October 1991

Control of Multiple Arm Systems With Rolling Constraints

Xiaoping Yun
University of Pennsylvania

R. Vijay Kumar
University of Pennsylvania, kumar@grasp.upenn.edu

Nilanjan Sarkar
University of Pennsylvania

Eric Paljug
University of Pennsylvania

Recommended Citation

Xiaoping Yun, R. Vijay Kumar, Nilanjan Sarkar, and Eric Paljug, "Control of Multiple Arm Systems With Rolling Constraints", . October 1991.

University of Pennsylvania Department of Computer and Information Science, Technical Report No. MS-CIS-91-79.

Control of Multiple Arm Systems With Rolling Constraints

Abstract

When multiple arms are used to manipulate a large object, it is necessary to maintain and control contacts between the object and effector(s) on one or more arms. The contacts are characterized by holonomic as well as nonholonomic constraints. This paper addresses the control of mechanical systems subject to nonholonomic constraints, rolling constraints in particular. It has been shown that such a system is always controllable, but cannot be stabilized to a single equilibrium by smooth feedback [l, 2]. In this paper, we show that the system is not input-state linearizable though input-output linearization is possible with appropriate output equations. Further, if the system is position-controlled (i.e., the output equation is a functions of position variables only), it has a zero dynamics which is Lagrange stable but not asymptotically stable. We discuss the analysis and controller design for planar as well as spatial multi-arm systems and present results from computer simulations to demonstrate the theoretical results.

Comments

University of Pennsylvania Department of Computer and Information Science, Technical Report No. MS-CIS-91-79.

Control Of Multiple Arm Sytems With Rolling Constraints

MS-CIS-91-79
GRASP LAB 278

Xiaoping Yun
Vijay Kumar
Nilanjan Sarkar
Eric Paljug

Department of Computer and Information Science
School of Engineering and Applied Science
University of Pennsylvania
Philadelphia, PA 19104-6389

October 1991

Acknowledgements:
This work was in part supported by Airforce grant AFOSR F49620-85-K-0018, Army/DAAG-29-84-K-0061, NSF-CER/DCR82-19196 Ao2, NASA NAG5-1045, ONR SB-35923-0, NIH grant NS-10939 -11 as part of Cerebro Vascular Research Center, NIH 1-RO1-NS-23636-01,NSF INT85-14199,NSF DMC85-17315, ARPA N0014-88-K-0632,NATO grant No.0224/85. by DEC Corp., IBM Corp., LORD Corp. and University of Pennsylvania Research Foundation.

Control of Multiple Arm Systems with Rolling Constraints

Xiaoping Yun, Vijay Kumar, Nilanjan Sarkar, and Eric Paljug
General Robotics and Active Sensory Perception
(GRASP) Laboratory
University of Pennsylvania
3401 Walnut Street, Room 301C
Philadelphia, PA 19104-6228

ABSTRACT

When multiple arms are used to manipulate a large object, it is necessary to maintain and control contacts between the object and effector(s) on one or more arms. The contacts are characterized by holonomic as well as nonholonomic constraints. This paper addresses the control of mechanical systems subject to nonholonomic constraints, rolling constraints in particular. It has been shown that such a system is always controllable, but cannot be stabilized to a single equilibrium by smooth feedback [1, 2]. In this paper, we show that the system is not input-state linearizable though input-output linearization is possible with appropriate output equations. Further, if the system is position-controlled (i.e., the output equation is a functions of position variables only), it has a zero dynamics which is Lagrange stable but not asymptotically stable. We discuss the analysis and controller design for planar as well as spatial multi-arm systems and present results from computer simulations to demonstrate the theoretical results.

1 Introduction

Most current manipulators perform tasks with their end effectors (e.g., grippers, hands, etc.) while manipulator links provide positioning of the end effectors. The class of objects which can manipulated by end effectors are limited to relatively small objects or objects with special features such as handles. A large object without special features (e.g., a cardboard box having dimensions on the order of the manipulator's size) can not easily be grasped by end effectors (which are normally much smaller than manipulators themselves). Having large end effectors is not a feasible solution since they in turn require large manipulators to support their own load. While a special-purpose end effector may be designed to grasp a specific object such as a cardboard box, the problem of manipulating large objects of arbitrary shape remains.

Human beings circumvent such problems by utilizing not only hands but also arms, bodies, and even legs for manipulation tasks, especially for transporting large objects. Salisbury and Townsend [3] proposed the concept of the whole arm manipulation which allows the contacts with the object to be on any part of the manipulator. However, it also poses a number of challenging problems such as arm design [3], distributed sensing, and control. The scope of this paper is confined to control issues concerning whole arm manipulation and grasping with multiple arms.

The main difference between the manipulation of small (graspable) objects and large objects is that in the latter, relative motion between the object and the effector is possible, and the contacts cannot transmit arbitrary forces/moments. In contrast, a small object can be lifted and transported with an end-effector employing a fixed grasp, that is one in which there is no relative motion between the end effector and the object, and the end effector can apply arbitrary forces and moments to the object. In the whole arm manipulation, however, the object may move (e.g., roll and/or slide) along the contact surfaces.

The kinematic constraint equations and transformations between cartesian (task-space) and local coordinates are presented in [4, 5, 6]. Control of sliding has been studied in [7]. But, the assumption here is that the contact forces are such that pure rolling (sticking) never occurs.

It is well-known that three-dimensional rolling constraint equations are nonholonomic. Dynamic modeling of mechanical systems with nonholonomic constraints is richly documented by work ranging from Neimark and Fufaev's comprehensive book [8] to more recent developments (see for example, [9]). However, the literature on control properties of such systems is sparse [2]. The interest in control of nonholonomic systems has been stimulated by the recent research in robotics. The dynamics of a wheeled mobile robot is nonholonomic [10], and so is a multi-arm system manipulating an object through the whole arm manipulation [11]. The dynamics of free-floating robots in space is nonholonomic. Here the nonholonomic constraint is the equation for conservation of angular momentum [12, 13].

Bloch and McClamroch [2] first demonstrated that a nonholonomic system cannot be feedback stabilized to a single equilibrium point by a smooth feedback. In a follow-up paper [14], they showed that the system is small-time locally controllable. Campion *et al* [1] showed that the system is controllable regardless of the structure of nonholonomic constraints. Barraquand and Latombe proved that a car towing up to two trailers is also controllable [15].

Motion planning of mobile robots has been an active topic in robotics in the past several years [16, 17, 10, 18]. Nevertheless, much less is known about the dynamic control of mobile robots with nonholonomic constraints and the developments in this area are very recent [19, 20, 21].

In this paper, we first formulate the control problem incorporating the dynamics of multiple arm systems with holonomic and nonholonomic constraints. We discuss several unique control properties of mechanical systems with nonholonomic constrains. Specifically, we show that such a system is not input-state linearizable. Nevertheless, the input-output linearization is still possible with properly chosen output equatoins. In particular, we investigate the input-output linearization and zero dynamics of the system with the output equations chosen for position control. It is shown that the system under position control is input-output linearizable and has a zero dynamics which is Lagrange stable but not asymptotically stable. These results are applied to a two-arm system in which two 6 degree-of-freedom arms manipulate a large object with arbitrary effectors attached to the sixth link. We derive the motion equations and the nonholonomic constraint equations, and present the controller design of the two-arm system. Finally, we conduct simulations with a planar, two-arm system in which the nonholonomic equations are integrable. Results from the simulation illustrate the effectiveness of the design method.

