does  $y_1 \in D(B)$ .

This proof follows closely proposition 3 in Arellano (2008). In particular, the result of this proposition is to show spreads are countercyclical even after the incorporation of an implicit bailout guarantee.

If  $y_2 \in D(b)$  then by definition

$$v^d(b, y_2) > v^c(b, y_2)$$

Therefore the following must hold

$$u(y_2) + \beta E[gv^0(0, y') + (1 - g)v^d(0, y')] > u(y_2 + b - q(b', y)b') + \beta E v^0(b', y')$$

If we can prove that

$$v^d(b, y_1) - v^c(b, y_1) > \overbrace{v^d(b, y_2) - v^c(b, y_2)}^{>0}$$

we are all set. Reordering this inequality we obtain

$$\begin{aligned} & u(y_2 + b - q(b^2, y)b^2) + \beta E v^0(b^2, y') - \{u(y_1 + b - q(b^1, y)b^1) + \beta E v^0(b^1, y')\} \\ & \geq u(y_2) + \beta E[gv^0(0, y') + (1 - g)v^d(0, y')] - u(y_1) - \beta E[gv^0(0, y') + (1 - g)v^d(0, y')] \end{aligned} \tag{B.1.1}$$



The RHS can be simplified to  $u(y_2) - u(y_1)$ . The first term of the LHS can be bounded from below in the following way because it is the maximum attainable value of the contract under  $y_2$

$$u(y_2 + b - q(b^2, y)b^2) + \beta Ev^0(b^2, y') \geq u(y_2 + b - q(b^1, y)b^1) + \beta Ev^0(b^1, y')$$

subtract  $u(y_1 + b - q(b^1, y)b^1) + \beta Ev^0(b^1, y')$  to both sides. The left hand side of this operation turns out to be the LHS of the inequality we are attempting to prove,

$$\begin{aligned} & u(y_2 + b - q(b^2, y)b^2) + \beta Ev^0(b^2, y') - \{u(y_1 + b - q(b^1, y)b^1) + \beta Ev^0(b^1, y')\} \\ & \geq u(y_2 + b - q(b^1, y)b^1) + \beta Ev^0(b^1, y') - \{u(y_1 + b - q(b^1, y)b^1) + \beta Ev^0(b^1, y')\} \end{aligned} \tag{B.1.2}$$

Finally the RHS of this last inequality is larger than  $u(y_2) - u(y_1)$  by the concavity of  $u(\cdot)$  and the fact that under default it must be that  $b + q(b^1, y)b^1 < 0$ . Therefore equation B.1.1 holds. □

**Proposition 4** (*Default sets are shrinking in post-bailout balance of debt*). For all  $\alpha_0 < \alpha_1$ , if default is optimal for  $b$  under parameter  $\alpha_0$ , for some states  $y$ , then default will be optimal for  $(b, y)$  under  $\alpha_1$ . That is  $D(b|\alpha_0) \subseteq D(b|\alpha_1)$ .

Post bailout balance of debt can go from zero to  $b$ . That is, the fraction that is not condoned  $1 - \alpha$  change the value of entering default. For notation define

$v_i^i(b, y) = v^i(b, y|\alpha_i)$  for  $i = d, c, 0$  and  $j = 0, 1$ . We know that

$$v_0^d(b, y) > v_0^c(b, y)$$

We want to show that the following holds as well,

$$v_1^d(b, y) > v_1^c(b, y)$$

Under the assumption of *iid* shocks and  $\theta = 0$  we can state the following

$$u(y) + \beta E[gv_0^0((1 - \alpha_0)b, y) + (1 - g)v_1^d(0, y')] > u(y + b - q(b'_0, y)b'_0) + \beta E v_0^0(b'_0, y') \quad (\text{B.1.3})$$

and we want to show that,

$$u(y) + \beta E[gv_1^0((1 - \alpha_1)b, y) + (1 - g)v_1^d(0, y')] > u(y + b - q(b'_1, y)b'_1) + \beta E v_1^0(b'_1, y') \quad (\text{B.1.4})$$

It is enough to prove that  $v_1^d(b, y) - v_1^c(b, y) > v_0^d(b, y) - v_0^c(b, y)$ . First consider the

case of  $g = 1$ . Then we know that

$$\begin{aligned}
v_o^d(b, y) &= u(y) + \beta E\{v_0^o((1 - \alpha_0)b, y')\} \\
&= u(y) + \beta E\{\max\{v_0^c((1 - \alpha_0)b, y'), v_0^d((1 - \alpha_0)b, y')\}\} \\
&= u(y) + \beta E\{\max\{\max_{b''} u(y' + (1 - \alpha_0)b + q(b'', y')b'') + \beta v_0^o(b'', y''), \\
&u(y) + \beta E v_0^d((1 - \alpha_0)^2 b, y'')\}\} \tag{B.1.5}
\end{aligned}$$

If substitute  $\alpha_1$  for  $\alpha_0$  for next period only, and noting that  $(1 - \alpha_1)b > (1 - \alpha_0)b$ , then it is clear that  $v^d(b, y|\alpha_0) < v^d(b, y|\alpha_1)$ . If we iterate forward, the same argument follows, B.1.4 under  $g = 1$  holds. Now consider  $g = 0$ . When bailout probability is zero, the haircut is no longer important. Taking a convex combination of these two cases, we obtain B.1.4.  $\square$

