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### Comments

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**Computational Aspects Of  
Color Constancy**

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# Computational Aspects of Color Constancy

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## Abstract

We examine color constancy algorithms based on finite-dimensional linear models of surface reflectance and illumination from a computational point of view. It is shown that, within finite-dimensional models, formulation and solution of color constancy are determined by the choice of basis functions, the number of spectral receptors and the spatial constraints. We analyze some algorithms with examples, and limitations of algorithms for applications on real images.

## 1 Introduction

The spectral power distribution (SPD) of the irradiance that reaches the eye is the product of the illuminant SPD and surface reflectance SPD. Color constancy is the ability in a visual system of reconstructing surface reflectance functions from received color signals which is the product of surface reflectance and illumination. Psychophysical experiments have shown that the human visual system perceives colors of objects relatively independent of the variations in the spectral distribution of the light. The processes that may explain this phenomenon have been intensively studied in human vision and several different solutions have been proposed [10] [13] [17] [12] [22] [7] [6].

Helmholtz [10] suggested that the memory of object color, viewed under white illumination, helps to assign stable colors to objects viewed under different illumination conditions. On the other hand, there are some approaches to finding ways of estimating the relative power distribution of illuminants in the scene which would then enable determination of the surface reflectances. Judd [13] proposed several solutions for finding the spectral characteristics of the illuminant directly, by observing the illuminant or the light scattered by dust or air, or alternatively to estimate the illumination from the highlights of glossy objects [13] [17]. Others proposed estimation of illuminant *SPD* using space-averaged surface reflectances [13] [12] [22].

There are some computational color constancy algorithms based on the finite-dimensional linear

models. With limited number of receptors, the algorithms need some spatial/spectral the conditions or constraints which render the separation of the illumination and surface reflectance mathematically possible [22] [7] [6]. Most of the theories (for an overview see [9]) have been proposed to explain and predict the human visual response with limited number of receptor classes. Therefore the formulations require some spectral or spatial constraints. In machine vision, however, the constraints can be partly reduced by using more spectral receptors. The goal is also to extract aspects of the spectral properties of object surfaces discounting various illuminations in order to provide useful information for image analysis such as recognition and identification of colored objects and image segmentation. [1]

In this paper we present an overview of general color constancy methods which are based on finite-dimensional linear models of illumination and surface reflectance. Emphasis is on the implementational feasibility for real images. A simple color constancy algorithm under restricted conditions employing a reference plate has been discussed elsewhere [2]. We begin OC in Section 2 with the representation of the spectral model for surface reflectance and illumination, and with the sensing of color images and the relationship between the reflectance, the illumination and the sensors. In Section 3 the possible algorithms for solving color constancy are described. We present some of our initial results and simulations in Section 4. Discussions on the limitations of algorithms are given in Sections 5 with suggestions for further studies.

## **2 Representation and Sensing**

First we introduce some notation:

<i>SYMBOL</i>	<i>SYMBOL DEFINITION</i>
$\lambda$	Wavelength
$S(\lambda)$	Surface reflectance spectral distribution
$E(\lambda)$	Illumination spectral power distribution
$I(\lambda)$	Color spectral distribution, $E(\lambda)S(\lambda)$
$S_i(\lambda)$	Basis function for surface reflectance
$E_j(\lambda)$	Basis function for illumination
$\rho_k$	Quantum catch or reception response of $k$ th filter
$Q_k(\lambda)$	Spectral response of $k$ th filter
$\sigma_i$	Scalar weight (coefficient) of surface reflectance basis
$\varepsilon_j$	Scalar weight (coefficient) of illumination basis
$n$	Number of basis functions for surface reflectance
$m$	Number of basis functions for illumination
$p$	Number of receptors.

Representation of color with finite-dimensional linear models has been a topic of many studies [19] [20] [7] [21]. There have been some approaches to obtaining characteristic basis functions by investigating many samples of daylight and surface reflectances [20] [19]. It has been suggested that, although the number of basis functions required to completely describe full spectra is essentially infinite, a small number of basis functions can provide good spectral approximations of most natural illuminants and surface reflectances.

For the finite-dimensional linear model, surface reflectance and illumination can be represented as a weighted sum of basis functions expressed as:

$$S(x, y, \lambda) = \sum_{i=0}^n \sigma_i(x, y) S_i(\lambda), \quad (1)$$

$$E(x, y, \lambda) = \sum_{j=0}^m \varepsilon_j(x, y) E_j(\lambda), \quad (2)$$

where  $S_i(\lambda)$  and  $E_j(\lambda)$  are the basis functions, and  $\sigma_i(x, y)$  and  $\varepsilon_j(x, y)$  are the scalar weighting factors at  $(x, y)$ .

The color image sensing is usually performed with a CCD camera using filters of different spectral responses. The measured color signal  $I(x, y, \lambda)$  is obtained as a product of the spectral power distribution (*SPD*) of illumination and the spectral response of the surface reflectance function, i.e.,

$$I(x, y, \lambda) = E(x, y, \lambda) S(x, y, \lambda). \quad (3)$$

With  $k$  different filters, the quantum catch or the measured signal from the camera is given by

$$\rho_k(x, y) = \int_{\lambda_1}^{\lambda_2} I(x, y, \lambda) \cdot Q_k(\lambda) d\lambda, \quad (4)$$

where  $Q_k(\lambda)$  and  $\rho_k(x, y)$  for  $k = 0, 1, \dots, p$  are the spectral response of the  $k$ -th filter and the camera output through the  $k$ -th filter at  $(x, y)$ , respectively. The wavelengths from  $\lambda_1 = 400 \text{ nm}$  to  $\lambda_2 = 700 \text{ nm}$  cover the range of the visible spectrum.

With  $n + m$  basis functions, the relationship between the sensor response, illumination and reflectance is given as

$$\rho = \Omega^\sigma \varepsilon = \Omega^\varepsilon \sigma \quad (5)$$

where the elements of  $\Omega^\sigma$  and  $\Omega^\varepsilon$  in the  $k$ -th row and  $j$ -th (or  $i$ -th) column are respectively

$$\Omega_{kj}^\sigma = \int_{\lambda_1}^{\lambda_2} \sum_{i=0}^n \sigma_i S_i E_j Q_k d\lambda, \quad (6)$$

$$\Omega_{ki}^\varepsilon = \int_{\lambda_1}^{\lambda_2} \sum_{j=0}^m \varepsilon_j E_j S_i Q_k d\lambda. \quad (7)$$



### 3 Color Constancy Algorithms

In psychological literature, the color constancy problem is usually defined in terms of *approximate color constancy*, which means that colors of objects are approximately constant with changes in the illuminant. Here we will adopt a more rigorous definition of the *color constancy* problem.

Given the sensor quantum catches at each location in the sensor array  $I(x, y, \lambda)$ , compute stable color descriptors for  $S(x, y, \lambda)$  discounting the illumination  $E(x, y, \lambda)$ .

In this section, we present an overview of various color constancy algorithms which are based on finite-dimensional linear models of illumination and surface reflectance. First we consider the case where there are no spatial constraints on the illuminant or the surface reflection. Later we analyze the situations where illumination or surface reflectances are spatially constrained. It is shown how the spatial-spectral relationship defines the local or global nature of the color constancy algorithm.

Computationally, the color constancy problem is to solve Equation 5 for  $\varepsilon$ 's and  $\sigma$ 's with the measurements  $\rho$ 's. The number of required measurements depends on the number of unknowns, the number of spatial points, the choice of basis functions and other spatial/spectral constraints. We mainly investigate the finite-dimensional case of  $m = n = 3$ , and the extension to higher dimensional cases is straightforward.

Since we are given the product  $E(\lambda)S(\lambda)$ ,  $E(\lambda)$  and  $S(\lambda)$  can be recovered only up to a multiplicative constant. This scaling ambiguity cannot be generally resolved, and thus we only solve for the spectral components of surface reflectance and illumination up to a multiplicative factor. For only obtaining spectral responses of illumination, we can normalize the illumination such that  $\varepsilon_0 = 1$ .

### 3.1 Color Constancy Using a Reference Object

While the general solution for color constancy with unknown illumination demands complex algorithms and some spectral and/or spatial constraints [22] [6], it is simple to remove the known illumination measured with a reference object in the controlled lighting environment. One assumption is that the objects of interest are illuminated by light sources of the same *SPD*, and the reference object is applicable. If we use a reference plate with known reflectance  $\sigma^{ref}$  (spectrally wide, e.g., white or grey), the *SPD* of illumination obtained from

$$\varepsilon^{ref} = (\mathbf{\Omega}^{\sigma^{ref}})^{-1} \rho \quad (8)$$

represents the spectral composition of the global illumination throughout the image area. With the normalized  $\varepsilon_{norm}^{ref}$ , the calculation of

$$\sigma = (\mathbf{\Omega}^{\varepsilon_{norm}^{ref}})^{-1} \rho \quad (9)$$

leads to the surface reflectance components under whitened illumination. This method is not sensitive to the choice of basis functions, but errors can arise when  $\varepsilon^{ref}$  cannot be approximated well with a small number of basis functions. Note that for many illumination sources with different colors or for the environments where the reference object cannot be applied, this calibration method is not effective.

