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Stabilizing Feedback Controllers for Robotic Assembly Problems

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Abstract

Assembly problems require that a robot with a few actuated degrees of freedom manipulate an environment with a greater number of unactuated degrees of freedom. Since the dynamical coupling between degrees of freedom in this setting is a function of their relative configuration, the motion of such systems is subject to constraints that preclude smooth feedback stabilization. In other words, in contrast to purely geometric motion planning problems, assembly planning cannot be carried out within the limits of traditional control theory. This paper explores the extent to which assembly planning and control may be effected by recourse to some other methodical means of generating stabilizing feedback controllers.

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Stabilizing Feedback Controllers For Robotic Assembly Problems

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Abstract

Assembly problems require that a robot with a few actuated degrees of freedom manipulate an environment with a greater number of unactuated degrees of freedom. Since the dynamical coupling between degrees of freedom in this setting is a function of their relative configuration, the motion of such systems is subject to constraints that preclude smooth feedback stabilization. In other words, in contrast to purely geometric motion planning problems, assembly planning cannot be carried out within the limits of traditional control theory. This paper explores the extent to which assembly planning and control may be effected by recourse to some other methodical means of generating stabilizing feedback controllers.

1   Introduction

As is the case for many planning problems involving physical objects, robotic assembly problems may be cast in a control theoretic setting [5]. Loosely speaking, say that a robotic task involves an assembly problem if the environment to be manipulated possesses more degrees of (unactuated) freedom than are available to the (actuated) robot system, and the specified goal state is to be achieved starting from arbitrary initial

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configurations (that is, all unactuated degrees of freedom must be exercised, in general, to complete the task). The unavailability of actuated degrees of freedom might result from limitations inherent in the robot's design (e.g., the PUMA has only six joints and the widget has twenty parts) or as a function of natural constraints imposed by the environment (e.g., the monkey's twenty degree of freedom hand has no bearing upon the banana's six degree of freedom state unless there is contact). A successful assembly plan must develop a sequence of manipulations none of whose single steps can achieve the goal yet each of whose concluding states brings the environment to a more favorable situation than the prior.

In a recent paper [4], the author reviews the principles underlying a program of robotics research that seeks to develop planning procedures by recourse to dynamical systems theory. Since physical machines are ultimately force or torque controlled dynamical systems, the specification of input torques must result in certain classes of vector fields. In this light, it makes sense to specify plans in the form of appropriately constructed sensor based feedback controllers whenever possible. This can be readily accomplished for planning problems involving purely geometric considerations [6], and seems to be possible for certain interesting tasks involving "dynamical dexterity" as well [3]. This paper will explore the extent to which control theoretic methods afford a solution to the assembly problem.

2 The One Degree of Freedom Unit Assembly Problem

To reduce the problem to simplest terms, consider a point unit mass body restricted to a single horizontal axis of motion, $b \in \mathbb{R}$. For simplicity, we will replace Newton's laws with Whitney's "generalized damper" dynamics [10] as is common in the assembly planning literature [8]. That is, assume sufficiently large friction forces are present that externally applied forces applied to this point mass will result in proportional changes in velocity. In particular, zero applied force is associated with zero velocity. Consider as well a point unit mass robot restricted to a single axis of motion parallel to that of the body, $r \in \mathbb{R}$. The robot is controlled by a force we are free to command, u_1, and its motion is governed by Newton's laws, $\ddot{r} = u_1$.

Let the robot have a "gripper" which, when closed, is capable of engaging the body, assuming the two point masses are aligned at the same relative positions on their respective axes. This variable will be denoted u_2.

It is desired to place the body at a specified goal location. The robot must start from an arbitrary location, relocate the body, and then return to a specified nest location. The problem at hand is to devise an autonomous strategy for the robot that will enable it to move toward the puck, "grab it", and place it in the arbitrarily designated new location,

and proceed to its nest.

2.1   Nonholonomy

Following loosely the treatment of Bloch and McClamroch [1], define the variables $q = [r, b]^{\mathrm{T}}$, where r denotes the robot's position and b that of the puck. The overall system may be written in the form

$$M(q)\ddot{q} = Bu_1$$
$$J(q, u_2)\dot{q} = 0$$

where M is the constant (diagonal) mass matrix, $B = [1, 0]^{\mathrm{T}}$, and the nonholonomic constraints are expressed as

$$J(q, u_2) = \begin{cases} [u_2, -1] & q = r \\ [0, -1] & q \neq r \end{cases}$$

i.e., the puck can be moved by the robot if and only if they are "touching" each other and the gripper is engaged.

A growing literature in robotics concerns open loop planning in the face of nonholonomic constraints (for example, consult the nice review by Murray and Sastry [9]). In the present case, problem at hand is entirely trivial if open loop strategies are permitted: simply let the robot approach and join the puck, then let the joint puck-robot mass be moved to the desired location, and finish by moving to the nest after releasing the puck.

Can such a policy be expressed as a feedback law rather than in terms of an open loop plan? Can the correctness of such feedback laws be proven in the usual sense of guaranteeing global asymptotic stability of the goal point? Can a a methdodical means be found of generating such feedback controllers and their proofs?

