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Essays on Nonlinear Macroeconomic Dynamics

Abstract

This dissertation consists of four essays that study topics in macroeconomics, finance and their interplay using nonlinear quantitative equilibrium models and state of the art econometric techniques. Chapter 1 proposes a general equilibrium model with financial intermediation and sovereign default risk to study the macroeconomic consequences of news regarding a future sovereign default. The model, estimated on Italian data, is used to measure the output losses of the 2010-2012 sovereign debt crisis, and to evaluate the effects of credit policies implemented by European authorities. Chapter 2 proposes a new class of time series model that can be used to measure nonlinearities in the data and to evaluate the fit of Dynamic Stochastic General Equilibrium (DSGE) models solved with high order perturbation. We first characterize this class, the Quadratic Autoregressive (QAR) model. We then show how the QAR model can be used as a diagnostic tool to assess whether a DSGE model is able to replicate the nonlinear behavior of a set of U.S. aggregate time series. Chapter 3 studies the determinants of medium term movements in the market value of U.S. corporations. We find that secular movements in the mean and volatility of TFP growth are strongly associated with these medium term fluctuations in asset prices. These empirical findings are then interpreted within a production based asset pricing model where the mean and volatility of aggregate productivity growth varies over time. We show that the model can rationalize a sizable elasticity of asset prices to the drivers of aggregate productivity. Chapter 4 proposes a method to identify Harrod-neutral technology shocks in the data in presence of input heterogeneity in the aggregate production function. We prove that, in a wide class of models, Harrod-neutral technology shocks are the only one consistent with a certain form of balanced growth. We then use this property to identify Harrod-neutral shocks using a state-space model. Monte Carlo simulations show that the proposed method performs very well in small samples.

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Luigi Bocola

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Luigi Bocola

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ABSTRACT

ESSAYS ON NONLINEAR MACROECONOMIC DYNAMICS

Luigi Bocola

Frank Schorfheide

This dissertation consists of four essays that study topics in macroeconomics, finance and their interplay using nonlinear quantitative equilibrium models and state of the art econometric techniques. Chapter 1 proposes a general equilibrium model with financial intermediation and sovereign default risk to study the macroeconomic consequences of news regarding a future sovereign default. The model, estimated on Italian data, is used to measure the output losses of the 2010-2012 sovereign debt crisis, and to evaluate the effects of credit policies implemented by European authorities. Chapter 2, co-authored with Boragan Aruoba and Frank Schorfheide, proposes a new class of time series model that can be used to measure nonlinearities in the data and to evaluate the fit of Dynamic Stochastic General Equilibrium (DSGE) models solved with high order perturbation. We first characterize this class, the Quadratic Autoregressive (QAR) model. We then show how the QAR model can be used as a diagnostic tool to assess whether a DSGE model is able to replicate the nonlinear behavior of a set of U.S. aggregate time series. Chapter 3, co-authored with Nils Gornemann, studies the determinants of medium term movements in the market value of U.S. corporations. We find that secular movements in the mean and volatility of TFP growth are strongly associated with these medium term fluctuations in asset prices. These empirical findings are then interpreted within a production based asset pricing model where the mean and volatility of aggregate productivity growth varies over time.

We show that the model can rationalize a sizable elasticity of asset prices to the drivers of aggregate productivity. Chapter 4, co-authored with Iourii Manovskii and Marcus Hagedorn, proposes a method to identify Harrod-neutral technology shocks in the data in presence of input heterogeneity in the aggregate production function. We prove that, in a wide class of models, Harrod-neutral technology shocks are the only one consistent with a certain form of balanced growth. We then use this property to identify Harrod-neutral shocks using a state-space model. Monte Carlo simulations show that the proposed method performs very well in small samples.

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Chapter 1

The Pass-Through of Sovereign Risk

1.1 Introduction

At the end of 2009, holdings of domestic government debt by banks in European peripheral countries - Greece, Italy, Portugal and Spain - were equivalent to 93% of banks' total equity. At the same time, these banks provided roughly three-quarters of external financing to domestic firms. Prior research has established that the sovereign debt crisis in these economies resulted in a substantial increase in the borrowing costs for domestic firms.¹ One proposed explanation of these findings is that the exposure to distressed government bonds hurts the ability of banks to raise funds in financial markets, leading to a pass-through of their increased financing costs into the lending rates payed by firms.² This view was at the core

¹See, for example, the evidence in [Klein and Stellner \(2013\)](#) and [Bedendo and Colla \(2013\)](#) using corporate bond data, the analysis of [Bofondi et al. \(2013\)](#) using Italian firm level data and [Neri \(2013\)](#) and [Neri and Ropele \(2013\)](#) for evidence using aggregate time series. See also [ECB \(2011\)](#).

²The report by [CGFS \(2011\)](#) discusses the transmission channels through which sovereign risk affected bank funding during the European debt crisis. For example, banks in the Euro area

of policy discussions in Europe and was a motive for major interventions by the European Central Bank (ECB).

I argue, however, that this view is incomplete. A sovereign default triggers a severe macroeconomic downturn and adversely affects the performance of firms. Consequently, as an economy approaches a sovereign default, banks perceive firms to be more risky. Because banks require fair compensation for holding this additional risk, firms' borrowing costs rise. If this mechanism is quantitatively important, policies that address the heightened liquidity problems of banks but do not reduce the increased riskiness of firms may prove ineffective in encouraging bank lending.

I formalize this mechanism in a quantitative model with financial intermediation and sovereign default risk. In the model, an increase in the probability of a sovereign default both tightens the funding constraints of banks (*leverage-constraint channel*) and raises the risks associated with lending to the productive sector (*risk channel*). I structurally estimate the model on Italian data with Bayesian methods. I find that the risk channel is indeed quantitatively important: it explains up to 47% of the impact of the sovereign debt crisis on the borrowing costs of firms. I then use the estimated model to assess the consequences of credit market interventions adopted by the ECB and to propose and evaluate alternative policies that are more effective in mitigating the implications of increased sovereign default risk.

My framework builds on a business cycle model with financial intermediation, in the tradition of [Gertler and Kiyotaki \(2010\)](#) and [Gertler and Karadi \(2011, 2013\)](#).

extensively use government bonds as collateral, and the decline in the value of these securities during the sovereign debt crisis reduced their ability to access wholesale liquidity. See also [Zole \(2013\)](#) and [Albertazzi et al. \(2012\)](#).

In the model, banks collect savings from households and use these funds, along with their own wealth (net worth), to buy long-term government bonds and to lend to firms. This intermediation is important because firms need external finance to buy capital goods. The model has three main ingredients. First, an agency problem between households and banks generates constraints in the borrowing ability of these latter. These constraints on bank leverage bind only occasionally, and typically when bank net worth is low. Second, financial intermediation is risky: bank net worth varies over time mainly because banks finance long-term risky assets with short-term risk-free debt. Third, the probability that the government defaults on its bonds and imposes losses on banks is time-varying and follows an exogenous stochastic process.

To understand the key mechanisms of the model, consider a scenario in which the probability of a future sovereign default rises. The anticipation of a “haircut” on government bonds depresses their market value and lowers the net worth of banks.³ This tightens their leverage constraints and has adverse consequences for financial intermediation: banks’ ability to collect funds from households decline, lending to the productive sector declines and so does aggregate investment. This is the conventional leverage-constraint channel in the literature.

However, even when the leverage constraints are *currently* not binding, a higher probability of a future sovereign default induces banks to demand higher compensation when lending to firms. This is the case because the sovereign default triggers a deep recession characterized by a severe decline in the payouts of firms. Thus, when the probability of a future sovereign default increases, banks have an incentive to deleverage in order to avoid these losses. More specifically, if the

³In the model, a haircut is the fraction of the principal that is reneged by the government in the event of a default.

sovereign default happens in the future, bank leverage constraints tighten because of the government haircut. This forces banks to liquidate their holdings of firms assets. The associated decline in their market value leads to a further deterioration in bank net worth, feeding a vicious loop. *Ex-ante*, forward-looking banks demand a premium for holding these claims because they anticipate that they will pay out little precisely when banks are mostly in need of wealth. The resulting risk premium is increasing in the probability of a sovereign default. This constitutes the risk channel.

I measure the quantitative importance of the leverage constraint channel and the risk channel by estimating the structural parameters of the model with Italian data from 1999:Q1 to 2011:Q4 using Bayesian techniques. The major empirical challenge is to separate these two propagation mechanisms since they have qualitatively similar implications for indicators of financial stress commonly used in the literature (e.g., credit spreads). I demonstrate that the Lagrange multiplier on bank leverage constraints is a function of observable variables, specifically of the TED spread (spread between the prime interbank rate and the risk free rate) and of the leverage of banks. I construct a time series for this multiplier and use it in estimation, along with output growth, to measure the cyclical behavior of the leverage constraint. In addition, I use credit default swap (CDS) spreads on Italian government bonds and data on holdings of domestic government debt by Italian banks to measure the time-varying nature of sovereign risk and the exposure of banks to that risk.⁴ The structural estimation is complicated by the fact that the model features time-variation in risk premia and occasionally binding financial constraints. I develop an algorithm for its global solution based on projections

⁴A CDS is a derivative used to hedge the credit risk of an underlying reference asset. CDS spreads on government securities are typically used in the literature as a proxy of sovereign risk, see [Pan and Singleton \(2008\)](#).

and sparse collocation, and I combine it with the particle filter to evaluate the likelihood function.

Having established the good fit of the model using posterior predictive analysis, I use it to answer two applied questions. First, I quantify the importance of the leverage-constraint channel and the risk channel for the propagation of sovereign credit risk to the financing premia of firms and output. I estimate that the increase in the probability of a sovereign default in Italy during the 2010:Q1-2011:Q4 period raised substantially firms' financing premia, with a peak of 100 basis points in 2011:Q4. This increase reflects both tighter constraints on bank leverage and increased riskiness of firms, with the risk channel explaining up to 47% of the overall effects. Moreover, the rise in the probability of a sovereign default had severe adverse consequences for the Italian economy: cumulative output losses were 4.75% at the end of 2011.

In the second set of quantitative experiments, I evaluate the effectiveness of a major unconventional policy adopted by the ECB in the first quarter of 2012 to address the crisis, the Longer Term Refinancing Operations (LTROs). I model the policy as a subsidized long-term loan offered to banks. Because of the inherent nonlinearities of the model, initial conditions matter for policy evaluation. Thus, I implement this intervention conditioning on the state of the Italian economy in 2011:Q4. I find that the effects of LTROs on credit to firms and output vary over the 2012:Q1-2014:Q4 window, but they are small and not significantly different from zero when we average over this time period. This is due to the fact that risk premia were sizable when the policy was enacted. Banks, thus, have little incentives to increase their exposure to firms and they mainly use LTROs to cheaply refinance their liabilities.

The lesson from the policy evaluation is that the success of unconventional policies, such as LTROs, crucially depends on current economic conditions, in particular on the relative importance of binding leverage constraints versus risk premia. The former prevents banks from undertaking otherwise profitable investment. Policies that relax these constraints have sizable effects on bank lending and capital accumulation. The latter, instead, signal that firms are forecasted to be a “bad asset” in the future and bank lending is less responsive to refinancing operations. In these circumstances, policies that insure banks from the downside risk of a sovereign default (for example through a large injection of equity or a floor on the price of government bonds) can achieve stimulative effects. These interventions lower the risk associated with lending to the private sector because they limit the contagion effects that occur in the event of a sovereign default through banks’ balance sheets. However, these stimulative effects should be weighed against the increased risk taking behavior that these policies are likely to bring and that I do not capture in my analysis.

Related Literature. This paper is related to several strands of the literature. Empirical studies document a strong link between sovereign spreads and private sector interest rates, both in emerging economies and more recently in southern European countries.⁵ Several authors recognize the importance of this relationship in different settings. For example, [Neumeyer and Perri \(2005\)](#) and [Uribe and Yue \(2006\)](#) suggest that sovereign spreads are a major driver of business cycles in emerging markets, while [Corsetti et al. \(2013\)](#) study the implications of the sovereign risk pass-through for fiscal policy. However, in these and related papers,

⁵For emerging market economies, [Durbin and Ng \(2005\)](#) and [Borensztein et al. \(2006\)](#) provide an empirical analysis of the “sovereign ceiling”, the practice of agencies to rate corporations below their sovereign. [Cavallo and Valenzuela \(2007\)](#) document the effects of sovereign spreads on corporate bonds spreads. See footnote 1 and 2 for evidence on southern European economies.

the reasons underlying the connection between sovereign spreads and private sector interest rate are not modeled. Part of the contribution of this paper to the literature is to microfound this link in a fully specified dynamic equilibrium model.

In doing so, my paper also relates to the literature covering the output costs of sovereign debt defaults, more precisely to papers studying the effects of defaults on domestic bondholders. Motivated by robust empirical evidence, [Gennaioli et al. \(2013b\)](#) and [Sosa Padilla \(2013\)](#) study the effects of sovereign defaults on domestic banks, and the impact that the associated output losses have on the government's incentives to default.⁶ My research is complementary to theirs: I take sovereign default risk as exogenous, but I explicitly model the behavior of private credit markets when sovereign risk increases. The novel insight of my paper is that the mere *anticipation* of a sovereign default can be recessionary because of its impact on the perceived riskiness of firms and on the funding constraints of exposed banks.⁷ While this exogeneity of sovereign default risk rules out important feedback effects between banks and sovereigns ([Uhlig, 2013](#); [Acharya et al., 2013](#)), it does allow for a transparent analysis of these transmission channels.⁸

This paper contributes to a growing literature on the aggregate implications of shocks to the balance sheet of financial intermediaries. In particular, I build on the modeling framework developed by [Gertler and Kiyotaki \(2010\)](#) and [Gertler](#)

⁶[Kumhof and Tanner \(2005\)](#) and [Gennaioli et al. \(2013a\)](#) document that banks are highly exposed to domestic government debt in a large set of countries. [Reinhart and Rogoff \(2011\)](#) and [Borensztein and Panizza \(2009\)](#) show that sovereign defaults typically occur simultaneously, or in close proximity, to banking crises.

⁷In an empirical study, [Yeyati and Panizza \(2011\)](#) point out that anticipation effects are key to understand the unfolding of sovereign debt crises. See also [Aguiar et al. \(2009\)](#) and [Dovis \(2013\)](#) for models where anticipation effects arise because of debt overhang problems.

⁸[Pancrazi et al. \(2013\)](#) and [Mallucci \(2013\)](#) are two contemporaneous papers studying the effects of sovereign credit risk on the funding costs of firms. Even though these authors model explicitly the incentives of the government to default on its debt, their production sector is static. As such, their analysis abstract from the effects that a sovereign default has on the perceived riskiness of firms, the key novel mechanism of this paper.

and Karadi (2011, 2013), where the limited enforcement of debt contracts generates endogenous constraints on intermediaries' leverage. Differently from these papers, my analysis studies how changes in the expectation of these constraints being binding in the future influence the choices of financial intermediaries regarding their lending behavior today.⁹ In my application, these phenomena arise because of shocks to the the default probability of government bonds, but the same logic could be applied to the study of other assets. My analysis uncovers two important phenomena. First, these changes in expectations can induce quantitatively sizable variation in risk premia. Brunnermeier and Sannikov (2013), He and Krishnamurthy (2012a,b) and Bianchi and Mendoza (2012) study related effects, but in the present context they emerge because of shocks to the volatility of an unproductive assets' payoff. Productive assets are affected because the balance sheet of banks generates contagion (e.g., produces correlation among the payoffs of different assets held by banks). Second, stabilization policies are state and size dependent in this environment.¹⁰ As explained earlier, these nonlinearities depend on the relative importance of currently binding leverage constraints and risk premia.

The measurement of these two latter components is therefore a key aspect of this paper. The construction of a model consistent indicator for the Lagrange multiplier on bank leverage constraints is novel, and it is related to the measurement of financial shocks in Jermann and Quadrini (2012). Methodologically, I draw from the literature on the Bayesian estimation and validation of dynamic equilibrium economies (Del Negro and Schorfheide, 2011a), more specifically of

⁹Technically, I capture these effects because I study the full nonlinear model rather than local approximation around a deterministic steady state.

¹⁰There are a number of papers that study unconventional monetary policy in related environments. See, for example, Curdia and Woodford (2010), Curdia and Woodford (2011), Del Negro et al. (2012) and Bianchi and Bigio (2013).

models where nonlinearities feature prominently (Fernández-Villaverde and Rubio-Ramírez, 2007a). The decision rules of the model are derived numerically using a projection algorithm. I use a Smolyak sparse grid (Krueger and Kubler, 2003), which sensibly reduces the curse of dimensionality.¹¹ I evaluate the likelihood function tailoring the auxiliary particle filter of Pitt and Shephard (1999) to the present application. To my knowledge, this is the first paper to estimate a model with occasionally binding financial constraints using global methods and nonlinear filters. However, there are other papers using related techniques for different applications (see Gust et al., 2013; Bi and Traum, 2012, 2013).

Finally, the shock to sovereign default probabilities considered in this paper is a form of time-varying volatility. As such, my research is related to the literature that studies how different types of volatility shocks influence real economic activity (Bloom, 2009; Bloom et al., 2012; Fernández-Villaverde et al., 2011). In particular, Rietz (1988) and Barro (2006) emphasize the role of large macroeconomic disasters in accounting for asset prices and Gourio (2012) studies how changes in the probability of these events affect risk premia and capital accumulation. The sovereign default studied in this paper can be seen as a potential source of macroeconomic disasters.¹²

Layout. The paper is organized as follows. Section 3.3 presents the model, while Section 1.3 discusses its main mechanisms using two simplified examples. Section 1.4 presents the estimation and an analysis of the model’s fit. Section 1.5 presents key properties of the estimated model that are useful to interpret

¹¹Christiano and Fisher (2000) is an early paper documenting the performance of projections in models with occasionally binding constraints. See also Fernández-Villaverde et al. (2012) for an application of the Smolyak sparse grid in a model where the zero lower bound constraint on nominal interest rate bind occasionally.

¹²Arellano et al. (2012), Gilchrist et al. (2013) and Christiano et al. (2013) study the real effects of a different form of time-varying volatility in models with financial frictions.

the two quantitative experiments, which are reported in Section 1.6. Section 4.5 concludes.

1.2 Model

I consider a neoclassical growth model enriched with a financial sector as in [Gertler and Kiyotaki \(2010\)](#) and [Gertler and Karadi \(2011\)](#). In this setting, I introduce long term government bonds to which financial intermediaries are exposed. These securities pay in every state of nature unless the economy is in a sovereign default—an event that can occur every period according to an exogenous and time-varying probability.

The model economy is populated by households, final good producers, capital good producers and a government. Each household is composed of two types of members: *workers* and *bankers*. Workers supply labor to final good firms in a competitive factor market, and their wages are made available to the household. Bankers manage the savings of other households and use these funds to buy government bonds and claims on firms. Bankers offer a risk free rate on households' savings. The perfectly competitive non-financial corporate sector produces a final good using a constant return to scale technology that aggregates capital and labor. Firms rent labor from households and buy capital from perfectly competitive capital good producers. Their capital expenses are financed by bankers. Finally, the government issues bonds and taxes households in order to finance government spending. In every period the government can default on its debt. This event is modeled as an exogenous stochastic process.

The key friction of the model is the limited enforcement of debt contracts between

households and bankers: bankers can walk away with the assets of their franchise, and households can recover only a fraction of their savings when this event occurs. This friction gives rise to constraints on the leverage of banks, with bank net worth being the key determinant of their borrowing capacity. When these incentive constraints bind, or are expected to bind in the future, credit to the productive sector declines. This has adverse consequences for capital accumulation. An increase in the probability of a future government default is recessionary because it adversely impacts the *current* and *expected* level of bank net worth, thereby influencing their lending behavior.

In the remainder of this section I describe the agents' decision problems, derive the conditions characterizing a competitive equilibrium, and sketch the algorithm used for the numerical solution of the model. In Section 1.3, I discuss the key mechanisms of interest. I denote by \mathbf{S} the vector collecting the current value for the state variables and by \mathbf{S}' the future state of the economy.

1.2.1 Agents and their Decision Problems

Households

A household is composed of a fraction f of workers and a fraction $1 - f$ of bankers. There is perfect consumption insurance between its members. Let $\Pi(\mathbf{S})$ be the net payments that bankers make to their own household, and let $W(\mathbf{S})$ be the wage that workers receive from supplying labor to final good firms. Households value consumption c and dislike labor l according to the flow utility $u(c, l)$, and they discount the future at the rate β . The problem for the household is that of making contingent plans for consumption, labor supply and savings b' so as to

maximize lifetime utility. Savings are deposited into financial intermediaries that are managed by bankers belonging to other households, and they earn the risk free return $R(\mathbf{S})$. Taking prices as given, a household solves

$$v_h(b; \mathbf{S}) = \max_{b' \geq 0, c \geq 0, l \in [0, 1]} \{u(c, l) + \beta \mathbb{E}_{\mathbf{S}}[v_h(b'; \mathbf{S}')]\},$$

$$c + \frac{1}{R(\mathbf{S})}b' \leq W(\mathbf{S})l + \Pi(\mathbf{S}) + b - \tau(\mathbf{S}),$$

$$\mathbf{S}' = \mathbf{\Gamma}(\mathbf{S}).$$

$\tau(\mathbf{S})$ denotes the level of lump sum taxes while $\mathbf{\Gamma}(\cdot)$ describes the law of motion for the aggregate state variables. Optimality is governed, at an interior solution, by the intra-temporal and inter-temporal Euler equations

$$u_l(c, l) = u_c(c, l)W(\mathbf{S}), \tag{1.1}$$

$$\mathbb{E}_{\mathbf{S}}[\Lambda(\mathbf{S}', \mathbf{S})R(\mathbf{S})] = 1, \tag{1.2}$$

where $\Lambda(\mathbf{S}', \mathbf{S}) = \beta \frac{u_c(c', l')}{u_c(c, l)}$. For the empirical analysis I will use preferences that are consistent with balanced growth, $u(c, l) = \log(c) - \chi \frac{l^{1+\nu-1}}{1+\nu-1}$, where ν parameterizes the Frisch elasticity of labor supply.

Bankers

A banker uses his accumulated net worth n and deposits b to buy government bonds and claims on firms.¹³ Let a_j be asset j held by a banker and let $Q_j(\mathbf{S})$ and $R_j(\mathbf{S}', \mathbf{S})$ be, respectively, the price of asset j and its realized returns next period on a unit of numeraire good invested in asset j . The banker's balance sheet

¹³A worker who becomes a banker this period obtains start-up funds from his households. These transfers will be specified at the end of the section.

equates total assets to total liabilities:

$$\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \leq n + \frac{b'}{R(\mathbf{S})}, \quad (1.3)$$

where subscript B refers to government bonds and K to firms' claims. A banker makes optimal portfolio choices in order to maximize the present discounted value of dividends paid to his own household. At any point in time there is a probability $1 - \psi$ that a banker becomes a worker in the next period. When this happens, the banker pays back a dividend to his own household.¹⁴ Bankers who continue running the business do not pay dividends, and they accumulate net worth. The objective of a banker is that of maximizing the expected discounted value of his terminal wealth. Net worth next period equals the difference between realized returns on assets and the payments promised to households.

$$n' = \sum_{j=\{B,K\}} R_j(\mathbf{S}', \mathbf{S})Q_j(\mathbf{S})a_j - b'. \quad (1.4)$$

Note that bad realizations of $R_j(\mathbf{S}', \mathbf{S})$ lead to reductions in bankers' net worth n' . This variation in net worth affects the ability of bankers to obtain funds from the household sector and, ultimately, their supply of credit to the firm. This occurs due to the limited enforcement of contracts between households and banks. At any point in time, a banker can walk away with a fraction λ of the project and transfer it to his own household. If he does, the depositors can force him into bankruptcy and recover a fraction $(1 - \lambda)$ of banks' assets. This friction defines an incentive constraint for the banker: the value of running his franchise must be higher than its outside option, $\lambda[\sum_j Q_j(\mathbf{S})a_j]$.

¹⁴When a banker exits, a worker replaces him so that their relative proportion does not change over time.

Taking prices as given, a banker solves the decision problem

$$\begin{aligned}
v_b(n; \mathbf{S}) &= \max_{a_B, a_K, b} \mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi)n' + \psi v_b(n'; \mathbf{S}')] \}, \\
n' &= \sum_{j=\{B, K\}} R_j(\mathbf{S}', \mathbf{S}) Q_j(\mathbf{S}) a_j - b', \\
\sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j &\leq n + \frac{b'}{R(\mathbf{S})}, \\
\lambda \left[\sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j \right] &\leq v_b(n; \mathbf{S}), \\
\mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}).
\end{aligned}$$

The following result further characterizes this decision problem.¹⁵

Result 1. *A solution to the banker's dynamic program is*

$$v_b(n; \mathbf{S}) = \alpha(\mathbf{S})n,$$

where $\alpha(\mathbf{S})$ solves

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')] R(\mathbf{S}) \}}{1 - \mu(\mathbf{S})}, \quad (1.5)$$

and the multiplier on incentive constraints satisfies

$$\mu(\mathbf{S}) = \max \left\{ 1 - \left[\frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')] R(\mathbf{S}) \} N}{\lambda [Q_K(\mathbf{S}) A_K + Q_B(\mathbf{S}) A_B]} \right], 0 \right\}, \quad (1.6)$$

where N , A_B and A_K are, respectively, aggregate bankers' net worth and aggregate bankers' holdings of government bonds and firms assets.

Proof. See Appendix A.1. □

¹⁵The problem is not well defined for negative values of net worth. When this happens, the government steps in and refinance the bank via lump sum taxation. At the same time, it issues a non-pecuniary punishment to the banker that is equivalent to the net worth losses.

This result clarifies that limited enforcement of contracts places an endogenous constraint on the leverage of the banker. Indeed, because of the linearity of the value function, the incentive constraint becomes

$$\frac{\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda}, \quad (1.7)$$

implying that bank leverage cannot exceed the time-varying threshold $\frac{\alpha(\mathbf{S})}{\lambda}$.¹⁶ Bank net worth is thus a key variable regulating financial intermediation in the model: when net worth is low, the leverage constraint is more likely to bind and this limits the amount of assets that a banker can intermediate.

The implications of this constraint for assets' accumulation can be better understood by looking at the Euler equation for risky asset j

$$\mathbb{E}_{\mathbf{S}} \left[\hat{\Lambda}(\mathbf{S}', \mathbf{S}) R_j(\mathbf{S}', \mathbf{S}) \right] = \mathbb{E}_{\mathbf{S}} \left[\hat{\Lambda}(\mathbf{S}', \mathbf{S}) R(\mathbf{S}) \right] + \lambda \mu(\mathbf{S}), \quad (1.8)$$

where $\hat{\Lambda}(\mathbf{S}', \mathbf{S})$ is the economy's pricing kernel, defined as

$$\hat{\Lambda}(\mathbf{S}', \mathbf{S}) = \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S}')]. \quad (1.9)$$

There are two main distinctions between this Euler equation and the one that would arise in a purely neoclassical setting. First, the presence of leverage constraints limits the ability of banks to arbitrage away differences between expected discounted returns on asset j and the risk free rate: this can be seen from equation (1.8), as the multiplier generates a wedge between these two returns. Second, the pricing kernel in equation (1.9) is not only a function of consumption growth as in

¹⁶Alternatively, we can interpret equation (1.7) as a collateral constraint. Indeed, using the balance sheet identity we write the leverage constraint as $b \leq \left[\frac{\alpha(\mathbf{S}) - \lambda}{\alpha(\mathbf{S})} \right] \sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j$. That is, bankers' debt cannot exceed a time varying fraction of the market value of their total assets.

canonical neoclassical models, but also of bank leverage. Indeed, as stated in equation (1.7), financial leverage is proportional to $\alpha(\mathbf{S})$ when $\mu(\mathbf{S}) > 0$. Adrian et al. (2013) provides empirical evidence in support of leverage-based pricing kernels for the U.S. economy and He and Krishnamurthy (2012b) discuss their asset pricing implications in endowment economies. If the leverage constraint never binds, i.e. $\mu(\mathbf{S}) = 0 \forall \mathbf{S}$, equation (1.8) collapses to the neoclassical benchmark.¹⁷

Result 1 also implies that banks heterogeneity in their net worth and asset holdings does not affect aggregate dynamics. Indeed, equation (1.8) suggests that assets returns depend on the dynamics of the multiplier $\mu(\mathbf{S})$ which, in turn, is a function of financial leverage (see equation (1.6)). Since this latter is identical across bankers when the constraint binds, $\mu(\mathbf{S})$ is independent on the cross-sectional distribution of bank net-worth: agents in the economy do not need to know this distribution when forecasting future prices, this making the numerical analysis of the model tractable. For future reference, it is convenient to derive an expression for the law of motion of aggregate net worth

$$N'(\mathbf{S}', \mathbf{S}) = \psi \left\{ \sum_{j=\{B,K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) A_j + R(\mathbf{S}) N \right\} + \omega \sum_{j=\{B,K\}} Q_j(\mathbf{S}') A_j. \quad (1.10)$$

Aggregate net worth equals the sum of the net worth accumulated by bankers who did not switch occupations today and the transfers that households make to newly born bankers. These transfers are assumed to be a fraction ω of the assets intermediated in the previous period, evaluated at current prices. In the empirical analysis, ω has the purpose of pinning down the level of financial leverage in a deterministic balanced growth path of the economy, and it will be a small number.

¹⁷Using equation (1.2), we can see that a solution to equation (1.5) is $\alpha(\mathbf{S}) = 1 \forall \mathbf{S}$ whenever $\mu(\mathbf{S}) = 0 \forall \mathbf{S}$.

Capital Good Producers

The capital good producers build new capital goods using the technology $\Phi\left(\frac{i}{K}\right) K$, where K is the aggregate capital stock in the economy and i the inputs used in production. They buy inputs in the final good market, and sell capital goods to final good firms at competitive prices. Taking the price of new capital $Q_i(\mathbf{S})$ as given, the decision problem of a capital good producer is

$$\max_{i \geq 0} \left[Q_i(\mathbf{S}) \Phi\left(\frac{i}{K}\right) K - i \right].$$

Anticipating the capital goods market clearing condition, the price for new capital goods is

$$Q_i(\mathbf{S}) = \frac{1}{\Phi'\left(\frac{I(\mathbf{S})}{K}\right)}, \quad (1.11)$$

where $I(\mathbf{S})$ is equilibrium aggregate investment.

For the empirical analysis, I specify the production function for capital goods as $\Phi(x) = a_1 x^{1-\xi} + a_2$, where ξ parametrizes the elasticity of Tobin's q with respect to the investment-capital ratio.

Final Good Producers

Final output y is produced by perfectly competitive firms that operate a constant returns to scale technology

$$y = k^\alpha (e^z l)^{1-\alpha}, \quad (1.12)$$

where k is the stock of capital goods, l stands for labor services, and z is a neutral technology shock that follows an AR(1) process in growth

$$\Delta z' = \gamma(1 - \rho_z) + \rho_z \Delta z + \sigma_z \varepsilon'_z, \quad \varepsilon'_z \sim \mathcal{N}(0, 1). \quad (1.13)$$

Labor is rented in competitive factor markets at the rate $W(\mathbf{S})$. Capital goods depreciate every period at the rate δ . Anticipating the labor market clearing condition, profit maximization implies that equilibrium wages and profits per unit of capital are

$$W(\mathbf{S}) = (1 - \alpha) \frac{Y(\mathbf{S})}{L(\mathbf{S})}, \quad Z(\mathbf{S}) = \alpha \frac{Y(\mathbf{S})}{K}, \quad (1.14)$$

where $Y(\mathbf{S})$ and $L(\mathbf{S})$ are equilibrium aggregate output and labor.

To purchase new capital goods, firms need external financing. At the beginning of the period, firms issue claims to bankers in exchange for funds. While these claims are perfectly state contingent and therefore correspond to equity holdings, I interpret them more broadly as privately issued paper such as bank loans. For each claim a_K bankers pay $Q_K(\mathbf{S})$ to firms.¹⁸ In exchange, they receive the realized return on a unit of the capital stock in the next period:

$$R_K(\mathbf{S}', \mathbf{S}) = \frac{(1 - \delta)Q_K(\mathbf{S}') + Z(\mathbf{S}')}{Q_K(\mathbf{S})}. \quad (1.15)$$

Realized returns to capital move over time because of two factors: variation in firms' profits and variation in the market value of corporate securities. These movements in $R_K(\mathbf{S}', \mathbf{S})$ induce variation in aggregate net worth, as equation (1.10) suggests.

¹⁸No arbitrage implies that the price of a unit of new capital equals in equilibrium the price of an IOU issued by firms, $Q_i(\mathbf{S}) = Q_K(\mathbf{S})$.

The Government

In every period, the government engages in public spending. Public spending as a fraction of GDP evolves as follows

$$\log(g)' = (1 - \rho_g) \log(g^*) + \rho_g \log(g) + \sigma_g \varepsilon'_g, \quad \varepsilon'_g \sim \mathcal{N}(0, 1). \quad (1.16)$$

The government finances public spending by levying lump sum taxes on households and by issuing long-term government bonds to financial intermediaries. Long term debt is introduced as in [Chatterjee and Eyigungor \(2013\)](#). In every period a fraction π of bonds matures. When this event happens, the government pays back the principal to investors. The remaining fraction $(1 - \pi)$ does not mature: the government pays the coupon ι , and investors retain the right to obtain the principal in the future. The average duration of bonds is therefore $\frac{1}{\pi}$ periods. I introduce risk of sovereign default by assuming that the government can default in every period and write off a fraction $D \in [0, 1]$ of its outstanding debt. The parameter D can be seen as the “haircut” that the government imposes on bondholders in a default. Denoting by $Q_B(\mathbf{S})$ the pricing function for government securities, tomorrow’s realized returns on a dollar invested in government bonds are

$$R_B(\mathbf{S}', \mathbf{S}) = [1 - d'D] \left[\frac{\pi + (1 - \pi) [\iota + Q_B(\mathbf{S}')] }{Q_B(\mathbf{S})} \right], \quad (1.17)$$

where d' is an indicator variable equal to 1 if the government defaults next period. Realized returns on government bonds vary over time and they affect the balance sheet of financial intermediaries. First, when the government defaults, it imposes a haircut on bondholders which has a direct negative effect on the net worth of bankers. Second, and to the extent that $\pi < 1$, $R_B(\mathbf{S}', \mathbf{S})$ is sensitive to variation

in the price of government securities: a decline in $Q_B(\mathbf{S}')$, for example, lowers the reselling value of government bonds and reduces the returns on holding government debt.

Denoting by B' the stock of public debt, the budget constraint of the government is given by

$$Q_B(\mathbf{S}) \left[\underbrace{B' - (1 - \pi)B[1 - dD]}_{\text{Newly issued bonds}} \right] = \underbrace{[\pi + (1 - \pi)\iota] B[1 - dD]}_{\text{Payments of principals and coupons}} + \underbrace{gY(\mathbf{S}) - \tau(\mathbf{S})}_{\text{Primary Deficit}}. \quad (1.18)$$

Taxes respond to past debt according to the law of motion

$$\frac{\tau(\mathbf{S})}{Y(\mathbf{S})} = t^* + \gamma_\tau \frac{B}{Y(\mathbf{S})},$$

where $\gamma_\tau > 0$.¹⁹ Finally, I assume that sovereign risk evolves exogenously. In every period the government is hit by a shock ε_d with a standard logistic distribution. The government defaults on its outstanding debt if ε_d is sufficiently large. In particular, d' follows

$$d' = \begin{cases} 1 & \text{if } \varepsilon'_d - s \geq 0 \\ 0 & \text{otherwise,} \end{cases} \quad (1.19)$$

with s being a Gaussian AR(1) process

$$s' = (1 - \rho_s) \log(s^*) + \rho_s s + \sigma_s \varepsilon_s. \quad (1.20)$$

This formulation allows us to study how the endogenous variables respond to variation in sovereign risk. In fact, the conditional probability of a sovereign de-

¹⁹This formulation guarantees that the government does not run a Ponzi scheme and that its intertemporal budget constraint is satisfied in every state of nature. See [Bohn \(1995\)](#) and [Canzoneri et al. \(2001\)](#).

fault is $p^d(\mathbf{S}) = \frac{e^s}{1+e^s}$: an increase in s is equivalent to an increase in the conditional probability that the government defaults tomorrow.

1.2.2 Market Clearing

Letting $f(\cdot)$ be the density of net worth across bankers, we can express the market clearing conditions as follows²⁰

- i) Credit market: $\int a_K(n; \mathbf{S})f(n)dn = K'(\mathbf{S})$.
- ii) Government bonds market: $\int a_B(n; \mathbf{S})f(n)dn = B'(\mathbf{S})$.
- iii) Market for households' savings: $\int b'(n; \mathbf{S})f(n)dn = b'(\mathbf{S})$.
- iv) Market for final goods: $Y(\mathbf{S})(1 - g) = C(\mathbf{S}) + I(\mathbf{S})$.

1.2.3 Equilibrium Conditions and Numerical Solution

Since the non-stationary technology process induces a stochastic trend in several endogenous variables, it is convenient to express the model in terms of detrended variables. For a given variable x , I define its detrended version as $\tilde{x} = \frac{x}{z}$.²¹ The state variables of the model are $\mathbf{S} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s, d]$. As I detail below, the variable \tilde{P} keeps track of aggregate bank net worth. The control variables $\{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ solve the residual equations (1.2), (1.5) and (1.8) (the last one for both assets).

²⁰Note that we have anticipated earlier the market clearing condition for the labor market and for the capital good market.

²¹The endogenous state variables of the model are detrended using the level of technology last period.

The endogenous state variables $[\tilde{K}, \tilde{B}, \tilde{P}]$ evolve as follows

$$\tilde{K}'(\mathbf{S}) = \left\{ (1 - \delta)\tilde{K} + \Phi \left[e^{\Delta z} \left(\frac{\tilde{Y}(\mathbf{S})(1 - e^g) - \tilde{C}(\mathbf{S})}{\tilde{K}} \right) \right] \tilde{K} \right\} e^{-\Delta z}, \quad (1.21)$$

$$\tilde{B}'(\mathbf{S}) = \frac{[1 - dD]\{\pi + (1 - \pi)[\iota + Q_B(\mathbf{S})]\}\tilde{B}e^{-\Delta z} + \tilde{Y}(\mathbf{S}) \left[g - \left(t^* + \gamma_\tau \frac{B}{\tilde{Y}(\mathbf{S})} \right) \right]}{Q_B(\mathbf{S})}, \quad (1.22)$$

$$\tilde{P}'(\mathbf{S}) = R(\mathbf{S})[Q_K(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_B(\mathbf{S})\tilde{B}'(\mathbf{S}) - \tilde{N}(\mathbf{S})]. \quad (1.23)$$

The state variable \tilde{P} measures the detrended *cum* interest promised payments of bankers to households at the beginning of the period, and it is necessary to keep track of the evolution of aggregate bankers' net worth. Finally, the exogenous state variables $[\Delta z, \log(g), s]$ follow, respectively, (1.13), (1.16) and (1.20), while d follows

$$d' = \begin{cases} 1 & \text{with probability } \frac{e^s}{1+e^s} \\ 0 & \text{with probability } 1 - \frac{e^s}{1+e^s}. \end{cases} \quad (1.24)$$

I use numerical methods to solve for the model decision rules. The algorithm for the global numerical solution of the model relies on projection methods (Judd, 1992; Heer and Maussner, 2009). In particular, let $x(\mathbf{S})$ be the function describing the behavior of control variable x . I approximate $x(\mathbf{S})$ using two sets of coefficients, $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}$. The law of motion for x is then described by

$$x(d, \tilde{\mathbf{S}}) = (1 - d)\gamma_0^{x'}\mathbf{T}(\tilde{\mathbf{S}}) + d\gamma_1^{x'}\mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s]$ is the vector of state variables that excludes d , and $\mathbf{T}(\cdot)$ is a vector collecting Chebyshev's polynomials. The coefficients $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}_x$

are such that the residual equations are satisfied for a set of collocation points $(d^i, \tilde{\mathbf{S}}^i) \in \{0, 1\} \times \tilde{\mathcal{S}}$. I choose $\tilde{\mathcal{S}}$ and the set of polynomial $\mathbf{T}(\cdot)$ using the Smolyak collocation approach. Krueger and Kubler (2003) and Krueger et al. (2010) provides a detailed description of the methodology. When evaluating the residual equations at the collocation points, I evaluate expectations by “precomputing integrals” as in Judd et al. (2011). Finally, I adopt Newton’s method to find the coefficients $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}_x$ satisfying the residual equations. Appendix A.2 provides a detailed description of the algorithm and discusses the accuracy of the numerical solution.

1.3 Two Simple Examples Illustrating the Mechanisms

Before moving on to the empirical analysis it is useful to describe the mechanisms that ties sovereign risk to the funding costs of firms and real economic activity. An increase in the probability of a future sovereign default lowers capital accumulation via two distinct channels. i) it tightens the leverage constraints of bankers, and ii) it increases the required premia for holding firms’ claims.

I illustrate these propagation mechanisms using two stylized versions of the model. We will see that a decline in current net worth tightens bankers’ leverage constraints (Section 1.3.1) and that bad *news* about future net worth leads to an increase in risk premia over firms’ assets (Section 1.3.2). I then discuss how sovereign risk interacts with these two mechanisms in the model described in the previous section. Finally, Section 1.3.3 explains why disentangling these two mechanisms provides important information for evaluating the effects of credit

policies in the model.

1.3.1 A Decline in Current Net Worth

I consider a deterministic economy with full depreciation ($\delta = 0$), no capital adjustment costs ($\xi = 0$) and no government. Moreover, I assume that the transfers to newly born bankers equal a fraction ω of current output, $N = \omega Y$ and that bankers live only one period ($\psi = 0$).

As in the neoclassical model with full depreciation and log utility, the saving rate is constant in this economy. Specializing equation (1.6) to this particular parametrization, we obtain an expression for the multiplier on incentives constraints

$$\mu = \frac{\lambda\sigma - \omega}{\lambda\sigma},$$

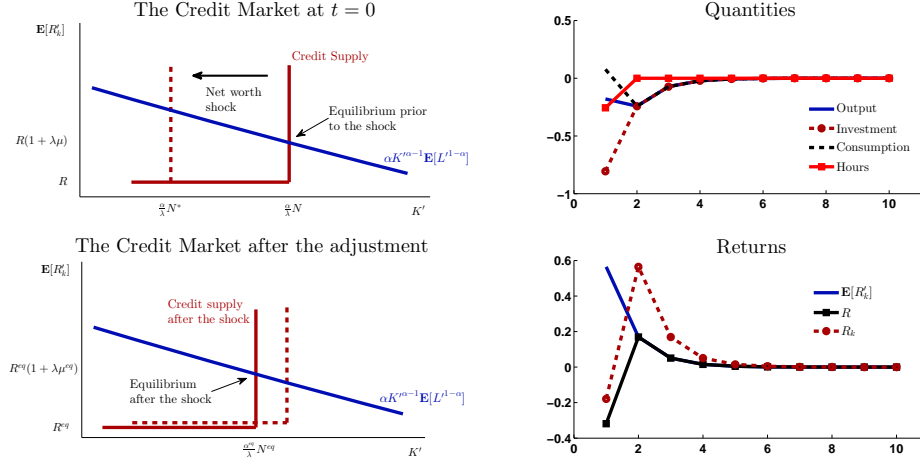
where σ is the saving rate. Using equation (1.8), we can solve for σ

$$\sigma = \min \left\{ \frac{\alpha\beta + \omega}{1 + \lambda}, \alpha\beta \right\}.$$

I assume that the leverage constraints are currently binding ($\lambda\beta\alpha > \omega$), and I analyze the implications of an unexpected transitory decline in the transfers to bankers. More specifically, I assume that at time $t = 1$ the transfer to bankers ω declines, then goes back to its previous level at $t = 2$, and no further changes occur at future dates. Agents do not expect such a change, but are perfectly informed about the path of the transfer from period $t = 1$ onward and they make rational choices based on this path. While analytical solutions for this example can be easily derived, I illustrate the transition to steady state using a numerical

example.²²

Figure 1.1: A Decline in Current Net Worth



Notes: The figure reports the transitional dynamics induced by a transitory and unexpected 5% decline in net worth. The parametrization adopted is $[\alpha = 0.33, \nu = \infty, \beta = 0.995, \lambda = 0.44, \omega = 0.10]$. The right panels report variables expressed as percentage deviations from their steady state.

The top left panel of Figure 1.1 plots the equilibrium in the credit market prior to the decline in ω . The supply of funds is derived from the bankers' optimization problem: if the leverage constraints were not binding, bankers would be willing to lend at the risk free rate R since this economy is non-stochastic. The supply of funds to firms is inelastic at $K' = \frac{\alpha}{\lambda}N$, the point at which the leverage constraint binds. The demand for credit is downward sloping and equal to the expected marginal product of capital, $\alpha K'^{\alpha-1} \mathbb{E}[L^{1-\alpha}]$. Since the leverage constraint binds, expected returns to capital equal $R(1 + \lambda\mu)$.

The unanticipated decline in ω tightens the leverage constraint ($\frac{\alpha}{\lambda}N^* < \frac{\alpha}{\lambda}N$), and the inelastic part of the supply schedule shifts leftward. The right panels of the figure describe adjustments for quantities and prices. The tightening of credit has adverse effects on capital accumulation. Consumption increases because agency

²²The qualitative behavior of this transition is robust across a wide range of parameter values.

costs makes savings in banks less attractive. The risk free rate declines in order to accommodate this rise in consumption. Aggregate hours falls because the low returns to savings make working unattractive. The decline in hours leads to a drop in output.

There are three important things to note about this example. First, consumption and output move in opposite directions conditional on a tightening of the leverage constraint of banks. This “comovement problem” arises frequently in neoclassical settings, see [Barro and King \(1984\)](#) for a general formulation and [Hall \(2011\)](#) and [Bigio \(2012\)](#) for specific analysis in models with financial frictions.²³ One way to restore comovement would be to allow the demand for labor to be directly affected by the tightness of bank leverage constraint. This could be done, for example, by introducing working capital constraint as in [Mendoza \(2010\)](#) and using preferences that mute the wealth effect on labor. While this extension is straightforward to pursue in the current set up, I focus on a benchmark real model for comparability with previous research. Second, variation in bank net worth is amplified in the full model because of endogenous response in Tobin’s q . This occurs if there are frictions in the production of capital goods, $\xi > 0$. [Brunnermeier et al. \(2013\)](#) provide a detailed discussion of these amplification effects in models with financial frictions. Third, the tightening of the leverage constraint induces negative comovement between bankers’ marginal value of wealth $\alpha = \frac{1}{1-\mu}$, and realized return on holding capital. When the constraint tightens, the former increases while the latter declines. This is intuitive: an additional unit of wealth for bankers is more valuable when the constraints are tight because it allows them to arbitrage away part of the difference between $\mathbb{E}[R'_K]$ and R . Moreover, the decline in credit

²³See also [Jaimovich and Rebelo \(2009\)](#), ? and ? for a discussion of related comovement problems in different environments.

to firms leads to a reduction in output per unit of capital, which translates into lower firms' profits. As we will see in the subsequent analysis, this negative comovement between R_K and α is the key mechanism that generates endogenous risk in the model.

An increase in the probability of a future sovereign default in the model of Section 3.3 triggers a decline in bank net worth and this may induce their leverage constraints to bind. An increase in s , in fact, leads to a decline in the market value of government bonds because investors anticipate a future haircut. Thus, current realized returns on government bond holdings decline. From equation (1.17) we can see that this effect is stronger the longer the maturity of bonds.²⁴ Low realized returns on bonds have a negative impact on bank net worth as we can see from equation (1.10). The parameters governing the exposure of banks to government bonds determine the quantitative importance of the elasticity of net worth to R_B . Thus, an increase in s can activate the process of Figure 1.1 through its adverse effects on bank net worth. I will refer to this mechanism as the *leverage-constraint* channel.

1.3.2 Bad News about Future Net Worth

Besides affecting the current net worth of financial intermediaries, sovereign credit risk acts as a bad news regarding their *future* wealth. As I will show in this section, this carries important consequences on the way banks discount risky assets. It is helpful at this stage to derive an equilibrium relation describing the pricing of assets in the economy of Section 3.3. From equation (1.8) and (1.5) we find that

²⁴The parameter π also has an indirect effect on the elasticity of R_B to the s -shock: when the maturity is longer, bond prices are more elastic to the sovereign risk shock.

expected returns to asset j equal

$$\mathbb{E}_{\mathbf{S}}[R_j(\mathbf{S}', \mathbf{S})] = R(\mathbf{S}) \left[1 + \frac{\lambda\mu(\mathbf{S})}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})]} \right] - \frac{R(\mathbf{S})\text{cov}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S}), R_j(\mathbf{S}', \mathbf{S})]}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})]}. \quad (1.25)$$

Equation (1.25) defines the cross-section of assets' returns. Expected returns to capital typically carry a risk premium represented by $\text{cov}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S}), R_K(\mathbf{S}', \mathbf{S})]$. In the model, this component is sensitive to news about how tight bank leverage constraints will be in the future.

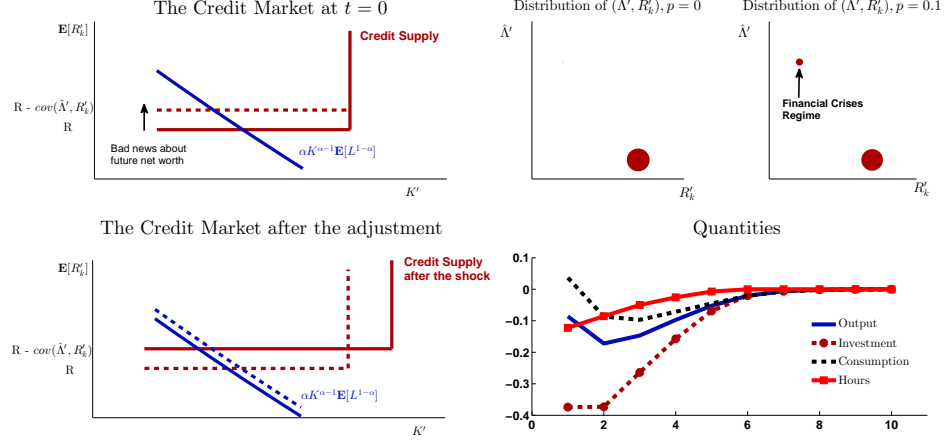
This can be illustrated with a simple modification of the previous set-up. I now allow ψ to be greater than 0.²⁵ Moreover, I assume that there are two regimes in the economy.

- i) “Normal times”: transfers are fixed at their steady state, and bank leverage constraints are not binding.
- ii) “Financial crises”: bankers are hit by the transitory decline in transfers described in the previous section.

I assume that the economy is currently in the normal time regime, and I denote by p the probability that in the next period it switches to a financial crisis regime. Once in a financial crisis, the economy experiences the temporary decline in ω described in the previous section. I assume that $p = 0$ at $t = 0$. In period $t = 1$, the economy experiences an unexpected increase in p to 0.1. In period $t = 2$, p returns to 0 and no further changes are anticipated. The agents are surprised by the initial increase in p , but they are aware of its future path from $t = 1$ onward, and they make rational choices based on this path. Figure 1.2 describes how the credit market and equilibrium quantities are affected by this increase in p .

²⁵The decision problem of bankers is static when $\psi = 0$.

Figure 1.2: *Bad News* about Future Net Worth



Notes: The figure reports the transitional dynamics induced by a transitory and unexpected increase in p from 0 to 0.10. The parametrization adopted is $[\alpha = 0.33, \nu = \infty, \beta = 0.995, \lambda = 0.44, \omega = 0.10, \psi = 0.95]$. The bottom right panel reports variables expressed as percentage deviations from their steady state.

The increase in p shifts the elastic component of the credit supply schedule upward because of a decline in $\text{cov}(\hat{\Lambda}', R'_k)$. The top right panels of the figure explain where this change in the covariance originates. The first panel plots the joint distribution for $(\hat{\Lambda}', R'_k)$ conditional on being in normal times when $p = 0$. This is a point distribution: the pricing kernel equals β while realized returns to capital are equal to β^{-1} . When p increases to 0.1, banks assign a higher probability of switching to the financial crises regime. As shown in the previous section, realized returns to capital are low in this state while bankers' marginal valuation of wealth is high. Capital is therefore a “bad” asset to hold during a financial crisis because it pays little precisely when bankers are most in need of wealth. For this reason, it commands a risk premium in normal times, and these premia are typically increasing in p .²⁶

A sovereign default in the model of Section 3.3 resembles the financial crisis

²⁶In this example this is true only when $p < 0.5$.

regime discussed here: banks suffer large balance sheet losses because of the haircut imposed by the government. Claims on firms pay off badly in this state because of low firm profits and the decline in their market value. These low payouts are highly discounted by banks because they are already facing large balance sheet losses, and their marginal valuation of wealth is high. When the likelihood of this event increases, banks have a precautionary incentive to deleverage because the economy is approaching a state where firms' claims are not particularly valuable, and this deleveraging results in a decline in capital accumulation. I will refer to this second mechanism through which sovereign credit risk propagates to the real economy as the *risk channel*.

1.3.3 Policy Relevance

While these two propagation mechanisms have similar implications for quantities and prices, they carry substantially different information. This can be seen by comparing the credit markets in Figure 1.1 and Figure 1.2. In Figure 1.1, excess returns over firms' claims arise because the constraints on bank leverage prevent profitable investment opportunities: if banks had an additional unit of wealth, they would invest it in firms' claims. In Figure 1.2, instead, excess returns reflect fair compensation for increased risk: bank leverage constraints are not binding, and there are no unexploited profitable opportunities.

This distinction has important implications for the evaluation of credit policies in the model. For example, it is reasonable to expect that an injection of equity to the banking sector may be more effective in stimulating banks' lending when these latter are facing tight constraints on their leverage, while their aggregate implications may be muted when risk premia are high. We will see in Section

1.5 that this intuition holds in the model. First though, I move to the empirical analysis.

1.4 Empirical Analysis

The model is estimated using Italian quarterly data (1999:Q1-2011Q4). This section proceeds in three steps. Section 1.4.1 describes the data used in estimation and discusses how they help identifying the mechanisms of interest. Section 1.4.2 illustrates the estimation strategy. I place a prior on parameters and conduct Bayesian inference. Because of the high computational costs involved in solving the model repeatedly, I adopt a two-step procedure. In the first step, I estimate a version of the model without sovereign default risk on the 1999:Q1-2009:Q4 subsample. In the second step, I estimate the parameters for the $\{s_t\}$ shock using a time series for sovereign default probabilities for the Italian economy. Section 1.4.3 presents an assessment of model fit based on posterior predictive checks for a set of sample moments computed from the data.

