



---

Publicly Accessible Penn Dissertations

---


Spring 5-17-2010

# Essays in Dynamic Corporate Finance

Michael Michaux

University of Pennsylvania, [michauxm@wharton.upenn.edu](mailto:michauxm@wharton.upenn.edu)

Follow this and additional works at: <http://repository.upenn.edu/edissertations>

 Part of the [Corporate Finance Commons](#), and the [Finance and Financial Management Commons](#)

---

## Recommended Citation

Michaux, Michael, "Essays in Dynamic Corporate Finance" (2010). *Publicly Accessible Penn Dissertations*. 151.  
<http://repository.upenn.edu/edissertations/151>

This paper is posted at Scholarly Commons. <http://repository.upenn.edu/edissertations/151>  
For more information, please contact [libraryrepository@pobox.upenn.edu](mailto:libraryrepository@pobox.upenn.edu).

---

# Essays in Dynamic Corporate Finance

## **Abstract**

My dissertation aims at understanding the financing and investment decisions of firms. It contains two chapters.

Chapter One studies the currency composition of debt for firms in emerging economies. Using a dataset of traded Mexican firms, I document two stylized facts about firms in the non-tradable sector: (i) they take on large amounts of dollar-denominated debt and (ii) their earnings are not sensitive to the exchange rate. I propose an explanation based on imperfect competition in the domestic goods market that reconciles these seemingly contradictory empirical facts. First I develop a stylized model of production and financing for firms in an open economy. I show that non-exporting firms are exposed to exchange rate risk because of the presence of exporters in the economy, and that they hedge their currency exposure using dollar debt. An extended model is used to quantify how much of the dollar debt in the data can be explained through this channel. A calibrated version of the model can account for all of the dollar debt observed in the data.

Chapter Two investigates the relationship between the investment decisions of firms and their cost of financing. Recent empirical work using panel data documents that, while the correlation of investment and Tobin's  $Q$  is low, the correlation of investment and credit spreads is high. We propose an explanation for these empirical findings, based on time-varying risk, i.e. stochastic volatility. In our model, firms finance investments using defaultable debt as well as equity issuance, and they are subject to standard profitability shocks as well as shocks to volatility. An increase in volatility leads to an increase in the probability of default and hence the credit spread, while reducing investment and increasing equity value. This shock hence generates a negative correlation between investment and credit spreads, and between investment and  $Q$ , helping the model match the data.

## **Degree Type**

Dissertation

## **Degree Name**

Doctor of Philosophy (PhD)

## **Graduate Group**

Finance

## **First Advisor**

Joao F. Gomes

## **Second Advisor**

Urban J. Jermann

## **Third Advisor**

Andrew B. Abel

---

**Keywords**

Capital Structure, Currency Composition of Debt, Exchange Rate Exposure, Optimal Investment, Q Theory, Credit Spreads

**Subject Categories**

Corporate Finance | Finance and Financial Management

**ESSAYS IN DYNAMIC CORPORATE FINANCE**

**Michael Michaux**

A DISSERTATION

in

Finance

For the Graduate Group in Managerial Science and Applied Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2010

Supervisor of Dissertation

Signature \_\_\_\_\_

Joao F. Gomes, James G. Campbell Professor of Finance

Graduate Group Chairperson

Signature \_\_\_\_\_

Eric Bradlow, Professor of Marketing, Statistics, and Education

Dissertation Committee

Joao F. Gomes, James G. Campbell Professor of Finance

Urban J. Jermann, Safra Professor of International Finance and Capital Markets

Andrew B. Abel, Ronald A. Rosenfeld Professor, Professor of Finance and Economics

Karen K. Lewis, Sondheimer Professor in International Economics and Finance

# ABSTRACT

## ESSAYS IN DYNAMIC CORPORATE FINANCE

Michael Michaux

Joao F. Gomes

My dissertation aims at understanding the financing and investment decisions of firms. It contains two chapters.

Chapter One studies the currency composition of debt for firms in emerging economies. Using a dataset of traded Mexican firms, I document two stylized facts about firms in the non-tradable sector: (i) they take on large amounts of dollar-denominated debt and (ii) their earnings are not sensitive to the exchange rate. I propose an explanation based on imperfect competition in the domestic goods market that reconciles these seemingly contradictory empirical facts. First I develop a stylized model of production and financing for firms in an open economy. I show that non-exporting firms are exposed to exchange rate risk because of the presence of exporters in the economy, and that they hedge their currency exposure using dollar debt. An extended model is used to quantify how much of the dollar debt in the data can be explained through this channel. A calibrated version of the model can account for all of the dollar debt observed in the data.

Chapter Two investigates the relationship between the investment decisions of firms and their cost of financing. Recent empirical work using panel data documents that, while the correlation of investment and Tobin's  $Q$  is low, the correlation of investment and credit spreads is high. We propose an explanation for these empirical findings, based on time-varying risk, i.e. stochastic volatility. In our model, firms finance investments using defaultable debt as well as equity issuance, and they are subject to standard profitability shocks as well as shocks to volatility. An increase in volatility leads to an increase in the probability of default and hence the credit spread, while reducing investment and increasing equity value. This shock hence generates a negative correlation between investment and credit spreads, and between investment and  $Q$ , helping the model match the data.

# Contents

<b>1</b>	<b>Pass-through, Exposure, and the Currency Composition of Debt</b>	<b>1</b>
1.1	Introduction	1
1.2	Stylized Model	4
1.2.1	Environment	5
1.2.2	Preferences and Demand	6
1.2.3	Production and Sales Decisions	7
1.2.4	Financing Decision	14
1.3	Empirical Evidence	17
1.3.1	Description of the Data	17
1.3.2	Currency Composition of Debt	19
1.3.3	Estimation of the Exchange Rate Sensitivities	21
1.3.4	Testable Model Implications	23
1.4	Quantitative Model	24
1.4.1	Preferences	25
1.4.2	Firm Problem	25
1.4.3	International Markets and Lenders	27
1.4.4	Dynamics of the Shocks	28
1.4.5	Recursive Formulation of the Firm Problem	28
1.4.6	Competitive Equilibrium	29

1.5	Numerical Results . . . . .	30
1.5.1	Calibration . . . . .	30
1.5.2	Production and Sales Policy . . . . .	33
1.5.3	Financing Policy . . . . .	34
1.6	Conclusion . . . . .	37
1.7	Appendix . . . . .	39
	<b>Bibliography</b>	<b>53</b>
<b>2</b>	<b>Stochastic Volatility, Credit Spreads, and the Q Theory of Investment</b>	
	<b>joint with François Gourio</b>	<b>57</b>
2.1	Introduction . . . . .	57
2.2	Two-Period Partial Equilibrium Example . . . . .	60
2.3	Quantitative Model . . . . .	69
2.3.1	Firm Problem . . . . .	70
2.3.2	Lenders . . . . .	71
2.3.3	Recursive Formulation of the Firm Problem . . . . .	71
2.4	Numerical Results . . . . .	72
2.4.1	Parameter Choice . . . . .	72
2.4.2	Basic Properties . . . . .	74
2.4.3	Predictability Properties . . . . .	75
2.5	Conclusion . . . . .	76
	<b>Bibliography</b>	<b>78</b>

# List of Tables

1.1	<b>Summary Statistics.</b> This table reports the summary statistics for the main variables. Panel A reports the raw data in millions of Mexican pesos. Panel B reports ratios. Panel C reports variables for local and exporting firms. The data consists of 99 traded Mexican firms, for the years 1999 to 2009. Source: <i>Bolsa Mexicana de Valores</i> . . . . .	18
1.2	<b>Regression Results.</b> Level of significance: * $p < 0.1$ , ** $p < 0.05$ , *** $p < 0.01$ , using robust standard errors. . . . .	20
1.3	<b>Regression Results.</b> These regressions use local firms only. Level of significance: * $p < 0.1$ , ** $p < 0.05$ , *** $p < 0.01$ , using robust standard errors. . . . .	22
1.4	<b>Regression Results.</b> These regressions use local firms only. Level of significance: * $p < 0.1$ , ** $p < 0.05$ , *** $p < 0.01$ , using robust standard errors. . . . .	24
1.5	<b>Statistics and AR(1) Estimates of the Real Exchange Rate Process.</b> Panel A reports the statistics for the real exchange rate process. Panel B reports the parameter estimates when the real exchange rate is assumed to follow an AR(1) process, given by $\log(e_{t+1}) = c + \rho_e \log(e_t) + \sigma_e \epsilon_t$ . The results are shown for two different time periods. The data frequency is quarterly. . . . .	31
1.6	<b>Mapping of Variables from the Model to Data.</b> . . . . .	32
1.7	<b>Target Moments for Calibration.</b> . . . . .	32
1.8	<b>Parameter Values.</b> . . . . .	33



1.9	<b>Summary Statistics of Monte carlo Simulations.</b> The economy is simulated for a panel of 100 firms over 200 periods (first 100 periods in the sample are dropped).	36
2.1	<b>Summary of the Comparative Statics.</b> Effect of an increase of each parameter on the endogenous variables.	65
2.2	<b>Parameter Values.</b> The model is calibrated at the quarterly frequency.	73
2.3	<b>Summary Statistics.</b> The summary statistics in the data are taken from Chen, Collin-Dufresne, and Goldstein (2008) and Eberly, Rebelo, and Vincent (2009). The model economy is simulated for a panel of 100 firms over 200 periods. Averages and volatilities for bond yields are quoted in basis points.	74
2.4	<b>Data.</b> Regressions results from Gilchrist and Zakrajsek (2009).	75
2.5	<b>Regression Results.</b> The regressions results in the data are taken from Gilchrist and Zakrajsek (2009). Regressions in the model are run on data simulated for a panel of 100 firms over 200 periods.	76

# List of Figures

1.1	<b>Exporting Firm’s Marginal Benefit Curves for the Domestic and Foreign Markets.</b> . . . . .	9
1.2	<b>Pass-through versus the Market Share.</b> The left panel plots the pass-through for the exporting firm. The right panel plots the pass-through for the local firm. . .	12
1.3	<b>Share of Foreign Debt versus the Share of Foreign Sales.</b> The data consists of Mexican firms traded on the Mexican Stock Market, for the years 1999 to 2009. Source: <i>Bolsa Mexicana de Valores</i> . . . . .	19
1.4	<b>Share of Foreign Debt versus the Share of Foreign Sales by Sector.</b> The data consists of Mexican firms traded on the Mexican Stock Market, for the years 1999 to 2009. Source: <i>Bolsa Mexicana de Valores</i> . . . . .	21
1.5	<b>Profits, Sensitivity of Profits, and Sales Policies.</b> The left figures plot profits $\pi_i$ and the sensitivity of profits to the exchange rate $d\pi_i/de$ as a function of the exchange rate $e$ and cost shock $z_i$ . The right figures plot the optimal sales policies: percentage of exports $x_i^*/(x_i + x_i^*)$ and total production $(x_i + x_i^*)$ as a function of the exchange rate $e$ and cost shock $z_i$ . Each row corresponds to a different values of the elasticity of substitution, $\rho \in \{0.25, 0.5, 0.75\}$ . . . . .	35
1.6	<b>Pass-through and Exposure.</b> The figure shows the pass-through $\eta$ (top row) and exposure $\delta$ (bottom row) as a function of the exchange rate $e$ and cost shock $z_i$ . Each column corresponds to a different values of the elasticity of substitution, $\rho \in \{0.25, 0.5, 0.75\}$ . . . . .	35

2.1	<b>Optimal Choice of Debt.</b> The figure shows the effects of debt $b$ on the firm financing options $s$ and $q * b$ , the yield spread, the probability of default, the equity value $V$ , and Tobin's $Q$ in the baseline model. The vertical red line shows the optimal debt choice. . . . .	64
2.2	<b>Comparative Statics.</b> The figure shows the effects of volatility $\sigma$ on the optimal firm policies $k$ and $b$ , the yield spread, the probability of default, the equity value $V$ , and Tobin's $Q$ in the baseline model. . . . .	66
2.3	<b>Comparative Statics.</b> The figure shows the effects of volatility $\sigma$ on the optimal firm policies $k$ and $b$ , the yield spread, the probability of default, the equity value $V$ , and Tobin's $Q$ in the model extended with a growth option. . . . .	68

# Chapter 1

## Pass-through, Exposure, and the Currency Composition of Debt

### 1.1 Introduction

Firms that contract debt in foreign currency rather than in domestic currency can create currency exposures via balance sheet mismatches. These currency exposures can potentially lead to more costly external financing, or even bankruptcy when the exchange rate depreciates sharply. Therefore the currency composition of corporate debt is an important capital structure decision.

In this paper I use a panel dataset of Mexican firms traded on the Mexican Stock Market in the last 10 years. I document two stylized facts about firms in the non-tradable sector: (i) they take on large amounts of dollar-denominated debt and (ii) their earnings, defined as sales minus costs and financing expenses, are not sensitive to the exchange rate. The first stylized fact is consistent with the emerging markets literature, where mounting empirical evidence shows that firms in emerging markets have a sizable amount of dollar-denominated debt<sup>1</sup>. The second empirical fact suggests that firms are managing their currency exposure.

Several explanations for the high dollar indebtedness have been offered in the theoretical literature. Schneider and Tornell (2004) argue that implicit guarantees by the government induce firms to borrow in dollars excessively. Expecting a bailout in the event of a large currency depreciation, firms

---

<sup>1</sup>See Bleakley and Cowan (2008), Allayannis and Ofek (2001), and Aguiar (2005)

overexpose themselves to exchange rate fluctuations. The implicit guarantees by the government provide some benefit to issuing large amounts of dollar debt. Caballero and Krishnamurthy (2001, 2003) argue that limited financial development in emerging markets led to firms taking on excessive dollar debt. Specifically they show that when financial constraints affect borrowing and lending, firms undervalue insuring against an exchange rate depreciation and therefore take on excessive dollar debt. Both of these theories are based on the existence of benefits to issuing dollar debt, which firms balance against the cost in the event of a large currency depreciation. In these models currency mismatches of assets and liabilities arise in equilibrium, and therefore firms' profits are exposed to fluctuations in the exchange rate. These theories are consistent with the first stylized fact of high dollar indebtedness. However they are in contradiction with the second stylized fact that earnings of Mexican firms are not exposed to the exchange rate.

In this paper I propose an explanation that reconciles the seemingly contradictory empirical facts that firms in the non-tradable sector issue large amounts of dollar debt but are hedged against currency risk. I develop a model of firms in an open economy and show that non-exporting firms are exposed to exchange rate risk because of the exporters in the economy, and that they hedge their currency exposure using dollar debt. In the model, high dollar indebtedness is a direct result of an optimal financing decision.

In the first section of the paper, I develop a stylized model of firms with imperfect competition in the domestic goods market. The model features a local firm (or non-exporting firm) and an exporting firm, each producing a differentiated good. The representative consumer in the domestic market chooses an optimal bundle of goods and prices clear in the goods market. Unlike the local firm, the exporting firm has the option to sell some of its production abroad. As the exchange rate fluctuates, the foreign price in the domestic currency fluctuates as well, which induces the exporting firm to adjust its domestic supply. Due to the imperfect competition, this affects all prices in the domestic goods market. Thus the local firm has a positive "pass-through," since changes in exchange

rates are passed onto the price of the non-tradable good in the domestic market. Both price and profits for the local firm vary with the exchange rate. If the domestic goods market were perfectly competitive, the exchange rate would not affect prices in the domestic market, and the local firm would not be affected by the exporter's decisions. This mechanism is present despite the fact that firms have similar costs and that there are no shocks to the demand for their product in the domestic market. The presence of pass-through in turn implies that firms' profits are potentially sensitive to the exchange rate. Managerial risk aversion creates a motive to smooth out dividends, so firms use dollar debt to hedge their exchange rate exposure. Therefore the model yields an optimal currency composition for debt.

In the next section, I use a panel dataset of Mexican firms traded on the Mexican Stock Market, called the Bolsa Mexicana de Valores (BMV). I explore empirically the determinants of the currency composition of debt, estimate the sales and earnings sensitivity to exchange rate, and test the implications of the model. Regressions of the currency composition of debt on firm characteristics show that local firms issue large fractions of their debt in dollars, and that industry dummies provide additional information beyond firms' characteristics. Regressions of sales and earnings on the exchange rate show that sales for local firms are sensitive to the exchange rate, but that their earnings are not. This is evidence that local firms hedge the exposure of their sales effectively. Additional regressions show that local firms in more concentrated industries (more imperfect goods market) and with smaller market shares have higher shares of foreign debt. These results are in agreement with the comparative statics of the model.

In the last section of the paper, I present a quantitative model that can be used to study the effect of the goods market imperfection on the amounts of foreign debt issued by firms. I incorporate a standard corporate finance model of the firm, similar in spirit to Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2005), and Gomes and Schmid (2009), into an industry equilibrium with imperfectly competitive goods market. The model is fully dynamic and is extended

to accommodate a continuum of firms. The heterogeneity between firms comes from an idiosyncratic cost shock similar in spirit to Melitz (2003) and Melitz and Ottaviano (2008), making the decision to export endogenous. I also add financial frictions in the form of costly equity issuances and defaultable debt. Bankruptcy costs are incurred upon default and the interest expense on debt can be deducted from taxable income. This extended model is used to quantify how much of the dollar debt in the data can be accounted for by this pass-through channel due to imperfect competition in the goods market. A calibrated version of the model can account for all of the dollar debt observed in the data.

The model in this paper is similar in spirit to the model of Bodnar, Dumas, and Marston (2002). The authors develop a model of exporting firms under imperfect competition to study the interaction between pass-through and exchange rate exposure. They study how changes in exchange rates affect the relative competitiveness of firms in the goods market through their production costs. In contrast, the model in this paper studies the domestic market and focuses on understanding how changes in exchange rates affect pass-through and exchange rate exposure of all firms in the domestic market. Local firms have a positive pass-through in the industry equilibrium, even though costs for all firms in this economy are similar. In addition this paper addresses the optimal currency composition of debt in such an economy, which is not considered in Bodnar, Dumas, and Marston (2002).

The paper is organized as follows. Section 1.2 presents and analyzes the stylized model. Section 1.3 describes the panel dataset of Mexican firms and tests the implications of the model. I present an extended model in Section 1.4. This model is calibrated and used to quantify the value of hedging using dollar debt in Section 1.5. Section 1.6 concludes the paper.

## 1.2 Stylized Model

This section presents a stylized model of production and financing for firms in an open economy. The model features imperfect competition in the domestic goods market and managerial risk aversion.

In this section I derive the pass-through policy and exchange rate exposure of firms, as well as the optimal currency composition of corporate debt.

### 1.2.1 Environment

There are 2 types of firms in the economy. Type 1 represents an *exporting firm* that sells goods in both the domestic and foreign markets. Type 2 represents a *local firm* that only sells goods in the domestic market. These two firms can be interpreted as two firms in a given sector of the economy where sectors are complements to one another. Alternatively, these firms can be thought of as two sectors of the economy: the non-tradable sector and the tradable sector (or export sector). The local firm can be thought as operating in a non-tradable sector of the economy or as a firm having large costs of exporting, making it optimal to not export. The decision to export will be endogenous in the extended model described in Section 1.4.

Time is discrete and there are 2 periods. The timing is as follows. In period 0, firms make their financing decision, and in period 1, make their production and sales decisions. This timing is chosen for clarity of exposition. The production decision could also be made in period 0 (along with the financing decision) at the cost of more complicated algebra, but the results would not be affected. The financing decision consists of choosing how much domestic (peso) and foreign (dollar) debt to issue. More formally, domestic debt for a firm of type  $i$  is denoted by  $b_{id} \geq 0$ , while foreign debt is denoted by  $b_{if} \geq 0$ . The debt is default free and is repaid in period 1. The debt is priced by risk-neutral investors, thus the debt price is equal to the common subjective discount rate  $\beta \in [0, 1]$  of investors. The production decision consists of choosing the quantity of goods to produce and the allocation of sales in the domestic and foreign markets. Each firm  $i$  produces a differentiated good.  $x_i \geq 0$  denotes sales in the domestic market while  $x_i^* \geq 0$  denotes sales in the foreign market. The good produced by firm  $i$  (or type  $i$ ) will be referred to as good  $i$  as each firm produces a single differentiated good.

Each good  $i$  sells at price  $p_i \geq 0$  in the domestic market and good 1 sells at price  $p_1^* \geq 0$  in the



foreign market. Each price is denominated in units of local currency, that is  $p_i$  is in pesos and  $p_1^*$  is in dollars. The domestic goods market is a noncooperative duopoly and domestic prices  $p_i$  satisfy the representative consumer optimal decision. The foreign market is perfectly competitive so the exporting firm is a price taker. In other words, the foreign price  $p_1^*$  is given and firm 1 can sell any quantity of goods at that price.

The uncertainty in this economy comes solely from the real exchange rate in period 1. It is defined as the price of 1 dollar in peso and is denoted  $e_1$ . The exchange rate in period 0 is set at  $\bar{e}$ . The exchange rate in period 1 is distributed on a bounded and non-negative support. Its expectation is equal to  $\bar{e}$  and its volatility is denoted  $\sigma_e$ .

### 1.2.2 Preferences and Demand

The utility function for the domestic consumer is CES, as in Dixit and Stiglitz (1977). The consumer's problem is given by,

$$U = \max_{x_1, x_2} [\alpha x_1^\rho + (1 - \alpha)x_2^\rho]^{\frac{1}{\rho}}, \quad (1.1)$$

$$\text{s.t.} \quad y = p_1 x_1 + p_2 x_2, \quad (1.2)$$

where  $y \geq 0$  is the total expenditure by the consumer,  $\alpha \in [0, 1]$  is a preference parameter, and  $\rho \in [0, 1]$  is a parameter that controls the substitutability between goods. The goods become perfect complements when  $\rho$  goes to 0, whereas they are perfect substitutes when  $\rho = 1$ .

The inverse demand functions in the domestic market are,

$$p_1(x_1, x_2) = \frac{\alpha x_1^{(\rho-1)} y}{\alpha x_1^\rho + (1 - \alpha)x_2^\rho}, \quad (1.3)$$

$$p_2(x_1, x_2) = \frac{(1 - \alpha)x_2^{(\rho-1)} y}{\alpha x_1^\rho + (1 - \alpha)x_2^\rho}. \quad (1.4)$$

The own and cross-price derivatives of these demand functions are negative, i.e.  $p_{ii} < 0$  and  $p_{ij} < 0$ .

This property means that an increase in the quantity of either good will lead to a decline in prices.