2 Dynamics of Mechanical Systems with Contact Constraints

2.1 Constraint Equations and Dynamic Equations of Motion

Consider a mechanical system with n generalized coordinates q subject to m bilateral constraints whose equations of motion are described by

$$M(q)\ddot{q} + V(q,\dot{q}) = E(q)\tau - J^T(q)\lambda \tag{1}$$

where $M(q)$ is the $n \times n$ inertia matrix, $V(q,\dot{q})$ is the n-dimensional vector of Coriolis, centripetal, and gravity forces, $E(q)$ is the $n \times r$ input transformation matrix [1]. τ is the r-dimensional input vector, $J(q)$ is the $m \times n$ Jacobian matrix, and λ is the vector of constraint forces. The m constraint equations of the mechanical system, in general, have the following form

$$C(q,\dot{q}) = \begin{bmatrix} C_1(q,\dot{q}) \\ C_2(q,\dot{q}) \\ \vdots \\ C_m(q,\dot{q}) \end{bmatrix} = 0 \tag{2}$$

If a constraint equation is in the form $C_i(q) = 0$, or can be integrated into this form, it is a holonomic constraint. Otherwise it is a kinematic (not geometric) constraint and is termed nonholonomic.

We assume that we have k holonomic and $m-k$ nonholonomic independent constraints, all of which can be written in the form of

$$A(q)\dot{q} = 0 \tag{3}$$

where $A(q)$ is an $m \times n$ dimensional matrix of full rank. Let $s_1(q)$, $\cdots$, $s_{n-m}(q)$ be a set of smooth and linearly independent vector fields in the null space of $A(q)$, i.e.,

$$A(q)s_i(q) = 0 \qquad\qquad i = 1,\ldots,n-m.$$

Let $S(q)$ be the full rank matrix made up of these vectors

$$S(q) = [s_1(q) \quad \cdots \quad s_{n-m}(q)] \tag{4}$$

and let Δ be the distribution spanned by these vector fields

$$\Delta = span\{s_1(q) \quad \cdots \quad s_{n-m}(q)\}$$

It follows that $\dot{q} \in \Delta$. Δ may or may not be involutive. For that reason, we let Δ^* be the smalled involutive distribution containing Δ. It is clear that $dim(\Delta) \leq dim(\Delta^*)$. There are three possible cases (as observed by Champion, *et al.* in [1]):

- If $k = m$, that is, all the constraints are holonomic, then Δ is involutive itself.
- If $k = 0$, that is, all the constraints are nonholonomic, then Δ^* spans the entire space.
- If $0 < k < m$, the k constraints are integrable and k components of the generalized coordinates may be eliminated from the motion equations. Now, $dim(\Delta^*) = n - k$.

2.2 Two-Body Contact

In this subsection, using the notations defined above we show the classic results that the constraint equations for two rigid bodies in the 2-dimensional space are always integrable (thus holonomic) and that those in the 3-dimensional space are nonholonomic.

---

[1] $E(q)$ is an identity matrix in most cases. However, if the generalized coordinates are chosen to be some variables other than the joint variables, or if there are passive joints without actuators, it is not an identity matrix.

2.2.1 Spatial Case

Consider two bodies in contact at a point p, as shown in Figure 1. We use S_1 and S_2 to denote the surfaces of the two bodies, respectively. Let S_{1p} be an open and connected subset of S_1 containing the point p. Then the pair (f_1, U_1) is called a coordinate system of S_{1p} if there exists an open subset U_1 of $\mathbf{R}^2$ and an invertible map $f_1 : U_1 \longrightarrow S_{1p}$ such that the partial derivatives $\frac{\partial f_1(\mathbf{u})}{\partial u}$ and $\frac{\partial f_1(\mathbf{u})}{\partial v}$ are linearly independent for all $\mathbf{u} = (u, v) \in U_1$. Let K_1, T_1, and M_1 denote, respectively, the curvature form, torsion form, and metric tensor of S_1 at point p relative to the coordinate system (f_1, U_1). All the notation for S_2 can be defined similarly. The contact point on S_1 (or S_2) is specified by the coordinates u_1 and v_1 (or u_2 and v_2). In order to completely specify the contact configuration we need a fifth variable ψ, which can be the angle between the tangent to the u_1-coordinate curve and that to the u_2-coordinate curve at the contact point, following any convenient convention for the sign of ψ. Thus

$$q = [u_1 \quad v_1 \quad u_2 \quad v_2 \quad \psi] \tag{5}$$

Let (v_x, v_y, v_z) be the relative translational velocity at the contact point, and $(\omega_x, \omega_y, \omega_z)$ the relative rotational velocity between the two bodies. The following equations for the contact kinematics of the two bodies have been derived by Montana [6]:

$$\begin{aligned}
\dot{\mathbf{u}}_1 &= M_1^{-1}(K_1 + \tilde{K}_2)^{-1}\left(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} - \tilde{K}_2 \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right) && (6)\\
\dot{\mathbf{u}}_2 &= M_2^{-1}R_\psi(K_1 + \tilde{K}_2)^{-1}\left(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} + K_1 \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right) && (7)\\
\dot{\psi} &= \omega_z + T_1 M_1 \dot{\mathbf{u}}_1 + T_2 M_2 \dot{\mathbf{u}}_2 && (8)\\
0 &= v_z && (9)
\end{aligned}$$

where

$$R_\psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ -\sin\psi & -\cos\psi \end{bmatrix} \qquad\qquad \tilde{K}_2 = R_\psi K_2 R_\psi$$

For the rolling contact, we have $v_x = 0$ and $v_y = 0$. Substituting them into Equations (6) and (7) and eliminating ω_x and ω_y, we obtain the rolling constraint equation

$$R_\psi M_1 \dot{\mathbf{u}}_1 - M_2 \dot{\mathbf{u}}_2 = 0 \tag{10}$$

It can be rewritten in the form of Equation (4)

$$A(q)\dot{q} = 0 \tag{11}$$

where

$$A(q) = [R_\psi M_1 \quad -M_2 \quad 0]$$

We choose the $S(q)$ matrix (defined in Equation (4)) as follows:

$$S(q) = [s_1(q) \quad s_2(q) \quad s_3(q)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ s_{31} & s_{32} & 0 \\ s_{41} & s_{42} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

where

$$\begin{bmatrix} s_{31} & s_{32} \\ s_{41} & s_{42} \end{bmatrix} = M_2^{-1} R_\psi M_1$$

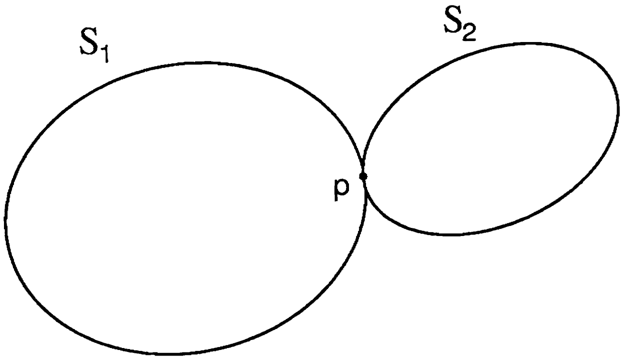

Figure 1: Two Rigid Bodies in Contact

We now compute the Lie Brackets

$$s_4(q) = [s_1(q), s_3(q)] = \frac{\partial s_3}{\partial q} s_1 - \frac{\partial s_1}{\partial q} s_3 = \begin{bmatrix} 0 \\ 0 \\ s_{34} \\ s_{44} \\ 0 \end{bmatrix}, \qquad s_5(q) = [s_2(q), s_3(q)] = \begin{bmatrix} 0 \\ 0 \\ s_{35} \\ s_{45} \\ 0 \end{bmatrix} \tag{13}$$

where

$$\begin{bmatrix} s_{34} & s_{35} \\ s_{44} & s_{45} \end{bmatrix} = -M_2^{-1} \frac{\partial R_\psi}{\partial \psi} M_1$$

Therefore, the distribution spanned by the vector fields $s_1(q)$, $s_2(q)$, and $s_3(q)$ is not involutive since $s_4(q)$ and $s_5(q)$ are not in the distribution. Further, $s_1(q)$ through $s_5(q)$ span the entire 5-dimensional configuration space. It follows from the result in the preceding subsection that the two rolling constraints are nonholonomic. Note that for pure rolling, that is, if the spin motion $\omega_z = 0$ in addition to v_x and v_y being zero), a similar approach shows that all three constraints are nonholonomic.

