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Abstract
This is an account of some recent work done with H. S. La [1] [2], based ultimately on the work of Fischler and Susskind [3] and Polchinski [4].

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BEYOND CONFORMAL FIELD THEORY

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This is an account of some recent work done with H. S. La [1] [2], based ultimately on the work of Fischler and Susskind [3] and Polchinski [4].

1. Credo

Since as Kastler has remarked, this is something of an ecumenical congress, and prone to heresies and inquisitions as only such congresses can be, I would like first of all to outline my prejudices and motivations rather than launching into a long unmotivated construction project.

String theory begins with the observation that certain mathematical constructions produce quantities which superficially resemble the scattering amplitudes of a relativistic quantum theory. These amplitudes moreover enjoy properties reminiscent of the perturbative unitarity rules for quantum field theory. Pursuing the analogy shows, however, that their short-"distance" behavior is significantly nicer than that of field theory amplitudes. As I will mention later, words like "distance" are not to be taken too seriously until we have some adequate notion of quantum spacetime. For now these words are defined by explicit reference to the picture of perturbing about flat spacetime — certainly not a tenable fundamental approach in any theory of gravity.

Just the same, the *long*-distance structure of perturbative string theory is singular, just as we know it must be if it is to describe scattering: amplitudes must have the usual kinematical poles and so on. Unraveling this desired singularity structure, and eliminating undesired singularities, will be my main project today.

What are the mathematical constructions alluded to above? We introduce an auxiliary structure into the problem, namely a compact 2-surface, and attempt to define a quantum field theory thereon. There are various motivations for this construction, all of them well known to this audience. I wish to stress, however, that we should not take these motivations too seriously. Rather, we should analyze *ab ovo* the utility of each element of the construction. I will touch only very briefly on some points chosen mainly on polemical grounds.

First of all, we need to introduce motions of field theory which make invariant sense on arbitrary compact 2-surfaces; the plane is not good enough. This is because successive orders of quantum-mechanical perturbation theory are all supposed to be given by exactly the same local 2d dynamics applied to increasingly complicated surfaces. Secondly, it is absolutely crucial that the local geometry of each surface must drop out.

One way to arrange this is to ask for a 2d *topological* field theory. This is not useful. In two dimensions, however, there is an alternative approach: we can ask for our field theory to be well-defined given only Σ as a complex manifold. Since all complex structures are the same locally this suits our requirements. Furthermore, one finds that the degenerations of Riemann surfaces correspond precisely to possible *long*-distance singularities in quantum field theory. The *short*-distance singularities have no such analogs. That is why they do not arise in string theory, *if* string theory is defined on shell by conformal field theories. Today we have a large kingdom of conformal field theory knowledge.

To repeat: the auxiliary constructions needed for string scattering amplitudes are 2d conformal field theories defined consistently on surfaces of all topologies. There is *no known role* for (a) conformally non-invariant 2d field theories, or (b) theories or the plane with no consistent extension to all genera, *i.e.* modular non-invariant theories. (Certainly there are extremely interesting conjectures that arbitrary 2d field theories play a role in an offshell continuation, though.)

What is more, all CFT's of interest to string theory are of a special form: they are all tensor products of an arbitrary CFT of central charge $c = 26$ with *one universal* CFT of central charge $c = -26$. The latter system plays a fundamental role in the geometry of string theory, as we will see hinted at in the sequel; it deserves further mathematical attention.[1] The physical idea I am alluding to here is of course called "Becchi-Rouet-Stora-Tyutin symmetry;" it plays a much more fundamental role in string geometry than its usual role as a machine implementing symplectic reduction. In this talk I will unfortunately have to suppress details of how this works, but see [1][2]

Having recited my creed, I will now try to show why and how it must be discarded. I do not know precisely what will replace it, but I will sketch an extremely suggestive calculation from [1][2]. Approximately, however, the new creed goes as follows.

It was a grave error to focus on individual fixed Riemann surfaces above. Returning to Polyakov's original heuristic principle, the local dynamics on Σ are expressed in terms of a 2-metric g and some "matter" (*i.e.* unspecified) field x. A large symmetry (2d general coordinate transformations) acts on both g and x, without prejudice. We choose to focus on g when we fix this gauge symmetry, leading to the *illusion* of a fixed nondynamical Riemann surface with a CFT defined thereon. The requirement of conformal invariance then says that for each *fixed* Σ no extra data about Σ are needed to define 'amplitudes'. But of course, our work is not done at this point. We must also complete our implementation of Polyakov's principle by integrating over conformal classes of Σ. Here *new* divergences occur — not unexpected, since the shape of Σ is itself a dynamical variable. Indeed, true amplitudes in general *cannot* be defined without choosing extra data on Σ; this is a failure of conformal invariance even though each CFT on fixed Σ is well defined!