1.4.1 Data

As discussed in the previous section, the transmission of sovereign risk to the real economy is the result of two key mechanisms. Their strength in the model is governed by three “parameters”: i) the elasticity of government bond returns to sovereign risk; ii) the elasticity of bank net worth to variation in realized returns on government bonds; iii) the macroeconomic implications of tighter leverage constraints for banks. The selection of the data aims to making the model consistent with three sets of facts that can empirically inform these aspects of the model.

First, I ensure that the time-varying nature of sovereign risk in the model is realistic. Indeed, the behavior of government bonds' prices in response to sovereign risk is partly determined by how persistent agents perceive these changes to be. For this purpose, I use credit default swaps (CDS) spreads on Italian government securities with a five-year maturity. This time series is available at daily frequencies starting in January 2003 from Markit. See Appendix [A.3.1](#) for further details.

Second, I measure the exposure of banks to this risk. I collect data on the exposure of the five largest Italian banks to domestic government debt obtained from the 2011 European Banking Authority stress test.²⁷ As detailed in Appendix [A.3.2](#), these data include holdings of domestic government securities, loans to central government and local authorities and other provisions, and these items are classified in terms of their maturity. I match this information with the end of 2010 consolidated balance sheet data obtained from Bankscope. This allows me to measure the size of the exposure of these five banks to the Italian government in terms of their total assets.²⁸

Third, I measure the cyclical behavior of the leverage constraint. The agency frictions studied in this paper are fairly abstract at this level of aggregation and they have poorly measured empirical counterparts. For this reason, I use the model's restrictions to relate the tightness of banks' leverage constraint to a set of observable variables. Result [2](#) in Appendix [A.3.3](#) shows that the Lagrange multiplier on the leverage constraint of banks can be expressed as a function of financial leverage (lev_t) and of the spread between a risk free security (R_t^f) that is

²⁷The five banks are: Unicredit, Intesa-San Paolo, MPS, BPI and UBI. Their total assets at the end of 2010 accounted for 82% of the total assets of domestic banking groups in Italy.

²⁸These data do not correct for the possibility that banks insured part of this debt via CDS. [Acharya and Steffen \(2013\)](#) impute the exposure of major banks to distressed sovereigns in the euro-area, finding that insurance via CDS is likely to be small.

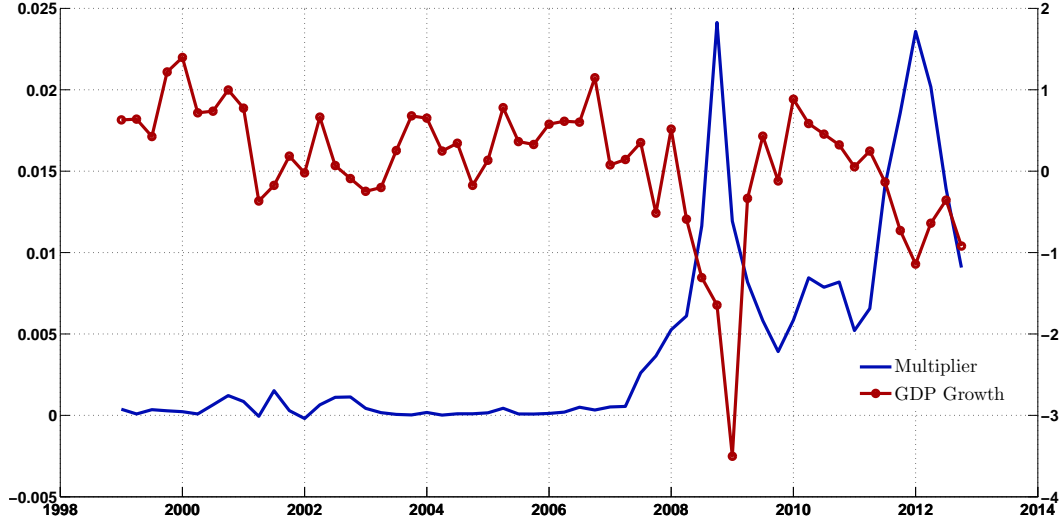
traded only by bankers and the risk free rate (R_t)

$$\mu_t = \frac{\left[\frac{R_t^f - R_t}{R_t} \right] \text{lev}_t}{1 + \left[\frac{R_t^f - R_t}{R_t} \right] \text{lev}_t}. \quad (1.26)$$

I use equation (1.26) to generate a time series for the multiplier μ_t . I measure R_t^f with the prime rate on interbank loans (EURIBOR). This is the natural rate to consider because we can interpret the model from Section 3.3 as having a frictionless interbank market of the type considered in Gertler and Kiyotaki (2010). The risk free rate R_t is matched with the yields on German government securities. The leverage of financial intermediaries is measured using the Italian flow of funds. Appendix A.3.3 describes in detail the steps involved in measuring μ_t . Figure 1.3 reports this time series along with GDP growth. Two main facts stand out from a visual inspection of the figure. First, the Lagrange multiplier is countercyclical, rising substantially in periods in which GDP growth is markedly below average. Second, it is very close to 0 until 2007:Q2. Thus, the constraints seem to bind only occasionally in our sample.

While these three sets of facts are important to identify the effects of interest, they are not informative for all model parameters. Thus, I complement this information with time series for the labor income share, the investment-output ratio, the government spending-output ratio and hours worked. Appendix A.3.4 provides detailed definitions and data sources.

Figure 1.3: Lagrange Multiplier on Leverage Constraint and GDP Growth: 1999:Q1-2012:Q4



Notes: The Lagrange multiplier on banks' leverage constraint is the solid line (left axis). The circled line is GDP growth (right axis). Appendix A.3 provides detailed information on data sources.

1.4.2 Estimation Strategy

I denote by $\theta \in \Theta$ the vector of model parameters. It is convenient to organize the discussion around the following partition, $\theta = [\theta_1, \theta_2]$

$$\theta_1 = \left[\mu^{bg}, \psi, \xi, \sigma_z, \rho_z, \gamma, \pi, g^*, \rho_g, \sigma_g, \gamma_t, \nu, \alpha, \frac{i^{bg}}{y^{bg}}, l^{bg}, lev^{bg}, R^{bg}, exp^{bg}, q_b^{bg}, adj^{bg} \right], \quad \theta_2 = [D, s^*, \rho_s, \sigma_s].$$

Conceptually, we can think of θ_1 as indexing a restricted version of the model without sovereign risk, while θ_2 collects the parameters determining the sovereign default process. I have reparametrized $[\lambda, \omega, \delta, \chi, \iota, \tau^*, a_1, a_2]$ with balanced growth values for, respectively, the Lagrange multiplier on leverage constraints (μ^{bg}), the leverage ratio (lev^{bg}), the investment-output ratio ($\frac{i^{bg}}{y^{bg}}$), worked hours (l^{bg}), the price of government securities (q_b^{bg}), the ratio of government securities held by bankers to their total assets (exp^{bg}) and the size of capital adjustment costs (adj^{bg}).

While a nonlinear analysis of the model is necessary to capture time variation

in risk premia and the fact that leverage constraints bind only occasionally, it complicates inference substantially since repeated numerical solutions of the model are computationally costly. I therefore estimate θ using a two-step procedure. In the first step, I infer θ_1 by estimating the model without sovereign risk on the 1999:Q1-2009:Q4 subsample using Bayesian methods. This restricted version of the model has fewer state variables and is easier to analyze numerically. Moreover, focusing on this restricted model should not substantially alter the inference over θ_1 because i) the 1999:Q1-2009:Q4 period was characterized by low sovereign risk for the Italian economy; and ii) the decision rules of the restricted model closely approximate those of the full model in this area of the state space. In the second step, I estimate θ_2 using a retrieved time series of sovereign default probabilities.

Estimating the Model without Sovereign Risk

The model without sovereign risk has five state variables $\mathbf{S}_t = [\hat{K}_t, \hat{P}_t, \hat{B}_t, \Delta z_t, g_t]$.

The parameters are

$$\theta_1 = \left[\underbrace{\mu^{bg}, \psi, \xi, \sigma_z, \rho_z, \gamma, \pi, g^*, \rho_g, \sigma_g, \gamma_t, \nu, \alpha, \frac{i^{bg}}{y^{bg}}}_{\tilde{\theta}_1}, \underbrace{l^{bg}, lev^{bg}, R^{bg}, exp^{bg}, q_b^{bg}, adj^{bg}}_{\theta_1^*} \right].$$

I construct the likelihood function of the model using time series for GDP growth and the Lagrange multiplier on banks' leverage constraint described earlier. As explained in the earlier section, the cyclical behavior of the model's financial friction is key to assess the impact of sovereign risk on the real economy: a likelihood-based approach guarantees a high degree of consistency between the model implied behavior for these variables and their data counterparts.

This choice has limitations. First, I am discarding potentially important infor-

mation as one could incorporate the components of the multiplier into the likelihood function: the risk free rate, the interbank rate and the leverage of banks. I verified though that the model is too restrictive to track the time series behavior of financial leverage in the earlier part of the sample, because structural shocks do not generate enough variation in asset prices when leverage constraints are far from binding.²⁹ Second, certain model parameters are only weakly affected by the information in the likelihood and their identification is problematic. For this reason, and prior to conduct full information inference, I determine a subset of θ_1 , θ_1^* , prior to the estimation using external information. Table 1.1 reports the numerical values for these parameters. I set $[\frac{i^{bg}}{y^{bg}}, lev^{bg}, l^{bg}, R^{bg}]$ to the sample average of their empirical counterparts while α is determined using the sample average of the labor income share. I use the information in Table A.1 in Appendix A.3.2 to determine $[exp^{bg}, \pi]$: holdings of government securities account for 8% of banks' total assets in the model, and the average maturity of those bonds is set to 23 months. I select $[g^*, \rho_g, \sigma_g]$ from the estimation of an AR(1) on the spending-output ratio over the 1999:Q1-2011:Q4 period. The remaining parameters in θ^* are determined through normalizations or previous research. I set the Frisch elasticity of labor supply to 2 and γ_τ to 0.5. The former is in the high range of the estimates obtained using U.S. data (Rios-Rull et al., 2012a), but it is not an uncommon value in the profession for the analysis of Real Business Cycle models. Since taxes are non-distortionary in the model, γ_τ has little implications for the model's endogenous variables other than debt. I set adjustment costs to zero in a balanced growth path while I normalize q_b^{bg} to 1 (bonds trade at par in a balanced growth path).

²⁹This aspect is related to one shortcoming of pure neoclassical models, namely their inability to generate volatility in asset prices. See for example Bocola and Gornemann (2013) for a discussion.

Table 1.1: **Parameters Determined with External Information**

Parameters					Source
$\frac{i^{bg}}{y^{bg}}$	lev^{bg}	l^{bg}	R^{bg}	α	OECD, EU-KLEMS, ECB, BoI
0.25	4.34	0.30	1.0034	0.30	
exp^{bg}	π				EBA, Bankscope
0.079	0.044				
e^g	ρ_g	σ_g			OECD
0.22	0.92	0.010			
q_b^{bg}	adj^{bg}	ν	γ_τ		Normalizations, Previous Research
1	0	2	0.5		

Notes: See Appendix A.3 for information on data sources.

I next turn to the estimation of $\tilde{\theta}_1 = [\mu^{bg}, \psi, \xi, \gamma, \rho_z, \sigma_z]$. Let $\mathbf{Y}_t = [\text{GDP Growth}_t, \mu_t]'$, and let $\mathbf{Y}^t = [\mathbf{Y}_1, \dots, \mathbf{Y}_t]'$. The model defines the nonlinear state space system

$$\begin{aligned} \mathbf{Y}_t &= f_{\tilde{\theta}_1}(\mathbf{S}_t) + \eta_t & \eta_t &\sim \mathcal{N}(\mathbf{0}, \Sigma) \\ \mathbf{S}_t &= g_{\tilde{\theta}_1}(\mathbf{S}_{t-1}, \varepsilon_t) & \varepsilon_t &\sim \mathcal{N}(\mathbf{0}, \mathbf{I}), \end{aligned}$$

where η_t is a vector of measurement errors and ε_t are the structural shocks.³⁰ Measurement errors, absent from the structural model, are included to help the evaluation of the likelihood function. I approximate the likelihood function of this nonlinear state space model using sequential importance sampling (Fernández-Villaverde and Rubio-Ramírez, 2007a).³¹ The posterior distribution of model parameters is

$$p(\tilde{\theta}_1 | \mathbf{Y}^T) = \frac{\mathcal{L}(\tilde{\theta}_1 | \mathbf{Y}^T) p(\tilde{\theta}_1)}{p(\mathbf{Y}^T)},$$

³⁰The functions $g_{\tilde{\theta}_1}(\cdot)$ and $f_{\tilde{\theta}_1}(\cdot)$ are approximated following the steps described in Appendix A.2 for a version of the model that does not feature sovereign credit risk. Since θ_1^* is fixed, I omit from the notation the dependence of decision rules on these parameters.

³¹I use the auxiliary particle filter of Pitt and Shephard (1999) which, in this application, substantially improves the efficiency of the likelihood evaluation. See Aruoba and Schorfheide (2013a) for a recent application to economics. I consider a diagonal matrix Σ where the nonzero elements are equal to 25% of the sample variance of $\{\mathbf{Y}_t\}$. Appendix A.4 provides a description of the evaluation of the model's likelihood function.

where $p(\tilde{\theta}_1)$ is the prior, $\mathcal{L}(\tilde{\theta}_1|\mathbf{Y}^T)$ the likelihood function and $p(\mathbf{Y}^T)$ the marginal data density. I characterize the posterior density of $\tilde{\theta}_1$ using the Random Walk Metropolis Hastings for DSGE models developed in [Schorfheide \(2000a\)](#) with an adaptive variance-covariance matrix for the proposal density. [Appendix A.4](#) provides a description of the estimation algorithm. [Table 1.2](#) reports the prior along with posterior statistics for $\tilde{\theta}_1$.

Table 1.2: **Prior and Posterior Distribution of $\tilde{\theta}_1$**

Parameter	Prior	Para 1	Para 2	Posterior Mean	90% Credible Set
$\mu^{bg} \times 100$	Uniform	0	∞	0.18	[0.13,0.21]
ψ	Uniform	0	1	0.97	[0.95,0.98]
ξ	Beta	0.5	0.25	0.42	[0.35,0.50]
$\gamma \times 400$	Normal	1.25	0.5	0.36	[0.06,0.75]
ρ_z	Beta	0.3	0.25	0.08	[0.04,0.14]
$\sigma_z \times 100$	Inverse Gamma	0.75	2	0.94	[0.84,1.06]

Notes: Para 1 and Para 2 list the mean and standard deviation for Beta and Normal distribution; and s and ν for the Inverse Gamma distribution, where $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The prior on γ is truncated at 0. Posterior statistics are computed using 10000 draws from the posterior distribution of model's parameters. The table reports equal tail probability 90% credible sets.

The prior on the TFP process is centered using presample evidence while I center ξ to 0.5, a conventional value in the literature. Priors on these three parameters are fairly diffuse. I choose uniform priors over μ^{bg} and ψ , implying that the shape of the posterior is determined by the shape of the likelihood. Regarding posterior estimates, the multiplier is estimated to be close to 0 in a deterministic balanced growth path while ψ is close to unity. This suggests that agency costs are fairly small on average in the model. This is not surprising given the time series behavior of μ_t in [Figure 1.3](#).³² Capital adjustment costs and the TFP process are in the range of what is typically obtained in the literature when using U.S. data.

³²One way of assessing the size of financial friction in the model is to ask how large the distortion is that they generate on returns to capital. The posterior mean of μ^{bg} tells us that this distortion is approximately equal to 18 basis points in a balanced growth path of the model.

Estimating Sovereign Risk

I next turn to the estimation of $\theta_2 = [D, s^*, \rho_s, \sigma_s]$. The empirical strategy consists of i) constructing a time series for the probabilities of a sovereign default and ii) using this time series to estimate θ_2 .

I accomplish the first task by exploiting the model's pricing equation. In fact, using equation (1.8) and equation (1.5), we can define the *risk neutral* measure as.³³

$$\hat{p}(\mathbf{S}'|\mathbf{S}) = \frac{R^f(\mathbf{S})p(\mathbf{S}'|\mathbf{S})\hat{\Lambda}(\mathbf{S}', \mathbf{S})}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})] + \lambda\mu(\mathbf{S})}.$$

After integrating the above expression over states \mathbf{S}' associated with a sovereign default next period, I obtain an expression for the actual probability of a sovereign default, p_t^d . This time series is related to its *risk neutral* counterpart, \hat{p}_t^d , as follows

$$p_t^d = \hat{p}_t^d \frac{\alpha_t(1 - \mu_t) + \lambda\mu_t}{R_t^f \mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]}. \quad (1.27)$$

Because of bankers' risk aversion, there is a wedge between these two probabilities, represented by the risk correction $\frac{\alpha_t(1-\mu_t)+\lambda\mu_t}{R_t^f \mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1}=1]}$. Equation (1.27) is important because it allows us to measure actual probabilities of sovereign default using empirical counterparts to risk neutral probabilities and the risk correction.

First, I obtain a time series for $\{\hat{p}_t^d\}$ using CDS spread on Italian government securities, up to a normalization of the haircut parameter D . I fix D to 0.45, consistent with the historical experience on recent sovereign defaults in emerging economies (Cruces and Trebesh, 2013).³⁴ While Pan and Singleton (2008) show

³³Note that $\hat{p}(\mathbf{S}'|\mathbf{S})$ is nonnegative and it integrates to 1. To see the last property, note that the return on a risk free security traded by bankers can be written as $R^f(\mathbf{S}) = \frac{\alpha(\mathbf{S})[1-\mu(\mathbf{S})]+\lambda\mu(\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})]}$ using equation (1.8) and equation (1.5).

³⁴Zettelmeyer et al. (2013) document a larger haircut (on average between 59% and 65%) in the Greek debt restructuring event of 2012. A value of D equal to 0.45 is conservative, and

that D could be estimated using information from the term structure of sovereign CDS spreads, their Monte Carlo analysis suggests that this parameter is typically poorly identified in small samples.

Second, I construct a time series for $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$, the conditional expectation of the pricing kernel in the event of a sovereign default. This is a difficult task because of the absence of a sovereign default in the sample. I indirectly use the model's restrictions to conduct this extrapolation. In particular, I approximate the object of interest as follows

$$\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1] \approx \mathbb{E}_t[\hat{\Lambda}_{t+1}] + \kappa \text{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}, \quad (1.28)$$

where $\kappa > 0$ is a hyperparameter. The idea underlying equation (D.10) is that the pricing kernel in the model is above its unconditional average in the event of a sovereign default because of banks' implicit risk aversion: κ parametrizes the number of standard deviations by which $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$ is above $\mathbb{E}_t[\hat{\Lambda}_{t+1}]$.

The terms $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}], \text{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}$ are generated using an empirical counterpart to the model's pricing kernel defined in equation (1.9). The pricing kernel, in turn, is a function of observables and model parameters estimated in the first step

$$\hat{\Lambda}_t = \beta e^{-\Delta \log(c_t)} [(1 - \psi) + \psi \lambda \text{lev}_t], \quad (1.29)$$

where Δc_t is consumption growth and lev_t is financial leverage. I use equation (1.29), the posterior mean for $[\beta, \psi, \lambda]$ and a time series for the conditional forecasts of $[\Delta c_{t+1}, \text{lev}_{t+1}]$ generated by a first order Bayesian Vector Autoregressive model to construct $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}], \text{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}$. I then select the hyperparameter κ with the

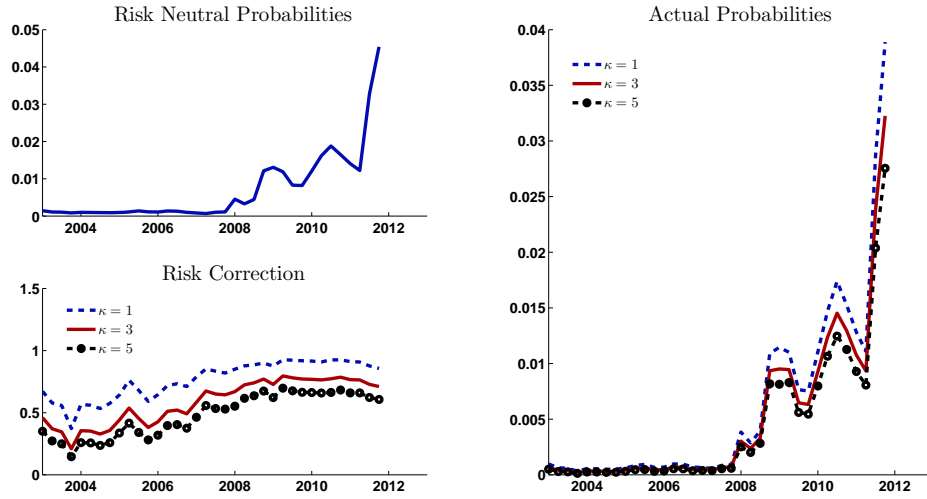
corrects for potential transfers that the government may give to its domestic bondholders.

help of the structural model. I consider a set of values $\kappa^i \in \{1, 3, 5\}$ and select the value that minimizes, in model simulated data, average root mean square errors for the approximation of $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$. This gives a value of $\kappa = 3$.

Third, I combine the retrieved time series for $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]\}$ with observations on banks' financial leverage, the multiplier and the prime interbank rate to generate the risk correction $\frac{\alpha_t(1-\mu_t)+\lambda\mu_t}{R_t^f \mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1}=1]}$. I make use of the fact that the marginal value of wealth for bankers is proportional to financial leverage when the constraint binds, and measure the risk correction as follows:

$$\frac{\lambda \text{lev}_t(1 - \mu_t) + \lambda \mu_t}{R_t^f \{\mathbb{E}_t[\hat{\Lambda}_{t+1}] + \kappa \text{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}}. \quad (1.30)$$

Figure 1.4: **Sovereign Default Probabilities**



Notes: The top left panel reports risk neutral probabilities of a sovereign default. The bottom right panel reports the risk correction, defined in equation (1.30). The right panel reports actual probabilities of a sovereign default, defined in equation (1.27).

Figure 1.4 plots $\{p_t^d\}$ along with its decomposition of equation (1.27) for the different values of κ . The top left panel reports the risk neutral probabilities, the bottom-left panel plots the risk correction and the right panel reports the time series for actual sovereign default probabilities. The estimates imply that roughly

30% of actual sovereign default probabilities in the sample is due to risk premia, consistent with the empirical evidence reported in Longstaff et al. (2011) for a group of developing countries.

I then use $\{p_t^d\}$ to estimate the parameters of the sovereign risk shock s_t . Indeed, the two are related in the model as follows

$$\log\left(\frac{p_t^d}{1-p_t^d}\right) = s_t, \quad s_t = (1-\rho_s)s^* + \rho_s s_{t-1} + \sigma_s \varepsilon_{s,t}, \quad (1.31)$$

where $\varepsilon_{s,t}$ is a standard normal random variable. I use the Kalman filter to evaluate the likelihood function of this linear state space model. Table 1.3 reports prior and posterior statistics for $[s^*, \rho_s, \sigma_s]$. As I do not have presample information, I consider fairly uninformative priors. Posterior statistics are computed from a canonical Random Walk Metropolis Hastings algorithm.

Table 1.3: **Prior and Posterior Distribution of $[s^*, \rho_s, \sigma_s]$**

Parameter	Prior	Para 1	Para 2	Posterior Mean	90% Credible Set
s^*	Normal	-7	5	-6.17	[-8.88,-3.35]
ρ_s	Beta	0.5	0.3	0.95	[0.87,0.98]
σ_s	Inverse Gamma	0.75	4	0.55	[0.44,0.70]

Notes: Para 1 and Para 2 list the mean and standard deviation for Beta and Normal distribution; and s and ν for the Inverse Gamma distribution, where $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. Posterior statistics are computed using 10000 draws from the posterior distribution of model's parameters. The table reports equal tail probability 90% credible sets.

1.4.3 Model Fit

In order to determine if the estimated model fits the time series described in the previous section, I verify whether model simulated trajectories for the multiplier, GDP growth and sovereign default probabilities resemble those observed in the

data. This is accomplished through posterior predictive checks.³⁵ I generate model implied densities for sample statistics and check how they compare with the same statistics computed from actual data.

First, I examine the performance of the model regarding GDP growth and the multiplier. I summarize their joint behavior using the following sample statistics: mean, standard deviation, first order autocorrelation, skewness, kurtosis and their correlation. These are collected in \mathcal{S} . The model implied densities for \mathcal{S} are generated using the following algorithm

Posterior Predictive Densities: Let θ^i denote the i 'th draw from the posterior density of the model's parameter. For $i = 1$ to M

- i) Conditional on θ^i simulate a realization for GDP growth and the multiplier of length $T=100$.³⁶ Let $\{\mathbf{Y}_t^i\}$ denote this realization.
- ii) Based on the simulated trajectories $\{\mathbf{Y}_t^i\}$, compute a set of sample statistics \mathcal{S}^i . \square

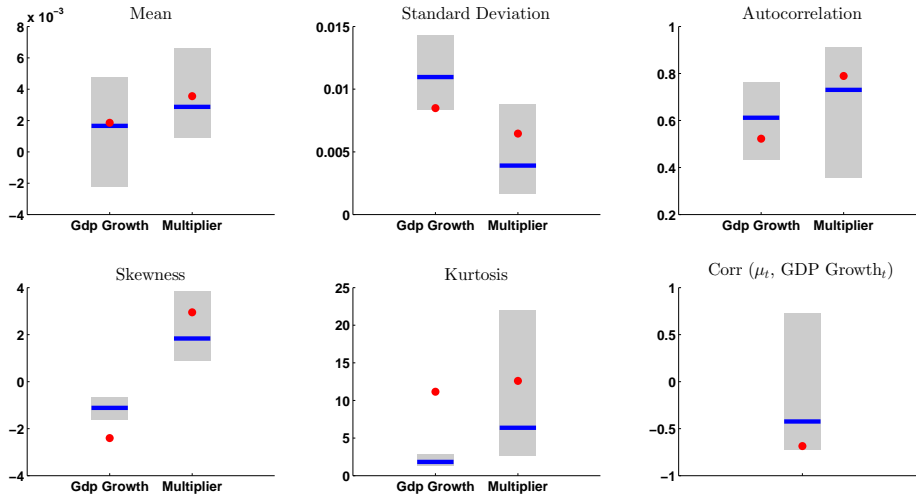
Given the draws $\{\mathcal{S}^i\}$, I use percentiles to describe the predictive density $p(\mathcal{S}(\cdot)|\mathbf{Y}^T)$. Figure 1.5 shows the 5th and 95th percentile of the model implied density (the box) along with its median (the bar) and their sample counterpart (the dot).

The model generates trajectories for the multiplier and GDP growth whose moments are in line with those observed in the data. The main discrepancy with the data is in the excess kurtosis for the GDP growth trajectory: the model is too restrictive to replicate this feature of the data. In addition, the model captures

³⁵See Geweke (2005a) for a general discussion of predictive checks in Bayesian analysis and Aruoba et al. (2013) for a recent application to the evaluation of estimated nonlinear Dynamic Stochastic General Equilibrium models.

³⁶These simulations are generated from the restricted model (no sovereign risk). Simulations are initialized at the ergodic mean of the state vector.

Figure 1.5: **Posterior Predictive Checks: Multiplier and GDP Growth**



Notes: Dots correspond to the value of the statistic computed from actual data. Solid horizontal lines indicate medians of posterior predictive distribution for the sample statistic and, the boxes indicate the equal tail 90% credible set associated with the posterior predictive distribution.

part of the left skewness of GDP growth. This derives from two properties: the amplification of the leverage constraint and the fact that it binds in recessions. In fact, GDP growth is more sensitive to structural shocks when the leverage constraint binds. Since these constraints are only occasionally binding, this amplification generates asymmetry in the unconditional distribution for GDP growth. Left skewness is then the result of GDP growth and the multiplier being negatively correlated. [Guerrieri and Iacoviello \(2013\)](#) discuss the asymmetry generated by occasionally binding credit constraints in a model of housing.

Second, I ask whether the behavior of sovereign default probabilities in the model is in line with what was observed in the data. The posterior predictive checks are reported in [Table 1.4](#). We can verify that the specification adopted to model time-variation in sovereign risk captures key features of the empirical distribution of sovereign default probabilities.

Overall, the results in this section suggest that: i) the cyclical behavior of the

Table 1.4: **Posterior Predictive Checks: Sovereign Default Probabilities**

Statistic	Data	Posterior Median	90% Credible Set
Median	0.07	0.25	[0.01,6.81]
Mean	0.53	0.53	[0.03,11.7]
Standard Deviation	0.76	0.63	[0.03,13.5]
Autocorrelation	0.91	0.83	[0.69,0.94]
Skewness	2.03	2.04	[0.96,3.78]
Kurtosis	7.37	7.23	[3.04,20.1]

Notes: Based on 1000 draws from the posterior distribution of $[s^*, \rho_s, \sigma_s]$. For each draw, I simulate the $\{s_t\}$ process for 100 periods. Statistics are computed on each of these 1000 samples. The table reports the posterior median and equal tail probability 90% credible set for the posterior predictive distributions.

leverage constraint in the estimated model is empirically reasonable; and that ii) agents in the model have beliefs about the time-varying nature of sovereign credit risk that closely track what was observed in the data.

1.5 Model Analysis

This section analyzes some properties of the estimated model that are important for the interpretation of the main experiments of this paper, which will be presented in Section 1.6. There are three key points that emerge from this analysis:

- i) A sovereign default leads to a deep decline in real economic activity. This occurs because the haircut on government bonds tightens the leverage constraints of banks and triggers a decline in aggregate investment (Section 1.5.1).
- ii) An increase in the probability of a sovereign default when the economy is in the non-default state leads to an increase in expected excess returns. This occurs through two mechanisms. First, sovereign credit risk tightens

the leverage constraint of banks (*leverage-constraint channel*). Second, it increases the required premia that banks demand for holding firms' assets (*risk channel*). This increase in the financing premia of firms is associated with a decline in capital accumulation and output (Section 1.5.2).

- iii) The aggregate effects of equity injections into the banking sector are highly state dependent, even if implemented at times of high financial stress.³⁷ These interventions are more successful in stimulating real economic activity in regions of the state space where leverage constraints are tight. Conversely, these policies have substantially weaker effects when risk premia on firms' assets are high (Section 1.5.3).

Since the aim of this section is purely illustrative, the model's parameters are fixed at their posterior mean.

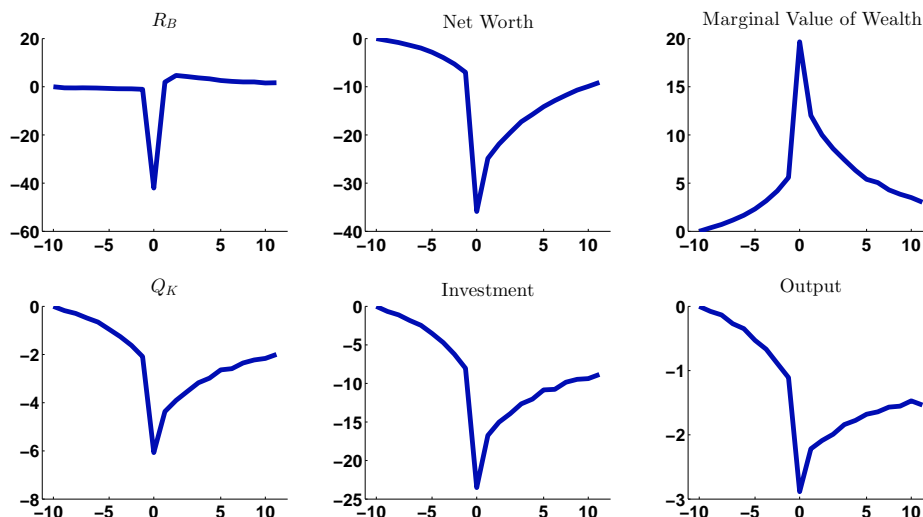
1.5.1 A Sovereign Default

Figure 1.6 shows the behavior of key model's variables around a typical sovereign default. I apply event study techniques to the simulated time series and report their average path around the default. The window covers 10 quarters before and after the event.

At $t = 0$ the government imposes a haircut on bondholders. As a consequence, bank net worth declines and the leverage constraint tightens, thus forcing them to reduce their holdings of firms' assets. This has adverse effects on aggregate investment and output: at $t = 0$, they are respectively 25% and 2.9% below their trend. From $t = 1$ onward, bank net worth recovers because excess returns

³⁷Periods of high financial stress will be defined as periods during which expected excess returns are above a threshold and output growth is below a threshold.

Figure 1.6: A Sovereign Default



Notes: The panels are constructed as follows. Simulate $M = 15000$ realizations of length $T = 300$. Each simulation is initialized at the ergodic mean of the state vector. For each realization, select time series around a sovereign default event. Net worth, output and investment are linearly detrended. The figure reports medians across the simulations. Returns on government bonds are expressed as deviations from their $t = -10$ value in annualized basis points. The other variables are expressed as percentage deviations from their $t = -10$ value.

are above average. This loosens the leverage constraint, and the economy slowly returns to its balanced growth path.

It is important to stress two important facts about a sovereign default in the model. First, the behavior of asset prices substantially amplifies this event (Kiyotaki and Moore, 1997; Mendoza, 2010). The tightening of the leverage constraint forces banks to restrict lending to firms. The associated decline in capital demand puts downward pressure on asset prices because of Tobin's Q and further depresses the net worth of banks. As we can see from the figure, the market value of firms is 6% below trend at $t = 0$, while bank net worth is roughly twice the size of the haircut imposed by the government. Second, as the bottom-right panel of the figure shows, the marginal value of wealth for bankers is high during a sovereign default.

It is also interesting to note that a sovereign default is preceded by a deep slow-down in real economic activity, which conforms with historical evidence on these episodes, see [Yeyati and Panizza \(2011\)](#). This observation is typically rationalized in the literature via a selection argument: equilibrium models of sovereign defaults predict that incentives for the government to renege on debt are high in bad economic times, see [Arellano \(2008\)](#) and [Mendoza and Yue \(2012\)](#) for example. In the model analyzed here, the “V” shape behavior of output around a default event occurs purely because of *anticipation* effects: increases in the probability of a future sovereign default are, in fact, recessionary. The next section explains why.

1.5.2 An Increase in the Probability of a Future Sovereign Default

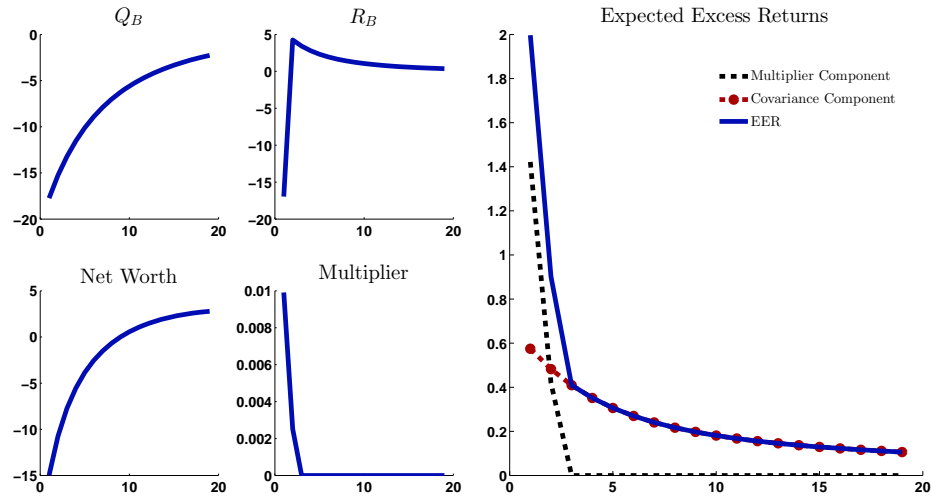
From equation (1.25) we obtain a decomposition of expected excess returns to capital into two pieces: the multiplier component and the covariance component.

$$\underbrace{\frac{\mathbb{E}_t[R_{K,t+1} - R_t]}{R_t}}_{\text{EER}_t} = \underbrace{\frac{\lambda\mu_t}{\alpha_t[1 - \mu_t]}}_{\text{Multiplier component}_t} - \underbrace{\frac{\text{cov}_t[\hat{\Lambda}_{t+1}, R_{K,t+1}]}{\alpha_t[1 - \mu_t]}}_{\text{Covariance component}_t}. \quad (1.32)$$

According to equation (1.32), expected excess returns can be high because of two distinct sources. First, banks face tight leverage constraints and this restricts the flow of funds to firms (Multiplier component). Second, banks require a premium for lending to firm because this intermediation is risky (Covariance component). Sovereign credit risk influences both of these components.

Figure 1.7 plots Impulse Response Functions (IRFs) to an s -shock when the

Figure 1.7: IRFs to an s -shock: Expected Excess Returns

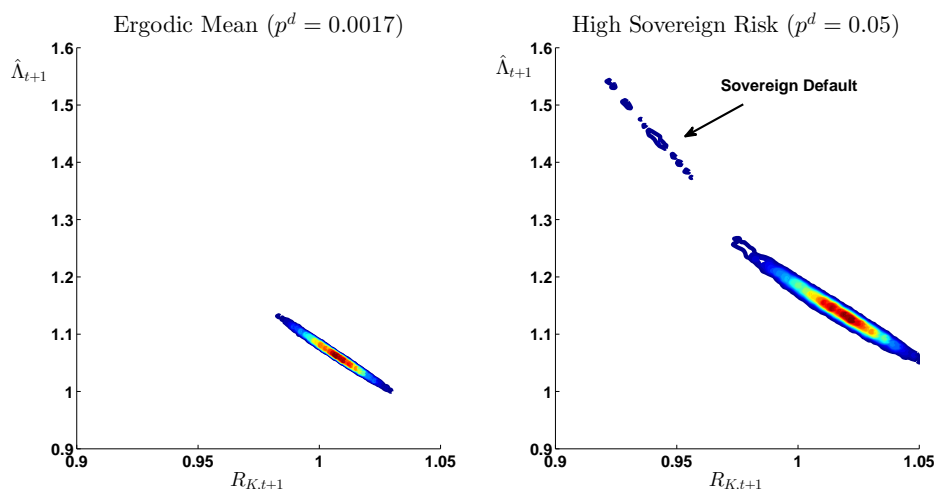


Notes: IRFs are computed via simulations initialized at the ergodic mean of the state vector. Q_B and Net Worth are expressed as percentage deviations from their ergodic mean value. Returns are reported in annualized basis points.

economy is at the ergodic mean. The initial impulse in s is such that the probability of a future sovereign default goes from 0.17% to 5%. This represents roughly a 6 standard deviations shock. The figure shows that this shock tightens the leverage constraint of banks. The price of government bonds declines by 18%, leading to a reduction in their realized returns of roughly the same magnitude. The net worth of banks declines by 15%. Because of this decline in net worth, the leverage constraints of banks start binding, as the behavior of the multiplier shows. Expected excess returns increase by 200 basis points in annualized terms on impact, 140 of which are attributable to the multiplier component. Also the covariance component respond to the s -shock: this risk channel explains 30% of the impact increase in expected excess returns.

Figure 1.8 explains why firms' are perceived to be riskier when a sovereign default approaches. The figure reports the joint probability density function (contour lines) for the next period pricing kernel and realized returns to capital. The

Figure 1.8: The s -shock and Risk Premia

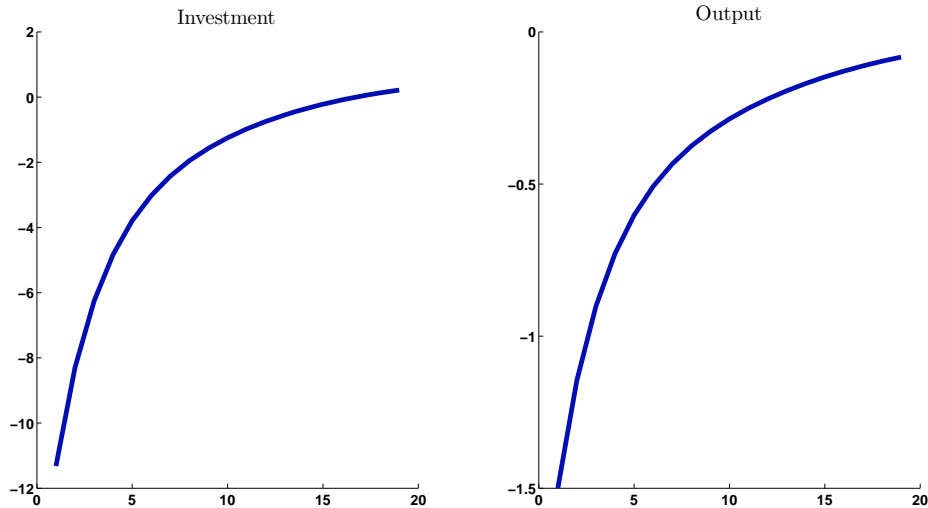


Notes: The left panel reports the joint density of $\{\hat{\Lambda}_{t+1}, R_{K,t+1}\}$ at the ergodic mean. This is constructed as follows. Simulate $M = 15000$ realizations for $\{\hat{\Lambda}, R_K\}$. Each simulation is initialized at the ergodic mean of the state vector and it has length $T = 2$. The contour lines are generated from a nonparametric density smoother applied to $\{\hat{\Lambda}_2^m, R_{K,2}^m\}_{m=1}^M$. The right panel reports the same information, at a different point in the state space. The procedure to construct the figure is the same as above, but the simulations are initialized as follows: i) s -shock is set so that $p_t^d = 0.05$; ii) the other state variable are set at their ergodic mean.

left panel reports it when the state vector is at its ergodic mean. The right panel reports the same object with the only exception that the probability of a sovereign default next period equals 5%. We can see from the left panel of the figure a clear negative association between realized returns to capital and the pricing kernel, suggesting that the model generates a non-trivial compensation for risk at the ergodic mean. As the economy approaches a sovereign default (right panel), these variables become more negatively associated. This motivates an increase in the compensation for holding claims on firms in their balance sheet. Intuitively, capital is a “bad” asset to hold during a sovereign default because the decline in its market value has adverse effects on bank net worth, and these balance sheet losses are very costly since banks’ marginal value of wealth is high. This makes the s -shock a priced risk factor for firms’ claims.

The rise in expected excess returns after an s -shock is associated with a decline in capital accumulation. Figure 1.9 reports the response of aggregate investment and output to the s -shock. The increase in the probability of a sovereign default leads to a decline in output and aggregate investment of, respectively, 1.5% and 12%. The mechanisms through which this happens are those described in Section 1.3.

Figure 1.9: IRFs to an s -shock: Quantities



Notes: IRFs are computed via simulations on linearly detrended data initialized at the ergodic mean of the state vector. The variables are expressed as percentage deviations from their ergodic mean.

1.5.3 An Injection of Equity into the Banking Sector

The distinction between the two propagation mechanisms studied in this paper is key for the assesment of credit policies. I illustrate this point by studying the effects of an equity injection into the banking sector. This type of interventions has been already studied in the literature, see for example [Gertler and Kiyotaki \(2010\)](#) and [He and Krishnamurthy \(2012a\)](#). I assume that at $t = 1$ the government transfers resources from households to banks using lump sum taxes. This policy

is not anticipated by agents, and no further policy interventions are expected in the future. The policy has the effect of changing the liability structure of banks, raising their net worth relative to their debt.

To make the experiment realistic, I implement the policy when the economy is in a “financial recession”. I define this as a state in which output growth is 1.5 standard deviations below average while expected excess returns are 1.5 standard deviations above average. Qualitatively, the results do not depend on these cut-offs. I denote by $\{\mathbf{S}_i^*\}_i$ a set of states variables that is consistent with this definition of financial recession.³⁸ For each element of $\{\mathbf{S}_i^*\}_i$, I compute the expected path for selected endogenous variables under the policy and without the intervention. The policy effects are reported as percentage differences between these two paths. In order to interpret the results, I define

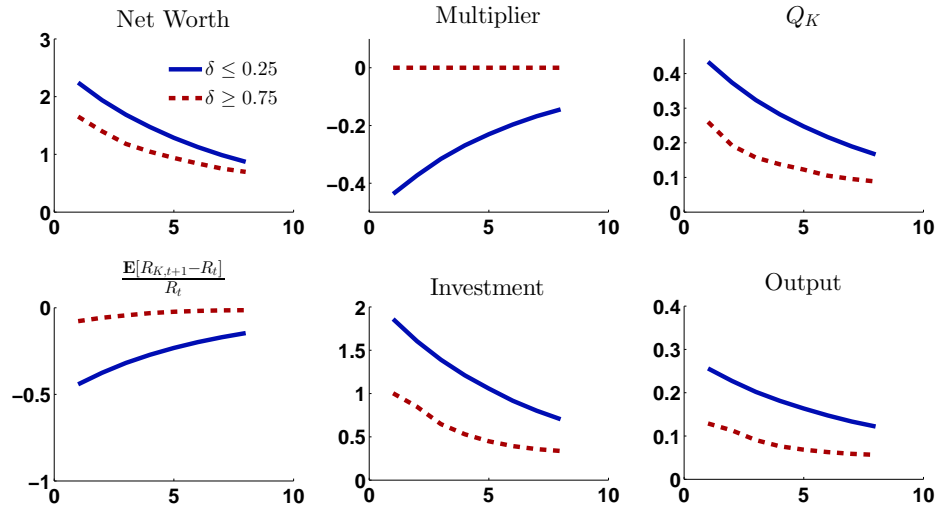
$$\delta_i = \frac{-\text{Covariance component}_i}{EER_i}, \quad (1.33)$$

where the Covariance component and EER are defined in equation (1.32). The variable $\delta_i \in [0, 1]$ gives us an indication of how risky are firms in state \mathbf{S}_i^* . In fact, when $\delta_i = 0$, expected excess returns exclusively reflects agency costs while $\delta_i = 1$ means that they reflect fair compensation for risk. Figure 1.10 plots two sets of results. The solid line reports policy responses conditioning on $\delta_i \leq 0.25$, while the dotted line conditions on $\delta_i \geq 0.75$.

The figure shows that equity injections are particularly effective in stimulating real economic activity when agency costs are large ($\delta \leq 0.25$). The policy relaxes bank leverage constraints and leads to an increase in capital accumulation.

³⁸Operationally, this set is constructed by simulating time series of length $T = 20000$ from the model and selecting $\{\mathbf{S}_i^*\}_i$ so that output growth and expected excess returns satisfy the threshold restrictions.

Figure 1.10: **An Injection of Equity into the Banking Sector**



Notes: The figure is constructed as follows: i) simulate the model for $T = 20000$ periods; ii) select state variables such that output growth is 1.5 standard deviations below average and expected excess returns 1.5 standard deviations above average; iii) for each of these states as initial condition, compute the expected path under the equity injection and in absence of the policy; iv) take the difference between these expected paths. The figure reports the effects of the policy on outcome variables when conditioning on different values of δ , see equation (1.33). Net Worth, Output, and Investment are linearly detrended in simulations. Returns are reported in basis points. The other variables as percentage changes.

This effect is reinforced by general equilibrium forces since the increase in capital demand pushes up the market value of firms, strengthening the balance sheet of banks and relaxing further their leverage constraint.

The same policy has substantially weaker effects in regions of the state space where firms' risk is high ($\delta \geq 0.75$). The red dotted line reports this case. We can observe that the response of investment and output to the equity injection is 2.5 times smaller with respect to the previous case. Moreover, the general equilibrium effects are substantially muted.

This state-dependence in the effects of equity injections has an intuitive explanation. Expected excess returns in the model can be high because of two reasons: tight leverage constraints (low δ -regions) and high risk premia (high δ -regions).

Leverage constraints prevents banks from undertaking otherwise profitable investment opportunities. Therefore, policies that relax their constraints stimulate investment because they facilitate the flow of funds from households to firms. A large value of δ , instead, indicates that the high excess returns we observe are fair compensation for risk: aggregate investment respond little to equity injections since these latter have only indirect effects on this risk.³⁹

1.6 Measurement and Policy Evaluation

I now turn to the two main quantitative experiments of this paper. In Section 1.6.1, I measure the effect of sovereign credit risk on the financing premia of firms and output, and I assess the contribution of the leverage-constraint channel and the risk channel. More specifically, I use the estimated model along with the particle filter to generate trajectories for variables of interest under the assumption that sovereign default probabilities in Italy were constraint over the sample. I then study the difference between these counterfactual trajectories and the actual trajectories in order to assess the impact of sovereign credit risk on the variables of interest. Section 1.6.2 proposes a quantitative assessment of the Longer Term Refinancing Operations (LTROs) implemented by the European Central Bank (ECB) in the first quarter of 2012. As we saw in the earlier section, the effects of policy interventions are state and size dependent due to the highly nonlinear nature of the model. Therefore, an integral part of the policy evaluation is to specify the “initial conditions”. I do so by estimating the state of the Italian economy in

³⁹By strengthening bank net worth, the policy provides a buffer when a sovereign default hits the economy. This dampens the effects of a sovereign default on realized returns to capital and lowers risk premia ex-ante. The size of the equity injection is thus an important determinant of the policy effects.

2011:Q4 using the particle filter. The evaluation of LTROs is conducted from an *ex-ante* perspective.

1.6.1 Sovereign Risk, Firms' Borrowing Costs and Output

What were the effects of sovereign credit risk on the financing premia of firms and on real economic activity in Italy? What was the relative strength of the leverage-constraint channel and the risk channel in driving this propagation? In order to answer these questions, I conduct a counterfactual experiment. First, I use the particle filter to extract the historical sequence of shocks for the Italian economy. Second, I feed the model with counterfactual trajectories for these shocks: these are equivalent to the estimated ones, with the exception that the innovations to s_t are set to 0 for the entire sample. I then compare the actual and counterfactual path for a set of the model's endogenous variables. Their difference reflects the effects of sovereign risk on the variables of interest. More specifically, I use the following algorithm

Counterfactual Experiment: Let θ^i denote the i 'th draw from the posterior distribution of the model's parameter. For $i = 1$ to M

- i) Conditional on θ^i , apply to $\{\mathbf{Y}_t = [\text{GDP Growth}_t, \mu_t, \pi_t^d]\}_{t=2003:Q1}^{2011:Q4}$ the particle filter and construct the densities $\{p(\mathbf{S}_t|\mathbf{Y}_t, \theta^i)\}_{t=2003:Q1}^{2011:Q4}$.
- ii) Sample N realizations of the state vector from $\{p(\mathbf{S}_t|\mathbf{Y}_t, \theta^i)\}_{t=2003:Q1}^{2011:Q4}$.
- iii) Feed into the model each of these realizations, $n \in N$, and generate a path for a set of outcome variables, $\{\mathbf{x}_t(i, n)\}_t$.
- iv) For each realization n , replace the sovereign risk shock with its unconditional

mean. Feed the model with this counterfactual realization of the state vector and collect in $\{\mathbf{x}_t^c(i, n)\}_t$ the implied outcome variables of interest.

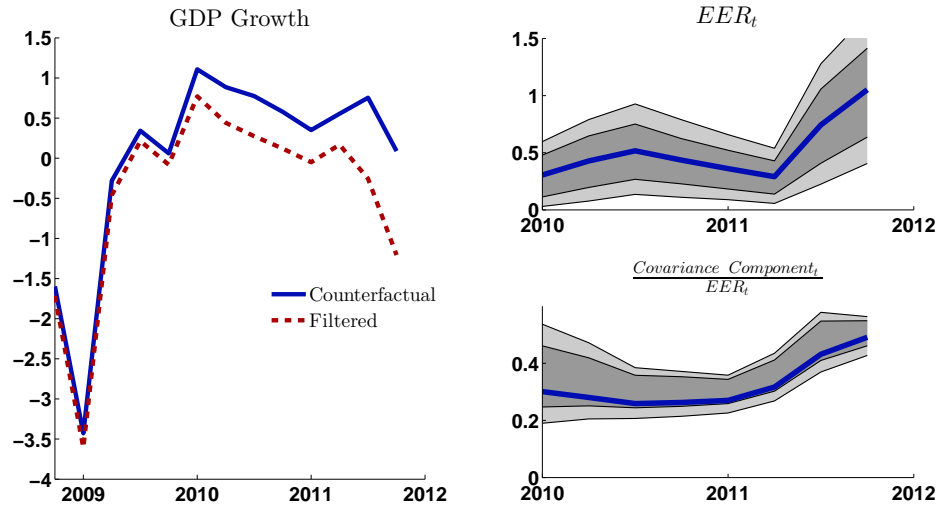
- v) The effect of sovereign credit risk for the outcome variable x is measured as $x_t^{\text{eff}}(i, n) = x_t(i, n) - x_t^c(i, n)$. \square

Regarding the specifics of this experiment, I select the parameters' draws by subsampling, picking 1 of every 100. Thus, $M = 100$. In the filtering state, I set measurement errors to 0.5% of the sample variance of $\{\mathbf{Y}_t\}_t$ and I use 500,000 particles. This implies that the filtered time series $\{\mathbf{Y}_t\}_t$ are essentially equivalent to the actual data. I first analyze the effect of sovereign risk on the financing costs of firms and on output. I then decompose these effects into the two transmission mechanisms.

The left panel of Figure 1.11 reports the filtered and counterfactual trajectories for GDP growth while the top-right panel reports the effects of sovereign risk on expected excess returns. The rise in sovereign risk in Italy over the 2010:Q1-2011:Q4 period led to an increase in the financing costs of firms and a decline in output growth. The model predicts that expected excess returns increased by 50 basis points on average over this period, with a peak of 100 basis points in the last quarter of 2011. GDP growth would have been on average 0.5422% higher throughout the 2010-2011 period if sovereign default probabilities were fixed at their unconditional mean.

The bottom-right panel of the figure reports the covariance component defined in equation (1.32) as a fraction of expected excess returns. The model shows that the risk channel played quantitatively a first order role in the propagation of sovereign credit risk in Italy, and its relevance grew over time: at the end of 2011:Q4, the

Figure 1.11: Sovereign Risk, Firms' Borrowing Costs and Output



Notes: The solid line in the left panel reports the posterior mean for the filtered GDP growth series, and the dotted line reports the posterior mean of its counterfactual. The solid lines in the right panels represent the posterior mean. The Dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets for the variables of interest.

covariance component explains on average 47% of the effects of sovereign risk on the financing premia of firms. Table 1.5 reports posterior statistics for variables of interest.

Table 1.5: Sovereign Risk, Firms' Borrowing Costs and Output: 2010:Q1-2011:Q4

Statistic	Posterior Mean	90% Credible Set
Cumulative Output Losses	4.7576	[2.0890,8.0290]
Average Expected Excess Returns	0.4768	[0.1633,1.0741]
Covariance Component	0.2619	[0.1940,0.5129]

Notes: Cumulative output losses: sum of GDP growth losses (difference between counterfactual and filtered GDP growth) over the 2010:Q1-2011:Q4 period. Average expected excess returns: average difference between filtered and counterfactual expected excess return, expressed in annualized basis points. Covariance component: fraction of expected excess returns explained by the covariance component.

