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Abstract

The surfaces of alluvial fans and river deltas (collectively fans) are often dissected by a small number of channels radiating from the fan apex. On long timescales, channels migrate via avulsion, the process of channel bed deposition and abandonment that often results in catastrophic flooding and loss of life on densely populated fans. We present results of an experimental fan that creates realistic channel patterns by avulsion. The avulsion cycle occurs with a period that is predictable from conservation of mass. Selection of a new flow path is inherently stochastic; however, once a network of 4–5 channels is established, flow oscillates among these channels indefinitely. We demonstrate that a directed random walk model with memory quantitatively reproduces these dynamics and limiting behavior, and is consistent with natural fans.

Disciplines

Earth Sciences | Environmental Sciences | Hydrology | Physical Sciences and Mathematics | Sedimentology

Flooding and flow path selection on alluvial fans and deltas

Meredith D. Reitz,[1] Douglas J. Jerolmack,[2] and John B. Swenson[3]

[1]   The surfaces of alluvial fans and river deltas (collectively fans) are often dissected by a small number of channels radiating from the fan apex. On long timescales, channels migrate via avulsion, the process of channel bed deposition and abandonment that often results in catastrophic flooding and loss of life on densely populated fans. We present results of an experimental fan that creates realistic channel patterns by avulsion. The avulsion cycle occurs with a period that is predictable from conservation of mass. Selection of a new flow path is inherently stochastic; however, once a network of 4–5 channels is established, flow oscillates among these channels indefinitely. We demonstrate that a directed random walk model with memory quantitatively reproduces these dynamics and limiting behavior, and is consistent with natural fans. **Citation:** Reitz, M. D., D. J. Jerolmack, and J. B. Swenson (2010), Flooding and flow path selection on alluvial fans and deltas, *Geophys. Res. Lett.*, *37*, L06401, doi:10.1029/2009GL041985.

1.   Introduction

[2]   On 18 August 2008, an embankment was breached on the Kosi River fan in Northern India. The channel, which had been filling slowly with sediment and was kept in place by artificial embankments, was unable to contain the modest discharge, and the resulting floodwaters displaced more than one million people and killed up to several thousand [*Sinha*, 2009]. The inundation pattern (Figure 1) shows that water principally filled previously abandoned channel paths, flooding areas more than 100 km away. As floodwaters receded, flow settled into a channel that had been abandoned ~100 yr prior [*Sinha*, 2009]. Flood hazard models on alluvial fans cannot predict such history-influenced flood risks because they do not account for channel reoccupation effects in flow switching due to avulsion [*Pelletier et al.*, 2005]. Although catastrophic, the avulsion cycle is a natural process that, over geologic time, constructs large fans such as those of the Kosi [*Parker et al.*, 1998; *Wells and Dorr*, 1987], Ganges-Brahmaputra [*Allison et al.*, 2003], Mississippi [*Coleman*, 1988; *Tornqvist et al.*, 1996; *Aslan et al.*, 2005], Rhine-Meuse [*Tornqvist*, 1994] and Yellow [*Slingerland and Smith*, 2004; *Wu et al.*, 2005] Rivers. Though alluvial fans and river deltas are fan types distinguished by differences in steepness, setting, and channel characteristics, both may be studied here as channelized distributary systems in which the avulsion process is driven primarily by large-scale deposition-induced channel routing. The dynamics of channel avulsions have a profound impact not only on the vast human and ecological communities that inhabit fans [*Neill and Deegan*, 1986; *Syvitski et al.*, 2005], but also on the generation of sedimentary deposits that act as hydrocarbon and water reservoirs [*Leeder*, 1978; *Mohrig et al.*, 2000].