Thus we have no choice: we must *spoil* explicit conformal invariance to *save* it overall; we must give away this kingdom to enter the next. In addition, we'll see that we cannot even work on a Riemann surface of any one fixed topology. Even though worldsheet gravity is almost trivial, it will lead to topology change just like any other quantum gravity. This is of course much more radical than merely relaxing conformal invariance on one fixed surface. Presumably this is a big hint about the true foundations of string theory.

Let's get specific.

[1] But see [5].

2. Shifts

To get our feet on the ground, we consider bosonic strings on flat spacetime. The techniques are general. Indeed it's one of the main points of the general CFT apparatus that the formalism is not tied to specific realizations (like sigma models, current algebra, *etc.*). We will use CFT even though our goal is to modify it. Also, as we will remark, all our constructions make sense only in the more refined setting of "superconformal field theory;" this is indeed the main focus of [1][2], where along the way we show how to avoid the famous 'ambiguity' problem of integrals over superspace.

The problem is very simple. A sigma model based on flat 26-dimensional spacetime is well-known to be conformally invariant on fixed Σ once we properly introduce ghost fields to fix reparametrization invariance. A small disturbance of flat $\mathbf{R}^{26}$ may or may not spoil this invariance. To compute the effect of such a change we can take the original theory's partition function and add to it the correlation function of the operater ψ corresponding to the desired deformation, integrated over Σ. If ψ has conformal weight $(h, \bar{h}) = (1,1)$ then $\langle\psi\rangle_\Sigma$ can be regarded as a (1,1)-form on Σ and the integral is well-defined without further choices.[2] Thus we maintain conformal invariance. As is well known this (1,1) condition is the linearized Einstein equation for the deformed metric on $\mathbf{R}^{26}$, $\Box h_{\mu\nu} = 0$, where the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

Consider now the integral over all Σ. Suppressing the unphysical "tachyon" pole (this happens automatically in a more realistic theory), we find an integral of the form

$$Z = \int \frac{d^2 q}{|q|^2} (\cdots)(\cdots) \quad . \tag{1}$$

Here q is a complex variable describing how close Σ is to being degenerate; $q \to 0$ produces the long-distance singularity alluded to above. The ellipses denote the *one-point* functions of *massless* fields on surfaces of genus g_1, g_2, respectively; $g = g_1 + g_2$ is the genus of Σ.

Clearly Z diverges if the "tadpole" amplitudes $(\cdots)$ are non-zero. The latter situation must obtain in an interesting theory, *i.e.* one in which supersymmetry breaks, by whatever means. So we have a problem.

From experience with quantum field theory our duty is now clear. We must *cut off* our q integral at some small value δ, then introduce *counterterms* on the lower-genus surfaces Σ_1, Σ_2 in such a way that the full $Z + Z^{c.t}$ is δ-independent. Of course we don't want this advice to apply too literally, since the goal is to get away from field theory.

Fortunately, the analogy to field theory dissipates almost at once. For one thing, the counterterms we introduce, interpreted as small changes ψ to the flat background, will all be *finite* — that's good. Of more immediate concern, though, is that our cutoff procedure at first seems to be ill-defined — that's bad. The point is that the choice of a coordinate q on the moduli space $\mathcal{M}_g$ is far from canonical, and without such a choice we don't know what it means to say a surface is "too pinched." We must inevitably *spoil conformal invariance* to get a cutoff. The trick is to do so in a wise way.

Before proceeding, let me give the bottom line: the correct counterterms will be finite, but they will be of slightly the *wrong* dimension, $(h, \bar{h}) \neq (1,1)$. Thus we *give up* conformal invariance at lower genus! To insert such states on $\Sigma_{1,2}$ we will again need a cutoff (=normal-ordering prescription). We now have *two* cutoffs, albeit of radically different-seeming sorts. One cuts off the moduli integral over all Σ; the other, the

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[2] Actually a broader class of states than this can be inserted. In the BRST formalism we can insert any ψ of total weight (including b, c) (0,0). See [6].

2d CFT on a given Σ. As remarked in the 'Credo', however, this dissimilarity is an illusion. In fact, we can choose these cutoffs consistently, in such a way that all dependence on the choice drops out for suitable ψ. Thus we save conformal invariance — and the blessings it brings — by letting go of it.

To define a cutoff, choose surfaces $\Sigma_{1,2}$ with marked points $P_{1,2}$ and a complex number q. Choose moreover local coordinates $z_{1,2}$ centered on $P_{1,2}$. Now glue Σ_1 to Σ_2 by the usual rule $z_1 = q/z_2$. Promoting $\Sigma_{1,2}$ to *families* of surfaces parametrized by $\vec{m}_{1,2}$ gives coordinates $\vec{m}_1$, $\vec{m}_2$, q for $\mathcal{M}_g$.