The market shares of the local and exporting firms will be important measures of size in this model. Define the market share of the exporting firm in the domestic market by  $\lambda$ . Using equation

(1.3), the market share can be expressed as,

$$\lambda \equiv \frac{p_1 x_1}{y} = \frac{\alpha x_1^\rho}{\alpha x_1^\rho + (1 - \alpha) x_2^\rho}. \quad (1.5)$$

Conversely the market share of the local firm is  $(1 - \lambda)$ . Note that the market share  $\lambda \in (0, 1)$ , as both firms are selling in the domestic market.

The own and cross reciprocals of elasticities of demand for both goods can be expressed as functions of the elasticity of substitution between goods and market shares as follows,

$$\begin{bmatrix} \frac{x_1}{p_1} \frac{\partial p_1}{\partial x_1} & \frac{x_2}{p_1} \frac{\partial p_1}{\partial x_2} \\ \frac{x_1}{p_2} \frac{\partial p_2}{\partial x_1} & \frac{x_2}{p_2} \frac{\partial p_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \rho(1 - \lambda) - 1 & -\rho(1 - \lambda) \\ -\rho\lambda & \rho\lambda - 1 \end{bmatrix}. \quad (1.6)$$

As market shares for both firms are never zero, a rise in the substitutability of goods (higher  $\rho$ ) raises the magnitude of the cross price reciprocal of elasticities and lowers the magnitude of the own price reciprocal of elasticities.

### 1.2.3 Production and Sales Decisions

For simplicity, firms incur no cost of production but they face a capacity constraint in their production denoted by  $\bar{x} > 0$ . Profits in local currency, denoted by  $\pi$ , are given by the solution to the production and sales problems. Firms choose their production and sales after they observe the exchange rate  $e$ . Firms decide on the scale of production ( $x + x^*$ ) and the quantities to sell in each market (i.e.  $x$  in the domestic market and  $x^*$  in the foreign market) such that they maximize revenues in local currency. The problem for the local firm is given by,

$$\begin{aligned} \pi_2(e) &= \max_{x_2} p_2(x_1, x_2)x_2, \\ \text{s.t.} \quad &x_2 \leq \bar{x}_2. \end{aligned}$$

This problem is trivial to solve. The local firm does not incur any production costs and its own elasticity of demand is greater than one, therefore it will want to sell as many units as possible. The domestic production for the local firm is thus  $x_2 = \bar{x}_2$ . The problem for the exporting firm is given

by,

$$\begin{aligned} \pi_1(e) = \max_{x_1, x_1^*} \quad & p_1(x_1, x_2)x_1 + ep_1^*x_1^*, \\ \text{s.t.} \quad & x_1 + x_1^* \leq \bar{x}_1. \end{aligned}$$

It is straightforward to see that the exporting firm will also produce at capacity as it incurs no production costs. Thus the allocation of sales is the only non-trivial decision the exporting firm has to make. The first-order condition for domestic sales  $x_1$  prescribes that the exporting firm should equalize the marginal benefit of selling one more unit in the domestic market to the marginal benefit in the foreign market. The following lemma formalizes this intuition.

**Lemma 1.2.1** (Export Decision). *Let  $\tilde{x}_1(e)$  be the solution to  $f(x_1, e) = 0$ , where  $f : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$  is defined by,*

$$f(x_1, e) = \rho(1 - \lambda(x_1, \bar{x}_2))p_1(x_1, \bar{x}_2) - ep_1^*.$$

*The function  $\tilde{x}_1(e)$  is decreasing in its argument,*

$$\frac{\partial \log(\tilde{x}_1)}{\partial \log(e)} = -\frac{1}{1 + \rho(2\lambda - 1)}.$$

**Proof:** See [Appendix A](#).

**Lemma 1.2.1** states that  $\tilde{x}_1$  is the quantity of good sold in the domestic market that equalizes marginal benefits of selling one more unit in domestic and foreign goods markets. When this level of domestic sales is feasible, that is when this level is lower than the production capacity, the exporting firm sells  $\tilde{x}_1$  domestically, and sells the remainder of the production ( $\bar{x}_1 - \tilde{x}_1$ ) in the foreign market. If this level is higher than the production capacity, the exporting firm will not sell in the foreign market. The exporting firm will sell all of its output in the domestic market and the economy will have no exports. [Figure 1.1](#) conveys this mechanism graphically. It depicts the marginal benefit curves for the the exporting firm in both the domestic and foreign markets. These curves graph the marginal benefit of selling one more unit in the goods market as a function of quantity supplied,  $x_1$ .

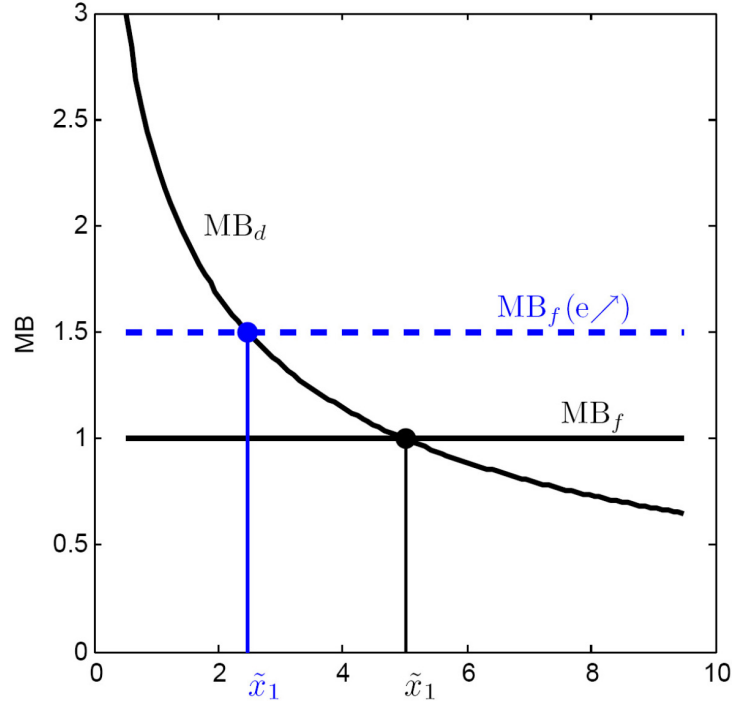


Figure 1.1: **Exporting Firm's Marginal Benefit Curves for the Domestic and Foreign Markets.**

**Lemma 1.2.1** also states that  $\tilde{x}_1$  is decreasing in the exchange rate  $e$ . It means that domestic sales fall when the peso depreciates. **Figure 1.1** shows the effect of an exchange rate increase (i.e. depreciation of the peso against the dollar). First note that the marginal benefit of selling one more unit in the foreign market is equal to the foreign price in peso. The foreign price in dollar is constant, thus when the exchange rate increases, the foreign price in peso increases as well. The marginal benefit curve for the foreign market shifts up (i.e. the dashed line) and intersects the domestic marginal benefit curve at a higher marginal benefit and a lower value for  $x_1$ . This illustrates why domestic sales decrease when the peso depreciates. As foreign prospects improve, the exporting firm reduces domestic sales to sell more of its production abroad. This substitution effect in the sales of the exporting firm is at the heart of the mechanism for the transmission of exchange rate fluctuations to the local firm. The following proposition formalizes the optimal sales policy for both firms.

**Proposition 1.2.2** (Optimal Sales Policy). *The optimal sales policies for the exporting firm are given by,*

$$\begin{aligned} x_1(e) &= \begin{cases} \tilde{x}_1(e) & \text{if } \tilde{x}_1 \leq \bar{x}_1, \\ \bar{x}_1 & \text{if } \tilde{x}_1 > \bar{x}_1, \end{cases} \\ x_1^*(e) &= (\bar{x}_1 - \tilde{x}_1(e)) \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}}. \end{aligned}$$

*The optimal sales policy for the local firm is given by,*

$$x_2 = \bar{x}_2.$$

**Proof:** See [Appendix A](#).

Following the exchange rate literature, pass-through is defined as the elasticity of price with respect to the exchange rate. It is denoted by  $\eta_i$  for firm  $i$  and formally expressed as follows,

$$\eta_i \equiv \frac{\partial \log p_i}{\partial \log e}.$$

Pass-through is a measure of the extent to which a firm “passes through” the changes in exchange rates onto the price it charges in the goods market. In this model, we are interested in understanding how much of the exchange rate changes are passed onto the prices of both goods in the domestic market. The dollar price of the exported good is independent of the exchange rate by assumption, so its pass-through is zero. The following proposition gives prices and pass-throughs for both firms in closed-form.

**Proposition 1.2.3** (Prices and Pass-through). *The price and pass-through for the exporting firm are given by,*

$$\begin{aligned} p_1(e) &= \begin{cases} \frac{\alpha \tilde{x}_1(e)^{(\rho-1)} y}{\alpha \tilde{x}_1(e)^\rho + (1-\alpha) \bar{x}_2^\rho} & \text{if } \tilde{x}_1 \leq \bar{x}_1, \\ \frac{\alpha \bar{x}_1^{(\rho-1)} y}{\alpha \bar{x}_1^\rho + (1-\alpha) \bar{x}_2^\rho} & \text{if } \tilde{x}_1 > \bar{x}_1, \end{cases} \\ \eta_1(e) &= \eta \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}}, \end{aligned}$$

where  $\eta \equiv \frac{1-\rho(1-\lambda)}{1-\rho(1-2\lambda)}$ .

The price and pass-through for the local firm are given by,

$$\begin{aligned}
 p_2(e) &= \begin{cases} \frac{(1-\alpha)\tilde{x}_2^{(\rho-1)}y}{\alpha\tilde{x}_1(e)^\rho+(1-\alpha)\tilde{x}_2^\rho}, & \text{if } \tilde{x}_1 \leq \bar{x}_1, \\ \frac{(1-\alpha)\tilde{x}_2^{(\rho-1)}y}{\alpha\tilde{x}_1^\rho+(1-\alpha)\tilde{x}_2^\rho}, & \text{if } \tilde{x}_1 > \bar{x}_1, \end{cases} \\
 \eta_2(e) &= (1-\eta) \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}}.
 \end{aligned}$$

**Proof:** See [Appendix A](#).

The pass-through for the exporting firm, denoted by  $\eta_1$ , is equal to  $\eta$ , which can be expressed as a function of two parameters: the elasticity of substitution  $\rho$  and the market share  $\lambda$ . It is interesting to note that  $\eta \in [1/2, 1]$ . This means that the price of good 1 in the domestic market will always respond to changes in the exchange rate. This result is intuitive and certainly not very surprising. Changes in the foreign market price (in peso) will affect the sales decisions of the exporting firm, and consequently affect the price of its good in the domestic market. However, the novel result is the fact that the sales decisions of the exporting firm affect prices of all goods in the domestic market. Therefore pass-through for the local firm, denoted by  $\eta_2$ , is non zero. It is equal to  $(1-\eta)$  and lies in the interval  $[0, 1/2]$ . This effect is due to the assumption of imperfect competition in the domestic goods market. If the domestic goods market were perfectly competitive, prices would not react to changes in the supply of any goods and, as a result, pass-through for all firms would be zero. Thus the exchange rate fluctuations are transmitted to the price of good 2 because of the domestic sales of firm 1, due to the imperfect competition in the domestic goods market. In the exchange rate exposure literature, pass-through generally results from a change in relative costs of production between firms. In this case a change in exchange rate leads to a change in the *opportunity cost* for the exporting firm. Thus this model highlights a channel for pass-through that links firms in the tradable sector to firms in the non-tradable sector.

[Figure 1.2](#) plots the pass-through for both firms versus the market share  $\lambda$ . A higher market share (higher  $\lambda$ ) leads to a lower pass-through for the exporting firm, but a higher pass-through for the local firm. This is due to the fact that if the exporter has a higher market share, a change in

its domestic sales will affect its own price less than it will affect the price of the local good. As a result, a larger exporting firm will be able to pass more of the exchange rate fluctuations onto the local firm.

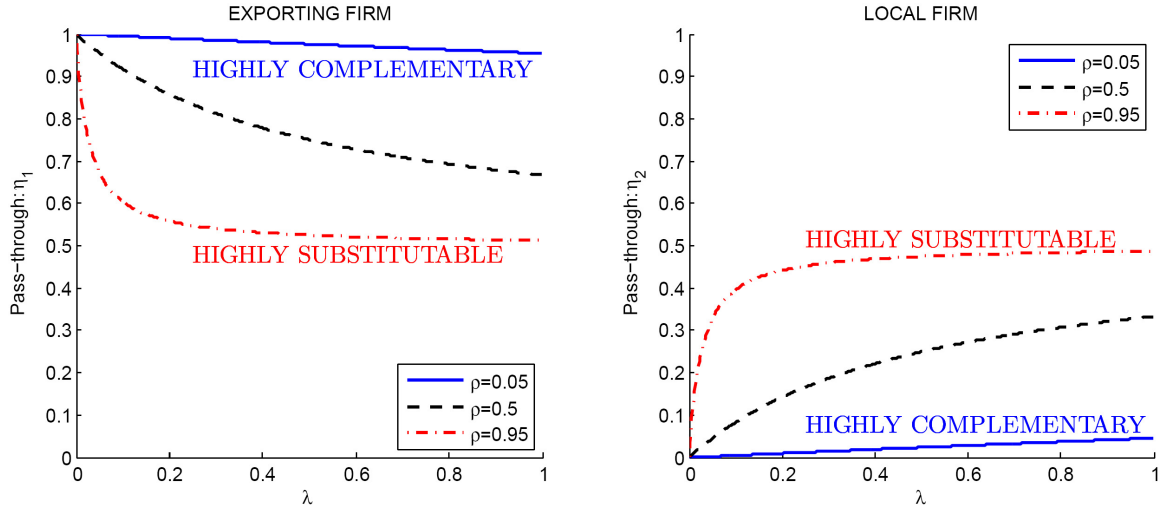


Figure 1.2: **Pass-through versus the Market Share.** The left panel plots the pass-through for the exporting firm. The right panel plots the pass-through for the local firm.

Figure 1.2 plots the pass-through for both firms for different values of the elasticity of substitution  $\rho$ . The solid (blue) line shows the case of  $\rho = 0.05$ , where goods are highly complementary. The dashed (red) line shows the case of  $\rho = 0.95$ , where goods are highly substitutable. These graphs show that a higher degree of substitution between goods (higher  $\rho$ ) leads to a lower pass-through for the exporting firm, but a higher pass-through for the local firm. As discussed earlier, this is due to the fact that an increase in substitutability raises the magnitude of the cross price elasticities and lowers the magnitude of the own price elasticities. Thus a change in domestic sales  $x_1$  by the exporting firm will affect the price of good 1 less and the price of good 2 more. The following proposition summarizes the comparative statics.

**Proposition 1.2.4** (Pass-through: Comparative Statics). *The comparative statics for the pass-through of the exporting firm are given by,*

$$\begin{aligned}\frac{\partial \log \eta_1}{\partial \log \lambda}(e) &= \frac{-\rho\lambda(1-\rho)}{(1-\rho(1-\lambda))(1+\rho(2\lambda-1))} \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}} \leq 0, \\ \frac{\partial \log \eta_1}{\partial \log \rho}(e) &= \frac{-\rho\lambda}{(1-\rho(1-\lambda))(1+\rho(2\lambda-1))} \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}} \leq 0.\end{aligned}$$

The comparative statics for the pass-through of the local firm are given by,

$$\begin{aligned}\frac{\partial \log \eta_2}{\partial \log \lambda}(e) &= \frac{1-\rho}{1+\rho(2\lambda-1)} \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}} \geq 0, \\ \frac{\partial \log \eta_2}{\partial \log \rho}(e) &= \frac{1}{1+\rho(2\lambda-1)} \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}} \geq 0.\end{aligned}$$

**Proof:** See [Appendix A](#).

The quantity of interest for the financing decision is not pass-through but rather the exchange rate exposure. It is defined as the elasticity of profits with respect to the exchange rate. It is denoted by  $\delta_i$  for firm  $i$  and formally expressed as  $\delta_i \equiv \frac{\partial \log \pi_i}{\partial \log e}$ . Profits and exchange rate exposures are given for both firms in the following proposition.

**Proposition 1.2.5** (Profits and Exchange Rate Exposure). *Profits and exchange rate exposure for the exporting firm are given by,*

$$\begin{aligned}\pi_1(e) &= \begin{cases} p_1 \tilde{x}_1 + ep_1^*(\bar{x}_1 - \tilde{x}_1), & \text{if } \tilde{x}_1 \leq \bar{x}_1, \\ p_1 \bar{x}_1, & \text{if } \tilde{x}_1 > \bar{x}_1, \end{cases} \\ \delta_1(e) &= \chi \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}}.\end{aligned}$$

where  $\chi \equiv \frac{ep_1^*(\bar{x}_1 - \tilde{x}_1)}{p_1 \tilde{x}_1 + ep_1^*(\bar{x}_1 - \tilde{x}_1)}$  is the share of foreign profits. Profits and exchange rate exposure for the local firm are given by,

$$\begin{aligned}\pi_2(e) &= p_2 \bar{x}_2, \\ \delta_2(e) &= (1-\eta) \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}}.\end{aligned}$$

**Proof:** See [Appendix A](#).

The exchange rate exposure for the exporting firm is equal to the share of its profits that is foreign. As exports become a larger proportion of total profits, the exchange rate exposure rises.



Specifically, the exposure asymptotes to 100% as exports become the main source of profits, i.e.  $\chi$  goes to 1 as  $\bar{x}_1$  goes to infinity. The converse holds true: as exports become a smaller portion of total profits, the exporting firm sees its exposure becoming very small. As the exporting firm is able to substitute sales between the domestic and foreign markets, its exchange rate exposure can potentially be very small. On the other hand, the local firm does not have the ability to do so. Because of the fixed scale of its operations, its exchange rate exposure is equal to its pass-through. As described previously, this means that the local firm can potentially have a large exchange rate exposure, as can be seen in [Figure 1.2](#).

#### 1.2.4 Financing Decision

One of the objectives of this model is to derive the optimal capital structure for firms that are exposed to exchange rate fluctuations, as described in the previous section. As the hedging literature has shown, imperfections in the financial markets, a convex tax schedule, or agency between managers and shareholders are factors that will induce firms to hedge their cashflows. Because the model presented in this section is parsimonious, the demand for hedging will be introduced by assuming that firms are run by risk-averse managers. Although it is not the only way to create a hedging behavior, it is an effective way to do so in a two periods setting. A more general economy with financing frictions is presented in the extended model described in [Section 1.4](#).

Firms are run by risk-averse managers, that is managers maximize the discounted sum of the utility of dividends. The utility function  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  satisfies (i)  $u' > 0$ , (ii)  $u'' < 0$ , and (iii) the Inada conditions. The managers have the same discount rate as the investors. The financing problem for both firms is given by,

$$\begin{aligned}
 V &= \max_{b_d, b_f} && u(d_0) + \beta \mathbb{E}[u(d_1)], \\
 \text{s.t.} &&& d_0 = \beta(b_d + \bar{e}b_f), \\
 &&& d_1 = \pi(e_1) - b_d - e_1 b_f.
 \end{aligned}$$

In period 0, firms issue domestic and foreign debt at the discount price  $\beta$ . In period 1, the exchange rate  $e_1$  is observed and firms produce  $\pi(e_1)$  and repay their domestic  $b_d$  and foreign debt  $b_f$ . The optimal debt satisfies the Euler equations,

$$\begin{aligned} u'(\beta(b_d + \bar{e}b_f)) &= \mathbb{E}[u'(\pi(e_1) - b_d - e_1b_f)], \\ \bar{e}u'(\beta(b_d + \bar{e}b_f)) &= \mathbb{E}[e_1u'(\pi(e_1) - b_d - e_1b_f)]. \end{aligned}$$

The Euler equations simply state that the manager wants to equalize marginal utilities of dividends across time, as well as across states. The optimal financing policy is similar for both firms, and given in the following proposition.

**Proposition 1.2.6** (Optimal Financing Policy). *If the volatility of the exchange rate is small, i.e.  $\sigma_e$  is in a neighborhood of zero, then the optimal financing policies can be approximated to a first order. The optimal financing policy for both firms is,*

$$\begin{aligned} b_d &\approx \frac{\pi(\bar{e})}{1 + \beta} - \bar{e} \frac{\partial \pi}{\partial e}(\bar{e}), \\ b_f &\approx \frac{\partial \pi}{\partial e}(\bar{e}). \end{aligned}$$

**Proof:** See [Appendix A](#).

First it is interesting to note that the degree of risk aversion does not enter the choice of debt. As long as the utility function is strictly concave, managers have an incentive to hedge cashflows regardless of the extent of their risk aversion. The optimal amount of domestic debt is such that the cashflow of the project in period 1 is distributed in both periods. The optimal amount of dollar debt is simply the expected sensitivity of profits to exchange rate. This result resembles the textbook prescription for firms to hedge their exchange rate economic exposure. Therefore firms use domestic debt to smooth out cashflows over time and use foreign debt to smooth cashflows across states of the world.

In order to relate the financing policies to the data, it is convenient to introduce a new variable:

the share of foreign debt. It is denoted by  $\omega$ , and defined as follows,

$$\omega \equiv \frac{\bar{e}b_f}{b_d + \bar{e}b_f}.$$

Using the expressions for exchange rate exposure in [Proposition 1.2.5](#), the following proposition yields the optimal share of foreign debt.

**Proposition 1.2.7** (Share of Foreign Debt). *The share of foreign debt for the exporting firm is,*

$$\omega_1 = (1 + \beta) \delta_1(\bar{e}) = (1 + \beta) \chi(\bar{e}) \mathbf{1}_{\{\bar{x}_1(\bar{e}) \leq \bar{x}_1\}}.$$

*The share of foreign debt for the local firm is,*

$$\omega_2 = (1 + \beta) \delta_2(\bar{e}) = (1 + \beta) (1 - \eta(\bar{e})) \mathbf{1}_{\{\bar{x}_1(\bar{e}) \leq \bar{x}_1\}}.$$

**Proof:** See [Appendix A](#).

The first result to notice is that shares of foreign debt for both firms can take values in a wide interval, i.e.  $\omega_1 \in [0, 1 + \beta] \subseteq [0, 2]$ , and  $\omega_2 \in [0, \frac{1+\beta}{2}] \subseteq [0, 1]$ . Therefore both firms can potentially take on large shares of foreign debt.

The share of foreign debt for the local firm increases with the substitutability of goods. If goods are more substitutable, the domestic consumer is more willing to substitute the exported good with the local good. This effect increases the sensitivity of profits to the exchange rate for the local firm. As its profits become more sensitive, the local firm takes on higher shares of foreign debt in order to hedge its profits against exchange rate movements.

