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Comments

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EXPLORATORY PROCEDURE FOR COMPUTER VISION

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Exploratory Procedure for Computer Vision

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Abstract

This paper deals with Exploratory Procedures for Computer Vision. The assumptions are that we have a mobile camera system with controllable focus, close/open aperture, and ability of recording its position, orientation and movement. Furthermore we assume an unknown and unstructured environment . For our analysis we consider two types of illumination sources: the point source and the extended sky-like source. The exploratory procedures determine the illumination energy, in some cases the illumination orientation, the albedo and the differentiation between the true 3D scene and its picture. The key idea is the mobile active observer.

1 Introduction

For past 15 years the Computer Vision Community has been addressing the problem known under the label: Shape from X. The problem is defined as follows: given one image (monocular view) how can we estimate and/or compute Shape, e.g., the surface slant and tilt from measuring different gradients: shading [8], texture [4], perspectivity [14] aerial cue citebib:sloan. Subsequently other authors have proposed improved algorithms [17], [15], [3], [19], [23], [7], but the principle is the same, that is, how to use so called monocular depth cues. It is well known that this problem as stated is ill-posed unless one brings to bear several constraints, i.e., *a priori* assumptions. Another approach to remove the ill-posedness is to make more controlled measurements in the spirit of Active Perception [5]. In particular, Aloimonos [2] has shown that under the assumption of the known movement of the observer, one can reduce the ill-posed problem into a well-posed problem for most of the formulations of Shape from X.

In this paper we wish to examine the following problem: given a mobile observer equipped with a monocular camera system (we can measure and control its position and orientation), we wish to establish from gray-scale measurements:

- 1. the energy and the orientation of the illumination,
- 2. the coefficient of reflectance of the material, i.e., the albedo,
- the decision whether a shaded appearance should be interpreted as 2D picture or 3D shaded surface.

These problems are important when the observer is in an unknown environment and has the need to explore and calibrate itself with respect to the current environment. This is not a paper on Shape from X, but rather a paper on Exploratory Procedures and Strategies that can be also viewed as a calibration procedure necessary before one can perform any further interpretation of the brightness measurements.

Pioneering work of Horn and his students [9], [24], [13] provides some results on the relationship between the camera, illumination and surface properties. Hence we shall build upon their results. The emphasis of our approach is on the exploration by a mobile observer.

Definition of the problem: As stated by Horn [12], the light source, observer/camera and the scene is a system described by many parameters: The point illumination has 2 orientation parameters (θ_i , ϕ_i), and if the source has some dimension, we have to consider the illumination from each part of the source (solid angle). The camera position can be represented by 2 parameters (θ_e , ϕ_e) for orthographic projection. For perspective projection, one more parameter for the distance from the object is added. The object surface in a position (2 parameters X and Y) has surface reflectance (ρ), and can be represented by its depth (Z) or equivalently by two angles of tilt and slant (or two gradients for surface normal). Now for an object point we have the reflected light that hits the retina (camera) $\delta L(\theta_e, \phi_e) = \rho \cdot \delta E(\theta_i, \phi_i) \cdot f(\theta_i, \phi_i; \theta_e, \phi_e)$, where ρ is the coefficient of reflectance. For the explanation of other symbols see the Figure 1. A standard scientific practice dictates that in order to compute the coefficient of reflectance from the measurement, we need all the other parameters be constant.

What follows is an outline of a sequence of exploratory procedures for estimating, the energy of the illumination source, the coefficient-of-reflectance, i.e., the albedo, and finally the recovery of the depth which will give us the decision making power to classify the 2D case from 3D case. This is all under the assumption of having control over the position, orientation of the light source and the camera.

2 Light and Albedo determination

Procedure for estimating energy of illumination source: One can make the camera directly aim at the illumination source and directly measure the energy (assuming that there is negligible absorption by the sensor). The test for the correct orientation of the camera is performed by creating a search space that samples different orientations with respect to the light source and searches for the maximum value. Confusion can occur if the environment contains a perfect mirror. In any cases we obtain the estimate of energy, but not the uniquely determined illumination orientation, which is similar to Pentland's result [18]. This measured intensity will be the estimate of the energy source. What is the advantage of estimating the illumination energy E versus the maximum scene radiance ?