2.   Results and Discussion

[3]   Avulsion and flow switching are difficult to study directly in natural systems because processes typically operate on timescales of centuries to millennia [*Tornqvist et al.*, 1996; *Aslan et al.*, 2005; *Tornqvist*, 1994]. To overcome this challenge, we performed a set of laboratory experiments in order to isolate the dynamics of the avulsion process during fan evolution (Figure 1). Parameters were chosen to simulate conditions of steep alluvial fans (Methods, auxiliary material).[4] After a transient initial phase, avulsion and channel migration followed a surprisingly regular cycle that was remarkably similar to that seen on the Kosi River fan (Figure 1): (1) flow abruptly collapses into a single channel (Figure 1a), focusing rapid deposition at the channel mouth; (2) slope decreases due to progradation until the channel can no longer transport the supplied load; then (3) within-channel deposition drives backfilling from the fan margin toward the apex (Figure 1b) resulting in flooding; (4) this produces a brief "finding phase" [*Martin et al.*, 2009] in which myriad potential flow paths are filled with water (Figure 1c); the cycle ends when a new path is selected and flow collapses back into a single channel, with the avulsion node located at the fan apex (Figure 1d). This avulsion cycle was similar to that seen in recently reported cohesive sediment experiments [*Edmonds et al.*, 2009; *Martin et al.*, 2009] but occurred without sediment cohesion or the deposition of mouth bars, indicating that decreasing channel slope due to progradation is sufficient to drive channel backfilling and avulsion.

[4]   Water discharge during experiments was constant, so that flooding resulted solely from the channel filling with sediment and breaching the river bank, as was also true for the 18 August 2008 Kosi River avulsion [*Sinha*, 2009]. Time series analysis shows that fluctuations in wet fraction (area fraction of fan covered by water) and mean fan margin position (r) were periodic (Figures 2 and S1), and both had characteristic timescales of 20 minutes over the first 80 minutes of the run. A decorrelation analysis of channel position revealed the same timescale (Figure S1), verifying that avulsion and channel filling drive key dynamics on the fan. The slow process of channel backfilling accounted for

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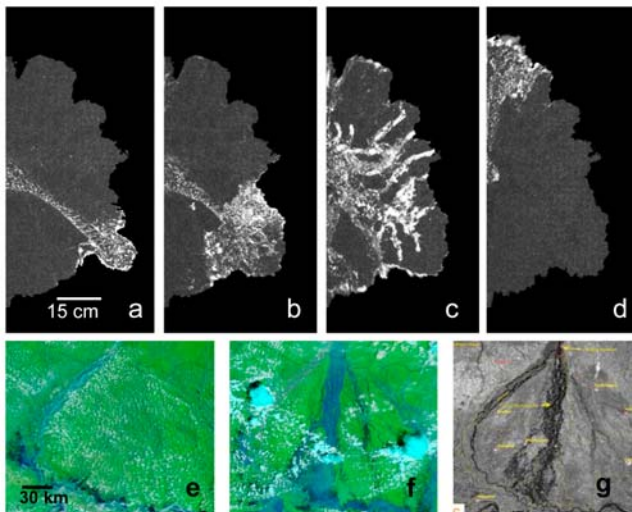
Figure 1. (a–d) A typical avulsion sequence from experiments, with white indicating location of water. See text for description. (e–g) August 2008 Kosi Fan avulsion. (e) Image acquired on 8 August 2008 before avulsion. (f) 2 September 2008 showing flood inundating multiple abandoned channels. (g) 9 October 2008, where flow has reoccupied a ~100-yr-old abandoned channel. Figures 1e and 1f are from the NASA Earth Observatory (http://earthobservatory.nasa.gov), and Figure 1g is from the Indian National Remote Sensing Agency (www.nrsa.gov.in).

the majority of the avulsion period (Figure 2b), suggesting that the characteristic avulsion timescale T_A may be estimated as the time to fill a channel belt of typical depth (h) and average wetted width (B) with bed material sediment at the imposed input rate (Q_s):