1.6.2 Longer Term Refinancing Operations

The ECB undertook several interventions in response to the euro-area sovereign debt crisis. Some of these policies were explicitly targeted toward easing the tensions in the market for bonds of distressed governments. The Security Markets Program (SMP) and the Outright Monetary Transactions (OMTs) fall within this category.⁴⁰ Other interventions, instead, had the objective of loosening the funding constraints of banks exposed to distressed government debt. The unconventional LTROs launched by the ECB in December 2011 and February 2012 were the most important in this class. Relative to canonical open market operations in Europe,⁴¹ these interventions featured a long maturity (36 months), a fixed-interest rate (1%) and special rules for the collateral that could be used by banks. Moreover, the two LTROs were the largest refinancing operations in the history of the ECB, as more than 1 trillion euros were lent to banks through these interventions.

A full assessment of the policy is beyond the scope of this paper. LTROs, in fact, are not sterilized and the real model considered here misses this aspect. Moreover, the policy may have resulted in a reduction of sovereign credit risk, and the analysis in this paper does not capture this effect either. However, we can use the model to ask whether the provision of liquidity to banks, by itself, stimulated lending. I model LTROs as a nonstationary version of the discount window lending considered in [Gertler and Kiyotaki \(2010\)](#). The government gives banks the option

⁴⁰In May 2010, the ECB started the SMP. Under the SMP, the ECB could intervene by buying, on secondary markets, the securities that it normally accepts as collateral. This program was extensively used for sustaining the price of government securities of southern European countries. The program was replaced by OMTs in August 2012. This latter program had two main differences compared with SMP: i) OMTs are *ex-ante* unlimited; ii) their approval is subject to a conditionality program from the requiring country.

⁴¹Open market operations in the euro-area are conducted through refinancing operations. These are similar to repurchase agreements: banks put acceptable collateral with the ECB and receive cash loans. Prior to 2008, there were two major types of refinancing operations: main refinancing operations (loans of a weekly maturity) and LTROs, with a three month maturity.

at $t = 1$ of borrowing resources up to a threshold \bar{m} . These resources are financed through lump sum taxes. The loans have a fixed interest rate R_m . Banks repay the loan (principal plus interest) at a future date T and no interests are paid between t and T . Finally, the government has perfect monitoring of banks, so that these liabilities do not count for their leverage constraint.⁴² Within the logic of the model, this intervention has the effect of relaxing the leverage constraint of banks, and it has a positive effect on their net worth. These two points are explained in Appendix A.5, along with a description of the numerical algorithm used to implement the policy.

The evaluation of LTROs is conducted using the following algorithm

Evaluating LTROs: Let θ^i denote the i 'th draw from the posterior distribution of the model's parameter. For $i = 1$ to M

- i) Conditional on θ^i , sample from $p(\mathbf{S}_{2011:Q4} | \mathbf{Y}^T, \theta^i)$ N realizations of the state vector.
- ii) For each $\{\mathbf{S}_{2011:Q4}^n\}_n$, simulate the model forward J times with and without the policy intervention.
- iii) For each outcome variable x , compute the difference between these two paths $x_t^{\text{eff}}(i, n, j) = x_t^{\text{lTRO}}(i, n, j) - x_t^{\text{no lTRO}}(i, n, j)$. Collect these paths in $\mathbf{x}_t^{\text{eff}}(i, n, j)$.

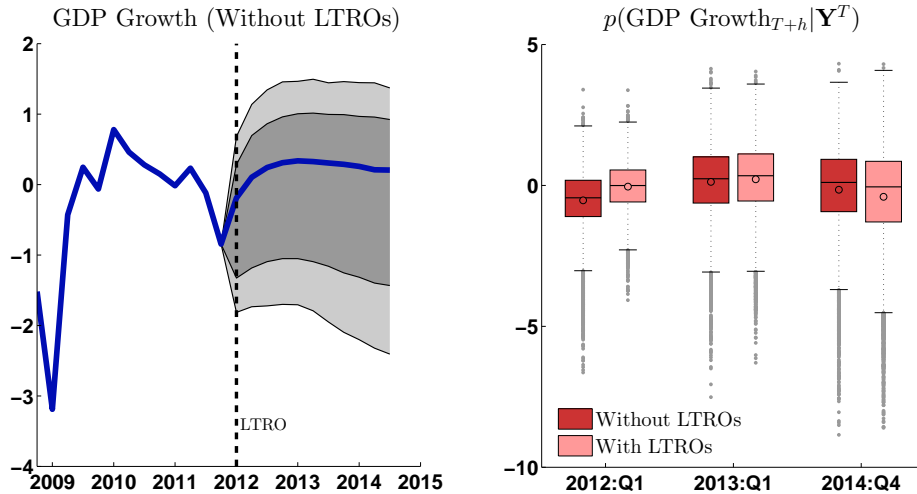
□

The density $p(\mathbf{S}_{t=2011:Q4} | \mathbf{Y}^T, \theta^i)$ is computed using the particle filter. The vector of variables $\{\mathbf{x}_t^{\text{eff}}(i, n, j)\}_t$ denotes the effect of the policy on variable x . The results of this experiment can be interpreted as an *ex-ante* evaluation of the policy, since

⁴²If that were not the case, the loans would perfectly crowd out households' deposits by construction: see [Gertler and Kiyotaki \(2010\)](#).

I am conditioning on retrospective estimates for the state vector in the 2011:Q4 period. In order to make the experiment more realistic, I calibrate the policy to the actual ECB intervention. I set $R_m = 1.00$, $T = 12$ and $\bar{m} = 0.1\hat{Y}^{ss}$.

Figure 1.12: Ex-Ante Assessment of LTROs



Notes: The solid line in the left panel is the conditional mean forecasts of the GDP growth time series from 2012:Q1 to 2014:Q4. The Dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets. The right panels reports the predictive densities for GDP growth with and without LTROs (box plots).

As a benchmark, I first discuss the forecasted path for GDP growth in absence of the policy. The left panel of Figure 1.12 reports the posterior median of the model’s forecast for GDP growth in absence of the policy along with its 60% and 90% credible set. The model predicts a “risky” recovery for GDP growth from the 2011:Q4 point of view. While on average GDP growth returns to its trend value by 2013, we can see a long left tail in these forecasts, especially in the early part of 2012. That is, the model indicates some probability that economic outcomes substantially worsen in the absence of the policy.⁴³

The right panel of Figure 1.12 shows how LTROs influence these forecasts. I

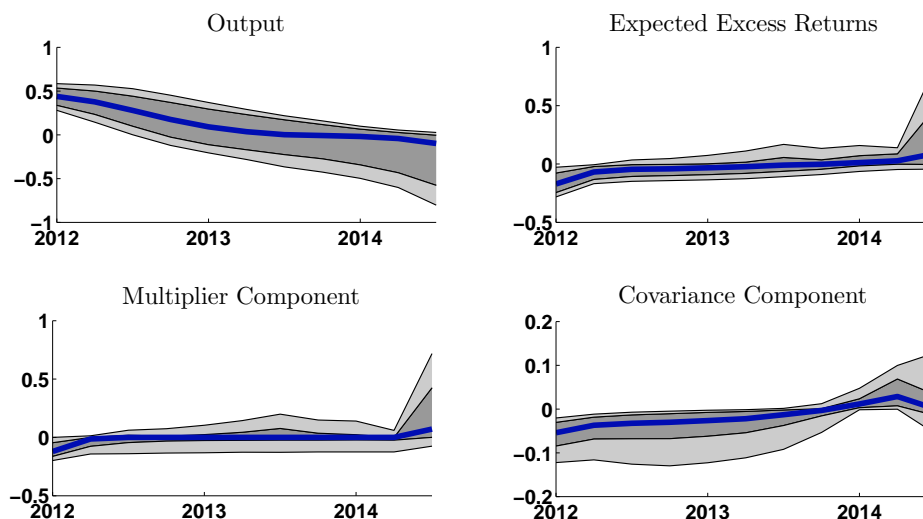
⁴³The long left tail is induced by two factors: i) the asymmetries induced by the leverage constraint; ii) the probability of a sovereign default.

use box plots to describe the predictive densities $p(\text{GDP Growth}_{T+h}|\mathbf{Y}^T)$ with and without LTROs for $h = \{1, 6, 11\}$. The box stands for the interquartile range, the line within the box is the median while the circle represents the mean. The refinancing operations have a clear positive effect on GDP growth in the first quarter of 2012. Indeed, the median forecast for GDP growth under the policy is 0.5% while in its absence is 0.16%. More strikingly, the policy removes most of the downside risk: the left tail of the predictive density for GDP growth in 2012:Q1 almost disappears. This happens because the policy increases the maturity of banks' liabilities, which makes their balance sheet less sensitive to adverse shocks. As time goes on and the repayment date approaches, though, GDP growth forecasts under LTROs become fairly similar to those in absence of this policy. At the scheduled repayment date, the predictive density for GDP growth is actually more left-skewed relative to that obtained in absence of the policy.

Figure 1.13 reports posterior statistics on the policy effects for the level of output, expected excess returns and their decomposition into multiplier and covariance components. The policy lowers expected excess returns on impact and most of these effects are due to looser funding constraints of banks. The covariance component is barely affected by refinancing operations at early stages because of the reasons discussed in Section 1.5.3.

These initial positive effects are reversed over time. Starting from 2013:Q1, the model places a probability of at least 20% on LTROs increasing the financing costs of firms and reducing the level of output. This result, which may appear paradoxical, is driven by the behavior of the model at the repayment stage. In 2014:Q4, banks need to repay the loans they took on and adverse net worth shocks

Figure 1.13: Effects of LTROs on Output and Expected Excess Returns



Notes: The solid line reports the posterior mean of the expected policy effects on the time series 2012:Q1 to 2014:Q4. The Dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets. Output is linearly detrended and expressed in percentages. The other variables are expressed in annualized basis points.

at that date are very costly for them. The anticipation of the repayment stage makes banks more cautious *ex-ante* and leads them to demand higher compensation for risk. This counteracts the initial positive effects of the policy. Under certain circumstances, this second effect may dominate and lead to an increase in expected excess returns and to a decline in output relative to the no-policy benchmark. Overall, these results suggest that refinancing operations are forecasted to be quite ineffective in stimulating real economic activity if we condition to empirically reasonable regions of the state space in 2011:Q4.

This result does not imply that refinancing operations are a bad policy instrument. Rather, that their effects depend on the economic environment in which they are implemented. In order to see this last point, I implement LTROs in a different region of the state space, drawn from the density $p(\mathbf{S}_{t=2008:Q3} | \mathbf{Y}^T, \theta^i)$. In contrast to the 2011:Q4 period, the model interprets the financial distress of 2008:Q3 as driven mainly by banks liquidity problems. Table 1.6 reports the ef-

fects of the policy on output and expected excess returns on impact and at the repayment stage. Two main differences stand out compared to the previous analysis. First, the policy has a substantially stronger effect on the financing premia of firms and on output when implemented in 2008:Q3. Expected excess returns decline on impact by 79 basis points while the level of output increases by 0.52%. Second, the downside risk at the repayment stage is substantially reduced. This can be seen by comparing the credible sets for output at the repayment stage for the two cases.

Table 1.6: **Effects of LTROs on Impact and at Repayment: 2008:Q3 vs. 2011:Q4**

Statistic	2008:Q3		2011:Q4	
	Impact	Repayment	Impact	Repayment
Output	0.52	0.01	0.34	-0.01
	[0.29,0.73]	[-0.24,0.15]	[0.11,0.56]	[-0.86,0.12]
Expected Excess Returns	-0.79	-0.01	-0.35	0.11
	[-1.30,-0.26]	[-0.39,0.25]	[-0.71,-0.01]	[-0.2054,2.14]

Notes: Posterior statistics on the effects of LTROs on output and expected excess returns on impact (period 1) and at the repayment stage (period 12). The first two columns initialize the state vector at $p(\mathbf{S}_{t=2008:Q3}|\mathbf{Y}^T, \theta^i)$. The last two columns initialize the vector at $p(\mathbf{S}_{t=2011:Q4}|\mathbf{Y}^T, \theta^i)$.

The reasons underlying this state dependence are related to the discussion of equity injections in Section 1.5.3. In 2008:Q3, agency costs are estimated to be high. This indicates that there are profitable investment opportunities in the economy and banks use the funds from the LTROs to lend to firms. General equilibrium, then, generates a positive loop: the market value of firms' claims is positively influenced by higher demand for capital, and this strengthens bank net worth. As a result, banks arrive at the repayment stage with a buffer that makes their balance sheet less sensitive to adverse shocks. These general equilibrium effects are, instead, muted when implementing the policy in 2011:Q4.

1.7 Conclusion

In this paper I have conducted a quantitative analysis of the transmission of sovereign credit risk to the borrowing costs of firms and real economic activity. I studied a model where banks are exposed to risky government debt and they are the main source of finance for firms. An increase in the probability of a sovereign default has negative effects on credit markets through two channels. First, by reducing the market value of government securities, higher sovereign risk reduces the net worth of banks and hampers their funding ability: their increased financing costs pass-through into the borrowing rates of firms (leverage-constraint channel). Second, an increase in the probability of a sovereign default raises the risks associated with lending to firms: if the default occurs in the future, in fact, claims on the productive sector will pay out little and banks will have to absorb these losses. I referred to this second mechanisms as the risk channel. The structural estimation of the model on Italian data suggests that the sovereign debt crisis significantly increased the financing premia of firms, with the risk channel explaining up to 47% of these effects. Moreover, the rise in the probability of a sovereign default had severe adverse consequences for the Italian economy: cumulative output losses were 4.75% at the end of 2011. In counterfactual experiments, I use the estimated model to evaluate the policy response adopted by the ECB, with particular emphasis on the LTROs of the first quarter of 2012. The model estimates that these interventions have minor effects on lending and output. This happens because risk premia, which were sizable when the policy was enacted, discourage banks' lending to firms. More generally, the analysis shows that the stabilization properties of these interventions are state dependent in the model, and their aggregate effects depend on the relative strength of the leverage-constraint channel and of

the risk-channel.

There are a number of dimensions in which the model could be extended. The most important is to allow sovereign default risk to respond to macroeconomic conditions. This could be done in different ways, for example by introducing distortionary taxation in the model and considering the optimal default policy of a Ramsey government. Incorporating these aspects would allow for a more complete evaluation of policy responses adopted by the ECB. A second extension would be that of considering an open economy. I believe this dimension would help the empirical identification of the mechanisms discussed in this paper, since they are likely to generate differential implications for international capital flows. While both of these issues are challenging, and require a substantial departure from this framework, they represent exciting opportunities for future work.

Abstracting from the current application, recent research advocates the use of indicators of credit spreads as observables when estimating quantitative models with financial intermediation. This paper adds to that by underscoring the importance of measuring the *sources* driving the movements in these indicators of financial stress. Understanding whether firms' financing premia during crises are high because of "frictions" in financial markets or because of fair compensation for increased risk is a key information for policy makers. Incorporating the nonlinearities emphasized in this paper in larger scale models used for policy evaluation is technically challenging. Moreover, given the policy relevance of these nonlinearities, there is a need for developing tools for their empirical validation in the data. I plan to address these issues in future work.

Chapter 2

Assessing DSGE Model

Nonlinearities

2.1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are now widely used for empirical research in macroeconomics, as well as for forecasting and quantitative policy analysis in central banks. In these models, decision rules of economic agents are derived from assumptions about agents' preferences and production technologies utilizing some fundamental principles such as optimization, rational expectations, and competitive equilibrium. In practice, this means that the functional forms and parameters of equations that describe the behavior of economic agents are tightly restricted by the equilibrium conditions. Consequently, a careful evaluation of the DSGE model-implied restrictions is an important aspect of empirical research.

Until recently, much of the research that estimates DSGE models used first-order approximations to the equilibrium decision rules. This made linear models such

as vector autoregressions (VARs) appropriate for evaluating the restrictions of the DSGE model. With the advance of the methods to estimate DSGE models using higher-order approximations, as developed in [Fernández-Villaverde and Rubio-Ramírez \(2007b\)](#), an important avenue of research has opened. The end of the Great Moderation also makes nonlinear models all the more relevant for empirical macroeconomics.¹ While there is a burgeoning literature on both the methods to solve nonlinear DSGE models and their applications, there does not seem to be an obvious nonlinear time series model to use to evaluate these DSGE models.

The objective of this paper is to develop a class of time series models that mimic nonlinearities of DSGE models and to use these models as a benchmark for the evaluation of a nonlinear DSGE model. Motivated by the popular second-order perturbation approximations of DSGE model dynamics, we consider autoregressive models that involve quadratic terms of lagged endogenous variables as well as interactions between current period innovations and lagged endogenous variables, which generate conditional heteroskedasticity. These time series models are derived from a perturbation solution to a nonlinear difference equation and have a recursively linear structure that makes it straightforward to characterize stability properties and derive moments. While multivariate extensions are possible, we focus in this paper on univariate specifications, that we refer to as QAR(p,q) models, where “Q” stands for quadratic.² In the empirical work, we use $p = q = 1$.

¹There are in principle two types of nonlinearities that can appear in a nonlinear DSGE model. First are (approximately) smooth nonlinearities, where decision rules display curvature and possibly asymmetries such as those that are generated by asymmetric loss or cost functions. Second are kinks in decision rules such as those that are generated by the zero lower bound on nominal interest rates. This paper is about the former. While the latter is also crucial and we intend to extend our work to address this type of nonlinearities, solving and estimating DSGE models with kinks in decision rules is very difficult. See, for example, [Gust et al. \(2012\)](#), [Aruoba and Schorfheide \(2013b\)](#), [Bocola \(2013\)](#), and references therein.

²The abbreviation QAR has previously been used for Quantile Autoregressions, see [Koenker and Xiao \(2006\)](#).

After documenting some of the theoretical properties of the QAR models, the first step of the empirical analysis is to fit QAR(1,1) models to growth of real gross domestic product (GDP), inflation, nominal wage growth, and interest rate data for the U.S. We start our sample in 1960 but consider various subsamples, using 1983 (the end of the Volcker era and the start of the Great Moderation) and 2007 (the end of the Great Moderation and the start of the Great Recession) as additional start and end points. We find three sets of important nonlinearities across the variables and samples we consider. First, GDP growth displays pronounced nonlinearities in the post-1983 samples with sharp output losses during recessions and relatively slow recoveries. Second, for inflation and wage growth the long samples that start in 1960 and extend beyond the 1990s exhibit high volatility in times of high inflation and wage growth, which is mainly driven by the observations in the 1970s. Finally, QAR estimates for interest rates imply an asymmetric behavior by the Federal Reserve in the post-1983 era; interest rates increase more gradually than they fall.

The second step of the empirical analysis consists of the estimation of a DSGE model. In our application we focus on the estimation and evaluation of a New Keynesian DSGE model with asymmetric price and wage adjustment costs, building on [Kim and Ruge-Murcia \(2009\)](#). This model can generate downward nominal wage and price rigidity and is interesting for several reasons. First, it is well known that in the absence of the zero-lower-bound (ZLB) constraint on nominal interest rates, unrealistically large shocks or degrees of risk aversion, New Keynesian DSGE models do not generate significant nonlinearities (see, for instance, [An \(2007\)](#)). However, once one allows for asymmetric adjustment costs, agents' decision rules can become strongly nonlinear. Thus, ex ante, to the extent that

there are nonlinearities in the data, the model may be able to deliver some of these.

Second, downward rigidity is a well-documented feature of nominal wage changes at the micro-level, e.g., [Gottschalk \(2005\)](#), [Barattieri et al. \(2010\)](#), and [Daly et al. \(2012\)](#). Third, there are a number of papers that have incorporated downward nominal wage rigidity into DSGE models to study its macroeconomic effects. For instance, [Kim and Ruge-Murcia \(2009\)](#) study optimal monetary policy in the presence of downward nominal wage rigidity. [Schmitt-Grohe and Uribe \(2013\)](#) use downward nominal wage rigidity to generate large output losses and a jobless recovery in a deflation (or liquidity-trap) equilibrium of a New Keynesian model with ZLB constraint. Thus, a careful evaluation of the nonlinearities that this mechanism generates is important.

In estimating the DSGE model, we use the same data set as in the estimation of the univariate QAR models and consider two samples, one long and one short, both of which end in 2007 to avoid using data where the ZLB starts to bind. By and large, the parameter estimates for the DSGE models are consistent with estimates that have been reported elsewhere in the literature. In particular, our estimates indicate asymmetries in the adjustment costs for both prices and nominal wages that make increases less costly than decreases.

The final, and most important step of the analysis is to conduct a posterior predictive check of the DSGE model that compares coefficient estimates obtained from data simulated from an estimated DSGE model to coefficient estimates obtained from actual data. The predictive check amounts to assessing how far the QAR estimates obtained from the actual data lie in the tails of the predictive distribution. The general conclusion is that the DSGE model does not generate very

strong nonlinearities except for inflation and nominal wage growth, both of which show conditional heteroskedasticity. This means that the asymmetric adjustment costs in prices and wages are able to deliver asymmetric behavior in inflation and nominal wage growth in line with the data but this asymmetry does not spill over neither to real GDP growth, nor to the policy instrument of the Federal Reserve.

Our work is related to several branches of the literature. There exists a large body of work on nonlinear time series models.³ However, none of these model classes seem to be directly useable for our purposes since either the nonlinearities do not match the nonlinearities of DSGE models solved with higher-order perturbation methods or the models have undesirable instability properties.

The proposed QAR family is most closely related to generalized autoregressions (GAR) discussed in [Mittnik \(1990\)](#) in the sense that the conditional mean of the dependent variable y_t is a polynomial function of its lags. Our QAR models also involve interactions between lagged dependent variables y_{t-j} and innovations u_t , which is a defining property of bilinear models and linear autoregressive conditional heteroskedasticity (LARCH) models, e.g., [Giraitis et al. \(2000\)](#). However, rather than simply augmenting a linear autoregressive model by quadratic terms and interactions between lagged endogenous variables and innovations, we derive its structure from a second-order perturbation approximation to the solution of a nonlinear difference equation along the lines of [Holmes \(1995\)](#). To the extent that both conditional mean and variance depend on quadratic functions of the innovations u_t our model is also related to the class of (G)ARCH-M models, e.g. [Engle et al. \(1987\)](#) and [Grier and Perry \(1996\)](#). Finally, the QAR model can

³These include regime switching models, e.g. [Hamilton \(1989\)](#) and [Sims and Zha \(2006\)](#), time-varying coefficient models, e.g. [Cogley and Sargent \(2002\)](#) and [Primiceri \(2005\)](#), threshold and smooth-transition autoregressive models, e.g. [Tong and Lim \(1980\)](#) and [Teräsvirta \(1994\)](#), bilinear models, e.g. [Granger and Andersen \(1978\)](#) and [Rao \(1981\)](#).

be viewed as a set of tight restrictions on the coefficients of a [Volterra \(1930\)](#) representation of a nonlinear time series.

There exists an abundant literature that develops methods to evaluate DSGE models based on comparisons with more flexible and densely parameterized time series models. However, much of the existing econometric work is based on linearized DSGE models. A natural benchmark for the evaluation of such models is provided by vector autoregressions (VARs) that relax the cross-coefficient restrictions. In fact, there exists an extensive literature that develops and applies methods to evaluate DSGE models based on comparisons with VARs, e.g., [Cogley and Nason \(1994\)](#), [Schorfheide \(2000b\)](#), [Christiano et al. \(2005a\)](#), [Del Negro et al. \(2007\)](#), and [Fernández-Villaverde et al. \(2007a\)](#).

In this paper we use so-called posterior predictive checks to evaluate a prototypical DSGE model. A general discussion of the role of predictive checks in Bayesian analysis can be found in [Lancaster \(2004\)](#) and [Geweke \(2005b\)](#). [Canova \(1994\)](#) is the first paper that uses predictive checks to assess implications of a DSGE model. While [Canova \(1994\)](#)'s checks were based on the prior predictive distribution, we use posterior predictive checks in this paper as, for instance, in [Chang et al. \(2007b\)](#). Finally, [Abbritti and Fahr \(2013\)](#) use a model with asymmetric wage adjustment costs and search and matching frictions to investigate the ability of the model to deliver nonlinearities, focusing on skewness and turning point statistics.

The remainder of the paper is organized as follows. In [Section 2.2](#) we review the structure of second-order perturbation approximations of DSGE models. The QAR model is developed in [Section 2.3](#). We discuss some of its theoretical properties as well as Bayesian inference. Estimates of the QAR model for U.S. data are presented in [Section 2.4](#). The DSGE model with asymmetric price and wage

adjustment costs is introduced in Section 2.5. The estimation and evaluation of the DSGE model is presented in Section 3.2. Finally, Section 4.5 concludes. An online Appendix contains detailed derivations of the properties of the QAR model, as well as details of the Markov chain Monte Carlo (MCMC) methods employed in this paper.

2.2 DSGE Model Nonlinearities

Most estimated nonlinear DSGE models are solved with perturbation methods because they can be efficiently applied to models with a large state space. A DSGE model solved by second-order perturbation can be generically written as

$$\begin{aligned}
c_{i,t} &= \psi_{1i}(\theta) + \psi_{2ij}(\theta)x_{j,t} + \psi_{3ij}(\theta)z_{j,t} \\
&\quad + \psi_{4ijk}(\theta)x_{j,t}x_{k,t} + \psi_{5ijk}(\theta)x_{j,t}z_{k,t} + \psi_{6ijk}(\theta)z_{j,t}z_{k,t} \\
x_{i,t+1} &= \zeta_{1i}(\theta) + \zeta_{2ij}(\theta)x_{j,t} + \zeta_{2ij}(\theta)z_{j,t} \\
&\quad + \zeta_{4ijk}(\theta)x_{j,t}x_{k,t} + \zeta_{5ijk}(\theta)x_{j,t}z_{k,t} + \zeta_{6ijk}(\theta)z_{j,t}z_{k,t} \\
z_{i,t+1} &= \xi_{2ij}(\theta)z_{j,t} + \xi_{3i}(\theta)\epsilon_{i,t+1},
\end{aligned} \tag{2.1}$$

where θ denotes the parameters of the model and the DSGE model variables are grouped into control variables $c_{i,t}$, e.g., consumption, endogenous state variables $x_{i,t}$, e.g., the capital stock, and exogenous state variables $z_{i,t}$, e.g., total factor productivity. The notation $a_{ijk}x_{j,t}x_{k,t}$ in (2.1) is shorthand for $\sum_{j=1}^n \sum_{k=1}^n a_{ijk}x_{j,t}x_{k,t}$. Since not all of the control and state variables are observable it is common to augment the system by a measurement equation of the form

$$y_{i,t} = A_{1i}(\theta) + A_{2ij}(\theta)c_{j,t} + A_{3ij}(\theta)x_{j,t} + A_{4ij}(\theta)z_{j,t} + e_{i,t}, \tag{2.2}$$

where the $e_{i,t}$'s are measurement errors. Typically, the vector of observables y_t is composed of a subset of the state and control variables such that the A matrices are simple selection matrices.

Nonlinear features may arise endogenously or exogenously. Curvature in utility functions, adjustment cost function, and production functions can generate nonlinear decision rules of households and firms endogenously. An example of an exogenous nonlinearity is stochastic volatility in the exogenous shocks that generate business cycle fluctuations. In (2.1) the endogenous nonlinearity is captured by the quadratic functions of x_t and z_t that appear in the law of motion of the control variables $c_{i,t}$ and the endogenous state variables $x_{i,t+1}$. The representation assumes that there are no exogenous nonlinearities as the exogenous states $x_{i,t}^{exo}$ evolve according to a linear autoregressive process.

The objective of this paper is to propose an econometric method to assess the empirical adequacy of the nonlinear terms in the DSGE model solution (2.1). The DSGE model generates cross-coefficient restrictions between the first-order terms and the higher-order terms which may or may not be correctly specified. In principle, one could try to estimate two versions of the state-space model given by (2.1) and (2.2): a restricted version that imposes the functional relationship between the low-dimensional DSGE model parameter vector θ and the state-space coefficients $\psi(\cdot)$, $\zeta(\cdot)$, and $\xi(\cdot)$ and an unrestricted version in which the ψ 's, ζ 's, and ξ 's are freely estimated. The discrepancy between the restricted and unrestricted estimates provides a measure of empirical adequacy. However, due to the large number of parameters and some inherent identification problems, the unrestricted estimation of the state-space system (2.1) and (2.2) is difficult to implement. In fact, even the literature that evaluates linearized DSGE models has by and large

abstained from trying to estimate unrestricted state-space representations.

A more common approach in the literature on linearized DSGE models is to compare properties of the DSGE model to properties of an unrestricted VAR. This comparison can take many different forms, e.g., assessing the discrepancy between unrestricted VAR coefficient estimates and the DSGE-model-implied VAR approximation as in [Smith \(1993\)](#), or the comparison of VAR and DSGE model impulse responses as in [Cogley and Nason \(1994\)](#) or [Christiano et al. \(2005a\)](#). Since our goal is to identify nonlinearities, a linear VAR would not be of any use. Instead, we compare parameter estimates of nonlinear autoregressive time series models obtained from actual U.S. data and data simulated from a nonlinear DSGE model. If the DSGE model is well specified, then the estimates of the auxiliary models ought to be very similar. This comparison is formalized as a Bayesian posterior predictive check. We proceed by providing a detailed description of the auxiliary time series model that is used for the DSGE model evaluation.

2.3 Quadratic Autoregressive Models

The most popular (and empirically successful) nonlinear time series models are those capturing time variation in the coefficients of linear time series models, e.g., Markov-switching models, time-varying coefficient models, GARCH models, stochastic volatility models. However, none of these models provide a good characterization of the nonlinearity generated endogenously by the DSGE model solution in [\(2.1\)](#). For this reason we develop a new class of nonlinear autoregressive time series models that are more closely tied to the DSGE model solution in [\(2.1\)](#).

We introduce the specification of a first-order quadratic autoregressive (QAR)

model in Section 2.3.1. We subsequently characterize some of its important properties in Section 2.3.2 and describe the implementation of posterior inference in Section 2.3.3. Section 2.3.4 provides generalizations of the basic specification and discusses the relationship of our QAR models to other nonlinear time series models.

2.3.1 Specification of the QAR(1,1) Model

The starting point is a perturbation approximation of the solution of a nonlinear difference equation of the form

$$y_t = f(y_{t-1}, \omega u_t), \quad u_t \stackrel{iid}{\sim} N(0, 1). \quad (2.3)$$

We assume that the process characterized by (2.3) has a unique deterministic steady state that solves the equation $y_* = f(y_*, 0)$. Following the literature on perturbation methods, e.g. Holmes (1995) and Lombardo (2011), we construct an approximate solution of the form

$$y_t^* = y_t^{(0)} + \omega y_t^{(1)} + \omega^2 y_t^{(2)}. \quad (2.4)$$

It turns out that this solution is second-order accurate in the sense that

$$y_t = y_t^* + \mathcal{O}_p(\omega^3) \quad (2.5)$$

as $\omega \rightarrow 0$.

To obtain y_t^* , we take a second-order Taylor expansion of the function f around

$y_t = y_*$ and $\omega = 0$:

$$\begin{aligned}
y_t - y_* &= f_y(y_{t-1} - y_*) + f_u \omega u_t \\
&+ \frac{1}{2} f_{y,y} (y_{t-1} - y_*)^2 + f_{y,u} (y_{t-1} - y_*) \omega u_t \\
&+ \frac{1}{2} f_{u,u} (\omega u_t)^2 + \text{higher-order terms},
\end{aligned} \tag{2.6}$$

where $f_{x,y}$ denotes the (x, y) 'th derivative of f evaluated at the point $(y_t = y_*, \omega = 0)$. Substituting (2.4) into (2.6) and neglecting terms of order $\mathcal{O}_p(\omega^3)$, one obtains:

$$\begin{aligned}
&y_t^{(0)} - y_* + \omega y_t^{(1)} + \omega^2 y_t^{(2)} \\
&= f_y \left(y_{t-1}^{(0)} - y_* + \omega y_{t-1}^{(1)} + \omega^2 y_{t-1}^{(2)} \right) + f_u \omega u_t \\
&+ \frac{1}{2} f_{y,y} \left(y_{t-1}^{(0)} - y_* + \omega y_{t-1}^{(1)} + \omega^2 y_{t-1}^{(2)} \right)^2 \\
&+ \frac{1}{2} f_{y,u} \left(y_{t-1}^{(0)} - y_* + \omega y_{t-1}^{(1)} + \omega^2 y_{t-1}^{(1)} \right) \omega u_t + \frac{1}{2} f_{u,u} \omega^2 u_t^2 + \mathcal{O}_p(\omega^3).
\end{aligned} \tag{2.7}$$

We set $y_t^{(0)} = y_{t-1}^{(0)} = y_*$ and then match terms of the same ω -order on the left-hand-side and the right-hand-side of (2.7) to obtain the laws of motion for $y_t^{(1)}$ and $y_t^{(2)}$:

$$\begin{aligned}
y_t^{(1)} &= f_y y_{t-1}^{(1)} + f_u u_t, \\
y_t^{(2)} &= f_y y_{t-1}^{(2)} + \frac{1}{2} f_{y,y} \left(y_{t-1}^{(1)} \right)^2 + \frac{1}{2} f_{y,u} y_{t-1}^{(1)} u_t + \frac{1}{2} f_{u,u} u_t^2.
\end{aligned}$$

Notice that $y_t^{(1)}$ follows an AR(1) process and that conditional on $y_t^{(1)}$ the dynamics of $y_t^{(2)}$ are also linear. Substituting the expressions for $y_t^{(1)}$ and $y_t^{(2)}$ into (2.4) and collecting terms, we obtain that a second-order accurate perturbation approx-

imation of the nonlinear difference equation (2.3) is given by the system:

$$\begin{aligned} y_t &= y_* + f_y(y_{t-1} - y_*) + \frac{1}{2}f_{y,y}(\omega y_{t-1}^{(1)})^2 + \left(f_u + \frac{1}{2}f_{y,u}y_{t-1}^{(1)}\right)\omega u_t + \frac{1}{2}f_{u,u}\omega^2 u_t^2 \\ y_t^{(1)} &= f_y y_{t-1}^{(1)} + f_u u_t \end{aligned} \quad (2.8)$$

We undertake a few additional modifications. We define $s_t \equiv \omega y_t^{(1)}$ and introduce the parameters

$$\phi_0 = y_*, \quad \phi_1 = f_y, \quad \tilde{\phi}_2 = \frac{1}{2}f_{y,y}, \quad \tilde{\gamma} = \frac{1}{2\omega}f_{y,u}, \quad \sigma = f_u\omega.$$

Moreover, we drop the term $\frac{1}{2}f_{u,u}\omega^2 u_t^2$ to obtain a conditional Normal distribution of y_t . Overall, this leads to the nonlinear state-space model:

$$\begin{aligned} y_t &= \phi_0 + \phi_1(y_{t-1} - \phi_0) + \tilde{\phi}_2 \tilde{s}_{t-1}^2 + (1 + \tilde{\gamma}\tilde{s}_{t-1})\sigma u_t \\ \tilde{s}_t &= \phi_1 \tilde{s}_{t-1} + \sigma u_t, \quad u_t \stackrel{iid}{\sim} N(0, 1). \end{aligned} \quad (2.9)$$

To complete the specification of the time series model we assume that the distribution of the initial values in period $t = -T_*$ have distribution F_{-T_*} , and that the innovations u_t are normally distributed:

$$(y_{-T_*}, \tilde{s}_{-T_*}) \sim F_{-T_*}, \quad u_t \stackrel{iid}{\sim} N(0, 1). \quad (2.10)$$

We refer to (2.9) as the QAR(1,1) model. The first “1” indicates the number of lags in the conditional mean function and the second “1” denotes how many lags interact with the innovation u_t .

It is convenient to reparameterize the QAR(1,1) model as follows. Define $\phi_2, \gamma,$

and s_t such that

$$\phi_2 = \tilde{\phi}_2 \frac{\sigma^2}{1 - \phi_1^2}, \quad \gamma = \frac{\sigma}{\sqrt{1 - \phi_1^2}} \tilde{\gamma}, \quad \text{and} \quad s_t = \frac{\sqrt{1 - \phi_1^2}}{\sigma} \tilde{s}_t. \quad (2.11)$$

Under the reparameterization the coefficients ϕ_2 and γ interact with standardized versions of s_{t-1}^2 and s_{t-1} , respectively. Thus, (2.9) becomes

$$\begin{aligned} y_t &= \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2 s_{t-1}^2 + (1 + \gamma s_{t-1}) \sigma u_t \\ s_t &= \phi_1 s_{t-1} + \sqrt{1 - \phi_1^2} u_t, \quad u_t \stackrel{iid}{\sim} N(0, 1). \end{aligned} \quad (2.12)$$

2.3.2 Important Properties of the QAR(1,1) Model

In order to appreciate two of the important implications of the recursively linear structure of the QAR(1,1) model given by (2.12) consider the alternative specification (omitting the constant term and the volatility dynamics) $y_t = \phi_1 y_{t-1} + \phi_2 y_{t-1}^2 + u_t$, $0 < \phi_1 < 1$ and $\phi_2 > 0$. It is straightforward to verify that this specification has two steady states, namely, $y_*^{(1)} = 0$ and $y_*^{(2)} = (1 - \phi_1)/\phi_2$. The second steady state arises as an artefact of the quadratic representation even if the underlying nonlinear model (2.3) only has a single steady state. Moreover, from writing $\Delta y_t = (-1 + \phi_1 + \phi_2 y_{t-1}) y_{t-1} + u_t$ notice that the system becomes explosive if a large shock has pushed y_{t-1} above $y_*^{(2)}$. This explosiveness can arise regardless of the value of ϕ_1 .

The multiplicity of steady states and the undesirable explosive dynamics have been pointed out in the context of second-order perturbation solutions of DSGE models by [Kim et al. \(2008\)](#) who proposed an ex-post modification of quadratic autoregressive equations to ensure that unwanted higher-order terms do not propa-

gate forward and generate explosive behavior not present in the underlying nonlinear model. This modification is called pruning in the literature.⁴ Our derivation of the QAR model in Section 2.3.1 automatically generates a recursively linear structure with a unique steady state and non-explosive dynamics for suitably restricted values of ϕ_1 . If the marginal distribution of s_{-T_*} is $N(0, 1)$, then the process s_t , $t \geq -T_*$, is strictly stationary under the restriction $|\phi_1| < 1$. In turn, the vector process $z_t = [s_{t-1}, s_{t-1}^2, u_t]'$ is strictly stationary and we can rewrite the law of motion of y_t in (2.9) as

$$y_t = \phi_0 + \phi_1(y_{t-1} - \phi_0) + g(z_t) = \phi_0 + \sum_{j=0}^{\infty} \phi_1^j g(z_{t-j}). \quad (2.13)$$

This representation highlights that y_t is a stationary process. Since $g(z_t)$ is a nonlinear function of u_t and its history, the process is, however, not linear in u_t anymore. In fact, under the assumption that y_t was initialized in the infinite past ($T_* \rightarrow -\infty$), we obtain the following representation:

$$y_t = \phi_0 + \sigma \sum_{j=0}^{\infty} \phi_1^j u_{t-j} + \sigma \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \left(\tilde{\gamma} \mathcal{I}\{l > j\} \phi_1^{l-j} + \tilde{\phi}_2 \sum_{k=0}^{\min\{j,l\}} \phi_1^{j+l-k} \right) u_{t-j} u_{t-l}. \quad (2.14)$$

(2.14) is a discrete-time Volterra series expansion, in which the Volterra kernels of order one and two are tightly restricted and the kernels of order larger than two are equal zero.⁵ The recursively linear structure also facilitates the computation of higher-order moments of y_t . Further details are provided in the appendix.

⁴ Lombardo (2011) constructs a pruned perturbation solution of a DSGE model directly rather than by ex-post adjustment. Lan and Meyer-Gohde (2013) solve DSGE models by constructing approximate second-order Volterra series expansions for the model variables, which also eliminates unwanted higher-order terms. Andreasen et al. (2013) derive the moments of observables from a general state-space representation for pruned DSGE models to facilitate moment-based estimation.

⁵ The infinite sequences of coefficients on terms $\{u_{t-j}\}_{j \geq 0}$, $\{u_{t-j}u_{t-l}\}_{j \geq 0, l \geq 0}$, $\{u_{t-j}u_{t-l}u_{t-k}\}_{j \geq 0, l \geq 0, k \geq 0}$, etc. are called Volterra kernels.

Impulse responses defined as

$$IRF_t(h) = \mathbb{E}_t[y_{t+h}|u_t = 1] - \mathbb{E}_t[y_{t+h}]$$

are state dependent. For instance, for $h = 1$ we obtain

$$IRF_t(0) = \sigma(1 + \gamma s_{t-1}), \quad IRF_t(1) = \sigma \left(\phi_1(1 + \gamma s_{t-1}) + 2\phi_1\phi_2\sqrt{1 - \phi_1^2}s_{t-1} \right). \quad (2.15)$$

Moreover, the model generates conditional heteroskedasticity. The conditional variance of y_t is given by

$$\mathbb{V}_{t-1}[y_t] = (1 + \gamma s_{t-1})^2 \sigma^2. \quad (2.16)$$

2.3.3 Posterior Inference for the QAR(1,1) Model

We estimate the QAR(1,1) model using Bayesian methods. Starting point is a joint distribution of data, parameters, and initial states:

$$p(Y_{0:T}, \theta, s_0) = p(Y_{1:T}|y_0, s_0, \theta)p(y_0, s_0|\theta)p(\theta),$$

where $p(Y_{1:T}|y_0, s_0, \theta)$ is a likelihood function that conditions on the initial values of y_0 and s_0 , $p(y_0, s_0|\theta)$ characterizes the distribution of the initial values, and $p(\theta)$ is the prior density of the QAR(1,1) parameters, and $\theta = [\phi_0, \phi_1, \phi_2, \gamma, \sigma^2]'$. Since for large values of $|s_{t-1}|$ the term $(1 + \gamma s_{t-1})$ in (2.12) may become close to zero or switch signs, we replace it by

$$\left((1 - \vartheta) \exp \left[\frac{\gamma}{1 - \vartheta} s_{t-1} \right] + \vartheta \right), \quad (2.17)$$

where $1 + \gamma s_{t-1}$ is the first-order Taylor series expansion of (2.17). The exponential transformation guarantees non-negativity of the time-varying standard deviation and the constant ϑ provides some regularization that ensures that the shock standard deviation is strictly greater than $\sigma \exp(\vartheta)$ in all states of the world.

It is convenient to factorize the likelihood function into conditional densities as follows:

$$p(Y_{1:T}|y_0, s_0, \theta) = \prod_{t=1}^T p(y_t|y_{0:t-1}, s_0, \theta).$$

Given s_0 and θ it is straightforward to evaluate the likelihood function iteratively. Conditional on s_{t-1} the distribution of y_t is normal. The equation for y_t in (2.12) can be solved for u_t to determine s_t , which completes iteration t . In addition to the likelihood function, we need to specify an initial distribution $p(y_0, s_0|\theta)$. We assume that the system was in its steady state in period $t = -T_*$, that is, $y_{-T_*} = \phi_0$ and $s_{-T_*} = 0$. Based on iterating the original system (2.9) forward we compute a mean and variance for (y_0, s_0) and assume that the initial values are normally distributed. Further details of this initialization are provided in the Appendix. Since the dimension of θ is small, we use a single-block random-walk Metropolis (RWM) algorithm to generate draws from the posterior of θ .

2.3.4 Further Discussion

The QAR(1,1) model in (2.8) has a straightforward generalization in which we include additional lag terms:

$$\begin{aligned} y_t &= \phi_0 + \sum_{l=1}^p \phi_{1,l}(y_{t-l} - \phi_0) + \sum_{l=1}^p \sum_{m=l}^p \tilde{\phi}_{2,lm} s_{t-l} s_{t-m} + \left(1 + \sum_{l=1}^q \tilde{\gamma}_l s_{t-l}\right) \sigma u_t \quad (2.18) \\ \tilde{s}_t &= \sum_{l=1}^p \phi_{1,l} s_{t-l} + \sigma u_t. \end{aligned}$$

We refer to (2.18) as QAR(p,q) model.⁶ As in the standard AR(p) model, the stationarity of y_t is governed by the roots of the lag polynomial $1 - \sum_{l=1}^p \phi_{1,l} z^l$. The quadratic terms generate an additional $p(p+1)/2$ coefficients in the conditional mean equation for y_t . Since the number of coefficients grows at rate p^2 , a shrinkage estimation method is required even for moderate values of p , in order to cope with the dimensionality problem. The QAR model can also be extended to the vector case, which is an extension that we are pursuing in ongoing research. The empirical analysis presented in Section 3.2 is based on the QAR(1,1) specification.

The QAR model is closely related but not identical to some of the existing nonlinear time series models. For $\gamma = 0$ the QAR(1,1) can be viewed as a pruned version of the generalized autoregressive model (GAR) discussed in Mittnik (1990) which augments the standard AR model by higher-order polynomials of the lagged variables. The conditional heteroskedasticity in (2.9) has a linear autoregressive structure. For $\phi_2 = 0$ our model is a special case of the LARCH model studied in Giraitis et al. (2000). Since the conditional variance of y_t can get arbitrarily close to zero, likelihood-based estimation of LARCH models is intrinsically difficult. We circumvent these difficulties by introducing the exponential transformation in (2.17).

Grier and Perry (1996, 2000) have estimated GARCH-M models on macroeconomic time series. GARCH-M models provide a generalization of the ARCH-M models proposed by Engle et al. (1987) and can be written as

$$\begin{aligned} y_t &= \phi_0 + \phi_1(y_{t-1} - \phi_0) + \phi_2(\sigma_t^2 - \sigma^2) + \sigma_t u_t \\ \sigma_t^2 - \sigma^2 &= \gamma_1(u_{t-1}^2 - \sigma^2) + \gamma_2(\sigma_{t-1}^2 - \sigma^2). \end{aligned}$$

⁶If we start with a p th order nonlinear difference equation in (2.3), then we can arrive at a QAR(p,p) model.

Under suitable parameter restrictions y_t can be expressed as a nonlinear function of u_t and its lags. As in the case of the QAR model, y_t depends on the sequence $\{u_{t-j}^2\}$. In addition, the term $\sigma_t u_t$ introduces interactions between u_t and u_{t-j}^2 , $j > 1$. However, coefficients on terms of the form $u_{t-j} u_{t-l}$, $j \neq l$ are restricted to be zero. From our perspective, the biggest drawback of the GARCH-M model is that nonlinear conditional mean dynamics are tied to the volatility dynamics: in the absence of conditional heteroskedasticity the dynamics of y_t are linear. The QAR model is much less restrictive in this regard: y_t can be conditionally homoskedastic ($\gamma = 0$) but at the same time have nonlinear conditional mean dynamics, that is, $\phi_2 \neq 0$.

2.4 QAR Empirics

We begin the empirical analysis by fitting the QAR(1,1) model to per capita output growth, nominal wage growth, GDP deflator inflation, and federal funds rate data.⁷ The choice of data is motivated by the DSGE model that is being evaluated subsequently. The DSGE model features potentially asymmetric wage and price adjustment costs and we will assess whether the nonlinearities generated by this DSGE model are consistent with the nonlinearities in U.S. data. We report parameter estimates for the QAR model in Section 2.4.1 and explore the properties of the estimated models in Section 2.4.2.

⁷ All series are quarterly and obtained from the FRED database of the Federal Reserve Bank of St. Louis. Output growth is the log difference of real GDP (GDPC96). We compute log differences of civilian noninstitutional population (CNP16OV) and remove a one-sided eight-quarter moving average to smooth population growth. The smoothed population growth series is used to obtain per capita GDP growth rates. Inflation is the log difference of the GDP deflator (GDPDEF). Nominal wage growth is the log difference of compensation per hour in the nonfarm business sector (COMPNFB). As interest rate we use quarterly averages of monthly effective federal funds rates (FEDFUNDS).

Table 2.1: **Estimation Samples and Pre-Samples**

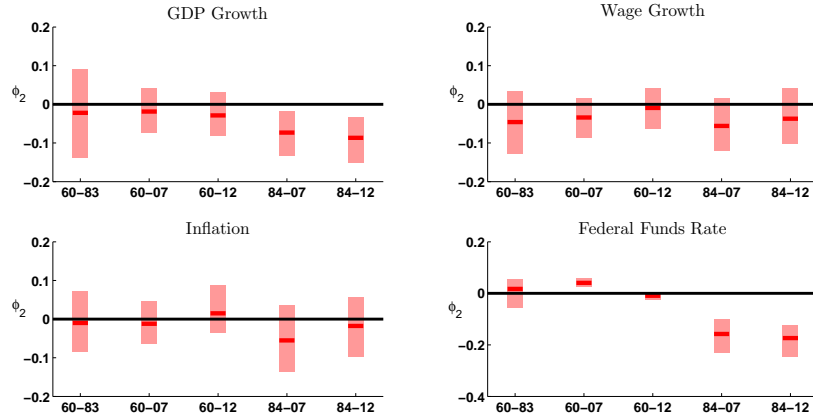
Sample	Estimation Sample	Pre-Sample for Prior
1	1960:Q1 - 1983:Q4	1955:Q1 - 1959:Q4
2	1960:Q1 - 2007:Q4	1955:Q1 - 1959:Q4
3	1960:Q1 - 2012:Q4	1955:Q1 - 1959:Q4
4	1984:Q1 - 2007:Q4	1955:Q1 - 1983:Q4
5	1984:Q1 - 2012:Q4	1955:Q1 - 1983:Q4

2.4.1 Estimation of QAR(1,1) Model on U.S. Data

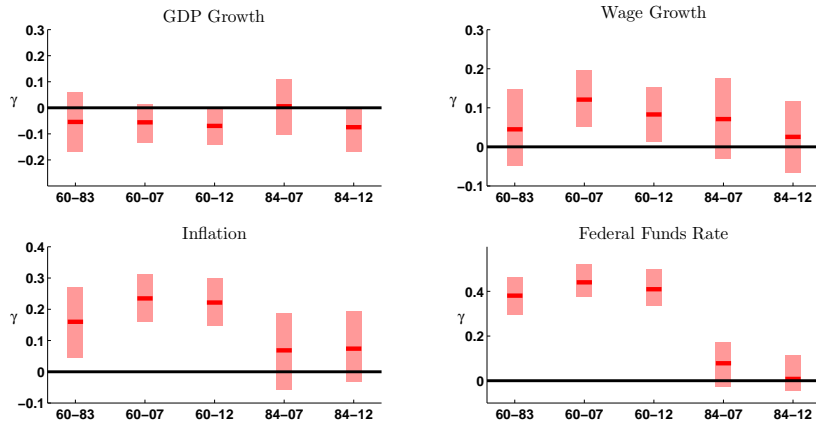
We estimate QAR(1,1) models for output growth, inflation, nominal wage growth, and interest rates using five different sample periods, which are summarized in Table 2.1. The longest sample spans the period from 1960:Q1 to 2012:Q4. This sample includes the high-inflation episode of the 1970s, the subsequent disinflation period, as well as the Great Recession of 2008-09. We then split this sample after 1983:Q4 into a pre-Great-Moderation sample that ranges from 1960:Q1 to 1983:Q4 and a post-Great-Moderation sample from 1984:Q1 to 2012:Q4. Since the 2008-09 recession involves large negative GDP growth rates which may be viewed as outliers, and federal funds rate has been at or near the lower bound of 0% since 2008, we consider two additional samples that exclude the Great Recession data and end in 2007:Q4.

To specify the prior distribution for the QAR parameters we use normal distributions for ϕ_0 , ϕ_2 , and γ . The autoregressive coefficient ϕ_1 is *a priori* also normally distributed, but the normal distribution is truncated to ensure stationarity of the QAR model. Finally, the prior distribution of σ is of the inverted gamma form. We use pre-sample information to parameterize the priors. The pre-sample periods for our five estimation samples are provided in the last column of Table 2.1. Throughout the estimation the tuning constant ϑ in (2.17) is fixed at $\vartheta = 0.1$. The prior distributions for ϕ_1 , the first-order autoregressive coefficient, are centered at

Figure 2.1: **Posterior Medians and Credible Intervals for QAR Parameters**
Parameter ϕ_2



Parameter γ



Notes: The solid bars indicate posterior medians and the shaded boxes delimit 90% equal-tail-probability credible intervals.

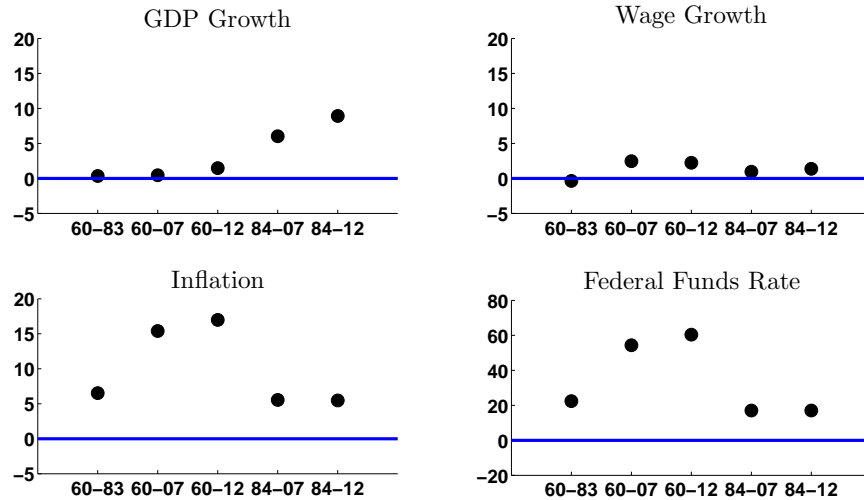
the pre-sample first-order autocorrelations of the four time series. The inverse Gamma distribution of σ is centered at the residual standard deviation associated with the pre-sample estimation of an AR(1) model. Finally, the prior mean of ϕ_0 is specified such that the implied $\mathbb{E}[y_t]$ of the QAR(1,1) model corresponds to the pre-sample mean of the respective time series. The priors for ϕ_2 and γ are centered at zero and have a standard deviation of 0.1. Further details are provided in the Appendix.

Figure 2.1 summarizes the posterior distributions of ϕ_2 and γ . In this figure the

boxes represent the 90% credible intervals and the solid bars indicate posterior medians. Detailed estimation results for the remaining QAR(1,1) parameters are tabulated in the Appendix. The ϕ_2 posteriors for GDP growth using the three samples starting in 1960 are essentially centered at zero with the 90% credible interval covering both positive and negative values. The γ posterior medians for the same samples are slightly negative, around -0.05, but the 90% credible sets also cover positive values, providing only some mild evidence for conditional variance dynamics. For the two post-Great Moderation samples the ϕ_2 estimates drop to about -0.1 and the credible set now excludes zero. The strongest evidence for nonlinearity in GDP growth is present in the 1984-2012 sample, which includes large negative growth rates of output during the Great Recession, in the form of $\phi_2 < 0$ and $\gamma < 0$. Nonlinearities in wages and inflation are reflected in positive estimates of γ . These nonlinearities are most pronounced for the 1960-2007 and the 1960-2012 samples. For the federal funds rate we obtain estimates of ϕ_2 near zero and estimates of γ of about 0.4 for samples that include the pre-1984 observations. For samples that start after the Great Moderation the pattern is reversed: the estimates of ϕ_2 are around -0.2 and the estimates of γ are close to zero. We will discuss the interpretation of these estimates in Section 2.4.2.