2.2.2 Planar Case

For two planar bodies (curves), the kinematic equations of contact, Equations (6) and (7), are reduced to

$$M_1(K_1 + K_2)\dot{u}_1 = -\omega_y - K_2 v_x \tag{14}$$
$$M_2(K_1 + K_2)\dot{u}_2 = -\omega_y + K_1 v_x \tag{15}$$

Once again, for the rolling constraint we set $v_x = 0$. Further if we eliminate ω_y from the above two equations, we obtain the rolling constraint for the two planar bodies in contact

$$M_1(K_1 + K_2)\dot{u}_1 - M_2(K_1 + K_2)\dot{u}_2 = 0 \tag{16}$$

Choosing the 2-dimensional configuration space which is locally defined by the coordinates of the two curves, we have

$$q = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The $A(q)$ matrix defining the rolling constraint $A(q)\dot{q} = 0$ is clearly

$$A(q) = [M_1(K_1 + K_2) \quad - M_2(K_1 + K_2)]$$

and the $S(q)$ matrix, which spanns the null space of $A(q)$, is

$$S(q) = \begin{bmatrix} M_2 \\ M_1 \end{bmatrix}$$

The distribution spanned by $S(q)$, a single vector field, is trivially involutive. Therefore we get the well-known result that the rolling constraint of the two planar bodies is holonomic.

2.3 Dynamics of Nonholonomic Systems

We now consider a mechanical system with the following motion and constraint equations

$$M(q)\ddot{q} + V(q,\dot{q}) = E(q)\tau - A^T(q)\lambda \tag{17}$$

$$A(q)\dot{q} = 0 \tag{18}$$

We assume, without loss of generality, that all the m constraint equations are nonholonomic. If $k \neq 0$, the k constraint equations can be used to eliminate k generalized coordinates, under the standard smoothness assumptions. With the matrix $S(q)$ being defined as in Equation (4), it follows that

$$A(q)S(q) = 0 \tag{19}$$

Noting Equation (19), we multiply the both sides of Equation (17) by $S^T(q)$ to eliminate the constraint force from the motion equations.

$$S^T(q)M(q)\ddot{q} + S^T(q)V(q,\dot{q}) = S^T(q)E(q)\tau \tag{20}$$

From the constraint equation (18), the constrained velocity is always in the null space of $A(q)$. It is possible to define $n-m$ velocites $\nu(t) = [\nu_1 \quad \nu_2 \cdots \nu_{n-m}]$ such that

$$\dot{q} = S(q)\nu(t) \tag{21}$$

These velocities need not be integrable but they can be regarded as being time derivatives of $n-m$ *quasi-coordinates*[2] $\mu_1, \mu_2, \ldots, \mu_{n-m}$. For example, we can choose the quasi-coordinates so that $\nu = \dot{\mu} = S^+\dot{q}$. Here S^+ is the generalized inverse of S.

Differentiating Equation (21) with respect to time, we obtain

$$\ddot{q} = S(q)\dot{\nu}(t) + \dot{S}(q)\nu(t) \tag{22}$$

Substituting Equation (22) into the motion equation (20), we have

$$S^T(q)M(q)S(q)\dot{\nu}(t) + S^T(q)M(q)\dot{S}(q)\nu(t) + S^T(q)V(q,\dot{q}) = S^T(q)E(q)\tau \tag{23}$$

At this point, we choose the the following state variable

$$x = \begin{bmatrix} q \\ \nu \end{bmatrix} \tag{24}$$

Using this state variable, the motion equation (23) is then written in the state space

$$\dot{x} = \begin{bmatrix} \dot{q} \\ \dot{\nu} \end{bmatrix} = \begin{bmatrix} S\nu \\ (S^TMS)^{-1}(-S^TM\dot{S}\nu - S^TV) \end{bmatrix} + \begin{bmatrix} 0 \\ (S^TMS)^{-1}S^TE \end{bmatrix}\tau \tag{25}$$

---

[2] See [22] for the definition of quasi-coordinates.

Assuming that the number of inputs is greater or equal to the degrees of freedom of the mechanical system, that it , $r \geq n-m$, and $(S^TMS)^{-1}S^TE$ has rank $n-m$, we may apply the following nonlinear feedback to simplify the state equation

$$\tau = ((S^TMS)^{-1}S^TE)^+[u - (S^TMS)^{-1}(-S^TM\dot{S}\nu - S^TV)] \tag{26}$$

where $(A)^+$ denotes the generalized inverse of matrix A. Applying this feedback, the state equation becomes

$$\begin{bmatrix} \dot{q} \\ \dot{\nu} \end{bmatrix} = \begin{bmatrix} S(q)\nu \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{27}$$

or simply

$$\dot{x} = f(x) + g(x)u \tag{28}$$

where $f(x)$ and $g(x)$ can be easily identified.

2.4 On the Control of Nonholonomic Systems

2.4.1 Controllability, Stabilization, and Linearization

The following properties of the system (27) have been established in [1, 2]

Theorem 1 *The nonholonomic system (27) is controllable.*

Theorem 2 *The equilibrium point $x = 0$ of the nonholonomic system (27) can be made Lagrange stable, but can not be made asymptotically stable by a smooth state feedback.*

The feedback linearization is a useful design technique for nonlinear systems. Unfortunately, the nonholonomic system (27) is not input-state linearizable. Nevertheless, the system is still input-output linearizable with proper output equations (see the next subsection).

Theorem 3 *The nonholonomic system (27) is not input-state linearization by a state feedback.*

Proof: The system has to satisfy two conditions: the strong accessibility condition and the involutivity condition [23, p. 179]. The strong accessibility condition is satisfied since the system is controllable.

Define a sequence of distributions

$$D_j = span\{L_f^i g \mid i = 0, 1, \ldots, j-1\}, \quad j = 1, 2, \ldots \tag{29}$$

Then the involutivity condition requires that the distribution D_1, D_2, $\ldots$, D_{2n-m} are all involutive. Note that the dimension of the state variable is $2n-m$. $D_1 = span\{g\}$ is involutive since g is constant. Next we compute

$$L_f g = [f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g = -\begin{bmatrix} S(q) \\ 0 \end{bmatrix}$$

Since the distribution spanned by the columns of $S(q)$ is not involutive for nonholonomic constraints, the distribution $D_2 = span\{g, L_f g\}$ is not involutive. Therefore, the system is not input-state linearizable.

2.4.2 Output Equations and Zero Dynamics

As shown above, the nonholonomic system is not input-state linearizable, but it may still be input-output linearizable if a proper set of output equations are chosen. Let us consider the position control of the system, i.e., the output equations are functions of position state variable q only. Since the degrees of freedom of the system is instantaneously $n-m$, we may have at most $n-m$ independent position components in output equations. Let the output equation be given by the following

$$y = h(q) = \begin{bmatrix} h_1(q) \\ \vdots \\ h_{n-m}(q) \end{bmatrix} \tag{30}$$

and let the $(n-m)\times n$ Jacobian matrix of the output be denoted by $J_h = \frac{\partial h}{\partial q}$. The necessary and sufficient condition for input-output linearization is that the decoupling matrix has full rank [24]. With the output equation (30), the decoupling matrix $\Phi(x)$ for the nonholonomic system is the $(n-m)\times(n-m)$ matrix

$$\Phi(x) = J_h(q)S(q) \tag{31}$$

For $\Phi(x)$ to be nonsingular, the rows of J_h can not be in the row space of $A(q)$.

Without loss of generality, we assume that the first $n-m$ rows of $S(q)$ are linearly independent. That is, if we partition $S(q)$ into $S_1(q)$ and $S_2(q)$ as follows

$$S(q) = \begin{bmatrix} S_1(q) \\ S_2(q) \end{bmatrix} \tag{32}$$

$S_1(q)$ is an $(n-m)\times(n-m)$ square matrix of full rank. We also partition q in accordance with the partition of $S(q)$

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{33}$$

where q_1 is $(n-m)$-dimensional and q_2 is m-dimensional. Using the partition of q, we have three blocks in the state space

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \nu \end{bmatrix} \tag{34}$$

Since S_1 is nonsingular, we may choose the first $n-m$ generalized coordinate as outputs, namely, $y = h(q) = q_1$. In this case, it is clear that the decoupling matrix is simply S_1. Therefore, the system is input-output linearizable.