The share of foreign debt for the local firm increases with the exporter's domestic market share. If the exporter has a higher market share, the high domestic supply of the exporter drives down the own reciprocal of price elasticity of good 1 (i.e. flatter price schedule  $p_1(x_1)$ ). The exporter's response to an exchange rate change will be higher in magnitude (i.e. more  $x_1$  are withdrawn for a given percentage price increase). As a result, the local firm will see a higher change in its price  $p_2$  and thus profit  $\pi_2$ . This effect increases the sensitivity of the local firm to the exchange rate.

## 1.3 Empirical Evidence

This section presents the panel dataset of Mexican firms that has been used. First I used simple regressions to understand the determinants of debt currency composition for the whole sample of firms. Then I estimate the sensitivity of sales and profits to the exchange rate for local firms. Finally I test specific implications of the model for local firms.

### 1.3.1 Description of the Data

The panel dataset has been constructed by gathering items from the balance sheets and the cash flow statements for a set of Mexican firms traded on the Mexican Stock Market (Bolsa Mexicana de Valores). The data items are recorded from the fourth quarter of 1999 to the second quarter of 2009. As it is customary, financial and insurance firms are excluded from the sample.

The dataset is composed of 99 firms, totaling 2,608 firm/quarter observations. It contains the following balance sheet items: book assets, book debt (including current liabilities and long term debt), decomposed into foreign (dollar-denominated) debt and domestic (peso-denominated) debt. The statements of cash flow contain quarterly figures for total sales, decomposed into foreign and domestic sales. These data items will allow the construction of the variables such as leverage (debt/assets), share of foreign debt, share of foreign sales, and asset turnover. The percentage of dollar debt will be the ratio of dollar denominated debt over total debt. The percentage of foreign sales is the ratio of foreign sales over total sales. Asset turnover is defined as the ratio of total sales over total assets. [Table 1.1](#) reports the summary statistics for these variables. Panel A reports the data in levels in millions of Mexican pesos, and panel B reports the variables relevant to the empirical analysis.

Firms are classified into different industries of operation. [Appendix B](#) lists all the firms traded on the BMV broken down by industry. The industries include the following: Materials, Manufacturing, Consumer Goods and Services, Consumer Durables, Health Care, and Telecommunication Services. Each of the 6 industries is broken down further into sectors, for a total of 15 sectors.

Panel A: Levels

Variable	Obs.	Mean	Std. Dev.	Min	Max
Assets	2,608	25,200	52,600	96.3	624,000
Sales	2,608	4,780	9,418	0	94,400
Domestic Sales	2,608	3,546	6,800	0	72,700
Foreign Sales	2,608	1,239	4,488	0	59,200
Debt	2,608	11,100	27,100	7.5	316,000
Domestic Debt	2,608	4,766	10,000	0	75,700
foreign Debt	2,608	6,354	19,500	0	276,000

Panel B: Ratios

Variable	Obs.	Mean	Std. Dev.	Min	Max
Share of Foreign Debt	2,608	0.37	0.32	0	1
Share of Foreign Sales	2,608	0.15	0.21	0	0.86
Debt/Assets	2,608	0.38	0.20	0.005	1.69
Total Sales/Assets	2,608	0.21	0.14	0	1.03
log(Assets)	2,608	15.83	1.63	11.47	20.25

Panel C: Local versus Exporting Firms

Firms	Local			Exporter		
Variable	Obs.	Mean	Std. Dev.	Obs.	Mean	Std. Dev.
Share of Foreign Debt	923	0.15	0.20	1,685	0.49	0.31
Share of Foreign Sales	923	0	0	1,685	0.24	0.22
Debt/Assets	923	0.30	0.18	1,685	0.43	0.20
Total Sales/Assets	923	0.23	0.17	1,685	0.20	0.11
log(Assets)	923	15.19	1.50	1,685	16.17	1.60

Table 1.1: **Summary Statistics.** This table reports the summary statistics for the main variables. Panel A reports the raw data in millions of Mexican pesos. Panel B reports ratios. Panel C reports variables for local and exporting firms. The data consists of 99 traded Mexican firms, for the years 1999 to 2009. Source: *Bolsa Mexicana de Valores*.

Figure 1.3 plots the share of foreign debt versus the share of foreign sales for each firm. As it has been documented empirically in the emerging markets literature, there is a positive correlation between the share of foreign debt and the share of foreign sales. This observation is consistent with the view that firms engage in hedging their currency exposure. In addition the graph shows that many non-exporters have sizable shares of foreign debt.

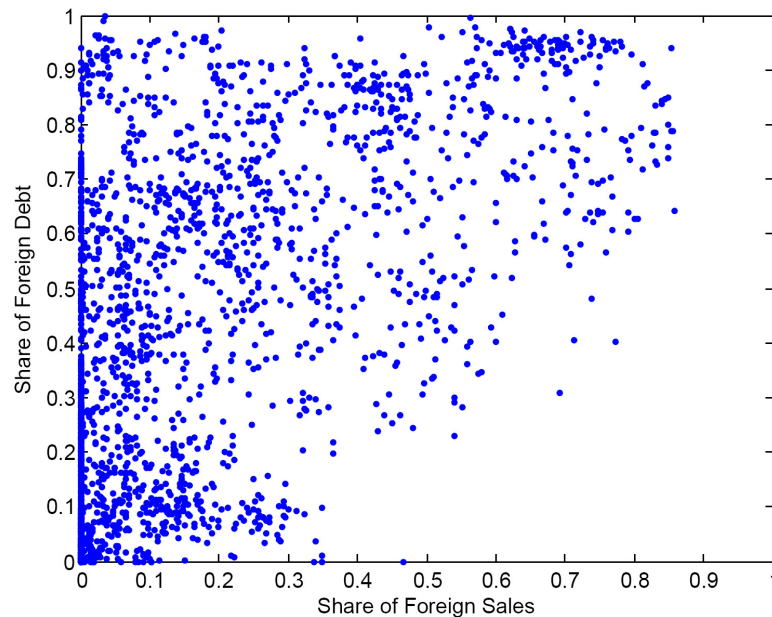


Figure 1.3: **Share of Foreign Debt versus the Share of Foreign Sales.** The data consists of Mexican firms traded on the Mexican Stock Market, for the years 1999 to 2009. Source: *Bolsa Mexicana de Valores*.

### 1.3.2 Currency Composition of Debt

A simple regression analysis is conducted in this subsection. I run regressions of firms' share of foreign debt on firms' characteristics. A natural regressor in this context is the share of foreign sales. In addition I use typical regressors in the empirical corporate finance literature, such as firm size, leverage, and asset turnover. A dummy variable that indicates whether firms have ADR listings in the NYSE is used to control for access to international markets. Industry dummies are used to control for the industry-specific effects. The regression results are summarized in Table 1.2.

Share of foreign Debt	(1)	(2)	(3)	(4)
Share of Foreign Sales	0.926*** (47.37)	0.789*** (39.23)	0.870*** (40.77)	0.805*** (37.30)
Debt/Assets		0.374*** (13.91)		0.336*** (13.26)
log(Assets)		0.012*** (4.27)		0.005* (1.75)
Sales/Assets		-0.521*** (-18.30)		-0.424*** (-11.67)
ADR Dummy		0.026** (2.05)		-0.021* (-1.65)
Constant	0.224*** (37.08)	0.018 (0.41)		
Industry Dummy	No	No	Yes	Yes
R-squared	0.384	0.494	0.495	0.546
N	2,608	2,608	2,608	2,608

Table 1.2: **Regression Results.** Level of significance: \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ , using robust standard errors.

The first main result from this regression analysis is that the share of foreign sales is statistically significant at the 1% level in all the regression specifications. This relationship is robust even when controlling for all firm characteristics, and industry and quarter dummies. This result confirms the broad result in the emerging markets empirical literature that exporting firms hedge the currency exposure that originates from their export activities. However the first regression (column 1) shows that the average share of foreign debt for firms with no exports is about 22%. This seems to be a puzzle as these firms have no direct exposure to the exchange rate.

Depending on the industry within which firms are operating, access to financing, sales, and cost of production may have different sensitivities to exchange rate fluctuations. These differences across industries will impact optimal debt composition in a systematic way. [Figure 1.4](#) plots the percentage of foreign debt versus the percentage foreign sales, for each industry. These plots indicate that the type of industry seems to play a part in determining the optimal debt composition. Firms in the materials and telecommunications industries tend to have more dollar debt than firms in the

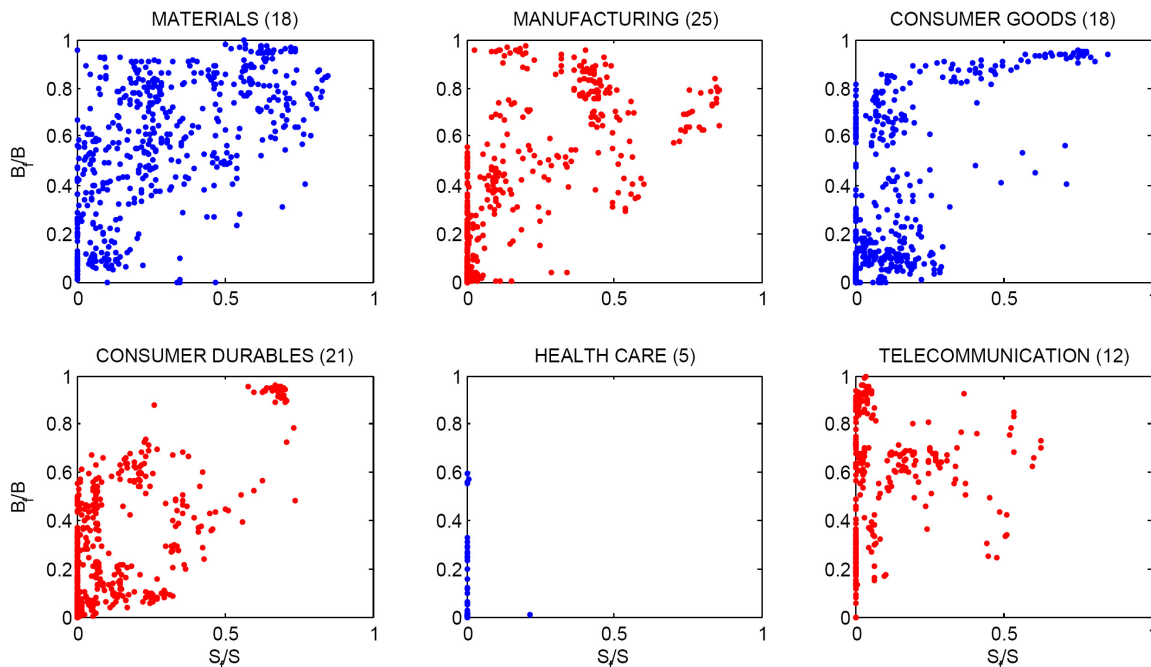


Figure 1.4: **Share of Foreign Debt versus the Share of Foreign Sales by Sector.** The data consists of Mexican firms traded on the Mexican Stock Market, for the years 1999 to 2009. Source: *Bolsa Mexicana de Valores*.

consumer durables and health care industries. The addition of industry dummies (i.e. columns 3 and 4 of Table 1.2) increases the explanatory power. This confirms the presence of the exchange rate sensitivity channel, acting at the industry level.

### 1.3.3 Estimation of the Exchange Rate Sensitivities

The time dimension of the panel permits the estimation of the foreign exchange rate exposure of firm earnings. The exchange rate sensitivity of sales and earnings can be estimated for each firm by running a regression of sales and earnings on the log of the exchange rate. Because accounting data are noisy, the exposure will not be estimated at the firm level, but rather firms will be pooled together. Firm fixed-effect regressions using only firms with no foreign sales are used to estimate the exposure of sales and earnings, defined as sales minus costs minus financing expenses, for local firms.



Sales and profits over a 10-year period have trends, thus instead of regressing the level of sales and earnings, their ratio over assets will be used as independent variables. The regression specification is given by,

$$Y_{it} = \alpha_i + \beta_e \log(e_t) + \beta_{GDP} \log(GDP_t) + \beta X_{it} + \varepsilon_{it}, \quad (1.7)$$

where the independent variable will be the ratio of sales over assets in the first regression (column 1 of [Table 1.3](#)) and the ratio of earnings over assets in the second regression (column 2 of [Table 1.3](#)). The aggregate states are included in the regressions: exchange rate,  $\log(e)$ , as well as the residuals of the Hodrick-Prescott filter of the logarithm of the Mexican GDP, denoted by  $\log(GDP)$ . In addition, firm characteristics  $X_{it}$  are used as control variables: leverage, size, and ADR dummy. In addition a quarter dummy is added to control for seasonality. [Table 1.3](#) reports the regression results for local firms. It can be seen that sales have a statistically significant positive sensitivity to the exchange rate, however earnings do not. These results are evidence that local firms are in fact hedging their exchange rate risk effectively, insulating their earnings from exchange rate fluctuations.

	(1)	(2)	(3)
	Sales/Assets	Operating Income/Assets	Earnings/Assets
$\log(e)$	0.067*** (6.34)	0.006* (1.65)	-0.009 (-1.13)
$\log(GDP)$	0.088*** (4.76)	0.036*** (5.15)	0.033** (2.32)
Debt/Assets	-0.006 (-0.62)	-0.021*** (-6.37)	-0.059*** (-10.04)
$\log(\text{Assets})$	-0.016*** (-5.92)	0.004*** (5.48)	0.006*** (6.50)
ADR Dummy	0.0008 (0.08)	-0.002 (-0.64)	-0.0007 (-0.18)
Firm Fixed Effect	Yes	Yes	Yes
Quarter Dummy	Yes	Yes	Yes
R-squared (within)	0.053	0.030	0.037
R-squared (between)	0.011	0.309	0.369
N	2,608	2,608	2,608

Table 1.3: **Regression Results.** These regressions use local firms only. Level of significance: \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ , using robust standard errors.

### 1.3.4 Testable Model Implications

The model argues that the presence of imperfect competition in the domestic goods market creates a link between exporting and local firms. I construct the Herfindahl index of each industry (and sector) and use it as a measure of market imperfection. The model also predicts that a higher market share will lead to a lower share of foreign debt for the local firm. I construct the market share of each local firm in order to test directly for this model implication.

I add both the market share and Herfindahl index variables to the regressions of share of foreign debt, using only local firms. The sign and significance of these variables are used to assess the presence and the magnitude of the effects predicted by the model. Using the panel dimension of the data, market share of firms and the Herfindahl index of each industry (and sector) can be constructed, for each quarter. The market share of firm  $i$  in industry  $j$ , at time  $t$  is defined as follows,

$$(\text{Market Share})_{it} = \frac{(\text{Domestic Sales})_{it}}{\sum_{i \in \text{industry}(j)} (\text{Domestic Sales})_{it}}, \quad (1.8)$$

where  $S_{d,it}$  is peso sales for firm  $i$  in the domestic market. The Herfindahl index for industry (or sector)  $j$  at time  $t$  is defined as follows,

$$(\text{Herfindahl Index})_{jt} = \sum_{i \in \text{industry}(j)} ((\text{Market Share})_{it})^2. \quad (1.9)$$

The model predicts that market imperfection will lead local firms to take on higher shares of foreign debt. Therefore, as market imperfection rises with the Herfindahl index, an increase in the Herfindahl index leads to an increase in the share of dollar debt for local firms. The estimated coefficient on the Herfindahl index should be positive. The model predicts that an increase in market share for the local firms will lead to a lower share of foreign debt, thus the estimated coefficient on the market share should be negative. [Table 1.4](#) reports the regression results for local firms. The sign of the estimated coefficients on the Herfindahl index and the market share are in line with what the model predicted. The statistical significance is stronger at the sector level, but still consistent

Share of foreign Debt	Predicted Sign	(1)	(2)	(3)
Debt/Assets		0.085 (1.38)	0.118 (1.49)	0.170* (1.76)
log(Assets)		0.006 (1.10)	0.023* (1.94)	0.027*** (3.43)
Sales/Assets		-0.407*** (-6.48)	-0.168* (-1.71)	-0.293*** (-4.20)
ADR Dummy		-0.019 (-0.44)	-0.010 (-0.20)	-0.010 (-0.22)
Constant		0.119 (1.27)	-0.399* (-1.81)	-0.249** (-2.00)
Market Share	(-)		-0.004 (-1.29)	-0.267*** (-3.35)
Herfindahl Index	(+)		0.377*** (3.69)	0.162* (1.68)
Level			Industry	Sector
N			6	15
R-squared		0.112	0.203	0.174
N		923	540	580

Table 1.4: **Regression Results.** These regressions use local firms only. Level of significance: \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ , using robust standard errors.

with the predictions at the industry level. These results are in accordance with the model.

## 1.4 Quantitative Model

This section presents an industry equilibrium of firms that make production and financing decisions. This model can be thought of as an extension of the stylized model presented in a previous section. The model is fully dynamic and has been extended to accommodate a continuum of firms. The heterogeneity between firms comes from an idiosyncratic cost shock similar in spirit to Melitz (2003), thus the decision to export is endogenous in this model. Financial frictions have been added, in the form of costly equity issuances and defaultable debt. Bankruptcy costs are incurred upon default and the interest expense on debt can be deducted from taxable income. These ingredients are standard in the corporate finance literature<sup>2</sup>. Such a framework can be used to quantify the effect of imperfect

<sup>2</sup>See Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2005), and Gomes and Schmid (2009)

competition in the domestic goods market on firms' share of foreign debt.

### 1.4.1 Preferences

Preferences are defined over a continuum of differentiated goods indexed by  $i \in \Omega$ . All domestic consumers share the same utility function given by,

$$U = \left( \int_{i \in \Omega} x_i^\rho di \right)^{\frac{1}{\rho}}, \quad (1.10)$$

where  $x_i$  represents the consumption level of each variety  $i$ . The parameter  $\rho \in [0, 1]$  governs the elasticity of substitution between each good. Let  $X$  denote the aggregate good,

$$X \equiv \left( \int_{i \in \Omega} x_i^\rho di \right)^{\frac{1}{\rho}}. \quad (1.11)$$

This consumer problem yields a set of domestic price schedules  $p_i$  (i.e. demand curves) for each variety  $i$ ,

$$p_i(x_i, X) = y x_i^{\rho-1} X^{-\rho}, \quad i \in \Omega, \quad (1.12)$$

where  $y$  is the total expenditure in the domestic market. Note that price of variety  $i$  is decreasing in the supply of both its own quantity and in the quantity of aggregate good supplied. The case of perfect substitutes is obtained by taking the limit of parameter  $\rho$  to one. The demand curve for the homogeneous good is simply  $p(X) = y X^{-1}$ .

Consumers in the foreign economy value good  $i$  according to an exogenous demand curve, denoted by  $p_i^*(x_i^*)$ . This economy is thus a small open economy exporting to a large open economy.

### 1.4.2 Firm Problem

The dynamic programming problem for firm  $i$  is to choose domestic debt  $\hat{b}'_{id}$ , foreign debt  $\hat{b}'_{if}$ , domestic and foreign sales  $x_i$  and  $x_i^*$ , and a default policy to maximize the present value of dividends. The exchange rate  $e$  and the idiosyncratic cost shock  $z_i$  are exogenous shocks.

Firms can choose the scale of the operation instantaneously but incur a convex cost of production. Firms can sell its production in the domestic market and in the foreign market. The sales are denoted

by  $x_i$  and  $x_i^*$ , respectively. Profits for firm  $i$  are given by the following program,

$$\pi_i(z_i, e) = \max_{x_i, x_i^*} p_i x_i + e p_i^* x_i^* - \frac{\eta_1}{\eta_2} \exp(z_i) (x_i + x_i^*)^{\eta_2} - f^* \mathbf{1}_{\{x_i^* > 0\}} - f,$$

where  $\eta_1 > 0$  and  $\eta_2 > 1$  are parameters of the cost function,  $f^*$  is a fixed cost paid if the firm exports,  $f$  is a fixed cost of operation, and  $p_i^*$  is the price (in foreign currency units) of the good sold in the foreign market.

Firms have access to international lenders and can borrow in the form of one-period debt contracts. Domestic debt is denominated in local currency units, while foreign debt is denominated in foreign currency units. The coupon rate for a newly issued domestic bond is denoted  $c'_{id}$ , while the coupon for the foreign bond is  $c'_{if}$ . Both bonds will be priced competitively by international investors assuming that markets are complete.

Profits are taxed at a rate  $\tau \in [0, 1]$  and interest expense can be deducted from taxable income. The dividend for the firm is defined as the sum of after-tax profits and new debt issues, net of debt repayment,

$$\tilde{d}_i = (1 - \tau)\pi_i(z_i, e) + \hat{b}'_{id} + e\hat{b}'_{if} - \hat{b}_{id}(1 + (1 - \tau)c'_{id}) - e\hat{b}_{if}(1 + (1 - \tau)c'_{if}),$$

Financing frictions are present in the form of costly equity issuances. Following Gomes (2001), equity can be issued at a proportional cost  $\lambda$ , that is dividend net of equity issuance is given by,

$$d_i = (1 + \lambda \mathbf{1}_{\{\tilde{d}_i < 0\}})\tilde{d}_i,$$

where  $\mathbf{1}_{\{\tilde{d}_i < 0\}}$  is the indicator function of strictly negative dividends, i.e.  $\mathbf{1}_{\{\tilde{d}_i < 0\}} = 1$  if  $\tilde{d}_i < 0$  (and 0 otherwise).

The value of the firm continuing operations is given by,

$$V_i(\hat{b}_{id}, \hat{b}_{if}, s) = \max_{\hat{b}'_{id}, \hat{b}'_{if}, x_i, x_i^*} d_i + \mathbb{E} \left[ m_d(s, s') \max \left( 0, V_i(\hat{b}'_{id}, \hat{b}'_{if}, s') \right) \right],$$

where  $m_d(s, s')$  is the stochastic discount factor that firms take as given. The vector  $s$  contains all the exogenous state variables in the economy. Specifically, the exchange rate  $e$  and the idiosyncratic

cost shock  $z_i$  are included in the exogenous state vector. The transition function for  $s$  will be described in detail in Section 1.4.4.