Figure 2.2 depicts log marginal likelihood differentials for the QAR(1,1) versus a linear autoregressive AR(1) model. The AR(1) models are estimated by setting $\phi_2 = \gamma = 0$ and using the same priors for ϕ_0 , ϕ_1 , and σ as in the estimation of the QAR(1,1) model. A positive value indicates evidence in favor for the nonlinear QAR(1,1). Under equal prior probabilities, the difference in log marginal data density between two models has the interpretation of log posterior odds. By and large, the marginal likelihood differentials favor the QAR(1,1) specification. The

Figure 2.2: **Log Marginal Data Density Differentials: QAR(1,1) versus AR(1)**



Notes: The figure depicts log marginal data density differentials. A positive number provides evidence in favor of the QAR(1,1) specification.

evidence in favor of the nonlinear specification is strongest for the federal funds rate. Marginal likelihood differentials range from 20 to 60. For output growth there is substantial evidence in favor of the QAR model for the post-Great Moderation samples, whereas for inflation large positive log marginal likelihood differentials are obtained for the 1960-2007 and the 1960-2012 samples. For wage growth the evidence in favor of the nonlinear specification is less strong: log marginal likelihood differentials are around 2.

2.4.2 Properties of the Estimated QAR Models

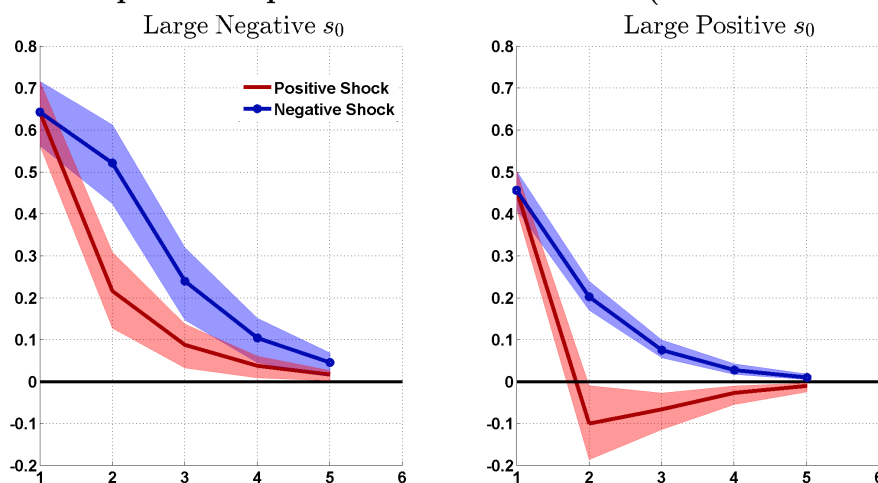
In this section we discuss what the nonlinearities we identified in the previous section mean for each variable. For ease of exposition, we focus on the subsample that “maximizes” the nonlinearities for each variable, which roughly corresponds to picking the subsample that has the largest marginal data density differential between the AR(1) and the QAR(1,1) models.

GDP Growth. Our results show that the posterior medians of ϕ_2 and γ for GDP growth are less than zero. The largest estimates (in absolute terms) are obtained for the 1984-2012 sample. As (2.16) shows, with a negative γ , the periods of below-mean growth (likely to be recessions) are also periods where volatility is higher, which is a well-known business cycle fact. A negative ϕ_2 , along with a negative γ also imply that the response to shocks is a function of the initial state s_0 . Using the formulas in (2.15), Figure 2.3 depicts the absolute responses of GDP growth to a negative and a positive one-standard deviation shock. In the left panel, we assume that the initial state s_0 takes on large negative values whereas the responses in the right panel condition on large positive s_0 's.⁸ This figure highlights that regardless of the initial state negative shocks are more persistent than positive shocks. Moreover, both shocks are more persistent in recessions. Combining these results, we deduce that multiple positive shocks are necessary to recover from a recession, while a small number of negative shocks can generate a recession. In other words output losses during recessions are sharp and recoveries are slow.

The impulse response findings are consistent with the time-series plot of GDP growth, which is provided in the top left panel of Figure 2.4. In this figure shaded areas indicate NBER recessions and the solid vertical line indicates the year 1984, which is the starting point of two of the five estimation samples. The unconditional mean of the variable is shown as a horizontal dashed line. Focusing on the post-1983 sample, the most extreme observations are all during recessions, confirming the effect of $\gamma < 0$. Looking at the quarters just prior to and just after NBER recessions, we see that the declines in GDP growth are always very sharp but the

⁸To obtain the s_0 for a given draw, we compute a two-period moving average to smooth the s_t series and use the minimum and the maximum values for this smoothed series.

Figure 2.3: Impulse Responses of GDP Growth (In Absolute Terms)



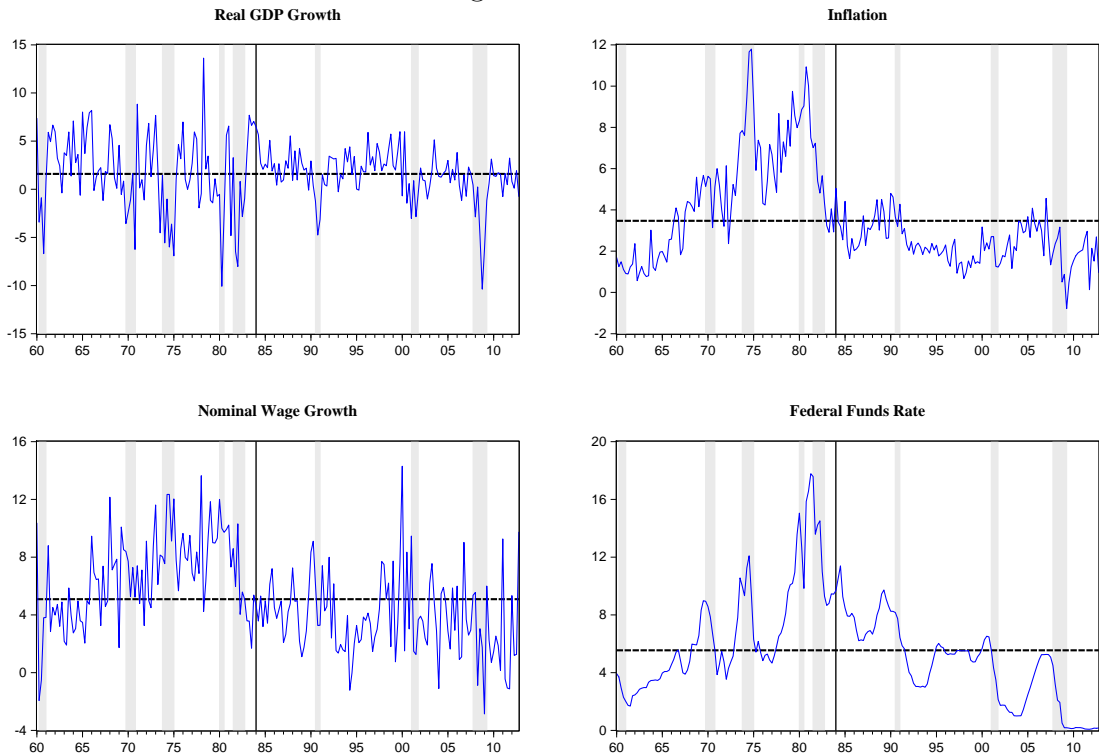
Notes: 1984-2012 sample. Solid and dashed lines correspond to median impulse responses to one-standard-deviation shocks and shaded bands represent 60% credible intervals (equal tail probability). To initialize the latent state s_0 we compute two-quarter moving averages based on the states associated with the estimated QAR model and calculate the minimum and the maximum of the smoothed series. For the left panel (large negative s_0) the initialization is based on the minimum and in the right panel (large positive s_0) it is based on the maximum.

recoveries, defined as getting back to and staying at pre-recession level, take much longer.

It is easy to see why the nonlinearities identified in the samples starting in 1984 are not as pronounced in the samples that start in 1960. First, prior to 1984 there are more episodes of large positive GDP growth rates. These are, in absolute terms, as large as the negative growth rates observed between 1960 and 2012. Thus, recessions are not necessarily periods of higher volatility. Second, the recoveries from recessions are as sharp as the entries, not displaying the clear asymmetry in the later sample. These findings explain why a linear AR(1) is a good description of GDP growth pre-1984.⁹

⁹Qualitatively, our results for GDP growth are in line with findings by Brunner (1997), who estimated three nonlinear models for real Gross National Product (GNP). Based on a sample from 1947 to 1990 the author obtained strong evidence of countercyclical volatility, that is recessions are periods of high volatility. Moreover, Brunner (1997) detects nonlinear conditional mean dynamics: according to the impulse responses the effects of a negative shock accumulate faster than those of a positive shock, in line with our findings. Similarly, McKay and Reis (2008) find that the brevity and violence of contractions and expansions are about equal in a sample

Figure 2.4: **Data**



Notes: All variables are in annualized percentage units. Shaded areas indicate NBER recessions and the dashed horizontal line represents the sample mean of the series.

Inflation and Wage Growth. The nonlinearities in the inflation dynamics are most pronounced in the 1960-2007 sample with $\gamma > 0$ and $\phi_2 = 0$. Once again referring to the conditional variance formula in (2.16), we conclude that periods of above-mean inflation are associated with high volatility. In fact, the top right panel of Figure 2.4 shows that the period from 1970 to 1980 has high and volatile inflation. A similar conclusion can be reached in the post-1983 sample but to a lesser degree. The bottom left panel of Figure 2.4 shows that nominal wage growth displays properties similar to inflation. In the 1960-2007 sample, which is also the relevant one for nominal wage growth, volatility tends to be high when the level is high. Because nominal wage growth is more volatile than inflation, and there are many large negative observations, the estimate of γ is smaller for the former than encompasses our longest sample, once again in line with our results.

series.

Federal Funds Rate. The bottom right panel of Figure 2.4 shows the plot of the federal funds rate. Based on the QAR(1,1) estimation results, there are two samples with strong nonlinearities. In the 1960-2007 sample, we find a positive γ . As was the case for inflation and nominal wage growth, this is due to the observations from late 1960s to mid 1980s, which are typically above the unconditional mean and thus volatility is higher when the level is higher. For the 1984-2012 sample we find $\phi_2 < 0$ and $\gamma = 0$. In this period the extreme observations are equally likely to be positive or negative and thus $\gamma = 0$ is reasonable. $\phi_2 < 0$ implies that interest rate fall faster than they rise. This seems to be consistent with Federal Reserve's operating procedures in the post-1983 sample and it can have two separate explanations. First, the Federal Reserve may have an asymmetric policy rule, in which reactions to deviations from targets may depend on the sign of the deviation. This can happen, for example, if the Federal Reserve is risk averse and wants to avoid recessions: when GDP growth falls, the central bank is willing to cut the policy rate quickly, but when GDP growth starts to improve, it is reluctant to increase the policy rate immediately. Second, the variables that the Federal Reserve track may have asymmetries themselves. Given our finding that $\phi_2 < 0$ for GDP in this sample, the second explanation is certainly reasonable. There is some evidence about the first explanation as well. For example [Dolado et al. \(2004\)](#) and [Cukierman and Muscatelli \(2008\)](#) estimate a non-linear Taylor rule using GMM and find that U.S. monetary policy is better characterized by a nonlinear policy rule after 1983, especially with respect to the reaction to output gap deviations.

To sum up, the estimation of QAR(1,1) models provides evidence of interesting

and substantial nonlinearities in the U.S. macroeconomic time series. For the two samples that start in 1960 and extend beyond the 1990s the nonlinearities are reflected in the run-up in inflation in the 1970s, with spill-overs to nominal wage growth and the federal funds rate. In the shorter post-1983 samples, there are two important nonlinearities – the asymmetries in GDP growth, which is particularly pronounced if the 2008-09 recession is included in the sample, and the federal funds rate. In the remainder of this paper we examine whether a DSGE model with asymmetric adjustment costs for prices and wage can possibly generate the nonlinearities documented in this section.

2.5 A DSGE Model with Asymmetric Price and Wage Adjustment Costs

By now there exists a large empirical literature on the estimation of New Keynesian DSGE models, including small-scale models such as the one studied in [Lubik and Schorfheide \(2004\)](#) and [Rabanal and Rubio-Ramírez \(2005\)](#), as well as variants of the [Smets and Wouters \(2007a\)](#) model. It turns out that in the absence of zero-lower-bound constraints on nominal interest rates, high degrees of risk aversion, large shocks, or exogenous nonlinearities such as stochastic volatility, these models do not generate strong nonlinearities, in the sense that first-order and higher-order perturbation approximations deliver very similar decision rules. In order to generate stronger nonlinearities that can be captured in higher-order perturbation approximations, we consider a model with potentially asymmetric price and wage adjustment costs that builds on [Kim and Ruge-Murcia \(2009\)](#).

The model economy consists of a final good producing firm, a continuum of inter-

mediate goods producing firms, a representative household, and a monetary as well as a fiscal authority. The model replaces Rotemberg-style quadratic adjustment cost functions by linear adjustment cost functions, which can capture downward (as well as upward) nominal price and wage rigidities. Our model abstracts from capital accumulation. In the subsequent empirical analysis we examine whether the asymmetric adjustment costs can generate the observed nonlinearities in inflation and wage growth and whether the effects of asymmetric adjustment costs translate into nonlinearities in GDP growth and the federal funds rate. In a nutshell, asymmetric price adjustments should lead to asymmetric quantity adjustments. To the extent that the central bank sets interest rates in response to inflation and output movements, nonlinearities in the target variables may translate into nonlinearities in the interest rate itself.

Final Good Production. The perfectly competitive, representative, final good producing firm combines a continuum of intermediate goods indexed by $j \in [0, 1]$ using the technology

$$Y_t = \left(\int_0^1 Y_t(j)^{1-\lambda_{p,t}} dj \right)^{\frac{1}{1-\lambda_{p,t}}}. \quad (2.19)$$

Here $1/\lambda_{p,t} > 1$ represents the elasticity of demand for each intermediate good. The firm takes input prices $P_t(j)$ and output prices P_t as given. Profit maximization implies that the demand for intermediate goods is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\lambda_{p,t}} Y_t. \quad (2.20)$$

The relationship between intermediate goods prices and the price of the final good

is

$$P_t = \left(\int_0^1 P_t(j)^{\frac{\lambda_{p,t-1}}{\lambda_{p,t}}} dj \right)^{\frac{\lambda_{p,t}}{\lambda_{p,t-1}}}. \quad (2.21)$$

Intermediate Goods. Intermediate good j is produced by a monopolist who has access to the following production technology:

$$Y_t(j) = A_t H_t(j), \quad (2.22)$$

where A_t is an exogenous productivity process that is common to all firms. Intermediate good producers buy labor services $H_t(j)$ at a nominal price of W_t . Moreover, they face nominal rigidities in terms of price adjustment costs. These adjustment costs, expressed as a fraction of the firm's revenues, are defined by the linex function

$$\Phi_p(x) = \varphi_p \left(\frac{\exp(-\psi_p(x - \pi)) + \psi_p(x - \pi) - 1}{\psi_p^2} \right), \quad (2.23)$$

where we let $x = P_t(j)/P_{t-1}(j)$ and π is the steady state inflation rate associated with the final good. The parameter ϕ_p governs the overall degree of price stickiness and ψ_p controls the asymmetry of the adjustment costs. Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its labor inputs $H_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left(\frac{P_{t+s}(j)}{P_{t+s}} \left(1 - \Phi_p \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} \right) \right) Y_{t+s}(j) - \frac{1}{P_{t+s}} W_{t+s} H_{t+s}(j) \right) \right]. \quad (2.24)$$

Here, $Q_{t+s|t}$ is the time t value of a unit of the consumption good in period $t + s$ to the household, which is treated as exogenous by the firm.

Labor Packers. Labor services used by intermediate good producers are supplied by a perfectly competitive labor packer. The labor packer aggregates the

imperfectly substitutable labor services of households according to the technology:

$$H_t = \left(\int_0^1 H_t(k)^{1-\lambda_w} dk \right)^{\frac{1}{1-\lambda_w}}. \quad (2.25)$$

The labor packer chooses demand for each type of labor in order to maximize his profits, taking as given input prices $W_t(k)$ and output prices W_t . Optimal labor demand is then given by:

$$H_t(k) = \left(\frac{W_t(k)}{W_t} \right)^{-\frac{1}{\lambda_w}} H_t. \quad (2.26)$$

Perfect competition implies that labor cost W_t and nominal wages paid to workers are related as follows:

$$W_t = \left(\int_0^1 W_t(k)^{\frac{\lambda_w-1}{\lambda_w}} dk \right)^{\frac{\lambda_w}{\lambda_w-1}}. \quad (2.27)$$

Households. Each household consists of a continuum of family members indexed by k . The family members provide perfect insurance to each other which equates their marginal utility in each state of the world. A household member of type k derives utility from consumption $C_t(k)$ relative to a habit stock. We assume that the habit stock is given by the level of technology A_t . This assumption ensures that the economy evolves along a balanced growth path even if the utility function is additively separable in consumption, real money balances, and leisure. The household member derives disutility from hours worked $H_t(k)$ and maximizes

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}(k)/A_{t+s})^{1-\tau} - 1}{1-\tau} - \chi_H \frac{H_{t+s}^{1+1/\nu}(k)}{1+1/\nu} \right) \right], \quad (2.28)$$

where β is the discount factor, $1/\tau$ is the intertemporal elasticity of substitution, χ_H is a scale factor that determines the steady state hours worked. Moreover ν is

the Frisch labor supply elasticity.

The household is a monopolist in the supply of labor services. As a monopolist, he chooses the nominal wage and labor taking as given the demand from the labor packer. We assume that labor market frictions induce a cost in the adjustment of nominal wages. Adjustment costs are payed as a fraction of labor income and they have the same linex structure assumed for prices

$$\Phi_w(x) = \varphi_w \left(\frac{\exp(-\psi_w(x - \gamma\pi)) + \psi_w(x - \gamma\pi) - 1}{\psi_w^2} \right), \quad (2.29)$$

where $x = W_t(k)/W_{t-1}(k)$ and $\gamma\pi$ is the growth rate of nominal wages where γ is the average growth rate of technology as we define below. Beside his labor choices, the household member faces a standard consumption/saving trade-off. He has access to a domestic bond market where nominal government bonds $B_t(k)$ are traded that pay (gross) interest R_t . Furthermore, he receives aggregate residual real profits $D_t(k)$ from the firms and has to pay lump-sum taxes T_t . Thus, the household's budget constraint is of the form

$$\begin{aligned} & P_t C_t(k) + B_t(k) + T_t \\ &= W_t(k) H_t(k) \left(1 - \Phi_w \left(\frac{W_t(k)}{W_{t-1}(k)} \right) \right) + R_{t-1} B_{t-1}(k) + P_t D_t(k) + P_t SC_t, \end{aligned}$$

where $SC_t(k)$ is the net cash inflow that household k receives from trading a full set of state-contingent securities. We denote the the Lagrange multiplier associated with the budget constraint by λ_t . The usual transversality condition on asset accumulation applies, which rules out Ponzi schemes.

Monetary and Fiscal Policy. Monetary policy is described by an interest rate

feedback rule of the form

$$R_t = R_t^*{}^{1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}}, \quad (2.30)$$

where $\epsilon_{R,t}$ is a monetary policy shock and R_t^* is the (nominal) target rate. Our specification of R_t^* implies that the central bank reacts to inflation and deviations of output growth from its equilibrium steady state γ :

$$R_t^* = r\pi^* \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\psi_2}. \quad (2.31)$$

Here r is the steady state real interest rate, π_t is the gross inflation rate defined as $\pi_t = P_t/P_{t-1}$, and π^* is the target inflation rate, which in equilibrium coincides with the steady state inflation rate. The fiscal authority consumes a fraction ζ_t of aggregate output Y_t , where $\zeta_t \in [0, 1]$ follows an exogenous process. The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus).

The model economy is perturbed by four exogenous processes. Aggregate productivity evolves according to

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t, \quad \text{where} \quad \ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}. \quad (2.32)$$

Thus, on average technology grows at the rate γ and z_t captures exogenous fluctuations of the technology growth rate. Define $g_t = 1/(1 - \zeta_t)$. We assume that

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \epsilon_{g,t}. \quad (2.33)$$

The inverse demand elasticity for intermediate goods evolves according to a first

order autoregressive processes in logs:

$$\ln \lambda_{p,t} = (1 - \rho_p) \ln \lambda_p + \rho_p \ln \lambda_{p,t-1} + \epsilon_{p,t}. \quad (2.34)$$

Finally, the monetary policy shock $\epsilon_{R,t}$ is assumed to be serially uncorrelated. The five innovations are independent of each other at all leads and lags and are normally distributed with means zero and standard deviations σ_z , σ_g , σ_p , and σ_R , respectively.

2.6 Estimation and Evaluation of DSGE Model

The estimation and evaluation of the DSGE model proceeds in three steps. In Section 2.6.1 the DSGE model is estimated for two samples: 1960-2007 and 1984-2007, using the same series that were studied in Section 2.4. In Section 2.6.2 we use posterior predictive checks that are based on posterior mean estimates of QAR(1,1) parameters to assess whether the nonlinearities captured in the second-order approximated DSGE model are commensurate with the nonlinearities captured by the QAR(1,1) model. Finally, we assess the effect of adjustment cost asymmetries on the model's ability to generate nonlinear inflation and wage growth dynamics in Section 2.6.3.

2.6.1 DSGE Model Estimation on U.S. Data

The second step in the empirical analysis consists of estimating the DSGE model based on the same data that was used to estimate the QAR(1,1) models in Section 2.4. The marginal prior distributions for the DSGE model parameters are summarized in Table ???. We use pre-sample evidence to quantify *a priori* beliefs

about the average growth rate of the economy, as well as average inflation and real interest rates. We use the same priors for both samples. The prior mean for τ implies a risk aversion coefficient of 2. Our prior for the Frisch labor supply elasticity covers some of the low values estimated in the microeconometrics literature as well as a value of 2 advocated in the real-business-cycle literature based on steady-state considerations. The prior for the price-adjustment-cost parameter φ_p is specified indirectly through a prior for the slope $\kappa(\varphi_p)$ of the New Keynesian Phillips curve. This prior encompasses values that imply an essentially flat as well as a fairly steep Phillips curve. The prior for the wage rigidity is directly specified on φ_w and spans values in the range from 0 to 30. The priors for the asymmetry parameters ψ_p and ψ_w are centered at zero (symmetric adjustment costs) and have a large variance, meaning that the asymmetries could potentially be strong. We do not restrict the signs of ψ_p and ψ_w , i.e., allowing *a priori* for both downward and upward price and wage rigidities. The priors for the monetary policy rule coefficients are centered at 1.5 (reaction to inflation), 0.2 (output growth), and 0.5 (interest rate smoothing). Finally, we use priors for the parameters associated with the exogenous shock processes then generate *a priori* reasonable magnitudes for the persistence and volatility of the observables.

The DSGE model presented in Section 2.5 is solved using a second-order approximation, which leads to a nonlinear state-space representation. We use the particle filter developed in Fernández-Villaverde and Rubio-Ramírez (2007b) to evaluate the likelihood function of the DSGE model. To facilitate the likelihood evaluation with the particle filter, the measurement equation contains mean-zero *iid* Gaussian measurement errors. The measurement error variances are set equal to 10% of the sample variances of GDP growth, inflation, interest rates, and nominal

Table 2.2: **Posterior Estimates for DSGE Model Parameters**

Parameter	Distribution	Prior		Posterior			
		Para (1)	Para (2)	1960:Q1-2007:Q4		1984:Q1-2007:Q4	
				Mean	90% Interval	Mean	90% Interval
$400 \left(\frac{1}{\beta} - 1 \right)$	Gamma	2.00	1.00	0.47	[0.08, 1.04]	1.88	[0.47, 3.01]
π^A	Gamma	3.00	1.00	3.19	[2.57, 3.84]	3.34	[2.44, 4.32]
γ^A	Gamma	2.00	1.50	2.04	[1.57, 2.77]	1.98	[1.59, 2.36]
τ	Gamma	2.00	1.00	4.83	[2.75, 7.28]	4.10	[2.35, 6.06]
ν	Gamma	0.50	1.00	0.37	[0.21, 0.52]	0.10	[0.05, 0.17]
$\kappa(\varphi_p)$	Gamma	0.30	0.20	0.02	[0.01, 0.04]	0.21	[0.12, 0.35]
φ_w	Gamma	15.0	7.50	18.7	[8.47, 38.1]	11.7	[5.34, 20.2]
ψ_w	Uniform	-200	200	67.4	[33.2, 99.5]	59.4	[21.7, 90.9]
ψ_p	Uniform	-300	300	150	[130, 175]	165	[130, 192]
ψ_1	Gamma	1.50	0.50	1.77	[1.51, 2.12]	2.57	[1.93, 3.26]
ψ_2	Gamma	0.20	0.10	1.41	[0.97, 1.85]	0.79	[0.42, 1.18]
ρ_r	Beta	0.50	0.20	0.81	[0.23, 0.72]	0.73	[0.64, 0.80]
ρ_g	Beta	0.80	0.10	0.95	[0.92, 0.98]	0.96	[0.94, 0.98]
ρ_z	Beta	0.20	0.10	0.48	[0.23, 0.72]	0.07	[0.01, 0.20]
ρ_p	Beta	0.60	0.20	0.89	[0.86, 0.94]	0.90	[0.76, 0.98]
$100\sigma_r$	InvGamma	0.20	2.00	0.17	[0.14, 0.21]	0.17	[0.12, 0.23]
$100\sigma_g$	InvGamma	0.75	2.00	0.88	[0.58, 1.29]	0.83	[0.49, 1.30]
$100\sigma_z$	Beta	0.75	2.00	0.44	[0.31, 0.62]	0.47	[0.38, 0.56]
$100\sigma_p$	Beta	0.75	2.00	2.62	[0.46, 7.23]	6.54	[4.56, 9.37]

Notes: $1/g$ is fixed at 0.85. Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; and s and ν for the Inverse Gamma distribution, where $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. The effective prior is truncated at the boundary of the determinacy region. As 90% credible interval we are reporting the 5th and 95th percentile of the posterior distribution.

wage growth. Posterior inference is implemented with a single-block RWM algorithm, described in detail in [An and Schorfheide \(2007\)](#). Theoretical convergence properties of so-called particle MCMC approaches are established in [Andrieu et al. \(2010\)](#).

Posterior summary statistics for the DSGE model parameters are reported in [Table 2.2](#). The most interesting and important estimates are the ones of the asymmetry parameters in the price and wage adjustment cost function. The wage and price rigidity estimates differ substantially across subsamples. For instance, the estimated slope of the New Keynesian Phillips curve is 0.02 for the 1960-2007 sample, whereas it increases to 0.2 for the post-1983 sample. Likewise, the estimated wage rigidity is larger over the long sample. The positive estimates of

ψ_p and ψ_w imply that it is more expensive to lower prices and wages than to raise them and that the asymmetry in prices is more pronounced than in wages. The asymmetry of the adjustment costs is more pronounced for prices ($\hat{\psi}_p$ equals 150 and 165, respectively) than for wages ($\hat{\psi}_w$ equals 67 and 59, respectively).

Compared to the estimates reported by [Kim and Ruge-Murcia \(2009\)](#) and [Abbritti and Fahr \(2013\)](#) who report estimates of $\hat{\psi}_w = 3,844$ and $\hat{\psi}_w = 24,700$, respectively, our estimates of the ψ_w 's are considerably smaller.¹⁰ In our experience such large values of ψ_w lead to a clear deterioration of the model's ability to track U.S. data. Moreover, the second-order solution of the DSGE model relies on a third-order approximation of the linex cost function which becomes very inaccurate for large values of ψ . In particular, we found that for values of ψ_w above 500 the the adjustment costs for large positive wage changes (that lie in the support of the ergodic distribution) would become negative due to the polynomial approximation of the linex function.

We estimate the risk-aversion parameter τ to be fairly large, around 4, and the Frisch labor supply elasticity to be fairly low, ranging from 0.1 to 0.4. The estimates of ν are in line with those reported in [Ríos-Rull et al. \(2012b\)](#). The policy rule coefficient estimates are similar to the ones reported elsewhere in the DSGE model literature. The coefficient ψ_1 on inflation is larger for the post-1983 sample, which is consistent with the view that after the Volcker disinflation the Federal Reserve Bank has responded more aggressively to inflation movements.

¹⁰[Kim and Ruge-Murcia \(2009\)](#) estimated their DSGE model Simulated Method of Moments (SMM). While they also used consumption and hours worked data in their estimation, the SMM objective function only includes second moments. The authors find that the covariance of consumption and hours worked, respectively, with wage growth plays a crucial role for their estimation. [Abbritti and Fahr \(2013\)](#) use a calibration approach to parameterize their model. Given their preferred calibration of the exogenous technology, discount-factor, and monetary-policy shocks, they find that a very large value of ψ_w is needed to match the volatility and skewness of wage growth observed in the data.

The government spending shock, which should be viewed as a generic demand shock, is the most persistent among the serially correlated exogenous shocks: ρ_g is approximately 0.95. The estimated autocorrelation ρ_z of technology growth shock, which generates most of the serial correlation in output growth rates, drops from 0.48 for the long sample to 0.07 for the post-1983 sample.

2.6.2 Posterior Predictive Checks

We proceed by examining whether QAR(1,1) parameter estimates obtained from data that are simulated from the estimated DSGE model are similar to the estimates reported in Section 2.4 computed from actual data. This comparison is formalized through a posterior predictive check. The role of posterior predictive checks in Bayesian analysis is discussed in the textbooks by Lancaster (2004) and Geweke (2005b) and reviewed in the context of the evaluation of DSGE models in Del Negro and Schorfheide (2011b). The posterior predictive checks is implemented with the following algorithm.

Posterior Predictive Check. Let $\theta^{(i)}$ denote the i 'th draw from the posterior distribution of the DSGE model parameter θ .

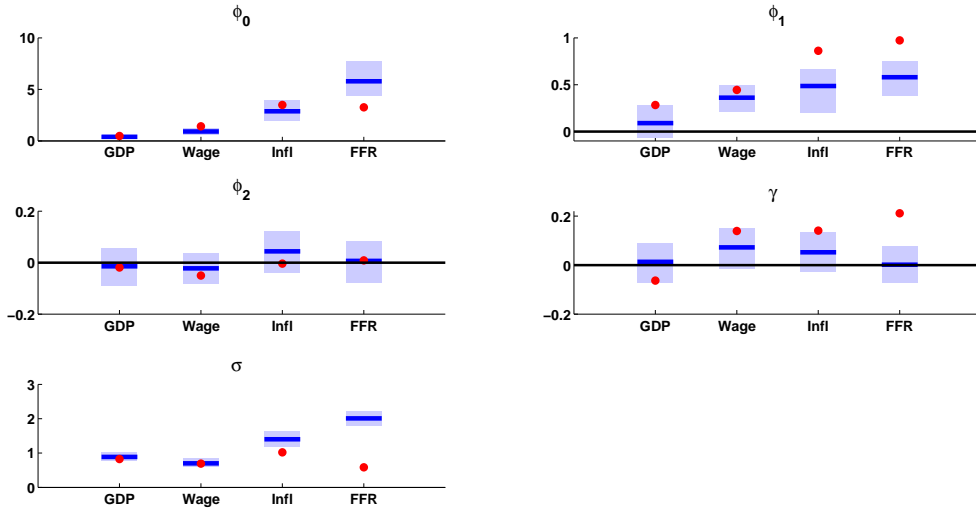
- i) For $i = 1$ to n :
- ii) Conditional on $\theta^{(i)}$ simulate a pre-sample of length T_0 and an estimation sample of size T from the DSGE model. The second-order approximated DSGE model is simulated using the pruning algorithm described in Kim et al. (2008). A Gaussian *iid* measurement error is added to the simulated data. The measurement error variance is identical to the one imposed during the estimation of the DSGE model. Denote the simulated data by $Y_{-T_0+1:T}^{(i)}$.

- iii) Based on the simulated trajectory $Y_{-T_0+1:T}^{(i)}$ estimate the QAR(1,1) model as described in Section 2.4.1. The prior for the QAR(1,1) parameters is elicited from the presample $Y_{-T_0+1:0}^{(i)}$ and the posterior is based on $Y_{1:T}^{(i)}$. Denote the posterior median estimates of the QAR parameters by $\mathcal{S}(Y_{-T_0+1:T}^{(i)})$.
- iv) The empirical distribution of $\{\mathcal{S}(Y_{-T_0+1:T}^{(i)})\}_{i=1}^n$ approximates the posterior predictive distribution of $\mathcal{S}|Y_{-T_0+1:T}$. Examine how far the actual value $\mathcal{S}(Y_{1:T})$, computed from U.S. data, lies in the tail of its predictive distribution. \square

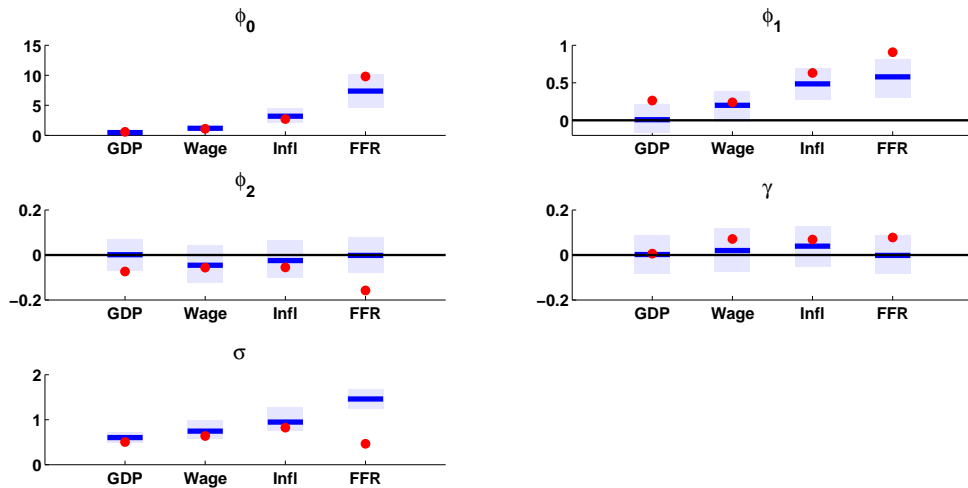
The predictive check is carried out for each QAR(1,1) parameter estimate separately. The results are summarized in Figure 2.5. The top panel corresponds to the 1960-2007 sample, whereas the bottom panel contains the results from the 1984-2007 sample. The red dots signify the posterior median estimates obtained from U.S. data and correspond to the horizontal bars in Figure 2.1. The blue rectangles delimit the 90% credible intervals associated with the posterior predictive distributions and the solid horizontal bars indicate the medians of the predictive distributions. The length of the credible intervals reflects both parameter uncertainty, i.e., the fact that each trajectory $Y_{-T_0+1:T}^{(i)}$ is generated from a different parameter draw $\theta^{(i)}$, and sampling uncertainty, meaning that if one were to hold the parameters θ fixed, the variability in the simulated finite-sample trajectories generates variability in posterior mean estimates. Because the posterior variance of the DSGE model parameters is fairly small, these intervals mostly capture sampling variability. Accordingly, they tend to be larger in the bottom panel (short sample) than in the top panel (long sample).

By and large the QAR parameter estimates for output growth, wage growth, and inflation from model-generated data are very similar to the ones obtained from

Figure 2.5: Predictive Checks Based on QAR(1,1) Estimates
1960-2007 Sample



1984-2007 Sample



Notes: Dots correspond to posterior median estimates from U.S. data. Solid horizontal lines indicate medians of posterior predictive distributions for parameter estimates and the boxes indicate 90% credible associated with the posterior predictive distributions.

actual data – in the sense that most of actual estimates do not fall far in the tails of their respective posterior predictive distributions. Only interest rates exhibit large discrepancies between actual and model-implied estimates of the QAR(1,1) parameters.

Overall, the estimated DSGE model does not generate very strong nonlinear-

ties. Posterior predictive distributions for $\hat{\phi}_2$ and $\hat{\gamma}$ typically cover both positive and negative values. The only exceptions are the predictive distributions for wage growth and inflation $\hat{\gamma}$ conditional on the 1960-2007 sample, which imply that $\hat{\gamma}$ is positive. Recall from Table 2.2 that for this sample we estimate sizeable adjustment costs ($\hat{\kappa} = 0.02$ and $\hat{\varphi}_w = 18.7$). Moreover, the asymmetry parameter estimates are substantially larger than zero: $\hat{\psi}_p = 150$ and $\hat{\psi}_w = 67.4$. The model-implied positive estimates of γ imply that high inflation and wage-growth rates are associated with high levels of volatility, which describes the experience of the U.S. economy in the 1970s and early 1980s. However, the nonlinear inflation and wage dynamics do not generate any spillovers to nonlinearities in GDP growth or the interest rate. Figure 2.5 indicates that the predictive distribution for the corresponding $\hat{\phi}_2$ and $\hat{\gamma}$ are centered at zero. For the 1984-2007 sample the overall magnitude of the estimated adjustment costs are smaller, which flattens the adjustment cost functions, makes the asymmetries less important for equilibrium dynamics, and shifts the predictive distribution for the inflation and wage growth $\hat{\gamma}$'s toward zero.

There are two types of nonlinearities present in the data that the estimated DSGE model does not predict. First, for the short sample $\hat{\phi}_2$ for GDP growth is negative, because the post-1983 sample exhibits pronounced drop in output growth during the recessions but does not feature positive growth rates of similar magnitudes in early parts of expansions. Second, the interest rate exhibits strong nonlinearities in the data, i.e., a large positive $\hat{\gamma}$ in the 1960-2007 sample and a large negative $\hat{\phi}_2$ in the 1984-2007 sample, that the DSGE model is unable to reproduce.

To sum up, of the nonlinearities we identified in Section 2.4, the only ones the

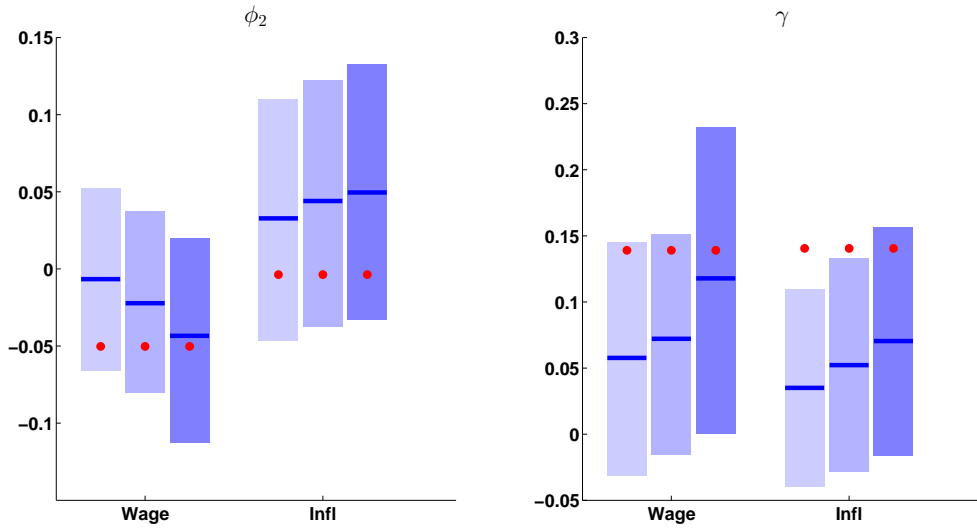
DSGE model seems to be able to deliver are the conditional heteroskedasticity in inflation and nominal wage growth. It is able to do so relying on the asymmetric adjustment costs which penalize downward adjustments more than upward adjustments. However, while ex-ante reasonable, these asymmetries in prices do not spill over to quantities. Moreover, since the interest-rate feedback rule in the model does not feature any asymmetries, which would result from the central bank having an asymmetric loss function, and since there are no asymmetries in GDP growth in the model, the policy instrument does not display the asymmetry we identified in the data.

2.6.3 The Role of Asymmetric Adjustment Costs

To further study the role of asymmetric adjustment costs in generating nonlinear wage and inflation dynamics we repeat the predictive checks based on $\hat{\phi}_2$ and $\hat{\gamma}$ for alternative choices of ψ_p and ψ_w . We focus on the 1960-2007 sample because the nonlinearities are more pronounced than in the post-1983 sample. For each draw $\theta^{(i)}$ from the posterior distribution of the DSGE model parameters, we replace $\psi_p^{(i)}$ and $\psi_w^{(i)}$ by alternative values $\bar{\psi}_p$ and $\bar{\psi}_w$. In particular, we consider an elimination of the asymmetries, i.e., $\bar{\psi}_p = \bar{\psi}_w = 0$ and an increase to $\bar{\psi}_p = \bar{\psi}_w = 300$. The results are plotted in Figure 2.6. A decrease of the asymmetry in the adjustment costs moves the predictive distributions of $\hat{\phi}_2$ and $\hat{\gamma}$ toward zero, whereas an increase shifts them further away from zero. Relative to the overall width of the predictive intervals the location shifts are fairly small. This highlights that a precise measurement of nonlinearities is very difficult using quarterly observations.

For nominal wage growth the increase in the asymmetry parameters essentially eliminates the gap between the median of the posterior predictive distributions

Figure 2.6: **Effect of Adjustment Costs on Nonlinearities**



Notes: 1960-2007 sample. Box plots of posterior predictive distribution for ϕ_2 and γ estimates for different parameter values of the adjustment cost functions. No Asymmetric Costs is $\psi_p = \psi_w = 0$ (light blue); High Asymmetric Costs is $\psi_p = \psi_w = 300$ (dark blue). Large Dots correspond to posterior median estimates based on U.S. data.

for $\hat{\phi}_2$ and $\hat{\gamma}$ and the estimates obtained from actual data, which are -0.05 and 0.14, respectively. For inflation the median of the predictive distributions for $\hat{\phi}_2$ and $\hat{\gamma}$ shift slightly upward, toward 0.05 and 0.06, respectively. This implies that the actual value of $\hat{\phi}_2$ lies further in the tail of the predictive distribution if ψ_w is increased, whereas the actual value of $\hat{\gamma}$ is less far in the tails. While an increase of ψ_w improves the outcome of the predictive check constructed from the QAR parameter estimates for nominal wage growth, judging from the overall posterior distribution, the increased asymmetries lead to a deterioration of fit in other dimensions of the model, which is why the posterior estimates for ψ_p and ψ_w are only about 150 and 68, respectively.

2.7 Conclusion

Building on the specification of generalized autoregressive models, bilinear models, and LARCH models, this paper uses a perturbation approximation of a nonlinear difference equation to obtain a new class of nonlinear time series models that can be used to assess nonlinear DSGE models. We use these univariate QAR(1,1) models to identify nonlinearities in the U.S. data and to construct predictive checks to assess a DSGE model ability to capture nonlinearities that are present in the data. The QAR(1,1) estimates obtained from U.S. data highlight nonlinearities in output growth, inflation, nominal wage growth, and interest rate dynamics. Output growth displays sharp declines and slow recoveries in the post-1983 sample. Inflation and nominal wage growth both display conditional heteroskedasticity in the 1960-2007 sample. Finally, downward adjustments in the federal funds rate seem to be easier than upwards adjustments in the post-1983 sample.

Among the nonlinearities identified through the estimation of the QAR models, the only ones that our estimated DSGE model seems to be able to capture, are the conditional heteroskedasticity in inflation and nominal wage growth. The model does so by relying on the asymmetric adjustment costs which penalize downward adjustments more than upward adjustments. The model is not able to generate the apparent nonlinearities in output growth and the federal funds rate.

The tools developed in this paper can be used to identify nonlinearities in any time series and doing this for other key series such as labor market variables in the U.S., and for key variables in other countries will be a useful exercise. The predictive checks simply require a simulation from the model and can be applied to any model, whether or not it is estimated, and should be a part of the toolbox for

researchers working with DSGE models. Finally, we leave multivariate extensions of the QAR model, where the main challenge is to cope with the dimensionality of the model, to future research.

Chapter 3

Risk, Economic Growth and the Market Value of U.S. Corporations

3.1 Introduction

During the postwar period, the U.S. economy experienced large movements in the value of firms. The market value of U.S. corporations relative to gross domestic product (henceforth, *value-output ratio*) went through a slump during the 1970s followed by a large increase throughout the 1980s and 1990s until the marked decline of the last decade.¹ Researchers have devoted a great effort to understanding the origins of these *medium-term fluctuations*.² A widespread view among them is that the value of corporations should be particularly sensitive to variables

¹We define the market value of the U.S. corporate sector to be the sum of outstanding equities and net debt liabilities. See Appendix D.5 for details in the construction of this series using *Flow of Funds* data.

²In this paper, we refer to medium-term fluctuations as the movements in the medium frequency component of a time series. As in Comin and Gertler (2006), the medium term consists of frequencies between 32 and 200 quarters.

that drive expectations of future corporate payouts and that influence the rate at which investors discount them. Within this context, research pioneered by [Barsky and De Long \(1993\)](#) and [Bansal and Yaron \(2004\)](#) suggests that economic fundamentals affecting the long-run growth of corporate payouts and its risk should be responsible for these large swings in the stock market.

Motivated by this view, recent studies have investigated the links between aggregate productivity and asset prices. While [Beaudry and Portier \(2006\)](#) and [Croce \(2012\)](#) have documented a strong sensitivity of stock prices to the mean of total factor productivity (TFP) growth, the theories mentioned above also suggest that variation in other moments of aggregate productivity may be relevant. Moreover, the documented empirical correlation between TFP growth and stock prices does not reveal a direction of causality.³ This latter concern is strengthened by the finding that productivity driven standard macroeconomic models are not able to generate the medium-term fluctuations in the value of firms when calibrated with a realistic TFP process (see [Boldrin and Peralta-Alva, 2009](#)).

In this paper, we present new evidence on the relation between the value of U.S. corporations and aggregate productivity. First of all, we document empirically that changes in the volatility of TFP growth are important in predicting the medium-term movements in the value of U.S. corporations. We fit a Markov Switching model to TFP growth, detecting large and infrequent shifts in the mean and volatility of this series throughout the postwar period.⁴ We then show that

³Changes in asset prices, for example, may feed-back into decisions of economic agents and therefore influence aggregate productivity. [Jermann and Quadrini \(2007\)](#) study an economy where increases in asset prices relax firms' credit constraints and endogenously generate an increase in measured TFP.

⁴In particular, we estimate that TFP was in a "high growth regime" between 1960Q1-1973Q1 and 1994Q1-2003Q4, while we estimate a "low volatility regime" during the period 1984Q1-2009Q3. These results are consistent with previous empirical analysis on the behavior of U.S. productivity; see for example [Kahn and Rich \(2007\)](#) and [Benigno et al. \(2011\)](#).

these shifts explain two-thirds of the medium-run variability in the value-output ratio measured using a band pass filter. In particular, a 1% increase in the conditional mean of productivity growth is associated with a 19% increase in the value-output ratio, while this indicator declines by 4% following a 1% increase in the standard deviation of TFP growth. The second contribution of this paper is to assess whether these elasticities can be interpreted as the response of asset prices to exogenous changes in the first two moments of TFP growth. We develop a stochastic growth model and show that, for reasonable calibrations, the model is consistent with a large response of the value-output ratio to shocks in the mean and volatility of TFP growth.

We build a stochastic growth model where households have Epstein-Zin preferences and where TFP growth is driven by two disturbances: a persistent Markov-Switching shock to its mean and a purely transitory shock. Consistent with our empirical analysis, we allow the volatility of the transitory component to vary over time. We assume a particular form of incomplete information: agents are aware of the underlying structure of the economy, and in every period they observe realized TFP growth, but they cannot tell whether movements in TFP growth come from the Markov-Switching mean or the transitory shock. We assume that they form beliefs about the mean of TFP growth using Bayes' rule.⁵ The induced movements in beliefs about the growth regime influence agents' views over future corporate payouts. Beside the standard neoclassical channel, our model features monopolistic rents in production. This is intended to capture variation in dividends unrelated to the marginal product of physical capital, e.g., organizational capital, patents, etc.. These are factors that previous research identifies as important

⁵The resulting filtering problem implies that the model shifts in the mean of productivity growth are difficult to detect in real time, a fact that is well documented for the U.S. economy (see [Edge et al., 2007](#)).

drivers of firms' valuation (see [Hall, 2001](#)). The interaction between incomplete information, monopolistic rents and Epstein-Zin preferences generates a strong sensitivity of the value-output ratio to the first two moments of TFP growth compared to a full information neoclassical benchmark. In the next two paragraphs, we briefly discuss the intuition underlying this result.

In the model, the behavior of the value of corporations conditional on a growth shock resembles *qualitatively* that of related production based asset pricing models, in particular [Croce \(2012\)](#). Indeed, under a plausible calibration of preferences, the model features a strong intertemporal substitution effect. A persistent increase in the mean of TFP growth is associated with expectations of a higher growth in corporate payouts, and households react to this change in expectations by demanding more assets. A higher demand for assets pushes up the value of corporations, therefore generating a positive association between TFP growth and asset prices. Our model, though, is less restrictive than [Croce \(2012\)](#) with respect to the *quantitative* association. In fact, and differently from a neoclassical setting, the value of corporations in our model is the sum of two components: the market value of the physical capital stock *and* the present value of rents that firms are expected to generate. As we will discuss in the paper, the latter component is an order of magnitude more sensitive to the mean of TFP growth than the former. This aspect greatly improves the model's ability to generate an empirically plausible behavior for prices and quantities relative to its neoclassical benchmark.⁶

The model also has implications for the effects of volatility on the value of corporations. As in other production-based long-run risk models, agents in our economy

⁶Indeed, absent monopolistic rents, strong frictions in the production of capital would be required by the model in order to generate a large elasticity of the value-output ratio to the mean of TFP growth. These frictions, while making asset prices more volatile, would reduce the relative volatility of investment to an empirical implausible level.

are strongly averse to long-run fluctuations in the growth rate of corporate payouts, and they therefore ask for a sizable compensation when holding shares. An increase in the uncertainty over the long-run component of firms' productivity growth would accordingly generate a reduction in asset prices through its effects on risk premia. In our model, this channel is triggered by the interactions between incomplete information and volatility. Indeed, an increase in the volatility of the *transitory* component of TFP growth adds more noise to the filtering problem that agents are solving in real time. This makes them more uncertain about the long-run properties of corporate payouts. Depending on the calibration considered, asset prices can be very sensitive to the volatility of TFP growth, a feature that a model with full information would miss.⁷

We document that, under plausible calibrations, the model generates business cycle statistics for real and financial variables that are in line with postwar U.S. observations. Moreover, we compute the model implied elasticities of the value-output ratio to the mean and volatility of TFP growth and compare them with our empirical estimates. We find that a 1% increase in the mean of TFP growth is associated with a 4% increase in the value-output ratio. At the same time, this indicator falls by 0.4% after a 1% increase in the standard deviation of productivity growth. This represents, respectively, 20% and 9% of the magnitude observed in the data. We also show that, for less conservative calibrations of the TFP process, the growth elasticity can be reconciled with our empirical estimates, while the model accounts for 60% of the sensitivity of the value-output ratio to the volatility of TFP growth.

Related Literature. The idea that variations in risk and economic growth

⁷See for example Naik (1994).

influence asset prices has a long tradition in economics, see for example [Malkiel \(1979\)](#), [Pindyck \(1984\)](#), [Barsky \(1989\)](#), [Barsky and De Long \(1993\)](#), [Bansal and Yaron \(2004\)](#) and references therein. In a recent paper, [Lettau et al. \(2008\)](#) present an endowment economy where shifts in the mean and volatility of consumption growth influence stock prices. Their model accounts for the 1990s “boom” in the U.S. stock market via a decline in the volatility of consumption growth, while they estimate that changes in the mean of consumption growth have small effects on stock prices. To the best of our knowledge, we are the first to look at more fundamental sources of variation in a general equilibrium model with production.

Our paper contributes to the production-based asset pricing literature ([Jermann, 1998](#); [Tallarini, 2000](#); [Boldrin et al., 2001](#); [Gourio, 2012](#)). Within this literature, our work is closely related to that of [Croce \(2012\)](#). He studies a neoclassical growth model with recursive preferences, adjustment costs and persistent variation in the mean of TFP growth. His model is consistent with the cyclical behavior of standard real and financial indicators of the U.S. economy. The analysis, though, is silent about the performance of the model regarding the elasticity of asset prices to the mean and volatility of TFP growth. One of our contributions is to show that two plausible mechanisms dramatically improve the model’s ability to capture this conditional behavior of asset prices: incomplete information and monopolistic rents.

Incomplete information is important in our model to generate a quantitatively meaningful association between volatility and asset prices. The friction we consider is not new in the literature, see for example [Kydland and Prescott \(1982a\)](#) and [Edge et al. \(2007\)](#). Relative to the existing literature, we point out an interesting interaction between learning and time-varying volatility. An increase in

the volatility of the transitory component of TFP growth dampens the ability of agents to learn about the persistent component of TFP growth.⁸ In a model with Epstein-Zin preferences, this endogenous variation in uncertainty over long-run growth has strong asset pricing implications. We find in addition that monopolistic rents greatly enhance the performance of standard exogenous growth models regarding the volatility of asset prices without impairing their ability to account for variations in quantities. A similar point has been made recently by [Comin et al. \(2009\)](#) and [Iraola and Santos \(2009\)](#) in a class of endogenous growth models.

We consider our empirical findings particularly relevant for the literature studying movements in asset prices over longer horizons. Several attempts have been put forth to explain the behavior of the U.S. stock market. Plausible explanations for the medium-term movements in the value of corporations include technological revolutions ([Greenwood and Jovanovic, 1999](#); [Laitner and Stolyarov, 2003](#); [Pastor and Veronesi, 2009](#)), variation in taxes and subsidies ([McGrattan and Prescott, 2005](#)), intangible investments ([Hall, 2001](#)) and the saving behavior of baby boomers ([Abel, 2003](#)). Our paper suggests that a successful theory should account for the *joint* evolution of productivity growth and asset prices, since these two series share common cycles in the medium run. This would help the profession with the task of measuring the contribution of each of these mechanisms.

Layout. The rest of the paper is organized as follows. In section [3.2](#) we document the medium-term association between productivity growth and the value-output ratio. Section [3.3](#) presents the model. In section [3.4](#) we calibrate the model,

⁸[Bullard and Singh \(2012\)](#) discuss a setting in which the opposite happens. They consider an RBC model with a similar signal extraction problem to ours, but they model an increase in the volatility of TFP growth by permanently increasing the gap between the two means of the TFP growth process. This change, while increasing the unconditional volatility of TFP growth, makes the signal extraction problem easier as agents can better distinguish between the two different growth regimes.

study its business cycle properties and analyze the behavior of the value-output ratio conditional on a persistent change in the mean and volatility of TFP growth. Section 4.5 concludes.