2.2   Feedback Stabilization is Impossible

Continuing to follow Bloch and McClamroch [1], consider the reduced representation of this system in the form

$$\begin{aligned} \dot{r}_1 &= r_2 \\ \dot{r}_2 &= u_1 \\ \dot{b} &= c(r_1, b, u_2)r_2 \end{aligned} \tag{1}$$

In the present case, note that c, the *coupling function* takes the form

$$c(r_1 - b, u_2) \triangleq J_2^{-1} J_1(q, u_2) = \begin{cases} u_2 & b - r_1 = 0 \\ 0 & b - r_1 \neq 0 \end{cases}$$

Notice that regardless of the form of c, the function,

$$f : (r_1, r_2, b, u) \mapsto \begin{bmatrix} r_2 \\ u \\ cr_2 \end{bmatrix},$$

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fails to be onto any neighborhood of the origin. For example $(0, 0, \epsilon)$ is not in the image of f for any $\epsilon \neq 0$. Thus, no smooth feedback controller is capable of stabilizing this system around an arbitrary isolated equilibrium state since it fails Brockett's test [2].

3 Stabilizing Feedback Controllers

To avoid any technical difficulties with existence or uniqueness of solutions, it is now expedient to replace $c : \mathbb{R}^2 \to \mathbb{R}$ in (1) by a smooth approximation. The only assumptions we require to maintain an essential representation of the assembly problem are

- $c(0, 1) \neq 0$
- $c(x, 0) \equiv 0$ for all $x \in \mathbb{R}$
- $|x| > C_0$ implies $c(x, u) \equiv 0$, for all $u \in \mathbb{R}$.

Roughly speaking, these conditions assure that manipulation is possible if and only if the robot is sufficiently close to the body and the gripper is engaged. For ease of exposition, assume that c takes values only in the unit interval, $0 \leq c \leq 1$, and, in particular, that $c(0, 1) = 1$.

3.1 Stabilization to a Manifold

Suppose it is merely desired that the robot come to rest in an arbitrary location after placing the body at some goal point, g. The task at hand is readily encoded by the zero set of the scalar valued function,

$$v \triangleq \frac{1}{2} r_2^2 + \frac{1}{2} (b - g)^2 + \frac{1}{2} (r_1 - b)^2, \tag{2}$$

which allows the robot to stop at the goal location of the body. The time derivative of v along the motion of the system,

$$\dot{v} = r_2 \left[u_1 + (b - g)c + (r_1 - b)(1 - c) \right],$$

can be made negative semi-definite by choosing

$$u_1 = -r_2 - (b - g)c - (r_1 - b)(1 - c).$$

According to LaSalle's invariance principle [7], the limit set of the resulting closed loop is contained within the largest positive invariant set lying in $\dot{v} \equiv 0$ — that is, the the plane $r_2 \equiv 0$. But the closed loop vector field evaluates to $[0, -c(b - g) - (1 - c)(r_1 - b), 0]^{\mathrm{T}}$ when $r_2 = 0$. Now suppose we adopt a gripper feedback law that (smoothly) satisfies

$$u_2 \triangleq = \begin{cases} 1 & |r_1 - b| = 0 \\ 0 & |r_1 - b| > D_0. \end{cases}$$

Note that no positive invariant set lies within the region of the plane where $|r_1 - b| > \min\{C_0, D_0\}$. Thus, as D_0 approaches zero (that is, as

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the gripper control policy, becomes discontinuous), the robot is guaranteed to bring the body to an increasingly small neighborhood around the goal point.

To the best of the author's knowledge there is no known obstruction to smooth feedback stabilization of the manifold whereon both $b = g$ and $r_2 = 0$. Yet the intuitively obvious policy described here does not work unless the gripper policy, u_2 is made discontinuous. It does have the virtue of arising "methodically" from a consideration of LaSalle's Invariance Principle, which, in turn, guarantees a proof of correctness. Can some better, non-intuitive smooth policy be found? Can it be found in a methodical fashion that admits a simple correctness proof?

3.2 Stabilization to a Point

Now consider the full one degree of freedom assembly problem that requires the robot to relocate the body at the goal point, g, and then return to its "nest" position, n. Assume now that the coupling law is discontinuous, that is $C_0 \rightarrow 0$. It is not hard to see that the following discontinuous control policy,

$$
\begin{aligned}
u_1 &= -r_2 - \{u_2[(b - g)c + (1 - u_2 c)(r_1 - b)] \\
&\quad + (1 - u_2)[r - n]\} \\
u_2 &= \left\{ \begin{array}{ll} 0 & |b - g| = 0 \\ 1 & |b - g| > 0, \end{array} \right.
\end{aligned}
$$

succeeds in stabilizing the system to the desired point, $(n, 0, g)$. Even the most casual glance at this policy shows that it is really a "computer program" that might be rendered: close gripper unless the body is properly located; move toward body if gripper is closed, and move toward goal if gripper is closed and robot is at body; move toward nest if gripper is open.

It would be interesting to find a formal proof of correctness for this controller.

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