To characterize the zero dynamics and achieve input-output linearization, we introdue a new state space variable z defined as follows

$$z = T(x) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} h(q) \\ L_f h(q) \\ q_2 \end{bmatrix} = \begin{bmatrix} q_1 \\ S_1(q)\nu \\ q_2 \end{bmatrix} \tag{35}$$

It is easy to verify that $T(x)$ is indeed a diffeomorphism (a valid state space transformation) by checking its Jacobian, which is computed below.

$$\frac{\partial T}{\partial x} = \begin{bmatrix} I & 0 & 0 \\ \frac{\partial(S_1\nu)}{\partial q_1} & \frac{\partial(S_1\nu)}{\partial q_2} & S_1 \\ 0 & I & 0 \end{bmatrix} \tag{36}$$

Since S_1 is of full rank, so is $\frac{\partial T}{\partial x}$. The inverse of the state space transformation is

$$x = T^{-1}(z) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_3 \\ S_1^{-1}(q)z_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_3 \\ S_1^{-1}(z_1, z_3)z_2 \end{bmatrix} \tag{37}$$

The system under the new state variable z is characterized by

$$\dot{z}_1 = \dot{q}_1 = S_1(q)\nu = z_2 \tag{38}$$

$$\dot{z}_2 = \frac{d(S_1\nu)}{dt} = \frac{\partial(S_1\nu)}{\partial x}\dot{x} = \frac{\partial(S_1\nu)}{\partial q}S(q)\nu + S_1(q)u \tag{39}$$

$$\dot{z}_3 = \dot{q}_2 = S_2(q)\nu = S_2(z_1, z_3)S_1^{-1}(z_1, z_3)z_2 \tag{40}$$

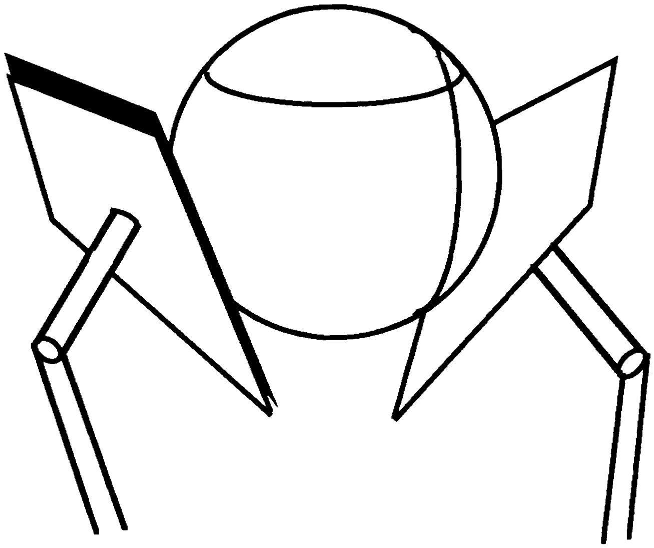

Figure 2: Two 6-DOF Arms Manipulating a Large Object

Utilizing the following state feedback

$$u = S_1^{-1}(q)(v - \frac{\partial(S_1\nu)}{\partial q}S(q)\nu) \tag{41}$$

we achieve input-output linearization as well as input-output decoupling by noting the observable part of the system

$$\begin{aligned} \dot{z}_1 &= z_2 & (42)\\ \dot{z}_2 &= v & (43)\\ y &= z_1 & (44) \end{aligned}$$

The unobservable zero dynamics of the system is (obtained by substituting $z_1 = 0$ and $z_2 = 0$)

$$\dot{z}_3 = 0 \tag{45}$$

which is Lagrange stable but not asymptotically stable.

3 A Two-Arm System

In this section, we will apply the results on nonholonomic systems described in the preceding section to a two-arm system shown in Figure 2.

Each arm has six degrees of freedom, and a flat-surface palm. The two arms manipulate a large object by supporting it with two palms. As shown in Section 2.2 the constraint equations characterizing two-body contacts in the three dimensional space is nonholonomic, this two-arm setup results in a nonholonomic system.

Let $X_i = [x_i \quad y_i \quad z_i \quad \theta_i \quad \phi_i \quad \psi_i]$ be the position and orientation of arm i in a fixed coordinate frame. Then the equations of motion of arm i are governed by

$$M_i(X_i)\ddot{X}_i + V_i(X_i, \dot{X}_i) = J_i^{-T}(X_i)\tau_i - \Gamma_{ai}\lambda_i \qquad i = 1,2 \tag{46}$$

where $M_i(X_i)$ is the inertia matrix of arm i, $V_i(X_i, \dot{X}_i)$ is the Coriolis, centripental, and gravity forces of arm i, J_i is the Jacobian of arm i, $\tau_i = [\tau_{i1} \cdots \tau_{i6}]^T$ is the input torques of arm i, $\lambda_i = [\lambda_{i\mathrm{n}} \quad \lambda_{i\mathrm{t}} \quad \lambda_{i\mathbf{b}}]^T$

is the constraint force, and Γ_{ai} is given by

$$\Gamma_{ai} = \begin{bmatrix} \mathbf{n}_{ix} & \mathbf{t}_{ix} & \mathbf{b}_{ix} \\ \mathbf{n}_{iy} & \mathbf{t}_{iy} & \mathbf{b}_{iy} \\ \mathbf{n}_{iz} & \mathbf{t}_{iz} & \mathbf{b}_{iz} \\ (S_{ei} \times \mathbf{n}_i)_x & (S_{ei} \times \mathbf{t}_i)_x & (S_{ei} \times \mathbf{b}_i)_x \\ (S_{ei} \times \mathbf{n}_i)_y & (S_{ei} \times \mathbf{t}_i)_y & (S_{ei} \times \mathbf{b}_i)_y \\ (S_{ei} \times \mathbf{n}_i)_z & (S_{ei} \times \mathbf{t}_i)_z & (S_{ei} \times \mathbf{b}_i)_z \end{bmatrix} = \begin{bmatrix} \mathbf{n}_i & \mathbf{t}_i & \mathbf{b}_i \\ S_{ei} \times \mathbf{n}_i & S_{ei} \times \mathbf{t}_i & S_{ei} \times \mathbf{b}_i \end{bmatrix} \tag{47}$$

In the above, **n** denotes the unit principal normal, **t** the unit tangent, and **b** the unit binormal. S_{ei} is the position vector from the center of palm i (where X_i is located) to the contact point (x_{ei}, y_{ei}, z_{ei}).

Let $X_o = [x_o \quad y_o \quad z_o \quad \theta_o \quad \phi_o \quad \psi_o]$ be the position and orientation of the mass center of the object. The motion equations of the object are

$$M_o \ddot{X}_o + V_o(\dot{X}_o) = \Gamma_{o1}\lambda_1 + \Gamma_{o2}\lambda_2 + G_o \tag{48}$$

where $M_o = diag\{m_o I_3, M_{or}\}$ is the 6×6 inertia matrix with m_o being the mass and M_{or} the 3×3 moment of inertia, $V(\dot{X}_o) = [0 \quad \omega \times M_{or}\omega]^T$ with $\omega = [\dot{\theta}_o \quad \dot{\phi}_o \quad \dot{\psi}_o]$, G_o is the gravity force, and Γ_{oi} is given by

$$\Gamma_{oi} = \begin{bmatrix} \mathbf{n}_i & \mathbf{t}_i & \mathbf{b}_i \\ r_i \times \mathbf{n}_i & r_i \times \mathbf{t}_i & r_i \times \mathbf{b}_i \end{bmatrix}$$

Here r_i is the vector from the mass center of the object (where X_o is located) to the contact point with palm i. Now if we define