E determines the maximum possible value of L and hence the dynamic range. If the scene contains a mirror like surface then the maximum radiance is equal to E. In all other cases, the maximum scene radiance will be substantially smaller than E and the dynamic range must be determined by the maximum radiance value.

Finally our goal is to estimate the albedos of different materials on the scene. It is true that for discriminatory purposes we only need to know the difference between the albedos of different materials. For the same surface orientation, $f(\theta_i, \phi_i; \theta_e, \phi_e)$ is a constant. E can be chosen as an arbitrary constant. The advantage of choosing E as the maximum possible illumination or as the maximum scene radiance (depending on the kind of scenes we observe) is that the range of albedos is normalized in every scene with respect to E and is between 0 and 1.

Procedure for estimating the coefficient-of-reflectance: The relationship for the coefficient-of-reflectance is equal to the ratio of the illumination energy to the reflected energy at a certain point, multiplied by a function called Bidirectional Reflectance Distribution Function (BRDF) [9] which depends on the geometry of the surface, the illumination source and the imaging device, The key to determine the coefficient is to know the illumination energy since the reflected energy is measured. The illumination energy is determined by the previous procedure. Following the analysis of Horn [12], for different geometries the relationships between illumination source and the surface are shown below: $f(\theta_i, \phi_i; \theta_e, \phi_e)$

Point source/specular surface	Point source/Lambertian surface
$\frac{\delta(\theta_e - \theta_i)\delta(\phi_e - \phi_i - \pi)}{\sin \theta_i \cos \theta_i}$	$rac{1}{\pi} \cdot \cos heta_i$
Extended (sky) source/specular	Extended (sky) source/Lambertian
1	1

Table I

We mean the extended sky-like source by the even illumination of the object all around. The function $f(\theta_i, \phi_i; \theta_e, \phi_e)$ is integrated over all the directions of the extended source/sky.

The next issue is how to decide which of the two different surfaces we are given. The necessary condition for this test is a mobile as oppose to a stationary observer.

Test for Lambertian surface can be made possible with the mobile observer. With a light source, different orientations of the object surface will result in different image radiance following the cosine law, but that image radiance is independent of the observing position. If the mobile observer provides us with two images with different viewing position on the same object, and if the correspondence between the two images is established, the corresponding points are of the same intensity for Lambertian. Image correspondence is easily provided in the case of orthographic projection. With known translation of the mobile observer, shifting the image with the same translation results in the same image intensities for Lambertian surface.

For specular surface, on the other hand, it is easy to see that the corresponding image points are not of the same intensity, since the image radiance depends on the viewing direction. Therefore any difference between the corresponding points indicates the presence of specular surface. As long as specular surface is not detected, we can treat the object as Lambertian in the given viewing direction even if the surface has some specular components.

In any of these tests the reflectance coefficient does not matter, since we are measuring only differences in image radiance as we move. (No correlation is needed, just the difference between the shifted first image and the second image). The above tests allow us to make the decision which of the relations shown in Table 1 one needs to use for determining the coefficient-of-reflectance. Note that a perfect specular surface has the coefficient-of-reflectance = 1 and so does the ideal Lambertian surface. We can determine the albedo of the surface only when we have extended sky-like illumination source.

Once we establish the coefficient-of-reflectance and the position of the observer, then we can investigate the structure of the object. Another way of establishing the coefficient-of-reflectance is by the use of the photometric stereo, developed by Woodham [24]. Now we present an example demonstrating the ability of mobile observer: a simple test of discrimination between a 3-D shaded object and a 2-D object with some variation of surface reflectance.

3 Exploring for the Consistency

So far we have assumed that the object under investigation is in fact either a Lambertian or a specular surface. As discussed in the previous section, specular surfaces are easily distinguishable. Any surface with slowly varying intensity may be interpreted as a Lambertian; the change of shading is solely due to the change of surface normal vector. Suppose instead the surface is a planar one on which the reflectance itself varies smoothly in such a way that the irradiance results in a shaded pattern. In other words, we are looking at the picture of shaded surface, not at the shaded surface itself.