Our problem is now that the same $\Sigma_{1,2}$, glued with the same q, will yield a *different* surface $\widetilde{\Sigma}$ if different $\widetilde{z}_{1,2}$ are chosen. Thus, the choice of cutoff amounts to choosing *two* things: a family $z_{1,2}$ of coordinates on $\Sigma_{1,2}$.

Now we begin to see the point: this *same* extra data $z_{1,2}$ are also just what's needed to insert a general background-shift state ψ, not necessarily of weight (1,1), onto $\Sigma_{1,2}$ respectively. We just cut out the disk $|z_1| = \delta$ and insert ψ there. Equivalently, we can cut out $|z_1| = 1$ and insert $\delta^{L_0+\bar{L}_0}\psi$. What's important is that z_1 is totally independent of anything on side 2, and vice versa.

What exactly happens when we change z_1 to $\widetilde{z}_1 = z_1 + \sum \epsilon_n z_1^{n+1}$? For one thing the insertion $\langle\psi\rangle_{\Sigma_1}$ changes by

$$\delta Z^{c.t.} = \langle \delta^{L_0+\bar{L}_0} \left(\sum_n \epsilon_n (L_n - \delta_{n,0}) \right) \psi \rangle_{\Sigma_1} \to \epsilon_0 \langle (L_0 - 1)\psi \rangle_{\Sigma_1} \ , \tag{2}$$

where the limit is for $\delta \to 0$ and ψ is nearly of weight (1,1). The cutoff changes, too. Remarkably, though, the change is extremely simple [2]: $\vec{m}_1$, $\vec{m}_2$, q change to $\vec{m}_1'$, $\vec{m}_2'$, q', where in particular

$$q' = q(1 + \epsilon_0) + \mathcal{O}(q^2). \tag{3}$$

Clearly changing the region of integration region of (1) from $\{|q| > \delta\}$ to $\{|q'| > \delta\}$ changes the integral by

$$\delta Z = \log(1 + \epsilon_0) \sum_a \langle\!\langle \phi_a \rangle\!\rangle_{\Sigma_1} \langle\!\langle \phi^a \rangle\!\rangle_{\Sigma_2} \ , \tag{4}$$

where ϕ_a runs over all massless states. Requiring the cutoff-dependences of (2), (4) to cancel now gives a conditions on ψ of the form

$$(L_0 - 1)\psi = \sum_a \phi_a \langle\!\langle \phi^a \rangle\!\rangle_{\Sigma_2} \ . \tag{5}$$

The LHS of (5) looks like a free wave operator acting on the shifted background state: *e.g.* it contains $\Box h_{\mu\nu}$ and so on. The RHS looks like a *source* for the wave equation. In fact, the equation (5) comes from a loop-corrected quantum action, as many authors have shown in various cases.

Our point here is that, whatever the interpretation of solutions to (5), it embodies the general precepts of the 'Credo': to obtain true, generalized conformal invariance of the full string system we must *destroy* naive conformal invariance in a very special way, one in which different surfaces *conspire* to cancel the anomaly. That this is possible at all comes from the remarkable geometrical fact (3) about moduli space.

The papers [1][2] were mainly concerned with the superconformal case, and specifically the heterotic string. Exactly the same sort of cutoff emerges as above, where we now choose a superconformal coordinate $\mathbf{z}=(z,\theta)$ near the attachment points. The key observation is that with such a cutoff one has only to perform a moduli integral over a supermanifold with boundary (namely $\{|q|>\delta\}$), thus sidestepping problems with integrals over noncompact supermanifolds.

Of course, the challenge is how to find a nonperturbative implementation of these principles. The answer is quite likely to be totally different in spirit from the CFT-inspired derivation above.

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Questions:

Rabin: Would you remind us of the distinction between a supermanifold with boundary, and the noncompact supermanifold resulting from deleting the boundary?

Nelson: The relevant definition is found in Manin [7] The essential point is that the pinching coordinate q constructed by sewing is a well-defined function on super moduli space $\widehat{\mathcal{M}}$ itself, not on ordinary moduli space $\mathcal{M}$, once the choices of $\mathbf{z}_1$, $\mathbf{z}_2$ are made. Thus $\{|q|>\delta\}$ leaves us with no doubt as to what the integration region looks like in the odd directions. Of course, we have now to check the dependence on $\mathbf{z}_{1,2}$, but this follows along the lines of the text.

Rabin: In principle, background fields (*e.g.* the metric) in string theory should be generated by condensates of string modes (*e.g.* the graviton), although the implementation of this principle is very unclear. Perhaps the difficulty in giving a nonperturbative formulation of the Fischler-Susskind mechanism is due to our ignorance on this point: you are correcting the background, but you don't know what corresponding correction to apply to the modes.

Nelson: I think that even the notion of correcting the background has got to give way to a background-independent formalism.