University of Pennsylvania Working Papers in Linguistics

Volume 22
Issue 1 *Proceedings of the 39th Annual Penn Linguistics Conference*

Article 28

1-1-2016

Continuations for Comparatives

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Abstract
I extend Barker and Shan's (2014) project of finding natural language applications for continuations to another domain in which it is useful, namely comparatives. I introduce existing analyses of comparatives, in particular Heim 2000, 2006, and Schwarzschild and Wilkinson 2002 and then demonstrate how these analyses can be implemented in the continuations framework.

Continuations for Comparatives

Todd Snider*

1 Introduction

In this paper, I continue the project of continuation semantics as carried out especially by Chris Barker and Chung-chieh Shan (2001–2014, in various permutations) by implementing an analysis of comparatives using continuations. I focus primarily on Heim's (2000, 2006) quantificational analysis, noting also how continuations can be used to implement the non-quantificational approach of 2002.

Due to space constraints, I presume the reader already has a command of the continuations literature, including both the general mechanisms and the motivations which underlie the project, as well as knowledge of comparatives. For a thorough introduction of both continuations and comparatives which presumes no such familiarity, see Snider 2015.

2 Continuations for Comparatives

In order to present the continuations implementation, I'll present a traditional quantificational account, namely that of Heim (2000, 2006). This account is by no means the only account of comparatives, of course, but it is a good example of a popular account, and one which relies on LF movement. Starting with a single example and then increasing in difficulty, I'll introduce the traditional account and then explain the corresponding moves we'll have to make to account for the same data in a continuations analysis.

2.1 Core Example

For our base example, consider (1).

(1) Mary is six inches taller than Bill is wide.

Heim's (2000) account gives a sentence like (1) an LF and truth conditions as in (2):

(2) a. $[6''$ -er than $[wh_1$ Bill is t_1 wide$]]_2$ Mary is t_2 tall
b. $\textsc{max}(\texttt{height}(m)) \geq \textsc{max}(\texttt{width}(b)) + 6''$

The logical form in (2a) involves two LF movements:[1] one of the comparative standard, and one within the comparative standard (which I'm referring to functionally, but which Heim calls the *than* clause). We'll look at each one in turn, introducing the continuation analysis additions as we go. I'll show that, using a continuation analysis of the comparative construction, we can derive the same truth conditions for (1) as Heim does (in 2b) without the need for any movement.

2.1.1 Movement of the Comparative Standard

Adjectives are assumed to (obligatorily) take a degree (of type d) to their left, and then return a property; the set of individuals (type et) that are that degree tall.[2] This applies to absolute adjectives with both explicit and implicit degree phrases (DegPs).

(3) a. John is tall.
b. John is **d** tall.
c. John is six feet tall.

*My thanks to Chris Barker, Miloje Despic, Molly Diesing, Mats Rooth, the Cornell Semantics Group, and the audience at PLC. All errors are my own.

[1] Heim (2006) adds a third movement; we'll return to this in Section 2.2.

[2] This then includes individuals taller than that degree.

The bare absolute adjective in (3a) is understood as having a contextually supplied DegP, as in (3b), while the DegP in (3c) is explicit. To keep this parallel, and in so doing require only one denotation for each adjective, Heim (2000, 2006) treats adjectives in comparative constructions the same way. Comparatives, after all, involve a degree-denoting piece: the comparative standard (the *than* clause). The comparative standard, then, being a DegP, should be in the same location, giving a sentence like (4a) a form like (4b).

(4) a. John is taller than six feet.
    b. John is [-er than six feet] tall.

This way, the denotation of the adjective *tall* remains the same: it takes a DegP (of type *d*) on its left; this one just happens to be a little bit more complex than the DegPs in (3a) and (3c).

To be more precise, Heim (2000) has the form in (4b) not as the LF of (3c), but as the nearly-surface form: as she puts it, "In those cases where [the comparative standard] is superficially discontinuous, we can attribute this to an (obligatory) superficial extraposition process which does not feed LF." (In an endnote, she then explains why extraposition is probably not the right analysis.) The important thing to note is that under this analysis the degree-denoting comparative standard is interpreted pre-adjectivally, in parallel with other degree phrases. This comparative standard, though, is of type $<< d,t >,t >$ for Heim (2000, 2006), so the entire *than* phrase must move from this (pre-surface structure) pre-adjectival position:

> "Being of type <dt,t>, these complex DegPs cannot be interpreted *in situ*, but must move for interpretability to a position above the adjective's subject (not necessarily above the surface subject, if there are lower covert subjects). The movement leaves a trace of type d and creates a λ-abstract of type <d,t>, which makes a suitable argument to the DegP." (Heim 2000:42)

It's also worth noting here that Heim treats *-er* as the morpheme containing the comparative operator, with *than* being a semantically vacuous phonological reflex on the comparative standard's DegP. For more on this assumption, see Snider 2015.