Obviously, the system will fail to discriminate two cases as long as it relies on the analysis conducted on a single image. Multiple images obtained from different views shall be used to disambiguate the situation. In principle, one can apply any of standard multi-frame techniques to reveal the 3-D structure. Among them are stereo, generalized stereo and motion analysis. Although such approach may provide us with more or less quantitative description of the 3-D surface, we would like to address the issues involving the exploratory nature of the problem. Namely, we are more interested in the control strategy to overcome disambiguous situation or to determine the optimal motion which reveals the maximal information unavailable previously.

We would like to use the camera motion as a visual cue since the surface curvature contributes the way that the shading changes in time due to the motion. A large motion must be prohibited since then a large effort is needed to establish the correspondence between images. A small motion produces a small change in the image plane, and the standard gradient method can be used to determine the flow component along the direction of the image gradient [11]. This "normal" flow v_n is evaluated from

$$\|\nabla I\| v_n - \frac{\delta I}{\delta t} = 0.$$
⁽¹⁾

The method to compute the actual flows from the normal flows will be sketched later.

We want to move the camera in the image plane, in such a way that the motion results in the maximal change of scene, since any difference in the scene is caused by the change of the surface structure relative to the camera. One reasonable strategy is to move along the direction of principal curvature of the surface. Such motion will result in the maximal change of surface slope. Unfortunately, however, the principal curvature is not available until one solves the shape from shading problem. For our purpose, it is sufficient to move our camera along the gradient of intensity since it can be shown that the surface curvature does not vanish wherever the gradient exists [17]. So far we have not considered the motion along the line of sight. We argue that this motion is not suitable to our testing procedure. We will justify this point later. In principle, we need the translational motion only since the rotational motion does not reveal the surface structure at all. However, the rotational motion is useful to keep the region of interest inside the fovea of camera. The amount of rotation to counter-balance the translation is determined by the flow vector at the center of image plane. This flow vector is readily available even before the full flow computation. It is identical to the normal flow since the direction of the motion coincides to the direction of the image gradient. The tracking capability plays a very important role in our testing procedure. Since the region of interest stays in the fovea of camera all the time, we can exercise a relatively large translation, increasing the sensitivity of the testing condition while keeping the absolute magnitude of image flow small so that Equation (1) holds. Bandyopadhyay reported the advantage of the tracking ability for a different purpose [6].

Consider the camera coordinate system (X, Y, Z) as adopted from [16] (Figure 2). The origin of this coordinate system is located at the vertex of perspective projection, and the Z-axis is directed along the center of the instantaneous field of view. The instantaneous rigid body motion of this coordinate system is specified in terms of the translational velocity $\mathbf{V} = (V_X, V_Y, V_Z)$ of its origin and its rotational velocity $\mathbf{\Omega} = (\Omega_X, \Omega_Y, \Omega_Z)$. These motion parameters can be controlled by the robotic manipulator which holds the camera. The 2-D image sequence is created by the perspective projection of the object onto a planar screen oriented normal to the Z-axis. The origin of the image coordinate system (x, y) on the screen is located in space at (X, Y, Z) = (0, 0, 1). Due to the camera's motion, a point P in space (located by position vector \mathbf{R}) moves with a relative velocity $\mathbf{U} = -(\mathbf{V} + \mathbf{\Omega} \times \mathbf{R})$. In component form we express the motion of P through space as

$$\dot{X} = -V_X - \Omega_Y Z + \Omega_Z Y, \quad (2.a)$$

$$\dot{Y} = -V_Y - \Omega_Z X + \Omega_X Z, \quad (2.b)$$

$$\dot{Z} = -V_Z - \Omega_X Y + \Omega_Y X. \quad (2.c)$$

(2)

At each instant, point P projects onto the screen as point p. The coordinates of p on the screen are given by

$$\begin{aligned} x &= X/Z, \quad (3.a) \\ y &= Y/Z. \quad (3.b) \end{aligned}$$

The corresponding image velocities of point p are simply $(v_x, v_y) = (\dot{x}, \dot{y})$, obtained by differentiating the image coordinates with respect to time and utilizing Equation (2),

$$v_{x} = \left\{ x \frac{V_{Z}}{Z} - \frac{V_{X}}{Z} \right\} + \left\{ x y \Omega_{X} - (1 + x^{2}) \Omega_{Y} + y \Omega_{Z} \right\}, \quad (4.a)$$

$$v_{y} = \left\{ y \frac{V_{Z}}{Z} - \frac{V_{Y}}{Z} \right\} + \left\{ (1 + y^{2}) \Omega_{X} - x y \Omega_{Y} - x \Omega_{Z} \right\}. \quad (4.b)$$
(4)

These equations define an instantaneous image flow field, assigning a unique 2-D image velocity \mathbf{v} to each direction (x, y) in the observer's field of view.