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_o \end{bmatrix}$$

we may write the motion equations of the two arms and the object together as

$$M(q)\ddot{q} + V(q, \dot{q}) = E(q)\tau + A^T(q)\lambda \tag{49}$$

where

$$E(q) = \begin{bmatrix} J_1^{-T}(q) & 0 \\ 0 & J_2^{-T}(q) \\ 0 & 0 \end{bmatrix} \qquad A^T(q) = \begin{bmatrix} -\Gamma_{a1}(q) & 0 \\ 0 & -\Gamma_{a2}(q) \\ \Gamma_{o1} & \Gamma_{o2} \end{bmatrix}$$

Let V_{ei} and V_{oi} be the velocity of the contact point on palm i and on the object, respectively. The constraint equation for maintaining contact (sliding condition) is that the normal velocities of the contact point on palm i and on the object be the same, i.e.,

$$(V_{oi} - V_{ei}) \cdot \mathbf{n}_i = 0 \tag{50}$$

Further, if rolling is maintained between palm i and the object, the tagential and binormal velocities of the two bodies at the contact point must be the same

$$\begin{aligned} (V_{oi} - V_{ei}) \cdot \mathbf{t}_i &= 0 \qquad (51) \\ (V_{oi} - V_{ei}) \cdot \mathbf{b}_i &= 0 \qquad (52) \end{aligned}$$

If we write the six constraint equations (50), (51), and (52) together in terms of variable q, we obtain

$$A(q)\dot{q} = 0 \tag{53}$$

The motion equation (49) and constraint equation (53) are now in the same form as ones discussed in section 2.4.1. Therefore, the results obtained there can be applied to the present two-arm system. In particular, we will use the state space representation, Equation (25).

Assuming rigid point contact at each palm, the closed mechanical chain formed by the two arms and the object has 12 DOF if rolling is always maintained or 16 DOF if sliding is allowed. In the former case, 12 parameters are needed to specify the configuration of the closed chain. However, there is one degree of freedom, namely the spin of the object about the axis joining the two contact points, can not be controlled. In the output equation, we may have 11 position components. Since the system has 12 inputs (six joint torques from each arm), using the surplus input we may control the critical contact force which is defined to be the projection of the interaction force along the line joining the two contact points [11]. The eleven position components in the output equation may be chosen as follows.

$$y = h(q) = [x_o \quad y_o \quad z_o \quad \phi_o \quad \psi_o \quad \theta_1 \quad \phi_1 \quad \psi_1 \quad \theta_2 \quad \phi_2 \quad \psi_2] \tag{54}$$

Since the spin motion can not be controlled, the matrix $\Gamma_o = [\Gamma_{o1} \quad \Gamma_{o2}]$ has rank 5. Consequently it can be shown that $S^T E$ in Equation (25) is of rank 11 while $n - m$ in this case is 12. Therefore the nonlinear feedback, Equation (26), used to simplify the state equation can not be employed. But we can precede with input-output linearization by differentiating the output equation twice as follows.

$$\dot{y} = \frac{\partial h}{\partial q}\dot{q} = \frac{\partial h}{\partial q} S(q)\nu = S_1(q)\nu \tag{55}$$

where $S_1(q)$ are the rows of $S(q)$ selected by $\frac{\partial h}{\partial q}$.

$$\ddot{y} = S_1\dot{\nu} + \dot{S}_1\nu = S_1(S^T M S)^{-1}(-S^T M \dot{S}\nu - S^T V) + S_1(S^T M S)^{-1} S^T E\tau + \dot{S}_1\nu \tag{56}$$

Since $S_1(S^T M S)^{-1} S^T E$ is of rank 11, by using the nonlinear feedback

$$\tau = (S_1(S^T M S)^{-1} S^T E)^{+}[u - S_1(S^T M S)^{-1}(-S^T M \dot{S}\nu - S^T V) - \dot{S}_1\nu] \tag{57}$$

we have the following input-output map

$$\ddot{y} = u \tag{58}$$

Therefere, the input-output is decoupled as well as linearized. The stability and performance of each decoupled subsystem can be achieved by designing a linear feedback. The zero dynamics of this system consists of two parts. The first part is characterized by Equation (45), which corresponds to the position variables of the constrained velocities. The other part is the uncontrolled spin motion of the object. The first part is Lagrange stable while the second part is unstable. Soft contacts are needed to make the overall system stable.

4 Examples

4.1 Manipulation with Two Planar Arms

In this section we consider a specific example of a system consisting of two 3R arms manipulating a circular object on a 2D plane, shown in Figure 3. The number of degrees of freedom of the system is 5 if rolling contact is maintained. Otherwise, this number is 7.

Following the notations defined in Section 3, the position and input variables for the planar example are

$$X_i = \begin{bmatrix} x_i \\ y_i \\ \phi_i \end{bmatrix} \qquad X_o = \begin{bmatrix} x_o \\ y_o \\ \phi_o \end{bmatrix} \qquad \tau_i = \begin{bmatrix} \tau_{i1} \\ \tau_{i2} \\ \tau_{i3} \end{bmatrix} \qquad i = 1, 2.$$

The matrices Γ_{ai} and Γ_{oi} are given by

$$\Gamma_{ai} = \begin{bmatrix} \mathbf{n}_{ix} & \mathbf{t}_{ix} \\ \mathbf{n}_{iy} & \mathbf{t}_{iy} \\ (S_{ei} \times \mathbf{n}_i)_z & (S_{ei} \times \mathbf{t}_i)_z \end{bmatrix} \tag{59}$$

$$\Gamma_{oi} = \begin{bmatrix} \mathbf{n}_{ix} & \mathbf{t}_{ix} \\ \mathbf{n}_{iy} & \mathbf{t}_{iy} \\ (r_i \times \mathbf{n}_i)_z & (r_i \times \mathbf{t}_i)_z \end{bmatrix} \tag{60}$$

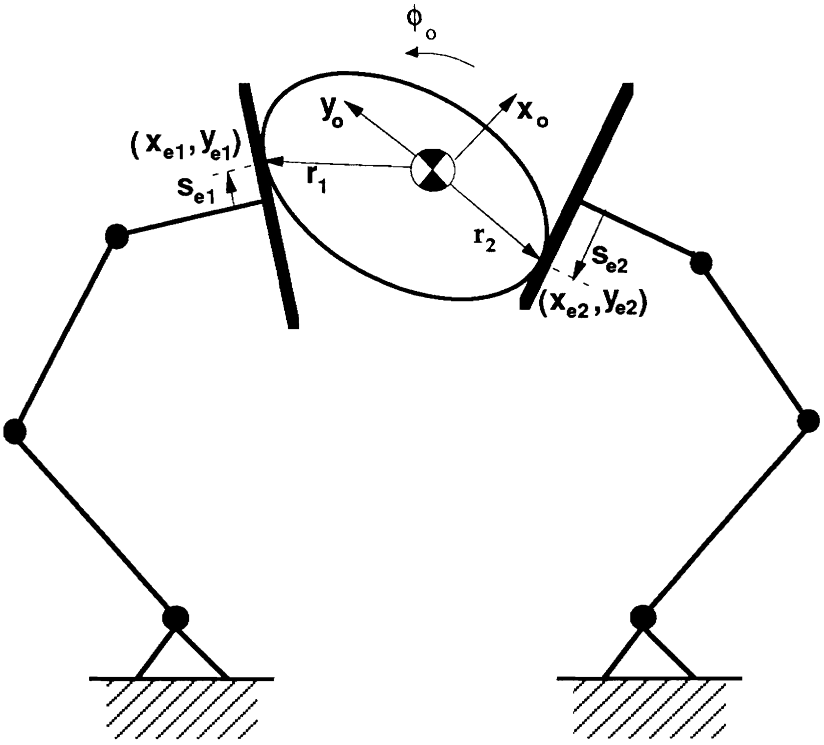

Figure 3: Planar 3-DOF Arms Manipulating a Circular Object

Then the motion equation and constraint equation of the planar system have the same form as Equations (49) and (53), with appropriate variables and matrices as defined above.