In the framework we're using, we can retain a single unified denotation form for adjectives without needing to posit movement (either pre-surface or post-surface). The comparative operator (*-er*) combines with the predicate before the predicate takes any arguments, and the comparative operator can introduce arguments internally within its denotation to the meaning of a predicate in whichever order it likes; nothing has to move for "interpretability" (type reasons). Our comparative operator, then, we can define as:

```
(("erthan" ((d\(e\t)) \ ((d\(e\t)) / d) )) (^ A (^ d (^ D (^ e (max A e >= d + D ))))))
```

This is our (new) lexical entry for the comparative operator in the parser associated with the Shan and Barker 2006 system. On the left is the phonological form (the PF or "spellout" of the morpheme), followed by the syntactic type (in the Shan and Barker 2006 system these are semantic types (*e*, *t*, and *d*) along with slashes) and then on the right the semantic denotation (with `^` standing in for λ). So, reading through the syntactic type, we can see that `erthan` first takes an adjective (of type *det*, represented as `(d\(e\t))`) on its left, then combines with a degree (of type *d*) on its right—this will be the comparative standard—, returning something also of type *det*, effectively standing in place of the original adjective. It then goes on to take on its left a degree (the differential) and individual that the adjective would have, before returning a truth-evaluable statement (of type *t*). The (maximum) degree to the subject has on the scale denoted by the adjective is compared to the sum of the comparative standard and the differential.

Following Heim (2000:42), the comparative returns a statement which includes the **maximum** operator, as defined in (5).

(5) $\max(P) := \iota d.P(d) = 1 \& \ \forall d'[P(d') = 1 \rightarrow d' \leq d]$ Heim 2000, (6)

max returns the maximum degree from some scale of degrees; this is intended to account for the notion that if John is six feet tall, then he's also five feet tall, and four feet tall, etc. Even though the scale of height has this property, it'd be false to say that Mary is taller than John, so it must be that we're not comparing just any values but the maximum values of each person's height; for this reason we need the **max** operator.

With our comparative operator and Heim's **max**, the system can easily handle the comparative standard's being located to the right of the adjective as opposed to the left, and can do so without needing an additional form for adjectives in comparative constructions. We can see just this part in action if we look at the derivation of a comparative even simpler than (1), something like (6a):

(6) a. John is one inch taller than six feet.

b. $\textsc{max}\ \{d : \text{tall(john,}d)\} \geq 6' + 1''$ (cf. Heim 2000, (7))

$= \texttt{height(j)} \geq 6' + 1''$

We should expect (6a) to have truth conditions matching the expression in (6b), and indeed we get just this sort of result: a full derivation for (6a) is in Figure 1.

```
1  edge      : 235 john is oneinch tall erthan sixfeet   (0 6)   t
2  semantics : (max (^ d (^ x (height x >= d))) j >= sixfeet + oneinch)
3  proofnet  : ((1 . t) (john (2 . e)) (is (((2 . e) \ (1 . t)) / ((3 . e) \ (4 . t)))) (oneinch
   d) (tall (d \ ((5 . e) \ (6 . t)))) (erthan ((d \ ((5 . e) \ (6 . t))) \ ((d \ ((3 . e) \ (4
   . t))) / d))) (sixfeet d))
4  derivation: ((L john) (is ((L oneinch) ((tall erthan) sixfeet))))
5
6  john is oneinch tall erthan sixfeet t = (max (^ d (^ x (height x >= d))) j >= sixfeet +
   oneinch)
7    john (1 / (e \ 1)) = (^ f (f j))
8      L ((2 / (1 \ 2)) / 1) = (^ x (^ f (f x)))
9      john e = j
10   is oneinch tall erthan sixfeet (e \ t) = (^ e (max (^ d (^ x (height x >= d))) e >= sixfeet
     + oneinch))
11     is ((e \ t) / (e \ t)) = (^ k k)
12     oneinch tall erthan sixfeet (e \ t) = (^ e (max (^ d (^ x (height x >= d))) e >= sixfeet
       + oneinch))
13       oneinch (1 / (d \ 1)) = (^ f (f oneinch))
14         L ((2 / (1 \ 2)) / 1) = (^ x (^ f (f x)))
15         oneinch d = oneinch
16       tall erthan sixfeet (d \ (e \ t)) = (^ D (^ e (max (^ d (^ x (height x >= d))) e >=
         sixfeet + D)))
17         tall erthan ((d \ (e \ t)) / d) = (^ d (^ D (^ e (max (^ d (^ x (height x >= d))) e
           >= d + D))))
18           tall (d \ (e \ t)) = (^ d (^ x (height x >= d)))
19           erthan ((d \ (e \ t)) \ ((d \ (e \ t)) / d)) = (^ A (^ d (^ D (^ e (max A e >= d +
             D)))))
20         sixfeet d = sixfeet
```

Figure 1: A derivation of *John is one inch taller than six feet*.