3.2 Productivity Growth and the Market Value of U.S. Corporations: Empirical Evidence

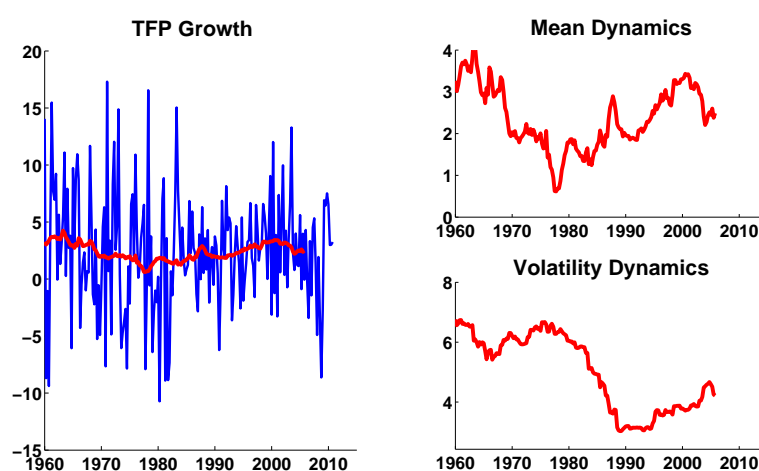
We begin by looking at simple indicators of time variation in the mean and volatility of productivity growth. We construct a quarterly series for *total factor productivity* (TFP) in the U.S. business sector using BLS and NIPA data. Our data cover the period from the first quarter of 1952 to the last quarter of 2010. In Figure 3.1, we plot 10 years centered rolling window estimates for the annualized mean (top-right panel) and standard deviation (bottom-right panel) of productivity growth. The left panel of the figure plots the TFP growth series.

The data show substantial time variation in the mean and volatility of TFP growth. In the top-right panel of Figure 3.1, we can observe the slowdown in growth during the late 1960s/early 1970s and the subsequent resurgence during the 1990s, facts that have been extensively discussed in narrative and econometric studies on the U.S. economy.⁹ Regarding volatility, the bottom-right panel of Figure 3.1 shows the drastic decline during the 1980s, consistent with the “great moderation” in macroeconomic aggregates after 1984 (see [Kim and Nelson, 1999](#); [McConnell and Quiros, 2000](#)). It is also important to notice that shifts in these two

⁹One issue of the *Journal of Economic Perspectives* is devoted to the productivity growth slowdown of the 1970s (*Volume 2, Number 4*) and another issue to the resurgence in productivity growth of the 1990s (*Volume 14, Number 4*). Econometric studies that have recently analyzed shifts in the trend growth rate of productivity include [Roberts \(2000\)](#), [French \(2001\)](#), [Kahn and Rich \(2007\)](#), [Croce \(2012\)](#) and [Benigno et al. \(2011\)](#).

series are *large* and *infrequent*. For instance, the standard deviation of productivity growth fluctuates very little during the period 1960-1980, and it experienced a sudden reduction of about 50% in the early 1980s. Similarly, productivity growth fell by 2% within a few years in the late 1960s and then fluctuated very little around a value of 1.5% for about 20 years.

Figure 3.1: **Growth and Volatility: Rolling Windows Estimates**



Note: The figure reports 10 years centered rolling windows estimates for the mean and volatility of TFP growth (right panel). For example, the mean of TFP growth in 1980 is calculated as the arithmetic mean of productivity growth within the period 1977.II-1983.II. The left panel reports the TFP growth series (blue line), along with the rolling windows estimates for the mean (red line). The TFP growth series is annualized.

3.2.1 Identifying Shifts in Growth and Volatility: A Markov-Switching Approach

We now describe a parametric model that we use to fit the shifts in the mean and volatility of TFP growth documented in the previous section. We model the growth rate of TFP as follows:

$$\begin{aligned}
\Delta Z_t &= \mu_t + \phi [\Delta Z_{t-1} - \mu_{t-1}] + \sigma_t \varepsilon_t \\
\mu_t &= \mu_0 + \mu_1 s_{1,t} \quad \mu_1 > 0 \\
\sigma_t &= \sigma_0 + \sigma_1 s_{2,t} \quad \sigma_1 > 0 \\
\varepsilon_t &\sim \mathcal{N}(0, 1) \quad s_{1,t} \sim \mathcal{MP}(P_\mu) \quad s_{2,t} \sim \mathcal{MP}(P_\sigma)
\end{aligned} \tag{3.1}$$

The variable μ_t captures the different “growth regimes” characterizing the post-war behavior of aggregate productivity. In particular, we assume that μ_t alternates between two regimes driven by the Markov process $s_{1,t} \in \{0, 1\}$ whose law of motion is governed by the transition matrix P_μ . Since $\mu_1 > 0$, we interpret $s_{1,t} = 1$ as the “high growth” regime during which productivity grows on average at the rate $\mu_0 + \mu_1$, while $s_{1,t} = 0$ implies that TFP grows at the rate μ_0 (“low growth”). Transitory fluctuations around μ_t are modeled via the white noise ε_t and an autoregressive component. The volatility of ε_t is allowed to fluctuate between a “high volatility” regime and a “low volatility” regime driven by the Markov process $s_{2,t} \in \{0, 1\}$.¹⁰

Markov-Switching models are commonly used in the literature to fit large and infrequent changes of the type observed in Figure 3.1, and they have already been used to fit changes in the trend growth rate of productivity (see French, 2001; Kahn and Rich, 2007).¹¹ Our approach is different from existing ones in that we do not model transitory fluctuations in the *level* of TFP. This is done mainly because the

¹⁰Our choice regarding the number of regimes is suggested by the nonparametric analysis in the previous section and is confirmed by formal posterior odds comparisons.

¹¹An equally plausible specification would be that of a random coefficients model in which variations in μ_t and σ_t are represented by continuous stochastic processes. This approach has been followed in a similar context by Cogley (2005). We have formally compared our specification with one in which μ_t follows an AR(1) process, and the marginal data density slightly favors our model. Results are available upon request.

process in equation (1) makes the general equilibrium model of Section 3.3 more tractable.

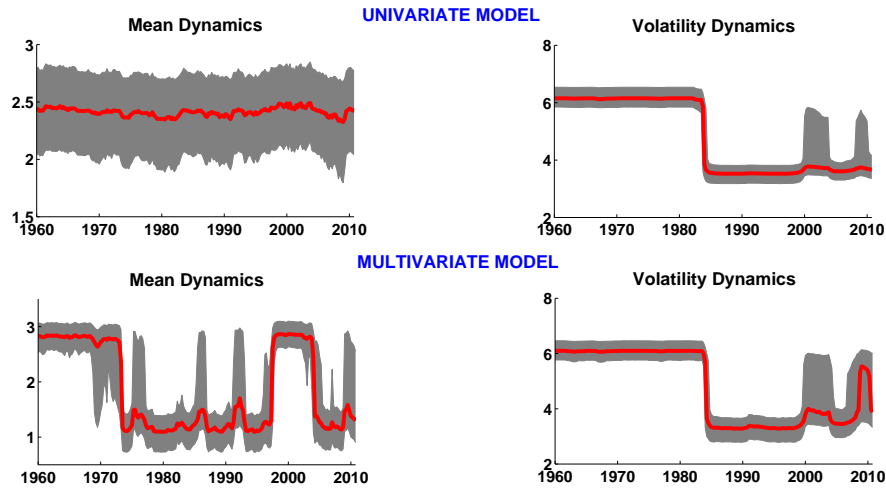
We estimate the model’s parameters using Bayesian methods. Appendix C.1.2 describes in detail the selection of the prior as well as the sampler adopted to conduct inference. Table 3.1 reports posterior statistics for the model’s parameters under the header *Univariate Model*, while the top panel of Figure 3.2 plots posterior estimates of μ_t and σ_t .

Table 3.1: Markov-Switching Model: Prior Choices and Posterior Distribution

Parameter	Prior	Univariate Model		Multivariate Model	
		Median	90% Credible Set	Median	90% Credible Set
μ_0	$\mathcal{N}(1.5, 1)$	1.68	[0.30, 2.67]	1.12	[0.59, 2.67]
μ_1	$\mathbf{1}(x > 0)\mathcal{N}(1, 1)$	1.19	[0.29, 2.48]	1.75	[0.32, 2.57]
σ_0	$\mathcal{IG}(5, 2)$	3.63	[3.40, 3.83]	3.30	[2.60, 3.99]
σ_1	$\mathcal{IG}(5, 2)$	2.51	[2.02, 2.79]	2.92	[2.00, 3.84]
ϕ	$\mathcal{N}(0, 1)$	0.10	[-0.01, 0.20]	0.04	[-0.06, 0.12]
$P_{1,1,\mu}$	$\mathbf{1}(x < 1)\mathcal{N}(0.98, 0.3)$	0.953	[0.543, 0.998]	0.971	[0.861, 0.997]
$P_{2,2,\mu}$	$\mathbf{1}(x < 1)\mathcal{N}(0.98, 0.3)$	0.975	[0.570, 0.998]	0.969	[0.890, 0.996]
$P_{1,1,\sigma}$	$\mathbf{1}(x < 1)\mathcal{N}(0.98, 0.3)$	0.995	[0.965, 0.999]	0.990	[0.901, 0.999]
$P_{2,2,\sigma}$	$\mathbf{1}(x < 1)\mathcal{N}(0.98, 0.3)$	0.992	[0.971, 0.998]	0.990	[0.954, 0.998]

The model clearly identifies movements in the volatility of productivity growth. From the top right panel of Figure 3.2, we can observe a decline in σ_t of 44% in the mid-1980s, with little uncertainty regarding this event. The model also identifies a slight increase in the volatility of TFP growth toward the end of the sample, although credible sets are large. On the contrary, shifts in the mean are poorly identified with this approach, as shown by the large credible sets on μ_t and on the parameters governing its behavior.

Figure 3.2: Growth and Volatility: Markov-Switching Model



Note: In the left panels, we report smoothed posterior estimates of μ_t . In particular, the solid line represents the posterior mean $\mathbb{E}[\mu_t | \mathcal{Z}^T]$, while the shaded area denotes a 60% pointwise credible set. The right panels report the same information for σ_t .

3.2.2 Identifying Shifts in Growth and Volatility: Multivariate Analysis

High uncertainty in our estimates for μ_t reflects the difficulties in detecting changes in the trend growth rate of TFP. As the left panel of Figure 3.1 shows, transitory fluctuations in TFP growth are large compared to the changes in the conditional mean that we wish to isolate, and this complicates the filtering problem significantly. A remedy suggested in the literature consists of introducing additional variables that carry information on μ_t . We follow this insight and augment the model in equation (3.1) as follows:

$$\begin{aligned}
\begin{bmatrix} \Delta Z_t \\ \Delta \mathbf{Y}_t \end{bmatrix} &= \begin{bmatrix} \mu_t \\ \mu_t \end{bmatrix} + \Phi \begin{bmatrix} \Delta Z_{t-1} - \mu_{t-1} \\ \Delta \mathbf{Y}_{t-1} - \mu_{t-1} \end{bmatrix} + \Sigma_t \mathbf{e}_t \\
\mu_t &= \mu_0 + \mu_1 s_{1,t} \\
\Sigma_t &= \Sigma_0 + \Sigma_1 s_{2,t} \\
\varepsilon_t &\sim \mathcal{N}(0, 1) \quad s_{1,t} \sim \mathcal{MP}(P_\mu) \quad s_{2,t} \sim \mathcal{MP}(P_\sigma)
\end{aligned} \tag{3.2}$$

This specification introduces a set of variables $\Delta \mathbf{Y}_t$ that share the same growth rate as TFP. This formulation is rooted in economic theory. Indeed, under balanced growth restrictions, equilibrium models predict that several economic ratios share a common trend with TFP. This justifies the introduction of $\Delta \mathbf{Y}_t$ into the analysis, as one can pool these time series with TFP growth in order to obtain a sharper estimate of μ_t . Following [Kahn and Rich \(2007\)](#), we include the growth rate of consumption per hour and compensation per hour in $\Delta \mathbf{Y}_t$.¹²

The law of motion for μ_t and Σ_t has the same Markov-Switching structure described in the previous section. For tractability and parsimony, we allow the variance of the innovations to have common switches while keeping their correlation structure constant over time.¹³ The model is estimated via Bayesian techniques as discussed in [Appendix C.1.3](#), and the results are reported under the header *Multivariate Model* in [Table 3.1](#) and in the bottom panel of [Figure 3.2](#). These estimates are consistent with the univariate analysis presented in the previous section. The multivariate approach allows us to identify the shifts in the mean of productivity growth more precisely. Credible sets on the parameters governing μ_t

¹²Consumption and wages are scaled by total hours in order to account for the unit root behavior of hours worked that, under preferences consistent with balanced growth, is unrelated to TFP dynamics. See [Chang et al. \(2007a\)](#) for a discussion of this issue.

¹³This is accomplished by reparametrizing Σ_t , see [Appendix C.1.3](#) for details.

are considerably tighter and this is reflected in increased precision of our estimates for μ_t . As Figure 3.2 shows, we estimate that the trend growth rate of TFP was about 3% in the periods 1960Q1:1973Q1 and 1997Q3:2004Q1, while growth was around 1.3% in the remaining periods.

3.2.3 Productivity Growth and the Market Value of U.S. Corporations

We now consider the relation between productivity growth and the market value of U.S. firms. Following Hall (2001), Wright (2004) and McGrattan and Prescott (2005), we use *Flow of Funds* data and define the market value of the U.S. corporate sector to be the sum of outstanding equities and net debt liabilities.¹⁴ This indicator has the advantage of including the market value of closely held firms, thus being a more reliable measure for trends in the value of firms relative to standard indicators that are based only on publicly held corporations.

We summarize the relation between the value-output ratio and our estimates of μ_t and σ_t via linear projections. Our benchmark specification is the following:

$$MV_t = \mathbf{a} + \mathbf{b}\hat{\mu}_t + \mathbf{c}\hat{\sigma}_t + e_t \quad (3.3)$$

Table 3.2 reports the results. From Column 3, we can observe a positive relation between trend growth and the value-output ratio. An increase of 1% in the trend growth rate of TFP is associated with an increase in the value-output ratio of 21%. Volatility, on the other hand, is negatively associated with the value of

¹⁴See Appendix D.5 for details on the calculation of this indicator.

U.S. corporations. An increase of 1% in the standard deviation of TFP growth is associated with a reduction in the value-output ratio of 12%.

Table 3.2: **Growth, Volatility and the Value of U.S. Corporations**

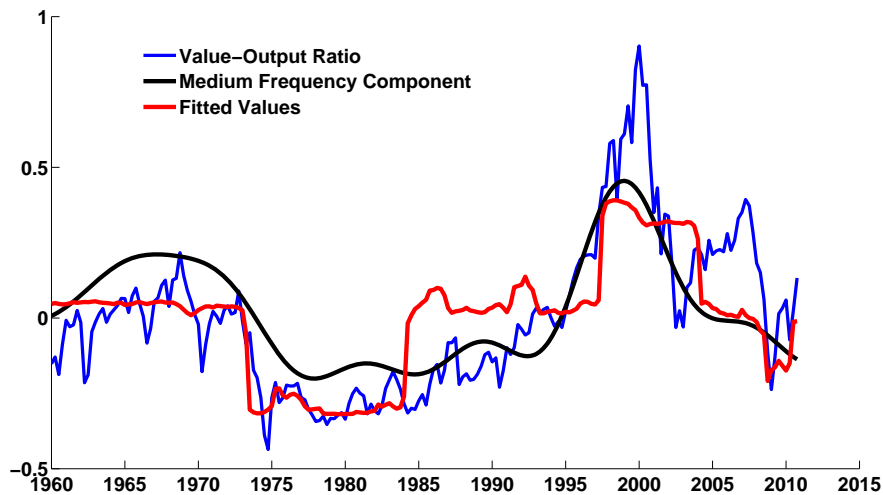
	[1]	[2]	[3]	[4]
Constant	-0.34 (0.04)	0.36 (0.07)	0.17 (0.05)	-0.13 (0.03)
$\hat{\mu}_t$	0.19 (0.02)		0.21 (0.02)	0.19 (0.01)
$\hat{\sigma}_t$		-0.07 (0.01)	-0.12 (0.01)	-0.04 (0.005)
R^2	0.23	0.13	0.56	0.68

Note: Column [1] reports the results of a linear projection of $\mathbb{E}[\mu_t|\mathcal{I}^T]$ on the value-output ratio. Column [2] reports the results of a linear projection of $\mathbb{E}[\sigma_t|\mathcal{I}^T]$ on the value-output ratio. Column [3] reports the results from the estimation of equation (3.3). Column [4] reports the results of a linear projection of $\mathbb{E}[\mu_t|\mathcal{I}^T]$ and $\mathbb{E}[\sigma_t|\mathcal{I}^T]$ on the medium frequency component of the the value-output ratio isolated using the band-pass filter between 32 and 200 quarters. The value-output ratio is demeaned prior to running the projections. Robust standard errors are in parentheses.

The linear projection also shows that the association between productivity growth and the value-output ratio is sizable. Indeed, fluctuations in the first two moments of productivity jointly predict more than half of the variation in the value-output ratio at quarterly frequencies. In order to gain more insights into this association, we plot in Figure 3.3 the value-output ratio along with the fitted values of the linear projection. We can verify that the decline in growth in the early 1970s is closely followed by a sharp decline in the value-output ratio and that the growth resurgence is associated with a boom in this indicator, while the subsequent decline in productivity growth is associated with a fall in the valuation of U.S. corporations. The great moderation occurred during a period of a rising value-output ratio, while the surge in aggregate volatility observed toward the end of the sample is associated with a decline in this indicator. Moreover, from the figure we can see that most of the association between productivity growth

and the value-output ratio occurs over horizons longer than the business cycle. The fitted values of equation (3.3) closely track the medium term component of the value-output ratio constructed using the band-pass filter (32-200 quarters). This point is confirmed by column [4] of Table 3.2, where we project the medium frequency component of the value-output ratio on $\hat{\mu}_t$ and $\hat{\sigma}_t$. Relative to column [3], the R^2 increases substantially, suggesting that the movements in TFP growth and volatility that we identify are mainly relevant for predicting the medium-term fluctuations in the value-output ratio.

Figure 3.3: Growth, Volatility and the Value of U.S. Corporations



Note: The blue solid line plots the value of corporations scaled by gross domestic product. The red solid line reports the fitted values of Equation (3.3). The black dotted line reports the medium frequency component of the value-output ratio isolated using the band-pass filter between 32 and 200 quarters.

3.3 Model

So far, we have documented a strong relationship between persistent innovations to the mean and standard deviation of TFP growth and medium-term fluctuations in the value-output ratio. As mentioned in the Introduction, this reduced form

association may have several interpretations. In what follows, we set up a quantitative model with the aim of measuring the fraction of this association that can be explained by exogenous variation in the mean and volatility of TFP growth. We consider a fairly standard growth model with four major ingredients:

- i) Markov-Switching regimes in the mean and volatility of technological growth
- ii) Recursive preferences
- iii) Capital adjustment costs and monopolistic rents
- iv) Incomplete information about the drivers of technological growth

In the model, infinitely lived households supply labor inelastically to firms and own shares of the corporate sector. They use their dividend and labor income to consume the final good and accumulate shares of the corporate sector. The final good is sold in a competitive market by firms that aggregate a set of imperfectly substitutable intermediate goods. Each variety is produced by an intermediate good firm using capital and labor. Those firms rent the capital stock and labor in competitive markets. They are monopolists in producing their variety. Capital services are supplied by capital producers in a competitive market. These firms own the capital stock and make optimal capital accumulation plans by maximizing the present discounted value of profits.

Below we describe the major ingredients of our model, while [Appendix C.2](#) contains a detailed account of the agents' decision problems and of the equilibrium concept adopted. In terms of notation, the level of variable X at time t is denoted by X_t . Even though every endogenous variable depends on the history of shocks, we keep the notation simple and omit this explicit dependence.

3.3.1 Preferences

Households have Epstein-Zin preferences over streams of consumption. Given a continuation value U_{t+1} and consumption c_t today, the agent's utility is given by:

$$U_t = ((1 - \beta)c_t^{\frac{1-\gamma}{\eta}} + \beta\mathbb{E}_t[(U_{t+1})^{1-\gamma}]^{\frac{1}{\eta}})^{\frac{\eta}{1-\gamma}}.$$

γ controls the degree of risk aversion, η is equal to $\frac{1-\gamma}{1-\frac{1}{\Psi}}$ and Ψ parametrizes the elasticity of intertemporal substitution in consumption. The operator $\mathbb{E}_t[\cdot]$ is interpreted as the expectation conditional on all the observations made by the agents up to period t .

3.3.2 Production

A fixed variety of intermediate goods is produced in the economy. Intermediate goods are indexed by $j \in [0, 1]$, and each variety is produced by an intermediate good producer. He uses capital services $k_{j,t}$ and labor services $l_{j,t}$ to produce $y_{j,t}$ units of the good according to the production function:

$$y_{j,t} = (e^{Z_t} l_{j,t})^{1-\alpha} k_{j,t}^{\alpha}.$$

Z_t is the log of TFP common to all firms. Intermediate goods are aggregated by final good producers into units of a final good using the production function

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}.$$

The final output is consumed by households or purchased by capital producers to invest in capital. In particular, if a capital producer with k_t units of capital invests i_t , his stock of capital in period $t + 1$ will increase by $G\left(\frac{i_t}{k_t}\right)k_t$. For the quantitative analysis, we parametrize $G(\cdot)$ as¹⁵

$$G(\cdot) = a(\cdot)^{1-\tau} + b.$$

Capital depreciates every period at the rate δ . Therefore, the stock of capital for a producer evolves according to the following law of motion:

$$k_t = (1 - \delta)k_t + G\left(\frac{i_t}{k_t}\right)k_t$$

3.3.3 Total Factor Productivity and Information Structure

We model the logarithm of TFP as a random walk with time varying drift and volatility.¹⁶

¹⁵This functional form is quite standard in the literature. [Jermann \(1998\)](#) points out that the inverse of τ is equal to the elasticity of the investment-capital ratio to Tobin's Q in a wide class of models.

¹⁶This is different from Section 3.2 in that we do not include a autoregressive component, as we did not find a strong contribution of this component in the estimation and as the omission simplifies the numerical solution of the model.

$$\Delta Z_t = \mu_t + \sigma_t \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(0, 1),$$

where the drift and the volatility follow the Markov processes:

$$\begin{aligned} \mu_t &= \mu_0 + \mu_1 s_{1,t} & \mu_1 &> 0 \\ \sigma_t &= \sigma_0 + \sigma_1 s_{2,t} & \sigma_1 &> 0. \end{aligned}$$

The variable $s_{j,t} \in \{0, 1\}$ is a state whose probabilistic law of motion is governed by the transition matrix \mathcal{P}_j .

Household and firms know the parameters governing the stochastic process and use it to form expectations about future periods. They are, however, imperfectly informed about the drivers of technological progress. In particular, we assume that they learn, at every point in time, the realization of TFP growth ΔZ_t while not observing its components μ_t and ε_t separately. Therefore, the information that agents can use to update their beliefs about the current state of the stochastic process is given by the history of TFP growth realizations, the state governing volatility and an unbiased Gaussian signal $g_t = \mu_t + \sigma_g e_t$ that they receive in every period. They are fully rational and form their beliefs about $s_{1,t}$ via Bayes' rule. We denote the probability that any agent assigns to being in growth regime $s_{1,t}$ in period t by $p_t(s_{1,t})$. Similarly, we label the likelihood that he attaches to being in state $s_{1,t+1}$ in period $t + 1$ by $p_{t+1|t}(s_{1,t+1})$. Bayes' rule implies the following updating equations:

$$p_{t+1|t}(s_{1,t+1}) = \frac{\sum_{i=1}^2 p_{t-1}(i) \mathcal{P}_1(s_{1,t}|i)}{\sum_{j=1}^2 \sum_{i=1}^2 p_{t-1}(i) \mathcal{P}_1(j|i)};$$

and

$$p_t(s_{1,t}) = \frac{p_{t|t-1}(s_{1,t})p_N(\Delta Z_t, g_t|s_{1,t}, s_{2,t})}{\sum_{i=1}^2 p_{t|t-1}(i)p_N(\Delta Z_t, g_t|i, s_{2,t})}$$

$p_N(\cdot|j, \hat{j})$ is the pdf of a two dimensional normal random variable with mean $\mu_0 + \mu_1 j$ and standard deviation $\sigma_0 + \sigma_1 \hat{j}$ for the first component and mean $\mu_0 + \mu_1 j$ and standard deviation σ_g for the second one. Both components are assumed to be independent. The first equation updates the beliefs about the state today into beliefs over the expected state tomorrow using the known probabilities of a state transition. The second equation captures how those probabilities are updated after observing the realizations of the growth rate and the signal. As we see, given the structure of the stochastic process considered, $(\Delta Z_t, s_{2,t}, g_t)$ are sufficient to update the beliefs of the household about the underlying state in the last period to the beliefs in the current period.

3.3.4 Equilibrium and Numerical Solution

We focus on a symmetric equilibrium in which all capital good producers initially own the same amount of capital. This assumption implies that capital good producers make the same investment choices and that intermediate good producers charge the same price and sell the same quantity to the final good producers. In appendix C.2, we argue that the equilibrium law of motion for aggregate variables can be derived from a planner's problem, which we describe below.

The planner maximizes lifetime utility of the representative household by selecting a sequence for investment, consumption, the capital stock and the value function $(i_t, c_t, k_{t+1}, V_{t+1})_{t=0}^{\infty}$,¹⁷ subject to the same information restriction as the

¹⁷These are functions of the realization of the stochastic process subject to the measurability restrictions implied by the information structure.

households, initial conditions and a function that maps the observed realizations of shocks into an aggregate capital stock $(\bar{k}_t)_{t=0}^\infty$.¹⁸

$$\max_{(i_t, c_t, k_{t+1}, V_{t+1})_{t=0}^\infty} V_0$$

$$\text{s.t. } c_t + i_t = \frac{\nu - 1}{\nu} Z_t k_t^\alpha + \frac{1}{\nu} Z_t \bar{k}_t^\alpha$$

$$V_t = [(1 - \beta)c_t^{\frac{1-\gamma}{\eta}} + \beta \mathbb{E}_t[V_{t+1}^{1-\gamma}]^{\frac{1}{\eta}}]^{\frac{\eta}{1-\gamma}}$$

$$k_{t+1} = (1 - \delta)k_t + G\left(\frac{i_t}{k_t}\right) k_t.$$

In addition, the choice of V_t has to be finite for all t . An equilibrium is fully characterized when $\bar{k}_t = k_t$. We solve the model numerically using global methods as described in Appendix C.3.

3.3.5 Asset Prices

We can express the market value of firms as the present discounted value of corporate payouts to households. In our economy, there are two types of firms making nonzero profits: the capital good producers and the intermediate good producers.

The per period profits of a capital good producer are given by:

¹⁸The aggregate capital is therefore measurable with respect to the households' information set and does not add new information to the signal extraction problem.

$$d_t^{cp} = r_t^k k_t - i_t,$$

where r_t^k stands for the return to capital. The per period profits of an intermediate good producer are given in equilibrium by:

$$d_t^{ip} = \frac{1}{\nu} y_t.$$

Profits are a fixed fraction of the revenues of an intermediate good producing firm. Both types of producers distribute these profits to households in every period. As a result, one can express the market value of these two types of firms as follows:

$$p_t^s = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} d_{t+j}^s \right] \quad s = \{cp, ip\}.$$

Here we denote by $\Lambda_{t,t+s}$ the stochastic discount factor of the household between period t and period $t + s$. The market value of the corporate sector is then the sum of these two components. For future reference, it is convenient to further characterize this object. Based on [Hayashi \(1982\)](#) it is easy to show that the equilibrium value of the corporate sector is given by

$$p_t = \frac{1}{(1-\tau)a} \left(\frac{i_t}{k_t} \right)^\tau k_{t+1} + \frac{1}{\nu} \mathbb{E}_t \left[\sum_{j=1}^{\infty} \Lambda_{t,t+j} y_{t+j} \right]. \quad (3.4)$$

Indeed, one can easily verify that in our model the equilibrium value of capital good producers can be expressed as the product of marginal Q and the capital stock. The decomposition of equation (3.4) has an intuitive interpretation. It tells us that, in equilibrium, the value of the corporate sector is the sum of two

components: the value of the capital stock and the present discounted value of monopolistic rents.¹⁹ In the next section, we will calibrate the model and study how these two components respond to fluctuations in μ_t and σ_t .

3.4 Risk, Economic Growth and the Value of Corporations

3.4.1 Calibration

We set a model period to be a quarter. The parameters of our model are:

$$\theta = [\delta, a, b, \tau, \beta, \Psi, \gamma, \mu_0, \mu_1, P_{0|0}^\mu, P_{1|1}^\mu, \sigma_0, \sigma_1, P_{0|0}^\sigma, P_{1|1}^\sigma, \alpha, \nu, \sigma_g]$$

The depreciation parameter δ is set to 0.025, leading to an annual depreciation rate of roughly 10%. We follow the literature in calibrating a and b so that the deterministic balanced growth path of our model coincides with that of an economy without adjustment costs (Van Binsbergen et al., 2010).²⁰ The parameter controlling the capital adjustment (τ) is set to 0.5, in the range of values considered in the literature. The discount factor β is set to 0.994, a value that implies an average annualized risk free-rate of 2.07%.²¹ Following Croce (2012), we set Ψ to 2 and γ to 10.²²

¹⁹Indeed, in our decentralization, the price of capital equals $\frac{1}{(1-\tau)a} \left(\frac{i_t}{k_t}\right)^\tau$.

²⁰We construct a deterministic balanced growth path around the average growth rate of TFP, which we denote by μ .

²¹We solve the model repeatedly for different values of β until the average risk-free rate computed on simulated data matches the target value. See Table 3.3 for additional details.

²²A value of 2 for the IES and a coefficient of relative risk aversion of 10 are, for example,

We use the empirical results in Section 3.2 to calibrate the parameters of the shock process Z_t .²³ In accordance with our estimates, we set $\mu_0 = 0.003$, $\mu_1 = 0.0045$, $\sigma_0 = 0.0082$ and $\sigma_1 = 0.0073$. As a benchmark, we restrict the transition matrices of the two Markov processes to be symmetric, and we assume that $\mathcal{P}_{j|j}^\mu = \mathcal{P}_{j|j}^\sigma$. Thus, the two transition matrices can be represented by a single parameter, denoted by ρ . We set $\rho = 0.99$, a value that implies an average state duration of 25 years. The unbiased signal in our model stands for all additional information that agents use to infer shifts in the mean of productivity growth. We calibrate its precision so that the average speed of learning is 16 quarters.²⁴ This number is consistent with the results in Edge et al. (2007) and Jorgenson et al. (2008).

We calibrate ν to 10, implying a markup of 10%. This value is in the range typically considered in the business cycle literature for the whole U.S. economy.²⁵ The remaining parameter to be calibrated is α . In our model, the labor income share is given by

$$\frac{w_t l_t}{y_t} = \frac{(\nu - 1)}{\nu} (1 - \alpha), \quad (3.5)$$

Because pure economic profits are treated as a reward to capital in U.S. national accounts, we can calibrate α by matching a labor income share of 70% in line with U.S. data. This strategy results in $\alpha = 0.22$.

consistent with the estimates obtained by Attanasio and Vissing-Jorgensen (2003).

²³In order to be consistent, we reestimate the model in Section 3.2 restricting the autoregressive component of TFP growth to be equal to zero.

²⁴We simulate the signal extraction problem 100,000 times. We keep the mean growth rate fixed in the high regime for 100 periods. We then switch the regime to the low state and count the number of periods it takes the filter to attach a probability of 0.9 to the low regime for the first time. We keep changing σ_g until the average time it takes over the simulations is 16. The resulting value for σ_g is 0.0074.

²⁵See for example Altig et al. (2011) and their references.

3.4.2 Unconditional Moments

Table 3.3 reports a set of model implied statistics for selected real and financial variables along with their data counterparts. For comparison, we also report the results for two natural benchmarks. We consider a version of our model in which agents have perfect information over the TFP process (Full Info) and a version in which intermediate firms operate in a competitive environment (No Rents).²⁶ Under the calibration considered, our model generates business cycle fluctuations for consumption, output and investment that are not too far from the data. In particular, we obtain that consumption growth is less volatile than output growth, while investment growth is more volatile, with relative magnitudes in line with data observations. The model predicts a high degree of comovement of consumption and investment growth with output growth. However it differs from the standard Real Business Cycle model in that we obtain a relatively small correlation between consumption and investment growth. This happens because changes in the beliefs over the trend growth rate of TFP induce differential movements in aggregate investment and consumption.²⁷ Finally, the model implies a small autocorrelation for the variables in growth rates, which is not surprising given its lack of a strong internal propagation mechanism.²⁸

The model is also consistent with the first two moments of the equity premium

²⁶We recalibrate β in each case to keep the risk-free rate at 2.07 in order to make it easier to contrast the three examples with regard to their asset pricing behavior.

²⁷Indeed, a shock to the growth rate of TFP induces offsetting wealth and substitution effects on the part of households. On the one hand, higher growth signals households' higher income in the future, which makes them more willing to reduce their savings and increase their consumption level today. On the other hand, higher TFP growth implies a higher reward to savings today, which motivates households to save more. Irrespective of which of these two effects prevails in equilibrium, consumption and investment growth move in opposite directions conditional on a TFP "growth" shock.

²⁸This problem is shared by many simple business cycle models as discussed in [Cogley and Nason \(1995\)](#).

Table 3.3: Model Implied Moments for Selected Variables

	$\sigma(\Delta y)$	$\frac{\sigma(\Delta c)}{\sigma(\Delta y)}$	$\frac{\sigma(\Delta i)}{\sigma(\Delta y)}$	$\rho(\Delta y, \Delta c)$	$\rho(\Delta y, \Delta i)$	$\rho(\Delta i, \Delta c)$	$\rho_{-1}(\Delta y)$	$\rho_{-1}(\Delta c)$	$\rho_{-1}(\Delta i)$
Data	1.00	0.79	2.69	0.70	0.69	0.60	0.34	0.23	0.50
Model	1.00	0.90	1.96	0.96	0.80	0.60	0.05	0.05	-0.04
Full Info	0.97	0.91	1.47	0.99	0.94	0.90	0.05	0.07	0.02
No Rents	0.90	0.89	1.94	0.89	0.82	0.47	0.06	0.05	-0.03

	$\mathbb{E}[R^e - R^f]$	$\sigma[R^e - R^f]$	$\rho_{-1}[R^e - R^f]$	$\mathbb{E}[R^f - 1]$	$\sigma[R^f - 1]$	$\rho_{-1}[R^f - 1]$
Data	4.49	15.89	0.02	2.07	2.6	0.61
Model	3.34	8.28	-0.06	2.07	1.63	0.87
Full Info	1.70	5.06	0.04	2.07	0.25	0.96
No Rents	3.52	5.00	0.43	2.07	2.12	0.87

Note: ΔX_t stands for the quarterly growth rate of variable X . R^e is the annualized gross return on equity, while R^f is the annualized gross return on a risk-free bond. We assume that equity is leveraged using a debt-to-equity ratio of 1. The data figure for the volatility of the value-output ratio stands for the standard deviation of the fitted values in equation (3.3). Means and standard deviations are reported in percentage terms. Model statistics are based on a long simulation (T=1000000). The data column is based on quarterly observations (1960Q1-2010Q4). Statistics on the equity premium and on the risk-free rate are calculated using annual data from 1948 to 2010, which we downloaded from Robert Shiller's website <http://www.econ.yale.edu/~shiller/data.htm>.

in postwar U.S. data. However, the mechanism through which we achieve a large and volatile equity premium differs from that of other production-based versions of the long-run risk model. Croce (2012), for example, generates a sizable equity premium by introducing a persistent random component into the growth rate of productivity. This leads to covariation at low frequencies between consumption growth and corporate payouts, therefore triggering the long run-risk channel discussed in Bansal and Yaron (2004). In our model, this channel is triggered by incomplete information. The Markov-Switching structure, in fact, imposes a trade-off between the persistence and the volatility of μ_t . Under complete information, the changes in the growth rate of corporate payouts would be too rare for agents to require large premia on stocks.²⁹ With incomplete information, though,

²⁹As we use realized postwar growth rates in output to discipline our calibration, the model cannot generate a high risk premium by triggering the rare disaster risk channel as in Gourio

what matters for the equity premium are the beliefs of agents regarding μ_t . In our model, these beliefs are more volatile than μ_t because the learning process is influenced by high frequency variations in TFP growth, in its volatility and in the signal. This generates additional risk from the perspective of investors. The resulting effect on prices stands out clearly when comparing the performance of our model to its full information benchmark since the latter generates a sensibly lower equity premium (1.70% versus 3.34% implied by our model).

While still falling short on the volatility of the equity premium in the data, the calibrated model substantially improves relative to the Full Information and the No Rents model. As will be argued in more depth in the next section, incomplete information raises the sensitivity of asset prices to σ_t , while monopolistic rents make them more sensitive to μ_t . A stronger response of asset prices to economic fundamentals contribute to raising the unconditional volatility of the equity premium. Finally, the model is able to generate the low volatility and high persistence of the risk-free rate observed in U.S. postwar data. The mechanisms through which this happens are well understood in the literature on the production-based long-run risk model, see for example [Croce \(2012\)](#).

3.4.3 Growth, Volatility and the Value of Corporations

In the previous section, we discussed the performance of the model in reproducing key unconditional moments. We now study the sensitivity of the value-output ratio to the first two moments of TFP growth. For this purpose, and in view of the analysis of Section 3.2, it is natural to study the model implied elasticities of the value-output ratio to the mean and volatility of TFP growth. In what

[\(2012\)](#).

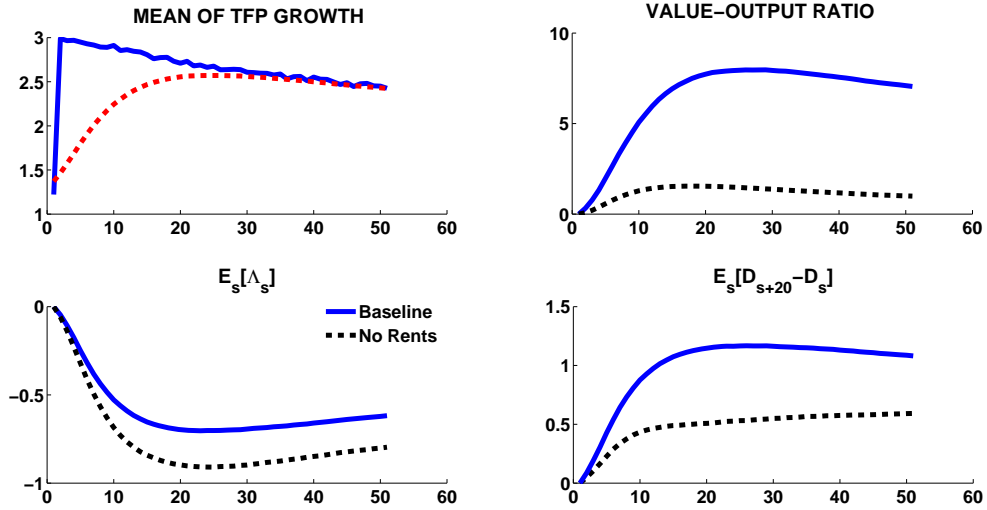
follows, we analyze the economic mechanisms through which μ_t and σ_t influence the value-output ratio by means of impulse response functions (IRFs) and through extensive sensitivity analysis. In Section 3.4.5, we will ask how far the model goes in matching quantitatively these elasticities.

Figure 3.4 shows IRFs of selected variables to a positive change in μ_t . The top panel reports the dynamics of TFP growth and the value-output ratio, while the bottom panel plots the response of the expected stochastic discount factor and the expected average 5-year growth rate of corporate payouts. The annualized growth rate of TFP increases by 1.5% and reverts back to trend thereafter. After the switch, agents slowly learn about the transition to the high-growth regime. Agents' beliefs about μ_t , represented by the dotted line in the top-left panel of the figure, steadily increase from quarter 1 to quarter 20, after which agents become almost sure that a change in regime has occurred. During this period, we observe a protracted increase in the value-output ratio. From a quantitative point of view, a 1.5% increase in TFP growth is associated, at peak, with an 8% increase in the value-output ratio.

The bottom panel of Figure 3.4 captures the mechanism through which higher economic growth induces an increase in the value-output ratio. As households learn about the switch to the high-growth regime, they anticipate higher consumption growth for an extended period of time. This positive wealth effect lowers the rate at which households discount corporate payouts, as the bottom-left panel of the figure shows. *Ceteris paribus*, the decline in the stochastic discount factor has a depressive effect on the value of corporations. However, higher TFP growth changes the expectations that households have regarding future corporate payouts. Indeed, the bottom-right panel of the figure shows that long-run expec-

tations regarding the growth rate of corporate payouts slowly increase after the switch to the high-growth regime. Households have thus an incentive to substitute from current to future consumption by acquiring more securities, generating upward pressures on the value of firms. Since under our calibration agents are not too averse to intertemporal substitution, the latter effect dominates and the value-output ratio rises after a switch to the high-growth regime.

Figure 3.4: IRFs to a Growth Switch



Note: IRFs are calculated via simulation techniques. We simulate $M = 25000$ different realization of length $T = 500$. Each simulation has the characteristic that the trend growth rate of TFP is in the low state between period 1 and period 400 and switches to the high regime in period 401. After that, the simulations are not restricted with regard to the mean. The volatility state is fixed to its low state throughout the simulations. The above figure report the mean across the Monte Carlo replications as percentages with respect to period 400 for the expected discount factor and the value output ratio. The growth rate of TFP and corporate payouts is reported in annualized terms. The average expected growth of corporate payouts over 20 periods is shown relative to the average dividends in period 400. The black dotted lines report the IRFs for a model with perfect competition in the markets for intermediate goods ($\nu = \infty$).

We can also observe that imperfect competition significantly raises the sensitivity of asset prices to μ_t . As we can see from the dotted line in Figure 3.4, the No Rents model implies a response of the value-output ratio of only 1% at peak, sensibly smaller with respect to that in our benchmark model. We can rationalize this difference across models by looking at the behavior of expected corporate payouts growth in the bottom-right panel of the figure. In our model, corporate payout

growth is more responsive to μ_t relative to what happens in the No Rents model. This result is best understood in terms of the decentralization of the economy discussed in Section 3.3. In the No Rents model, the value of corporations equals the value of capital good firms, while in our model the value of corporations also includes the market value of intermediate good producers. These two sectors of the economy differ in terms of their competitiveness. Capital good producers are identical to each other, while intermediate good firms have traits that partly shield them from competitive pressures. Once the growth rate of TFP increases, capital good producers have an incentive to invest. As more producers invest, the marginal product of existing capital for every firms declines, therefore eroding part of the profits induced by higher TFP growth. Firms operating in the intermediate good sector, instead, are not affected by these competitive pressures. Thus, their payouts growth is more responsive to changes in μ_t .

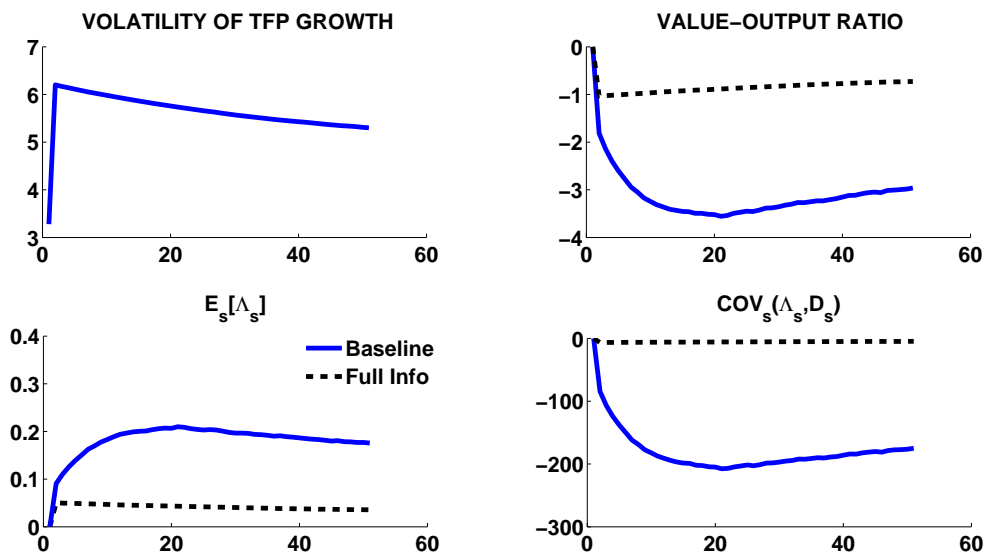
Figure 3.5 reports the response of the value-output ratio when the economy transits from the low to the high volatility regime. Higher volatility of TFP growth is associated with declining asset prices. In particular, the value-output ratio in our model falls by 3.5%. The bottom panel of Figure 3.5 captures the major trade-off that higher volatility brings. An increase in σ_t is associated by agents with more aggregate risk. As individuals are risk averse, they have stronger incentives to demand shares in order to insure consumption fluctuations. The expected stochastic discount factor, therefore, increases. However, households also realize that corporate shares are now riskier securities. Indeed, as the bottom-right panel of Figure 3.5 shows, the covariance between the stochastic discount factor and equity returns declines. Therefore, households have an incentive to substitute corporate shares with current consumption, and this puts downward pressure on

share prices. Since the IES is sufficiently large in our economy, this latter effect dominates, resulting in a negative association between volatility and asset prices. It is also clear from the figure that incomplete information is the key model element governing the sensitivity of asset prices to σ_t . Indeed, in the full information model, the switch to the high volatility regime is associated with a 1% decline in the value-output ratio, 3.5 times smaller with respect to our benchmark specification. This happens because a change in the volatility is perceived differently by the agents in the two models. In the full information set-up, the increase in the volatility of the transitory component of TFP growth has almost no influence on risk, since the stochastic discount factor is hardly affected by transitory TFP growth shocks. In our model, instead, an increase in σ_t makes learning over μ_t more difficult and raises agents' uncertainty over long-run growth. As a result, households demand a higher compensation to hold assets whose expected discounted payouts are strongly influenced by μ_t . This variation in risk premia, absent in the full information model, is the major driver of the response of the value-output ratio to σ_t .

3.4.4 Sensitivity Analysis

We now briefly discuss the sensitivity of the results presented in the previous section to our calibration. In order to do so, we construct the model implied elasticities of the value-output ratio to μ_t and σ_t and study how these elasticities are affected when changing some key parameters of the model. Let θ' be a given value for our parameter vector. Conditional on θ' , we simulate realizations of length \mathbf{T} for μ_t , σ_t and for the value-output ratio. Given these simulated series,

Figure 3.5: IRFs to a Volatility Switch



Note: IRFs are calculated via simulation techniques. We simulate $M = 25000$ different realization of length $T = 500$. Each simulation has the characteristic that the volatility of TFP is in the low state between period 1 and period 400 and switches to the high regime in period 401. After that, the simulations are not restricted with regard to the volatility. The mean growth rate is fixed to its high state throughout the simulations. The above figure reports the mean across the Monte Carlo replications as percentages with respect to period 400 for all series but the volatility. The black dotted lines report the IRFs for a model with perfect information ($\sigma_g = 0$).

we run the following linear projection:

$$\frac{p_t}{y_t} = \mathbf{a} + \mathbf{b}\hat{\mu}_t + \mathbf{c}\hat{\sigma}_t + e_t.$$

The coefficients of these linear projections, $\{\mathbf{b}(\theta'), \mathbf{c}(\theta')\}_{\theta'}$, are the model counterparts of the elasticities computed in Section 3.2 using U.S. data.³⁰ Moreover, they are an interesting object to base our sensitivity analysis on since they give information on the *sign* and *size* of the association between the value-output ratio, economic growth and volatility.

³⁰In our simulations, $\hat{\mu}_t$ and $\hat{\sigma}_t$ are the retrospective estimates of households. These differ, in principle, from what an econometrician would obtain by estimating the system in (2) using data simulated from our model. We have verified, though, that in practice the two produce almost identical quantitative results. We have therefore decided to use households' retrospective estimates when calculating the model implied elasticities, since it substantially reduces the computational burden of the procedure.

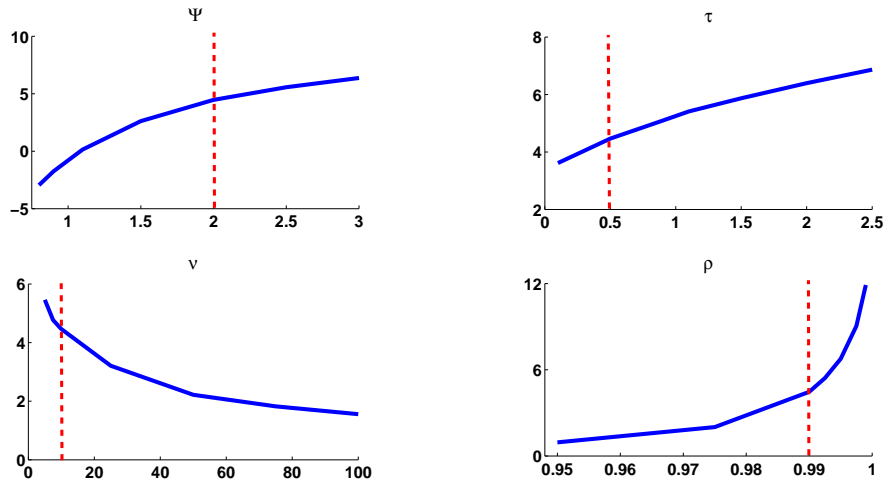
We organize our discussion around four key parameters: i) the elasticity of intertemporal substitution (Ψ); ii) the elasticity of marginal Q with respect to the investment-capital ratio (τ); iii) the elasticity of substitution between intermediate goods (ν); and iv) the persistence of the Markov processes (ρ). Figure 3.6 reports the value of \mathbf{b} when varying these four parameters one at a time, with the red dotted line marking our benchmark calibration.

The top-left panel shows that \mathbf{b} is increasing in Ψ . This is in line with our discussion in the previous section. Indeed, we have seen how a switch from the low- to the high-growth regime brings in offsetting wealth and substitution effects on households. As agents become less averse to intertemporal substitution (Ψ increases), the substitution effect becomes stronger, and their demand of corporate shares becomes more sensitive to fluctuations in μ_t . As a result, a 1% increase in the trend growth rate of TFP is associated with a stronger increase in the value-output ratio. Notice that when Ψ is sufficiently small, \mathbf{b} becomes negative. In these situations, the wealth effect dominates the substitution effect, leading to a negative association between economic growth and the value of corporations.

The next two panels of Figure 3.6 report the sensitivity of \mathbf{b} with respect to τ and ν . We can see that a higher τ is associated with a stronger response of asset prices to fluctuations in μ_t , while a higher ν is associated with a smaller \mathbf{b} . When τ is large, adjusting capital is more costly from the perspective of capital good producers, who then have less incentives to invest. Thus, after a shift in μ_t , their profits on existing capital are eroded less from the process of capital accumulation. This implies that the value of capital good firms is more sensitive to μ_t : when τ equals 2.5, a 1% increase in the trend growth rate of the economy is associated with a 7% increase in the value-output ratio, almost double with

respect to what we obtain in our benchmark calibration. A similar phenomenon occurs when decreasing ν . Indeed, we have seen that the present value of rents is the most volatile component of asset prices in our model. As ν declines, the share of this component on the total value of corporation increases, thus raising the sensitivity of the value-output ratio to μ_t .

Figure 3.6: Sensitivity Analysis: Elasticity of Value-Output Ratio to μ_t



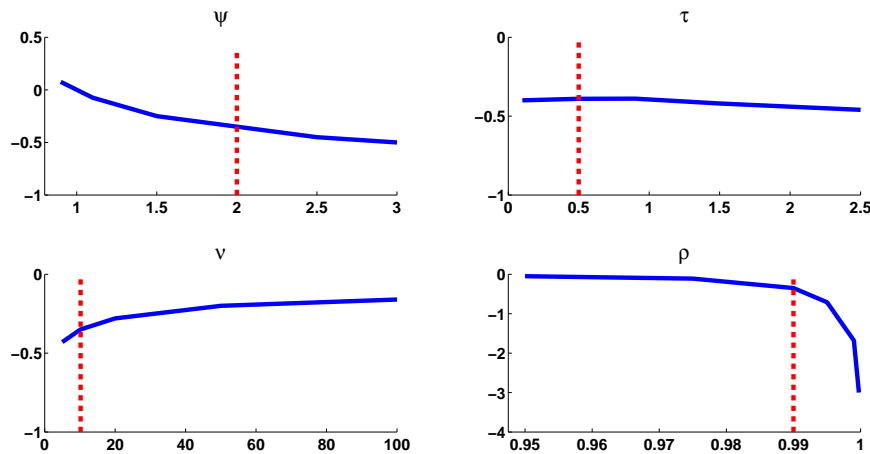
Note: For each point in the parameter space, the elasticity of the value output ratio to μ_t is calculated according to the procedure in Section 3.4.4. In the simulation, \mathbf{T} is set to 25000000. The red dotted line marks the benchmark calibration.

The last panel of the Figure shows the sensitivity of \mathbf{b} with respect to ρ . This is by far the most important parameter in determining quantitatively the response of the value-output ratio to fluctuations in economic growth. When the switch from the low to the high TFP growth regime is perceived to be almost permanent ($\rho = 0.999$), a 1% increase in μ_t is associated with a 12% increase in the value-output ratio. On the contrary, \mathbf{b} is almost 0 when ρ is equal to 0.95. This is in line with our previous discussion. When ρ is high, households expect the growth rate of corporate payouts to be high for a long period of time. Since households are forward looking, they have now stronger incentives to buy corporate shares,

and this raises the response of the value-output ratio. Notice also that \mathbf{b} is highly nonlinear in ρ around our benchmark parametrization. Even a small increase in this parameter results in the elasticity of the value-output ratio to double or triple with respect to our benchmark calibration.

Figure 3.7 reports the same experiment for the elasticity of the value-output ratio to σ_t . The Figure confirms the above discussion. Higher Ψ is associated with a decline in \mathbf{c} . Asset prices are marginally more sensitive to volatility fluctuations when the supply of capital is less elastic (τ is large) or when monopolistic rents are more relevant (ν small). Again, ρ is the most important parameter governing the elasticity of the value-output ratio to σ_t . As ρ passes from 0.99 to 0.999, the absolute value of \mathbf{c} increases by almost 10 times.