### 1.4.3 International Markets and Lenders

This model assumes the existence of complete markets. This assumption allows the consideration of only one pricing kernel, in this case the domestic one. In other words, the foreign pricing kernel is given by,

$$m_f(s, s') = m_d(s, s') \frac{e'}{e}.$$

There is a continuum of competitive lenders. The emerging economy is small in comparison to the international markets, thus the international lenders pricing kernel  $m_f(s, s')$  is taken as exogenous. Any debt contract has to satisfy the international lenders' Euler equation. Risky dollar debt  $\hat{b}'_{if}$  will be supplied to firm  $i$  at a coupon rate  $c'_{if}$  such that,

$$\hat{b}'_{if} = \mathbb{E} \left[ m_d(s, s') \frac{e'}{e} \left\{ (1 + c'_{if}) \hat{b}'_{if} \mathbf{1}' + \xi \frac{e'(1 + c'_{if}) \hat{b}'_{if}}{(1 + c'_{id}) \hat{b}'_{id} + e'(1 + c'_{if}) \hat{b}'_{if}} \frac{\pi'_i + f^* + f}{e'} (1 - \mathbf{1}') \right\} \right],$$

where  $\mathbf{1}'$  is the indicator function for the decision to continue operations, that is if  $V'_i \geq 0$ . The bankruptcy costs are captured by the parameter  $\xi \in [0, 1]$ , which represents the percentage of current profits that is recovered by debtholders after the firm is sold or reorganized. Similarly lenders will supply risky peso debt  $\hat{b}'_{id}$  to firm  $i$  at a coupon rate  $c'_{id}$  such that,

$$\hat{b}'_{id} = \mathbb{E} \left[ m_d(s, s') \left\{ (1 + c'_{id}) \hat{b}'_{id} \mathbf{1}' + \xi \frac{(1 + c'_{id}) \hat{b}'_{id}}{(1 + c'_{id}) \hat{b}'_{id} + e'(1 + c'_{if}) \hat{b}'_{if}} (\pi'_i + f^* + f) (1 - \mathbf{1}') \right\} \right].$$

Define  $b$  to be debt inclusive of interest payment and net of tax shield subsidy,

$$b_{id} \equiv (1 + (1 - \tau)c_{id}) \hat{b}_{id}, \quad b_{if} \equiv (1 + (1 - \tau)c_{if}) \hat{b}_{if}.$$

The pricing equations can be rewritten as follows,

$$\begin{aligned} \hat{b}'_{if} &= \frac{\mathbb{E} \left[ m_d(s, s') \frac{e'}{e} \left\{ \frac{1}{1-\tau} b'_{if} \mathbf{1}' + \xi \frac{e' b'_{if}}{b'_{id} + e' b'_{if}} \frac{\pi'_i + f^* + f}{e'} (1 - \mathbf{1}') \right\} \right]}{1 + \frac{\tau}{1-\tau} \mathbb{E} \left[ m_d(s, s') \frac{e'}{e} \mathbf{1}' \right]}, \\ \hat{b}'_{id} &= \frac{\mathbb{E} \left[ m_d(s, s') \left\{ \frac{1}{1-\tau} b'_{id} \mathbf{1}' + \xi \frac{b'_{id}}{b'_{id} + e' b'_{if}} (\pi'_i + f^* + f) (1 - \mathbf{1}') \right\} \right]}{1 + \frac{\tau}{1-\tau} \mathbb{E} \left[ m_d(s, s') \mathbf{1}' \right]}. \end{aligned}$$

#### 1.4.4 Dynamics of the Shocks

The exchange rate process is the only source of uncertainty at the aggregate level. The exchange rate is exogenous from the point of view of the firms. This paper does not take a stand on how it is determined. All firms face idiosyncratic shocks to their cost, denoted by  $z_i$ .

In addition to these shocks, the exchange rate from the previous period is also required. Thus the vector of exogenous state, denoted by  $s$ , is defined as  $s \equiv (z_i, e, e_{-1})$ .

#### 1.4.5 Recursive Formulation of the Firm Problem

For computational reasons, it is convenient to rewrite the problem in terms of the firm's total debt  $b$  and its share of foreign debt  $\omega$ , instead of the domestic debt  $b_d$  and the foreign debt  $b_f$ . Define,

$$\omega' \equiv \frac{eb'_f}{b'_d + eb'_f} \quad \text{and} \quad b' \equiv b'_d + eb'_f.$$

Dividend can be rewritten as follows,

$$d_i = (1 + \lambda \mathbf{1}_{\{d_i < 0\}}) \left\{ (1 - \tau) \pi_i(z_i, e) + q'_i b'_i - \left( 1 - \omega_i + \frac{e}{e_{-1}} \omega_i \right) b_i \right\}.$$

where the discount price of total debt  $q'_i$  is given in equation (1.18).

Similarly, the coupon rates can be written in terms of total debt and foreign share,

$$c'_{if} = \frac{1}{1 - \tau} \left( \frac{1 + \frac{\tau}{1 - \tau} \mathbb{E} \left[ m_d(s, s') \frac{e'}{e} \mathbf{1}' \right]}{\mathbb{E} \left[ m_d(s, s') \frac{e'}{e} \left\{ \frac{1}{1 - \tau} \mathbf{1}' + \xi \frac{\frac{e'}{e}}{1 - \omega'_i + \frac{e'}{e} \omega'_i} \frac{\pi'_i + f^* + f}{b'_i} (1 - \mathbf{1}') \right\} \right]} - 1 \right), \quad (1.13)$$

$$c'_{id} = \frac{1}{1 - \tau} \left( \frac{1 + \frac{\tau}{1 - \tau} \mathbb{E} [m_d(s, s') \mathbf{1}']}{\mathbb{E} \left[ m_d(s, s') \left\{ \frac{1}{1 - \tau} \mathbf{1}' + \xi \frac{1}{1 - \omega'_i + \frac{e'}{e} \omega'_i} \frac{\pi'_i + f^* + f}{b'_i} (1 - \mathbf{1}') \right\} \right]} - 1 \right). \quad (1.14)$$

The firm's problem can now be stated in terms of the total debt and foreign share.

**Problem 1.4.1** (Recursive Formulation of Firm  $i$ 's Problem). *Given the price schedules  $p_i(x_i, X)$ ,*

*$c'_{if}(b'_i, \omega'_i, s)$ , and  $c'_{id}(b'_i, \omega'_i, s)$ , firm  $i$  solves the following program,*

$$V_i(b_i, \omega_i, s) = \max_{b'_i, \omega'_i} d_i + \mathbb{E} [m_d(s, s') \max(0, V_i(b'_i, \omega'_i, s'))], \quad (1.15)$$

subject to,

$$d_i = (1 + \lambda \mathbf{1}_{\{d_i < 0\}}) \left\{ (1 - \tau) \pi_i(z_i, e) + q'_i b'_i - \left( 1 - \omega_i + \frac{e}{e-1} \omega_i \right) b_i \right\}, \quad (1.16)$$

$$\pi_i(z_i, e) = \max_{x_i, x_i^*} p_i x_i + e p_i^* x_i^* - \frac{\eta_1}{\eta_2} \exp(z_i) (x_i + x_i^*)^{\eta_2} - f^* \mathbf{1}_{\{x_i^* > 0\}} - f, \quad (1.17)$$

where the discount price of total debt issued is,

$$q'_i = \frac{1 - \omega'_i}{1 + (1 - \tau) c'_{id}} + \frac{\omega'_i}{1 + (1 - \tau) c'_{if}}. \quad (1.18)$$

### 1.4.6 Competitive Equilibrium

Now that the problems for the representative consumer, the international lenders, and firms have been defined, the competitive equilibrium can be formally stated.

**Definition 1.4.2** (Recursive Competitive Equilibrium). *A recursive competitive equilibrium consists of pricing functions  $p_i(x_i, X)$ ,  $c'_{id}(b'_i, \omega'_i, s)$ , and  $c'_{if}(b'_i, \omega'_i, s)$ , value functions  $V_i(b_i, \omega_i, s)$ , and optimal decision rules  $g_{i,b'}$ ( $b_i, \omega_i, s$ ),  $g_{i,\omega'}$ ( $b_i, \omega_i, s$ ),  $g_{i,x}$ ( $s$ ), and  $g_{i,x^*}$ ( $s$ ), such that:*

1. **Consumer's Optimization:** Price functions  $p_i(x_i, X)$  satisfy the following conditions:

$$p_i(x_i, X) = y x_i^{\rho-1} X^{-\rho}, \quad i \in \Omega,$$

where

$$X = \left( \int_{i \in \Omega} x_i^\rho di \right)^{\frac{1}{\rho}}.$$

2. **International Lenders:** Coupon schedules  $c'_{id}(b'_i, \omega'_i, s)$ , and  $c'_{if}(b'_i, \omega'_i, s)$  satisfy the following Euler equations (1.13) and (1.14).
3. **Firms' Optimization:** Value functions  $V_i(b, \omega, s)$  solve the Bellman equation (1.15), subject to constraints (1.16), (1.17), and (1.18). The associated optimal decision rules for firm  $i$  are denoted by:  $b'_i = g_{i,b'}$ ( $b, \omega, s$ ),  $\omega'_i = g_{i,\omega'}$ ( $b, \omega, s$ ),  $x_i = g_{i,x}$ ( $s$ ), and  $x_i^* = g_{i,x^*}$ ( $s$ ).



4. **Aggregate Consistency:** The aggregate good  $X(e)$  is consistent with the actual law of motion implied by the optimal decision rules  $g_{i,x}(s)$  and the transition matrix for  $s$ ,

$$X(e) = \left( \int_{z_i} g_{i,x}(z_i, e)^{\rho} dG(z_i) \right)^{\frac{1}{\rho}},$$

where  $G$  is the unconditional distribution of idiosyncratic shocks  $z_i$ .

There are no closed-form expressions for this equilibrium. However numerical techniques can be employed to solve it. The next section will present numerical solutions for this economy.

## 1.5 Numerical Results

First I present the calibration strategy for the exogenous shocks in the model: the exchange rate process and the idiosyncratic shock process. Then I choose a reasonable set of model parameters based on various studies in the corporate finance literature. Finally I present some simulation results of the model.

### 1.5.1 Calibration

#### Exchange Rate Process

The real exchange rate series is constructed from the nominal exchange rate and the inflation data. The details are shown in [Appendix C](#). The statistical properties of the real exchange rate are summarized in Panel A of [Table 1.5](#). For simplicity, the real exchange rate process is assumed to follow an AR(1) process in the logarithm,

$$\log(e_{t+1}) = c + \rho_e \log(e_t) + \sigma_e \epsilon_t. \tag{1.19}$$

The estimated parameters are shown in the panel B of [Table 1.5](#). The process [\(1.19\)](#) is transformed into a discrete Markov chain using the Tauchen and Hussey (1991) procedure, using 11 points.

Panel A: Statistics

	$\mu(e_t)$	$\sigma(e_t)$	$\rho(e_t, e_{t-1})$
Estimates (1994-2009)	3.4158	0.7412	0.9017
Estimates (1999-2009)	3.0432	0.2305	0.7038

Panel B: AR(1) Estimates

	$c$	$\sigma_e$	$\rho_e$
Estimates (1994-2009)	0.3359	0.3205	0.9017
Estimates (1999-2009)	0.9015	0.1638	0.7038

Table 1.5: **Statistics and AR(1) Estimates of the Real Exchange Rate Process.** Panel A reports the statistics for the real exchange rate process. Panel B reports the parameter estimates when the real exchange rate is assumed to follow an AR(1) process, given by  $\log(e_{t+1}) = c + \rho_e \log(e_t) + \sigma_e \epsilon_t$ . The results are shown for two different time periods. The data frequency is quarterly.

### Cost Parameters

The variables in the model are mapped to data according to [Table 1.6](#). The various cost parameters:  $\eta_1$ ,  $\eta_2$ ,  $f$ , and  $f^*$  and the preference parameters:  $y$  and  $p_i^*$  are chosen such that the first moments of the simulations match their data counterpart, given in [Table 1.7](#). The deterministic steady state of this economy is used for the purpose of this calibration exercise. The exchange rate and the cost shocks are set to their steady state value, that is  $\bar{e} = \bar{z} = 0$ . The steady state production and sales policies are solved as a function of the model parameters. These expressions are used to match 3 first moments of the data: (Domestic Sales/Assets), (Foreign Sales/Assets), and (Cost/Assets). The details of the algebra can be found in [Appendix D](#). The parameter values for the cost parameters are reported in [Table 1.8](#).

The cost process  $z_i$  is parameterized along the lines of Gomes (2001) and Hennessy and Whited (2005). I assume the cost process to be a random walk and have a high volatility  $\sigma_e = 0.3$ . These numbers are standard in the literature for this type of exercise. The shock process is represented by a discrete Markov chain using 11 points. The parameters are reported in [Table 1.8](#).

Data	Model
Real Side:	
Total Assets	$x_i + x_i^*$
Total Sales	$p_i x_i + e p_i^* x_i^*$
Domestic Sales	$p_i x_i$
Foreign Sales	$e p_i^* x_i^*$
Cost	$\frac{\eta_1}{\eta_2} \exp(z_i)(x_i + x_i^*)^{\eta_2} + f^* + f$
Profits	$\pi_i = p_i x_i + e p_i^* x_i^* - (\frac{\eta_1}{\eta_2} \exp(z_i)(x_i + x_i^*)^{\eta_2} + f^* + f)$
Profit Margin	$\pi_i / (p_i x_i + e p_i^* x_i^*)$
Market Share	$p_i x_i / \int_i p_i x_i dG(z_i)$
Financing Side:	
Total Debt	$b'_i$
Domestic Debt	$(1 - \omega'_i) b'_i$
Foreign Debt	$\omega'_i b'_i$
Corporate Spread	$(q'_i)^{-1} - \beta^{-1}$

Table 1.6: Mapping of Variables from the Model to Data.

Data	Mean	Std. Dev.
Total Sales/Assets	0.212	0.136
Domestic Sales/Assets	0.183	0.141
Foreign Sales/Assets	0.029	0.046
Cost/Assets	0.191	0.132
Profits/Assets	0.022	0.023
Profit Margin	0.105	0.388
Market Share	0.119	0.217

Table 1.7: Target Moments for Calibration.

## Institutional Parameters

Cost of equity issuances are chosen to be 15%. This figure seems reasonable as Hennessy and Whited (2008) estimated these issuance costs to be about 6% using the simulated method of moments with US data. It is certainly plausible to believe that equity markets in Mexico have more informational asymmetry than in the US. The average tax rate is chosen to be 25%. Bankruptcy costs are assumed to correspond to a dead-weight cost of 20% of current production, thus the recovery rate is set to  $\xi = 0.8$ . Lastly the pricing kernel used to discount firms' cashflows and price the debt is chosen to be equal to the subjective discount rate  $\beta$ , which is chosen to correspond to a 3% yearly risk-free interest rate. The set of parameters used to solve the model are summarized in [Table 1.8](#).

Preference	$\beta = 0.992$	Risk free rate
	$\rho \in \{0.25, 0.5, 0.75\}$	Substitutability of goods
	$y = 3.6$	Consumer expenditure
	$p_i^* = 0.145$	Foreign prices
Technology	$\eta_1 = 0.006$	Production cost parameter
	$\eta_2 = 2$	Production cost parameter
	$f = 2.1$	Fixed cost of operation
	$f^* = 0.6$	Fixed cost of exporting
Institution	$\lambda = 0.15$	Linear cost of issuing equity
	$\xi = 0.8$	Recovery rate in event of bankruptcy
	$\tau = 0.25$	Average corporate tax rate
Exchange Rate	$\bar{e} = 1$	Long-run mean for $e$
	$\rho_e = 0.7$	Autocorrelation for $e$
	$\sigma_e = 0.15$	Volatility for $e$
Idiosyncratic Cost	$\bar{z} = 1$	Long-run mean of $z_i$
	$\rho_z = 0$	Autocorrelation of $z_i$
	$\sigma_z = 0.3$	Volatility of $z_i$

Table 1.8: **Parameter Values.**

### 1.5.2 Production and Sales Policy

Cost shocks are the only idiosyncratic shocks in these numerical examples. The foreign prices  $p_i^*$  are constant. [Figure 1.5](#) shows profits  $\pi_i(z_i, e)$ , sensitivity of profits  $d\pi_i(z_i, e)/de$ , domestic  $x_i(z_i, e)$  and foreign  $x_i^*(z_i, e)$  sales as a function of the exchange rate  $e$  and idiosyncratic cost shock  $z_i$ , for

different values of the elasticity of substitution. These graphs will give some intuition about the basic margins in the equilibrium sales decisions. Total production and exports are decreasing in the cost. This result is fairly intuitive. As the firm becomes more productive, i.e. cost decreases, it can produce more units. As more units are being sold, domestic prices decrease up to the point where the foreign price becomes more attractive. Total production and exports are increasing in the exchange rate. This result follows a similar intuition. As the exchange rate appreciates, foreign prices (in peso) increase and thus exporting becomes more profitable. This will drive firms to produce more and export more. As expected, profits follow the monotonicity properties of total production, i.e. decreasing in the cost and increasing in the exchange rate. Because profits are convex in both arguments, it follows that the sensitivity of profits to  $e$  is also decreasing in the cost and increasing in the exchange rate. Thus the most productive firms have the highest sensitivity of profits to the exchange rate. This property will be crucial for the optimal capital structure.

Pass-through and exposure are two statistics central to the financing decisions of firms. [Figure 1.6](#) shows the pass-through  $\eta(z_i, e)$  and exposure  $\delta(z_i, e)$  as a function of exchange rate  $e$  and cost shock  $z_i$ . Both pass-through and exposure are decreasing in the cost and increasing in the exchange rate. However the interesting feature is that firms can have no exports but strictly positive exposure. An increase in the elasticity of substitution leads to an increase in both pass-through and exposure for less productive firms, but a decrease in the pass-through for the more productive firms. This result confirms the basic intuition from the stylized model. A higher elasticity of substitution will link firms more strongly and thus firms with no foreign sales will be exposed to the exchange rate fluctuations.

### 1.5.3 Financing Policy

The cost and preference parameters have been selected so as to match data moments on the real side. The institution parameters have been chosen following previous work in the corporate finance

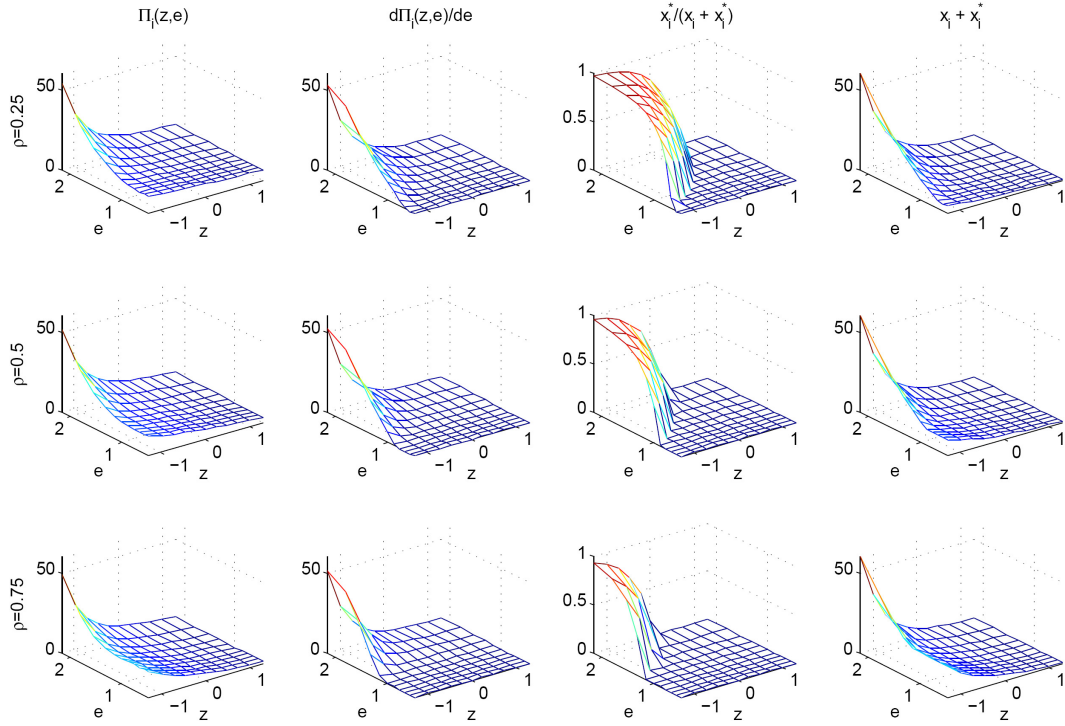


Figure 1.5: **Profits, Sensitivity of Profits, and Sales Policies.** The left figures plot profits  $\pi_i$  and the sensitivity of profits to the exchange rate  $d\pi_i/de$  as a function of the exchange rate  $e$  and cost shock  $z_i$ . The right figures plot the optimal sales policies: percentage of exports  $x_i^*/(x_i + x_i^*)$  and total production  $(x_i + x_i^*)$  as a function of the exchange rate  $e$  and cost shock  $z_i$ . Each row corresponds to a different values of the elasticity of substitution,  $\rho \in \{0.25, 0.5, 0.75\}$ .

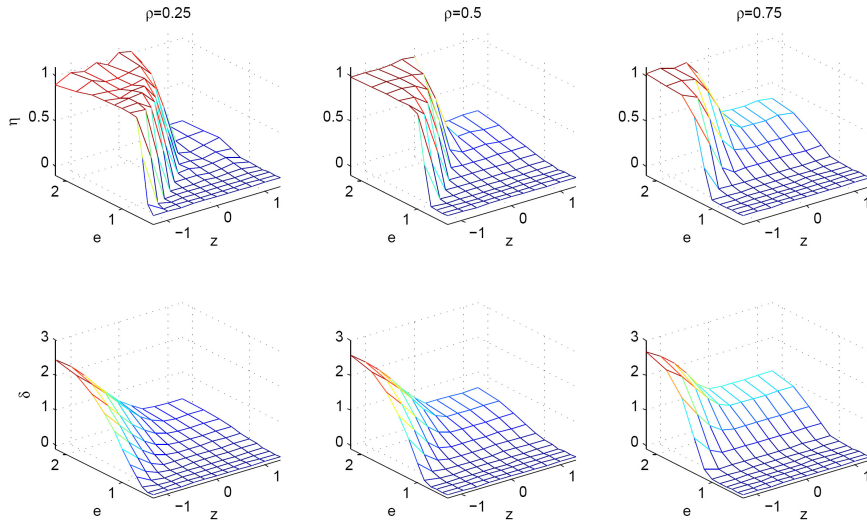


Figure 1.6: **Pass-through and Exposure.** The figure shows the pass-through  $\eta$  (top row) and exposure  $\delta$  (bottom row) as a function of the exchange rate  $e$  and cost shock  $z_i$ . Each column corresponds to a different values of the elasticity of substitution,  $\rho \in \{0.25, 0.5, 0.75\}$ .

literature. The parameters of the idiosyncratic cost process have been chosen such that the trade-off between tax benefits and expected cost of default implies an average leverage in the simulations that matches the mean in the data for (Debt/Assets). The model has not been calibrated to match the quantity of dollar debt observed in the data. Therefore the share of dollar debt that comes out of the model can be thought as an over-identifying restriction for the model.