There are some comments worth making about this derivation, but first, let me explain what we're looking at here. The first line, labeled "edge", has the input string, namely the sentence in (6a), as well as its length (4 words) and its output type (t). Line 2, labeled "semantics", has the truth conditions of the sentence. (I'll comment on the contents of this line in a moment.) Line 3, "proofnet", describes the internal workings of the parser, whose inner workings are not crucial to understand right now. On line 4, "derivation", however, we can see the derivation which leads to this parse. The parentheses indicate order of operations, the order in which Function Application has taken place. In this line, `L` is Lift.

Below that is the derivation itself, step by step. The first line on which any lexical item appears is its 'leaf node', and that line lists (from left to right) its phonological form, syntactic type, and semantic denotation. Looking at the line 9, for instance, we can see the introduction of *John* into the derivation. The phonological form is `john`, its type is e, and its denotation is the individual (abbreviated as) `j`. Here I recommend reading from the bottom up, just as one might with a derivation tree. In this particular one, we see the comparative combine with the predicate (*tall*), then the comparative standard (six feet),[3] then the differential (one inch), and finally the subject (John).

[3]Represented here as a single lexical entry, `sixfeet`, just to keep the derivation short. Another grammar which separates numerals (like *six*) and measure terms (like *feet*) exists for any interested readers. That dis-

The astute reader may have noticed that the truth conditions listed on line 2 don't match exactly with the expression in (6b) that we were hoping for. This is a side effect of using **max** in this parser as opposed to on paper. The way to read this is as though **max**, embedded in the denotation of the comparative, could access the adjective's measure function, allowing us to make the following simplification (see full discussion in Snider 2015):

(7) $\max(\lambda d \lambda x.[\text{height}(x) \geq d])(j) \geq 6' + 1''$
$= \lambda x.[\text{height}(x)](j) \geq 6' + 1''$
$= \text{height}(j) \geq 6' + 1''$

And this fully simplified expression is, of course, what we were looking for as in (6b).

Now we've got a comparative operator, Heim's maximum operator (and the means to simplify it), which is enough to handle the placement of the comparative standard in an example like (4a) without needing the sort of movement in (4b). The only other thing we'll need to handle (4a) is a contextual variable to take the place of the covert differential that must be there. We can call it `d`, and if its semantics, the contextual differential it picks out, is handled by an assignment function (or something akin to one), then we can handle examples like (4a).

2.1.2 Movement within the Comparative Standard

A sentence like (1), repeated here for convenience, unlike (4a), also needs an additional movement under Heim's (2000, 2006) analysis.

(1) Mary is six inches taller than Bill is wide.

To understand why, it'll be useful to take a moment to look at comparative standards as a class. Comparative standards appear to be able to take a number of different forms, even in a single construction, like that of (1).

(8) a. Mary is taller than [six feet].
b. Mary is taller than [John].
c. Mary is taller than [Bill is (tall)].

Looking at the sentences in (8), we see a variety of things in the *than* phrase: we see what looks like a DegP in (8a), what looks like a DP in (8b), and what looks like a CP in (8c). If we consider the bracketed material in these sentences to be so varied, we're mising an important generalization about these comparative standards. In fact, all of these denote a degree, something of type d. And note further that the sentence-like structure is not truly a full sentence of type t; it's not just any such sentence that can fit in this position, and this structure cannot contain an explicit degree in the pre-adjectival position where one would otherwise put a degree:

(9) * John is taller than Mary is five feet tall.

What is needed, then, is a way to interpret a structure like [*Bill is tall*] as itself a degree-denoting phrase.

In the classical account, the degree-ness of these CP-like clauses (and the infelicity of explicit degrees in them) is handled via *wh-* movement. As Heim (2000:51) puts it:

> "Following standard practice, I take *than*-clauses to be derived by *wh*-movement of a covert operator from the degree-argument position of an adjective. The trace is interpreted as a variable over degrees. The *wh*-clause as a whole may be treated in analogy with a free relative, as a definite description of a maximal degree.
>
> (39) $[\![\textit{wh}_1 \textit{the bed is } t_1 \textit{long}]\!] = \max\{\text{ d: long(the bed,d)}\}$ "

tinction, however, makes no difference to the computation of the comparative other than to lead to somewhat longer derivations, and so I've simplified things for the purposes of this project.

Under Heim's analysis, it's this *wh*-movement which allows us to interpret the CP-like structure as a degree, and it's the presence of the trace of the covert operator which blocks sentences like (9).