Note that all the constraint equations for the planar system are, in principle, integrable as shown in Section 2.2. However, it is still productive to use the formulation in Section 2. This is because of two reasons. First, for a general case, it is not easy to integrate the constraint equations. For example, even without the rolling constraint, Equations 14 and 15 cannot be integrated to obtain the local coordinates u_1 and u_2, unless the curvatures of the object and the effectors are constants [4]. Secondly, in order to control the contact conditions, it is often desirable to control the arc length variables, and although an expression for the derivative of the arc length is available, an analytical expression is not available.

Therefore we use the framework developed in the preceding sections for nonholonomic systems. Clearly theorems specific to nonholonomic systems (e.g., Theorems 1, 2 and 3) are not applicable here.

We first discuss the case in which the rolling constraint is absent. In other words, the motion is characterized by a combination of rolling and sliding (also called roll-slide in Reference [4]). We assume here that the contacts are frictionless (or with very low friction) so that the possibility of "jamming" or sticking is eliminated as in Reference [7] and roll-slide motion is practical.

The number of degrees of freedom in the system is 7 and the number of inputs is 6. Thus, if 7 output variables are chosen, there is one degree of freedom which cannot be controlled. If the object is circular, this uncontrolled degree of freedom is the spin motion of the object. Since our emphasis is on the control of rolling and sliding, we choose to control the position of the object (x_o and y_o), the orientation of the two palms (ϕ_1 and ϕ_2), and the arc length of the contact trajectory on each of the two palms (S_{e1} and S_{e2}). The control of the arc lengths is important since, typically, we would like to keep the contact point at or near the center of each palm. However, first we must express these as functions of the generalized coordinates q. This poses problems since we only have analytical expressions for the derivatives of S_{e1} and S_{e2}. These expressions are of the form [4]:

$$\dot{S}_{ei} = R(\dot{\phi}_i - \dot{\phi}_o) + (V_{oi} - V_{ei}) \cdot \mathbf{t}_i \tag{61}$$

where R is the radius of the circular object. Therefore, we choose the following output equation:

$$y = h(x) = [x_o \quad y_o \quad \phi_1 \quad \phi_2 \quad \int_0^t \dot{S}_{e1} dt \quad \int_0^t \dot{S}_{e2} dt] \tag{62}$$

Because we only have an analytical expression for the derivatives of S_{e1} and S_{e2}, the integration is needed in the output equation to obtain the values for S_{e1} and S_{e2}. However, the input-output linearization can be carried out in the same manner as shown in Section 3.

If the same mechanical system is considered with the rolling constraint, the system has 5 degrees of freedom and now we have one surplus input. The problem of resolving such redundancies has been treated in different ways. The focus in references such as [25, 26, 27, 28, 29] is on the control of closed chain dynamics, while the redundancy in actuation is resolved through an *ad hoc* scheme such as a pseudo-inverse decomposition [30]. The problem of static indeterminacy (redundancy), and optimal solutions of the problem of distribution of forces have been studied for multifingered grippers [5, 31] and for legged locomotion systems [32, 33]. These methods are suited to control in a quasi-static framework. In our previous work, we demonstrated the benefits of utilizing the surplus inputs to control the *critical contact forces* [11, 34].

The critical contact forces is merely a vector of minimum set-points for force components that are critical for prehension. For example, when manipulating an object with two rigid, convex surfaces as shown in Figure 3, we have two frictional point contacts, with contact forces F_1 and F_2. The critical contact force is given by:

$$\begin{aligned} F_c &= \min\{e_{12} \cdot F_1, -e_{12} \cdot F_2\} \\ &= \frac{e_{12} \cdot F_1 - e_{12} \cdot F_2 - \mid e_{12} \cdot F_1 + e_{12} \cdot F_2 \mid}{2} \end{aligned}$$

where e_{12} is the unit vector along the line joining the two points of contact. Clearly, if a rolling contact is desired, then $F_{c,desired}$ is selected to have a sufficiently large value in order to prevent slip.

Thus, in this case we have the following output equation:

$$y = h(x, \tau) = [x_o \quad y_o \quad z_o \quad \phi_1 \quad \phi_2 \quad F_c] \tag{63}$$

Note that h is a function of both x and τ since F_c is directly related to τ. The controller design technique for systems with position and force in output equations was presented in [11]. It uses an *extended state space formulation* [34], in which the state space is enlarged to include the actuator torque. This introduces an integrator into the force control subsystem and enables dynamic force control. The controller design for the system is accomplished using a nonlinear feedback which linearizes and decouples the system as explained in References [11, 35].

4.2 Results from Computer Simulations

The simulation results are presented in Figures 4 through 7. For the case of rolling constraint, the planned trajetory of the object is a straight line in the X-Y plane as shown in the left plot of Figure 4. The actual position is initially off from the desired one, but converges to the desired trajectory in less than 0.2 seconds. The plot to the right in the same figure shows the desired and actual trajectories of the critical contact force F_c. The orientation of the two palms is planned in such a way that the force applied to the object by each palm is kept at the center of the friction cone. Figure 5 depicts the trajectories of ϕ_1 and ϕ_2. Though they track the desired trajectory closely, there is a lag in the response.

For the case where rolling and sliding are present, the planned trajectory of the object is to follow a circle in the X-Y plane. The actual trajectory which has an initial offset, follows the path accurately after the overshoot (see the left plot in Figure 6). The main objective of this simulation is to demonstrate the control of sliding. For this purpose, the desired trajectory for S_{e1} and S_{e2} is such that the contact points are slided from their initial locations to the center of the palms ($S_{e1} = 0$ and $S_{e2} = 0$ in this case) while the object is tracking the global circular trajectory. This is successfully achieved in the simulation as shown in Figure 7. The plot to the right in Figure 6 shows how the uncontrollable variable ϕ_o behaves as a result of zero dynamics.

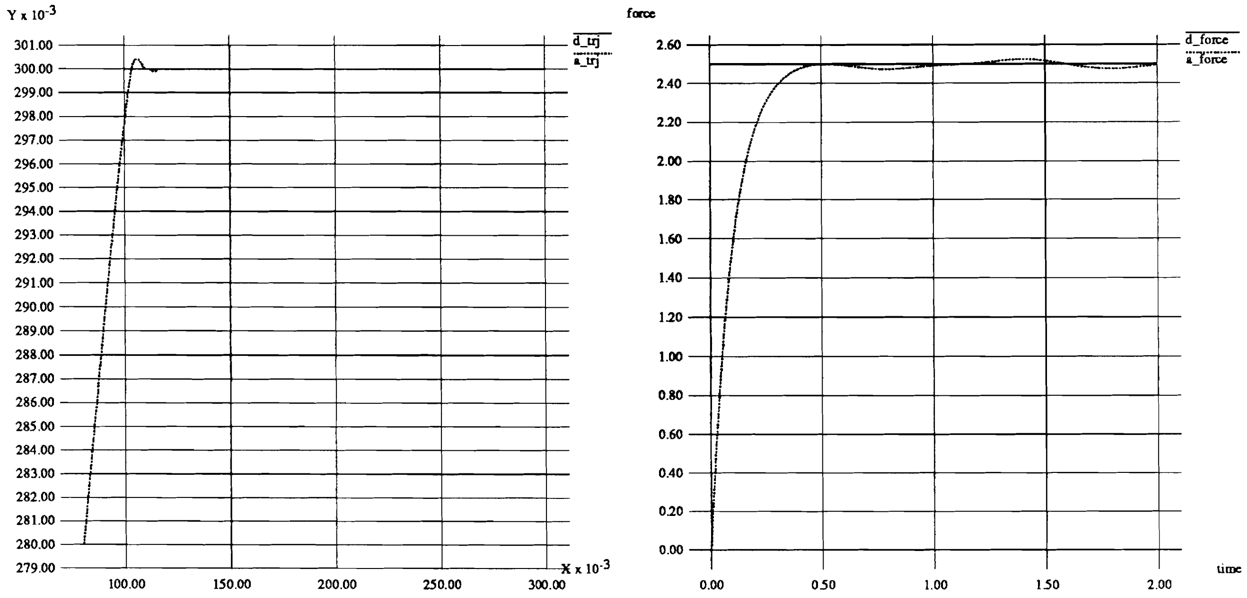

Figure 4: Cartesian X-Y Trajectory of the Object (left) and the Trajectory of the Critical Contact Force F_c (right) for the Rolling Constraint