### 3.2 Pointwise Color Constancy

In this subsection we investigate the color constancy with no spatial assumptions on the illuminant or the surface reflectance, but with an assumption that the representation of both illumination and surface reflectance is limited to three basis functions. The generalization to more number of basis functions is straightforward.

The basic limitations of the color constancy algorithms come from the errors in finite-dimensional

approximation. The measure of goodness of an algorithm is how gracefully the solution degrades as the approximation breaks down. Choice of basis functions is important not only for good spectral approximation of surface reflectance and illumination, but also for solving color constancy problems. It determines the number of independent equations in Equation 5 and thus the solvability of the equations [5].

### 3.2.1 Nine-Receptor Case

The finite-dimensional model of image irradiance  $I(\lambda)$  is rewritten as:

$$I(\lambda) = [\sigma_0 S_0(\lambda) + \sigma_1 S_1(\lambda) + \sigma_2 S_2(\lambda)][\varepsilon_0 E_0(\lambda) + \varepsilon_1 E_1(\lambda) + \varepsilon_2 E_2(\lambda)]. \quad (10)$$

The actual number of independent terms in Equation 10 depends on basis functions. If the basis functions are chosen such that all the functions  $E_j(\lambda)S_i(\lambda)$  are linearly independent, we can have nine independent measurement. It is known [11] that the first three of Cohen's reflectance basis functions, multiplied with the first three of Judd's illumination functions, form a set of nine independent terms in  $\varepsilon_j \sigma_i$  product-pair unknowns. Figure 1 shows (a) the first three basis functions for reflectance by Cohen and (b) the first three basis functions for illumination by Judd.

With nine different  $E_j(\lambda)S_i(\lambda)$ 's, we have a system of nine independent linear equations.

$$\rho = \mathbf{\Omega}' [\varepsilon_i \sigma_j] \quad (11)$$

The elements of the matrix  $\mathbf{\Omega}'$  in the  $k$ -th row and  $l$ -th column are given as:

$$\Omega'_{kl} = \int_{\lambda_1}^{\lambda_2} (S_i(\lambda)E_j(\lambda))_l Q_k(\lambda) d\lambda. \quad (12)$$

for  $l = 0, 1, 2, \dots, 9$ . In Appendix, we will show how to determine if the matrix  $\mathbf{\Omega}'$  is well conditioned.

In order to solve the set of linear equations (11) we need to obtain independent measurements through nine different receptors. From the nine measurements, we can solve a system of 9 different linear equations for 9  $\varepsilon_j \sigma_i$  product pairs. When the 3-dimensional model does not approximate the

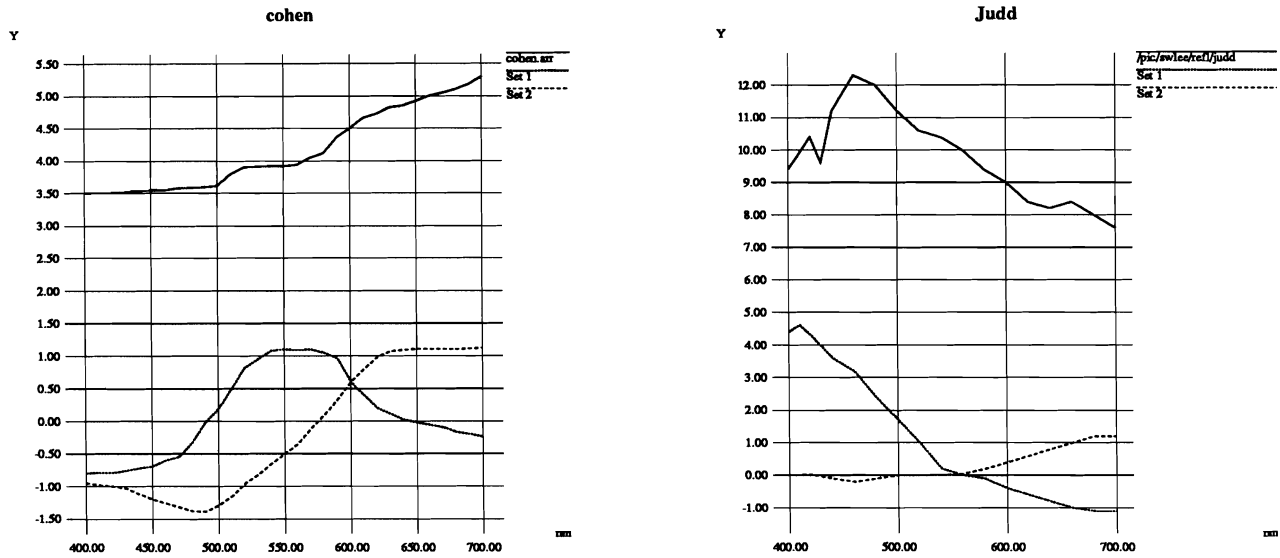


Figure 1: (a) Three basis functions for reflectances by Cohen. (b) Three basis functions for illuminations by Judd.

illumination and reflectance well, error can arise due to the neglected higher-order terms. In this case, a minimum-error solution can be obtained.

### 3.2.2 Five-Receptor Case

Using a finite-dimensional linear model which is limited to three basis functions for the illumination and three basis functions for reflectance, we actually deal only with five unknowns, that is,  $\sigma_0$ ,  $\sigma_1$ ,  $\sigma_2$ ,  $\varepsilon_1$  and  $\varepsilon_2$  (with  $\varepsilon_0 = 1$ ). This means that we may need only five independent equations (five measurements), but in this case we have to solve in general a system of non-linear equations.

As shown by Yuille [5], the number of independent terms in Equation 10 depends on the basis functions and the filter functions. If the number of independent terms is less than nine and greater than or equal to five with some basis functions, we have to deal in general with non-linear equations. If there are less than five independent terms, pointwise color constancy cannot be achieved and

other (spatial or spectral) constraints have to be employed.

With same basis functions for surface reflectance and illumination, the number of independent equations is less than or equal to six. An example of such a case is presented in Section 7, where the illumination and surface reflectance are modeled with three Fourier basis functions, respectively, and only five independent equations are available. When the scene is illuminated by the reflected lights coming from another object (inter-reflections) [2], spectral characteristics of the illumination (*secondary illumination*) are very similar to those of the reflectance and the number of independent equations is reduced. Having the basis functions for illumination very similar to those for surface reflectance, we can get at most five independent equations.

### 3.3 Color Constancy Using Spatial Constraints

In this subsection we discuss the color constancy problems in situations where illumination or surface reflectances are spatially constrained, and show how the spatial-spectral relationship defines the local or global nature of color constancy. The advantage of including the spatial domain, in addition to the spectral, is twofold. First, we can reduce the number of filters that are necessary to solve the problem. Second, spatial measurements can be used to overconstrain the system of equations, thus making the method less sensitive to noise.

Several spatial assumptions have been proposed [8] [6] [5]. Basically they differ only in the size of the region, across which the spectral energy distribution of the light is assumed to be constant. The size of the region can vary from two neighboring image elements to the whole image. Since illumination and surface reflectance, from a computational point of view, represent two interchangeable components, these assumptions mean that in a localized area only one component can change at a time. The relationship between the number of spatial points  $q$ , which have the same illumination, but have distinctively different surface reflectance, and the number of basis functions  $m, n$  and receptor classes  $p$  is given as follows [5]:

$$q \geq \frac{m-1}{p-n}. \quad (13)$$

An important result which follows from Equation 13 is that the number of basis functions for reflection must always be less than the number of sensor classes:  $p > n$ .

### 3.3.1 Two-point Color Constancy

This approach was suggested by Yuille [5]. It is proposed to pick points on both sides of a material edge close enough in order to ensure the same illumination. If we include another point ( $q = 2$ ) of different reflectance ( $\sigma'_0, \sigma'_1, \sigma'_2$ ) and same illumination, four measurements ( $p = 4$ ) are necessary for eight equations and for eight unknowns ( $\sigma_0, \sigma_1, \sigma_2, \sigma'_0, \sigma'_1, \sigma'_2, \varepsilon_1$ , and  $\varepsilon_2$ ). Even with an extra point, we cannot avoid non-linear formulation.

Under the assumption that the color of surface reflectances is constant, solutions obtained at the boundaries can be propagated to the regions away from the boundaries. However, in the images where color edges are not present evenly over the image area, the region of color constancy calculation is limited and thus determination of spectral components far from the edges would be difficult. Note that color constancy using two points is inevitably related to the problems of color edge detection [18].

### 3.3.2 Three-point Color Constancy

The three-point ( $q = 3$ ) color constancy algorithm was first proposed by Maloney [8] for four receptors ( $p = 4$ ). The difficulty in using this algorithm in practical applications lies in the method of finding three points with the same illumination, but with distinctively different surface reflection. Points near junctions where more than three different surfaces meet would be appropriate for input points in the algorithm, but such regions do not frequently appear in real images.

### 3.3.3 Multi-point Color Constancy

Using more points than three under same illumination, we can improve the results by reducing errors due to the finite-dimensional models [8]. Again the problem is how to find the points of different reflectances under same illumination.

Other approach is to put more serious assumptions on the illuminations and reflectances that illumination varies smoothly in space and reflectances are *Mondrian* (two-dimensional with patches of uniform reflectance and of sharp discontinuities). In this case we can use 3 measurements and many *lightness algorithms* either with receptor primaries (*RGB*) or with  $\varepsilon$ 's and  $\sigma$ 's. [12] [14] [15] [16]

## 4 Simulations and Experimental Results

In this section we will present some results on synthetic and real images. The major concern is how well the solutions of color constancy are approximated with finite-dimensional linear models of reflectances and illuminations.