Because our aim is to account for comparatives without this sort of movement, we need to account for the degree-type nature of the standard, even when it appears to be sentence-like. And, as this is precisely the part of the derivation that under Heim's (2000, 2006) account requires *wh*-movement, the same sort of movement which is theorized to account for quantifier raising for scope at LF, it's this part that we can account for using continuations. Just as quantifiers made use of continuation-sensitive material which has some local type but which took scope at some other type, we'll want something similar for the comparative standards within comparative structures.

To fill this need, I introduce a new covert operator for the pre-adjectival position of comparative standards, just the same place as Heim's *wh-* trace. Because it's in this pre-adjectival position, its local type should be d. What we need it to do, however, is to allow us to interpret the sentence-looking comparative standard as a degree. Translating this need into our continuation type language, our new operator will take scope over the entire comparative standard (the *than* phrase) which is of type t, but it will return something of type d, allowing us to interpret the standard as denoting a degree. Its definition, then, looks like this:

```
((”?d” (d // (d \\ t))) (^ k (max k)))
```

I named the operator `?d` so as to invoke the idea of 'the issue of d': the (unknown) answer to the question *How d is x?*, in our example (1), the degree of Bill's width (which is unspecified in the comparative). Reading through the lexical entry, it's phonologically null (but has a name for the parser's sake), and it has the syntactic type $d /\!\!/ (d \setminus\!\!\setminus t)$ (read the double slashes as fat slashes): its local type is d, it takes scope at t, and its return type is d.

Let's try to understand what's really going on here. Within our (often elided) comparative standard, we have a nearly complete sentence: *Bill is* [ ] *wide*. It would be a full sentence (of type t) except that it's missing a degree phrase (of type d) somewhere specific inside of it. This continuation is the argument of our scope taker, our new operator, which should return a degree, something of type d, so its type is $(d /\!\!/ (d \setminus\!\!\setminus t))$: an expression which needs a continuation of type $d \setminus\!\!\setminus t$ surrounding it in order to return a d.

Unlike the generalized quantifiers Shan and Barker discuss (type $t /\!\!/ (e \setminus\!\!\setminus t)$), this operator's scope-taking input type (t) does not match its output type (d). This is still kosher in the world of continuations, following Wadler 1994. See also discussion in Barker and Shan 2014 (Section 18.4).

Turning back to the lexical entry, we can see that semantically, `?d` is a **max**-imized identity function: it passes the content up, but embeds that content under a **max** operator. This allows us to interpret the comparative standard in (1) not just as any old degree of Bill's width, but the maximal degree of Bill's width: in common parlance, Bill's width.

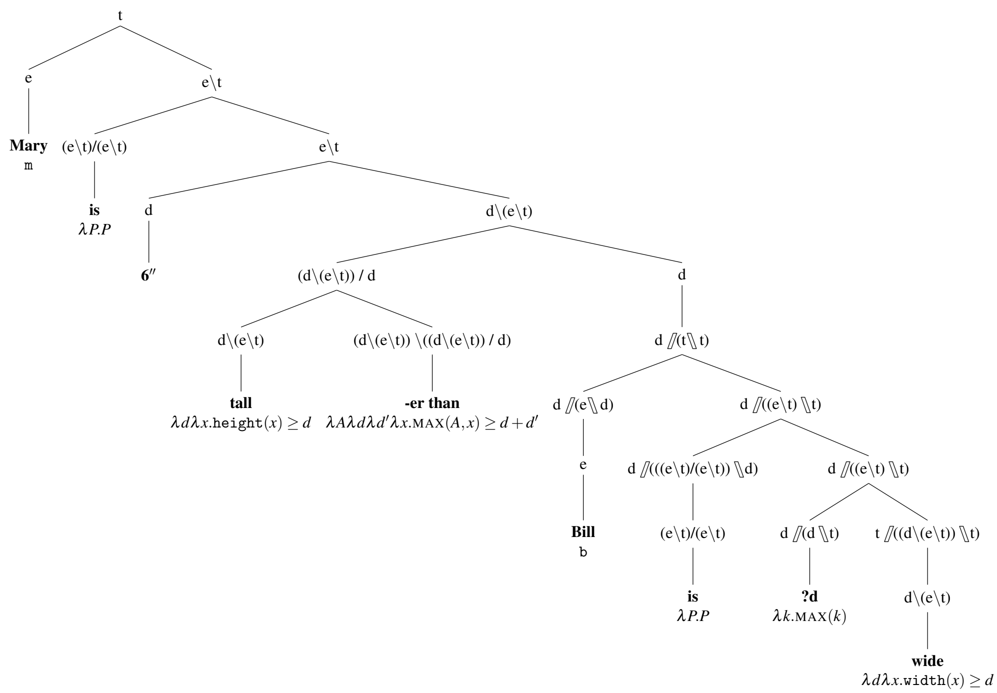
In action, the comparative standard from our example (1), *Bill is wide*, as represented in (10), returns the correct semantics for the embedded degree-denoting sentence-like phrase.