The economy is simulated for a panel of 100 firms over 200 periods. The first 100 periods in the simulated sample is dropped in order to mitigate the effect of the initial conditions. Summary statistics of the firms' optimal policies along with the data counterpart are given in [Table 1.9](#). As can be seen, most of the calibrated moments are reasonably close the data moments. The share of the foreign debt in the model is close to 40%, whereas it is about 37% in the data. This quantitative exercise shows that the model can in fact rationalize the aggregate quantities for firms, both on the real side and the financing side.

Firms	All			Local		
Variable	Data	Model $\rho = 0.5$		Data	Model $\rho = 0.5$	
Real Side:						
Total Sales/Assets	$\mu$ 21.2%	22.4%		$\mu$ 22.8%	17.3%	
	$\sigma$ 13.6%			$\sigma$ 16.9%		
Domestic Sales/Assets	$\mu$ 18.3%	19%		$\mu$ 22.8%	17.3%	
	$\sigma$ 14.1%			$\sigma$ 16.9%		
Foreign Sales/Assets	$\mu$ 3%	3.4%		$\mu$ 0%	0%	
	$\sigma$ 4.6%			$\sigma$ 0%		
Profits/Assets	$\mu$ 2.2%	1.8%		$\mu$ 2.2%	0.9%	
	$\sigma$ 2.3%			$\sigma$ 2.3%		
Profit Margin	$\mu$ 10.5%	8%		$\mu$ 9.8%	3.9%	
	$\sigma$ 38.8%			$\sigma$ 62.1%		
Financing Side:						
Debt/Assets	$\mu$ 38.3%	55.5%		$\mu$ 29.9%	64%	
	$\sigma$ 20%			$\sigma$ 17.9%		
Foreign Debt/Assets	$\mu$ 16.3%	24.8%		$\mu$ 4.6%	26.3%	
	$\sigma$ 18.1%			$\sigma$ 7.8%		
Share of Foreign Debt	$\mu$ 36.7%	44.3%		$\mu$ 14.9%	37.4%	
	$\sigma$ 31.9%			$\sigma$ 20.3%		

Table 1.9: **Summary Statistics of Monte carlo Simulations.** The economy is simulated for a panel of 100 firms over 200 periods (first 100 periods in the sample are dropped).

## 1.6 Conclusion

In this paper I propose a model of production and financing decisions for firms in an open economy. I show that exchange rate fluctuations can potentially affect not only the profits of firms in the tradable sector, but also the profits of firms in the non-tradable sector as well. This effect is due to the imperfect competition in the domestic goods market. In addition, I also derive the model implications for the optimal denomination of corporate debt. This framework reconciles the puzzling fact that firms in emerging markets with no exports take on large shares of their debt in dollars.

First I develop a stylized model that highlights a channel by which the export decision of the exporting firm impacts the prices of all domestic goods. An implication of this channel is that local firms can potentially have a high exchange rate exposure. The motive to hedge these high currency exposures leads firms to use large amounts of dollar debt. Although it may seem that firms are taking on excessive dollar debt, they are in fact hedging the exchange rate sensitivity of their profits. The model predicts that the level of exchange rate fluctuations that is passed on to the local firm is increasing in the substitutability of the goods in the economy, and decreasing in the market share of the local firm.

Using a panel of traded Mexican firms for the last ten years, I show that the sales of local firms are sensitive to the exchange rate, but that their earnings, defined as sales minus costs minus financing expenses, are not. This is evidence that local firms are effectively using financing to hedge the sensitivity of their sales. I also find support for the comparative statics of the model. Local firms (i) in more concentrated goods markets and (ii) with smaller market shares hold higher shares of dollar debt.

In the last section I develop a quantitative model that is used to study the effect of the goods market imperfection on the amount of foreign debt issued by local firms. The stylized model is augmented to include a continuum of firms -different cost shocks- and imperfect financial markets, and is made fully dynamic. Firms can issue defaultable debt and costly equity. Optimal leverage



is a result of the tradeoff between tax shield and expected distress costs, and the optimal share of foreign debt arises from the motive to hedge profits. The model parameters are chosen to match moments of the data. The calibrated model accounts for the average amount of dollar debt observed in the Mexican firms data.

## 1.7 Appendix

### Appendix A – Proofs

#### Proof of Lemma 1.2.1:

Using the fact that both firms produce at capacity, the optimization problem for the exporting firm is simply,

$$\pi_1(e) = \max_{x_1} p_1(x_1, \bar{x}_2)x_1 + ep_1^*(\bar{x}_1 - x_1).$$

The first order condition is,

$$\frac{\partial \pi_1}{\partial x_1} = p_1(x_1, \bar{x}_2) \left( 1 + \frac{x_1}{p_1} \frac{\partial p_1}{\partial x_1}(x_1, \bar{x}_2) \right) - ep_1^* = 0.$$

Recall from matrix (1.6) that the own elasticity for good 1 is  $p_{1,1} = \rho(1 - \lambda) - 1$ , thus the first order condition is,

$$\rho(1 - \lambda(x_1, \bar{x}_2))p_1(x_1, \bar{x}_2) - ep_1^* = 0.$$

The equilibrium condition can be restated as  $f(x_1, e) = 0$  where the function  $f$  is defined as,

$$f(x_1, e) = \rho(1 - \lambda(x_1, \bar{x}_2))p_1(x_1, \bar{x}_2) - ep_1^*.$$

Let  $\tilde{x}_1(e)$  be the solution of  $f(x_1, e) = 0$ . The following formulae will be useful for the proof,

$$\log(1 - \lambda(x_1, \bar{x}_2)) = \log(1 - \alpha) + \rho \log \bar{x}_2 - \log(\alpha x_1^\rho + (1 - \alpha)\bar{x}_2^\rho),$$

$$\log(p_1(x_1, \bar{x}_2)) = \log \alpha + (\rho - 1) \log x_1 + \log y - \log(\alpha x_1^\rho + (1 - \alpha)x_2^\rho).$$

Note that,

$$\frac{\partial \log(\alpha x_1^\rho + (1 - \alpha)x_2^\rho)}{\partial \log e} = \frac{\frac{\partial \log x_1}{\partial \log e} \alpha \rho x_1^{\rho-1}}{\alpha x_1^\rho + (1 - \alpha)x_2^\rho} = \rho \lambda \frac{\partial \log x_1}{\partial \log e}.$$

Using the definition of  $f$  yields the following condition,

$$\rho(1 - \lambda(\tilde{x}_1, \bar{x}_2))p_1(\tilde{x}_1, \bar{x}_2) = ep_1^*.$$

Taking the logarithm on both sides yields,

$$\log\left(\frac{\rho(1-\alpha)\alpha\bar{x}_2^\rho y}{p_1^*}\right) + (\rho-1)\log\tilde{x}_1 - 2\log(\alpha\tilde{x}_1^\rho + (1-\alpha)\bar{x}_2^\rho) = \log e.$$

Differentiating both sides with respect to  $\log(e)$  yields,

$$\frac{\partial \log \tilde{x}_1}{\partial \log e} (\rho - 1 - 2\rho\lambda) = 1.$$

This implies that  $\tilde{x}_1$  is decreasing in  $e$ , as  $\lambda \in (0, 1)$  and  $\rho \in [0, 1]$ ,

$$\frac{\partial \log \tilde{x}_1}{\partial \log e} = -\frac{1}{1 + \rho(2\lambda - 1)} \leq 0.$$

**Proof of Proposition 1.2.2:**

Neither firms incur costs of production, so firms produce at capacity. This result yields the optimal sales policy for firm 2,

$$x_2 = \bar{x}_2.$$

As shown in Lemma 1,  $\tilde{x}_1$  is the level of domestic sales that equates the marginal benefits between the domestic and foreign markets. Thus if the exporting firm has not reached capacity, the optimal policy will be to sell  $\tilde{x}_1$  in the domestic market and sell the rest of the production in the foreign market. Formally,

$$\begin{aligned} x_1(e) &= \tilde{x}_1(e), \\ x_1^*(e) &= \bar{x}_1 - \tilde{x}_1(e). \end{aligned}$$

On the other hand, if  $\tilde{x}_1$  is greater than the production capacity  $\bar{x}_1$ , the exporting firm will sell all of its production in the domestic market, as the marginal benefit of selling in the domestic market is higher than in the foreign market, i.e.,

$$\begin{aligned} x_1(e) &= \bar{x}_1, \\ x_1^*(e) &= 0. \end{aligned}$$

**Proof of Proposition 1.2.3:**

Prices in the domestic market are obtained directly by plugging the optimal policies  $x_1(e)$  and  $x_2$  from Lemma 1.2.1 into equations (1.3) and (1.4),

$$p_1(e) = \begin{cases} \frac{\alpha \tilde{x}_1(e)^{(\rho-1)} y}{\alpha \tilde{x}_1(e)^\rho + (1-\alpha) \bar{x}_2^\rho} & \text{if } \tilde{x}_1 \leq \bar{x}_1, \\ \frac{\alpha \bar{x}_1^{(\rho-1)} y}{\alpha \bar{x}_1^\rho + (1-\alpha) \bar{x}_2^\rho} & \text{if } \tilde{x}_1 > \bar{x}_1, \end{cases}$$

$$p_2(e) = \begin{cases} \frac{(1-\alpha) \bar{x}_2^{(\rho-1)} y}{\alpha \tilde{x}_1(e)^\rho + (1-\alpha) \bar{x}_2^\rho}, & \text{if } \tilde{x}_1 \leq \bar{x}_1, \\ \frac{(1-\alpha) \bar{x}_2^{(\rho-1)} y}{\alpha \bar{x}_1^\rho + (1-\alpha) \bar{x}_2^\rho}, & \text{if } \tilde{x}_1 > \bar{x}_1. \end{cases}$$

The logarithm of prices, for  $\tilde{x}_1 \leq \bar{x}_1$ , are,

$$\log p_1(e) = \log \alpha + (\rho - 1) \log \tilde{x}_1(e) + \log y - \log(\alpha \tilde{x}_1(e)^\rho + (1 - \alpha) \bar{x}_2^\rho),$$

$$\log p_2(e) = \log(1 - \alpha) + (\rho - 1) \log \bar{x}_2 + \log y - \log(\alpha \tilde{x}_1(e)^\rho + (1 - \alpha) \bar{x}_2^\rho),$$

and are constants if  $\tilde{x}_1 > \bar{x}_1$ .

Recall from the proof of Lemma 1 that,

$$\frac{\partial \log(\alpha x_1^\rho + (1 - \alpha) x_2^\rho)}{\partial \log e} = \rho \lambda \frac{\partial \log x_1}{\partial \log e}.$$

The elasticity of prices, for  $\tilde{x}_1 \leq \bar{x}_1$ , are given by,

$$\frac{\partial \log p_1}{\partial \log e}(e) = -(1 - \rho(1 - \lambda)) \frac{\partial \log x_1}{\partial \log e},$$

$$\frac{\partial \log p_2}{\partial \log e}(e) = -\rho \lambda \frac{\partial \log x_1}{\partial \log e}.$$

Using the expression for the elasticity of  $\tilde{x}_1(e)$  from Lemma 1, pass-throughs for both firms are given by,

$$\eta_1(e) \equiv \frac{\partial \log p_1}{\partial \log e}(e) = \frac{1 - \rho(1 - \lambda)}{1 + \rho(2\lambda - 1)},$$

$$\eta_2(e) \equiv \frac{\partial \log p_2}{\partial \log e}(e) = \frac{\rho \lambda}{1 + \rho(2\lambda - 1)}.$$

Alternatively,  $\eta_1 = \eta$  and  $\eta_2 = (1 - \eta)$ , with  $\eta \equiv \frac{1 - \rho(1 - \lambda)}{1 + \rho(2\lambda - 1)}$ . For the case where  $\tilde{x}_1 > \bar{x}_1$ , the logarithm of prices are constants and thus elasticities are zero.

**Proof of Proposition 1.2.4:**

The logarithm of pass-throughs for both firms are given by,

$$\log \eta_1(e) = \log(1 - \rho(1 - \lambda)) - \log(1 + \rho(2\lambda - 1)),$$

$$\log \eta_2(e) = \log(\rho) + \log(\lambda) - \log(1 + \rho(2\lambda - 1)).$$

Note that,

$$\begin{aligned} \frac{\partial \log(1 - \rho(1 - \lambda))}{\partial \log \lambda} &= \frac{\rho\lambda}{1 - \rho(1 - \lambda)}, \\ \frac{\partial \log(1 - \rho(1 - \lambda))}{\partial \log \rho} &= \frac{-\rho(1 - \lambda)}{1 - \rho(1 - \lambda)}, \end{aligned}$$

and that,

$$\begin{aligned} \frac{\partial \log(1 + \rho(2\lambda - 1))}{\partial \log \lambda} &= \frac{2\rho\lambda}{1 + \rho(2\lambda - 1)}, \\ \frac{\partial \log(1 + \rho(2\lambda - 1))}{\partial \log \rho} &= \frac{\rho(2\lambda - 1)}{1 + \rho(2\lambda - 1)}. \end{aligned}$$

The comparative statics for the pass-through of the local firm are given by,

$$\begin{aligned} \frac{\partial \log \eta_1}{\partial \log \lambda}(e) &= \frac{-\rho\lambda(1 - \rho)}{(1 - \rho(1 - \lambda))(1 + \rho(2\lambda - 1))} \leq 0, \\ \frac{\partial \log \eta_1}{\partial \log \rho}(e) &= \frac{-\rho\lambda}{(1 - \rho(1 - \lambda))(1 + \rho(2\lambda - 1))} \leq 0. \end{aligned}$$

The comparative statics for the pass-through of the exporting firm are given by,

$$\begin{aligned} \frac{\partial \log \eta_2}{\partial \log \lambda}(e) &= \frac{1 - \rho}{1 + \rho(2\lambda - 1)} \geq 0, \\ \frac{\partial \log \eta_2}{\partial \log \rho}(e) &= \frac{1}{1 + \rho(2\lambda - 1)} \geq 0. \end{aligned}$$

**Proof of Proposition 1.2.5:**

Profits are obtained directly by plugging the optimal policies  $x_1(e)$  and  $x_2$  from Lemma 1.2.1 and prices from Proposition 1.2.3 into the definition of profits,

$$\begin{aligned}\pi_1(e) &= \begin{cases} p_1 \tilde{x}_1 + ep_1^*(\bar{x}_1 - \tilde{x}_1), & \text{if } \tilde{x}_1 \leq \bar{x}_1, \\ p_1 \bar{x}_1, & \text{if } \tilde{x}_1 > \bar{x}_1, \end{cases} \\ \pi_2(e) &= p_2 \bar{x}_2,\end{aligned}$$

The logarithm of profits for the exporting firm, for  $\tilde{x}_1 \leq \bar{x}_1$ , is given by,

$$\log \pi_1(e) = \log (p_1 \tilde{x}_1 + ep_1^*(\bar{x}_1 - \tilde{x}_1)).$$

The elasticity of profits for the exporting firm, for  $\tilde{x}_1 \leq \bar{x}_1$ , is given by,

$$\frac{\partial \log \pi_1}{\partial \log e}(e) = \frac{p_1 \tilde{x}_1 \left( \frac{\partial \log \tilde{x}_1}{\partial \log e} + \frac{\partial \log p_1}{\partial \log e} \right) - ep_1^* \tilde{x}_1 \frac{\partial \log \tilde{x}_1}{\partial \log e} + ep_1^*(\bar{x}_1 - \tilde{x}_1)}{p_1 \tilde{x}_1 + ep_1^*(\bar{x}_1 - \tilde{x}_1)}.$$

Using the fact that  $\frac{\partial \log p_1}{\partial \log e} = \frac{\partial \log p_1}{\partial \log \tilde{x}_1} \frac{\partial \log \tilde{x}_1}{\partial \log e}$  and the equilibrium condition  $f(\tilde{x}_1(e), e) = 0$  yields,

$$\left\{ p_1 \left( 1 + \frac{\partial \log p_1}{\partial \log \tilde{x}_1} \right) - ep_1^* \right\} \tilde{x}_1 \frac{\partial \log \tilde{x}_1}{\partial \log e} = \left\{ p_1 \left( 1 + \frac{\partial \tilde{x}_1}{\partial p_1} \frac{\partial p_1}{\partial \tilde{x}_1} \right) - ep_1^* \right\} \tilde{x}_1 \frac{\partial \log \tilde{x}_1}{\partial \log e} = 0.$$

Therefore the elasticity of profits for the exporting firm, for  $\tilde{x}_1 \leq \bar{x}_1$ , is,

$$\frac{\partial \log \pi_1}{\partial \log e}(e) = \frac{ep_1^*(\bar{x}_1 - \tilde{x}_1)}{p_1 \tilde{x}_1 + ep_1^*(\bar{x}_1 - \tilde{x}_1)}.$$

The logarithm of profits for the local firm is simply,

$$\log \pi_2(e) = \log p_2 + \log \bar{x}_2.$$

Therefore the elasticity of profits for the local firm, for  $\tilde{x}_1 \leq \bar{x}_1$ , is,

$$\frac{\partial \log \pi_2}{\partial \log e}(e) = \frac{\partial \log p_2}{\partial \log e}(e) = \eta_2(e) = 1 - \eta.$$

If  $\tilde{x}_1 > \bar{x}_1$ , profits for both firms are constant and therefore exposures are zero.

Exchange rate exposure for both firm are given by,

$$\begin{aligned}\delta_1(e) &= \chi \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}}, \\ \delta_2(e) &= (1 - \eta) \mathbf{1}_{\{\tilde{x}_1 \leq \bar{x}_1\}}.\end{aligned}$$

where  $\chi \equiv \frac{ep_1^*(\bar{x}_1 - \tilde{x}_1)}{p_1\bar{x}_1 + ep_1^*(\bar{x}_1 - \tilde{x}_1)}$ .

**Proof of Proposition 1.2.6:**

The first order conditions are given by,

$$\begin{aligned}u'(\beta(b_d + \bar{e}b_f)) &= \mathbb{E}[u'(\pi(e_1) - b_d - e_1b_f)], \\ \bar{e}u'(\beta(b_d + \bar{e}b_f)) &= \mathbb{E}[e_1u'(\pi(e_1) - b_d - e_1b_f)].\end{aligned}$$

Assume that the volatility of the exchange rate is small, i.e.  $\sigma_e \ll 1$ . Given the exchange rate dynamics, a Taylor series for the profit function can be exploited,

$$\pi(e_1) = \pi(\bar{e}) + (e_1 - \bar{e})\frac{\partial\pi}{\partial e}(\bar{e}) + O((e_1 - \bar{e})^2).$$

Therefore,

$$\pi(e_1) - b_d - e_1b_f = \{\pi(\bar{e}) - b_d - \bar{e}b_f\} + (e_1 - \bar{e})\left\{\frac{\partial\pi}{\partial e}(\bar{e}) - b_f\right\} + O((e_1 - \bar{e})^2).$$

A Taylor series for the marginal utility function is,

$$u'(\pi(e_1) - b_d - e_1b_f) = u'(\bar{\pi}) + (e_1 - \bar{e})\left(\frac{\partial\pi}{\partial e}(\bar{e}) - b_f\right)u''(\bar{\pi}) + O((e_1 - \bar{e})^2).$$

where  $\bar{\pi} \equiv \pi(\bar{e}) - b_d - \bar{e}b_f$  is a constant. The utility is strictly concave, i.e.  $u'' < 0$ . Combining the first order conditions yields the following equilibrium condition,

$$0 = \mathbb{E}[(e_1 - \bar{e})u'(\pi(e_1) - b_d - e_1b_f)].$$

Using the Taylor series approximation simplifies the previous expression to,

$$0 = \mathbb{E}[(e_1 - \bar{e})^2]\left(\frac{\partial\pi}{\partial e}(\bar{e}) - b_f\right).$$

The equilibrium condition yields the following rule for the optimal dollar debt,

$$b_f = \frac{\partial \pi}{\partial e}(\bar{e}).$$

The Taylor series for the marginal utility function becomes,

$$u'(\pi(e_1) - b_d - e_1 b_f) = u'(\bar{\pi}) + O((e_1 - \bar{e})^2).$$

Plugging the previous approximation into the first order condition for  $b_d$  yields,

$$u'(\beta(b_d + \bar{e}b_f)) = \mathbb{E} [u'(\bar{\pi}) + O((e_1 - \bar{e})^2)] = u'(\bar{\pi}) + \mathbb{E} [O((e_1 - \bar{e})^2)] \approx u'(\bar{\pi}).$$

Taking the inverse marginal utility function on both sides yields,

$$\beta(b_d + \bar{e}b_f) = \bar{\pi} = \pi(\bar{e}) - b_d - \bar{e}b_f.$$

Rearranging the previous expression yields the desired result,

$$b_d = \frac{\pi(\bar{e})}{1 + \beta} - \bar{e} \frac{\partial \pi}{\partial e}(\bar{e}).$$



**Proof of Proposition 1.2.7:**

The share of foreign debt is given by,

$$\omega \equiv \frac{\bar{e}b_f}{(b_d + \bar{e}b_f)} = \frac{\bar{e} \frac{\partial \pi(\bar{e})}{\partial e}}{\frac{\pi(\bar{e})}{1+\beta}} = (1 + \beta) \frac{\bar{e}}{\pi(\bar{e})} \frac{\partial \pi(\bar{e})}{\partial e} = (1 + \beta) \frac{\partial \log \pi(\bar{e})}{\partial \log e} = (1 + \beta)\delta.$$

The share of foreign debt for the exporting firm is thus,

$$\omega_1 = (1 + \beta) \chi(\bar{e}) \mathbf{1}_{\{\bar{x}_1(\bar{e}) \leq \bar{x}_1\}}.$$

The share of foreign debt for the local firm is thus,

$$\omega_2 = (1 + \beta) (1 - \eta(\bar{e})) \mathbf{1}_{\{\bar{x}_1(\bar{e}) \leq \bar{x}_1\}}.$$

## Appendix B – Mexican Firms Listed on the Mexican Stock Market

### Panel A: Materials

Ticker Symbol	Company's Name
AHMSA	ALTOS HORNOS DE MEXICO, S.A. DE C.V.
AUTLAN	COMPAIA MINERA AUTLAN, S.A.B. DE C. V.
CEMEX	CEMEX, S.A.B. DE C.V.
CMOCTEZ	CORPORACION MOCTEZUMA, S.A.B. DE C.V.
CODUSA	CORPORACION DURANGO, S.A.B. DE C.V.
COLLADO	G COLLADO, S.A.B. DE C.V.
CONVER	CONVERTIDORA INDUSTRIAL, S.A.B. DE C.V.
CYDSASA	CYDSA, S.A.B. DE C.V.
FRES	FRESNILLO PLC
GCC	GRUPO CEMENTOS DE CHIHUAHUA, S.A.B. DE C.V.
GMEXICO	GRUPO MEXICO, S.A.B. DE C.V.
ICH	INDUSTRIAS CH, S.A.B. DE C.V.
MEXCHEM	MEXICHEM, S.A.B. DE C.V.
PENOLES	INDUSTRIAS PEOLES, S.A.B. DE C. V.
POCHTEC	GRUPO POCHTECA, S.A.B. DE C.V.
QBINDUS	Q.B. INDUSTRIAS, S.A. DE C.V.
SIMEC	GRUPO SIMEC, S.A.B. DE C.V.
TEKCHEM	TEKCHEM, S.A.B. DE C.V.
TS	TENARIS S.A.
VITRO	VITRO, S.A.B. DE C.V.