### 4.1 Pointwise Color Constancy with Five Receptors

Let us assume that surface reflectance and illumination can be well approximated with the first three Fourier basis functions:

$$S_0(\lambda) = E_0(\lambda) = 1, \quad S_1(\lambda) = E_1(\lambda) = \sin\lambda, \quad S_2(\lambda) = E_2(\lambda) = \cos\lambda. \quad (14)$$

It follows that irradiance  $I(\lambda)$  is:

$$I(\lambda) = \gamma_0 I_0(\lambda) + \gamma_1 I_1(\lambda) + \gamma_2 I_2(\lambda) + \gamma_3 I_3(\lambda) + \gamma_4 I_4(\lambda), \quad (15)$$

where

$$I_0(\lambda) = 1, \quad I_1(\lambda) = \sin\lambda, \quad I_2(\lambda) = \cos\lambda, \quad I_3(\lambda) = \sin 2\lambda, \quad I_4(\lambda) = \cos 2\lambda. \quad (16)$$

With  $\varepsilon_0 = 1$ , the normalized system of equations has the following form:

$$\gamma_0 = \sigma_0 + \frac{1}{2}(\varepsilon_1\sigma_1 + \varepsilon_2\sigma_2) \quad (17)$$

$$\gamma_1 = (\varepsilon_1\sigma_0 + \sigma_1) \quad (18)$$

$$\gamma_2 = (\sigma_2 + \varepsilon_2\sigma_0) \quad (19)$$

$$\gamma_3 = \frac{1}{2}(\varepsilon_1\sigma_2 + \varepsilon_2\sigma_1) \quad (20)$$

$$\gamma_4 = \frac{1}{2}(-\varepsilon_1\sigma_1 + \varepsilon_2\sigma_2) \quad (21)$$

The system is non-linear, and there are two important problems related to the solution. The first one is whether the solution exists, and the second is whether it is unique. Because the equations describe a physical phenomenon, we expect that the solution exists.

Let  $\beta$  denote  $\frac{1}{\varepsilon_0\sigma_0} = \frac{1}{\sigma_0}$ . From the above equations we can derive a cubic equation for  $\beta$  as

$$\beta^3(2\gamma_1\gamma_2\gamma_3 - \gamma_1^2(\gamma_0 + \gamma_4) - \gamma_2^2(\gamma_0 - \gamma_4)) + \beta^2((\gamma_1^2 + \gamma_2^2) + 4(\gamma_0 + \gamma_4)(\gamma_0 - \gamma_4) - 4\gamma_3^2) + \beta(-8\gamma_0) + 4 = 0. \quad (22)$$

It can be shown that there are 3 positive solutions for  $\beta$  and the smallest one is the correct value.

With  $\beta$  value obtained, the quadratic equation

$$x^2 - \beta\gamma_1x + \beta\gamma_0 - \beta\gamma_4 - 1 = 0, \quad (23)$$

gives us either  $\varepsilon_1$  or  $\sigma_1/\sigma_0$ , and

$$y^2 - \beta\gamma_2y + \beta\gamma_0 + \beta\gamma_4 - 1 = 0, \quad (24)$$

gives us either  $\varepsilon_2$  or  $\sigma_2/\sigma_0$ . For each picture element in the color image, we have the pair of separated values for  $(\varepsilon_1, \varepsilon_2)$  and  $(\sigma_1/\sigma_0, \sigma_2/\sigma_0)$ . However, because of symmetry, we do not know which one is illumination and which one is surface reflectance.

Figures 2, 3, 4 and 5 show the results of simulation. We generated the illumination, and for the surface reflectance, we used a picture obtained through a CCD camera which contains realistic



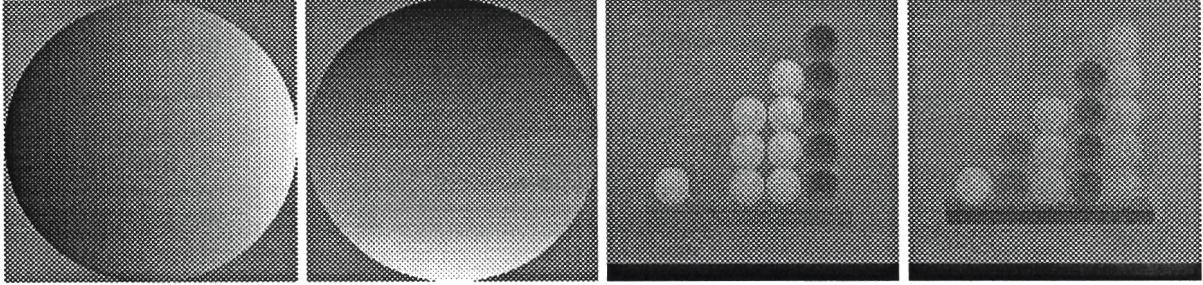


Figure 2: Genenated illumination (a) 1 (b) 2 Surface reflectance (c) 1 (d) 2

noises. Figure 2 (a) and (b) show  $\varepsilon_1$  and  $\varepsilon_2$  of the illumination, respectively, and Figure 2 (c) and (d) show  $\sigma_1$  and  $\sigma_2$  of the surface reflectance, respectively. The spectral coefficients  $\gamma_1, \gamma_2, \gamma_3$  and  $\gamma_4$  of the multiplied color image are shown in Figure 3 (a), (b), (c) and (d), respectively.

Figure 4 (a) and (b) again show  $\varepsilon_1$  and  $\varepsilon_2$  of the illumination only in the object regions, respectively, and Figure 4 (c) and (d) show the normalized surface reflectance  $\sigma_1/\sigma_0$  and  $\sigma_2/\sigma_0$ , respectively. The solutions of pointwise color constancy  $x_1$  and  $x_2$  are shown in Figure 5 (a) and (b), respectively, and  $y_1$  and  $y_2$  are shown in Figure 5 (a) and (b), respectively. As shown in Figure 5, the reflectance and the illumination are correctly decomposed pointwise, but they are not properly arranged for reflectance and illumination. Therefore we see that as long as surface reflectance and illumination are strictly confined to three dimensions, solutions of the five-point formulation provides good separation of reflectance and illumination.

The only possible way to resolve color constancy at a point would be to choose different basis functions for illumination and different basis functions for reflectance. Otherwise, it is not possible to solve the problem without any further information or some additional assumptions. One source of information is the knowledge of the spatial arrangement of illumination and reflectance signals. Two solutions obtained at each point can be grouped into two sets on the basis of spectral proximity. This is successful if the spectral properties of the illumination and spectral properties of the reflectance

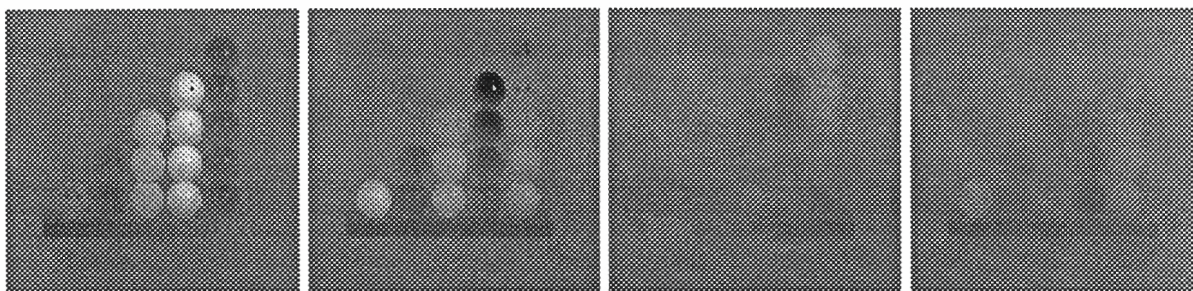


Figure 3: Color image (a) 1 (b) 2 (c) 3 (d) 4

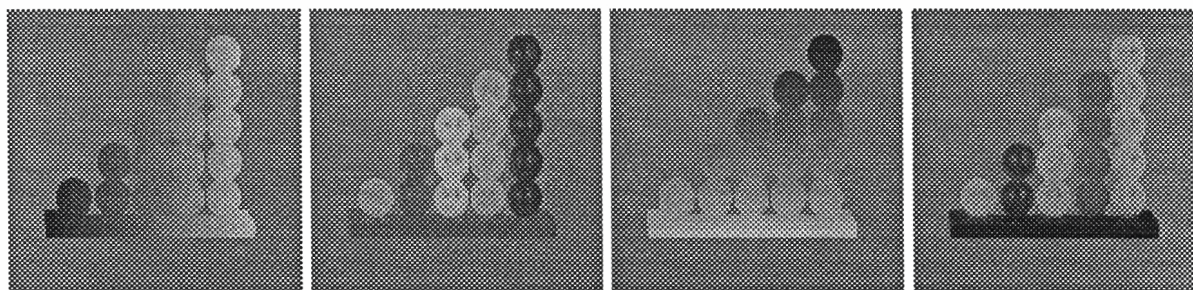


Figure 4: (a) Generated illumination 1 (b) Normalized surface reflectance 1 (c) Generated illumination 2 (d) Normalized surface reflectance 2