(10) a. ...[Bill is wide].
b. `Bill is ?d wide`

For a complete derivation tree and parser derivation for (10), see Snider 2015.

With this in hand, we can now turn to the full derivation of our core example (1). The sentence will have a type tree as in (11):

(11)



Making the entire derivation tree in this format would be quite crowded and hard to read. Instead, we can see the whole process in the parser's derivation in Figure 2.

```
edge       : 535 mary is sixinches tall erthan bill is ?d wide        (0 9)    t
semantics  : (max (^ d (^ x (height x >= d))) m >= (max (^ r (width b >= r))) + sixinches)
proofnet   : ((1 . t) (mary (2 . e)) (is (((2 . e) \ (1 . t)) / ((3 . e) \ (4 . t)))) (sixinches d) (tall (d \ ((5 . e) \ (6 . t))))
(erthan ((d \ ((5 . e) \ (6 . t))) \ ((d \ ((3 . e) \ (4 . t))) / d))) (bill (7 . e)) (is (((7 . e) \ (8 . t)) / ((9 . e) \ (10 .
t)))) (?d (d // (d \\ (11 . t)))) (wide (d \ ((9 . e) \ (10 . t)))))
derivation: ((L mary) (is ((L sixinches) ((tall erthan) (D ((S (U (L bill))) ((S (U is)) ((S ((S (U L)) ?d)) (U wide)))))))))

mary is sixinches tall erthan bill is ?d wide t = (max (^ d (^ x (height x >= d))) m >= (max (^ r (width b >= r))) + sixinches)
  mary (1 / (e \ 1)) = (^ f (f m))
    L ((2 / (1 \ 2)) / 1) = (^ x (^ f (f x)))
    mary e = m
  is sixinches tall erthan bill is ?d wide (e \ t) = (^ e (max (^ d (^ x (height x >= d))) e >= (max (^ r (width b >= r))) +
  sixinches))
    is ((e \ t) / (e \ t)) = (^ k k)
    sixinches tall erthan bill is ?d wide (e \ t) = (^ e (max (^ d (^ x (height x >= d))) e >= (max (^ r (width b >= r))) +
    sixinches))
      sixinches (1 / (d \ 1)) = (^ f (f sixinches))
        L ((2 / (1 \ 2)) / 1) = (^ x (^ f (f x)))
        sixinches d = sixinches
      tall erthan bill is ?d wide (d \ (e \ t)) = (^ D (^ e (max (^ d (^ x (height x >= d))) e >= (max (^ r (width b >= r))) + D)))
        tall erthan ((d \ (e \ t)) / d) = (^ d (^ D (^ e (max (^ d (^ x (height x >= d))) e >= d + D))))
          tall (d \ (e \ t)) = (^ d (^ x (height x >= d)))
          erthan ((d \ (e \ t)) \ ((d \ (e \ t)) / d)) = (^ A (^ d (^ D (^ e (max A e >= d + D)))))
        bill is ?d wide d = (max (^ r (width b >= r)))
          D (1 / (1 // (t \\ t))) = (^ k (k (^ x x)))
          bill is ?d wide (d // (t \\ t)) = (^ c (max (^ r (c (width b >= r)))))
            bill ((1 // (2 \\ 3)) / (1 // ((e \ 2) \\ 3))) = (^ R (^ c (R (^ r (c (r b))))))
              S (((4 // (2 \\ 5)) / (3 // (1 \\ 5))) / (4 // ((2 / 1) \\ 3))) = (^ L (^ R (^ c (L (^ l (R (^ r (c (l r)))))))))
              bill (1 // ((2 / (e \ 2)) \\ 1)) = (^ f (f (^ f (f b))))
                U ((2 // (1 \\ 2)) / 1) = (^ x (^ f (f x)))
                bill (1 / (e \ 1)) = (^ f (f b))
                  L ((2 / (1 \ 2)) / 1) = (^ x (^ f (f x)))
                  bill e = b
            is ?d wide (d // ((e \ t) \\ t)) = (^ c (max (^ r (c (^ x (width x >= r))))))
              is ((1 // ((e \ t) \\ 2)) / (1 // ((e \ t) \\ 2))) = (^ R (^ c (R (^ r (c r)))))
                S (((4 // (2 \\ 5)) / (3 // (1 \\ 5))) / (4 // ((2 / 1) \\ 3))) = (^ L (^ R (^ c (L (^ l (R (^ r (c (l r)))))))))
                is (1 // (((e \ t) / (e \ t)) \\ 1)) = (^ f (f (^ k k)))
                  U ((2 // (1 \\ 2)) / 1) = (^ x (^ f (f x)))
                  is ((e \ t) / (e \ t)) = (^ k k)
              ?d wide (d // ((e \ t) \\ t)) = (^ c (max (^ r (c (^ x (width x >= r))))))
                ?d ((d // (1 \\ 2)) / (t // ((d \ 1) \\ 2))) = (^ R (^ c (max (^ r (R (^ r1 (c (r1 r))))))))
                  S (((4 // (2 \\ 5)) / (3 // (1 \\ 5))) / (4 // ((2 / 1) \\ 3))) = (^ L (^ R (^ c (L (^ l (R (^ r (c (l r)))))))))
                  ?d (d // ((1 / (d \ 1)) \\ t)) = (^ c (max (^ r (c (^ f (f r))))))
                    RULE ((1 // ((2 / (3 \ 2)) \\ 4)) / (1 // (3 \\ 4))) = (^ R (^ c (R (^ r (c (^ f (f r)))))))
                      S (((4 // (2 \\ 5)) / (3 // (1 \\ 5))) / (4 // ((2 / 1) \\ 3))) = (^ L (^ R (^ c (L (^ l (R (^ r (c (l
                      r)))))))))
                      RULE (1 // (((2 / (3 \ 2)) / 3) \\ 1)) = (^ f (f (^ x (^ f (f x)))))
                        U ((2 // (1 \\ 2)) / 1) = (^ x (^ f (f x)))
                        L ((2 / (1 \ 2)) / 1) = (^ x (^ f (f x)))
                    ?d (d // (d \\ t)) = (^ k (max k))
                wide (1 // ((d \ (e \ t)) \\ 1)) = (^ f (f (^ d (^ x (width x >= d)))))
                  U ((2 // (1 \\ 2)) / 1) = (^ x (^ f (f x)))
                  wide (d \ (e \ t)) = (^ d (^ x (width x >= d)))
```