Figure 3.7: Sensitivity Analysis: Elasticity of Value-Output Ratio to σ_t



Note: See Figure 3.6

3.4.5 Posterior Predictive Analysis

After having analyzed the economic mechanisms governing the relation between growth, volatility and asset prices, we now assess the model's quantitative per-

formance along this dimension. For this purpose, we will ask how far the model implied elasticities \mathbf{b} and \mathbf{c} are from the ones we estimated for the U.S. economy (Column 3 of Table 3.2). Because of the extreme sensitivity of these elasticities to the value chosen for the persistence of μ_t and σ_t , we will rely on posterior predictive analysis. In particular, let $\theta_1 = [\mu_0, \mu_1, P_{0|0}^\mu, P_{1|1}^\mu, \sigma_0, \sigma_1, P_{0|0}^\sigma, P_{1|1}^\sigma]$ and let θ_{-1} be the vector collecting the remaining structural parameters of our model, fixed at their calibration values. Given a series of posterior draws for the TFP process parameters, $\{\theta_1^m\}_{m=1}^M$, one can calculate $\{\mathbf{b}^{\text{model}}(\theta_1^m, \theta_{-1}), \mathbf{c}^{\text{model}}(\theta_1^m, \theta_{-1})\}_{m=1}^M$ and use those values to characterize the posterior distribution of the model implied value-output ratio elasticities. Because of the high computational burden involved when solving our equilibrium model repeatedly, we evaluate the model implied elasticities at $\{\theta_1^m\}_{m=1}^M$ using the following procedure:³¹

Posterior Draws for Model Implied Elasticities Let $\{\theta_1^m\}_{m=1}^M$ be a set of posterior draws for the TFP growth process.

- i) Given $\{\theta_1^m\}_{m=1}^M$ we obtain bounds on each parameter so that all elements of the sequence lie in the set defined by those bounds. We denote this set by Θ_1 .
- ii) We compute the Smolyak collocation points for Θ_1 as described in Krueger and Kuebler (2003). We denote these collocation points by $\{\theta_1^s\}_{s=1}^S$.
- iii) For each element in $\{\theta_1^s\}_{s=1}^S$, we compute the model implied elasticities $\mathbf{b}(\theta_1^s, \theta_{-1})$ and $\mathbf{c}(\theta_1^s, \theta_{-1})$ using the simulation procedure described in Section 3.4.4.

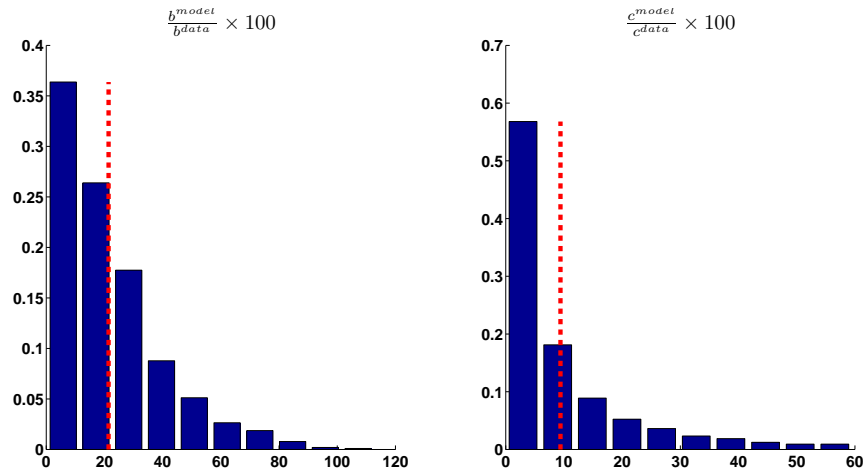
³¹In order to reduce the dimensionality of the problem, we estimate the system in (2) by imposing symmetry on the transition matrices and by fixing $\mu_0 + \frac{1}{2}\mu_1$ and $\sigma_0 + \frac{1}{2}\sigma_1$ to their sample means. Thus, θ_1 is a 4-dimensional object.

iv) We fit a polynomial through the computed $\{\mathbf{b}^{\text{model}}(\theta_1^s, \theta_{-1}), \mathbf{c}^{\text{model}}(\theta_1^s, \theta_{-1})\}_{s=1}^S$.

We then use this polynomial to evaluate the model implied elasticities at the sequence $\{\theta_1^m\}_{m=1}^M$.

Our exercise consists of assessing how far the coefficients of the linear projection in Table 3.2, obtained from actual U.S. data, lie in the tails of the model implied distributions for the same objects.

Figure 3.8: **Growth, Volatility and the Value of U.S. Corporations: Model vs. Data**



Note: The histogram reports the model implied elasticities of the value-output ratio to the mean and volatility of TFP growth relative to their data counterparts. The red dotted lines report the posterior mean of these statistics. The figures are constructed using 50000 draws from the posterior distribution of θ_1 .

The left panel of Figure 3.8 reports the posterior distribution of $\frac{b^{\text{model}}}{b^{\text{data}}} \times 100$. A value of this statistic equal to 100 tells us that the model predicts the same elasticity estimated in the data, while values smaller than 100 would imply a weaker association between economic growth and the value-output ratio with respect to what we have estimated in the data. We verify from the figure that the model is broadly consistent with the data along this dimension. Indeed, on average the model captures 20% of the relation between economic growth and the value-output ratio (red vertical line in the Figure). Moreover, we can also see that for reasonable

parametrizations of the TFP process,³² the model is able to deliver elasticities of the value-output ratio to the trend growth rate of TFP that are consistent with the estimates in Table 3.2.

The right panel of Figure 3.8 plots the posterior distribution of $\frac{c^{\text{model}}}{c^{\text{data}}} \times 100$. The graph shows that the model is less successful in accounting for the association between the value-output ratio and the volatility of TFP growth. On average, in fact, it predicts that the value-output ratio falls by 0.4% following a 1% increase in the standard deviation of TFP growth, roughly 10% of what we have estimated in the data. Moreover, the histogram shows that the model can account for at most 60% of the association between the value-output ratio and the volatility of TFP growth.

3.5 Conclusion

In this paper we have uncovered a striking association between the first two moments of TFP growth and the value of corporations in postwar U.S. data. Persistent fluctuations in the mean and volatility of TFP growth predict two-thirds of the medium-term variation in the value-output ratio. This indicator rises strongly after an increase in the trend growth rate of TFP, while it declines substantially following an increase in the volatility of TFP growth. A possible explanation for this association, suggested elsewhere in the literature, is that movements in aggregate productivity influence investors' expectations of future corporate payouts as well as the rate at which they discount them. This explanation is put under scrutiny by us. We developed a general equilibrium model with production fea-

³²By reasonable parametrization, we mean regions of the parameter space that have positive mass in the posterior distribution.

turing Markov-Switching fluctuations in the mean and volatility of TFP growth, incomplete information, capital adjustment costs, monopolistic competition and recursive preferences. Under plausible calibrations, the model is consistent with the behavior of several U.S. real and financial indicators during the postwar period. It accounts on average for roughly 20% (9%) of the association between the mean (volatility) of TFP growth and the value-output ratio. For reasonable parametrizations of the TFP process, the model predicts an elasticity of the value-output ratio to economic growth that is in line with the data, while it predicts an elasticity of the value-output ratio to the volatility of TFP growth that is 60% of the data observation.

It is important to stress the *ex-post* nature of our analysis. This has at least two important implications. First of all, our estimates for the TFP process are retrospective and this may contribute to muting some of the channels analyzed in this paper. This is surely the case for the implications of incomplete information. Indeed, while a two-state process for the mean of TFP growth fits postwar U.S. data well, agents may still consider states that never occurred during this period when forming their expectations. If that was the case, a model restricted to two states necessarily bounds the amount of perceived risk over long-run growth, which dampens the response of risk premia to a change in the volatility of TFP growth. This particular aspect may explain why the model does not generate a strong sensitivity of the value-output ratio to second moments. Secondly, agents in our model have perfect knowledge of the parameters governing the growth and volatility regimes. This assumption rules out important sources of history dependence. With uncertainty on the persistence of the Markov processes, for example, agents' perception about these parameters depends on previous realizations of the

process. Our approach may thus misrepresent how agents interpreted these fluctuations when solving their decision problem in real time. This may severely affect our results, since the sensitivity of asset prices to shifts in the first two moments of TFP growth depends crucially on their perceived duration.

While difficult to discipline empirically, we believe that an analysis that relaxes these two types of restrictions would further enhance our understanding of the medium-term movements in the value of corporations.

Chapter 4

Identifying Neutral Technology Shocks

4.1 Introduction

The objective of this paper is to propose a method to identify neutral labor-augmenting technology shocks in the data. Classic results, starting with [Uzawa \(1961\)](#), establish that these shocks drive the long-run economic behavior along the balanced growth path. They are also the key driving force inducing fluctuations in real business cycle (RBC) models pioneered by [Kydland and Prescott \(1982b\)](#), [Long and Plosser \(1983\)](#), and play a quantitatively important role in New Keynesian models, e.g., [Smets and Wouters \(2007b\)](#).¹ Moreover, the relationships between various economic variables and neutral technology shocks identified in the data are routinely used to assess model performance and to distinguish between competing models. For example, the empirical finding that aggregate hours

¹In most models the production function is such that the labor augmenting or Harrod-neutral technology shocks are isomorphic to the Hicks-neutral shocks that do not affect the marginal rate of substitution between any factors of production.

worked fall in response to a technology shock called into question the usefulness of the RBC model for interpreting aggregate fluctuations.

However, the methods used in the literature to identify the technology shocks are not designed to measure *neutral* technology shocks. Consider, for example, the classic Solow residual accounting procedure. Suppose output is produced with effective labor input $L_t^e = G(L_1, \dots, L_n, t)$ aggregating various labor inputs and, for simplicity, a single capital input according to the following constant returns to scale production function:

$$Y = F(K, Z G(L_1, \dots, L_n, t)), \quad (4.1)$$

where Z represents the labor-augmenting neutral technology shock we are interested in identifying. Note that the labor aggregator is allowed to depend on time to capture non-neutral changes in technology, e.g., changes in relative productivity or substitutability of various labor inputs. Such changes are thought key for understanding various issues in macro and labor economics. For example, the vast literature on skill biased technical change rationalizes the simultaneous increase in supply and in the relative wages of college educated workers since the 1970s through the change in the relative productivity of these workers in aggregate production (e.g., [Katz and Murphy \(1992\)](#), [Acemoglu \(2002\)](#)).² Thus, as emphasized by [Solow \(1957\)](#), one must allow for the possibility that the neutral technology parameter is only one of many technological parameters that can change over time. Differentiating the production function with respect to time and dividing by Y we

²The alternative interpretation of the evidence in [Krusell et al. \(2000\)](#) also relies on non-neutral change in the parameters governing relative productivity of the investment good sector.

obtain the Solow residual:

$$(1 - \omega_K) \frac{\dot{Z}}{Z} + \frac{\partial F / \partial t}{F} = \frac{\dot{Y}}{Y} - \omega_K \frac{\dot{K}}{K} - \sum_{j=1}^n \omega_{L_j} \frac{\dot{L}_j}{L_j}, \quad (4.2)$$

where, assuming that factors are paid their marginal products, ω_i represents income share of factor i . Clearly, as emphasized in the original [Solow \(1957\)](#) article, the residual equals neutral plus non-neutral technology changes. Hence its other name - the total factor productivity. As variables, such as total hours worked, may react either positively or negatively to a non-neutral shock, their response to an innovation in the Solow residual is difficult to interpret. Unfortunately, the growth accounting methodology is not designed to identify the contribution of only the neutral shock, to which models have a robust prediction regarding the response of endogenous variables. It also provides no possibility to ascertain the relative importance of neutral and biased technological innovations in driving aggregate economic dynamics.

An alternative approach to identifying neutral technology shocks is based on the assumption put forward by [Gali \(1999\)](#) that only technology shocks have a long-run effect on labor productivity (output per hour). He implemented this idea in a business cycle context using structural vector autoregressions (SVAR) identified with long-run restrictions following [Blanchard and Quah \(1989\)](#). Unfortunately, this approach is also not designed to identify neutral technology shocks. Denote by L the total sum of hours worked. Then, using [\(4.1\)](#), output per hour can be written as

$$\log \left(\frac{Y_t}{L_t} \right) = \log(Z_t) + \log \left(\frac{L_t^e}{L_t} \right) + \log \left(F \left(\frac{K_t}{Z_t L_t^e}, 1 \right) \right). \quad (4.3)$$

Note that $\frac{K_t}{Z_t L_t^e}$ is stationary in most models (consistent with the stationary interest rate in the data). However, the long-run changes in productivity can be induced either by persistent technology shocks or by persistent changes in the effective labor input per hour worked. In particular, L^e/L could change in the long run either due to the persistent changes in worker composition (e.g., changes in female labor force participation and their distribution across occupations), changes in the effective units of labor supplied by various labor inputs (e.g., expecting longer careers, women invest more in human capital through on-the-job training) or changes in the production function parameters that govern the relative productivity or substitutability of various labor inputs (e.g., an increase in the relative productivity of females due to an increase in the demand for tasks in which they have a comparative advantage or due to directed changes in technology induced by an increase in their labor force participation). Any of these changes affecting labor productivity in the long run will be interpreted as a technology shock by this methodology. The existing literature provides no guidance on how the neutral technological changes can be isolated.³

These observations lead us to propose a method for estimating neutral technology shocks. To do so, we assume a constant returns to scale aggregate production

³The econometric issues underlying this approach have been intensely discussed in the literature (e.g., [Faust and Leeper \(1997\)](#), [Chari et al. \(2008\)](#), [Christiano et al. \(2006\)](#), [Fernandez-Villaverde et al. \(2007b\)](#)). Instead, we question the long-run restriction itself. Indeed, *any* shock that affects the composition of factors of production or their relative productivity in the long run will have a long-run effect on labor productivity and will be erroneously interpreted as a neutral technology shock by this methodology. Several related critiques of this approach appeared in the literature. [Shea \(1998\)](#) suggests that if low-productivity firms are destroyed in recessions, there might be a long run effect on productivity. [Uhlig \(2004\)](#) argues that permanently changing social attitudes to workplace, whereby workers substitute leisure activities at home with leisure activities at work, will result in mis-measurement of effective work hours and affect measured productivity in the long run. [Francis and Ramey \(2005, 2009\)](#) note that changes in capital taxes or low frequency movements in age composition of population also may have a long-run effect on labor productivity. [Fisher \(2006\)](#) imposes additional restrictions to separate neutral from investment-specific shocks.

function and exploit the rich implications of Uzawa's characterization of neutral technology on a balanced growth path. We do not assume the economy to be on the balanced growth path but instead use a weak conditional form of this assumption. We only require that the impulse responses to a permanent neutral technology shocks have the standard balanced growth properties in the long run. This is sufficient to identify the neutral technology shock because we are able to prove that no other shock (to non-neutral technology, preferences, etc.) satisfies these restrictions.

To implement this identification strategy we use a state-space model for a set of variables for which we know the long run effect of neutral technology. These macroeconomic variables can be represented as a sum of a neutral technology shock, which is treated as one of the unobserved components driving the system, and an unobserved state. For example, the log of the wage of workers of a particular type is written as the sum of the neutral technology shock and an unobserved component that is partially idiosyncratic to that worker type (in a competitive framework representing the derivative of the production function with respect to that labor input).

We do not require orthogonality among the state variables, an assumption commonly used to identify these types of models although inconsistent with typical economic models. Instead we prove that the conditional balanced growth restrictions are sufficient to identify neutral technology shocks in the resulting system of equations collecting various macroeconomic time-series using filtering/smoothing techniques. Since we do not treat the technology shock as a residual, our method does not require to specify an explicit function that aggregates heterogeneous labor

and capital inputs.⁴ Instead, all this information is summarized in the unobserved states which we identify without the need to specify the structure behind the dynamics of these states. Moreover, our method does not require the parameters of this function to be invariant over time. The identification methodology relies on a testable assumption on the time series process for the neutral technology, e.g., AR(1) and other unobserved states, e.g., VAR(1). This process is only required to provide a good statistical approximation and does not have to be consistent with a structural model since we do not need to assign a structural interpretation to the other shocks affecting the economy.

To assess the small sample properties of the proposed method, we conduct a Monte Carlo study using samples drawn from estimated benchmark business cycle models. We consider the RBC and the New-Keynesian models with worker heterogeneity. We find that the proposed method is successful in identifying neutral technology shocks in the data generated by the models and does not confound neutral technology with other disturbances such as non-neutral technology, preference shifts or wage markup shocks.

The paper is organized as follows. In Section 4.2 we develop the method to recover neutral technology shocks and establish the sufficient conditions for identification. In Section 4.3 we illustrate the implementation and evaluate the performance of the proposed method in an estimated RBC model. In Section 4.4 we assess the performance of the proposed method in small samples drawn from an estimated medium scale DSGE model with multiple sources of real and nominal rigidities and numerous exogenous shocks. Finally, in Section ?? we apply

⁴This is in contrast to attempts to identify neutral technology shocks by fully specifying the production function and all the associated inputs as in e.g., [Nadiri and Prucha \(2001\)](#) and [Dupuy \(2006\)](#). The data requirements underlying this approach seem prohibitive.

our method in the data and estimate a quarterly technology series for the US. We also describe and analyze the sequence of identified shocks and document its co-movement with other economic aggregates. Section 4.5 concludes.

4.2 Identifying Neutral Technology Shocks

In this section we propose a method to estimate Harrod-neutral technology shocks. Section 4.2.1 provides a characterization of these type of shocks. We prove that Harrod-neutral technical change is the only type of shock that can induce balanced growth on a set of macroeconomic variables. We next show how we can use this property to identify neutral technology shocks from the data using benchmark time series models. Section 4.2.2 present the time series model we use while Section 4.2.3 formally proves that the long run restrictions implied by balanced growth are sufficient to identify Harrod-neutral technology shocks. Section 4.2.4 discusses several issues related to the practical implementation of our approach.

4.2.1 Identification: Theory

In this section we build on this classic result and show how to use the insights from Uzawa's theorem (see Acemoglu (2009) for an excellent treatment) on technological progress in the long-run to identify Harrod-neutral technology shocks. Suppose that aggregate output Y_t is produced as follows

$$Y_t = F(K_{1,t}, \dots, K_{J,t}, Z_t L_{1,t}, \dots, Z_t L_{M,t}; \theta_t), \quad (4.4)$$

where $K_{j,t}$ represent capital input of type j , $L_{m,t}$ represents labor inputs of type m , Z_t is Harrod-neutral technology progress and θ_t is a vector collecting other non-

neutral technological changes. We assume that F is constant return to scale in capital and labor inputs. Our methodology does not require any further restriction on the aggregate production function.

We make the following *conditional balanced growth assumption*, anticipating that the implementation of the identification methodology in the data will use a state-space model and thus identify the shock through its impulse response.

Assumption 1 (conditional balanced growth assumption). *A time T exists such that the impulse response of a variable X_t to a Harrod-neutral innovation ϵ_0^Z (to Z_0) of x percent at time 0,*

$$IR_t^X(x) = E_0[X_t \mid \log(\epsilon_0^Z) = x] - E_0[X_t]$$

equals

$$IR_t^X(x) = (e^{g_X x} - 1)E_0[X_t]$$

for all $t \geq T$, where $g_X x$ is the percent increase in X . If $g_X = g$ for output, for all capital inputs, and for all types of investment and of consumption, then labor inputs $X = L_m$ do not respond in the long-run to a neutral shock, $g_X = g_{L_m} = 0$.

This assumption guarantees convergence of the impulse response for all variables. In addition, it assumes a linearity property of the long-run response to a neutral technology shock (i.e. a shock of size $2x$ has exactly twice the effect of a shock of size x). History dependence of the impulse response function is not ruled out in the short run, nor it is for all other economic shocks. Moreover, this assumption tells us that if Harrod-neutral shocks induce a common trend in output, capital inputs, investments and consumption, then they do not influence labor inputs in the long run.

We now demonstrate that Harrod-neutral technology shocks are the only one that can induce a certain pattern of long run responses for a set of macroeconomic variables. This property will be then used to identify this technical change from aggregate data. Before stating the main theorem, though, we prove a useful result

Lemma 1. *Suppose the conditional balanced growth assumption holds for variables $X_t, X_{1,t}, \dots, X_{H,t}$ where $X_t = \sum_{h=1}^H X_{h,t}$. The long-run response for variable X is g_X and for the components $X_h > 0$ equal to g_{X_h} . Then*

$$g_X = g_{X_1} = \dots g_{X_h} = \dots = g_{X_H}.$$

The proof is in Appendix D.1.1. Lemma 1 tells us that, for a shock to have a well defined long run effect on a variable X , it must have the same long run effect on its components.

Theorem 1. *Suppose the conditional balanced growth assumption holds. Then a permanent Harrod-neutral technological shock is the only shock with the following (balanced-growth) properties for some time T . An innovation which increases the level of the shock by x percent at time 0 implies for all $t \geq T$*

- An increase in aggregate output Y by x percent, $IR_t^Y(x) = (e^x - 1)E_0[Y_t]$
- An increase in investment I_j by x percent, $IR_t^{I_j}(x) = (e^x - 1)E_0[I_{j,t}]$
- An increase in capital K_j by x percent, $IR_t^{K_j}(x) = (e^x - 1)E_0[K_{j,t}]$
- An increase in aggregate consumption C by x percent, $IR_t^C(x) = (e^x - 1)E_0[C_t]$
- No effect on labor inputs L_m , $IR_t^{L_m}(x) = 0$
- No effect on the marginal product of capital F_{K_j} , $IR_t^{F_{K_j}}(x) = 0$

- An increase in the marginal product of labor F_{L_m} by x percent, $IR_t^{F_{L_m}}(x) = (e^x - 1)E_0[F_{L_m,t}]$

The proof in Appendix [D.1.2](#) follows the steps in the proof of Uzawa's theorem in [Acemoglu \(2009\)](#).

The conditions in the theorem rule out non-neutral technical change. For example, they rule out investment-specific shocks ([Greenwood et al., 1997](#)), which are often modeled as a non-neutral shock to the technology for producing capital equipment. This shock has long run effects on the capital equipment to capital structures ratio and the capital output ratio, which is inconsistent with the above properties.

The theorem is not limited, however, to distinguishing between different types of technical change. Instead, it characterizes Harrod-neutral technology shocks and thus tells them apart from any other economic shock, e.g. preference shocks, government expenditure shocks or wage mark-up shocks. The logic is as follows. If output and capital increase by the same percentage rate then constant returns to scale imply that effective labor input has to increase by the same percentage rate. Because labor inputs is assumed not to change in the long run, the productivity of labor has to increase by the same percentage term, i.e. it has to be a Harrod-neutral technological change.

4.2.2 Implementation: The State Space Model

In this section we show how we can implement the conditional balance growth restrictions of the previous section using a benchmark time series model. To this aim, we assume that we observe a vector time series \mathbf{D}_t , collecting growth rates

of a set of macroeconomic variables. Without loss of generality, we write \mathbf{D}_t as the sum of two components

$$\mathbf{D}_t = \Delta Z_t \mathbf{1}_n + \tilde{\mathbf{S}}_t, \quad (4.5)$$

where ΔZ_t is the growth rate of the neutral technology series (in logs and $\mathbf{1}_n$ is the n -dimensional vector of ones), and $\tilde{\mathbf{S}}_t$ is a vector of states. Both ΔZ_t and $\tilde{\mathbf{S}}_t$ are unobserved. Clearly, any macroeconomic time-series can be written this way. Two examples for \mathbf{D}_t with a clear economic interpretation, are output growth $\Delta \log(Y_t)$ and the growth rate of competitive wages for a worker of type m , $\Delta \log(W_{m,t})$:⁵

$$\Delta \log(Y_t) = \Delta Z_t + \Delta \log \left[F \left(\frac{K_{1,t}}{Z_t}, \dots, \frac{K_{J,t}}{Z_t}, L_{1,t}, \dots, L_{M,t}; \theta_t \right) \right], \quad (4.6)$$

$$\Delta \log(W_{m,t}) = \Delta Z_t + \Delta \log \left(\frac{\partial F}{\partial L_{m,t}} \right). \quad (4.7)$$

Thus, the unobserved state $\tilde{\mathbf{S}}_t$ is $\Delta \log \left[F \left(\frac{K_{1,t}}{Z_t}, \dots, \frac{K_{J,t}}{Z_t}, L_{1,t}, \dots, L_{M,t}; \theta_t \right) \right]$ for output and for wages it is equal to $\Delta \log \left(\frac{\partial F}{\partial L_{m,t}} \right)$.

Since we treat the second component as an unobserved state variable, we do not have to make any assumptions on the shape of the production function. Instead our approach consists in restricting the time-series behavior of $\mathbf{S}_t = [\Delta Z_t, \tilde{\mathbf{S}}_t]$ and in exploiting the factor structure of the system in (4.5). In particular, we propose to estimate the technology series $\{\Delta Z_t\}_{t=0}^T$ in a three steps procedure:

- i) Assume a time series model for the behavior of $[\Delta Z_t, \tilde{\mathbf{S}}_t]$, indexed by the vector of parameters Λ .
- ii) Estimate the parameters' vector Λ .

⁵Wages are competitive here for illustrative purposes only. Our method does not assume that wages are competitive.

- iii) Conditional on the estimation of $\mathbf{\Lambda}$ and given a time series for \mathbf{D}_t , we estimate the realization of ΔZ_t using smoothing techniques.

For concreteness, suppose that ΔZ_t is an univariate AR(1) process with persistence parameter given by ϕ_{zz} and innovation variance given by r_{zz}^2 (which we normalize to one), while the unobserved states follow a VAR(1). None of the results discussed in this section depend on this parametrization, and richer dynamics can be allowed for by introducing additional lags and moving average terms. Under these assumptions we can express the dynamics of \mathbf{D}_t in state space form:

$$\begin{aligned}
\begin{array}{c} n \times 1 \\ \boxed{\mathbf{D}_t} \\ \mathbf{D}_t \end{array} &= \begin{array}{c} n \times (n+1) \\ \boxed{\mathbf{1} \quad \mathbf{I}} \\ \mathbf{B} \end{array} \begin{array}{c} (n+1) \times 1 \\ \boxed{\begin{array}{c} \Delta Z_t \\ \tilde{\mathbf{S}}_t \end{array}} \\ \mathbf{S}_t \end{array} \\
\begin{array}{c} (n+1) \times 1 \\ \boxed{\begin{array}{c} \Delta Z_t \\ \tilde{\mathbf{S}}_t \end{array}} \\ \mathbf{S}_t \end{array} &= \begin{array}{c} (n+1) \times (n+1) \\ \boxed{\begin{array}{cc} \phi_{zz} & \mathbf{0}' \\ \mathbf{\Phi}_{\tilde{\mathbf{S}}z} & \mathbf{\Phi}_{\tilde{\mathbf{S}}\tilde{\mathbf{S}}} \end{array}} \\ \mathbf{\Phi} \end{array} \begin{array}{c} (n+1) \times 1 \\ \boxed{\begin{array}{c} \Delta Z_{t-1} \\ \tilde{\mathbf{S}}_{t-1} \end{array}} \\ \mathbf{S}_{t-1} \end{array} + \begin{array}{c} (n+1) \times (n+1) \\ \boxed{\begin{array}{cc} r_{zz} & \mathbf{0}' \\ \mathbf{R}_{\tilde{\mathbf{S}}z} & \mathbf{R}_{\tilde{\mathbf{S}}\tilde{\mathbf{S}}} \end{array}} \\ \mathbf{R} \end{array} \begin{array}{c} n \times 1 \\ \boxed{\begin{array}{c} e_{z,t} \\ \tilde{\mathbf{e}}_t \end{array}} \\ \mathbf{e}_t \end{array} \quad (4.8) \\
\mathbf{e}_t &\stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}_{n+1}, \mathbf{I}_{n+1})
\end{aligned}$$

ΔZ_t is assumed to be an exogenous process in the above system. In particular, $\tilde{\mathbf{S}}_{t-1}$ does not affect current technology once we condition on Z_{t-1} , this explaining the zeros in the transition matrix $\mathbf{\Phi}$. Moreover, the zero restrictions on the \mathbf{R} matrix tell us that the first element of the \mathbf{e}_t vector has to be interpreted as an innovation to technology. Notice that we allow for contemporaneous correlation among the innovations to ΔZ_t and $\tilde{\mathbf{S}}_t$ since we do not restrict $\mathbf{R}_{\tilde{\mathbf{S}}z}$ to be zero. This is particularly relevant in our application since technology shocks are likely

to affect $\tilde{\mathbf{S}}_t$.⁶ Because of this correlation, the state space model is not identified without further restrictions. Fortunately, as we show in the next section, we can use Theorem 1 to impose a set of restrictions that are sufficient to identify the parameters of the model. In terms of notation, we will refer to $D_{j,t}$ as the j^{th} element of the measurement vector \mathbf{D}_t while to $S_{j,t}$ as the j^{th} element of the state vector $\mathbf{S}_t = [\Delta Z_t, \tilde{\mathbf{S}}_t]'$. We denote by \mathbf{e}_t the vector $[e_{z,t}, \tilde{\mathbf{e}}_t]'$.

4.2.3 Identification of the State Space Model

We include in the vector of observable variables, \mathbf{D}_t , the growth rates of output, investment and hours as well as of the wages of two groups of workers, (s)killed and (u)skilled:

$$\mathbf{D}_t = (\Delta \log(Y_t), \Delta \log(I_t), \Delta \log(L_t), \Delta \log(W_{s,t}), \Delta \log(W_{u,t}))'. \quad (4.9)$$

From the discussion in Section 4.2.1 we know that these variables are sufficient to distinguish Harrod-neutral technology shocks from other economic disturbances. Clearly, one could incorporate in \mathbf{D}_t more variables with known balanced growth restrictions: this would sharpen identification at the cost of increasing the complexity of the model.

We now formally define identifiability of the state space model

Definition 1. Let $\mathbf{\Lambda}$ and $\hat{\mathbf{\Lambda}}$ be two parameterizations of the system in (4.8).

These are observationally equivalent if $\mathbf{\Gamma}_{\mathbf{D}}(\tau, \mathbf{\Lambda}) = \mathbf{\Gamma}_{\mathbf{D}}(\tau, \hat{\mathbf{\Lambda}})$ for all $\tau \in \mathbb{N}$, where

$\mathbf{\Gamma}_{\mathbf{D}}(\tau, \mathbf{\Lambda})$ is the τ^{th} order autocovariance of \mathbf{D}_t under $\mathbf{\Lambda}$.

Definition 2. The state space model in (4.8) is identifiable from the autoco-

⁶Both of the examples discussed earlier share this characteristic. A shock to technology affects labor inputs, this generating correlation between ΔZ_t and $\tilde{\mathbf{S}}_t$.

variances of \mathbf{D}_t at $\mathbf{\Lambda} = (\mathbf{\Phi}, \mathbf{R})$ if for any admissible parametrization $\hat{\mathbf{\Lambda}} = (\hat{\mathbf{\Phi}}, \hat{\mathbf{R}})$ we have that $\mathbf{\Lambda}$ and $\hat{\mathbf{\Lambda}}$ are observationally equivalent if and only if $\mathbf{\Phi} = \hat{\mathbf{\Phi}}$ and $\mathbf{R}\mathbf{R}' = \hat{\mathbf{R}}\hat{\mathbf{R}}'$

In what follows, we show how the restrictions brought by the conditional balanced growth assumption are sufficient to guarantee the identification of $\mathbf{\Lambda}$. Prior to that, we make an additional technical assumption

Assumption 2. *i) The matrix \mathbf{R} is invertible.*

ii) $(-1, 1, \dots, 1)'$ is not an eigenvector with eigenvalue ϕ_{zz} of the matrix $\tilde{\mathbf{\Phi}}$.

In Appendix D.1.3 we prove that this assumption implies that the state space representation in (4.8) is minimal, i.e. the dimension of the state vector \mathbf{S}_t can not be reduced. This assumption allows us to cast our problem within the literature of identification of minimal state space systems (Hannan and Diestler, 1988).

Lemma 2. *Let Assumption 2 hold. Then, the state space model in (4.8) is minimal.*

Given minimality of the state space in (4.8), lack of identification is known to be represented by linear transformations of the state vector through invertible matrices \mathbf{T} and \mathbf{U} with $\mathbf{U}\mathbf{U}' = \mathbf{I}$ (see Proposition 1-S in Komunjer and Ng (2011)). In fact, consider defining the state vector $\hat{\mathbf{S}}_t = \mathbf{T}^{-1}\mathbf{S}_t$ and the innovation vector as $\hat{\mathbf{e}}_t = \mathbf{U}^{-1}\mathbf{e}_t$. Then, one can rewrite the system in (4.8) as:

$$\begin{aligned}\tilde{\mathbf{D}}_t &= \hat{\mathbf{B}}\hat{\mathbf{S}}_t \\ \hat{\mathbf{S}}_t &= \hat{\mathbf{\Phi}}\hat{\mathbf{S}}_{t-1} + \hat{\mathbf{R}}\hat{\mathbf{e}}_t\end{aligned}\tag{4.10}$$

where the new matrices $(\hat{\mathbf{B}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{R}})$ are related to the original one as follows:

$$\begin{aligned}\hat{\mathbf{B}} &= \mathbf{B}\mathbf{T} \\ \hat{\mathbf{\Phi}} &= \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T} \\ \hat{\mathbf{R}} &= \mathbf{T}^{-1}\mathbf{R}\mathbf{U}\end{aligned}\tag{4.11}$$

Clearly, the observationally equivalent parametrization must satisfy the restrictions made on $(\mathbf{B}, \mathbf{\Phi}, \mathbf{R})$, narrowing the set of admissible (\mathbf{T}, \mathbf{U}) matrices. In what follows we provide a characterization of this set for the system in (4.8).

First of all, notice that since the matrix \mathbf{B} is known, one needs to have $\hat{\mathbf{B}} = \mathbf{B}$. This implies that the matrix \mathbf{T} has the form:

$$\mathbf{T} = \begin{bmatrix} 1 + \kappa_1 & -\kappa_2 & \dots & -\kappa_n \\ -\kappa_1 & 1 + \kappa_2 & \dots & \kappa_n \\ \dots & \dots & \dots & \dots \\ -\kappa_1 & \kappa_2 & \dots & 1 + \kappa_n \end{bmatrix} \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 - \frac{\kappa_1}{\kappa} & \frac{\kappa_2}{\kappa} & \dots & \frac{\kappa_n}{\kappa} \\ \frac{\kappa_1}{\kappa} & 1 - \frac{\kappa_2}{\kappa} & \dots & -\frac{\kappa_n}{\kappa} \\ \dots & \dots & \dots & \dots \\ \frac{\kappa_1}{\kappa} & \frac{\kappa_2}{\kappa} & \dots & 1 - \frac{\kappa_n}{\kappa} \end{bmatrix}\tag{4.12}$$

for some scalars $\kappa_1, \dots, \kappa_n$ and for $\kappa = 1 + (\sum_{l=1}^n \kappa_l)$. What this means is that if $\kappa_1 = \kappa_2 = \kappa_3 = \dots \kappa_n = 0$ all the parameters of the system in (4.8) are identified and $\mathbf{T} = \mathbf{I}$: we are then able to identify correctly the parameters $\mathbf{\Phi}$ and $\mathbf{\Sigma} = \mathbf{R}\mathbf{R}'$, only the ordering of $\tilde{\mathbf{e}}_t$ would not be identified.

In general, we can easily verify that the state vector associated with the \mathbf{T}^{-1} matrix becomes:

$$\hat{\mathbf{S}}_t = \begin{bmatrix} (1 - \frac{\kappa_1}{\kappa})Z_t + \sum_{l=2}^n \frac{\kappa_l}{\kappa}S_{l,t} \\ \frac{\kappa_1}{\kappa}Z_t + S_{2,t} - \sum_{l=2}^n \frac{\kappa_l}{\kappa}S_{l,t} \\ \dots \\ \frac{\kappa_1}{\kappa}Z_t + S_{n,t} - \sum_{l=2}^n \frac{\kappa_l}{\kappa}S_{l,t} \end{bmatrix}.\tag{4.13}$$

This parametrization needs to satisfy the restrictions on the transition equation

in (4.8), namely that the first element of $\hat{\mathbf{S}}_t$ follows an AR(1) with innovations given by the first element of $\hat{\mathbf{e}}_t$. This cannot be ruled out given the assumptions made so far, i.e. without further restrictions, the system in (4.8) is not identified. This is where we use Theorem 1 which states that the balanced growth properties identify the neutral technology process.

The balanced growth restrictions for output, investment, hours, skilled and unskilled wages can be written as⁷

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{1 - \rho_z} \mathbf{B}(\mathbf{I} - \mathbf{\Phi})^{-1} \mathbf{R}_{1:(n+1),1}. \quad (4.14)$$

Thus, one can express the long run effect of neutral technology on the variables in \mathbf{D} as a function of the parameters in the matrices $\mathbf{\Phi}$ and \mathbf{R} and restrict it to be equal to 0 or 1. For example the first row of the restriction in (4.14) states that the the long-run response of output to a unit increase in $\varepsilon_{z,t}$ equals 1. Similarly rows 2, 4 and 5 restrict the long-run response of investment, high skilled and low skilled wages to be of the same magnitude as well (again scaled by $\frac{1}{1-\rho_z}$). Row 3 requires the long-run response of hours to be 0.

As Theorem 1 shows, these long-run restrictions uniquely identify the neutral shock, that is $\mathbf{U}(1, 0, \dots, 0)' = (1, 0, \dots, 0)'$. This implies that the first column of \mathbf{U} equals $(1, 0, \dots, 0)'$ and using that $\mathbf{U}\mathbf{U}' = \mathbf{I}$ then implies that the first row

⁷ ΔZ_t follows an AR(1) with persistence parameter ρ_z . A one standard deviation error to the innovation (which we normalized to one) of the growth rate accumulates to a long-run change of $\frac{1}{1-\rho_z}$ in the level of z . As the balanced growth restrictions apply to changes in the level of z , the term $\frac{1}{1-\rho_z}$ multiplies the long-run effect on V .

equals $(1, 0, \dots, 0)$.

Theorem 2. [Identification] *Consider the state space model (4.8) with \mathbf{D} including the logs of output, investment, hours worked, skilled and unskilled wages as in (4.9) and with balanced growth restrictions (4.14). Then the parameters Φ and $\mathbf{R}\mathbf{R}'$ are identified. In particular $\kappa_1 = \kappa_2 = \dots = \kappa_n = 0$. Furthermore the neutral technology shock is identified, i.e.*

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & & \tilde{\mathbf{U}} & \\ 0 & & & \end{bmatrix}. \quad (4.15)$$

The proof is in Appendix D.1.4.

4.2.4 Discussion

In this section we discuss how we estimate the model, how to obtain impulse-responses and the choice of the time-series model that we use to implement our identification procedure. We also discuss how additional restrictions on the state space can be imposed.

Estimation

Because of the linear-gaussian structure of the state space model, we can evaluate the likelihood function using the Kalman filter. The model parameters are then estimated by maximum likelihood. Conditional on the estimated parameters, we can apply the Kalman smoother and obtain retrospective estimates of Harrod-neutral technical change, $\{p(\Delta Z_t | \mathbf{D}^T)\}_{t=1}^T$. See Durbin and Koopman (2001) for an extensive discussion of these methodologies.

Impulse Response Functions

Impulse Response Functions (IRFs) to a neutral technology shock for variables included in the data vector \mathbf{D}_t can be easily computed using the estimated parameters and the state space model in (4.8). We may be also interested in computing IRFs for variables \mathbf{x}_t that do not enter the measurement equation. In this case we proceed by using the estimated technology innovations of $\{e_{z,t}\}_{t=1}^T$. We project $\{e_{z,t}\}_{t=1}^T$ and its lags onto \mathbf{x}_t ,

$$\mathbf{x}_t = \alpha + \beta(L)e_{z,t} + \varepsilon_t,$$

where $\beta(L)$ are polynomials in the lag operator and they represent the IRFs. OLS delivers consistent estimates of these parameters to the extent that $e_{z,t}$ is exogenous. This assumption is natural if we think of \mathbf{x}_t as being generated by an underlying equilibrium model and we are willing to assume orthogonality of its structural shocks.

Choice of Time-Series Model

Our procedure requires to specify a parametric time series model for key macroeconomic variables. Because of its generality, we focus here on a linear state space model, but in principle our analysis could be carried using other linear or nonlinear time series models. As in the SVAR literature, we need to make several specification choices regarding the number of macroeconomic time series to include in the model and the law of motion of the state variables.

The dimension of the state space \mathbf{S}_t may be limited by the curse of dimensionality. First, the number of parameters increases in the lag length of the VAR for \mathbf{S}_t .

This problem, common to the SVAR literature, can be partly circumvented with the use of shrinkage methods that are becoming popular in applied time series econometrics (Del Negro and Schorfheide, 2010). However, because of the exogeneity restrictions on Z_t , we can adopt a more flexible specification for its law of motion without imposing much burden on the estimation. For example, suppose we assume a more general ARMA(p,q) for neutral technology. Then, the number of unknown parameters associated with the technology process equals $(n+1)p+q$ with n being the dimension of \mathbf{S}_t . Second, given a DGP for the vector \mathbf{S}_t , the number of parameters to be estimated steeply increases in the number of variables in the measurement equation. For the example described in Section 4.3, the number of parameters to be estimated equals $2+2n(n+1)+s(2+n)$, where n is the dimension of the vector \mathbf{D}_t and s is the dimension of \mathbf{S}_t . This limits the number of variables, and associated balanced growth restrictions, that can be allowed for.

The Monte Carlo exercise in the next section is supposed to shed lights on these issues. We will see that a parsimonious specification of the state space model considered in this section performs well when data are simulated from reasonably calibrated business cycle models.

Using Additional Theoretical Restrictions

The method proposed in this paper can easily accommodate additional restrictions implied by economic theory. While these restrictions are not strictly necessary, they may help sharpening identification of neutral technology shocks especially when dealing with short samples. A popular identification scheme in the SVAR literature are sign restrictions as in Uhlig (2005). These can be easily incorporated in our set up: for example, we could set $R_{\Delta w_j, z} > 0$ to restrict the neutral technol-

ogy shock to have a positive impact effect on wages. Other types of information regarding the properties of neutral technology shocks can be easily implemented by appropriate restriction on the state space form. Aside from these identification schemes, the state space model considered here can incorporate external information without imposing excessive burden to the estimation. For example, suppose that we have a robust method to identify other types of structural structural shocks, say a government spending shock $\{e_{g,t}\}_{t=1}^T$ which we know a priori to be orthogonal to neutral technology shocks. Then, we could proceed in two steps: i) Add $\{e_{g,t}\}_{t=1}^T$ to the list of observables in the measurement equation; ii) add an additional state variables in \mathbf{S}_t that selects one of the non-technology reduced form innovations; iii) restrict the matrix \mathbf{R} so that $e_{z,t}$ and $e_{g,t}$ are orthogonal. Importantly, this does not result in additional parameters to be estimated, but it helps the identification of the neutral shock. See also [Stock and Watson \(2012\)](#) for a discussion of the role of external information (“instruments”) for the identification of structural shocks in dynamic factor models.

4.3 An Example: A Simple RBC Model

We now illustrate the proposed procedure by means of an example. We study the basic RBC model with two types of labor, a useful benchmark due to its transparency and widespread use. We use this example to illustrate how our method for measuring neutral technology shocks can be applied in practice. Using data simulated from the calibrated model we study the relation between identified technology shocks and the true structural disturbances. In particular, we consider the small sample performance of our method and contrast it with the performance of an SVAR with long run restrictions on labor productivity and with Solow residu-

als. The transparency of the model allows us to isolate the reasons for the poor performance of the latter two methods in recovering neutral technology shocks.

4.3.1 The Real Business Cycle Model with Heterogeneous Labor

We consider a frictionless RBC model with worker heterogeneity. Agents of type $j = \{u, s\}$ (unskilled of measure u and skilled of measure $1 - u$) value consumption, c_t , and dislike labor, h_t , according to a type-dependent utility function

$$U_j(c_t, h_t) = \log(c_t) - e^{A_t} b_j \frac{h_t^{1+\nu_j^{-1}}}{1 + \nu_j^{-1}}. \quad (4.16)$$

A_t is a shock to the disutility of labor parameterizing the labor wedge, commonly found to play an important role in business cycle accounting. We allow the elasticity of labor supply, ν_j , to differ across the two demographic groups. Because of this, aggregate productivity in our model will vary over the cycle due to endogenous changes in the skill compositions of the labor input. Firms in the economy have access to the production function

$$Y_t = K_t^\alpha (e^{Z_t} L_t^e)^{1-\alpha}, \quad (4.17)$$

where L_t^e , the effective labor input, is an aggregator of low and high-skilled labor

$$L_t^e = L_{s,t}^{\phi_t} L_{u,t}^{1-\phi_t}.$$

Note that observed labor input (total hours) equals $L_t = L_{s,t} + L_{u,t}$, where unskilled labor input equals $L_{u,t} = u h_{u,t}$ and skilled labor input $L_{s,t} = (1 - u) h_{s,t}$. The relative productivity of skilled workers, ϕ_t , changes over time. This is one source

of non-neutral technical change in the model. The accumulation equation for investment is expressed as

$$K_{t+1} = (1 - \delta)K_t + I_t q_t, \quad (4.18)$$

where q_t represents the current state of the technology for producing new capital goods, a second source of non-neutral technical change. Capital depreciates in every period at rate δ . The resource constraint equals

$$Y_t = I_t q_t + C_t + g_t Y_t, \quad (4.19)$$

where g_t is the fraction of final good devoted to government spending.

The laws of motion for economic shocks are standard:⁸

$$\Delta Z_t = \gamma + \rho_z \Delta Z_{t-1} + \sigma_z \varepsilon_{z,t}, \quad (4.20)$$

$$A_t = \rho_a A_{t-1} + \sigma_a \varepsilon_{a,t}, \quad (4.21)$$

$$\log(\phi_t) = (1 - \rho_\phi) \phi^* + \rho_\phi \log(\phi_{t-1}) + \sigma_\phi \varepsilon_{\phi,t}, \quad (4.22)$$

$$\log(g_t) = (1 - \rho_g) g^* + \rho_g \log(g_{t-1}) + \sigma_g \varepsilon_{g,t}, \quad (4.23)$$

$$\log(q_t) = \rho_q \log(q_{t-1}) + \sigma_q \varepsilon_{q,t}. \quad (4.24)$$

Firms hire labor and rent capital from households at competitive factor prices, produce the final good and sell it to households in a competitive market. Households use labor and capital income to finance their consumption and saving choices. The equilibrium law of motion for the model's endogenous variables is defined by a set

⁸The only novel process here is the one for the skill-biased technical change. Although the specification we use permits $\phi_t > 1$, this event has almost zero measure in all our simulations. We could use a logistic function to preclude that. However, since we study a linearized version of the model, nothing would prevent the linearized shock to be larger than 1.

of conditions that describes the optimal behavior of agents, and the evolution of shocks. Since these equations are standard in the literature, we avoid repeating them here.

To ensure stationarity, certain model's endogenous variables need to be normalized. We have estimated the model with an unrestricted persistence of the preference shock process and found that, to match the high persistence in hours worked, it is estimated to be unit root.⁹ Given this, we restrict $\rho_a = 1$, and scale hours worked by a type j household by $e^{-\frac{\nu_j}{1+\nu_j}At}$, while the other model's variables by $e^{Z_t e^{[\phi^* \frac{\nu_s}{1+\nu_s} + (1-\phi^*) \frac{\nu_u}{1+\nu_u}]At}}$.

4.3.2 Identifying Neutral Technology Shocks: Setup

The choice of the variables added to the state vector is guided by the balanced growth restrictions. We therefore consider the growth rate of output, $\Delta \log(Y_t)$, the growth rate of wages of skilled workers, $\Delta \log(w_{s,t})$, and unskilled workers, $\Delta \log(w_{u,t})$, the growth rate of labor productivity, $\Delta \log(Y_t/L_t)$ and investment, $\Delta \log(I_t)$. As described earlier, we interpret each of these times series as the sum of two unobserved components: the growth rate in neutral technological component (common to all variables) and a residual component (specific to each variable). Thus, defining $\mathbf{D}_t = [\Delta \log(Y_t), \Delta \log(w_{s,t}), \Delta \log(w_{u,t}), \Delta \log(Y_t/L_t), \Delta \log(I_t)]'$ the vector of observables, and by \mathbf{S}_t the vector collecting these unobserved components, we can write the measurement equation as

$$\mathbf{D}_t = \underbrace{\begin{bmatrix} \mathbf{1} & \mathbf{I} \end{bmatrix}}_{\mathbf{B}} \mathbf{S}_t. \quad (4.25)$$

⁹As in [Gali \(2005\)](#) and [Chang et al. \(2007c\)](#), among others.

Notice that, under this formulation, ΔZ_t is the first entry of the state vector. Next we must chose the model for the time series behavior of the state vector \mathbf{S}_t . In the Monte Carlo analysis, we will restrict to the simple VAR(1) model used in Section 4.2.2:

$$\mathbf{S}_t = \underbrace{\begin{bmatrix} \rho_z & \mathbf{0}' \\ \Phi_{\tilde{s}z} & \Phi_{\tilde{s}\tilde{s}} \end{bmatrix}}_{\Phi} \mathbf{S}_{t-1} + \underbrace{\begin{bmatrix} r_{zz} & \mathbf{0}' \\ \mathbf{R}_{\tilde{s}z} & \mathbf{R}_{\tilde{s}\tilde{s}} \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} e_{z,t} \\ \tilde{\mathbf{e}}_t \end{bmatrix}. \quad (4.26)$$

We restrict ΔZ_t to be exogenous with respect to the other unobserved states: this is achieved with the “zeros” restrictions on the matrix Φ and \mathbf{R} . However, note that we are not ruling out correlation among the idiosyncratic states.

Balanced Growth Restrictions

The balanced growth restrictions discussed in the earlier section can be easily implemented in this time series model. For example, consider a one standard innovation increase in neutral technology. Under balanced growth restrictions, we know that the effect of this shock on the level of output is equal to $\frac{\sigma_z}{1-\rho_z}$ in the long run. In the time series model described earlier, the long run effect of the first element of \mathbf{S}_t (which we label neutral technological growth) on the level of output equals to:¹⁰

$$\lim_{m \rightarrow \infty} IR_{t+m}^{\log(Y)}(e_{z,t} = 1) = [1, 0, 0, 0, 0, 0] \mathbf{B}(\mathbf{I} - \Phi)^{-1} \mathbf{R}_{1:(n+k),1}.$$

Hence, the balanced growth restriction for output consist in equating the above

¹⁰This expression derives from the fact that the long run effect of a shock on variable x equals the cumulative effect on its growth rates.

expression to $\frac{\sigma_z}{1-\rho_z}$. Similar balanced growth restriction can be derived for the other variables. More specifically, we restrict output per hour and investment to have the same long run effect of output: this implies, among other things, that hours worked and the investment-output ratio are not affected in the long run by a neutral technology shock. Similarly, we restrict neutral technology shocks to have the same long run effect on the two wages (e.g., relative wages are not affected by a neutral technology shock in the long run).

From a technical point of view, these are restrictions on the Φ and \mathbf{R} matrices that, as discussed in the earlier section, are sufficient to identify the neutral technology shock.

4.3.3 Estimation and Results

The calibration is standard and described in Appendix D.3. We simulate 100 realizations from the calibrated model, with each sample being composed of 250 quarters. For each realization, we estimate the state space model discussed in Section 4.3.2, and we study the properties of the retrieved neutral technology innovations. In addition, we compute technology innovations using the Solow residual accounting and using SVAR with long-run restrictions as in Gali (1999).

We assess the accuracy of each procedure using the R^2 of the following linear regressions:

$$\varepsilon_{j,t}^{true} = \alpha + \beta \varepsilon_{z,t}^{identified} + \eta_t \quad j \in \{z, a, \phi, g, q\}, \quad (4.27)$$

where $\{\varepsilon_{z,t}^{identified}\}$ are the technology shocks identified according to the procedure and $\{\varepsilon_{j,t}^{true}\}$ is structural innovation j in the model economy. These statistics have

a clear interpretation. Indeed, a method that perfectly identifies the technology innovations would yield an $R^2 = 1$ with $\varepsilon_{z,t}^{true}$ as the dependent variable and an $R^2 = 0$ for the other structural innovations.¹¹ These R^2 are calculated for each of the Monte Carlo replications.

Table 4.1: **True vs. Identified Technology Shocks: RBC Model**

	$\varepsilon_{z,t}$	$\varepsilon_{a,t}$	$\varepsilon_{\phi,t}$	$\varepsilon_{g,t}$	$\varepsilon_{q,t}$
BHM	0.94	0.00	0.01	0.00	0.00
Gali	0.73	0.10	0.09	0.00	0.03
Solow	0.62	0.00	0.26	0.00	0.00

Notes: Each column contains R^2 from the regression of the structural innovation $\varepsilon_{j,t}$, $j \in \{z, a, \phi, g, q\}$ on technology shock $\varepsilon_{z,t}$, identified using the procedure in each row. Results are based on a Monte Carlo studies with 30 replications. BHM refers to the method proposed in this paper as specified in Section 4.3.2. Gali refers to the technology shock identified following the procedure in Gali (1999). The Solow residual is calculated applying Jorgenson correction for labor composition effects.

The results reported in Table 4.1 imply that the method proposed in this paper performs very well. The identified neutral technology shocks are closely related to the true neutral technology shocks used when simulating the model (median $R^2 = 0.94$), and are not systematically related to other structural shocks in the model. In contrast, the technology shocks identified using the other two methods are less closely related to the true neutral technology shocks and systematically pick up other structural disturbances. We now use this simple model to better understand the reasons for their shortcomings.

Using SVAR with long-run restrictions to identify technology shocks

The second row of Table 4.1 indicates that retrieving technology innovations using an SVAR with long run restrictions yields a median R^2 of only 0.73.¹² The reason

¹¹Note that these are theoretical benchmarks. Even if we observed the actual process for neutral technology, the R^2 of the $\varepsilon_{z,t}^{true}$ equation would be below 1 because of sampling errors in the estimation of (ρ_z, σ_z) , which are needed to measure the innovations $\varepsilon_{z,t}^{identified}$.

¹²Specifically, we follow Gali (1999) and estimate a VAR(4) on the growth rate of labor productivity and hours worked, and identify technology innovations as the unique shock having

is that an SVAR with long run restrictions on labor productivity interprets any low frequency variation in labor productivity as a neutral technology shock. Indeed, labor productivity can be decomposed as

$$\log\left(\frac{Y_t}{L_t}\right) = Z_t + \alpha \log\left(\frac{K_t}{e^{Z_t} L_{e,t}}\right) + \log\left(\frac{L_{e,t}}{L_t}\right).$$

The idea underlying the use of an SVAR with long-run restrictions to identify z is that the second term $\alpha \log\left(\frac{K_t}{e^{Z_t} L_{e,t}}\right)$ is stationary and thus is not affected by any shock in the long-run. If labor is homogeneous, the third term $\log\left(\frac{L_{e,t}}{L_t}\right)$ is zero so that only neutral technology affects output per hour in the long-run.