**Proposition 5** (*Default sets are shrinking on  $g$  for large  $\theta$  and expanding on  $g$  for small  $\theta$ ): For  $g_L \leq g_H < \bar{g}$ , (i) if default is optimal for  $b$  in some states  $y$ , with  $g_H$ , then default will be optimal in the same states with  $g_L$  when  $\theta$  is high, (ii) if default is optimal for  $b$  in some states  $y$ , with  $g_L$ , then default will be optimal in the same states with  $g_H$  when  $\theta$  is low.*

The proof of this proposition hinges radically on other parameter values. In particular the remainder of payment after bailout  $(1 - \alpha)$  plays a critical role in shaping the force of the bailout. Start from the observation that under  $g_H$ , and

states  $(b, y)$  default is optimal. We need to determine the effect on the value of default under  $g_L$ . In particular,

$$\begin{aligned} \frac{\partial v^d(b, y)}{\partial g} &= \beta E [v^0((1 - \alpha)b, y')] - \beta E [\theta v^0(0, y') + (1 - \theta)v^d(0, y')] \\ &\quad + g\beta E \left\{ \frac{\partial v^0}{\partial g}((1 - \alpha)b, y') - \theta \frac{\partial v^0}{\partial g}(0, y') - (1 - \theta) \frac{\partial v^d}{\partial g}(0, y') \right\} \end{aligned}$$

If we approximate this derivative around  $g = 0$  then the last term becomes second order and the proposition result follows from the sign of the following expression

$$E \left\{ v^0((1 - \alpha)b, y') - [\theta v^0(0, y') + (1 - \theta)v^d(0, y')] \right\}$$

which we can express in the following way,

$$E \left\{ \theta [v^0((1 - \alpha)b, y') - v^0(0, y')] + (1 - \theta) [v^0((1 - \alpha)b, y') - v^d(0, y')] \right\} \quad (\text{B.1.6})$$

If  $\theta = 1$  then this expression is

$$E[v^0((1 - \alpha)b, y') - v^0(0, y')]$$

which we know is negative because the value function is increasing in assets. Therefore if the exogenous probability of re-entry into financial markets is high, higher bailout probability lowers the value of default, shrinking the default set. On the

opposite extreme case consider what happens if  $\theta = 0$  . Then the result would be

$$E[v^0((1 - \alpha)b, y') - v^d(0, y')]$$

We want to argue that this expression is positive. We prove this by contradiction.

Assume it is negative. Note that by definition, if  $\alpha = 1$  this amounts to

$$E[v^0(0, y') - v^d(0, y')] \geq 0$$

which is a contradiction. Furthermore, assume now that if  $\alpha = 0$ . Then we have that this reduces to

$$\max\{v^c(b, y'), v^d(b, y')\} \geq v^d(0, y')$$

Clearly, if default is optimal, then  $v^d(b, y') \geq v^d(0, y')$  which we know is a contradiction. Now, if honoring the contract is optimal then

$$v^c(b, y') > v^d(b, y') \geq v^d(0, y')$$

but then default was not optimal to begin with for particular  $y' = y$ . This is a contradiction. Therefore the last term in B.1.6 is positive or zero, which means that when the exogenous probability of re-entering markets is small, increasing the bailout probability induces moral hazard because the value of default is higher, and the set of default expanding. □

**Proposition 6** (*Bailout sets are expanding in income*). For all  $Y_1 < Y_2$ , if  $Y_1 \in F(B)$ , then  $Y_2 \in F(B)$

For completion consider the value of bailing out and not bailing out  $\mathcal{P}$ .

$$\begin{aligned} V^B(B, Y) &= u(A Y - \lambda) + \beta E \int_{Y'} V^0((1 - \alpha)B, Y') dF Y' \\ V^{NB}(B, Y) &= u(A Y) + \beta E \int_{Y'} V^0(0, Y') dF Y' \end{aligned}$$

We know from the first assumption that for income  $Y_1$  bailout is an optimal choice.

Hence,

$$u(A Y_1 - \lambda) + \beta E \int_{Y'} V^0((1 - \alpha)B, Y') dF Y' > u(A Y_1) + \beta E \int_{Y'} V^0(0, Y') dF Y'$$

If B.1.7 holds, then we have that  $Y_2 \in F(B)$

$$\begin{aligned} &u(A Y_2 - \lambda) + \beta E \int_{Y'} V^0((1 - \alpha)B, Y') dF Y' \\ &\quad - \left\{ u(A Y_1 - \lambda) + \beta E \int_{Y'} V^0((1 - \alpha)B, Y') dF Y' \right\} > \\ &u(A Y_2) + \beta E \int_{Y'} V^0(0, Y') dF Y' - \left\{ u(A Y_1) + \beta E \int_{Y'} V^0(0, Y') dF Y' \right\} \end{aligned}$$

The right hand side of this inequality can be further simplified to  $u(A Y_2) - u(A Y_1)$  and the left hand side to  $u(A Y_2 - \lambda) - u(A Y_1 - \lambda)$ . We know by the concavity of

$u(\cdot)$ , and  $Y_2 > Y_1$ , that B.1.7 holds, and  $Y_2 \in F(B)$ .

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