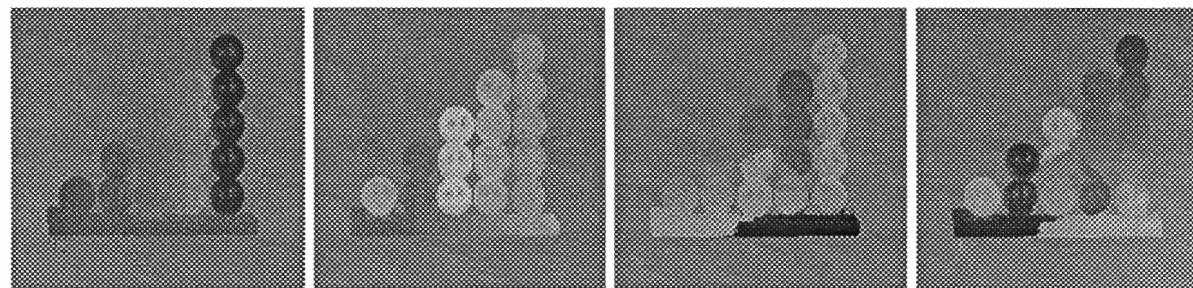


Figure 5: Results of separation (a) x1 (b) x2 (c) y1 (d) y2

do not change abruptly at the same point. After classifying the spectral values into two groups, we can determine what is light and what is reflectance.

## 4.2 Experiments

We have performed some experiments for color constancy with 5 filters. Figure 6 shows the 5 filters we used in which the spectral responses of camera lens, CCD receptor and IR-cutoff filter shown in Figure 7 are included.

We use a matte object of the distinct colors of red, green, blue, yellow, white and grey under daylight (3:00 PM, May 3, 1989, Philadelphia, Pennsylvania), and specular reflection is avoided. Figure 8 shows 6 curves of spectral approximation with Fourier basis functions.

Figure 9 (a), (b), Figure 10 (a), (b) and Figure 11 (a), (b) show the results of separation obtained from the nonlinear equations described in the above, for white, grey, red, green, blue and yellow, respectively. As can be seen in the figures, although the 5 Fourier bases approximates the spectra well, the separated curves show neither any common illumination nor correct approximations of surface reflectances. It can be easily seen that the separation fails due to the perturbation of higher-order-dimensional terms.

## 4.3 Finite-Dimensional Approximation of Real Reflectances and Illuminations

In order to better understand the finite-dimensional linear approximation of natural surface reflectances and illuminations, we examined some real reflectance and illumination functions. We used some color plates with spectrophotometric measurements shown in Figure 12. Figure 13, Figure 14 (a), (b) and Figure 15 (a), (b) show the power spectra in Fourier domain for white, red, green, blue and yellow plates, respectively. Figure 16, Figure 17 (a), (b) and Figure 18 (a), (b) show the magnitude spectra in Fourier domain for white, red, green, blue and yellow plates, respectively. It can be seen in the figures that, although the first three terms take most of the

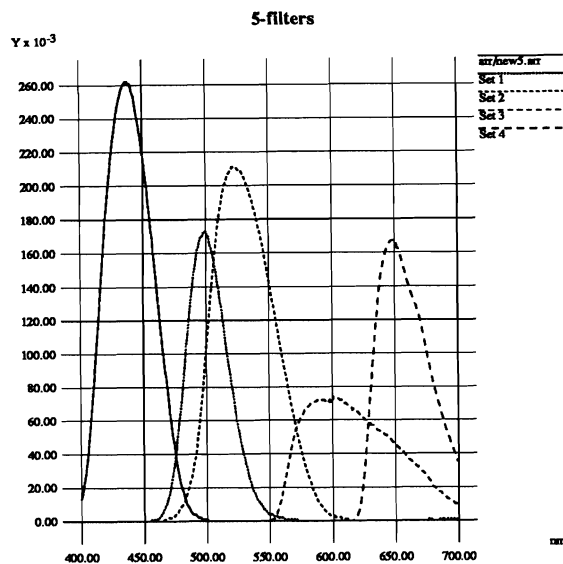


Figure 6: Spectral response of 5 filters

power, there are many higher-order terms in the magnitude spectra. In the computation of color constancy, therefore, errors can be caused by the neglect of the higher-order coefficients, since the coefficients of magnitude spectrum are used for the calculation.

Figure 19 shows the approximations of surface reflectances up to second-order terms (five coefficients) and they are similar to the ones we experimentally obtained using five filters. Figure 20 shows the data of a light from a tungsten bulb that we measured using a monochromator and a calibrated photo-detector, and Figure 21 (a) and (b) show the magnitude spectrum and the approximation with 5 coefficients.

In order to compare the above results with the optimal case, we computed the principal components of the surface reflectances by Karhunen-Loeve (K-L) transformation (i.e., principal axes transformation). The K-L transformation applied on the distribution of illumination and surface reflectance irradiance can provide us with the optimal basis functions in the statistical sense. Figure 22 (a) shows the three principal components, and Figure 22 (b), Figure 23 (a),(b), and Figure 24

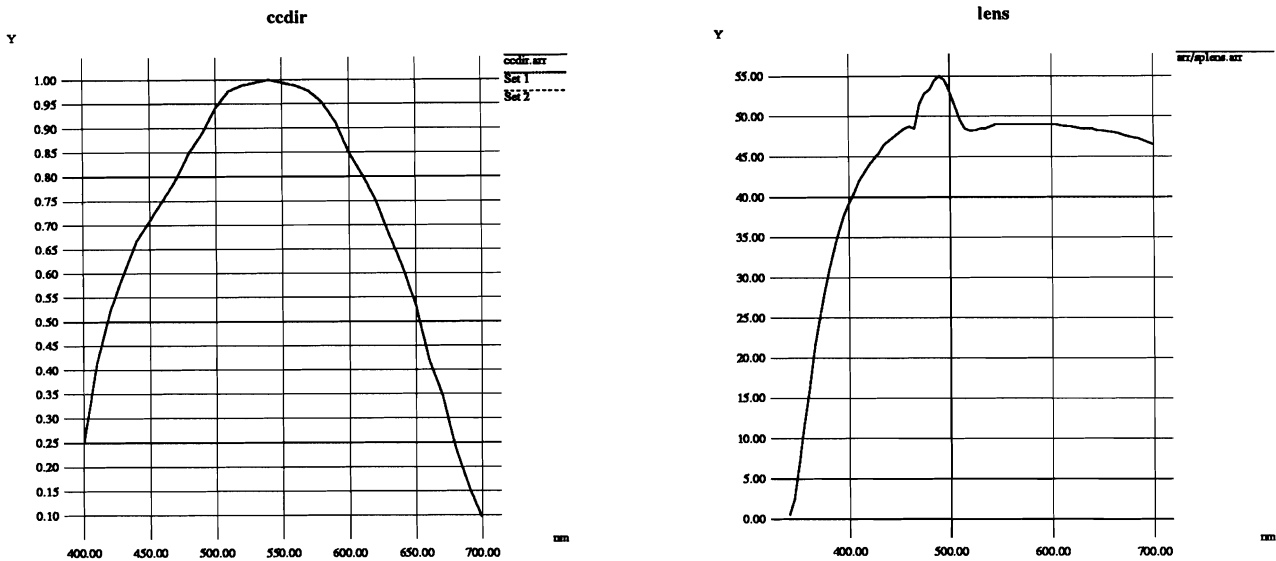


Figure 7: (a) Spectral responses of IR filter and CCD receptor. (b) Spectral response of camera lens

(a), (b) show the magnitude spectra on K-L domain of white, red, green, blue and yellow, respectively. Note that the basis functions are not ordered by the magnitude of their eigenvalues in the K-L domain. In this optimal case, most of the coefficients are very small except the three or four. Note that the optimal basis functions can be obtained by choosing the principal components of large values, only when we know the spectral data of all the possible objects. Although Cohen's basis functions for surface reflectances are derived from many spectral data, they do not always effective in approximating reflectances in small dimensions.