Figure 2: A derivation of *Mary is six inches taller than Bill is wide*.

For our core example sentence (1), then, we can derive the same truth conditions as Heim's (2000) in (2b) without needing either *wh-* movement within the comparative standard or type-mismatch movement of the entire comparative standard to widest scope. Instead of the LF as in (2a) (repeated here), we end up with the much simpler interpretation form in (12).

(2a) [6″ -er than [wh_1 Bill is t_1 wide]]$_2$ Mary is t_2 tall — HEIM 2000
(12) Mary is 6″ tall -er than Bill is `?d` wide — CONTINUATION ANALYSIS

This continuation-analysis form, which is not coincidentally quite close to the surface form, differs from the surface form only in the presence of our new scoping operator which, though covert, seems to block the presence of explicit degree phrases. And, building on the work of Shan and Barker (2006), we can do so in a way that also has further explanatory power regarding preferences for left-to-right interpretation (but without sacrificing the ability to derive ambiguous alternative readings). For some additional benefits, including unifying the semantics of variety of shapes of comparatives, see Snider 2015.

2.2 Quantificational Comparative Standards

The analysis of comparatives met a considerable challenge when attempting to account for sentences with quantificational comparative standards, that is, comparative standards which include a quantified element, such as (13).

(13) Mary is 6″ taller than every boy is.

Why should this sort of sentence pose a challenge? First, there is a question of types: The comparative standard in (4a), *six feet*, is plainly a degree, of type *d*, and I've argued (following the prevailing opinion in the literature) that the standard *Bill is wide*, as in (1), should also be interpreted as degree-denoting. But how can the comparative standard of (13), presuming it's [*every boy is* [ ] *tall*], be understood as a degree? Unless all of the boys are the exact same height, presumably that standard should denote a set of degrees (or perhaps a plurality of degrees, or some other complex type), or at least not a simple type *d*.

Second, putting type issues aside, there is an issue of scope-taking. Under quantificational theories (like that of Heim (2000, 2006)), both the comparative standard and the *wh* variable-over-degrees within the comparative standard move at LF. And, as is known, sentences with multiple quantificational elements should exhibit scope ambiguity, with multiple possible scopal configurations of those quantificational elements. In a sentence like (13), then, with multiple quantificational elements (the quantifier *every* and the quantificationally-bound degree denoted by the comparative standard), we should expect there to be multiple possible scopal orderings and thus multiple possible interpretations.

(13) only has one interpretation available, however. This (single) available interpretation is clear: for each boy (in the salient set of boys), Mary is taller than that boy. In other words, Mary is taller than the tallest boy (and every other boy as well). This is the wide-scope universal reading.

(13) Mary is 6″ taller than every boy is.

1. For every boy *x*, Mary is taller than the height *d* of *x*. — $\forall > -er$[4]
2. Mary is taller than *d*, where *d* is the height that every boy has. — $*{-er} > \forall$

The other interpretation that we might have otherwise predicted is a bit trickier, but one can imagine it: there is some height *d* such that every boy has that height; such that that height is true of the entire set of boys. Remembering the way these scales are understood to work, the only (maximal) height that could fit that criterion is the (maximal) height of the *shortest* boy. The tallest height that all of the boys share is the highest height of the shortest boy, making this interpretation of (13) equivalent to that of *Mary is 6″ taller than the shortest boy*, which has much weaker truth conditions. Following

[4] The comparative isn't actually the thing scoping, but this makes for a shorter more parallel schema than any other possible label.