$$T_A(t) = \frac{hBr(t)}{Q_s}. \tag{1}$$

[5] Note that, for a prograding fan, the avulsion period is predicted to increase as fan radius grows. We calculate an avulsion period ranging from 20 min to 45 min over the duration of the run (Figure S2), in excellent agreement with timescales calculated for the first 80 minutes. Results agree with earlier findings that avulsion frequency increases with deposition rate [*Bryant et al.*, 1995], and confirm a proposed hypothesis [*Jerolmack and Mohrig*, 2007] that avulsion frequency is determined principally by the ratio of bed deposition rate to channel depth. The avulsion period of equation (1) sets the pace of fan evolution; a dynamic similarity exists between fans in the laboratory and the field when scaled by this timescale. Observed avulsion period for the progradational Late Holocene Mississippi Delta is $T_A \approx$ 1300 yr [*Aslan et al.*, 2005], while predicted $T_A \approx$ 1500 yr ($h =$ 25 m, $B =$ 1.5 km, $r =$ 400 km, $Q_s = 10^7$ m³/yr [*Aslan et al.*, 2005]). Observed $T_A \approx$ 25 yr for the Kosi River alluvial fan [*Wells and Dorr*, 1987] is also close to the predicted value $T_A \approx$ 30 yr ($h =$ 2 m, $B =$ 2 km, $r =$ 150 km, $Q_s =$ 2 × 10⁷ m³/yr [*Nayak*, 1996]).

[6] Although selection of new channel paths following flooding was unpredictable, it exhibited a coherent limiting behavior. Once a small number of channels were created,

subsequent avulsions consistently returned flow to previously established paths rather than create new ones (Figures 4a and 4c) - even when previous channels appeared to have been entirely filled with sediment (Figure S3). Over timescales much longer than the avulsion period, an "active channel set" emerged, consisting of 4 to 5 channels that were periodically revisited by flow (Figures 3a and 4c), providing the first experimental confirmation of numerical predictions [*Jerolmack and Paola*, 2007]. Statistics of channel migration were demonstrably nonrandom (Figure S4) and showed a strong tendency for flow to reoccupy abandoned channels (Figures 3 and 4). Non-channelized areas filled in slowly during flooding events (Figures 1b and 1c). Channels in the active channel set were never completely erased over the duration of the experiment. These channels often appeared as relict floodplain scars in images of the fan surface (Figure S5) but displayed little topographic relief, significantly less than the depth of active channels. Thus abandoned channel bottoms were not significant topographic lows, yet they acted as attractors for future flow. Channel reoccupation dynamics and resultant network structure were consistent with observed behavior of avulsion-affected fans [*Wu et al.*, 2005; *Wells and Dorr*, 1987; *Allison et al.*, 2003; *Tornqvist et al.*, 1996; *Aslan et al.*, 2005; *Slingerland and Smith*, 2004].

[7] Here we present a minimal model that captures the essential limiting behaviors of avulsion-dominated fans, in which channel paths are directed random walks in a system with memory. At each model time step an avulsion occurs at the apex of the fan, and channel path is chosen as a directed random walk that is terminated when it either (a) reaches the fan margin, or (b) collides with a previous flow path. Abandoned channels cannot persist indefinitely; in our
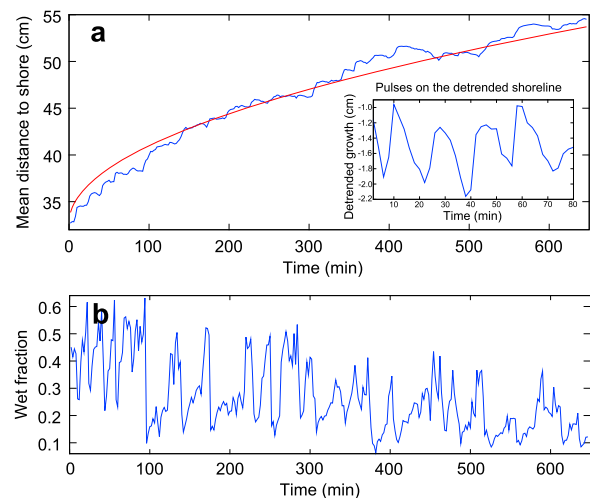