## 5 Discussion

We examined some cases of color constancy with finite-dimensional linear models of surface reflectances and illuminations. Besides the lightness algorithms, the algorithm which requires the least number of receptors is the one with four filters proposed by Maloney and Wandell [6] which demands three points in space having different spectral reflectance functions, but sharing the same

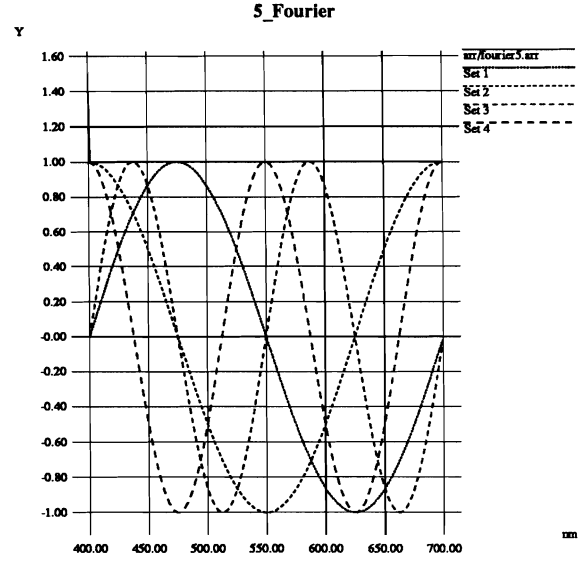
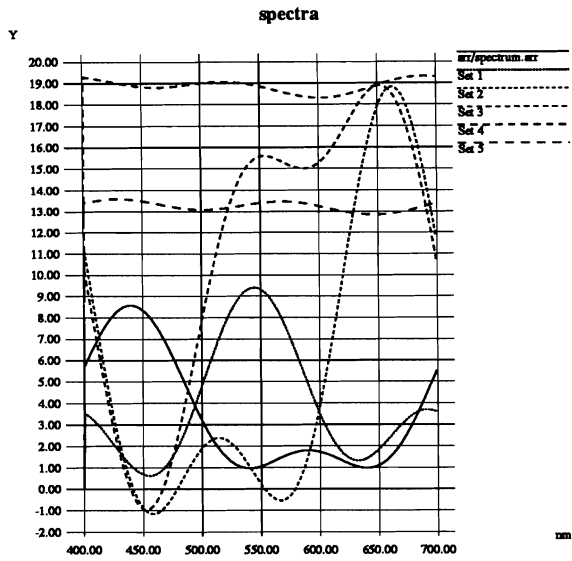


Figure 8: (a) Spectral approximation with (b) 5 Fourier bases

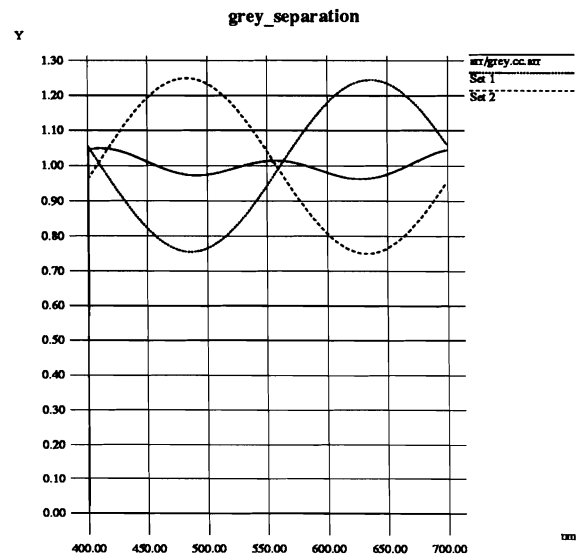
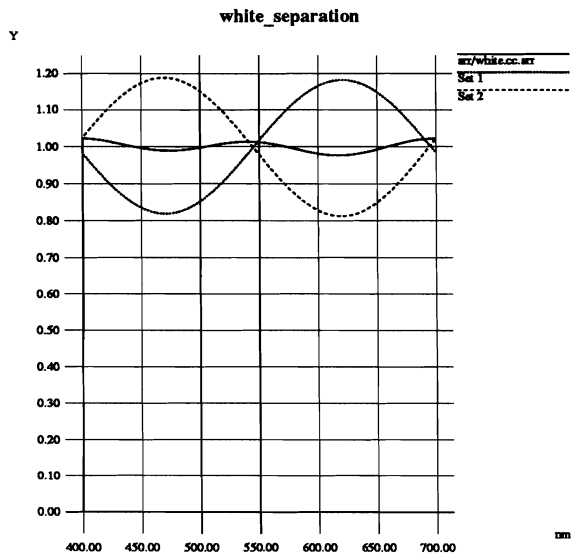


Figure 9: (a) Separation of white (b) grey

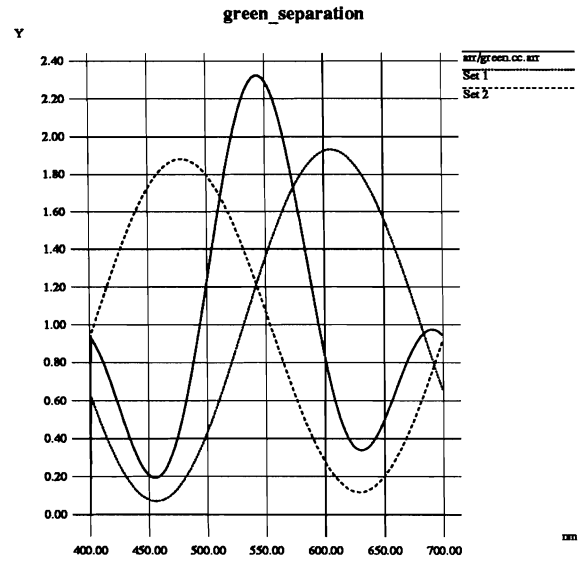
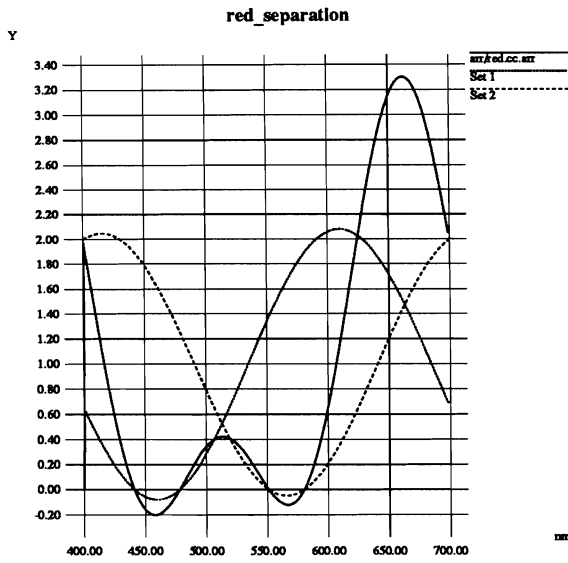


Figure 10: (a) Separation of red (b) green

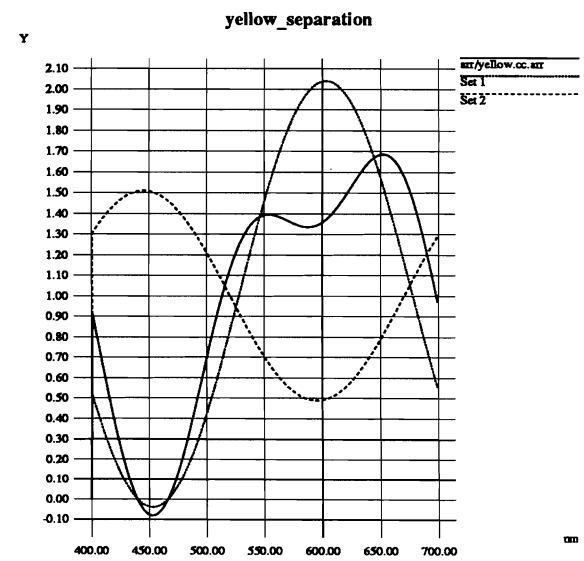
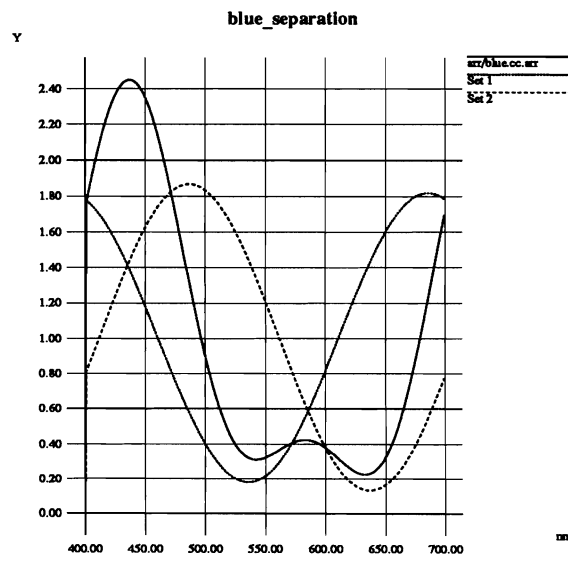