Fleisher (2015b), I'll call this a MIN reading, as opposed to the MAX reading we seem to obligatorily get in this case.

To explain this behavior, we need to explain the exceptional wide-scope of the universal quantifier, provide another mechanism for comparative scope-taking, or do something else. Schwarzschild and Wilkinson (2002) argued forcefully that these sorts of sentences are problematic, showing additionally that there was similarly problematic behavior of other elements in *than* clauses, such as verbs as modals, which aren't usually considered to undergo LF movement, which nevertheless get wide scope readings. Schwarzschild and Wilkinson (2002) proposed that rather than degrees, gradable adjectives denote relations between individuals and *intervals*; compare (14a) and (14b).

(14) a. $[\![\text{tall}]\!] = \lambda d_d \lambda x_e.\text{height}(x) \geq d$ DEGREE
b. $[\![\text{tall}]\!] = \lambda D_{<d,t>} \lambda x_e.\text{height}(x) \in D$ INTERVAL

In the degree semantics we were using above, an adjective denotes the $\geq$ relation between an individual and the degree they hold on a scale. On an interval semantics, an adjective denotes the relationship between an individual and a set of degrees. This allows the *than* clause in (13) to pick out not a particular degree of height that all boys have, but instead to pick out an interval that contains all of their (maximum) heights. If Mary's height exceeds this interval (or, really, if the interval denoted by Mary's height properly *contains* this boys' height interval), then she must be taller than every boy, including the tallest boy; which gives us the MAX reading we want.

In response to the arguments presented in Schwarzschild and Wilkinson 2002, as well as some other data about modals, Heim (2006) introduces a new scope-taking operator to her system: Π, a "point-to-interval" ("p-i") operator.[5] Π turns degree predicates into generalized quantifiers over degrees: interval predicates. Importantly to Heim's analysis, though, "the Π-phrase, being a generalized quantifier over degrees, could move up and be interpretable in higher positions", allowing it to take non-local scope; this, in turn, allows Heim to account for scopal ambiguities that arise with modals within *than* clauses. A comparative standard like (15a) is analyzed as having an LF as (15b) derived in (15c), interpreted as (15d), and the same for (16).

(15) a. [Bill is wide] (cf. Heim 2006, (41))
b. $wh\,\lambda 1\,[\,[\Pi t_1]\lambda 2\,[\text{ Bill is } t_2 \text{ wide}]\,]$
c. [Bill is $\Pi\,wh$ wide]
$[\Pi\,wh]\lambda 2\,[\text{Bill is } t_2 \text{ wide}]$
$wh\,\lambda 1\,[\,[\Pi t_1]\lambda 2\,[\text{ Bill is } t_2 \text{ wide}]\,]$
d. $\lambda P_{<d,t>}.\text{Bill's width} \in P$

(16) a. [every boy is tall]
b. $wh\,\lambda 1[\text{every boy } \lambda 2\,[\,[\Pi t_1]\lambda 3\,[t_2 \text{ is } t_3 \text{ tall}]\,]\,]$
c. [every boy is $\Pi\,wh$ tall]
$[\Pi\,wh]\lambda 3\,[\text{every boy is } t_3 \text{ tall}]$
$\text{every boy } \lambda 2\,[\,[\Pi\,wh]\lambda 3\,[t_2 \text{ is } t_3 \text{ tall}]\,]$
$wh\,\lambda 1[\text{every boy } \lambda 2\,[\,[\Pi t_1]\lambda 3\,[t_2 \text{ is } t_3 \text{ tall}]\,]\,]$
d. $\lambda D.\forall x[\text{girl}(x) \rightarrow \text{height}(x) \in D]$

In (16b), the quantifier *every* has wider scope than Π, which gives the MAX reading for (13). When Π scopes above a comparative-standard-internal quantifier, though, we get the MIN reading.

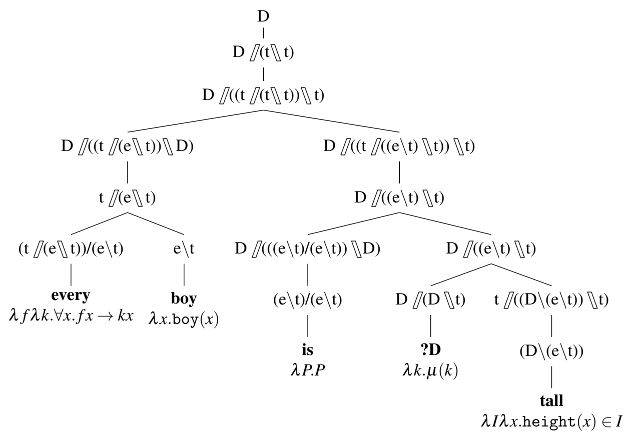
Whether one prefers handling quantificational comparative standards by moving entirely to intervals (à la Schwarzschild and Wilkinson 2002) or by raising degrees to intervals when necessary (e.g., through the Π operator, à la Heim 2006), the same can be accomplished in a continuations analysis which handles this scope-taking without movement. (For more on the comparison between these approaches, see Snider 2015.)