Since we know the camera motion \mathbf{V} and $\mathbf{\Omega}$, we could determine the object distance by solving Equation (4) at each point. Further structural information such as the slope and the curvature may by obtained by differentiating the distance Z. There are two problems in this approach. First, it is not easy to maintain the camera motion precisely; while the translational motion is relatively easy to control, the pure rotational motion is not. Since the center of robot end-effector does not coincide with the center of camera coordinates, the rotation of camera must involve both translation and rotation of the end-effector. The exact amount of translation and rotation of end-effector may be determined by the *dynamic* camera calibration [1]. It was reported that a small error in the calibration often results in a large error in positioning after a few sequences of motion. Secondly, even after one controls the camera motion accurately, the image acquisition and the camera motion must be synchronized perfectly so that the motion parameters at the time of image acquisition are to be known. Because of the nature of the problem, the camera may move from one location to another without knowing its exact motion, although its approximate motion is known. Under this circumstance, we would like to find some properties of flow field which reveal the surface structure only.

Suppose the surface under investigation is approximated locally as a quardric one;

$$Z = Z_0 + pX + qY + C_1 X^2 + C_2 XY + C_3 Y^2,$$
(5)

where p and q are the surface gradient, and C_1 , C_2 and C_3 are the curvature parameters. Substituting Z in Equation (4) with Equation (5), and expanding it near

the center of the image coordinates [21];

$$v_{x} = \left[-\frac{V_{x}}{Z_{0}} - \Omega_{Y}\right] + \left[\frac{V_{z}}{Z_{0}} + p\frac{V_{x}}{Z_{0}}\right]x + \left[q\frac{V_{x}}{Z_{0}} + \Omega_{Z}\right]y + \left[-p\frac{V_{z}}{Z_{0}} + V_{X}C_{1} - \Omega_{Y}\right]x^{2} + \left[-q\frac{V_{z}}{Z_{0}} + V_{X}C_{2} + \Omega_{X}\right]xy + \left[V_{X}C_{3}\right]y^{2} + \dots \quad (6.a)$$

$$v_{y} = \left[-\frac{V_{Y}}{Z_{0}} + \Omega_{X}\right] + \left[p\frac{V_{Y}}{Z_{0}} - \Omega_{Z}\right]x + \left[\frac{V_{z}}{Z_{0}} + q\frac{V_{Y}}{Z_{0}}\right]y + \left[V_{Y}C_{1}\right]x^{2} + \left[-p\frac{V_{z}}{Z_{0}} + V_{Y}C_{2} - \Omega_{Y}\right]xy + \left[-q\frac{V_{z}}{Z_{0}} + V_{X}C_{3} + \Omega_{X}\right]y^{2} + \dots \quad (6.b)$$

$$(6)$$

Notice that the curvature is revealed by the second-order terms of image motion, while the first-order terms are determined by the surface slope only. In fact, one can show that the image flow generated by an arbitrary motion of planar surface $Z = Z_0 + pX + qY$ is modeled exactly as second-order polynomials [22];

$$v_{x} = \left[-\frac{V_{X}}{Z_{0}} - \Omega_{Y} \right] + \left[\frac{V_{Z}}{Z_{0}} + p\frac{V_{X}}{Z_{0}} \right] x + \left[q\frac{V_{X}}{Z_{0}} + \Omega_{Z} \right] y + \left[-p\frac{V_{Z}}{Z_{0}} - \Omega_{Y} \right] x^{2} + \left[-q\frac{V_{Z}}{Z_{0}} + \Omega_{X} \right] xy,$$

$$v_{y} = \left[-\frac{V_{Y}}{Z_{0}} + \Omega_{X} \right] + \left[p\frac{V_{Y}}{Z_{0}} - \Omega_{Z} \right] x + \left[\frac{V_{Z}}{Z_{0}} + q\frac{V_{Y}}{Z_{0}} \right] y + \left[-p\frac{V_{Z}}{Z_{0}} - \Omega_{Y} \right] xy + \left[-q\frac{V_{Z}}{Z_{0}} + \Omega_{X} \right] y^{2}.$$
(7)

The image motion in the polynomial form as in Equations (6) can be obtained from two image frames, by fitting the second-order polynomial to each component flow vectors [22]. This procedure involves solving a linear least-squares problem, and is very efficient computationally.