Figure 2. (a) Growth of mean fan margin position r of the experimental fan with time t. The trend follows the relation $r \sim t^{1/2}$ predicted from mass conservation [*Swenson et al.*, 2000]. Superimposed on this trend are periodic pulses, shown in inset figure of detrended $r(t)$, that result from the avulsion sequence. (b) Fluctuation of the wet fraction f with time. Troughs represent times of strong channelization, followed by gradual increases in f due to backfilling, and peaks in f corresponding to flooding phases (similar to fluctuation seen by *Kim and Jerolmack* [2008], but at a different timescale).
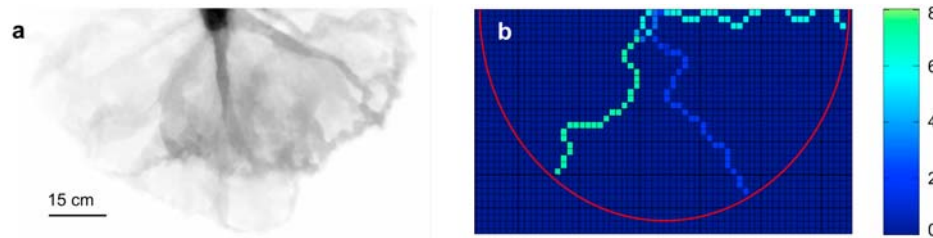
Figure 3. (a) Map showing cumulative duration of inundation by water on the experimental fan surface over 300 minutes (~$15T_A$). About two channel paths are visibly dominant over this time. (b) Map showing cumulative number of times any cell has been visited by flow over 15 timesteps ($15T_A$) of our numerical model ($n = 25$). A small number of dominant channel paths are visible in a manner similar to the experimental image.

simple treatment of floodplain processes, flow paths that have not been visited for n number of timesteps are erased ("annealed") (Methods). We explored the dynamical effects as n was varied from 1 to infinity (i.e., no memory of old channels to perfect memory). We find that the average number of active channels increases slowly with annealing time (Figures S6b and S6e). In the limit of long annealing time ($n > \sim 10$) the model achieves a dynamic steady state in which flow oscillates among a set of 3 to 5 channels (Figures 4b and 4d), in good agreement with our experiments and with avulsion-dominated fans in nature. In addition, for large n we find that the probability of reoccupying an old channel after avulsion – as opposed to creating a new flow path – approaches unity (Figures S6c and S6f). Model results (Figures S6a and S6d) suggest that our laboratory fan should have an annealing time of 25 T_A, or about 500 minutes, which is comparable to the duration of the experiment. We did not see significant annealing of active channels throughout a run, providing additional support for the model. Qualitative fan patterns of reoccupation and flow switching for this annealing time also correspond well with our experimental results (Figures 3b, 4b, and 4d).

[8] Our model predicts that the competition between channel reactivation by reoccupation and channel erasure by annealing determines the statistics of the channel network. Avulsion frequency is dictated by the supply rate of coarse bed material sediment that fills channels [*Jerolmack and Mohrig*, 2007; *Martin et al.*, 2009], while annealing results from floodplain deposition of fine sediment (i.e., washload) [*Aslan and Autin*, 1999; *Aalto et al.*, 2008]. We expect annealing to be more rapid in lowland deltas due to the presence of mud, leading to less persistent channel networks. We predict that annealing should be slower on steep alluvial fans where floodplain sedimentation is minimal, conditions consistent with our experiments (Figure S7). Our results demonstrate that in slow-annealing settings fans develop a semi-permanent network of channels that accumulate and distribute sediment. The mean recurrence interval of avulsion and the mean rate of fan growth are deterministic functions of applied boundary conditions, while channel switching is predictable only in a statistical sense. Experimental channels are abandoned because local fan margin progradation decreases river slope and hence transport capacity, causing channels to fill with sediment. Abandoned channels persist as attractors for flow, how-
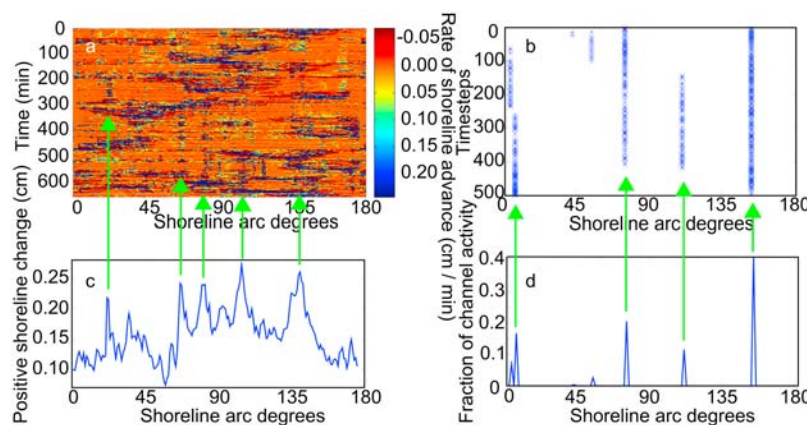
Figure 4. (a) Arc-length locations (x-axis) of fan margin progradation rate through time (y-axis) over duration of the experiment. Slow to fast progradation is visualized as orange to blue color. Vertical packets of rapid progradation indicate persistence of a channel at one fan margin location, and horizontal lines show brief, widespread deposition of a flooding event. Averaging progradation rate at each fan margin point through time (c) reveals an active channel set (arrows) that persists throughout the run. (b) Similar plot for numerical model over 500 timesteps ($n = 25$). Because model incorporates channelization stage of the avulsion sequence but not flooding stage, vertical packets are apparent but horizontal lines are not. (d) Numerical model results showing fraction of run time that channels spent at each fan margin position, revealing an active channel set similar to experiments.