Accommodating the Schwarzschild and Wilkinson 2002 analysis requires changing the denotations for gradable adjectives to replace entries that denote degrees (as we saw in (14a)) with those that denote intervals (as in (14b)), as well as replacing the comparative operator and maximality operator with equivalents that deal with intervals as opposed to degrees (for both, see Schwarzschild

[5] Heim attributes the idea for this operator and its name to Schwarzschild 2004.

and Wilkinson 2002:23). Even in this analysis, though, the comparative standard in the *than* clause still involves abstraction (now over intervals), as we still need to interpret *every boy is tall* not as a denoting a truth value but as denoting an interval (the interval containing the heights of every boy). We can use the what is basically the same `?d` operator defined above, albeit now with Schwarzschild and Wilkinson's (2002) μ maximality operator as opposed to Heim's (2000) **max**:

(17)



The type tree in (17) is as above, only now with *D* (the type of an interval, a set of degrees) and with the modified lexical entries for `?D` and *wide*.

To accommodate Heim's (2006) analysis, we need only introduce a weaker version of the Π operator, one which simply raises predicates of degrees to predicates of sets of degrees (intervals). The extra mobility of Π is unnecessary under a continuation implementation: both quantifiers and our new quantifiers-over-degrees take scope via the same mechanism, continuations, and so can interact freely without additional mechanisms. The interpretive system has free access to LIFT, which as we've seen can allow different elements to scope over one another, not just those elements which encode continuation-sensitivity lexically. There is a preference order for such scope-taking, but this order is not fixed. Two continuation-level functions can combine in either order (the right over the left only if the right is at a higher continuation level). Unfortunately, this strength is also a weakness, one shared by Heim's (2006) account.

The important remaining caveat in the Heim 2006 account is that it relies upon an additional constraint on the scopal possibilities of Π. Without such a constraint, the flexibility available to the system in the relative orderings of the quantifier and the quantifier-over-degrees in a sentence like (13) allows for both the licit wide-scope universal interpretation and the wide-scope degree interpretation which is unattested. Takahashi (2006) named this constraint the **Heim-Kennedy constraint**, and its scope, limits, and motivations (including attempts to reduce it to a general economy principle) have been much discussed in the literature (Takahashi 2006, Lassiter 2012, Mayr and Spector 2012, Fleisher 2015a, among others). As understood by Alrenga and Kennedy (2014:26), the constraint stipulates that "If the scope of a quantificational DP contains the trace of a DegP, it also contains that DegP itself." This constraint is sufficient to rule out a number of unattested scope combinations that the theory allows to be generated, but it is quite stipulative, in that it is a constraint on a particular type of phrase (not heads in general, or functional projections in general), and it remains unclear why it should be that a degree phrase and a quantificational DP should have to stand in this particular relation to one another.

We can capture the Heim-Kennedy constraint within the continuations framework by imposing a restriction on the combinatory possibilities of the SCOPE operation. This means adding another clause to the SCOPE rule which composes continuations. This new criterion would be the only to make reference in its formulation to the character of the combinants over which SCOPE operates: the other criteria care about the relations between the combinants (identity and function-argument compatibility, to be exact), but they don't make reference to the specific types of those combinants. The type-sensitive nature of this restriction, however, may not be unique to the Heim-Kennedy constraint, in that there may be other phenomena which require SCOPE to be sensitive to particular phrase types, as Charlow (2015) has argued for the wide scoping of indefinites.

The elimination or at least explanation of this constraint continues to elude the field. In the meantime, we can represent it formally within the continuation semantics framework.

3 Conclusion

In this paper, I have adopted and introduced the work on continuations in natural language by Barker and Shan and extended their project to comparatives, which have often been analyzed as involving post-surface movement to LF for type-interpretability reasons. I've shown that the formalism described by Barker and Shan is capable of handling the difficulties posed by comparatives when supplemented by a few continuation-sensitive operators which I introduced. This is the case whether one prefers a theory of comparatives that is quantificational (Heim 2000, 2006) or not (Schwarzschild and Wilkinson 2002), based on intervals or degrees, as demonstrated here. For additional extensions of this work, discussing the non-upward entailing differentials highlighted by Fleisher (2015b), the relationship between the comparative and the superlative, and cross-linguistic variation, all as they relate to a continuation semantics analysis, see Snider 2015.

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