If labor inputs are heterogeneous, the third term is not zero and will be moved by shocks other than Z_t . There are two key sources inducing such movements in the simple model studied in this section. Preference shocks induce changes in the share of hours worked by skilled and unskilled workers due to different labor supply elasticities of the two groups. This moves labor productivity at low frequencies and leads the SVAR procedure to erroneously interpret the innovations in preference shocks as technology shocks. This explains the R^2 of 0.1 in the regression of the preference shock on the technology shock identified using this method. In the next Section we will study a richer model with more shocks and will observe that *any* shock that induces persistent changes in labor composition will be interpreted as a technology shock in this context.

Even for a counterfactually constant share of hours worked by skilled and unskilled individuals, persistent changes in the relative productivity of skilled workers induced by the skill-biased shock ϕ_t will also induce low-frequency movements in the effective labor input per hour worked. This explains the R^2 of 0.09 in the

a long run effect on labor productivity.

regression of the skill-bias shock on the technology shock identified using this method. Thus, any such non-neutral shocks will also be identified as technology shocks by this methodology.

Finally, as is well known from the work of [Fisher \(2006\)](#), without additional restrictions this method confounds neutral and non-neutral investment-specific shocks.

The Solow Residual

The Solow residual explains on average only 62% of the variation in actual neutral technology because it is a composite of neutral and non-neutral technological change. This is clear from Equation (4.2) in the Introduction, which specializes to

$$(1 - \alpha) \frac{\dot{Z}}{Z} + \dot{\phi}(1 - \alpha) \log \left(\frac{L_s}{L_e} \right) = \frac{\dot{Y}}{Y} - \alpha \frac{\dot{K}}{K} - \phi(1 - \alpha) \frac{\dot{L}_s}{L_s} - (1 - \phi)(1 - \alpha) \frac{\dot{L}_u}{L_u}$$

in this model. Thus, the Solow residual picks up the non-neutral skill premium shock ϕ and explains on average 26% of its variation. The difference between how much of the skill premium shock is picked up by the Solow residual and how much is picked up by the SVAR with long run restrictions depends on the persistence of the skill premium shock. As the persistence of this shock is estimated to be less than one in this model, its contribution to the Solow residual - which is independent of this persistence - is larger.

Note that in contrast to the SVAR procedure, the Solow residual we compute does not pick up the effects of labor composition induced by the preference shocks. The reason is that when calculating the Solow residual we applied Jorgenson's cor-

rection for labor composition effects pioneered by [Jorgenson and Griliches \(1967\)](#). The key idea underlying this correction is to disaggregate the labor force into categories based on education, age, gender, etc. Then, in computing total effective labor input, each hour is weighted by the observed average wage of the group it belongs to, assumed to coincide with the marginal product of that labor input. Then, adding an additional worker with, say, a college degree would account for more of an increase in output than would adding a worker with a high school diploma. While this procedure corrects for pure changes in composition, in [Appendix D.2](#) we show that it does not correct for the biased changes in technology affecting the relative productivity of labor inputs. Of course, it was never intended to do so as the growth accounting literature was not interested in measuring neutral technological innovations.¹³

4.4 Monte Carlo Analysis using a New Keynesian Model

In this section we assess the performance of our method in a Monte Carlo study using a calibrated benchmark New Keynesian business cycle model. We use a medium-scale model with price and wage rigidities, capital accumulation, investment adjustment costs, variable capital utilization, and habit formation. The model is based on [Christiano et al. \(2005b\)](#) and [Smets and Wouters \(2007b\)](#). We

¹³We computing the Solow residual we assumed that the parameter α is known and all inputs are observed. More realistically though, suppose that the aggregator L_t^e features richer heterogeneity, for example three groups l, m, h , $L_t^e = L_{h,t}^{\phi_{1,t}} L_{m,t}^{\phi_{2,t}} L_{l,t}^{1-\phi_{1,t}-\phi_{2,t}}$ but the researcher can distinguish only two groups. This misclassification worsens the ability of the Solow residual to identify technology shocks substantially whereas our methodology is immune to such misclassifications. The result in [Table 4.1](#) assume a correct classification and thus the Solow residual performs better than it will likely do in real data where misclassification is present.

enrich this setting with labor heterogeneity. As in the RBC model analyzed in the previous section, workers can be of two types: low and high skilled. They are distinguished by their marginal productivity, defined through the production function in equation (4.17), and by their Frisch elasticity of labor supply $\nu_l \neq \nu_h$. Appendix D.4 contains the full description of the model. In addition to the economic shocks that were present in the RBC model, this model incorporates monetary policy shocks, price markup shocks, wage markup shocks and shocks to the discount factor of households. There are nine economic disturbances in total.

After calibrating the model to match the behavior of post-1984 U.S. business cycles, we apply our procedure on simulated data and compare our estimates with the true neutral technology series. The economic significance of deviations between the actual and estimated technology series is assessed by comparing our estimated impulse response functions to their theoretical counterparts. We repeat this exercise for technology series estimated using the SVAR with long run restrictions and the Solow residual accounting procedure. Finally, we perform various robustness checks by varying the parameter estimates of our benchmark calibration.

4.4.1 Calibration

Most of the model's parameters associated to preferences and technology are fixed to conventional values used in the literature. In particular, we use the estimates (posterior mean) reported by [Schorfheide et al. \(2010\)](#), who consider a version of the model studied here without wage markup shocks and labor heterogeneity. The parameters associated to labor heterogeneity come from our analysis of the RBC model, while those governing the economy's structural shocks are calibrated through moment matching. In particular, denote the parameters governing the

structural shocks by θ , and let $\mathbf{m}_{\mathbf{T}}$ be a vector of sample moments for selected time series of length \mathbf{T} computed using US data. We denote by $\mathbf{m}_{\mathbf{T}}(\theta)$ their model counterpart when the vector of structural parameter is θ . θ is chosen to minimize a weighted distance between model and data moments:

$$\min_{\theta_2} [\mathbf{m}_{\mathbf{T}} - \hat{\mathbf{m}}(\theta)]' \mathbf{W}_{\mathbf{T}} [\mathbf{m}_{\mathbf{T}} - \hat{\mathbf{m}}(\theta)],$$

where $\mathbf{W}_{\mathbf{T}}$ is a diagonal matrix whose nonzero elements are the inverse of the variance of the corresponding moment. The empirical moments included in the vector $\mathbf{m}_{\mathbf{T}}$ are standard measures of cyclical variation and comovement for post 1984 quarterly US data. The time series used are the growth rate in GDP, private non-durable consumption, private nonresidential investment, total hours worked in the business sector, total hours of low and high skilled individuals in the business sector, nominal wages for these two demographic groups, labor productivity, and an inflation series constructed using the GDP deflator and the Federal Funds Rate. For each of these time series, we compute the sample standard deviation, the first order autocorrelation and the cross-correlation with GDP growth. We collect these sample moments in the vector $\mathbf{m}_{\mathbf{T}}$. The associated model's moments are calculated via a Monte Carlo procedure. In particular, for each θ , we solve for the policy functions using first order perturbation. We next simulate a realization of length \mathbf{T} for the model's counterparts of the above time series and calculate the vector $\hat{\mathbf{m}}_{\mathbf{T}}(\theta)$. We repeat this procedure $M = 300$ times, each time changing the seed used in the simulation. We then take the (component wise) median of $\hat{\mathbf{m}}(\theta)$ across the Monte Carlo replications.

Table [A-3](#) summarizes the procedure used for the calibration of our model and reports numerical values for the structural parameters. Table [A-4](#) reports the fit

of our model in terms of the calibration targets. We can verify that the calibrated model is consistent along many dimensions with the behavior of aggregate time series at business cycle’s frequencies, although certain features of the data are missed.

4.4.2 Identifying Technology Shocks in Model-Generated Data

Suppose that data on output, capital and hours worked etc. have been generated from the New Keynesian model described above and assume that a researcher identifies technology using the methodology proposed in this paper (i.e., estimates the state space model discussed in Section 4.3.2), as the Solow residual or using a SVAR with long run restrictions on labor productivity. Is the researcher correctly backing-out the actual realization of technology shocks in the economy? To answer this question, we perform a simple exercise. Given the parametrization of our model in Table A-3, we simulate $M = 300$ realizations of length $T = 250$ for the model’s variables, and calculate the series of “technology” innovations identified using the three methods.¹⁴

In order to assess the accuracy of each procedure, as in Section 4.3.3, we consider the R^2 of the following linear regressions:

$$\varepsilon_{j,t}^{true} = \alpha + \beta \varepsilon_{z,t}^{identified} + \eta_t \quad j \in \{z, a, \phi, g, q, \beta, r, p, w\},$$

where $\{\varepsilon_{z,t}^{identified}\}$ are the identified technology shocks and $\{\varepsilon_{j,t}^{true}\}$ is structural

¹⁴When calculating the Solow residual we use the true parameter α rather than estimating it. Moreover, we assume that the level of capital utilization is observed by the researcher. Therefore, the only source of discrepancy between neutral technology shocks and the solow residual is coming from the time variation in the non neutral technological parameter.

innovation j in the model economy. The results are presented in Table 4.2.

As in our analysis of the simple RBC model, we find that the shocks identified using the SVAR with long run restrictions or as the Solow residuals have little structural interpretation, whereas the proposed method is recovering neutral technology shocks very well. Indeed, the median R^2 of our method equals 0.93. For comparison, retrieving technology innovations using SVAR with long run restrictions yields a median R^2 of 0.52, while the Solow residual explains on average 23% of the variation in actual neutral technology. As discussed in Section 4.3.3, these two methods are not well suited to identify neutral technology shocks in models with heterogeneous inputs. Indeed, the SVAR with long run restrictions on labor productivity interprets any low frequency variation in labor productivity as a neutral technology shock, while the Solow residual is a composite of neutral and non-neutral skill-biased technical change. This generates biased estimates of the neutral technology shock, as is clear from Table 4.2. The SVAR procedure systematically picks up changes in the composition of the labor force induced by preference and other shocks, which drive labor productivity at low frequencies in the model. The Solow residual, instead, explains on average 42% of the variation in the skill premium shock. The procedure proposed in this paper is not subject to these problems and provides a correct identification of the neutral technology shock.

Notice also that the correlation between the Solow residual and the Long-Run shock to productivity is quite high (0.58) and of similar magnitude to the empirical one reported by Gali (2004). Table 4.3 suggests, therefore, that a high correlation between these two series is not necessarily a sign of the robustness for either one of the two procedures.

Table 4.2: **True vs. Identified Technology Shocks: NK Model**

Method	$\varepsilon_{z,t}$	$\varepsilon_{a,t}$	$\varepsilon_{\phi,t}$	$\varepsilon_{g,t}$	$\varepsilon_{q,t}$	$\varepsilon_{\beta,t}$	$\varepsilon_{r,t}$	$\varepsilon_{p,t}$	$\varepsilon_{w,t}$
BHM	0.93	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00
Gali	0.52	0.08	0.14	0.00	0.03	0.02	0.00	0.00	0.04
Solow	0.23	0.00	0.42	0.00	0.00	0.00	0.00	0.00	0.00

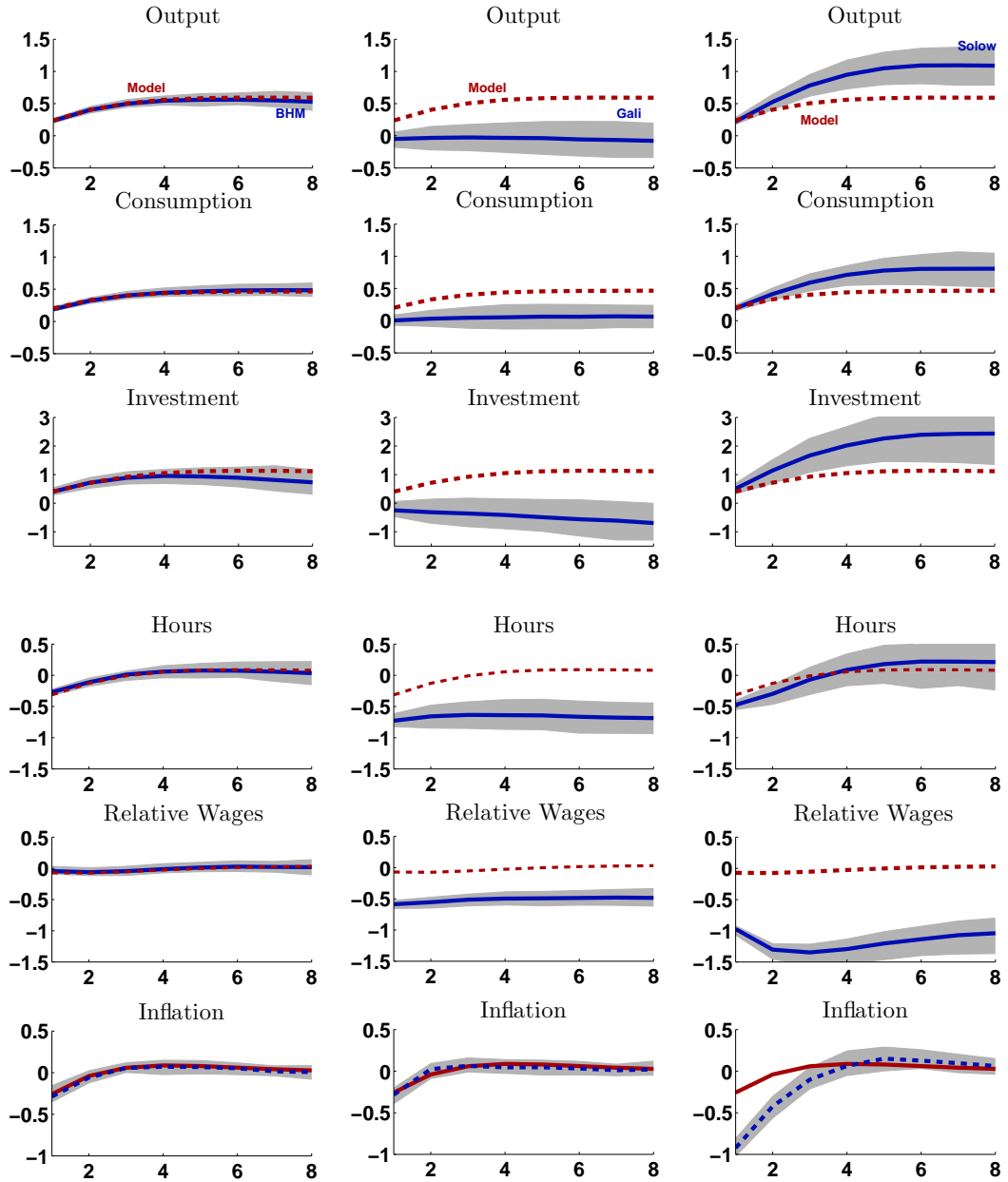
Notes: Each column contains R^2 from the regression of the structural innovation $\varepsilon_{j,t}$, $j \in \{z, a, \phi, g, q, \beta, r, p, w\}$ on technology shock $\varepsilon_{z,t}$, identified using the procedure in each row. Results are based on a Monte Carlo studies with 300 replications. BHM refers to the method proposed in this paper as specified in Section 4.3.2. Gali refers to the technology shock identified following the procedure in Gali (1999). The Solow residual is calculated applying Jorgenson correction for labor composition effects.

4.4.3 Impulse Response Functions

In the previous section we assessed the quality of our method as well as of the other two methods (Gali and Solow) by considering the correlation between the true technology series and the identified ones. While indicative of the various biases induced by the three methods, these correlations do not provide information on the economic importance of these biases. In this Section we complement this evidence by computing impulse responses to identified technology shocks. For all three identified technology series we compute the impulse response of key model variables - output, consumption, investment, hours, relative wages of skilled and unskilled and inflation - and compare it to the impulse response for the true technology series. Figure 4.1 shows the results. The response of each of these variables is reported in a separate row of the figure. The three columns report results for, respectively, our method, SVAR with long run restriction on labor productivity and the Solow residual. In each panel, the dashed line reports the true impulse response while the solid line the estimated one, with the shaded area marking the 90% confidence interval for the estimated impulse response.¹⁵

¹⁵The estimated impulse response and their confidence interval are constructed as via a Monte Carlo simulation. Specifically, for $n = 1 : N$, we i) apply the three procedures on time series simulated from the model; ii) collect the series of estimated technology innovations for the three procedures; iii) compute impulse response as described in Section 4.2.4. The figure reports the pointwise median and 90% confidence interval across these Monte Carlo simulations.

Figure 4.1: Impulse Responses to Identified Technology Shocks



As already suggested by the high correlation between our identified technology series and the true technology series, we find in the first column of the figure that the true and estimated impulse response are very similar as well. With the exception of investment, we can verify that our estimated impulse response functions track very closely their model counterpart. Moreover, the true impulse response always fall in the 90% confidence interval of our estimator. This is clearly not the case for the SVAR approach: the estimates of the the response of the model's variables to a neutral technology shock are, in fact, very imprecise. From our previous discussion we know that SVARs with long run restrictions on labor productivity misinterpret low frequency variation in labor supply with a neutral technology shock. Specifically, a decline in labor supply moves measured output per worker up, and it is interpreted by this procedure as a technology improvements. Not surprisingly, the response of hours to this innovation is biased downward relative to its response to the true technology shock. Because of that, the response of output, consumption and investment is also biased downward: in our numerical simulations, a researcher using SVARs with long run restrictions would conclude that neutral technology shocks are unimportant for business cycle fluctuations, as these variables hardly move conditional on an increase in the identified neutral shock. The response to the Solow residual for real variables are more in line with the true impulse response functions. This reflects the fact that, under our parametrization, skill premium shocks are fairly unimportant for business cycle dynamics. The pattern, though, is that the a positive skill bias shock raises the Solow residual. This shock, in the model, lowers worked hours, increase output and its components and lowers inflation as it decreases firms' marginal costs. These biases can be observed by comparing the true and estimated impulse response functions in the third column of the figure.

Beside the average behavior, the figure also documents that our method significantly improves in the precision of estimates for the impulse response. Confidence interval are, in fact, significantly tighter relative to the other two approaches. This is the result that the technology series we identify is, on average, less noisy with respect to the other methods.

4.4.4 Sensitivity

We now assess whether these results are sensitive to the particular parameterization we used. To do so we vary the value of each potentially relevant estimated parameter to the upper or lower boundary of its 95% confidence interval. For each resulting parameterization we report the R^2 of the following linear regressions:

$$\varepsilon_{z,t}^{true} = \alpha + \beta\varepsilon_{z,t}^{identified} + \eta_t,$$

where $\{\varepsilon_{z,t}^{identified}\}$ are the identified technology shocks and $\{\varepsilon_{z,t}^{true}\}$ is structural neutral technology innovation. The results are presented in Table 4.3.

Table 4.3 shows results for those parameters where we obtained a different R^2 from either increasing it to the lower or upper bound of its confidence interval. In addition we report results for “technology” parameters such as κ (capital adjustment), γ_u (capital utilization), the persistence of the price of new investment ρ_q and of the skill shock, ρ_ϕ that may be thought of easily confoundable with neutral technology. We find that this is not the case. This conclusion remains also if we for example increase the persistence of the price of investment q to $\rho_q = 0.99$. Similarly changing the parameters governing the stickiness of prices and wages, σ_w , σ_p , ρ_w and θ_w does not alter our conclusions. Although the R^2 slightly moves,

Table 4.3: **Sensitivity to Parameters in New Keynesian Model**

Parameter	BHM		Gali		Solow	
	down	up	down	up	down	up
$\theta_w \in \{0.15, 0.35\}$	0.95	0.87	0.46	0.54	0.23	0.23
$h \in \{0.58, 0.72\}$	0.92	0.92	0.53	0.50	0.23	0.23
$\gamma_u \in \{0.14, 0.41\}$	0.93	0.93	0.55	0.50	0.23	0.23
$\kappa \in \{0.84, 3.91\}$	0.93	0.93	0.52	0.50	0.23	0.23
$\rho_w \in \{0.02, 0.04\}$	0.93	0.90	0.52	0.46	0.23	0.23
$\rho_q \in \{0.31, 0.46\}$	0.93	0.93	0.52	0.52	0.23	0.23
$\rho_\phi \in \{0.04, 0.12\}$	0.93	0.93	0.52	0.46	0.23	0.24
$\sigma_w \in \{0.27, 0.62\}$	0.93	0.93	0.50	0.53	0.23	0.23
$100 \times \sigma_p \in \{0.09, 0.21\}$	0.93	0.92	0.51	0.51	0.23	0.23

Notes: Each column contains the R^2 from the regression of the structural neutral technology innovation $\varepsilon_{z,t}$ on the technology shock $\varepsilon_{z,t}$, identified using the procedure in the respective column. The column “down” refers to lowering the respective parameter to the lower bound of its confidence band and the column “up” refers to the increase to the upper bound. Results are based on a Monte Carlo study with 300 replications. BHM refers to the method proposed in this paper as specified in Section 4.3.2. Gali refers to the technology shock identified following the procedure in Gali (1999). The Solow residual is calculated applying Jorgenson correction for labor composition effects.

we checked that this change is inconsequential for the impulse responses to the neutral technology identified using our proposed method.

4.5 Conclusion

Standard methods for identifying technology shocks in the data do not identify neutral technology in models with heterogeneous inputs. In particular, the presence of worker heterogeneity invalidates the key identification assumption in Gali (1999) because not only technology, but virtually all persistent shocks have a long run effect on productivity in such models. The identification of neutral technology shocks using the Solow residual accounting procedure is also biased if the effects of factor heterogeneity and non-neutral technical changes are not explicitly accounted for.

Yet, most models have clear predictions for the dynamic responses of variables

to neutral technology shocks only. Thus, to evaluate such models it is desirable to be able to separate neutral technology shocks from the multitude of other shocks in the data and to compare the conditional response of variables to these neutral shocks in the data to the responses implied by the models. As existing measures of technology in the data confound neutral technology with non-neutral technology shocks or even with non-technology shocks, such a comparison would not be informative on the empirical performance of a model.

In this paper we therefore propose a method to identify neutral technology in the data. We use Uzawa's classic characterization on balanced growth, to show that imposing balanced growth properties on long-run impulse responses uniquely identifies neutral technology shocks. We implement this identification in the data using an identified state-space model and establish in Monte Carlo simulations that neutral technology is very well recovered in business cycle models including the New Keynesian one. In particular small samples do not lead our methodology to confound neutral technology neither with non-neutral technology shocks nor with non-technology shocks, such as wage markup shocks or preference shifts.

In future research we plan to apply our method to identify neutral technology shocks in U.S. data. In particular, we will describe and analyze the sequence of identified shocks and document its co-movement with other economic aggregates. Finally, we will hopefully be able to provide conclusive answers to some of the classic questions in macroeconomics.

Appendix A

Appendix to “The Pass-Through of Sovereign Risk”

A.1 Derivation of Results 1

Combine equation (1.3) and (1.4) to eliminate the demand for deposits from the decision problem of the banker. The decision problem is then

$$\begin{aligned} v_b(n; \mathbf{S}) &= \max_{a_B, a_K} \mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi)n' + \psi v_b(n'; \mathbf{S}')] \}, \\ n' &= \sum_{j=\{B, K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) a_j + R(\mathbf{S}) n, \\ \lambda \left[\sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j \right] &\leq v_b(n; \mathbf{S}), \\ \mathbf{S}' &= \Gamma(\mathbf{S}). \end{aligned}$$

Guess that the value function is $v(n, \mathbf{S}) = \alpha(\mathbf{S})n$. Necessary and sufficient

conditions for an optimum are

$$\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')] [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] \} = \lambda \mu(\mathbf{S}) \quad j = \{B, K\}, \quad (\text{A.1})$$

$$\mu(\mathbf{S}) \left(\alpha(\mathbf{S})n - \lambda \sum_{j=\{B, K\}} Q_j(\mathbf{S})a_j \right) = 0. \quad (\text{A.2})$$

Substituting the guess in the dynamic program, and using the law of motion for n' , we obtain

$$v_b(n, \mathbf{S}) = \max_{a_B, a_K} \left\{ \sum_{j=\{B, K\}} \mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')] [R_j(\mathbf{S}') - R(\mathbf{S})] \} Q_j(\mathbf{S})a_j \right\} + \mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')] \} R(\mathbf{S})n.$$

Note that the first term on the right hand side of the above equation equals $\mu(\mathbf{S})\alpha(\mathbf{S})n$. Indeed, when the leverage constraint does not bind ($\mu(\mathbf{S}) = 0$), expected discounted returns on assets held by bankers equal the discounted risk free rate by equation (A.1). This implies that the term equals 0. When the constraint binds ($\mu(\mathbf{S}) > 0$), instead, this term can be written as

$$\lambda \mu(\mathbf{S}) \sum_{j=\{B, K\}} Q_j(\mathbf{S})a_j.$$

Using the complementary slackness condition in (A.2), we can rewrite the expression $\lambda \mu(\mathbf{S}) \sum_{j=\{b, k\}} Q_j(\mathbf{S})a_j$ as $\mu(\mathbf{S})\alpha(\mathbf{S})n$. Thus, the value function under

the guess takes the following form:

$$\alpha(\mathbf{S})n = \mu(\mathbf{S})\alpha(\mathbf{S})n + \mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi\alpha(\mathbf{S}')] \} R(\mathbf{S})n.$$

Solving for $\alpha(\mathbf{S})$, we obtain

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi\alpha(\mathbf{S}')] \} R(\mathbf{S})}{1 - \mu(\mathbf{S})}.$$

The guess is verified if $\mu(\mathbf{S}) < 1$. From equation (A.2) we obtain:

$$\mu(\mathbf{S}) = \max \left\{ 1 - \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi\alpha(\mathbf{S}')] \} R(\mathbf{S})n}{\lambda \left(\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \right)}, 0 \right\} < 1.$$

Finally, notice that financial leverage equals across bankers whenever $\mu(\mathbf{S}) > 0$. This implies that $\frac{n}{\lambda \sum_{j=\{B,K\}} Q_j a_j}$ is equal to $\frac{N}{\lambda [\sum_{j=\{B,K\}} Q_j A_j]}$ when the constraint binds.

A.2 Numerical Solution

A.2.1 Equilibrium Conditions

The state variables of the model are $\mathbf{S} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s, d]$. The control variables $\{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ solve the residual equations

$$E_{\mathbf{S}} \left[\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} R(\mathbf{S}) \right] - 1 = 0, \quad (\text{A.3})$$

$$E_{\mathbf{S}} \left\{ \beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} [(1 - \psi) + \psi \alpha(\mathbf{S}')] \left[\frac{(1 - \delta) Q_K(\mathbf{S}') + \alpha \frac{\tilde{Y}(\mathbf{S}')}{\tilde{K}'(\mathbf{S}')} e^{\Delta z'}}{Q_K(\mathbf{S})} \right] \right\} - \lambda \mu(\mathbf{S}) = 0, \quad (\text{A.4})$$

$$E_{\mathbf{S}} \left\{ \beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} [(1 - \psi) + \psi \alpha(\mathbf{S}')] [1 - d'D] \left[\frac{\pi + (1 - \pi) [\iota + Q_b(\mathbf{S}')] }{Q_B(\mathbf{S})} \right] \right\} - \lambda \mu(\mathbf{S}) = 0, \quad (\text{A.5})$$

$$\alpha(\mathbf{S}) - \frac{(1 - \psi) + \psi R(\mathbf{S}) E_{\mathbf{S}} \left[\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} \alpha(\mathbf{S}') \right]}{1 - \mu(\mathbf{S})} = 0, \quad (\text{A.6})$$

where $Q_K(\mathbf{S})$ is the market value of the capital stock and the multiplier $\mu(\mathbf{S})$ is given by

$$\mu(\mathbf{S}) = \max \left\{ 1 - \left[\frac{E_{\mathbf{S}} \left\{ \beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} [(1 - \psi) + \psi \alpha(\mathbf{S}')] R(\mathbf{S}) \right\} \tilde{N}(\mathbf{S})}{\lambda [Q_K(\mathbf{S}) \tilde{K}'(\mathbf{S}) + Q_B(\mathbf{S}) \tilde{B}(\mathbf{S})]} \right], 0 \right\}. \quad (\text{A.7})$$

The endogenous state variables $[\tilde{K}, \tilde{B}, \tilde{P}]$ evolve as follows

$$\tilde{K}'(\mathbf{S}) = \left\{ (1 - \delta) \tilde{K} + \Phi \left[e^{\Delta z} \left(\frac{\tilde{Y}(\mathbf{S})(1 - e^g) - \tilde{C}(\mathbf{S})}{\tilde{K}} \right) \right] \tilde{K} \right\} e^{-\Delta z}, \quad (\text{A.8})$$

$$\tilde{B}'(\mathbf{S}) = \frac{[1 - dD]\{\pi + (1 - \pi)[l + Q_B(\mathbf{S})]\}\tilde{B}e^{-\Delta z} + \tilde{Y}(\mathbf{S}) \left[g - \left(t^* + \gamma_\tau \frac{B}{\tilde{Y}(\mathbf{S})} \right) \right]}{Q_B(\mathbf{S})}, \quad (\text{A.9})$$

$$\tilde{P}'(\mathbf{S}) = R(\mathbf{S})[Q_k(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_b(\mathbf{S})\tilde{B}'(\mathbf{S}) - \tilde{N}(\mathbf{S})]. \quad (\text{A.10})$$

The state variable \tilde{P} measures the detrended *cum* interest deposits that bankers pay to households at the beginning of the period and it is sufficient to keep track of the evolution of aggregate bankers' net worth. Indeed, the aggregate net worth of banks can be expressed as

$$\begin{aligned} \tilde{N}(\mathbf{S}) = \psi \left\{ \left[Q_K(\mathbf{S}) + \alpha \frac{\tilde{Y}(\mathbf{S})}{\tilde{K}} e^{\Delta z} \right] \tilde{K} + [1 - dD][\pi + (1 - \pi)[l + Q_B(\mathbf{S})]] \tilde{B} - \tilde{P} \right\} \\ + \omega [Q_K(\mathbf{S})\tilde{K} + Q_B(\mathbf{S})\tilde{B}]. \end{aligned} \quad (\text{A.11})$$

Using the intratemporal Euler equation of the household, we can express detrended output as

$$\tilde{Y}(\mathbf{S}) = \left[\chi^{-1} \frac{(\tilde{K}e^{-\Delta z})^\alpha}{\tilde{C}(\mathbf{S})} \right]^{\frac{1-\alpha}{\alpha+\nu-1}} (Ke^{-\Delta z})^\alpha. \quad (\text{A.12})$$

The exogenous state variables $[\Delta z, \log(g), s]$ evolve as follows

$$\Delta z' = (1 - \rho_z)\gamma + \rho_z \Delta z + \sigma_z \varepsilon_z, \quad (\text{A.13})$$

$$g' = (1 - \rho_g)g^* + \rho_g g + \sigma_g \varepsilon_g, \quad (\text{A.14})$$

$$s' = (1 - \rho_s)s^* + \rho_s s + \sigma_s \varepsilon_s, \quad (\text{A.15})$$

while d follows

$$d' = \begin{cases} 1 & \text{with probability } \frac{e^s}{1+e^s} \\ 0 & \text{with probability } 1 - \frac{e^s}{1+e^s}. \end{cases} \quad (\text{A.16})$$

It will be convenient to express detrended state and control variables as log-deviations from their deterministic steady state. I denote this transformation for variable x as \hat{x} .

A.2.2 Algorithm for Numerical Solution

I approximate the control variables of the model using piece-wise smooth functions, parametrized by $\gamma = \{\gamma_{d=0}^x, \gamma_{d=1}^x\}_{x=\{\hat{C}, \hat{\alpha}, \hat{Q}_b, \hat{R}\}}$. The law of motion for a control variable x is described by

$$x(d, \hat{\mathbf{S}}) = (1 - d)\gamma_{d=0}^x \mathbf{T}(\hat{\mathbf{S}}) + d\gamma_{d=1}^x \mathbf{T}(\hat{\mathbf{S}}), \quad (\text{A.17})$$

where $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}, \Delta\hat{z}, \hat{g}, \hat{s}]$ and $\mathbf{T}(\cdot)$ is a vector collecting Chebyshev's polynomials.¹ Define $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\})$ to be a 4×1 vector collecting the left hand side of the residual equations (A.3)-(A.6) for the candidate solution γ^c evaluated at $\{d, \hat{\mathbf{S}}\}$. The numerical solution of the model consists in choosing γ^c so that $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\}) = 0$ for a set of collocation points $\{d, \hat{\mathbf{S}}^i\} \in \{0, 1\} \times \mathcal{S}$.

The choice of collocation points and of the associated Chebyshev's polynomials follows Krueger et al. (2010). The rule for computing conditional expectations when evaluating $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\})$ follows Judd et al. (2011). To give an example of this latter, suppose we wish to compute $\mathbb{E}_{d, \mathbf{S}^i}[y(d', \hat{\mathbf{S}}')]$, where y is an integrand of interest.² Given a candidate solution γ^c , we can compute y at every collocation

¹The shocks are expressed as deviation from their mean.

²For example, y could be $\frac{e^{-\Delta z'}}{C(\mathbf{S}^i)}$ in equation (A.3).

point using the model's equilibrium conditions. Next, we can construct an implied policy function for y , $\{\gamma_{d=0}^y, \gamma_{d=1}^y\}$, via a Chebyshev's regression. Using the law of total probability, the conditional expectation of interest can be expressed as

$$\begin{aligned} \mathbb{E}_{d, \mathbf{S}^i}[y(d', \hat{\mathbf{S}}')] &= (1 - \text{Prob}\{d' = 1 | \hat{\mathbf{S}}^i\}) \mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma_{d=0}^y \mathbf{T}(\hat{\mathbf{S}}')] + \\ &+ \text{Prob}\{d' = 1 | \hat{\mathbf{S}}^i\} \mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma_{d=1}^y \mathbf{T}(\hat{\mathbf{S}}')], \end{aligned} \tag{A.18}$$

where $\text{Prob}\{d' = 1 | \hat{\mathbf{S}}^i\} = \frac{e^{s^i}}{1+e^{s^i}}$. Judd et al. (2011) propose a simple procedure to evaluate integrals of the form $\mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma_{d=1}^y \mathbf{T}(\hat{\mathbf{S}}')]$. In proposition 1 of their paper, they show that, under weak conditions, the expectation of a polynomial can be calculated via a linear transformation \mathcal{I} of the coefficient vector γ_d^y , where \mathcal{I} depends exclusively on the deep parameters of the model. The authors provide general formulas for the transformation \mathcal{I} .

The algorithm for the numerical solution of the model goes as follows

Step 0.A: Defining the grid and the polynomials. Set upper and lower bounds on the state variables $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}, \Delta z, g, s]$. Given these bounds, construct a μ -level Smolyak grid and the associated Chebyshev's polynomials $\mathbf{T}(\cdot)$ following Krueger et al. (2010).

Step 0.B: Precomputing integrals. Compute \mathcal{I} using Judd et al. (2011) formulas.

Step 1: Equilibrium conditions at the candidate solution. Start with a guess for the model's policy functions γ^c . For each $(d, \hat{\mathbf{S}}^i)$, use γ^c and equation (A.17) to compute $\{\hat{C}(d, \hat{\mathbf{S}}^i), \hat{a}(d, \hat{\mathbf{S}}^i), \hat{Q}_b(d, \hat{\mathbf{S}}^i), \hat{R}(d, \hat{\mathbf{S}}^i)\}$. Given the control variables, solve for the endogenous state variables next period

using the model's equilibrium conditions. Given the value of control and state variables, compute the value of every integrand in equations (A.3)-(A.6) at $(d, \hat{\mathbf{S}}^i)$. Collect these integrands in the matrix \mathbf{y} .

Step 2: Evaluate conditional expectations. For each $d = \{0, 1\}$, run a Chebyshev regression for the integrand in \mathbf{y} , and denote by γ_d^y the implied policy function for an element $y \in \mathbf{y}$. Conditional expectations are calculated using equation (A.19) and the matrix \mathcal{I} .

Step 3: Evaluate residual equations. Given conditional expectations, compute the multiplier using equation (A.7). Evaluate the residual equations $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}^i\})$ at every collocation point $(d, \hat{\mathbf{S}}^i)$. The dimension of the vector of residuals equals 4 times the cardinality of the state space. Denote by r the Euclidean norm for this vector.

Step 4: Iteration. If $r \leq 10^{-20}$, stop the algorithm. Else, update the guess and repeat Step 1-4. \square

The specific for the algorithm are as follows

Choice of bounds. The bounds on $[\Delta\hat{z}, \hat{g}]$ are +/- 3 standard deviations from their mean. The bounds on \hat{s} are larger and set to $[-5, +5]$. The bounds on the endogenous state variables $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}]$ are set to +/- 4.5 their standard deviations in the model without sovereign risk. The standard deviation is calculated by simulating a third order perturbation of the model without sovereign risk. A desirable extension of the algorithm is to select different bounds depending on whether the economy is in a default state or not.

Smolyak Grid. For tractability, I choose $\mu = 3$ for the Smolyak grid. This

implies that I have 389 distinct points in \mathcal{S} .

Precomputation of Integrals. I use Gaussian numerical quadrature for computing the matrix \mathcal{I} .

Iterative Algorithm. I find the zeros of the residual equation using a variant of the Newton algorithm. To speed up computations, numerical derivatives are computed in parallel.

I denote the model solution by γ^* .

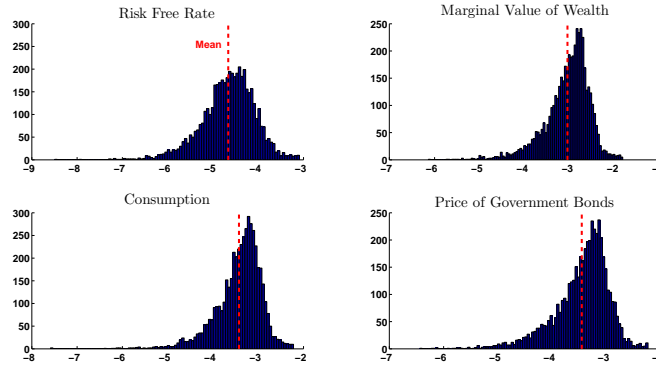
A.2.3 Accuracy of Numerical Solution

I check the accuracy of the numerical solution by computing the errors of the residual equations (Judd, 1992). More specifically, I proceed as follows. First, I simulate the model forward for 5000 periods. This gives a simulation for the state variable of the model $\{d_t, \hat{\mathbf{S}}_t\}_{t=1}^{5000}$. Second, for each pair $(d_t, \hat{\mathbf{S}}_t)$, I calculate the errors of the residual equations $\mathcal{R}(\gamma^*, \{d_t, \hat{\mathbf{S}}_t\})$. As an example, let's consider equation (A.4). Then, the residual error at $(d_t, \hat{\mathbf{S}}_t)$ for this equation is defined as

$$1 - \frac{\mathbb{E}_{d_t, \hat{\mathbf{S}}_t} \left\{ \beta \frac{\tilde{C}(d_t, \hat{\mathbf{S}}_t)}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} [(1 - \psi) + \psi \alpha(\mathbf{S}')] \left[\frac{(1 - \delta) Q_K(\mathbf{S}') + \alpha \frac{\tilde{Y}(\mathbf{S}')}{K'(d_t, \hat{\mathbf{S}}_t)} e^{\Delta z'}}{Q_K(d_t, \hat{\mathbf{S}}_t)} \right] \right\}}{\lambda \mu(d_t, \hat{\mathbf{S}}_t)},$$

where the model's policy functions are used to generate the value for endogenous variables at $d_t, \hat{\mathbf{S}}_t$. Note that, by construction, the residual errors are zero at the collocation points. This residual equation errors provide a measure of how large are the discrepancy between the decision rule derived from the numerical algorithm and those implied by the model's equilibrium condition in other points of the state space. Following standard practice, I report the decimal log of the absolute value

Figure A.1: Residual Equations Errors

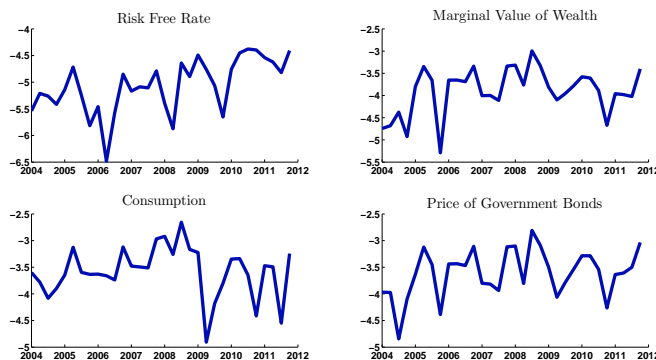


Notes: The histograms reports the residual equations errors in decimal log basis. The dotted line marks the mean residual equation error.

of these residual errors. Figure A.1 below reports the density (histogram) of those errors.

On average, residual equation errors are in the order of -4.75 for the risk free rate, -3.5 for consumption and the price of government securities and -3 for the marginal value of wealth. These numbers are comparable to values reported in the literature for models of similar complexity, and they are still very reasonable. Figure A.2 reports residual equation errors for a sequence of states $\{\mathbf{S}_t\}_{t=2004:Q1}^{2011:Q4}$ extracted using the particle filter (See Section 1.4). The figure shows that residual equation errors are reasonable in empirically relevant region of the state space.

Figure A.2: Residual Equations Errors: Empirically Relevant Region



A.3 Data Source

A.3.1 Credit Default Swap (CDS) Spread

Daily CDS spreads on 5 years Italian government securities (RED code: 4AB951). The restructuring clause of the contract is CR (complete restructuring). The spread is denominated in basis points and paid quarterly. The source is Markit, accessed from the Wharton Research Data Services.

A.3.2 Banks' exposure to the Italian government

The European Banking Authority (EBA) published information on holdings of government debt by European banks participating to the 2011 stress test. Five Italian banks were in this pool: Unicredit, Intesa-San Paolo, Monte dei Paschi di Siena (MPS), Banco Popolare (BPI) and Unione di Banche Italiane (UBI). Results of the stress test for each of these five banks are available at <http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2011/results>. I measure exposure of each bank to Italian central and local government as gross direct long exposure (accounting value gross of specific provisions). This information is available by maturity of financial instrument, and it reflects positions as of 31st of December 2010. I match these data on exposure with end of 2010 total financial assets for each of the five institutions. This latter information is obtained using consolidated banking data from Bankscope, accessed from the Wharton Research Data Service. Table [A.1](#) reports these information.

Table A.1: **Exposure to Domestic Sovereign by Major Italian Banks: End of 2010**

	3Mo	1Yr	2Yr	3Yr	5Yr	10Yr	15Yr	Tot.	Tot. Assets
Intesa	17.71	9.86	2.82	5.16	8.16	6.64	9.77	60.15	658.76
Unicredit	17.97	10.14	3.04	6.21	4.47	6.39	0.87	49.07	929.49
MPS	5.67	4.99	4.00	3.58	1.45	3.75	9.02	32.47	240.70
BPI	3.89	1.65	1.14	3.64	0.78	0.40	0.25	11.77	134.17
UBI	1.33	3.56	0.30	0.31	0.72	2.53	1.76	10.54	129.80
Total	46.57	30.02	11.30	18.9	13.76	19.71	21.67	164.00	2092.99

Notes: Data is reported in billions of euros.

A.3.3 Construction of the Multiplier

Result 2. *In equilibrium, the multiplier on the incentive constraint of bankers is a function of financial leverage and of the spread between a risk free security traded by bankers and the risk free rate*

$$\mu_t = \frac{\left[\frac{R_t^f - R_t}{R_t} \right] lev_t}{1 + \left[\frac{R_t^f - R_t}{R_t} \right] lev_t}. \quad (\text{A.19})$$

Proof. Since the asset is risk free, one has that $cov_t(\hat{\Lambda}_{t+1}, R_{t+1}^f) = 0$. Therefore, using equation (1.25) in the main text, one has:

$$\left[\frac{R_t^f - R_t}{R_t} \right] = \frac{\lambda}{\alpha_t} \frac{\mu_t}{1 - \mu_t}.$$

Equation (A.19) follows from the fact that $\frac{\alpha_t}{\lambda}$ equals financial leverage when

$\mu_t > 0$.

□

Essentially, Result 2 tells us that the agency friction can be interpreted as a markup on financial intermediation. To measure this markup, one needs to focus on returns on assets, traded only by bankers, that have the same risk properties of households' deposits. I use the prime interbank rate to measure R_t^f since the model of Section 3.3 can be interpreted as having a frictionless interbank market as in Gertler and Kiyotaki (2010). The time series used in the construction of the multiplier are the following

Financial Leverage: The definition of financial leverage in the model is banks' equity divided by the market value of total assets. I use quarterly data from the Italian flow of funds (*Conti Finanziari*) to construct these two time series. First, I match banks in the model with Monetary and Financial Institutions (MFIs). This category includes commercial banks, money market funds and the domestic central bank. I use balance sheet information for the Bank of Italy to exclude the latter from this pool. Second, I construct a time series for bank equity as the difference between "total assets" and total debt liabilities. This latter is defined as "total liabilities" minus "shares and other equities" (liabilities) and "mutual fund shares" (liabilities). Financial leverage is the ratio between equity and total assets. Data can be downloaded at <http://bip.bancaditalia.it/4972unix/>. See Bartiloro et al. (2003) for a description of the Italian flow of funds.

TED Spread: The prime interbank rate (EURIBOR 1yrs.) is obtained from the ECB Statistical Data Warehouse, under *Market Indexes* in the section *Monetary and Financial Statistics*. The frequency of observation is monthly. Data can be downloaded at <http://sdw.ecb.europa.eu/>. I match the model's risk free rate with the yields on German bonds with a 1 year maturity. A monthly time series

for this latter is obtained from the time series database of the Deutsche Bundesbank. Data can be downloaded at http://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/time_series_databases.html. I construct a quarterly measure of the TED spreads by averaging the computed series over three months.

A.3.4 Other Time Series

GDP growth: Real GDP growth is the growth rate relative to previous quarter of real gross domestic product (*B1_EG*). Data are quarterly, 1980:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Consumption growth: Consumption growth is the growth rate relative to previous quarter of real private final consumption expenditure (*P31S14_S15*). Data are quarterly, 1991:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Spending-Output Ratio: Spending-output ratio is general government final consumption expenditure divided by gross domestic product. Both series are seasonally adjusted and in volume estimates. Data are quarterly, 1991:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Investment-Output Ratio: Investment-output ratio is gross capital formation divided by gross domestic product. Both series are seasonally adjusted and in volume estimates. Data are quarterly, 1991:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Labor income share: I obtain annual data (1970-2007) for and labor compensation (*LAB*) and value added (*VA*) from EU KLEMS database. Labor share is

defined as $\frac{LAB}{VA}$. Data can be downloaded at <http://www.euklems.net/>.

Worked hours: Average numbers of hours worked per year by person engaged. I scale the series by $(24 - 8) \times 7 \times 52$. Data are annual (1970-2007), and obtained from EU KLEMS. Data can be downloaded at <http://www.euklems.net/>.

A.4 Estimating the Model without Sovereign Risk

The model without sovereign risk has five state variables $\mathbf{S}_t = [\hat{K}_t, \hat{P}_t, \hat{B}_t, \Delta z_t, g_t]$. Let \mathbf{Y}_t be a 2×1 vector of observables collecting output growth and the time series for the multiplier on the leverage constraint. The state-space representation is

$$\mathbf{Y}_t = f_{\tilde{\theta}}(\mathbf{S}_t) + \eta_t \quad \eta_t \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (\text{A.20})$$

$$\mathbf{S}_t = g_{\tilde{\theta}}(\mathbf{S}_{t-1}, \varepsilon_t) \quad \varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}). \quad (\text{A.21})$$

The first equation is the measurement equation, with η_t being a vector of Gaussian measurement errors. The second equation is the transition equation, which represents the law of motion for the model's state variables. The vector ε_t represents the innovation to the structural shocks Δz_t and g_t . The function $f_{\tilde{\theta}}(\cdot)$ and $g_{\tilde{\theta}}(\cdot)$ are generated using the numerical procedure described in Appendix A.2 applied to the model without sovereign risk. I characterize the posterior distribution of $\tilde{\theta}$ using full information Bayesian methods. I denote by $p(\tilde{\theta})$ the prior on $\tilde{\theta}$. In what follows, I provide details on the likelihood evaluation and on the posterior sampler adopted.

A.4.1 Likelihood Evaluation

Let $\mathbf{Y}^t = [\mathbf{Y}_1, \dots, \mathbf{Y}_t]$, and denote by $p(\mathbf{S}_t | \mathbf{Y}^{t-1}; \theta)$ the conditional distribution of the state vector given observations up to period $t - 1$. The likelihood function for

the state-space model of interest can be expressed as

$$\mathcal{L}(\mathbf{Y}^T|\theta) = \prod_{t=1}^T p(\mathbf{Y}_t|\mathbf{Y}^{t-1}; \theta) = \prod_{t=1}^T \left[\int p(\mathbf{Y}_t|\mathbf{S}_t; \theta) p(\mathbf{S}_t|\mathbf{Y}^{t-1}; \theta) d\mathbf{S}_t \right]. \quad (\text{A.22})$$

While the conditional density of \mathbf{Y}_t given \mathbf{S}_t is known and Gaussian, there is no analytical expression for the density $p(\mathbf{S}_t|\mathbf{Y}^{t-1}, \theta)$. I use the auxiliary particle filter of [Pitt and Shephard \(1999\)](#) to approximate this density via a set of pairs $\{\mathbf{S}_t^i, \tilde{\pi}_t^i\}_{i=1}^N$. This approximation is then used to estimate the likelihood function.

Step 0: Initialization. Set $t = 1$. Initialize $\{\mathbf{S}_0^i, \tilde{\pi}_0^i\}_{i=1}^N$ from the model's ergodic distribution and set $\tilde{\pi}_0^i = \frac{1}{N} \forall i$.

Step 1: Prediction. For each $i = 1, \dots, N$, draw $\mathbf{S}_{t|t-1}^i$ values from the proposal density $g(\mathbf{S}_t|\mathbf{Y}^t, \mathbf{S}_{t-1}^i)$.

Step 2: Filtering. Assign to each $\mathbf{S}_{t|t-1}^i$ the particle weight

$$\pi_t^i = \frac{p(\mathbf{Y}_t|\mathbf{S}_{t|t-1}^i; \theta) p(\mathbf{S}_t|\mathbf{S}_{t-1}^i; \theta)}{g(\mathbf{S}_t|\mathbf{Y}^t, \mathbf{S}_{t-1}^i)}.$$

Step 3: Sampling. Rescale the particles $\{\pi_t^i\}$ so that they add up to unity, and denote these rescaled values by $\{\tilde{\pi}_t^i\}$. Sample N values for the state vector with replacement from $\{\mathbf{S}_{t|t-1}^i, \tilde{\pi}_t^i\}_{i=1}^N$. Call each draw $\{\mathbf{S}_t^i\}$. If $t < T$, set $t = t + 1$ and go to Step 1. Else, stop. \square

The likelihood function of the model is then approximated as

$$\mathcal{L}(\mathbf{Y}^T|\theta) \approx \frac{1}{N} \left(\prod_{t=1}^T \left[\frac{1}{N} \sum_{i=1}^N p(\mathbf{Y}_t|\mathbf{S}_{t|t-1}^i; \theta) \right] \right).$$

Regarding the tuning of the filter, I set $N = 20000$. The matrix Σ is diagonal,

and the diagonal elements equal 25% of the variance of the observable variables. The choice for the proposal density $g(\mathbf{S}_t|\mathbf{Y}^t, \mathbf{S}_{t-1}^i)$ is more involved. I sample the structural innovations ε_t from $\mathcal{N}(m_t, \mathbf{I})$. Then, I use the model's transition equation (A.21) to obtain $\mathbf{S}_{t|t-1}^i$. The center for the proposal distribution for ε_t is generated as follows:

- Let $\bar{\mathbf{S}}_{t-1}$ be the mean for $\{\mathbf{S}_{t-1}^i\}$ over i .
- Set m_t to the solution of this optimization program

$$\operatorname{argmin}_{\varepsilon} \{ [\mathbf{Y}_t - f_{\bar{\theta}}(g_{\bar{\theta}}(\bar{\mathbf{S}}_{t-1}, \varepsilon))]' [\mathbf{Y}_t - f_{\bar{\theta}}(g_{\bar{\theta}}(\bar{\mathbf{S}}_{t-1}, \varepsilon))] + \varepsilon' \boldsymbol{\Sigma}^{-1} \varepsilon \}.$$

The first part of the objective function pushes ε toward values such that the state vector can rationalize the observation \mathbf{Y}_t . The second part of the objective function imposes a penalty for ε that are far away from their high density regions. I verify that this proposal density results in substantial efficiency gains relative to the canonical particle filter, especially when the model tries to fit extreme observations for \mathbf{Y}_t .

A.4.2 Posterior Sampler

I characterize the posterior density of $\tilde{\theta}$ using a Random Walk Metropolis Hastings with proposal density given by

$$q(\tilde{\theta}^p | \tilde{\theta}^{m-1}) \sim \mathcal{N}(\tilde{\theta}^{m-1}, c\mathbf{H}).$$

The sequence of draws $\{\tilde{\theta}^m\}$ is generated as follows

- i) Initialize the chain at $\tilde{\theta}^1$.

ii) For $m = 2, \dots, M$, draw $\tilde{\theta}^p$ from $q(\tilde{\theta}^p | \tilde{\theta}^{m-1})$. The jump from $\tilde{\theta}^{m-1}$ to $\tilde{\theta}^p$ is accepted ($\tilde{\theta}^m = \tilde{\theta}^p$) with probability $\min\{1, r(\tilde{\theta}^{m-1}, \tilde{\theta}^p | \mathbf{Y}^T)\}$, and rejected otherwise ($\tilde{\theta}^m = \tilde{\theta}^{m-1}$). The probability of accepting the draw is

$$r(\tilde{\theta}^{m-1}, \tilde{\theta}^p | \mathbf{Y}^T) = \frac{\mathcal{L}(\mathbf{Y}^T | \tilde{\theta}^p) p(\tilde{\theta}^p)}{\mathcal{L}(\mathbf{Y}^T | \tilde{\theta}^{m-1}) p(\tilde{\theta}^{m-1})}.$$

First, I run the chain for $M = 10000$ with \mathbf{H} being the identity matrix and $c = 0.001$. The chain is initialized from an estimation of the model using the Method of Simulated Moments.³ I drop the first 5000 draws, and I use the remaining draws to initialize a second chain and to construct a new candidate density. This second chain is initialized at the mean of the 5000 draws. Moreover, the variance-covariance matrix \mathbf{H} is set to the empirical variance-covariance matrix of these 5000 draws. The parameter c is fine tuned to obtain an acceptance rate of roughly 60%. I run the second chain for $M = 20000$. Posterior statistics are based on the latter 10000 draws.

³The moments used in this step are: i) mean, standard deviation and autocorrelation for GDP growth and the multiplier; ii) correlation between GDP growth and the multiplier.