Figure 11: (a) Separation of blue (b) yellow

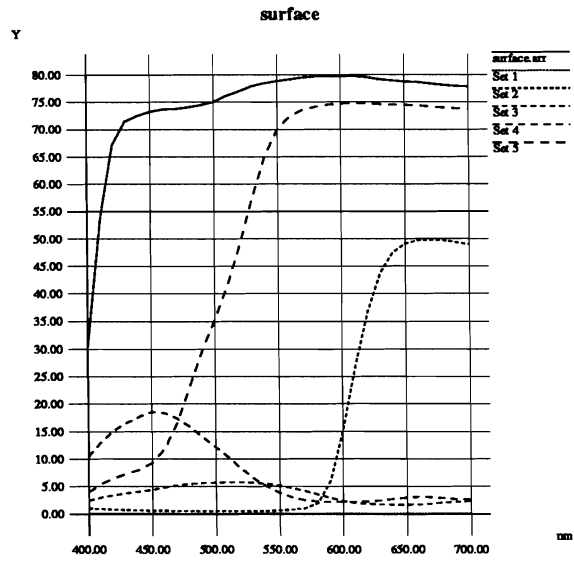


Figure 12: Measured spectra of 6 surfaces

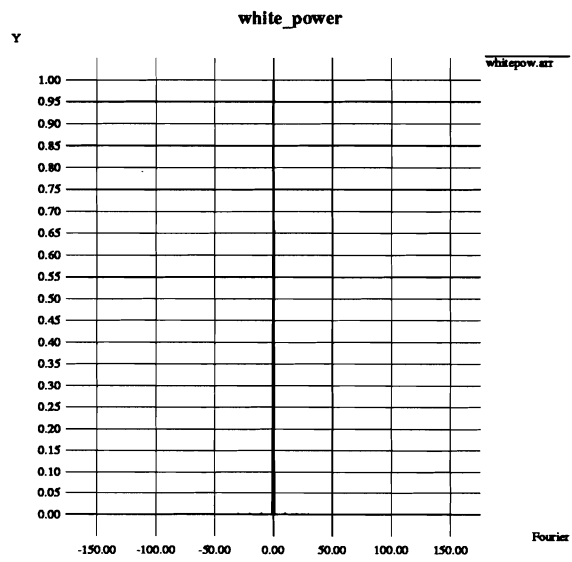


Figure 13: Power spectrum in Fourier domain, White



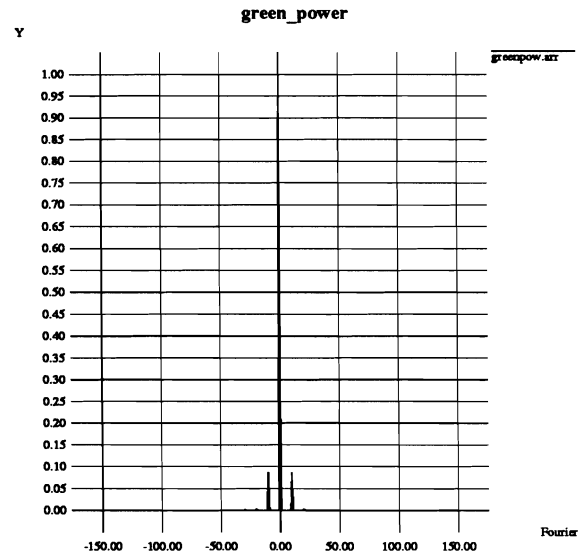
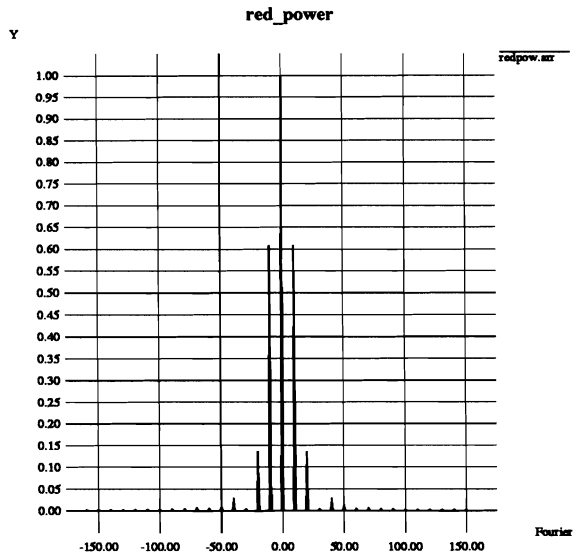


Figure 14: Power spectrum in Fourier domain (a) Red (b) Green

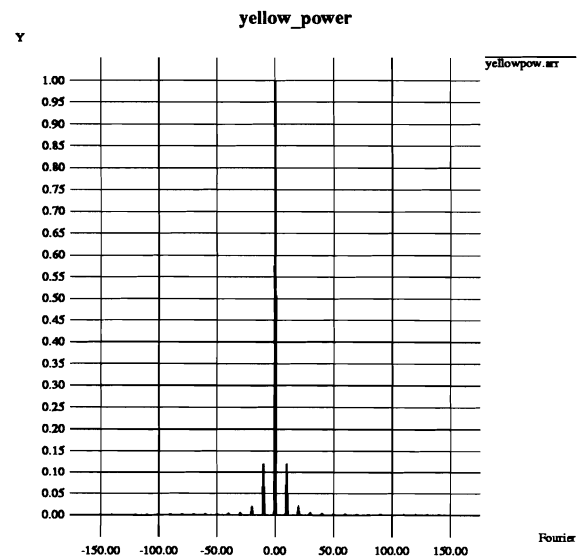
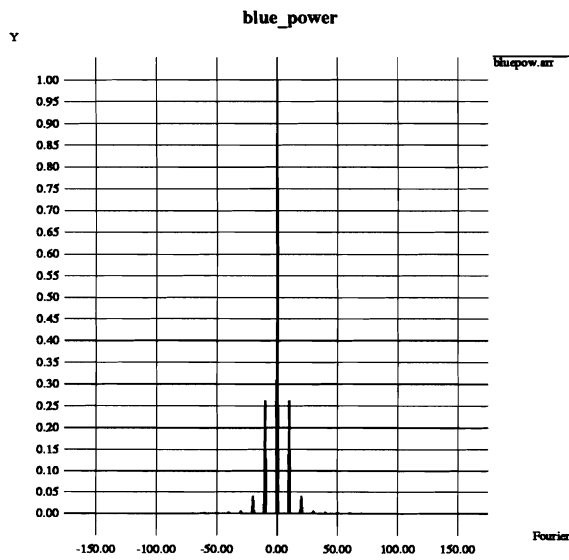