### Panel B: Manufacturing

Ticker Symbol	Company's Name
ACCELSA	ACCEL, S.A.B. DE C.V.
ALFA	ALFA, S.A.B. DE C.V.
ARA	CONSORCIO ARA, S.A.B. DE C.V.
ASUR	GRUPO AEROPORTUARIO DEL SURESTE, S.A.B. DE C.V.
CERAMIC	INTERNACIONAL DE CERAMICA, S.A.B. DE C.V.
CICSA	CARSO INFRAESTRUCTURA Y CONSTRUCCIN, S.A.B. DE C.V.
DINE	DINE, S.A.B. DE C.V.
GAP	GRUPO AEROPORTUARIO DEL PACIFICO, S.A.B. DE C.V.
GCARSO	GRUPO CARSO, S.A.B. DE C.V.
GEO	CORPORACION GEO, S.A.B. DE C.V.
GISSA	GRUPO INDUSTRIAL SALTILLO, S.A.B. DE C.V.
GMD	GRUPO MEXICANO DE DESARROLLO, S.A.B.
GMDR	GMD RESORTS, S.A.B.
HOGAR	CONSORCIO HOGAR, S.A.B. DE C.V.
HOMEX	DESARROLLADORA HOMEX, S.A.B. DE C.V.
ICA	EMPRESAS ICA, S.A.B. DE C.V.
IDEAL	IMPULSORA DEL DESARROLLO Y EL EMPLEO

Panel B: Manufacturing (continued)

Ticker Symbol	Company's Name
KUO	GRUPO KUO, S.A.B. DE C.V.
LAMOSA	GRUPO LAMOSA, S.A.B. DE C.V.
OMA	GRUPO AEROPORTUARIO DEL CENTRO NORTE
PASA	PROMOTORA AMBIENTAL, S.A.B. DE C.V.
PINFRA	PROMOTORA Y OPERADORA DE INFRAESTRUCTURA
PYP	GRUPO PROFESIONAL PLANEACION Y PROYECTOS
SARE	SARE HOLDING, S.A.B. DE C.V.
TMM	GRUPO TMM, S.A.
URBI	URBI DESARROLLOS URBANOS, S.A.B. DE C.V.

Panel C: Consumer Goods & Services

Ticker Symbol	Company's Name
ALSEA	ALSEA, S.A.B. DE C.V.
ARISTOS	CONSORCIO ARISTOS, S.A. DE C.V.
CIDMEGA	GRUPE, S.A.B. DE C.V.
CIE	CORPORACION INTERAMERICANA DE ENTRETENIMIENTO
CMR	CMR, S.A.B. DE C.V.
CNCI	UNIVERSIDAD CNCI, S.A. DE C.V.
EDOARDO	EDOARDOS MARTIN, S.A.B. DE C.V.
ELEKTRA	GRUPO ELEKTRA, S.A. DE C.V.
GFAMSA	GRUPO FAMSA, S.A.B. DE C.V.
GMARTI	GRUPO MARTI, S.A.B.
GOMO	GRUPO COMERCIAL GOMO, S.A. DE C.V.
GPH	GRUPO PALACIO DE HIERRO, S.A.B. DE C.V.
HILASAL	HILASAL MEXICANA S.A.B. DE C.V.
IASASA	INDUSTRIA AUTOMOTRIZ, S.A. DE C.V.
LIVEPOL	EL PUERTO DE LIVERPOOL, S.A.B. DE C.V.
POSADAS	GRUPO POSADAS, S.A.B. DE C.V.
REALTUR	REAL TURISMO S.A. DE C.V.
SANLUIS	SANLUIS CORPORACION, S. A. DE C. V.
VASCONI	GRUPO VASCONIA S.A.B.

Panel D: Consumer Durables

Ticker Symbol	Company's Name
AGRIEXP	AGRO INDUSTRIAL EXPORTADORA, S.A. DE C.V.
ARCA	EMBOTELLADORAS ARCA, S.A.B. DE C.V.
BACHOCO	INDUSTRIAS BACHOCO, S.A.B. DE C.V.
BAFAR	GRUPO BAFAR, S.A. DE C.V.
BIMBO	GRUPO BIMBO, S.A.B. DE C.V.
COMERCI	CONTROLADORA COMERCIAL MEXICANA, S.A.B. DE C.V.
CONTAL	GRUPO CONTINENTAL, S.A.B.

Panel D: Consumer Durables (continued)

Ticker Symbol	Company's Name
FEMSA	FOMENTO ECONMICO MEXICANO, S.A.B. DE C.V.
GAM	GRUPO AZUCARERO MXICO, S.A. DE C.V.
GEUPEC	GRUPO EMBOTELLADORAS UNIDAS, S.A.B. DE C.V.
GIGANTE	GRUPO GIGANTE, S.A.B. DE C.V.
GMACMA	GRUPO MAC MA, S.A.B. DE C.V.
GMODELO	GRUPO MODELO, S.A.B. DE C.V.
GMODERN	GRUPO LA MODERNA, S.A.B. DE C.V.
GRUMA	GRUMA, S.A.B. DE C.V.
HERDEZ	GRUPO HERDEZ, S.A.B. DE C.V.
KIMBER	KIMBERLY-CLARK DE MEXICO S.A.B. DE C.V.
KOF	COCA-COLA FEMSA, S.A.B. DE C.V.
MASECA	GRUPO INDUSTRIAL MASECA, S.A.B. DE C.V.
MINSA	GRUPO MINSA, S.A.B. DE C.V.
NUTRISA	GRUPO NUTRISA, S. A. DE C. V.
SAVIA	SAVIA, S.A. DE C.V.
SORIANA	ORGANIZACION SORIANA, S.A.B. DE C.V.
WALMEX	WAL-MART DE MEXICO, S.A.B. DE C.V.

Panel E: Health Care

Ticker Symbol	Company's Name
BEVIDES	FARMACIAS BENAVIDES, S.A.B. DE C.V.
FRAGUA	CORPORATIVO FRAGUA, S.A.B. DE C.V.
LAB	GENOMMA LAB INTERNACIONAL, S.A.B. DE C.V.
MEDICA	MEDICA SUR, S.A.B. DE C.V.
SAB	GRUPO CASA SABA, S.A.B. DE C.V.

Panel F: Telecommunication Services

Ticker Symbol	Company's Name
AMX	AMERICA MOVIL, S.A.B. DE C.V.
AXTEL	AXTEL, S.A.B. DE C.V.
CABLE	EMPRESAS CABLEVISION, S.A. DE C.V.
CEL	GRUPO IUSACELL, S. A. DE C. V.
MAXCOM	MAXCOM TELECOMUNICACIONES, S.A.B. DE C.V.
MEGA	MEGACABLE HOLDINGS, S.A.B. DE C.V.
QUMMA	GRUPO QUMMA, S.A. DE C.V.
RCENTRO	GRUPO RADIO CENTRO, S.A.B. DE C.V.
TELECOM	CARSO GLOBAL TELECOM, S.A.B. DE C.V.
TELINT	TELMEX INTERNACIONAL, S.A.B. DE C.V.
TELMEX	TELEFONOS DE MEXICO, S.A.B. DE C.V.
TLEVISA	GRUPO TELEVISA, S.A.
TVAZTCA	TV AZTECA, S.A. DE C.V.

## Appendix C – Real Exchange Rate

The nominal and real exchange rates are denoted by  $\tilde{e}$  and  $e$ , respectively. Let  $\tilde{R}_e$  and  $R_e$  be the gross rate of return on the nominal and real exchange rates, respectively. Denote the inflation in country  $i$  by  $\pi^i$ , for  $i = \{MEX, USA\}$ . The following no-arbitrage condition must hold,

$$\tilde{R}_e = R_e \left( \frac{1 + \pi^{USA}}{1 + \pi^{MEX}} \right).$$

The real exchange rate series  $\{e_t\}$  can be computed using data on nominal exchange rate  $\{\tilde{e}_t\}$  and inflation  $\{\pi_t^{MEX}, \pi_t^{USA}\}$  as follows,

$$\frac{e_{t+1}}{e_t} = \frac{\tilde{e}_{t+1}}{\tilde{e}_t} \frac{1 + \pi_{t+1}^{USA}}{1 + \pi_{t+1}^{MEX}}.$$

Taking the log on both sides yields,

$$\Delta \log(e_{t+1}) = \Delta \log(\tilde{e}_{t+1}) + \log(1 + \pi_{t+1}^{USA}) - \log(1 + \pi_{t+1}^{MEX}).$$

## Appendix D – Calibration of the Cost Parameters

The economy is set at its deterministic steady state, that is  $e = \bar{e}$  and  $z_i = \bar{z}$  for all firms. Assume that there are  $N$  firms in this economy. Because all firms have the same level of productivity and have the same cost structure, they will choose the same policies, i.e.  $x_i = x$  and  $x_i^* = x^*$ . That also means that the domestic market share for all firms is identical  $\lambda_i = \lambda$ , and all prices are equal  $p_i = p$ .

The first order conditions for problem 1.17 are given by,

$$\begin{aligned}\frac{\partial \pi_i}{\partial x_i} &= \rho(1 - \lambda_i)p_i - \eta_1(x_i + x_i^*)^{\eta_2 - 1} = 0 \\ \frac{\partial \pi_i}{\partial x_i^*} &= \bar{e}p_i^* - \eta_1(x_i + x_i^*)^{\eta_2 - 1} = 0\end{aligned}$$

Recall that the price for good  $i$  is  $p_i(x_i, X) = y x_i^{\rho - 1} X^{-\rho}$ , where

$$X = \left( \int_i x_i^\rho di \right)^{\frac{1}{\rho}} = \left( x_i^\rho \int_i di \right)^{\frac{1}{\rho}} = x_i.$$

Thus the price can be rewritten as follows,

$$p_i = y x_i^{\rho - 1} X^{-\rho} = y x_i^{-1}.$$

Subtracting  $\text{FOC}(x_i)$  from  $\text{FOC}(x_i^*)$  leads to,

$$p_i = \frac{\bar{e}p_i^*}{\rho(1 - \lambda_i)}.$$

Using the two previous formulae for the price yields the domestic sales,

$$x_i = \frac{\rho y (1 - \lambda_i)}{\bar{e}p_i^*}. \quad (1.20)$$

$\text{FOC}(x_i^*)$  yields the total production,

$$x_i + x_i^* = \left( \frac{\bar{e}p_i^*}{\eta_1} \right)^{\frac{1}{\eta_2 - 1}}. \quad (1.21)$$

Foreign sales are thus,

$$x_i^* = \left( \frac{\bar{e}p_i^*}{\eta_1} \right)^{\frac{1}{\eta_2 - 1}} - \frac{\rho y (1 - \lambda_i)}{\bar{e}p_i^*}. \quad (1.22)$$

Production costs are,

$$\text{Cost} \equiv \frac{\eta_1}{\eta_2} (x_i + x_i^*)^{\eta_2} + f^* + f = \frac{\eta_1}{\eta_2} \left( \frac{\bar{e}p_i^*}{\eta_1} \right)^{\frac{\eta_2}{\eta_2-1}} + f^* + f. \quad (1.23)$$

The ratio of domestic sales to total production is,

$$\frac{p_i x_i}{x_i + x_i^*} = y \left( \frac{\bar{e}p_i^*}{\eta_1} \right)^{-\frac{1}{\eta_2-1}}. \quad (1.24)$$

The ratio of foreign sales to total production is,

$$\frac{\bar{e}p_i^* x_i^*}{x_i + x_i^*} = \bar{e}p_i^* \left\{ \left( \frac{\bar{e}p_i^*}{\eta_1} \right)^{\frac{1}{\eta_2-1}} - \frac{\rho y (1 - \lambda_i)}{\bar{e}p_i^*} \right\} \left( \frac{\bar{e}p_i^*}{\eta_1} \right)^{-\frac{1}{\eta_2-1}}. \quad (1.25)$$

The ratio of costs to total production is,

$$\frac{\frac{\eta_1}{\eta_2} (x_i + x_i^*)^{\eta_2} + f^* + f}{x_i + x_i^*} = \frac{\bar{e}p_i^*}{\eta_2} + (f^* + f) \left( \frac{\bar{e}p_i^*}{\eta_1} \right)^{-\frac{1}{\eta_2-1}}. \quad (1.26)$$

# Bibliography

- [1] Michael Adler and Bernard Dumas. Exposure to currency risk: Definition and measurement. *Financial Management*, 13(2):41–50, 1984.
- [2] Mark Aguiar. Investment, devaluation, and foreign currency exposure: The case of Mexico. *Journal of Development Economics*, 78(1):95 – 113, 2005.
- [3] George Allayannis and Jane Ihrig. Exposure and markups. *Review of Financial Studies*, 14(3):805–35, 2001.
- [4] George Allayannis and Eli Ofek. Exchange rate exposure, hedging, and the use of foreign currency derivatives. *Journal of International Money and Finance*, 20(2):273 – 296, 2001.
- [5] Hoyt Bleakley and Kevin Cowan. Corporate dollar debt and depreciations: Much ado about nothing? *The Review of Economics and Statistics*, 90(4):612–626, 08 2008.
- [6] Gordon M. Bodnar, Bernard Dumas, and Richard C. Marston. Pass-through and exposure. *The Journal of Finance*, 57(1):199–231, 2002.
- [7] Gordon M. Bodnar and William M. Gentry. Exchange rate exposure and industry characteristics: evidence from Canada, Japan, and the USA. *Journal of International Money and Finance*, 12(1):29 – 45, 1993.
- [8] Gordon M. Bodnar, Gregory S. Hayt, and Richard C. Marston. 1998 Wharton Survey of Financial Risk Management by US Non-Financial Firms. *Financial Management*, 27(4):70–91, 1998.



- [9] Gordon M. Bodnar and M. H. Franco Wong. Estimating exchange rate exposures: Issues in model structure. *Financial Management*, 32(1):35–67, 2003.
- [10] Ricardo J. Caballero and Arvind Krishnamurthy. International and domestic collateral constraints in a model of emerging market crises. *Journal of Monetary Economics*, 48(3):513 – 548, 2001.
- [11] Ricardo J. Caballero and Arvind Krishnamurthy. Excessive dollar debt: Financial development and underinsurance. *The Journal of Finance*, 58(2):867–893, 2003.
- [12] Thomas F. Cooley and Vincenzo Quadrini. Financial markets and firm dynamics. *The American Economic Review*, 91(5):1286–1310, 2001.
- [13] Kevin Cowan, Erwin Hansen, and Luis Oscar Herrera. Currency mismatches, balance sheet effects and hedging in chilean non-financial corporations. (346), December 2005.
- [14] Peter M. DeMarzo and Darrell Duffie. Corporate incentives for hedging and hedge accounting. *The Review of Financial Studies*, 8(3):743–771, 1995.
- [15] Avinash K. Dixit and Joseph E. Stiglitz. Monopolistic competition and optimum product diversity. *The American Economic Review*, 67(3):297–308, 1977.
- [16] Kathryn M.E. Dominguez and Linda L. Tesar. Exchange rate exposure. *Journal of International Economics*, 68(1):188 – 218, 2006.
- [17] Kenneth A. Froot and Paul D. Klemperer. Exchange rate pass-through when market share matters. *The American Economic Review*, 79(4):637–654, 1989.
- [18] Kenneth A. Froot, David S. Scharfstein, and Jeremy C. Stein. Risk management: Coordinating corporate investment and financing policies. *The Journal of Finance*, 48(5):1629–1658, 1993.
- [19] Sebastian Galiani, Eduardo Levy Yeyati, and Ernesto Schargrotsky. Financial dollarization and debt deflation under a currency board. *Emerging Markets Review*, 4(4):340 – 367, 2003.

- [20] Christopher Geczy, Bernadette A. Minton, and Catherine Schrand. Why firms use currency derivatives. *The Journal of Finance*, 52(4):1323–1354, 1997.
- [21] Joao F. Gomes. Financing investment. *The American Economic Review*, 91(5):1263–1285, 2001.
- [22] Joao F. Gomes and Lukas Schmid. Levered returns. *The Journal of Finance*, 2009.
- [23] John R. Graham. How big are the tax benefits of debt? *The Journal of Finance*, 55(5):1901–1941, 2000.
- [24] John R. Graham and Daniel A. Rogers. Do firms hedge in response to tax incentives? *The Journal of Finance*, 57(2):815–839, 2002.
- [25] Christopher A. Hennessy and Toni M. Whited. Debt dynamics. *The Journal of Finance*, 60(3):1129–1165, 2005.
- [26] Philippe Jorion. The exchange-rate exposure of u.s. multinationals. *Journal of Business*, 63(3):331–45, July 1990.
- [27] Richard C. Marston. The effects of industry structure on economic exposure. *Journal of International Money and Finance*, 20(2):149 – 164, 2001.
- [28] Marc J. Melitz. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725, 2003.
- [29] Marc J. Melitz and Gianmarco I.P. Ottaviano. Market size, trade, and productivity. *Review of Economic Studies*, 75(1):295–316, 2008.
- [30] Shehzad L. Mian. Evidence on corporate hedging policy. *The Journal of Financial and Quantitative Analysis*, 31(3):419–439, 1996.
- [31] Deana R. Nance, Jr. Smith, Clifford W., and Charles W. Smithson. On the determinants of corporate hedging. *The Journal of Finance*, 48(1):267–284, 1993.

- [32] Sangeeta Pratap, Ignacio Lobato, and Alejandro Somuano. Debt composition and balance sheet effects of exchange rate volatility in Mexico: a firm level analysis. *Emerging Markets Review*, 4(4):450 – 471, 2003.
- [33] Martin Schneider and Aaron Tornell. Balance sheet effects, bailout guarantees and financial crises. *The Review of Economic Studies*, 71(3):883–913, 2004.
- [34] Clifford W. Smith and Rene M. Stulz. The determinants of firms' hedging policies. *The Journal of Financial and Quantitative Analysis*, 20(4):391–405, 1985.
- [35] Rene M. Stulz. Optimal hedging policies. *The Journal of Financial and Quantitative Analysis*, 19(2):127–140, 1984.
- [36] Rene M. Stulz. Managerial discretion and optimal financing policies. *Journal of Financial Economics*, 26(1):3 – 27, 1990.
- [37] Rohan Williamson. Exchange rate exposure and competition: evidence from the automotive industry. *Journal of Financial Economics*, 59(3):441 – 475, 2001.

## Chapter 2

# Stochastic Volatility, Credit Spreads, and the Q Theory of Investment

joint with François Gourio

### 2.1 Introduction

According to the neoclassical theory of investment (Abel (1979), Hayashi (1982)), Tobin's Q, the ratio of the firm value to its capital stock, is a sufficient statistic for the firm's optimal investment. However, this model is empirically rejected: Tobin's Q is weakly correlated with investment; moreover, cash flow enters significantly and reduces further the economic significance of Q.

Recent empirical work shows that bond yields, on the other hand, are strongly correlated with investment, both in the cross-section (Gilchrist and Zakrajsek (2008)), and in the time series (Philippon (2009)). This result is surprising, because in a standard investment model, there is no reason why one of the two asset prices - stocks or bonds - should correlate more with Q: a positive shock to profitability increases *both* the stock price and the bond price, i.e. decrease the yield or spread, as well as increase investment. That is true even if the firm never uses the equity market for financing – the equity value, which is the expected present discounted value of dividends, *still* reflects the higher profitability, a point emphasized by Gomes (2001).

In this paper, we propose a model of financing and investment for firms that attempts to explain

*both* the weak correlation of Tobin's Q with investment, and the higher correlation of bond yields with investment. Following Gomes (2001), Hennessy and Whited (2005) and Gomes and Schmid (2010), our model augments the standard neoclassical model with financing frictions. Specifically, we assume that firms fund their investment by issuing equity or defaultable debt. The firm-specific probability of default leads to a firm-specific interest rate, which exactly compensates investors for the risk of default. An increase in profitability leads, in this model, to an increase in investment, as well as a rise in the stock price and in the bond price (i.e. a reduction in the firm-specific interest rate or yield). As is well known, this model thus generates a positive correlation between investment and Q, and between investment and bond yield. The correlation between Q and investment is high, contrary to the data.

Our contribution is to introduce an additional source of shocks in the model, namely shocks to firm-level volatility. When volatility goes up, the option value of default goes up, leading the equity value to go up while the bond value goes down. At the same time, investment falls because of the higher cost of financing (and possibly because of the higher uncertainty in itself<sup>1</sup>). Hence, investment is correlated with bond prices but not with stock prices.

While a large empirical literature documents that stock returns are heteroskedastic, there is relatively little structural modeling which incorporates this mechanism. We find the stochastic volatility appealing, since firms evolve over time and may become more risky, e.g. because they introduce face new competitors due to a technological innovation. Other real-world examples include the introduction of a new product, or the entry of a new market.<sup>2</sup>

The unconditional correlation, in our model, is a weighted average of the correlation conditional on profitability shocks, and the correlation conditional on volatility shocks. Both shocks lead to a

---

<sup>1</sup>This is the effect emphasized by the real options literature, e.g. Pindyck (1989), Bloom, Bond and Van Reenen (2007), and others.

<sup>2</sup>Some of these examples may reflect an endogenous choice by the firm, rather than an exogenous shock to volatility. However many of these choices are likely determined by other considerations than a choice of volatility, weakening the endogeneity problem. Still, it would be interesting to consider a model where firms decide on their volatility.

positive correlation of bond prices and investment, hence the model predicts that this correlation is high. But only the profitability shock generates a positive correlation between stock prices and investment, hence the correlation of stock prices and investment is low. Hence, our model can replicate the patterns of correlations in the data.

An interesting empirical implication of the model regards the correlation between a firm's bond return and its stock return. In the standard model (i.e. with constant volatility), bond returns and stock returns are highly correlated. In our model, volatility shocks induce a negative correlation. We plan to use this as a test of our model. Preliminary results suggests that this correlation is indeed negative for firms close to default.