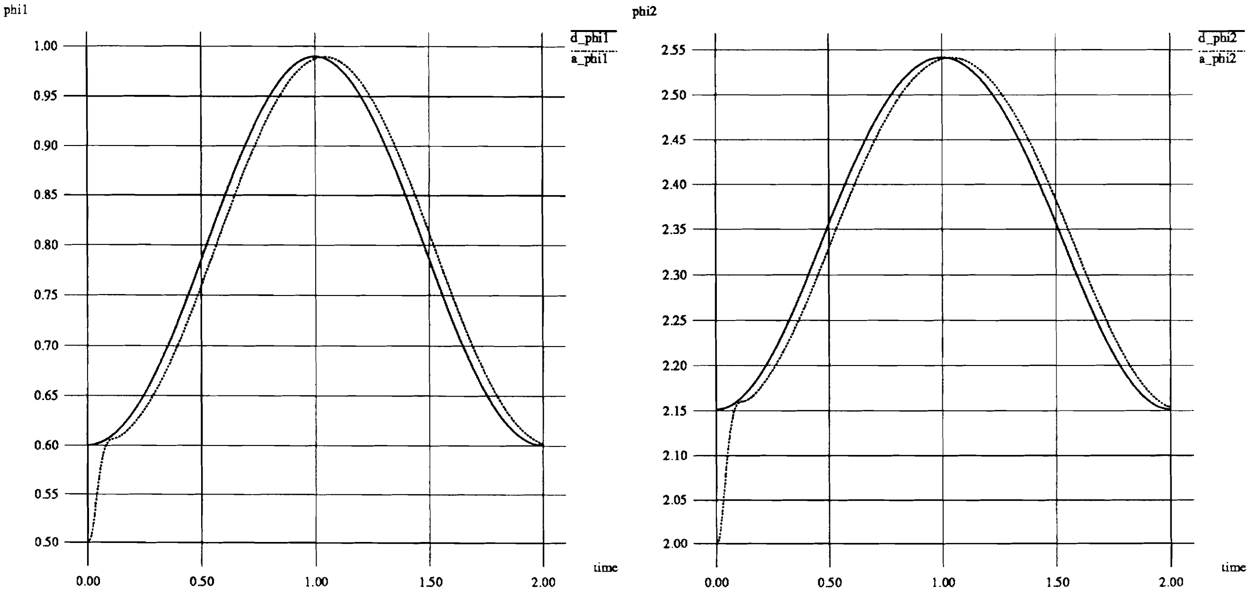

Figure 5: Trajectories of ϕ_1 (left) and ϕ_2 (right) for the Rolling Constraint

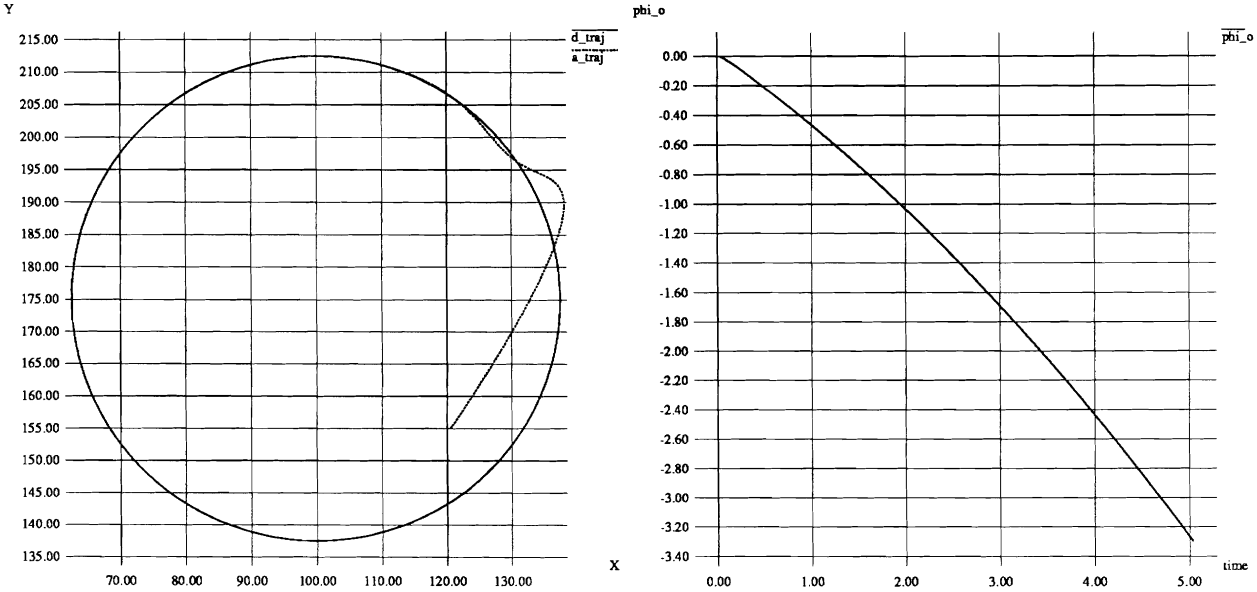

Figure 6: Cartesian X-Y Trajectory of the Object (left) and the Trajectory of Orientation $\phi_o(t)$ of the Object (right) for the Sliding Constraint

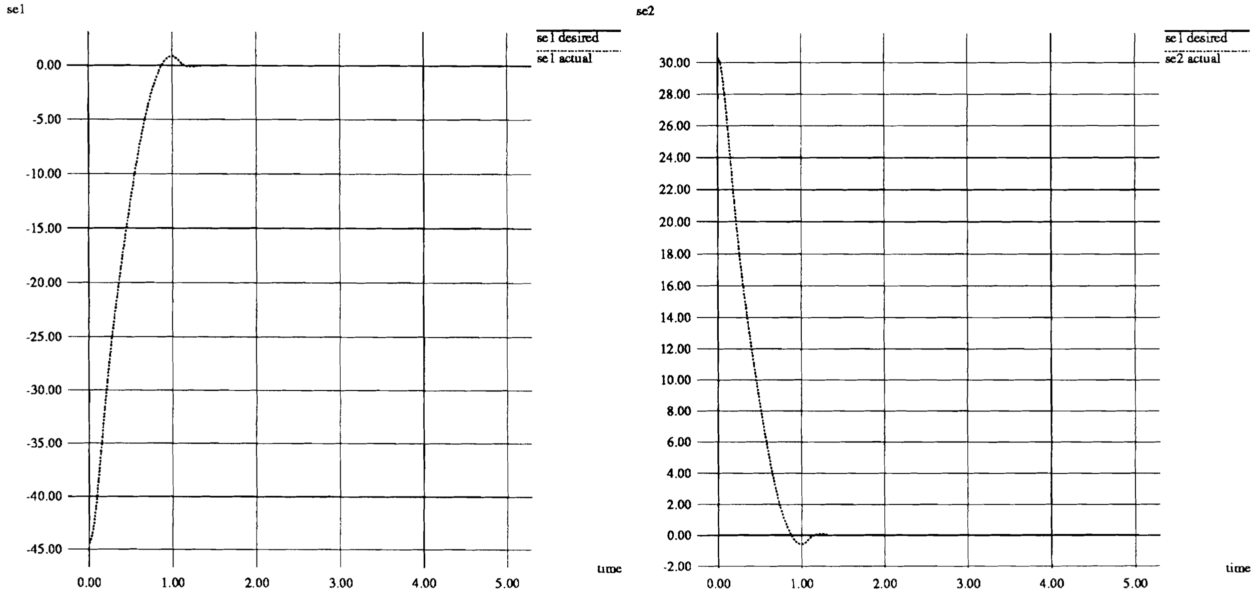

Figure 7: Trajectories of S_{e1} (left) and S_{e2} (right) for the Sliding Constraint

5 Concluding Remarks

In this paper, we studied the properties of mechanical systems subject to rolling and sliding constraints. In particular, we showed that mechanical systems with nonholonomic constraints are not input-state linearizable. With only position variables in the output equation, we characterized the zero dynamics of the system and derived a nonlinear feedback for input-output linearization. In the second half of the paper, we confined ourselves to a two-arm system in which two arms manipulate an object with their palms. The contact constraints between the object and the palms are nonholonomic. It is demonstrated that the result from the early sections, the state space formulation of the problem in particular, provides an useful methodology to treat this type of systems. Finally, simulation results are presented to illustrate that rolling and sliding can be effectively controlled.