Figure 15: Power spectrum in Fourier domain (a) Blue (b) Yellow

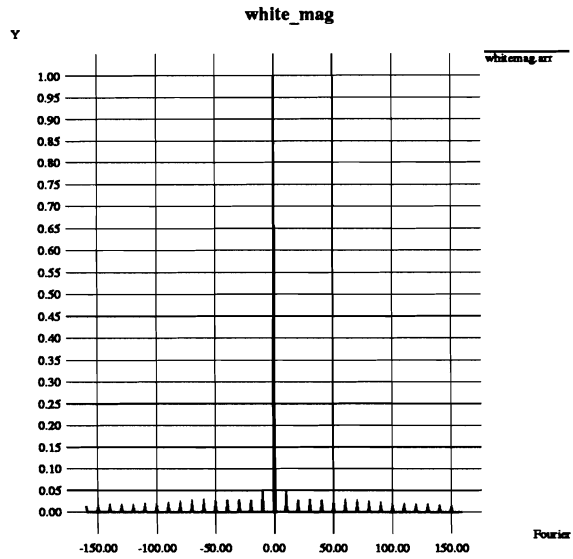


Figure 16: Magnitude spectrum in Fourier domain, White

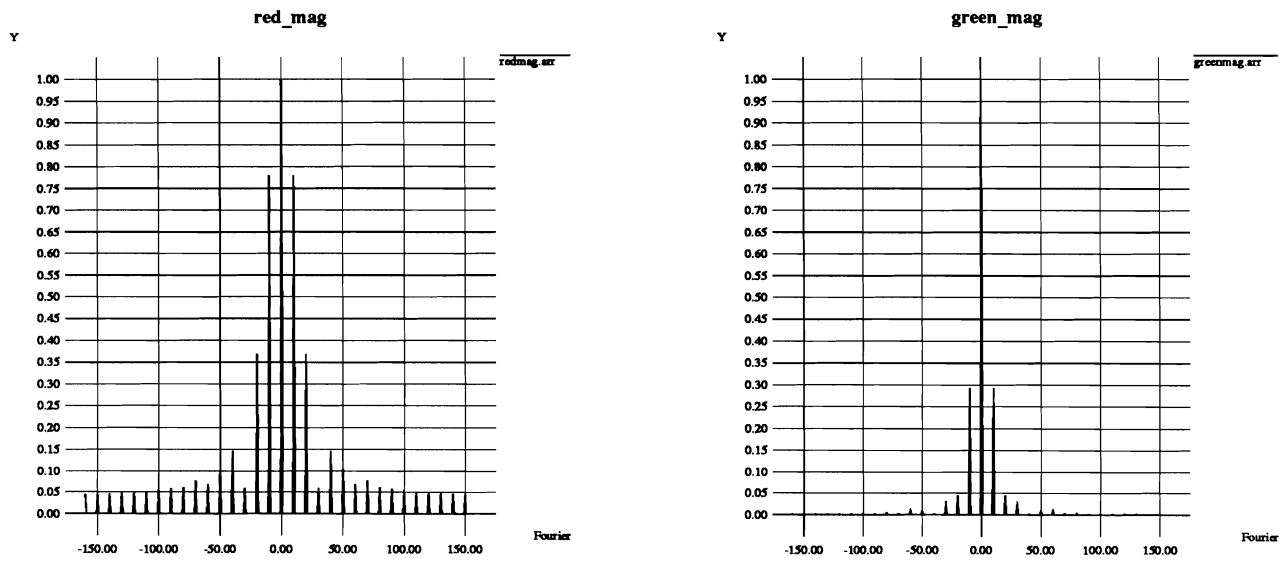


Figure 17: Magnitude spectrum in Fourier domain (a) Red (b) Green

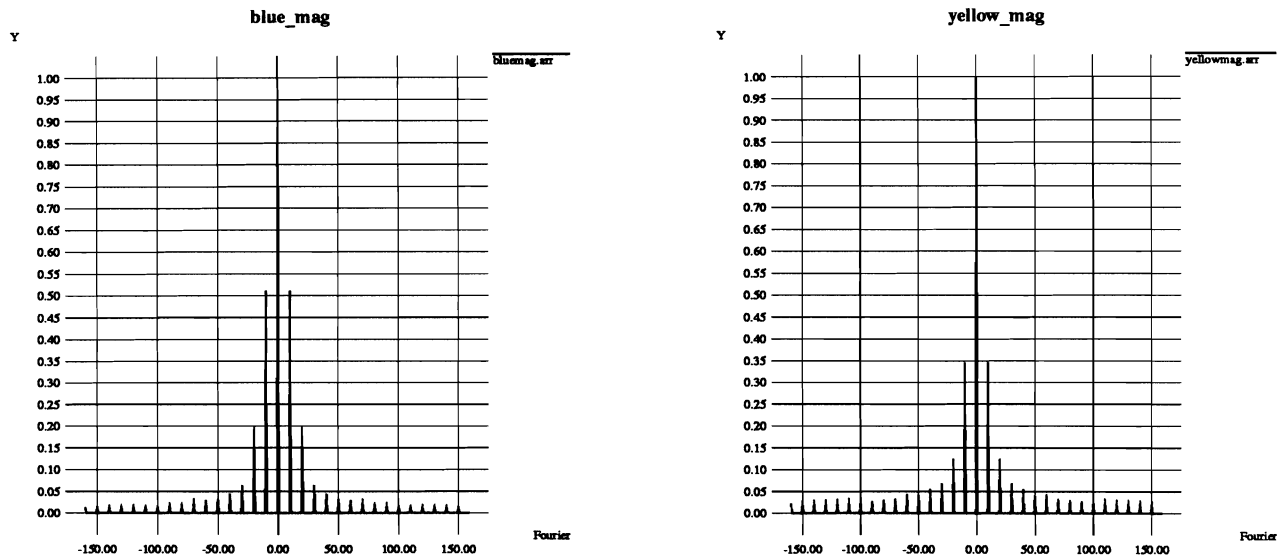


Figure 18: Magnitude spectrum in Fourier domain (a) Blue (b) Yellow

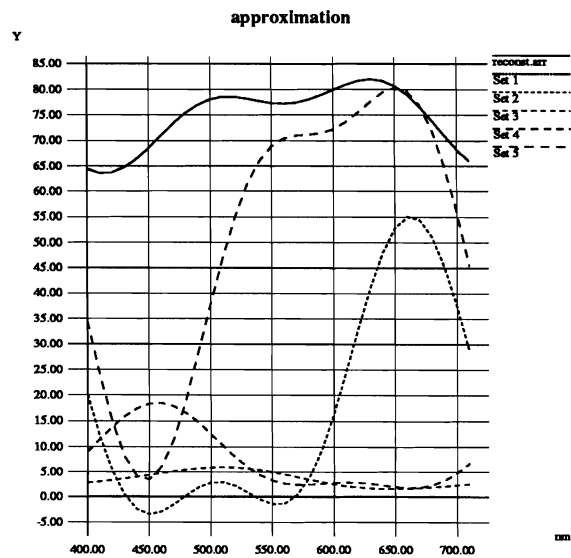


Figure 19: Approximation using 5 Fourier Coefficients

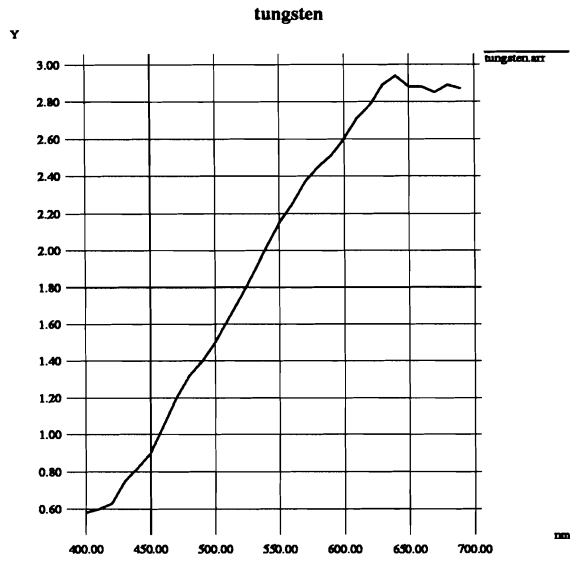


Figure 20: Measured spectrum of tungsten bulb light

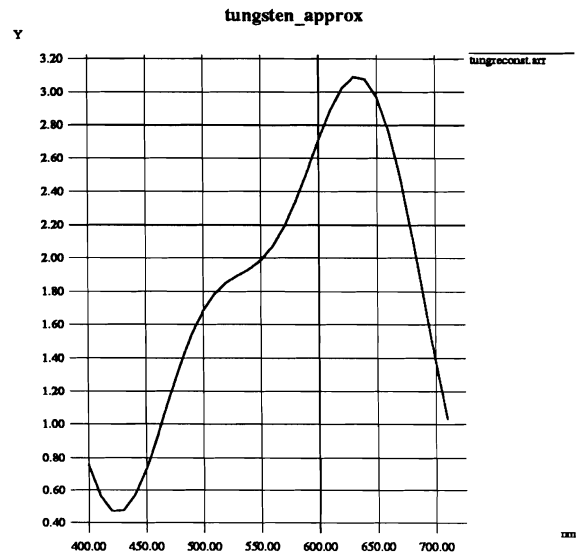
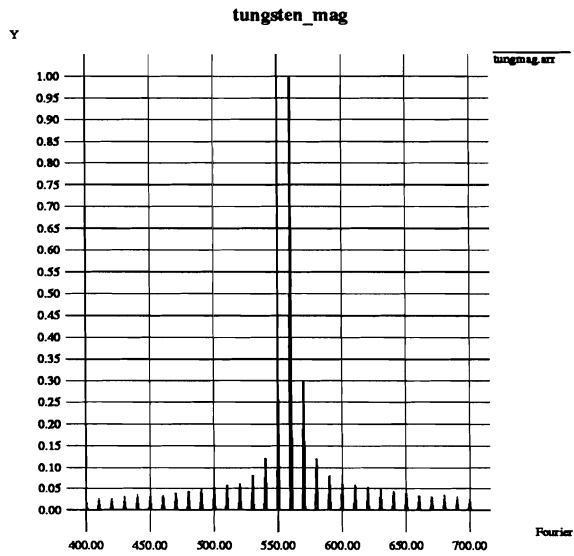


Figure 21: (a) Fourier magnitude spectrum (b) Approximation using 5 Fourier Coefficients

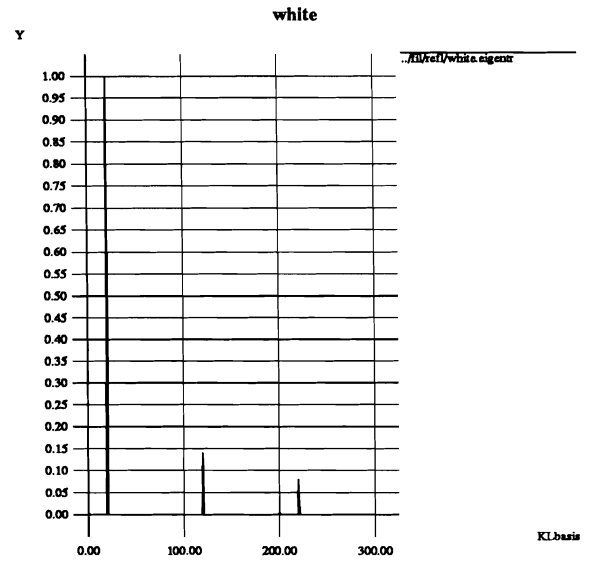
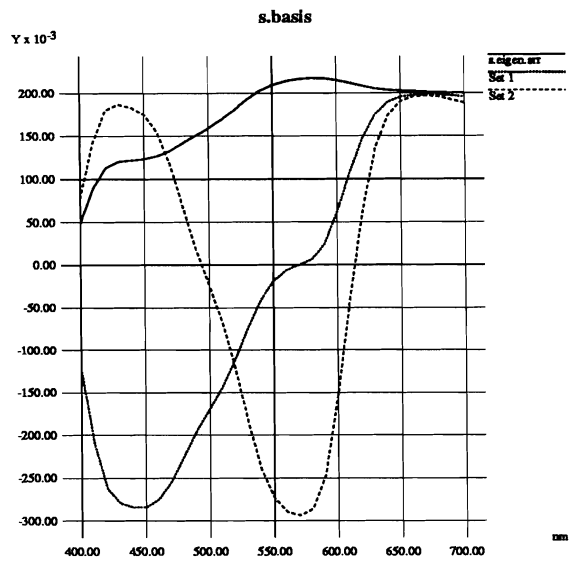


Figure 22: (a) 3 principal components. (b) Magnitude spectrum in K-L domain, White