ever, because they represent coherent downslope (steepest-descent) paths; after sufficient progradation and/or aggradation elsewhere on the fan, these paths become favorable again.

[9] Our directed random walk model obviously neglects many processes that are crucial for accurate modeling of alluvial fans and deltas. That this model succeeds in capturing the essential stochastic dynamics of channel migration implies that important aspects of channel behavior are a generic result of nonlinear, threshold dynamics with memory. This notion is supported by qualitative similarities with earthquake dynamics where accumulated strain energy from slow tectonic motion is rapidly released during rupture, which intermittently alternates among a network of faults [*Sammis and Smith*, 1999].

3. Methods Summary

3.1. Experiments

[10] Our experimental basin was $1 \times 1 \times 3$ m, with a constant water depth of 0.03 m. Sediment and water feed rates were constant at 10^{-6} m³/s and 10^{-4} m³/s, respectively. Sediment and water were mixed in a funnel above the basin and directed to the basin edge by a vertical tube. Sediment mixture by volume was 20% granite chips, 2 mm in diameter with a density of 3 g/mL, and 80% acrylic sand, 0.3 mm in diameter with a density of 1.15 g/mL. Other mixtures were also used and generated qualitatively similar behavior. Fan growth was by fan margin progradation in a quasi-radial pattern, with accompanying fan surface deposition to maintain equilibrium transport slope; radially-averaged behavior was well predicted by one-dimensional mass conservation models [*Parker et al.*, 1998; *Swenson et al.*, 2000] (Figure 2a). We observed a strong first-order grain size sorting as the fan grew, in which coarse grains were extracted first to make a steep upper fan, while fine grains deposited downstream to produce a bipartite fan profile (Figure S8). The sharp coarse-to-fine sediment transition, analogous to the shoreline of a delta, is referred to as the "fan margin." Channel dynamics are described for this coarse upper fan portion, after an initial "spin-up" time of 3 hr required for strong channelization to occur. Typical channel dimensions were 3–4 cm width, 0.5–1 cm depth. The comparison of the morphodynamics resulting from these thin, laminar flows to their natural, turbulent counterparts is physically justified by the similarity in transport dynamics between laminar and turbulent flows [*Lajeunesse et al.*, 2010].

3.2. Numerical Model

[11] Fan topography may be characterized as a cone sloping downstream [*Parker et al.*, 1998] with superimposed random elevation fluctuations on the floodplain [*Sun et al.*, 2002]. Thin floodplain flows are friction dominated, in which case flow direction follows a steepest-descent path [*D'Alpaos et al.*, 2005]. Under these conditions, channel route selection may be modeled as a directed random walk [*Dodds and Rothman*, 2000]. Diffusive [*Pizzuto*, 1987] and/or random [*Jerolmack and Paola*, 2007] floodplain processes eventually erase - or "anneal" - the memory of a particular channel completely. We implement these processes in a two-dimensional cellular

model of a semi-circular fan with a fixed fan margin (auxiliary material).

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