References

[1] G. Campion, B. d'Andrea-Novel, and G. Bastin. Controllability and state feedback stabilization of non holonomic mechanical systems. In *The International Workshop in Adaptive and Nonlinear Control*, Grenoble, France, 1990.

[2] Anthony Bloch and N. H. McClamroch. Control of mechanical systems with classical nonholonomic constraints. In *Proceedings of 28th IEEE Conference on Decision and Control*, pages 201–205, Tampa, Florida, December 1989.

[3] J. K. Salisbury, W. Townsend, B. Eberman, and D. DiPietro. Preliminary design of a whole arm manipulation system. In *Proceedings of 1988 International Conference on Robotics and Automation*, pages 254–260, Philadelphia, PA, April 1988.

[4] C. Cai and B. Roth. On the planar motion of rigid bodies with point contact. *Mechanism and Machine Theory*, 21(6):453–466, 1986.

[5] J. Kerr and B. Roth. Analysis of multifingered hands. *Int. J. of Robotics Research*, 4(4), 1986.

[6] David J. Montana. The kinematics of contact and grasp. *The International Journal of Robotics Research*, 7(3):17–32, June 1988.

[7] L. Cai and A. A. Goldenberg. An approach to force and position control of robot manipulators. In *Proceedings of 1990 International Conference on Robotics and Automation*, pages 86–91, Cincinnati, OH, May 1990.

[8] Ju I. Neimark and N. A. Fufaev. *Dynamics of Nonholonomic Systems.* American Mathematical Society, Providence, RI, 1972.

[9] S. K. Saha and J. Angeles. Dynamics of nonholonomic mechanical systems using a natural orthoganal complement. *Transactions of the ASME, Journal of Applied Mechanics*, 58:238–243, March 1991.

[10] J. Barraquand and Jean-Claude Latombe. On nonholonomic mobile robots and optimal maneuvering. In *Proceedings of Fourth IEEE International Symposium on Intelligent Control*, Albany, NY, September 1989.

[11] V. Kumar, X. Yun, E. Paljug, and N. Sarkar. Control of contact conditions for manipulation with multiple robotic systems. In *Proceedings of 1991 International Conference on Robotics and Automation*, Sacramento, CA, April 1991.

[12] Z. Vafa and S. Dubowsky. On the dynamics of manipulators in space using the virtual manipulator approach. In *Proceedings of 1987 International Conference on Robotics and Automation*, pages 579–585, Raleigh, North Carolina, 1987.

[13] Y. Nakamura and R. Mukherjee. Nonholonomic path planning of space robots. In *Proceedings of 1989 International Conference on Robotics and Automation*, pages 1050–1055, Scottsdale, AZ, 1989.

[14] Anthony Bloch, N. H. McClamroch, and M. Reyhanoglu. Controllability and stability properties of a nonholonomic control system. In *Proceedings of 29th IEEE Conference on Decision and Control*, pages 1312–1314, Honolulu, Hawaii, December 1990.

[15] J. Barraquand and Jean-Claude Latombe. *Controllability of Mobile Robots with Kinematic Constraints.* Technical Report STAN-CS-90-1317, Stanford University, Stanford, CA, June 1990.

[16] J. P. Laumond. Finding collision-free smooth trajectories for a non-holonomic mobile robot. In *10th International Joint Conference on Artificial Intelligence*, pages 1120–1123, Milano, Italy, 1987.

[17] Z. Li and J. F. Canny. *Robot Motion Planning with Nonholonomic Constraints.* Technical Report Memo UCB/ERL M89/13, Electronics Research Laboratory, University of California, Berkeley, CA, February 1989.

[18] G. Lafferriere and H. Sussmann. Motion planning for controllable systems without drift. In *Proceedings of 1991 International Conference on Robotics and Automation*, pages 1148–1153, Sacramento, CA, April 1991.

[19] B. d'Andrea-Novel, G. Bastin, and G. Campion. Modelling and control of non holonomic wheeled mobile robots. In *Proceedings of 1991 International Conference on Robotics and Automation*, pages 1130–1135, Sacramento, CA, April 1991.

[20] C. Samson and K. Ait-Abderrahim. Feedback control of a nonholonomic wheeled cart in cartesian space. In *Proceedings of 1991 International Conference on Robotics and Automation*, pages 1136–1141, Sacramento, CA, April 1991.

[21] C. Canudas de Wit and R. Roskam. Path following of a 2-dof wheeled mobile robot under path and input torque constraints. In *Proceedings of 1991 International Conference on Robotics and Automation*, pages 1142–1147, Sacramento, CA, April 1991.

[22] L. Meiroritch. *Methods of Analytical Dynamics.* McGraw-Hill, 1976.

[23] H. Nijmeijer and A. J. van der Schaft. *Nonlinear Dynamic Control Systems.* Springer-Verlag, New York, 1990.

[24] A. Isidori. *Nonlinear Control Systems: An Introduction.* Springer-Verlag, Berlin, New York, 1985.

[25] Y. F. Zheng and J. Y. S. Luh. Optimal load distribution for two industrial robots handling a single object. In *Proceedings of 1988 International Conference on Robotics and Automation*, pages 344–349, Philadelphia, PA, April 1988.

[26] Yan-Ru Hu and A. A. Goldenberg. An adaptive approach to motion and force control of multiple coordinated robot arms. In *Proceedings of 1989 International Conference on Robotics and Automation*, pages 1091–1096, Scottsdale, Arizona, May 1989.

[27] S.A. Schneider and R.H. Cannon. Object impedance control for cooperative manipulation. In *Proceedings of the IEEE International Conference on Robotics and Automation*, Scottsdale, AZ, May 1989.

[28] M. A. Unseren and A. J. Koivo. Reduced order model and decoupled control architecture for two manipulators holding an object. In *Proceedings of 1989 International Conference on Robotics and Automation*, pages 1240–1245, Scottsdale, Arizona, May 1989.

[29] J. Wen and K. Kreutz. Motion and force control for multiple cooperative manipulators. In *Proceedings of the IEEE International Conference on Robotics and Automation*, Scottsdale, AZ, May 1989.

[30] K. Salisbury and B. Roth. Kinematics and force analysis of articulate mechanical hands. *ASME Journal of Mechanisms, Transmissions and Automation in Design*, 105:35–41, March 1983.

[31] Z. Li and S. Sastry. Task oriented optimal grasping by multifingered robot hands. *IEEE Transactions of Robotics and Automation*, 4(1), February 1988.

[32] D.E. Orin and F.T. Cheng. General dynamic formulation of the force distribution equations. In *Advanced Robotics 1989, Proceedings of the Fourth International Conference on International Robotics*, pages 525–545, Columbus, Ohio, June 1989.

[33] V. Kumar and K.J. Waldron. Force distribution in walking vehicles. *ASME Journal of Mechanical Design*, July 1990.

[34] Xiaoping Yun. Coordination of two-arm pushing. In *1991 IEEE International Conference on Robotics and Automation*, Sacramento, CA, April 1991.

[35] Eric Paljug, Xiaoping Yun, and Vijay Kumar. Control of rolling contacts in multiple robotic manipulation. In *Proceedings of the International Conference on Advanced Robotics*, pages 591–596, Pisa, Italy, 1991.