### **Organization of the Paper**

The rest of the introduction reviews the related literature. Section 2.2 studies a simple two period example which helps clarify the intuition. Section 2.3 presents our quantitative model. Section 2.4 calibrates it and studies its implications numerically. Section 2.5 concludes.

### **Literature Review**

Our paper is directly related to the vast literature on the Q-theory of investment and the cash flow sensitivity (e.g. Fazzari, Hubbard and Petersen (1988), Gilchrist and Himmelberg (1995), Gomes (2001)). This literature documents the empirical significance of cash flows and its interpretation. One conclusion from this literature is that, while there are many potential mechanisms that break the theoretical link between Q and investment, such as fixed costs, financing constraints, or decreasing return to scales, there are very few quantitative models which replicate the failure of the investment regressions, and most researchers appeal to measurement error in Q (e.g., Erickson and Whited (2001), Eberly, Rebelo, and Vincent (2008)).

Second, there is a small literature on the relation between investment and uncertainty. This literature emphasizes that models with real options or equivalently fixed cost or irreversibility imply

a negative effect of uncertainty on investment (see e.g. Bernanke (1983), Dixit and Pindyck (1994), Caballero (1991), Pindyck (1993), Bloom, Bond and Van Reenen (2007)). In our model, the effect of uncertainty occurs through a different mechanism: debt becomes a less efficient means of finance when uncertainty increases. Empirical evidence suggests a significant effect of uncertainty on investment (Leahy and Whited (1996), Guiso and Parigi (1999)) but the channel through which the effect operates is not established.

Recently several studies have considered the possibility that idiosyncratic risk varies over time (Bloom (2009), Bachmann and Bayer (2009)), however our point of departure is somewhat different since we consider idiosyncratic increases in firm volatility, i.e. volatility shocks are not correlated across firms.

The corporate finance literature has emphasized the endogeneity of volatility, known as the risk-shifting problem (equity holders may choose to increase volatility). Our model has exogenous increases in the volatility of the technology shocks, but firms respond to these changes by altering capital, hence the total volatility of profits is endogenous. But because debt is one-period, the volatility is known by debtholders when they decide to buy the firm's bonds. Hence there is no risk-shifting problem in this setup.

## 2.2 Two-Period Partial Equilibrium Example

This section provides a simple model to illustrate the effect of an increase in risk on investment, equity value, Tobin Q and bond yields. There are two time periods. At time 1, the firm buys capital  $k$ , which is financed using equity issuance  $s$  and debt: the firm issues a debt with face value  $b$ , with market price  $q(k, b)$ . The budget constraint is,

$$\chi q(k, b)b + s = k. \tag{2.1}$$

The parameter  $\chi > 1$  reflects the tax shield effect (i.e. interest expenses are deductible from corporate income). To simplify we assume in this section that the deduction is done at issuance: for

each dollar of debt issued, the firm receives a subsidy  $(\chi - 1)\$$ .

At time 2, the firm produces, and generates a profit  $\pi = zk^\alpha$ , where  $z$  is an idiosyncratic shock, which is distributed according to a cumulative distribution function  $H$ . We denote by  $h$  the corresponding probability distribution function.

The firm will default if its profits are not large enough to repay its debt, i.e. if  $z < z^*$ , where the threshold  $z^*$  is defined through the condition,

$$z^*k^\alpha = b. \tag{2.2}$$

If the firm does default, the absolute priority rule applies: equity holders receive nothing, while bondholders share the firm profits, net of proportional bankruptcy costs. We denote by  $\theta$  the recovery rate, i.e.  $1 - \theta$  is bankruptcy costs.

The market price of debt is the expected discounted payoff to debtholders. Assuming that investors are risk-neutral and have a discount factor  $\beta$ , we have,

$$q(k, b) = \beta \left( \int_{z^*}^{\infty} dH(z) + \int_0^{z^*} \theta \frac{zk^\alpha}{b} dH(z) \right). \tag{2.3}$$

In this formula, the first term is the repayment of the face value in the non-default states, and the second term is the recovery of profits, divided across all the bondholders in the event of default.

The firm equity value is the expected discounted payoff to equity holders, i.e. the expected profits net of debt repayment, in non-default states:

$$V = \beta \int_{z^*}^{\infty} (zk^\alpha - b) dH(z). \tag{2.4}$$

The firm picks  $k$ ,  $b$ ,  $s$ , and  $z^*$  to maximize its present discounted value,  $V - s$ , subject to equations (2.1), (2.2) and (2.3).<sup>3</sup> This problem can be written as,

$$\max_{k, b, z^*} \left\{ \beta \int_{z^*}^{\infty} (zk^\alpha - b) dH(z) - k + \chi q(k, b)b \right\},$$

---

<sup>3</sup>We assume in this example that there are no equity issuance costs.



subject to,

$$\begin{aligned} q(k, b) &= \beta \left( \int_{z^*}^{\infty} dH(z) + \int_0^{z^*} \theta \frac{zk^\alpha}{b} dH(z) \right), \\ z^* k^\alpha &= b. \end{aligned}$$

We can rewrite this by substituting out  $b$ :

$$\max_{k, z^*} \left\{ \beta \int_{z^*}^{\infty} k^\alpha (z - z^*) dH(z) - k + \chi \beta \left( z^* k^\alpha \int_{z^*}^{\infty} dH(z) + \int_0^{z^*} \theta z k^\alpha dH(z) \right) \right\},$$

or

$$\max_{k, z^*} \left\{ \beta k^\alpha E(z) + \beta (\chi - 1) z^* k^\alpha \int_{z^*}^{\infty} dH(z) + \beta (\theta \chi - 1) k^\alpha \int_0^{z^*} z dH(z) - k \right\}. \quad (2.5)$$

The first term in this expression is the expected discounted operating profit, i.e.  $\beta k^\alpha E(z)$ . The second term reflects the expected tax shield benefits in non-default states, since  $z^* k^\alpha = b$  is the debt issued. The third term reflects the expected bankruptcy costs, net of tax shield benefits, in default states. The last term is the cost of investment.

It is easy to check that if there are no bankruptcy costs,  $\theta = 1$ , then it is optimal to finance with debt only. Inversely, if there is no tax shield,  $\chi = 1$ , then an all-equity financing is optimal. We assume that  $\chi \theta < 1$ , i.e. bankruptcy costs are larger than the tax shield effect conditional on default. This assumption is necessary to generate the standard trade-off.

The program (2.5) can be solved by writing the two first order conditions, with respect to  $k$  and  $z^*$ , which determine the optimal investment and financing (intuitively, we can think of  $z^*$  as leverage, since it is the ratio of debt to expected output,  $z^* = b/k^\alpha$ ). First, we have,

$$1 = \beta \alpha k^{\alpha-1} \left\{ E(z) + (\chi - 1) z^* (1 - H(z^*)) + (\theta \chi - 1) \int_0^{z^*} z dH(z) \right\}. \quad (2.6)$$

In the case with  $\chi = \theta = 1$ , we obtain the usual user cost rule: the expected marginal product of capital is equal to the cost of capital,  $1 = \beta \alpha k^{\alpha-1} E(z)$ . When  $\theta < 1$  or  $\chi > 1$ , the user cost needs to be adjusted to reflect expected bankruptcy costs (which increase the user cost) and the tax shield (which decreases it).

The second first-order condition, with respect to  $z^*$ , yields, after rearrangement,

$$((1 - H(z^*)) (\chi - 1) = \chi (1 - \theta) z^* h(z^*) . \quad (2.7)$$

This equation determines the optimal probability of default  $H(z^*)$ , and the leverage  $z^* = b/k^\alpha$ . The left side is the marginal benefit of leverage, which is the higher tax shield in non-default states, while the right side is the marginal cost of leverage, i.e. the increase in expected bankruptcy costs, which depends on the probability of having  $z$  close to  $z^*$ . Under some regularity conditions on the distribution  $h$ , equation (2.7) determines a unique default cutoff  $z^*$ , given the parameters  $\chi$  and  $\theta$  and the distribution  $h$ .<sup>4</sup> Equation (2.6) then determines the investment choice  $k$ .

We define  $Q$  as the total market value of the firm, divided by its capital stock:

$$\begin{aligned} Q &= \frac{\text{market value(equity)} + \text{market value(debt)}}{k}, \\ &= \frac{V + \chi q(k, b)b}{k}. \end{aligned}$$

Note that we include the subsidy received, i.e. we count the total debt finance raised by the firm in period 1.<sup>5</sup>

To understand the model, [Figure 2.1](#) illustrates the optimal choice of debt. More precisely, we present some firm outcomes, if the firm borrows an amount  $b$ , and makes the optimal investment  $k^*$ . The optimal level of debt  $b^*$  is denoted with a vertical line. Before picking  $b^*$ , however, the firm contemplates financing with a different amount  $b \neq b^*$ , and this figure illustrates the consequences of choosing a different debt level  $b$ . As illustrated in the first panel, as the firm increases its debt issuance, it simultaneously reduces the equity issuance, since the investment is fixed at  $k^*$  in this experiment. The second panel illustrates a ‘‘Laffer curve’’: as the face value of debt increases, the

---

<sup>4</sup>The technical condition (which we assume from now on) is that the function  $z \rightarrow \frac{zh(z)}{1-H(z)}$  is increasing. Bernanke, Gertler and Gilchrist (1999) make the same assumption in the context of a somewhat different model, and note that most distributions satisfy this assumption, e.g. the log-normal distribution satisfies it. In our numerical example, we will use the log-normal distribution.

<sup>5</sup>In practice the tax shield affects future profits and thus equity value, which is why we need to include it in the firm value. Also, contrary to the empirical literature, we use the market value of debt rather than the book value. It is the theoretically cleaner definition, and the quantitative difference turns out not to be very important.

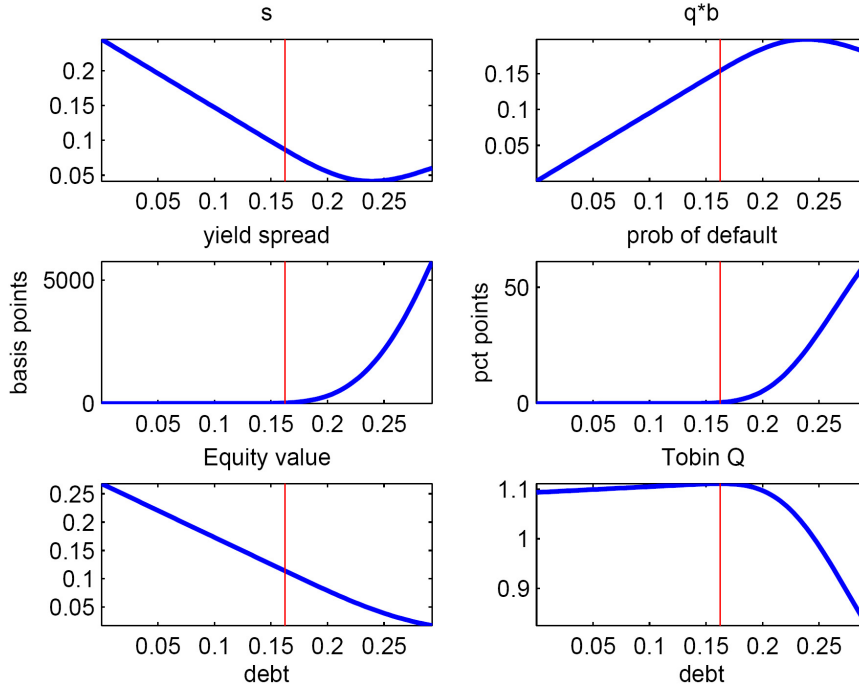


Figure 2.1: **Optimal Choice of Debt.** The figure shows the effects of debt  $b$  on the firm financing options  $s$  and  $q * b$ , the yield spread, the probability of default, the equity value  $V$ , and Tobin’s  $Q$  in the baseline model. The vertical red line shows the optimal debt choice.

amount actually raised first increases nearly one-for-one, then less than one-for-one, and finally falls as default becomes more likely. Default implies bankruptcy costs which will reduce the value available to creditors, hence at some point a decrease in the total amount raised. Obviously the firm never decides to increase debt beyond the maximum of this curve. Indeed, the third and fourth panel document that the probability of default, and hence the credit spread, has a “hockey stick” pattern as debt is increased, and the firm limits its debt issuance to remain safely left of the sharp increase. The last panel shows that the firm actually picks the debt level to maximize  $Q$ : ex-ante, the firm’s managers decide on financial policy to maximize the total firm value (i.e. we can think of the manager as deciding on debt and equity issues so as to maximize the value he can raise by selling the firm in both debt and equity markets).

We can now use this “toy model” to perform comparative statics. As is well known, an increase in the recovery rate  $\theta$  reduces the expected bankruptcy costs, and hence, according to equation

(2.7), lead to higher leverage  $z^*$ , and, according to equation (2.6), to a higher capital stock. The probability of default  $H(z^*)$  also rises, leading to a rise in the spread. The equity value falls as firms substitute debt for equity. Similarly, an increase in the tax shield parameter  $\chi$  reduces the user cost of capital and hence leads, according to equation (2.6), to a higher capital stock, higher debt, as well, a higher leverage  $z^*$ , and hence a higher probability of default, higher spreads and higher equity value. Last, an increase in the mean of  $z$  leads to a higher capital, equity value, and debt, but does not affect  $z^*$  or the probability of default. Table 2.1 summarize the result of the comparative statics.

	$k$	$b$	$z^*$	Pr(default)	Spread	$V$
$\theta$	+	+	+	+	+	-
$\chi$	+	+	+	+	+	+
Mean of $H$	+	+	0	0	0	+
Risk of $H$	-	-	+	+	+	+

Table 2.1: **Summary of the Comparative Statics.** Effect of an increase of each parameter on the endogenous variables.

We now turn to the main experiment of this section, the effect of a mean-preserving spread of the distribution  $H$ . Because the analysis depends on the exact shape of the distribution function, no analytical result is available, but we consider some numerical examples. Specifically, we assume that  $z$  is log-normally distributed, with mean  $-\frac{1}{2}\sigma^2$  and variance  $\sigma^2$ . As a result, an increase in  $\sigma$  is a mean-preserving spread of  $H$ .<sup>6</sup>

Figure 2.2 shows the responses of the endogenous variables as we increase  $\sigma$ . The specific pa-

<sup>6</sup>For any  $\sigma$ , we have  $E(z) = 1$  and we can obtain simple formula for the integrals appearing in these formulas. In particular, we use the result that if  $\log z$  is  $N(\mu, \sigma)$ , then the cumulative distribution function of  $z$  is  $H(z) = \Phi\left(\frac{\log z - \mu}{\sigma}\right)$ , where  $\Phi$  is the standard normal CDF. Moreover the probability distribution function is,

$$h(z) = \frac{1}{\sigma z} \phi\left(\frac{\log z - \mu}{\sigma}\right) = \frac{1}{\sigma z} \frac{1}{\sqrt{2\pi}} \exp\left(-\left(\frac{\log z - \mu}{\sigma}\right)^2\right),$$

where  $\phi$  is the standard normal PDF. Last, we have for all  $x$ ,

$$\int_0^x zh(z)dz = \Phi(\gamma - \sigma) E(z)$$

with  $\gamma = \frac{\log x - \mu}{\sigma}$ .

parameter values used for this example are:  $\beta = .95, \alpha = .9, \theta = .5, \chi = 1.03$ , but the results appear extremely robust to changes in these parameter values. Intuitively, the increase in volatility increases the probability of default, and hence the expected bankruptcy costs, making debt less attractive. This leads the firm to reduce its leverage, which lowers the probability of default, but does not completely offset the effect of a higher  $\sigma$ . The higher probability of default naturally pushes up the yield on the debt. Last, the higher bankruptcy costs increase the user cost, which generates a reduction in the capital stock. This numerical example suggests that shocks to volatility can generate the pattern of correlations needed to match the empirical evidence outlined in the introduction.

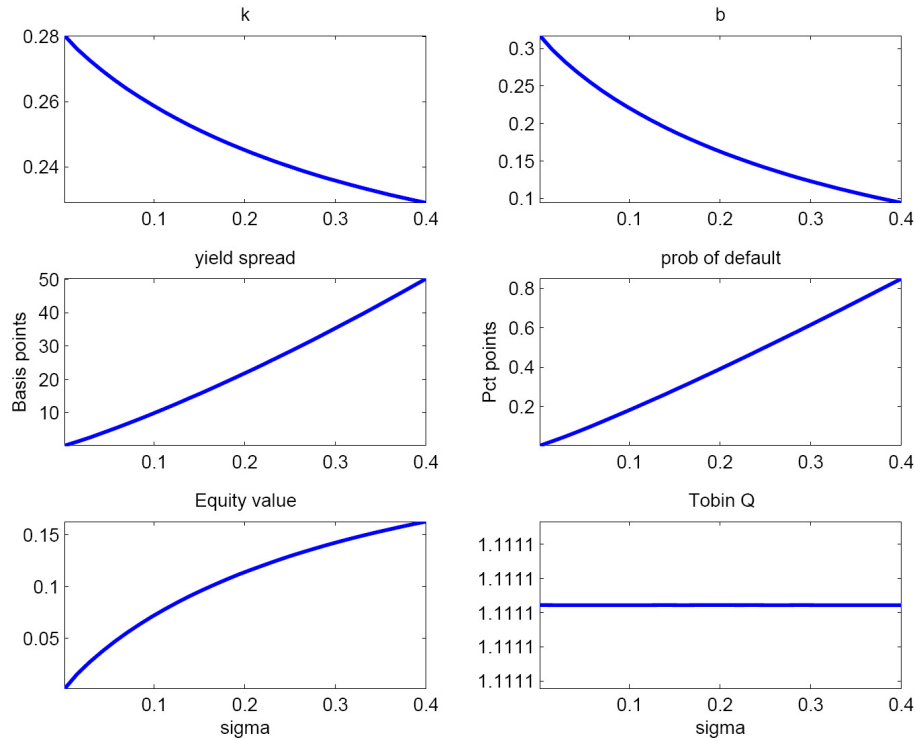


Figure 2.2: **Comparative Statics.** The figure shows the effects of volatility  $\sigma$  on the optimal firm policies  $k$  and  $b$ , the yield spread, the probability of default, the equity value  $V$ , and Tobin's  $Q$  in the baseline model.

There is however one dimension in which the model is failing: as shown in the bottom-right panel of [Figure 2.2](#), Tobin's  $Q$  is unaffected by the volatility  $\sigma$ . Indeed, it is easy to show that  $Q = \frac{1}{\alpha}$ . Intuitively, as  $\sigma$  rises, the firm value falls, and  $k$  falls, but these two quantities fall proportionately,

and hence their ratio  $Q$  is unchanged. Formally, the user cost equation (2.6) can be written as,

$$1 = \beta \alpha k^{\alpha-1} N,$$

where  $N = E(z) + \text{expected tax shield} - \text{expected bankruptcy costs}$ . Tobin's  $Q$  is actually,

$$Q = \frac{\beta k^\alpha N}{k},$$

and hence  $Q = \frac{1}{\alpha}$ , the expected average profit rate. This feature of the model, however, is highly specific to this example: the marginal value of capital is proportional to the average value, which is why  $Q$  is invariant to all parameters except  $\alpha$ . We now consider a simple extension of the example with a growth option, which breaks the proportionality of the average and marginal value of capital.

### Extension with Growth Option

We extend the model in a minimal way, by assuming that the value of operations at time 2 is not  $zk^\alpha$  but  $zk^\alpha + G(z)$ , where  $G$  is a growth option, i.e.  $G(z)$  is the expected present discounted value of future profits if  $z$  is realized.<sup>7</sup> We assume that  $G'(z) > 0$ . In our quantitative model (see Section 2.3), the growth option will be endogenously determined by the future profits of the firm.

With this modification, the firm will now default if  $z < z^*$  where

$$z^* k^\alpha + G(z^*) = b,$$

and the bond price is,

$$q(k, b) = \beta \left( \int_{z^*}^{\infty} dH(z) + \int_0^{z^*} \theta \frac{zk^\alpha + G(z)}{b} dH(z) \right),$$

where we assume that the growth option can also be recovered at rate  $\theta$ .<sup>8</sup> The firm equity value is,

$$V = \beta \int_{z^*}^{\infty} (zk^\alpha - b + G(z)) dH(z).$$

---

<sup>7</sup>To make  $G$  endogenous, we may write a three-period model, e.g. at time 2 the firm can buy capital to take advantage of the realized  $z$ .

<sup>8</sup>Alternative assumptions are possible and do not affect the main result significantly.

The firm picks  $k, b, s, z^*$  to maximize its present discounted value,  $V - s$ , which leads to two first-order conditions. First,

$$1 = \beta \alpha k^{\alpha-1} \left\{ E(z) + (\chi - 1) z^* (1 - H(z^*)) + (\theta \chi - 1) \int_0^{z^*} z dH(z) \right\},$$

which is the same as equation (2.6), since the marginal value of capital is unchanged – the only effect of the growth option is to change the default region. Second, we have,

$$(\chi - 1) (1 - H(z^*)) k^{*\alpha} = \chi (1 - \theta) (z^* k^{*\alpha} + G(z^*)) h(z^*).$$

Note that this equation simplifies to the condition (2.7) if we assume  $G(z) = 0$ .

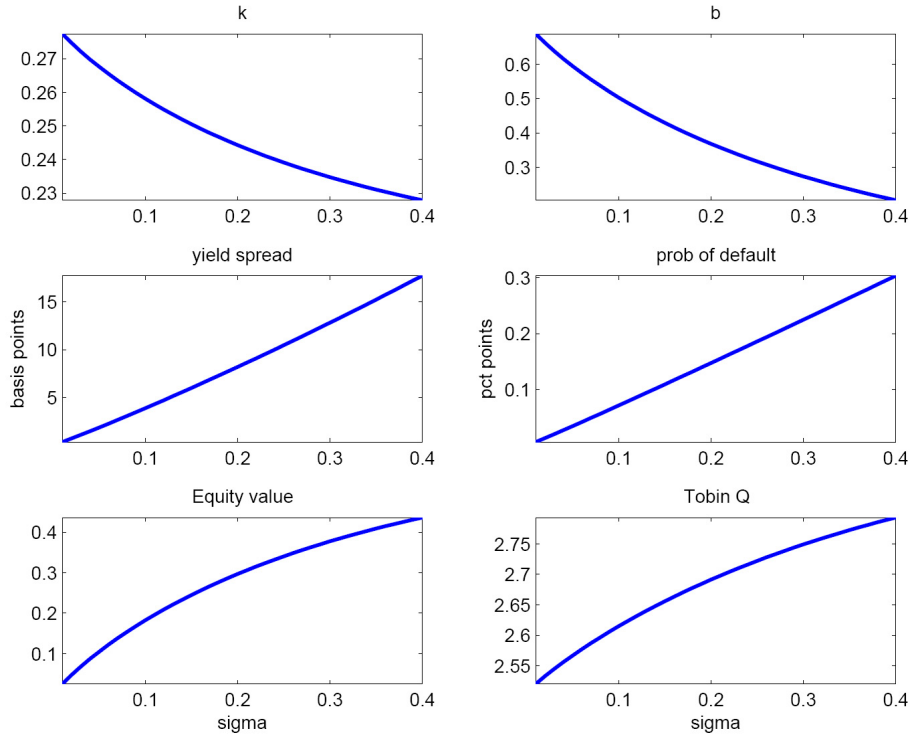


Figure 2.3: **Comparative Statics.** The figure shows the effects of volatility  $\sigma$  on the optimal firm policies  $k$  and  $b$ , the yield spread, the probability of default, the equity value  $V$ , and Tobin's  $Q$  in the model extended with a growth option.