By comparing Equations (6) and (7) one can see that the y^2 term of v_x is determined completely by the translational motion and surface curvature. They are independent of the rotational motion. Similar observation is made at the x^2 term of v_y . Let the y^2 term of v_x be v_x^{02} and the x^2 term of v_y be v_y^{20} . Then the total curvature is given as

$$C_1 + C_2 = \frac{v_x^{02}}{V_X} + \frac{v_y^{20}}{V_Y} \tag{8}$$

Rather than evaluating the curvature, we simply test the condition;

$$v_x^{02} + v_y^{20} \ge \epsilon. \tag{9}$$

If this condition is satisfied, we conclude that the surface is curved, and at the same time it is a Lambertian. Otherwise, the surface is planar, and the shading is due to the change of the reflectance. The tolerance ϵ in Equation (9) is determined by the least-squares error in the course of image motion recovery. The larger the error is, the less accurate is the quality of the recovered image motion, thereby loosing the ability to discriminate the curved surfaces from the planar ones.

4 Experimental Results and Discussions

Experiments are performed to demonstrate the efficacy of simple plane/curvature test. We used a round object as in Figure 3 and the picture of the same round object as a 3-D uniform reflectance object and a shaded 2-D planar one, respectively. Both objects do not possess any specular reflection, and a single illumination source at a long distance is used to simulate a point source. The camera held by a robot is directed downward to the object, and simple translational movement is generated perpendicular to the camera axis. As mentioned before, it is not necessary to measure the amount of translational motion. A simple calibration procedure is taken for **V**. To insure the perspectivity, we use a wide angle camera (f=8.5 mm) and keep the camera close to the object.

In Figures 4(a) and 4(b), two frames of sampled images of the 3-D object are shown, and in Figures 5(a) and 5(b) are shown two image frames of the shaded 2-D object. The image flow recovered from Figures 4(a) and 4(b) is computed as

$$v_x = 4.119586 + 0.001779x + 0.031398y - 0.000327x^2 + 0.000110xy - 0.005391y^2$$
$$v_y = 0.948992 - 0.007141x - 0.024631y + 0.000008x^2 + 0.000072xy + 0.000127y^2$$
$$v_x^{02} + v_y^{20} = 0.005399$$

The normal flows and the full flows are shown in Figures 4(c) and 4(d), respectly. The image flow from Figures 5(a) and 5(b) is

$$v_x = 5.785596 + 0.000694x - 0.001483y - 0.000239x^2 - 0.000035xy - 0.000044y^2$$

$$v_y = 0.155736 - 0.002326x + 0.003212y - 0.000013x^2 + 0.000221xy - 0.000025y^2$$

$$v_x^{02} + v_y^{20} = 0.000057$$

Again, Figures 5(c) and 5(d) show the normal flows and the full flows, respectively. The testing procedure described in Section 3 interprets images correctly.

5 Conclusion

We have presented here exploratory or calibration procedures for computer vision. We make two assumptions: First one is the existence of an mobile observer/camera equipped with controllable aperture and focus. The observer can also record position, orientation and movement of itself. However, this observer is in an unknown environment. The second assumption is about the illumination, that is, we assume either a point source or an extended sky-like source. We have investigated what procedures/strategies have to be built in for the observer to determine visually different materials and and to determine whether it sees 3D scene or just a 2D picture. The results indicate that one can uniquely determine all the above with a mobile observer. This is important especially in the application of robotics in unknown environments such as in the space, underwater and hazardous environments.

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Figure 1. Object, light and camera



Figure 2. The image plane and the camera coordinates.



Figure 3. The object used in the experiment.



Figure 4. Experiment on the curved surface.

1.4

(a) frame 1 (b) frame 2 (c) normal flows (d) full flows