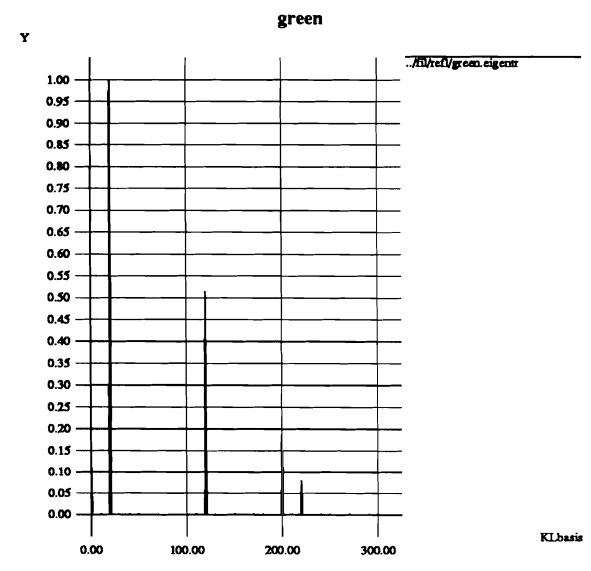
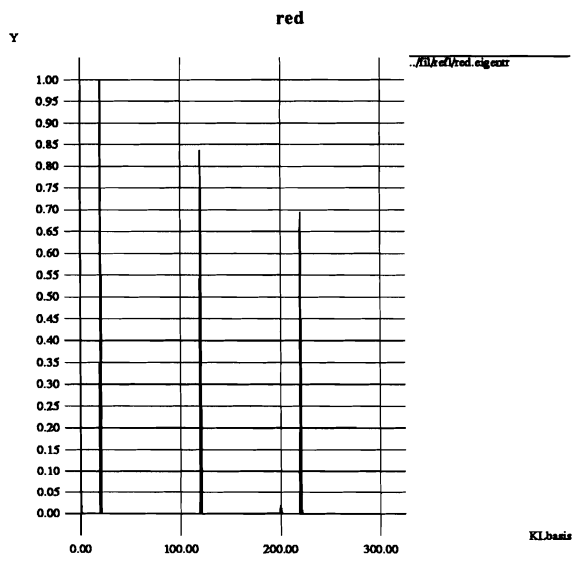


Figure 23: Magnitude spectrum in K-L domain (a) Red (b) Green

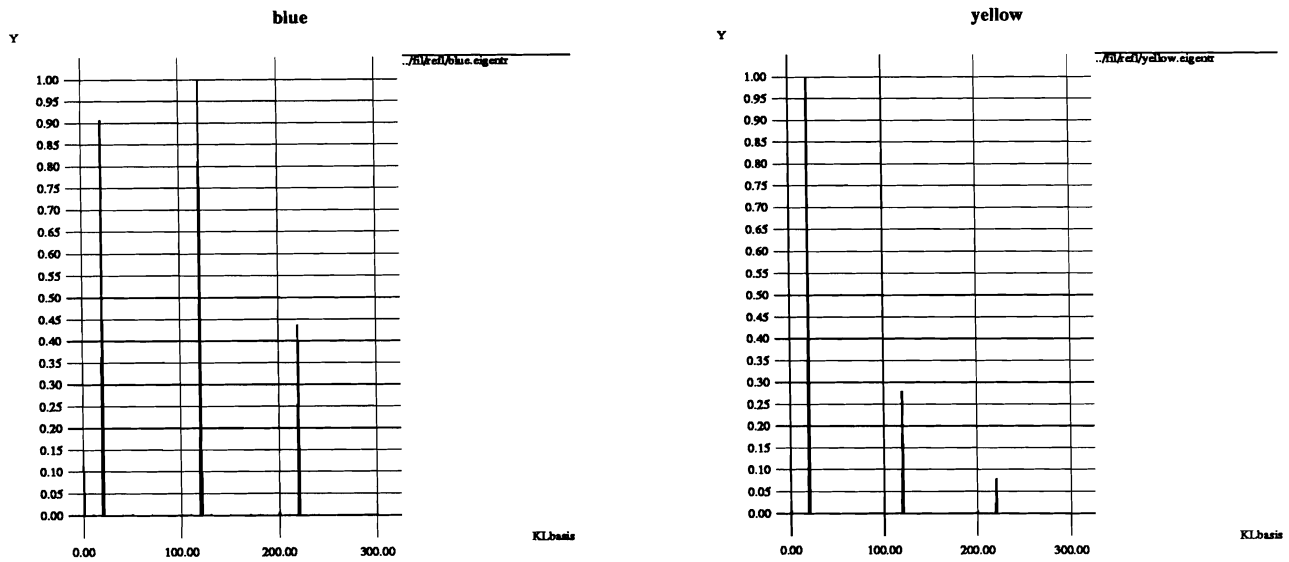


Figure 24: Magnitude spectrum in K-L domain (a) Blue (b) Yellow

illumination. As mentioned earlier, however, the question is where to find such points. Those points can be found near the junctions of three or more regions under uniform illumination. These constraints are severe in many applications with real images, and it is not clear how the results can be used to solve color constancy for other points. This method has the advantage that a small number of receptors is required.

Both the color constancy algorithm with four filters and two points [5] and the pointwise color constancy with five filters involve solving non-linear equations and require same basis functions for reflectance and illumination. Surface reflectances of artificial color may be represented by three basis functions since they are produced mostly by three or four color pigment vectors, and it is reported that small number of Fourier bases can well approximate artificial surface reflectances (e.g., Munsell chips) [7]. However we see that for color constancy using non-linear equations they are not good enough to be used as basis functions for reflectance and illumination.

Despite that a large number of independent measurements are needed with nine narrow band

filters, the pointwise algorithm using nine filters has the advantage that the basis functions can be separately optimized for surface reflectance and illumination, and that a system of linear equations can be solved up to the solutions  $\varepsilon_j \sigma_i$ . However choosing the optimal basis functions requires studying many natural reflectances and illuminants.

Our future studies include more quantitative error analyses for various color constancy algorithms. As mentioned earlier, the measure of good algorithm is how gracefully the solutions degrade as the approximation breaks down. It can be expected that the algorithms of non-linear equations are very sensitive to the errors in the finite-dimensional approximation and to any errors in measurements. In controlled lighting environments, we found that the method using a reference object is insensitive to small errors [2], and the nine-filter formulation has the advantage of solving linear equations up to  $\varepsilon \sigma$  product-pairs.

The choice of basis functions requires investigation of many natural reflectances and illuminations. Since illumination and surface reflectance are different in nature, it is difficult to determine a small number of common basis functions to be used for the four or the five filters non-linear algorithms. The functions by Judd can be used well for daylights, however it has not yet been studied whether they can well represent other light sources such as tungsten and fluorescent lights or the lights reflected from other surfaces.

## 6 Conclusion

In this report we present an overview on the computational aspects of color constancy. The choice of basis functions determines the formulation of color constancy algorithms which also depend on the number of receptors and spatial points. Color constancy methods based on nonlinear equations are sensitive to errors in the finite-dimensional representation of reflectance and illumination. It is shown that the Fourier basis functions are not appropriate for use with this type of color

constancy algorithm. Development of color constancy algorithms based on the finite-dimensional representation should be directed towards better basis functions and robustness against modeling errors.

## Appendix

The transformation from the sensor space  $\rho$  into the space subtended by functions  $I(\lambda)$  is characterized by a linear system. Let us denote it by

$$\Omega\gamma = \rho \quad (25)$$

where the components of the vector  $\gamma$  are combinations of coefficients of basis functions for reflectance  $\sigma_i$  and illumination  $\varepsilon_i$ .  $\Omega$  is the matrix, whose elements depend on the choice of basis functions and filter functions, and components of the vector  $\rho$  are sensor measurements.

There is uncertainty in the elements of matrix  $\Omega$  caused by the error in estimation of filter functions. The values of the vector  $\rho$  contain digitization errors and sensory noise. The robustness analysis follows the arguments about the conditionality of linear systems [23]. Let us change the matrix  $\Omega$  for  $\delta\Omega$  and vector  $\rho$  for  $\delta\rho$ . The new solution will be  $\gamma + \delta\gamma$ .

$$(\Omega + \delta\Omega)(\gamma + \delta\gamma) = (\rho + \delta\rho) \quad (26)$$

What we are interested in is the relative change of  $\|\delta\gamma\| / \|\gamma\|$  relative to the changes  $\|\delta\Omega\| / \|\Omega\|$  and  $\|\delta\rho\| / \|\rho\|$ . From equation (26) we have

$$(I + \Omega^{-1}\delta\Omega)\delta\gamma = \Omega^{-1}(\delta\rho - \delta\Omega\gamma). \quad (27)$$

Applying an vector norm on the equation (27) we get



$$(1 - \|\Omega^{-1}\| \|\delta\Omega\|) \|\delta\gamma\| \leq \|\Omega^{-1}\| (\|\delta\rho\| + \|\delta\Omega\| \|\gamma\|). \quad (28)$$

The final result is

$$\frac{\|\delta\gamma\|}{\|\gamma\|} \leq \frac{\|\Omega\| \|\Omega^{-1}\|}{1 - \|\delta\Omega\| \|\Omega^{-1}\|} \left( \frac{\|\delta\Omega\|}{\|\Omega\|} + \frac{\|\delta\rho\|}{\|\rho\|} \right). \quad (29)$$

Let us denote the product

$$\|\Omega\| \|\Omega^{-1}\| = \text{cond}(\Omega). \quad (30)$$

If  $\text{cond}(\Omega)$  is small, then small perturbations in the data have little influence on the solution.

The lower bound of  $\text{cond}(\Omega)$  is

$$\text{cond}(\Omega) \geq 1. \quad (31)$$

The  $\text{cond}(\Omega)$  depends on the choice of basis functions and filter functions.

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