Figure 2.3 illustrates the effect of an increase in  $\sigma$  on the firm value in this extended model.<sup>9</sup>

As in the example above,  $k$  and  $b$  fall, as expected bankruptcy costs rise, and the probability of

<sup>9</sup>The parameters are as in the first example. We set  $G(z) = \gamma z$  with  $\gamma = .4$

default, the spread and the equity value all increase. The key difference is that this model now also generates an increase in Tobin  $Q$  in response to the increase in  $\sigma$ .

To understand this, we can compute  $Q$ , which after some algebra is,

$$\begin{aligned} Q &= \frac{\beta k^\alpha \left( \int_0^\infty z dH(z) + (\chi - 1) z^* \int_{z^*}^\infty dH(z) + (\theta\chi - 1) \int_0^{z^*} z dH(z) \right)}{k} \\ &\quad + \frac{\beta \int_0^\infty G(z) dH(z) + (\chi - 1) \beta G(z^*) \int_{z^*}^\infty dH(z) + \beta (\theta\chi - 1) \int_0^{z^*} G(z) dH(z)}{k} \\ &= \frac{1}{\alpha} + \frac{\beta \int_0^\infty G(z) dH(z) + (\chi - 1) \beta G(z^*) \int_{z^*}^\infty dH(z) + \beta (\theta\chi - 1) \int_0^{z^*} G(z) dH(z)}{k}, \end{aligned}$$

i.e.  $Q$  equals the expected profit rate  $\frac{1}{\alpha}$ , plus the ratio of the expected value of the growth option and its tax shield, net of bankruptcy costs, to the capital stock. An increase in  $\sigma$ , to a first order, does not affect this expected value, but it does reduce the capital stock and hence increases  $Q$ . Because the average and marginal value of capital are not proportional any more, the model can generate the intuitive result for all four key variables (capital, yields, equity value and Tobin  $Q$ ).

This example has highlighted the key mechanism of the model. We now turn to a fully dynamic model with more realistic features, to provide a quantitative evaluation of the strength of the mechanism.

## 2.3 Quantitative Model

This section presents a partial equilibrium dynamic model of investment and financing decisions. This model is an extension of the static model presented in [Section 2.2](#). Firms are ex-ante identical but become heterogeneous ex-post because of different idiosyncratic realizations of productivity and volatility shocks. As in the static model, firms can borrow in the form of a one-period defaultable debt. Bankruptcy costs are incurred upon default and the interest expense on debt can be deducted from taxable income. The model builds on the recent corporate finance literature<sup>10</sup>, and more specifically is an extension of the recent study of Gomes and Schmid (2009). The main difference is that we explicitly introduce stochastic volatility shocks in order to quantify the effect of stochastic

---

<sup>10</sup>See Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2005).



volatility on investment, credit spreads, and Tobin's  $Q$ .

### 2.3.1 Firm Problem

The dynamic programming problem for a firm is to choose capital  $k'$ , debt  $\hat{b}'$ , and a default policy to maximize the present value of dividends. The idiosyncratic productivity shock  $z$  and volatility shock  $\sigma$  are exogenous shocks. Firms use physical capital as input to their production function. Operating profits are denoted by  $\pi(k, z)$  and are strictly increasing and strictly concave in capital, that is  $\pi_k > 0$  and  $\pi_{kk} < 0$ . Capital depreciates at rate  $\delta$ , thus investment in capital follows the equation,

$$i = k' - (1 - \delta)k.$$

Firms have access to lenders and can borrow in the form of one-period debt contracts. The coupon rate for a newly issued bond is denoted  $c'$ . Bonds are priced competitively. Profits are taxed at a rate  $\tau \in [0, 1]$  and interest expense can be deducted from taxable income. The dividend for the firm is defined as the sum of after-tax profits and new debt issues, net of debt repayment, investment and investment cost,

$$\tilde{d} = (1 - \tau)\pi(k, z) + \hat{b}' - \hat{b}(1 + (1 - \tau)c) - i - \phi(i, k).$$

where  $\phi(i, k)$  is the cost of investing  $i$  from a capital level  $k$ . Financing frictions are present in the form of costly equity issuances. Following Gomes (2001), equity can be issued at a proportional cost  $\lambda$ , that is dividend net of equity issuance is given by,

$$d = (1 + \lambda \mathbf{1}_{\{\tilde{d} < 0\}})\tilde{d},$$

where  $\mathbf{1}_{\{\tilde{d} < 0\}}$  is the indicator function of strictly negative dividends, i.e.  $\mathbf{1}_{\{\tilde{d} < 0\}} = 1$  if  $\tilde{d} < 0$  (and 0 otherwise). The value of the firm continuing operations is given by,

$$V(k, \hat{b}, s) = \max_{k', \hat{b}'} d + \beta \mathbb{E} \left[ \max \left( 0, V(k', \hat{b}', s') \right) \right],$$

where  $\beta \in [0, 1]$  is the discount factor that firms take as given. The vector  $s$  contains all the exogenous state variables in the economy. Specifically, the idiosyncratic productivity shock  $z$  and volatility shock  $\sigma$  are included in the exogenous state vector.

### 2.3.2 Lenders

There is a continuum of competitive risk-neutral lenders. Any debt contract has to satisfy the lenders' Euler equation. Risky debt  $\hat{b}'$  is supplied to firms at a coupon rate  $c'$  such that,

$$\hat{b}' = \mathbb{E} \left[ \beta \left\{ (1 + c') \hat{b}' \mathbf{1}_{\{V' \geq 0\}} + \xi (\pi(k', z') + (1 - \delta)k') (1 - \mathbf{1}_{\{V' \geq 0\}}) \right\} \right].$$

In this equation,  $c'$  is the coupon rate, and  $\xi$  is the recovery rate upon default. We assume that only current profits and the capital stock can be recovered.

For computational reasons, it is convenient to rewrite debt to be inclusive of interest payment and net of tax shield subsidy,

$$b \equiv (1 + (1 - \tau)c) \hat{b}.$$

Following Gomes and Schmid (2009), the pricing equations can be rewritten as follows,

$$\hat{b}' = \frac{\beta \mathbb{E} \left[ \frac{1}{1-\tau} b' \mathbf{1}_{\{V' \geq 0\}} + \xi (\pi(k', z') + (1 - \delta)k') (1 - \mathbf{1}_{\{V' \geq 0\}}) \right]}{1 + \frac{\tau}{1-\tau} \beta \mathbb{E} [\mathbf{1}_{\{V' \geq 0\}}]}.$$

and the coupon rates can be written,

$$c'(k', b', s) = \frac{1}{1 - \tau} \left( \frac{1 + \frac{\tau}{1-\tau} \beta \mathbb{E} [\mathbf{1}_{\{V' \geq 0\}}]}{\beta \mathbb{E} \left[ \frac{1}{1-\tau} \mathbf{1}_{\{V' \geq 0\}} + \xi \frac{\pi(k', z') + (1-\delta)k'}{b'} (1 - \mathbf{1}_{\{V' \geq 0\}}) \right]} - 1 \right). \quad (2.8)$$

### 2.3.3 Recursive Formulation of the Firm Problem

The firm problem is stated formally in the following.

**Problem 2.3.1** (Recursive Formulation of the Firm Problem). *Given the coupon schedule  $c'(k', b', s)$ , firms solve the following program,*

$$V(k, b, s) = \max_{k', b'} d + \beta \mathbb{E} [\max(0, V(k', b', s'))],$$

subject to,

$$d = (1 + \lambda \mathbf{1}_{\{\bar{d} < 0\}}) \left( (1 - \tau) \pi(k, z) + \frac{1}{1 + (1 - \tau) c'(k', b', s)} b' - b - i - \phi(i, k) \right), \quad (2.9)$$

$$i = k' - (1 - \delta)k. \quad (2.10)$$

## 2.4 Numerical Results

This section describes our approach to solve the model in [Section 2.3](#). There are no closed-form expressions for this equilibrium, therefore numerical techniques are employed to solve it. We explain the choice of the key parameters of the model and describe the numerical approach.

### 2.4.1 Parameter Choice

The model is calibrated at the quarterly frequency. [Table 2.2](#) reports the parameters we use for the numerical exercise. The preference parameter  $\beta$  (i.e. the subjective discount rate) is set to generate a 2% yearly risk free rate.

The productivity process  $z$  is parameterized along the lines of Gomes (2001) and Hennessy and Whited (2005). The productivity process  $z$  is restricted to follow a first order autoregressive process with normal innovations. Specifically,

$$z' = \rho_z z + \sigma \epsilon, \quad (2.11)$$

where  $\epsilon_{t+1}$  are independently and identically distributed shocks drawn from a standard normal distribution. In order to match the persistence of firms' output, the productivity process has to be persistent, that is we set  $\rho_z = 0.97$ . These numbers are somewhat standard in the literature for this type of exercise. The productivity process is represented by a discrete Markov chain using 50 points. The stochastic volatility process is assumed to be a Markov chain with 3 states  $\sigma \in \{\sigma_L, \sigma_M, \sigma_H\}$  and transition matrix  $\Gamma_{\sigma\sigma'}$ .

Firm profits exhibit decreasing returns in physical capital and the firm must pay a fixed cost of

operation  $f$  each period. Profits are assumed to take the following functional form,

$$\pi(k, z) = e^z k^\alpha - f, \quad (2.12)$$

where  $\alpha \in (0, 1)$ .

Following the Q theory literature,<sup>11</sup> the capital adjustment cost function  $\phi(i, k)$  is assumed to be quadratic in investment rates, given by,

$$\phi(i, k) = \frac{1}{2} \theta_1 \left( \frac{i}{k} - \delta \right)^2 k \mathbf{1}_{\{i/k < \delta\}} + \frac{1}{2} \theta_2 \left( \frac{i}{k} - \delta \right)^2 k \mathbf{1}_{\{i/k > \delta\}}, \quad (2.13)$$

where  $\theta_1$  and  $\theta_2$  are parameters which capture the asymmetry between the cost of investing and disinvesting. This formulation has been previously considered by Zhang (2005).

Cost of equity issuances are chosen to be 25%. This figure is reasonable as Hennessy and Whited (2008) estimated these issuance costs to be about 6% using the simulated method of moments. The average tax rate is chosen to be 20%. Bankruptcy costs are assumed to correspond to a dead-weight cost of 40% of current capital stock, thus the recovery rate is set to  $\xi = 0.6$ . The set of parameters used to solve the model is summarized in [Table 2.2](#).

Preference	$\beta = 0.995$	Risk free rate
	$\alpha = 0.4$	Production parameter
	$\theta_1 = 1$	Adjustment cost parameter
Technology	$\theta_2 = 0.1$	Adjustment cost parameter
	$f = 5$	Fixed cost of operation
	$\delta = 0.03$	Capital depreciation rate
	$\lambda = 0.25$	Linear cost of issuing equity
Institution	$\xi = 0.6$	Recovery rate in event of bankruptcy
	$\tau = 0.2$	Average corporate tax rate
	$\rho_z = 0.97$	Autocorrelation of $z$
	$\sigma_L = 0.1$	Low Volatility of $z$
Uncertainty	$\sigma_M = 0.25$	Medium Volatility of $z$
	$\sigma_H = 0.4$	High Volatility of $z$

Table 2.2: **Parameter Values.** The model is calibrated at the quarterly frequency.

In order to understand the effect of the stochastic volatility in this environment, we will contrast

<sup>11</sup>For example, see Eberly, Rebelo and Vincent (2009).

the results between a model with deterministic volatility and a model with stochastic volatility. The level of volatility used in the deterministic is equal to the long-run average volatility of the stochastic volatility Markov chain. Given the parameters of our computations, the model with deterministic volatility has  $\sigma = 0.25$ .

## 2.4.2 Basic Properties

The model parameters have been selected so as to match moments of the data. The moments of the data are taken from Eberly, Rebelo, and Vincent (2009). Their sample of firms are taken from Compustat in the period 1981-2003, and they focus on the top quartile of firms sorted by size of the capital stock in 1981. The book leverage statistics are taken from Chen, Collin-Dufresne, and Goldstein (2008).

Variable		Data	Model Deterministic $\sigma$	Model Stochastic $\sigma$
Real Side:		Mean		
<i>Tobin's Q</i>	Time-series Average	1.298	1.452	1.485
	Time-series Volatility	0.625	0.606	0.626
	Serial Correlation	0.838	0.514	0.471
<i>i/k</i>	Time-series Average	0.150	0.079	0.084
	Time-series Volatility	0.055	0.363	0.348
	Serial Correlation	0.600	0.048	0.027
Financing Side:		Mean		
<i>b/k</i>	Time-series Average	0.450	0.518	0.525
	Time-series Volatility	0.090	0.155	0.152
	Serial Correlation	N.A.	0.935	0.931
<i>c - r<sup>f</sup></i>	Time-series Average (bps)	109	31	57
	Time-series Volatility (bps)	41	46	80
	Serial Correlation	N.A.	0.008	0.017

Table 2.3: **Summary Statistics.** The summary statistics in the data are taken from Chen, Collin-Dufresne, and Goldstein (2008) and Eberly, Rebelo, and Vincent (2009). The model economy is simulated for a panel of 100 firms over 200 periods. Averages and volatilities for bond yields are quoted in basis points.

Our model economy is solved and simulated for a panel of 100 firms over 200 periods. Summary statistics of the firms' optimal policies along with the data counterpart are given in [Table 2.3](#).

Although the model is solved at the quarterly frequency, all moments are reported at the annual frequency. Most of the moments in the model economy are reasonably close the data moments. This quantitative exercise shows that the model can in fact rationalize the aggregate quantities for firms, both on the real side and the financing side. However it is important to note that the credit spreads produced in the model are not as high as in the data. The model falls short for the deterministic volatility model (31 basis points) and even for the stochastic volatility model (57 basis points), compared to 109 basis points found in the data.

### 2.4.3 Predictability Properties

In this subsection, we investigate the performance of the model to replicate some standard corporate finance regressions. We are interested in understanding how stochastic volatility impacts the co-movements between investment rate and corporate bond yields.

The results for the regressions of investment on  $Q$  and spreads were provided to us by Gilchrist and Zakrajsek. These authors assembled firm-level data on investment, Tobin  $Q$  and credit spreads by merging the standard Compustat database with data on individual bond issues. The credit spread of a firm is defined as the weighted average of the spread on each of its bond issues, where the weights are the market value of each issue. Spreads are observed at the monthly frequency, and the annual spread is the time-series average of the monthly spreads. The data covers the period 1983-2006 and is an unbalanced panel with 799 firms. Their results are summarized in [Table 2.4](#).

Variable	$i/k$			$\log(i/k)$		
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(c)$	-0.035 (0.005)		-0.034 (0.005)	-0.297 (0.024)		-0.277 (0.025)
$\log(Q)$		0.051 (0.016)	0.0015 (0.0018)		0.241 (0.049)	0.156 (0.046)
$R^2$ (within)	0.054	0.064	0.062	0.208	0.146	0.203

Table 2.4: **Data.** Regressions results from Gilchrist and Zakrajsek (2009).

Regression results for the model economy along with the data counterpart are given in [Table 2.5](#).

The first result is that credit spreads explain about 5% of the variation in investment rates in the simulated data for both models, as in the data. However the magnitude of the regression coefficients is much higher than in the data. This result is mitigated in the model with stochastic volatility. The second result is that the addition of  $Q$  in the regressions improves the fit dramatically, from an R-squared of about 5% to 70% for the deterministic volatility model, and to 60% for the stochastic volatility model. In contrast,  $Q$  does not improve the explanatory power much beyond credit spreads in the data, from an R-squared of 5.4% to 6.2%. Again this problem is mitigated in the model with stochastic volatility. Overall the model with stochastic volatility performs better than the model with deterministic volatility.

Variable	Data		Model		Model	
			Deterministic $\sigma$		Stochastic $\sigma$	
$\log(c)$	-0.035 (0.005)	-0.034 (0.005)	-0.166 (0.026)	-0.045 (0.015)	-0.115 (0.017)	-0.047 (0.011)
$\log(Q)$		0.002 (0.002)		0.357 (0.008)		0.322 (0.009)
$R^2$	0.054	0.062	0.046	0.704	0.047	0.601

Table 2.5: **Regression Results.** The regressions results in the data are taken from Gilchrist and Zakrajsek (2009). Regressions in the model are run on data simulated for a panel of 100 firms over 200 periods.

## 2.5 Conclusion

We introduced stochastic volatility in a standard model of corporate finance and investment. We showed that, consistent with intuition, this additional shock can generate higher correlations between credit spreads and investment, and weaker correlations between  $Q$  and investment, as in the data. However, the correlation between spreads and investment is still too small, and the correlation between  $Q$  and investment is still too high, compared to the data.

In future work, we plan to introduce some extensions which might magnify the effect of stochastic

volatility. First, we may introduce risk-aversion through an exogenous stochastic discount factor. Second, we may introduce long-term debt. Third, we may consider different specifications for adjustment costs. We also plan to study empirically the correlation of firm-level stock returns and bond returns, to measure directly the amount of stochastic volatility in the data.

A possible extension of the project is to consider the macroeconomic implications of the model, when the shock to volatility is correlated across firms. A lower volatility, such as the “Great Moderation”, would lead firms to take on more debt, and increase leverage and investment. In the process, they might become more sensitive to productivity (or other) shocks.



# Bibliography

- [1] Andrew Abel. Optimal investment under uncertainty. *The American Economic Review*, 73(1):228–233, 1983.
- [2] Andrew Abel and Janice Eberly. How q and cash flow affect investment without frictions: An analytic explanation. *Mimeo, Wharton*, 2008.
- [3] Ruediger Bachmann and Christian Bayer. Firm-Specific Productivity Risk Over the Business Cycle: Facts and Aggregate Implications. *SSRN eLibrary*, 2009.
- [4] Ben S. Bernanke. Irreversibility, uncertainty, and cyclical investment. *The Quarterly Journal of Economics*, 98(1):85–106, 1983.
- [5] Nicholas Bloom. The impact of uncertainty shocks. *Econometrica*, 77(3):623–685, 05 2009.
- [6] Nicholas Bloom, Stephen Bond, and John van Reenen. Uncertainty and investment dynamics. *The Review of Economic Studies*, 74(2):391–415, 2007.
- [7] Ricardo J. Caballero. On the sign of the investment-uncertainty relationship. *The American Economic Review*, 81(1):279–288, 1991.
- [8] Ricardo J. Caballero. Chapter 12 aggregate investment. In John B. Taylor and Michael Woodford, editors, *Handbook*, volume 1, Part 2 of *Handbook of Macroeconomics*, pages 813 – 862. Elsevier, 1999.

- [9] Thomas F. Cooley and Vincenzo Quadrini. Financial markets and firm dynamics. *The American Economic Review*, 91(5):1286–1310, 2001.
- [10] Russell Cooper and Joo Ejarque. Financial frictions and investment: requiem inq. *Review of Economic Dynamics*, 6(4):710 – 728, 2003. Finance and the Macroeconomy.
- [11] A.K. Dixit, R.S. Pindyck, and G.A. Davis. *Investment under uncertainty*. Princeton University Press Princeton, NJ, 1994.
- [12] Janice Eberly, Sergio Rebelo, and Nicolas Vincent. Investment and value: A neoclassical benchmark. *Mimeo, Kellogg*, 2009.
- [13] Timothy Erickson and Toni M. Whited. Measurement error and the relationship between investment and q. *Journal of Political Economy*, 108(5):1027–1057, October 2000.
- [14] Simon Gilchrist and Charles P. Himmelberg. Evidence on the role of cash flow for investment. *Journal of Monetary Economics*, 36(3):541 – 572, 1995.
- [15] Simon Gilchrist and John C. Williams. Investment, capacity, and uncertainty: a putty-clay approach. *Review of Economic Dynamics*, 8(1):1 – 27, 2005.
- [16] Simon Gilchrist and Egon Zakrajsek. Investment and the cost of capital: New evidence from the corporate bond market. *SSRN eLibrary*, 2007.
- [17] Joao F. Gomes. Financing investment. *The American Economic Review*, 91(5):1263–1285, 2001.
- [18] Joao F. Gomes and Lukas Schmid. Levered returns. *The Journal of Finance*, 2009.
- [19] L. Guiso and G. Parigi. Investment and demand uncertainty. *Quarterly Journal of Economics*, 114(1):185–227, 1999.
- [20] Fumio Hayashi. Tobin’s marginal q and average q: A neoclassical interpretation. *Econometrica*, 50(1):213–224, 1982.

- [21] Christopher A. Hennessy and Toni M. Whited. Debt dynamics. *The Journal of Finance*, 60(3):1129–1165, 2005.
- [22] John V. Leahy and Toni M. Whited. The effect of uncertainty on investment: Some stylized facts. *Journal of Money, Credit and Banking*, 28(1):64–83, 1996.
- [23] Martin Lettau and Sydney Ludvigson. Time-varying risk premia and the cost of capital: An alternative implication of the q theory of investment. *Journal of Monetary Economics*, 49(1):31–66, 2002.
- [24] R.S. Pindyck. Irreversible investment, capacity choice, and the value of the firm. *The American Economic Review*, 78(5):969–985, 1988.
- [25] R.S. Pindyck. A note on competitive investment under uncertainty. *The American Economic Review*, 83(1):273–277, 1993.
- [26] James H. Stock and Mark W. Watson. New indexes of coincident and leading economic indicators. *NBER Macroeconomics Annual*, 4:351–394, 1989.
- [27] Lu Zhang. The value premium. *The Journal of Finance*, 60(1):67–103, 2005.