A.5 Policy Experiments

A.5.1 Refinancing Operations

It is instructive to first consider the stationary problem. The government allows bankers to borrow up to \bar{m} at the fixed interest rate R_m , and this intervention is financed through lump-sum taxation. Moreover, these loans are not subject to limited enforcement problems. The decision problem of the banker becomes

$$\begin{aligned}
 v_b(n; \mathbf{S}) &= \max_{a_B, a_K, b} \mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi)n' + \psi v_b(n'; \mathbf{S}')] \}, \\
 n' &= \sum_{j=\{B, K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) a_j + \\
 &\quad + [R_m - R(\mathbf{S})] m - R(\mathbf{S}) b, \\
 \sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j &= n + b, \\
 \lambda \left[\sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j - m \right] &\leq v_b(n; \mathbf{S}), \\
 m &\in [0, \bar{m}], \\
 \mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}).
 \end{aligned}$$

Assuming that $m \geq 0$ does not bind,⁴ the first order condition with respect to m is

$$\mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi) + \psi \frac{\partial v_b(n'; \mathbf{S}')}{\partial n'} \right] \right\} [R(\mathbf{S}) - R_m] + \lambda \mu(\mathbf{S}) = \chi(\mathbf{S})$$

It can be showed, following a similar logic of Result 1, that $v_b(n; \mathbf{S}) = \alpha(\mathbf{S})n + x(\mathbf{S})$, with $x(\mathbf{S}) \geq 0$. The leverage constraint becomes

⁴This is not a restriction, as the policy considered involves an R_m substantially below R , meaning that bankers are willing to accept the loan.

$$\frac{\sum_j Q_j(\mathbf{S})a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda} + \frac{x(\mathbf{S})}{\lambda} + \bar{m}$$

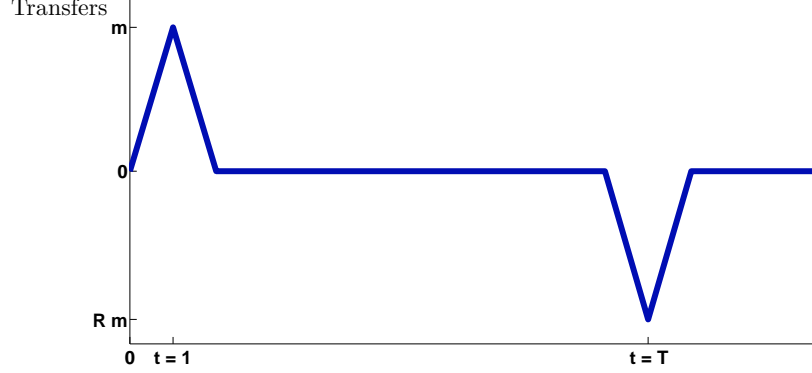
Notice that refinancing operations have two main direct effects on banks. First, they represent an implicit transfer to banks. Indeed, to the extent that ($R_m < R(\mathbf{S})$), banks benefit from the policy as their debt is subsidized. This has a positive effect on the net worth of banks relative to what would happen in the no-policy case. Second, the policy relaxes the leverage constraint of banks. This happens because of two distinct reasons: i) the loan from the government does not enter in the computation of the constrained level of leverage (the \bar{m} component); and ii) the value function of bankers increase as a result of the subsidized loan, this lowering the incentives of the banker to walk away.

A.5.2 Longer Term Refinancing Operations (LTROs)

The LTROs are a non-stationary version of the refinancing operations discussed above. The government allows banker to borrow up to \bar{m} in period $t = 1$, and they receive the principal and the interest in a later period T . Figure A.3 describes the timing of transfers between government and banks under LTROs.

I assume that the policy was unexpected by agents. At time $t = 1$, agents are perfectly informed about the time path of the loans and they believe that the policy will not be implemented in the future. Note that the decision rules under LTROs are time dependent: the dynamics at $t = 1$ will be different from those at $t = T - 1$ as in the latter case we are getting closer to the repayment stage and banks will have a different behavior. In order to solve for the path of model's decision rules, I follow a backward induction procedure. From period $t = T + 1$

Figure A.3: **Timing of LTROs**



onward, the decision rules are those in absence of policy. Thus, at $t = T$, agents use those decision rules to form expectations. By solving the equilibrium conditions under this assumption and the repayment of the loan, we can obtain decision rules for $c_T(\mathbf{S})$, $R_T(\mathbf{S})$, $\alpha_T(\mathbf{S})$, $Q_{B,T}(\mathbf{S})$. At $t = T - 1$ we proceed in the same way, this time using $c_T(\mathbf{S})$, $R_T(\mathbf{S})$, $\alpha_T(\mathbf{S})$, $Q_{B,T}(\mathbf{S})$ to form expectations. More specifically, the policy functions in the transition $\{c_t(\mathbf{S}), R_t(\mathbf{S}), \alpha_t(\mathbf{S}), Q_{b,t}(\mathbf{S})\}_{t=1}^T$, are derived as follows:

- i) **Period T :** Solve the model using $\{c(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), q(\mathbf{S})\}$ to form expectations. The multiplier is modified as follows

$$\mu_T(\mathbf{S}) = \max \left\{ 1 - \frac{\mathbb{E}_{\mathbf{S}'} \{ \Lambda_{T+1}(\mathbf{S}') [(1 - \psi) + \psi \alpha_{T+1}(\mathbf{S}')] \} R_T(\mathbf{S}) (N' - m)}{\lambda (Q_{b,T}(\mathbf{S}) B'_T + Q_{k,T}(\mathbf{S}) K'_T)}, 0 \right\}$$

Denote the solution by $\{c_T(\mathbf{S}), R_T(\mathbf{S}), \alpha_T(\mathbf{S}), q_T(\mathbf{S})\}$.

- ii) **Period $t = T - 1, \dots, 1$:** Solve the model using the previous iteration policy functions to form expectations. \square

Appendix B

Appendix to “Assessing DSGE Model Nonlinearities”

B.1 QAR(1,1) Model

This section shows how to derive important moments for the QAR(1,1) model given by

$$y_t = \phi_1 y_{t-1} + \phi_2 s_{t-1}^2 + (1 + \gamma s_{t-1}) \sigma u_t, \quad u_t \sim iidN(0, 1) \quad (\text{B.1})$$

$$s_t = \phi_1 s_{t-1} + \sigma u_t, \quad |\phi_1| < 1 \quad (\text{B.2})$$

by exploiting the recursively linear structure of the model. The model corresponds to (2.9) in the main text. To simplify the presentation, we dropped the tildes for ϕ_2 , γ , and s .

B.1.1 Moments

We now derive the time-invariant mean and autocovariances for y_t , assuming the process is stationary and was initialized in the infinite past. Due to the recursively linear structure of the model we begin with the derivation of the moments of s_t .

Moments of s_t . The process s_t in (B.2) is linear and has a moving average representation of the form

$$s_t = \sigma \sum_{j=0}^{\infty} \phi_1^j u_{t-j}.$$

The mean and the autocovariances of s_t are given by

$$\mathbb{E}[s_t] = 0, \quad \mu_{s^2} = \mathbb{E}[s_t^2] = \frac{\sigma^2}{1 - \phi_1^2}, \quad \mathbb{E}[s_t s_{t-h}] = \phi_1^h \mu_{s^2}.$$

Since the innovations u_t are *iid* standard normal variates, we obtain the following third and fourth moments:

$$\mathbb{E}[s_t^3] = \sum_{j=0}^{\infty} \phi_1^{3j} \mathbb{E}[u_{t-j}^3] = 0, \quad \mathbb{E}[s_t^4] = \sum_{j=0}^{\infty} \phi_1^{4j} \mathbb{E}[u_{t-j}^4] = \frac{3\sigma^4}{1 - \phi_1^4}.$$

Mean of y_t . Taking expectations on both sides of (B.1) we obtain

$$\mathbb{E}[y_t] = \phi_1 \mathbb{E}[y_{t-1}] + \phi_2 \mu_{s^2} + (1 + \gamma \mathbb{E}[s_{t-1}]) \sigma \mathbb{E}[u_t] = \phi_1 \mathbb{E}[y_t] + \frac{\phi_2 \sigma^2}{1 - \phi_1^2}.$$

Here we used the expression for μ_{s^2} obtained previously as well as the fact that u_t and s_{t-1} are independent. In turn,

$$\mu_y = \mathbb{E}[y_t] = \frac{\phi_2 \sigma^2}{(1 - \phi_1)(1 - \phi_1^2)}. \quad (\text{B.3})$$

Variance of y_t . Consider the centered second moment of y_t :

$$\begin{aligned}
\mathbb{V}[y_t] &= \mathbb{E} [(\phi_1(y_{t-1} - \mu_y) + \phi_2(s_{t-1}^2 - \mu_{s^2}) + \sigma(1 + \gamma s_{t-1})u_t)^2] \\
&= \mathbb{E} [\phi_1^2(y_{t-1} - \mu_y)^2 + \phi_2^2(s_{t-1}^2 - \mu_{s^2})^2 + \sigma^2(1 + \gamma s_{t-1})^2 u_t^2 \\
&\quad + 2\phi_1\phi_2(y_{t-1} - \mu_y)(s_{t-1}^2 - \mu_{s^2}) + 2\phi_2\sigma(s_{t-1}^2 - \mu_{s^2})(1 + \gamma s_{t-1})u_t \\
&\quad + 2\phi_1\sigma(1 + \gamma s_{t-1})(y_{t-1} - \mu_y)u_t] \\
&= \phi_1^2\mathbb{E}[(y_{t-1} - \mu_y)^2] + \phi_2^2\mathbb{E}[(s_{t-1}^2 - \mu_{s^2})^2] + \sigma^2(1 + \gamma^2\mu_{s^2}) \\
&\quad + 2\phi_1\phi_2\mathbb{E}[(y_{t-1} - \mu_y)(s_{t-1}^2 - \mu_{s^2})].
\end{aligned}$$

The time-invariant solution is

$$\mathbb{V}[y_t] = \frac{1}{1 - \phi_1^2} \left[\phi_2^2 \mathbb{V}[s_t^2] + \sigma^2(1 + \gamma^2\mathbb{E}[s_t^2]) + 2\phi_1\phi_2 \text{Cov}[y_t, s_t^2] \right],$$

where

$$\begin{aligned}
\text{Cov}[y_t, s_t^2] &= \mathbb{E} [(\phi_1(y_{t-1} - \mu_y) + \phi_2(s_{t-1}^2 - \mu_{s^2}) + (1 + \gamma s_{t-1})\sigma u_t) \\
&\quad \times (\phi_1^2(s_{t-1}^2 - \mu_{s^2}) + 2\phi_1\sigma s_{t-1}u_t + \sigma^2(u_t^2 - 1))] \\
&= \phi_1^3\mathbb{E}[(y_{t-1} - \mu_y)(s_{t-1}^2 - \mu_{s^2})] + \phi_1^2\phi_2\mathbb{E}[(s_{t-1}^2 - \mu_{s^2})^2] \\
&\quad + 2\phi_1\gamma\sigma^2\mu_{s^2},
\end{aligned}$$

which implies

$$\text{Cov}[y_t, s_t^2] = \frac{1}{1 - \phi_1^3} \left[\phi_1^2\phi_2\mathbb{V}[s_t^2] + 2\phi_1\gamma\sigma^2\mathbb{E}[s_t^2] \right].$$

Interestingly,

$$\begin{aligned} Cov[y_t, s_t] &= \mathbb{E}[(\phi_1(y_{t-1} - \mu_y) + \phi_2(s_{t-1}^2 - \mu_{s^2}) + (1 + \gamma s_{t-1})\sigma u_t)(\phi_1 s_{t-1} + \sigma u_t)] \\ &= \phi_1^2 Cov[y_{t-1}, s_{t-1}] + \sigma^2 \end{aligned}$$

All other terms drop out because $\mathbb{E}[u_t] = \mathbb{E}[s_t] = \mathbb{E}[s_t^3] = 0$. Thus, solving for the time-invariant solution leads to the “first-order” variance expression

$$Cov[y_t, s_t] = \mathbb{E}[s_t^2] = \frac{\sigma^2}{1 - \phi_1^2}.$$

Autocovariances of y_t . Consider $\mathbb{E}[(y_t - \mu_y)(y_{t-1} - \mu_y)]$:

$$\begin{aligned} Cov[y_t, y_{t-1}] &= \mathbb{E}[(\phi_1(y_{t-1} - \mu_y) + \phi_2(s_{t-1}^2 - \mu_{s^2}) + (1 + \gamma s_{t-1})\sigma u_t)(y_{t-1} - \mu_y)] \\ &= \phi_1 \mathbb{V}[y_{t-1}] + \phi_2 Cov[y_{t-1}, s_{t-1}^2]. \end{aligned}$$

In general, higher-order autocovariances can be computed recursively:

$$\begin{aligned} Cov[y_t, y_{t-h}] &= \mathbb{E}[(\phi_1(y_{t-1} - \mu_y) + \phi_2(s_{t-1}^2 - \mu_{s^2}) + (1 + \gamma s_{t-1})\sigma u_t)(y_{t-h} - \mu_y)] \\ &= \phi_1 Cov[y_{t-1}, y_{t-h}] + \phi_2 Cov[y_{t-1}, s_{t-1}^2]. \end{aligned}$$

The term $Cov[y_{t-h}, s_{t-1}^2]$ can also be calculated recursively:

$$\begin{aligned} Cov[y_{t-h}, s_{t-1}^2] &= \mathbb{E}[(y_{t-h} - \mu_y)(\phi_1^2(s_{t-2} - \mathbb{E}[s_{t-2}^2]) + 2\phi_1 s_{t-2}\sigma u_{t-1} + \sigma(u_{t-1})^2 - 1)] \\ &= \phi_1^2 Cov[y_{t-h}, s_{t-2}^2]. \end{aligned}$$

B.1.2 Initialization and Identification

In order to compute the likelihood function recursively, it is necessary to initialize s_0 . We write the joint distribution of observables, initial state, and parameters as:

$$p(Y_{0:T}, \theta, s_0) = p(Y_{1:T}|y_0, s_0, \theta)p(y_0, s_0|\theta)p(\theta)$$

and use MCMC methods to generate draws from the posterior

$$p(\theta, s_0|Y_{0:T}) \propto p(Y_{1:T}|y_0, s_0, \theta)p(y_0, s_0|\theta)p(\theta).$$

We will approximate the distribution of (y_0, s_0) using a normal distribution

$$\begin{bmatrix} y_0 \\ s_0 \end{bmatrix} \Big| \theta \sim N \left(\begin{bmatrix} \mu_y \\ \mu_s \end{bmatrix}, \begin{bmatrix} \Sigma_{yy} & \Sigma_{ys} \\ \Sigma_{sy} & \Sigma_{ss} \end{bmatrix} \right). \quad (\text{B.4})$$

The moments of this normal distribution are calculated as follows. We will assume that the system was in its steady state in period $t = -T_*$, i.e. $s_{-T_*} = 0$ and $y_{-T_*} = \phi_0$. In principle, T_* could be infinite, but this will create some problems if $\phi_1 = 1$. In order to simplify the time subscripts a bit, we shift the time index by T_* periods. Starting from $s_0 = 0$ and $y_0 = \phi_0$ we will calculate the first and second moments of y_t , s_t , and s_t^2 recursively, starting with

$$\begin{aligned} \mathbb{E}[s_0] &= 0, \quad \mathbb{E}[y_0] = \phi_0, \quad \mathbb{V}[s_0] = 0, \quad \mathbb{V}[y_0] = 0, \\ \text{Cov}[y_0, s_0] &= 0, \quad \text{Cov}[y_0, s_0^2], \quad \mathbb{V}[s_0^2] = 0. \end{aligned} \quad (\text{B.5})$$

The process for s_t is linear autoregressive of order one and we obtain

$$\mathbb{E}[s_t] = \phi_1 \mathbb{E}[s_{t-1}], \quad \mathbb{V}[s_t] = \phi_1^2 \mathbb{V}[s_{t-1}] + \sigma^2. \quad (\text{B.6})$$

Since the innovations ϵ_t are *iid* standard normal variates, we obtain that the third moment is zero:

$$\mathbb{E}[s_t^3] = \sum_{j=0}^{t-1} \phi_1^{3j} \mathbb{E}[\epsilon_{t-j}^3] = 0.$$

Now consider

$$\begin{aligned} \mathbb{V}[s_t^2] &= \mathbb{E}[(s_t^2 - \mathbb{V}[s_t])^2] & (B.7) \\ &= \mathbb{E}[(\phi_1^2(s_{t-1}^2 - \mathbb{V}[s_{t-1}]) + 2\phi_1 s_{t-1} \sigma \epsilon_t + \sigma^2(\epsilon_t^2 - 1))^2] \\ &= \phi_1^4 \mathbb{V}[s_{t-1}^2] + 4\phi_1^2 \sigma^2 \mathbb{V}[s_{t-1}] + 2\sigma^4. \end{aligned}$$

A formula for the mean of y_t is obtained by taking expectations of the observation equation:

$$\mathbb{E}[y_t] = \phi_0(1 - \phi_1) + \phi_1 \mathbb{E}[y_{t-1}] + \phi_2 \mathbb{V}[s_{t-1}]. \quad (B.8)$$

The covariance between y_t and s_t is given by

$$\begin{aligned} \text{Cov}[y_t, s_t] &= \mathbb{E}[(y_t - \mathbb{E}[y_t])s_t] & (B.9) \\ &= \mathbb{E}[(\phi_1(y_{t-1} - \mathbb{E}[y_{t-1}]) + \phi_2(s_{t-1}^2 - \mathbb{E}[s_{t-1}^2]) + (1 + \gamma s_{t-1})\sigma \epsilon_t)(\phi_1 s_{t-1} + \sigma \epsilon_t)] \\ &= \phi_1^2 \text{Cov}[y_{t-1}, s_{t-1}] + \sigma^2. \end{aligned}$$

All other terms drop out because the first and third moments of s_{t-1} and ϵ_t are equal to zero. The covariance between y_t and s_t^2 is given by

$$\begin{aligned} \text{Cov}[y_t, s_t^2] &= \mathbb{E}[(y_t - \mathbb{E}[y_t])(s_t^2 - \mathbb{V}[s_t])] & (B.10) \\ &= \mathbb{E}[(\phi_1(y_{t-1} - \mathbb{E}[y_{t-1}]) + \phi_2(s_{t-1}^2 - \mathbb{V}[s_{t-1}]) + (1 + \gamma s_{t-1})\sigma \epsilon_t) \\ &\quad \times (\phi_1^2(s_{t-1}^2 - \mathbb{V}[s_{t-1}]) + 2\phi_1 \sigma s_{t-1} \epsilon_t + \sigma^2(\epsilon_t^2 - 1))] \\ &= \phi_1^3 \text{Cov}[y_{t-1}, s_{t-1}^2] + \phi_1^2 \phi_2 \mathbb{V}[s_{t-1}^2] + 2\phi_1 \gamma \sigma^2 \mathbb{E}[s_{t-1}^2]. \end{aligned}$$

The variance of y_t can be computed as follows:

$$\begin{aligned}
\mathbb{V}[y_t] &= \mathbb{E}[(\phi_1(y_{t-1} - \mathbb{E}[y_{t-1}]) + \phi_2(s_{t-1}^2 - \mathbb{V}[s_{t-1}]) + \sigma(1 + \gamma s_{t-1})\epsilon_t)^2] \quad (\text{B.11}) \\
&= \phi_1^2 \mathbb{V}[y_{t-1}] + \phi_2^2 \mathbb{V}[s_{t-1}^2] + \sigma^2(1 + \gamma^2 \mathbb{V}[s_{t-1}]) \\
&\quad + 2\phi_1\phi_2 \text{Cov}[y_{t-1}, s_{t-1}^2].
\end{aligned}$$

We can iterate Equations (B.6) to (B.11) forward for T_* periods to obtain the moments for the initial distribution of (y_0, s_0) in (B.4).

Note that for $\gamma = \phi_2 = 0$ s_0 and y_0 become perfectly correlated conditional on θ since for a linear model $y_0 = s_0 + \phi_0$. This may affect our posterior sampler when we include s_0 into the parameter vector. To avoid the singularity we add a small constant to the covariance matrix of (y_0, s_0) .

B.1.3 MCMC Implementation

The RWM algorithm mentioned in Section 2.3.3 is used to implement the posterior inference. Using a preliminary covariance for the proposal distribution in the RWM algorithm that is constructed from the prior variance of the QAR parameters we generate an initial 100,000 draws from the posterior. Based on the last 50,000 draws we compute a covariance matrix that replaces the preliminary covariance matrix of the proposal distribution. We then continue the chain, generating an additional 60,000 draws, retaining the last 50,000 to construct summary statistics for the posterior.

B.1.4 Detailed Estimation Results

Table A-1: **Prior Distribution for QAR(1,1) Model, Samples Starting in 1960**

	GDP Growth	Wage Growth	Inflation	Fed Funds Rate
ϕ_0	$N(0.48, 2)$	$N(1.18, 2)$	$N(2.38, 2)$	$N(2.50, 2)$
ϕ_1	$N^\dagger(0.36, 0.5)$	$N^\dagger(-0.02, 0.5)$	$N^\dagger(0.00, 0.5)$	$N^\dagger(0.66, 0.5)$
σ	$IG(1.42, 4)$	$IG(0.82, 4)$	$IG(1.87, 4)$	$IG(0.58, 4)$
ϕ_2	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$
γ	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$

Notes: (\dagger) The prior for ϕ_1 is truncated to ensure stationarity. The IG distribution is parameterized such that $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$.

Table A-2: **Prior Distribution for QAR(1,1) Model, Samples Starting in 1984**

	GDP Growth	Wage Growth	Inflation	Fed Funds Rate
ϕ_0	$N(0.43, 2)$	$N(1.58, 2)$	$N(4.38, 2)$	$N(6.08, 2)$
ϕ_1	$N^\dagger(0.28, 0.5)$	$N^\dagger(0.34, 0.5)$	$N^\dagger(0.85, 0.5)$	$N^\dagger(0.94, 0.5)$
σ	$IG(1.33, 4)$	$IG(0.88, 4)$	$IG(1.83, 4)$	$IG(1.45, 4)$
ϕ_2	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$
γ	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$	$N(0, 0.1)$

Notes: (\dagger) The prior for ϕ_1 is truncated to ensure stationarity. The IG distribution is parameterized such that $p_{IG}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$.

Table A-3: **Posterior Estimates for QAR(1,1) Model, 1960:Q1-1983:Q4**

Data	ϕ_0	ϕ_1	ϕ_2	γ	σ	s_0
GDP	0.42 [0.11 , 0.69]	0.28 [0.11 , 0.46]	-0.02 [-0.14 , 0.09]	-0.05 [-0.17 , 0.06]	1.16 [0.91 , 1.53]	1.42 [1.02 , 1.85]
WAGE	1.75 [1.49 , 1.98]	0.41 [0.23 , 0.58]	-0.05 [-0.13 , 0.04]	0.04 [-0.05 , 0.15]	0.52 [0.40 , 0.68]	0.89 [0.63 , 1.15]
INFL	4.24 [2.28 , 5.84]	0.87 [0.80 , 0.95]	-0.01 [-0.08 , 0.07]	0.16 [0.04 , 0.27]	1.52 [1.08 , 2.12]	-1.97 [-4.68 , 0.79]
FFR	4.84 [0.86 , 6.75]	0.92 [0.88 , 0.96]	0.02 [-0.05 , 0.05]	0.38 [0.30 , 0.47]	0.62 [0.41 , 1.00]	-1.56 [-4.21 , 0.14]

Notes: We report posterior means and 90% equal-tail-probability credible sets in brackets.

Table A-4: **Posterior Estimates for QAR(1,1) Model, 1960:Q1-2007:Q4**

Data	ϕ_0	ϕ_1	ϕ_2	γ	σ	s_0
GDP	0.48 [0.33 , 0.63]	0.29 [0.16 , 0.41]	-0.02 [-0.07 , 0.04]	-0.06 [-0.13 , 0.01]	0.69 [0.58 , 0.82]	1.37 [1.19 , 1.56]
WAGE	1.41 [1.25 , 1.59]	0.44 [0.33 , 0.55]	-0.03 [-0.09 , 0.02]	0.12 [0.05 , 0.20]	0.48 [0.40 , 0.57]	1.22 [1.00 , 1.42]
INFL	3.51 [2.74 , 4.47]	0.85 [0.79 , 0.91]	-0.01 [-0.06 , 0.05]	0.23 [0.16 , 0.31]	1.06 [0.81 , 1.38]	-1.31 [-2.90 , 0.31]
FFR	2.96 [2.16 , 4.16]	0.96 [0.95 , 0.97]	0.04 [0.02 , 0.06]	0.44 [0.37 , 0.52]	0.28 [0.22 , 0.42]	-0.74 [-1.27 , 0.45]

Notes: We report posterior means and 90% equal-tail-probability credible sets in brackets.

Table A-5: **Posterior Estimates for QAR(1,1) Model, 1960:Q1-2012:Q4**

Data	ϕ_0	ϕ_1	ϕ_2	γ	σ	s_0
GDP	0.45 [0.28 , 0.60]	0.33 [0.22 , 0.44]	-0.03 [-0.08 , 0.03]	-0.07 [-0.14 , 0.00]	0.68 [0.58 , 0.81]	1.41 [1.19 , 1.61]
WAGE	1.29 [1.12 , 1.46]	0.43 [0.32 , 0.53]	-0.01 [-0.06 , 0.04]	0.08 [0.01 , 0.15]	0.54 [0.46 , 0.63]	1.31 [1.11 , 1.50]
INFL	3.23 [2.55 , 4.16]	0.84 [0.78 , 0.90]	0.02 [-0.04 , 0.09]	0.22 [0.15 , 0.30]	1.09 [0.87 , 1.36]	-1.26 [-2.82 , 0.22]
FFR	3.54 [2.29 , 5.06]	0.96 [0.94 , 0.97]	-0.01 [-0.02 , 0.00]	0.41 [0.33 , 0.50]	0.22 [0.13 , 0.37]	0.43 [-0.94 , 1.47]

Notes: We report posterior means and 90% equal-tail-probability credible sets in brackets.

Table A-6: **Posterior Estimates for QAR(1,1) Model, 1984:Q1-2007:Q4**

Data	ϕ_0	ϕ_1	ϕ_2	γ	σ	s_0
GDP	0.57 [0.44, 0.70]	0.26 [0.10 , 0.44]	-0.07 [-0.13 , -0.02]	0.01 [-0.10 , 0.11]	0.25 [0.20 , 0.32]	1.06 [0.91,1.21]
WAGE	1.09 [0.93,1.21]	0.24 [0.06,0.42]	-0.06 [-0.12,0.02]	0.07 [-0.03,0.17]	0.41 [0.32,0.53]	0.10 [-0.09,0.29]
INFL	2.72 [2.30,3.13]	0.63 [0.48,0.78]	-0.06 [-0.14,0.04]	0.07 [-0.06,0.19]	0.68 [0.52,0.89]	2.42 [1.76,2.93]
FFR	9.80 [8.68,11.56]	0.91 [0.87,0.93]	-0.16 [-.23,-.10]	0.08 [-0.03,0.17]	0.22 [0.15,0.32]	0.79 [-0.26,1.64]

Notes: We report posterior means and 90% equal-tail-probability credible sets in brackets.

Table A-7: **Posterior Estimates for QAR(1,1) Model, 1984:Q1-2012:Q4**

Data	ϕ_0	ϕ_1	ϕ_2	γ	σ	s_0
GDP	0.53 [0.38 , 0.66]	0.36 [0.22 , 0.52]	-0.09 [-0.15 , -0.03]	-0.07 [-0.17 , -0.00]	0.28 [0.23 , 0.35]	1.09 [0.87 , 1.28]
WAGE	0.98 [0.83 , 1.14]	0.18 [0.02 , 0.36]	-0.04 [-0.10 , 0.04]	0.03 [-0.06 , 0.12]	0.48 [0.38 , 0.60]	0.20 [0.03 , 0.37]
INFL	2.51 [2.12 , 2.93]	0.63 [0.48 , 0.77]	-0.02 [-0.10 , 0.06]	0.07 [-0.03 , 0.19]	0.76 [0.61 , 0.97]	2.54 [1.80 , 3.00]
FFR	10.00 [8.72 , 11.43]	0.92 [0.90 , 0.94]	-0.17 [-0.25 , -0.12]	0.01 [-0.05 , 0.11]	0.19 [0.15 , 0.29]	1.00 [0.05 , 1.40]

Notes: We report posterior means and 90% equal-tail-probability credible sets in brackets.

B.2 The DSGE Model

B.2.1 First-Order Conditions

Intermediate Goods Producers. Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its labor inputs $H_t(j)$ and the price $P_t(j)$ to maximize the present value of future profits. After using the production function to substitute our $Y_t(j)$ from the present value of future profits in (2.24) (see main text) we can write the objective function of the firm as

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s Q_{t+s|t} \left(\frac{P_{t+s}(j)}{P_{t+s}} \left(1 - \Phi_p \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} \right) \right) A_{t+s} H_{t+s}(j) - \frac{1}{P_{t+s}} W_{t+s} H_{t+s}(j) \right) \right]. \quad (\text{B.12})$$

This objective function is maximized with respect to $H_t(j)$ and $P_t(j)$ subject to

$$A_{t+s} H_{t+s}(j) = \left(\frac{P_t(j)}{P_t} \right)^{-1/\lambda_{p,t}} Y_{t+s}.$$

We use $\mu_{t+s} \beta^s Q_{t+s|t}$ to denote the Lagrange multiplier associated with this constraint. Setting $Q_{t|t} = 1$, the first-order condition with respect to $P_t(j)$ is given by

$$\begin{aligned} 0 = & \frac{1}{P_t} \left(1 - \Phi_p \left(\frac{P_t(j)}{P_{t-1}(j)} \right) \right) A_t H_t(j) - \frac{P_t(j)}{P_t P_{t-1}(j)} \Phi'_p \left(\frac{P_t(j)}{P_{t-1}(j)} \right) A_t H_t(j) \\ & - \frac{\mu_t}{\lambda_{p,t} P_t} \left(\frac{P_t(j)}{P_t} \right)^{-1/\lambda_{p,t}-1} Y_t + \beta \mathbb{E}_t \left[Q_{t+1|t} \frac{P_{t+1}^2(j)}{P_{t+1} P_t^2(j)} \Phi'_p \left(\frac{P_{t+1}(j)}{P_t(j)} \right) A_{t+1} H_{t+1}(j) \right]. \end{aligned} \quad (\text{B.13})$$

Taking first-order conditions with respect to $H_t(j)$ yields

$$\frac{W_t}{P_t} = \frac{P_t(j)}{P_t} \left(1 - \Phi_p \left(\frac{P_t(j)}{P_{t-1}(j)} \right) \right) A_t - \mu_t A_t. \quad (\text{B.14})$$

Households. The first-order condition with respect to consumption is given by

$$P_t \lambda_t = \left(\frac{C_t(k)}{A_t} \right)^{-\tau} \frac{1}{A_t}. \quad (\text{B.15})$$

We define

$$Q_{t+1|t} = \frac{\lambda_{t+1}P_{t+1}}{\lambda_t P_t}. \quad (\text{B.16})$$

Using this definition, the first-order condition for bond holdings becomes

$$1 = \beta \mathbb{E}_t \left[Q_{t+1|t} \frac{R_t}{\pi_{t+1}} \right]. \quad (\text{B.17})$$

Member k is a monopolistic competitor with respect to his wage choice. Taking into account the demand for labor of type k the relevant portion of the utility function for the wage decision is

$${}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\cdots - \chi_H \frac{1}{1+1/\nu} \left(\frac{W_{t+s}(k)}{W_{t+s}} \right)^{-(1+1/\nu)/\lambda_w} H_t^{1+1/\nu} \right) \right],$$

The relevant portion of the budget constraint after substituting $H_{t+s}(k)$ by the labor demand schedule is

$$\cdots = W_{t+s}(k) \left(\frac{W_{t+s}(k)}{W_{t+s}} \right)^{-1/\lambda_w} H_{t+s} \left(1 - \Phi_w \left(\frac{W_{t+s}(k)}{W_{t+s-1}(k)} \right) \right) + \cdots,$$

where the demand for aggregated labor services H_{t+s} is taken as given. Taking first-order conditions with respect to $W_t(k)$ yields

$$\begin{aligned} 0 &= \frac{\chi_H}{\lambda_w W_t} \left(\frac{W_t(k)}{W_t} \right)^{-\frac{1+1/\nu}{\lambda_w} - 1} H_t^{1+1/\nu} + \lambda_t \left(\frac{W_t(k)}{W_t} \right)^{-1/\lambda_w} H_t \left(1 - \Phi_w \left(\frac{W_t(k)}{W_{t-1}(k)} \right) \right) \\ &\quad - \frac{\lambda_t}{\lambda_w} \frac{W_t(k)}{W_t} \left(\frac{W_t(k)}{W_t} \right)^{-1/\lambda_w - 1} H_t \left(1 - \Phi_w \left(\frac{W_t(k)}{W_{t-1}(k)} \right) \right) \\ &\quad - \lambda_t \frac{W_t(k)}{W_{t-1}(k)} \left(\frac{W_t(k)}{W_t} \right)^{-1/\lambda_w} H_t \Phi'_w \left(\frac{W_t(k)}{W_{t-1}(k)} \right) \\ &\quad + \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{W_{t+1}^2(k)}{W_t^2(k)} \left(\frac{W_{t+1}(k)}{W_{t+1}} \right)^{-1/\lambda_w} H_{t+1} \Phi'_w \left(\frac{W_{t+1}(k)}{W_t(k)} \right) \right]. \end{aligned} \quad (\text{B.18})$$

B.2.2 Equilibrium Relationships

We consider the symmetric equilibrium in which all intermediate goods producing firms, as well as households, make identical choices when solving their optimization problem. Therefore, we can drop the index k and j . In slight abuse of notation let $\Delta X_t = X_t/X_{t-1}$ and $\pi_t = \Delta P_t$. We use $w_t = W_t/P_t$ to denote the real wage. Since the non-stationary technology process A_t induces a stochastic trend in output, consumption and real wages, it is convenient to express the model in terms of detrended variables $y_t = Y_t/A_t$, $c_t = C_t/A_t$ and $\tilde{w}_t = w_t/A_t$.

Intermediate Goods Producers. Using the above notation, multiplying (B.13) by P_t , and replacing Y_t by $A_t y_t$ we can simplify the first-order condition for $P_t(j)$ as follows

$$0 = (1 - \Phi_p(\pi_t))A_t y_t - \pi_t \Phi'_p(\pi_t)A_t y_t - \frac{\mu_t}{\lambda_{p,t}}A_t y_t + \beta \mathbb{E}_t [Q_{t+1|t} \pi_{t+1} \Phi'_p(\pi_{t+1})A_{t+1} y_{t+1}].$$

Dividing by $A_t y_t$ and replacing A_{t+1}/A_t by $\gamma \exp(z_{t+1})$ we obtain

$$0 = (1 - \Phi_p(\pi_t)) - \pi_t \Phi'_p(\pi_t) - \frac{\mu_t}{\lambda_{p,t}} + \beta \mathbb{E}_t [Q_{t+1|t} \pi_{t+1} \Phi'_p(\pi_{t+1}) \Delta y_{t+1} \gamma \exp(z_{t+1})].$$

We proceed by rewriting (B.14) as

$$\tilde{w}_t = (1 - \Phi_p(\pi_t)) - \mu_t. \tag{B.19}$$

Households. In terms of detrended consumption we can express $Q_{t+1|t}$ as

$$Q_{t+1|t} = \left(\frac{c_{t+1}}{c_t} \right)^{-\tau} \frac{1}{\gamma} \exp(-z_{t+1}). \tag{B.20}$$

The consumption Euler equation remains unchanged:

$$1 = \beta \mathbb{E}_t \left[Q_{t+1|t} \frac{R_t}{\pi_{t+1}} \right]. \quad (\text{B.21})$$

We now divide (B.18) by λ_t and replace λ_t by $c_t^{-\tau}/(A_t P_t)$:

$$\begin{aligned} 0 = & \frac{\chi_H}{\lambda_w} \frac{1}{\tilde{w}_t} c_t^\tau H_t^{1+1/\nu} + H_t (1 - \Phi_w(\pi_t \Delta w_t)) - \frac{1}{\lambda_w} H_t (1 - \Phi_w(\pi_t \Delta w_t)) \\ & - \pi_t \Delta w_t H_t \Phi'_w(\pi_t \Delta w_t) + \beta \mathbb{E}_t \left[Q_{t+1|t} \pi_{t+1} \Delta w_{t+1}^2 H_{t+1} \Phi'_w(\pi_{t+1} \Delta w_{t+1}) \right]. \end{aligned}$$

Aggregate Resource Constraint. The aggregate production function (in terms of detrended output) is

$$y_t = H_t. \quad (\text{B.22})$$

The intermediate goods producers' dividend payments to the households are given by

$$D_t = (1 - \Phi_p(\pi_t)) Y_t - w_t H_t. \quad (\text{B.23})$$

Combining the household budget constraint and the government budget constraint and detrending all variables leads to aggregate resource constraint

$$c_t + \zeta y_t = (1 - \Phi_p(\pi_t)) y_t - \tilde{w}_t y_t \Phi_w(\pi_t \Delta w_t),$$

where $\Delta w_t = \Delta \tilde{w}_t \gamma \exp(z_t)$.

The model economy has a unique steady state in terms of the detrended variables that is attained if the innovations $\epsilon_{R,t}$, $\epsilon_{g,t}$, and $\epsilon_{z,t}$ are zero at all times. The steady state inflation π equals the target rate π^* and

$$R = \frac{\gamma}{\beta} \pi^*, \quad \mu = \lambda_p, \quad c = \left(\frac{(1 - \lambda_p)(1 - \lambda_w) g^{-\frac{1}{\nu}}}{\chi_H} \right)^{\frac{1}{\tau+1/\nu}}, \quad y = g\tilde{c}, \quad H = y, \quad \tilde{w} = (1 - \lambda_p).$$

B.2.3 Posterior Simulator

We first estimate a log-linearized version of the DSGE model using the Random-Walk Metropolis (RWM) algorithm described in [An and Schorfheide \(2007\)](#). Using the same covariance matrix for the proposal distribution as for the linearized DSGE model, we then run the RWM algorithm based on the likelihood function associated with the second-order approximation of the DSGE model. The covariance matrix of the proposal distribution is scaled such that the RWM algorithm has an acceptance rate of approximately 50%. We use 80,000 particles to approximate the likelihood function of the nonlinear DSGE model, while the variance of measurement errors is set to 10% of the sample variance of the observables. We generate 120,000 draws from the posterior distribution of the nonlinear DSGE model. The summary statistics reported in [Table 2.2](#) in the main paper are based on the last 100,000 draws of this sequence.

Table A-8: **Posterior Estimates for DSGE Model Parameters: Linear Model**

Parameter	1960:Q1-2007:Q4		1984:Q1-2007:Q4	
	Mean	90% Interval	Mean	90% Interval
$400 \left(\frac{1}{\beta} - 1 \right)$	0.48	[0.06, 1.01]	1.31	[0.60, 2.17]
π^A	3.46	[2.94, 3.97]	2.80	[2.33, 3.29]
γ^A	1.86	[1.39, 2.34]	1.88	[1.53, 2.24]
τ	6.54	[4.37, 9.24]	4.78	[2.57, 8.70]
ν	0.09	[0.06, 0.13]	0.08	[0.03, 0.15]
$\kappa(\varphi_p)$	0.01	[0.01, 0.02]	0.18	[0.09, 0.30]
φ_w	62.33	[44.48, 83.14]	14.89	[6.15, 25.88]
ψ_w			N/A	
ψ_p			N/A	
ψ_1	1.45	[1.24, 1.68]	2.67	[2.10, 3.30]
ψ_2	0.80	[0.54, 1.09]	0.76	[0.41, 1.11]
ρ_r	0.77	[0.73, 0.82]	0.71	[0.61, 0.79]
ρ_g	0.97	[0.96, 0.98]	0.96	[0.93, 0.98]
ρ_z	0.26	[0.10, 0.41]	0.07	[0.01, 0.19]
ρ_p	0.99	[0.98, 0.99]	0.93	[0.87, 0.98]
$100\sigma_r$	0.18	[0.14, 0.22]	0.18	[0.13, 0.25]
$100\sigma_g$	0.65	[0.44, 0.95]	0.76	[0.39, 1.34]
$100\sigma_z$	0.75	[0.64, 0.85]	0.47	[0.37, 0.56]
$100\sigma_p$	15.28	[12.66, 18.18]	7.63	[5.96, 9.48]

Notes: As 90% credible interval we are reporting the 5th and 95th percentile of the posterior distribution.

Appendix C

Appendix to “Risk, Economic Growth and the Market Value of U.S. Corporations”

C.1 Empirical Analysis

C.1.1 Data Definition and Sources

TFP Growth

Quarterly data on output growth and hours growth for the U.S. Business Sector are from the BLS, respectively Series id PRS85006042 and PRS85006032. We construct an annual series for the growth rate of the U.S. Business Sector capital stock using NIPA Tables. In particular, from Table 6.2, we calculate the growth rate of real private fixed assets for the following sectors:

- Corporate

- Sole Proprietorships
- Partnerships

We use Table 7.2A and 7.2B from NIPA to construct the growth rate of real fixed assets of government sponsored enterprises (GSE). We construct the growth rate of real fixed assets for the Business Sector as follows:

$$\Delta k_{bus} = \theta_{corp} \Delta k_{corp} + \theta_{propr} \Delta k_{propr} + \theta_{part} \Delta k_{part} + \theta_{gse} \Delta k_{gse},$$

where θ_j is the share of subsector j in total fixed assets of the Business Sector.¹

Next, we use the following formula to calculate TFP growth for the Business Sector:²

$$\Delta z_t = \frac{\Delta y_t - \alpha \Delta k_t - (1 - \alpha) \Delta l_t}{1 - \alpha}$$

We choose a value of α equal to 0.30, as it is customary in the macroeconomic literature.³

¹That is, we take fixed assets measured at current cost in sector j , divide by the same figure for the entire Business Sector, and average over the time period considered.

²In order to reconcile the frequency of output and hours with those of capital, we linearly interpolate the growth rate of capital and convert it at quarterly frequencies.

³We also considered a version of TFP growth with time-varying factor shares. Data on labor shares for the Business Sector are from BLS, Series id PRS85006173. Our results did not change, both from a qualitative and quantitative standpoint.

Market Value of U.S. Corporations

Our indicator of value is the sum of two components, the value of equities and the value of net debt for the U.S. corporate sector. We use data from the Federal Reserve's Flow of Funds Accounts to obtain these two time series. In particular:

- Market value of Corporate Equities: We take the data from Table L.213 of the Flow of Funds (“Market Value of Domestic Corporations”).
- Net Debt: We construct a net debt series for all domestic sectors issuing corporate equities. The sectors issuing corporate equities can be obtained from Table F.213 of the Flow of Funds. These are:
 - Non Financial Corporate Business (Table L.201).
 - Domestic Financial Corporations (Tables L.110, L.114, L.115, L.121, L.126, L.122, L.127, L.128, L.129).⁴

We define net debt as the difference between total debt liabilities and total debt assets, where “debt” includes any financial instrument that is not corporate equity, mutual funds holdings that are equity and the equity component of “miscellaneous claims.”⁵ We then aggregate to obtain the net debt series. Notice that the net debt series computed consists of instruments that are recorded mainly at book value in the Flow of Funds.⁶

⁴Domestic financial corporations issuing equities are, in order: i) U.S. chartered depository institutions; ii) property-casualty insurance companies; iii) life insurance companies; iv) close-end funds and exchange traded funds; v) REITS; vi) government-sponsored enterprises; vii) brokers and dealers; viii) holding companies; and ix) funding companies.

⁵We follow the procedure described in the online appendix of [McGrattan and Prescott \(2005\)](#) in order to deduce the equity component of mutual funds holdings and miscellaneous claims. For this purpose, we use Flow of Funds Tables L.229, L.230, L.231.

⁶[Hall \(2001\)](#) proposes a procedure to correct for this issue. [McGrattan and Prescott \(2005\)](#) show that correcting for this issue results in only minor changes to the sample 1960-2001 (see Figure A.3 in their online appendix).

C.1.2 Univariate model

We model TFP growth as follows:

$$\Delta Z_t = \mu_t + \phi [\Delta Z_{t-1} - \mu_{t-1}] + \sigma_t \varepsilon_t$$

$$\mu_t = \mu_0 + \mu_1 s_{1,t}$$

$$\sigma_t = \sigma_0 + \sigma_1 s_{2,t}$$

$$\varepsilon_t \sim \mathcal{N}(0, 1) \quad s_{1,t} \sim \mathcal{MP}(P_\mu) \quad s_{2,t} \sim \mathcal{MP}(P_\sigma)$$

We collect in $\theta = [\mu_0, \mu_1, \sigma_0, \sigma_1, \phi, P_{1,1,\mu}, P_{2,2,\mu}, P_{1,1,\sigma}, P_{2,2,\sigma}]$ the parameters to be estimated. We use Bayesian methods to conduct inference over θ . Given prior information on the parameters, represented by the distribution $p(\theta)$, and given the likelihood function $p(\{\Delta Z_t\}_{t=2}^T | \theta, \Delta Z_1)$, the posterior distribution of θ is found using Bayes' rule:

$$p(\theta | \{\Delta Z_t\}_{t=1}^T) = \frac{p(\theta) p(\{\Delta Z_t\}_{t=1}^T | \theta)}{p(\{\Delta Z_t\}_{t=1}^T)}$$

The parametrization of the prior distribution is given in Table 3.1. Functional forms are chosen for tractability. We center the prior on μ_0 and μ_1 so that, on average, the growth rate of TFP is 2% at an annual level. In the low-growth regime, TFP growth is 60% of the high-growth regime. We center the prior on

σ_0 and σ_1 so that the the standard deviation of TFP growth is on average 4%. In the low volatility regime, the standard deviation of TFP growth is on average 60% that of the high volatility regime. We center the prior of ϕ at zero, reflecting beliefs of low autocorrelation of TFP growth. Finally, the prior on the transition probabilities is centered so that the expected duration of a regime is approximately 15 years. A better way of interpreting our prior is to look at Table A-1, which reports prior predictive checks. Essentially, our prior is that TFP growth has very low autocorrelation, with small and very persistent time variation in the mean. Moreover, we can verify from the table that our prior is quite diffused, as the standard deviations of the statistics show.

We use a Random Walk Metropolis Hastings (RWMH) algorithm to sample from the posterior of θ . We follow common practice in choosing the following proposal density:

$$\theta^{proposal} \sim \mathcal{N}(\theta^i, c\mathbf{H}^{-1}),$$

where θ^i is the state of the Markov Chain at iteration i and \mathbf{H}^{-1} is the inverse Hessian of the log-posterior density evaluated at the posterior mode. The scaling factor c is chosen so that our RWMH algorithm has an acceptance rate of approximately 30%. We generate $N = 100000$ draws and discard the first 20000 when computing posterior statistics.⁷

⁷We perform several tests confirming that our choice of N yields an accurate posterior approximation.

Table A-1: **Prior Predictive Checks**

Statistic	MS-Model
Mean(ΔZ_t)	2.00 (1.10)
Stdev(ΔZ_t)	4.42 (4.50)
Acorr(ΔZ_t)	0.05 (0.53)
Expected Duration of Regimes	10 (80)

Note: Prior Predictive Checks are calculated as follows: 1) generate a random draw of the model's parameters θ^m from $p(\theta)$; 2) given θ^m , use the Markov-Switching model to compute a realization ($T = 10000$) for ΔZ_t ; 3) compute statistics on the generated sample; 4) repeat this procedure $M = 10000$ times and report mean and standard deviation (in parenthesis) of each statistic computed in 3). Expected duration of a regime is reported in years.

C.1.3 Multivariate Model

The multivariate analysis is based on the following model:

$$\begin{aligned} \begin{bmatrix} \Delta Z_t \\ \Delta \mathbf{Y}_t \end{bmatrix} &= \begin{bmatrix} \mu_t \\ \mu_t \end{bmatrix} + \Phi \left(\begin{bmatrix} \Delta Z_{t-1} - \mu_{t-1} \\ \Delta \mathbf{Y}_{t-1} - \mu_{t-1} \end{bmatrix} \right) + \Sigma_t \mathbf{e}_t \\ \mu_t &= \mu_0 + \mu_1 s_{1,t} \\ \Sigma_t &= \Sigma_0 + \Sigma_1 s_{2,t} \end{aligned}$$

$$\varepsilon_t \sim \mathcal{N}(0, 1) \quad s_{1,t} \sim \mathcal{MP}(P_\mu) \quad s_{2,t} \sim \mathcal{MP}(P_\sigma)$$

We include in $\Delta \mathbf{Y}_t$ the growth rate of aggregate consumption per hour and the growth rate of real compensation per hour in the Non Farm Business Sector. We follow [Cogley and Sargent \(2005\)](#) in parametrizing the matrix Σ_t as follows:

$$\Sigma_t = \mathbf{B} \hat{\Sigma}(s_{2,t})$$

with \mathbf{B} being a lower triangular matrix with ones on the main diagonal and $\hat{\Sigma}(s_{2,t})$ a diagonal matrix whose (j, j) element evolves as follows:

$$\sigma_{j,t} = \sigma_{j,0} + \sigma_{j,1} s_{2,t}$$

Given these restrictions, the parameters to be estimated are 25. As in the previous section, we use Bayesian methods to estimate the above model. In particular, we sample from the posterior distribution of the model's parameter using a Metropolis-within-Gibbs algorithm. Our posterior simulator has four main steps:

- i) Sample $\{s_{1,t}, s_{2,t}\}_{t=1}^T$ given the data and the model's parameters using the Kim-Hamilton smoother (Kim and Nelson, 1999);
- ii) Sample Φ conditional on $\{s_{1,t}, s_{2,t}\}_{t=1}^T$ and the other model's parameters from a standard linear Bayesian regression with conjugate priors;
- iii) Sample the lower diagonal elements of \mathbf{B} conditional on $\{s_{1,t}, s_{2,t}\}_{t=1}^T$ and the other model's parameters from a system of unrelated regressions with conjugate priors, see Cogley and Sargent (2005);
- iv) Sample the parameters $[\mu_0, \mu_1, \{\sigma_{j,0}, \sigma_{j,1}\}_{j=1}^3, P_{1,1,\mu}, P_{2,2,\mu}, P_{1,1,\sigma}, P_{2,2,\sigma}]$ using a Metropolis step, with proposal density constructed in the same way as in Appendix C.1.2

The prior for the parameters governing μ_t and $\sigma_{j,t}$ is the same as the one described in the previous section, while the priors on the remaining parameters are fairly diffuse. We generate 100000 draws from the posterior and discard the first 20000 when computing posterior statistics.

C.2 Equilibrium and Auxiliary Planner's Problem

In this appendix, we are going to

- formally define an equilibrium for our model;
- characterize some of its properties that are useful for the computation;
- describe the Auxiliary Planner's Problem utilized in the numerical solution;
- explain how we choose the state variables during computation to minimize

the computational burden.

C.2.1 Equilibrium

An equilibrium of our economy are sequences (depending on realizations of the stochastic process) of quantities ⁸

$$(k_t, (k_{j,t})_{j \in [0,1]}, i_t, (l_{j,t})_{j \in [0,1]}, (d_{j,t})_{j \in [0,1]}, (y_{j,t})_{j \in [0,1]}, (\hat{y}_{j,t})_{j \in [0,1]}, (\omega_{j,t})_{j \in [0,1]}, y_t, c_t, \omega_t)_{t=0}^{\infty},$$

prices $(r_t, w_t, (\mathcal{P}_{j,t})_{j \in [0,1]}, (p_{j,t})_{j \in [0,1]}, \bar{p}_t, p_t^k, d_t)_{t=0}^{\infty}$, value functions $(V_t)_{t=0}^{\infty}$ and discount factors $(\Lambda_{0,t})_{t=0}^{\infty}$ such that

- $(V_t)_{t=0}^{\infty}, ((\omega_{j,t})_{j \in [0,1]}, \omega_t)_{t=0}^{\infty}, (c_t)_{t=0}^{\infty}$ solves the household's problem given $(w_t, (d_{j,t})_{j \in [0,1]}, (\mathcal{P}_{j,t})_{j \in [0,1]}, p_t^k, d_t)_{t=0}^{\infty}$:

$$\max_{(\tilde{V}_t)_{t=0}^{\infty}, ((\tilde{\omega}_{j,t})_{j \in [0,1]})_{t=0}^{\infty}, (\tilde{c}_t)_{t=0}^{\infty}, (\tilde{d}_t)_{t=0}^{\infty}} \tilde{V}_0$$

$$\text{s.t. } \tilde{V}_t = [(1 - \beta)\tilde{c}_t^{\frac{1-\gamma}{\eta}} + \beta \mathbb{E}_t[\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{\eta}}]^{1-\gamma}$$

$$\tilde{c}_t + \int \mathcal{P}_{j,t} \tilde{\omega}_{j,t+1} dj + \omega_{t+1} p_t^k = w_t + \int (\mathcal{P}_{j,t} + d_{j,t}) \tilde{\omega}_{j,t} dj + \omega_t (d_t + p_t^k)$$

and a no Ponzi condition for ω and finiteness for \tilde{V}_t ;

- $\forall j (d_{j,t}, k_{j,t}, l_{j,t}, p_{j,t})_{t=0}^{\infty}$ solve intermediate good producer j 's problem given $(r_t, w_t, p_{i,t})_{i \in [0,1] \setminus \{j\}}, (\bar{p}_t)_{t=0}^{\infty}, (\Lambda_{0,t})_{t=0}^{\infty}$ and $(\hat{y}_{j,t})_{j \in [0,1]}$:

$$\max_{(\tilde{d}_{j,t}, \tilde{k}_{j,t}, \tilde{l}_{j,t}, \tilde{p}_{j,t})_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \tilde{d}_{j,t}$$

$$\text{s.t. } \tilde{d}_{j,t} = \hat{y}_{j,t}(\tilde{p}_{j,t}) \frac{\tilde{p}_{j,t}}{\bar{p}_t} - w_t \tilde{l}_{j,t} - r_t \tilde{k}_{j,t}$$

⁸ $\hat{y}_{j,t}$ is the demand function for good j and not a number.

- $(k_{t+1}, i_t)_{t=0}^{\infty}$ solves the capital good producers problem given r_t :

$$\max_{(\tilde{i}_t, \tilde{k}_{t+1})_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} r_t \tilde{k}_t - \tilde{i}_t$$

$$\tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + G \left(\frac{\tilde{i}_t}{\tilde{k}_t} \right)$$

- $\forall t (y_t, y_{j,t})_{j \in [0,1]}$ given $\bar{p}_t, (p_{j,t})_{j \in [0,1]}$ solve the final good producers problem

$$\max_{\tilde{y}_t, (\tilde{y}_{j,t})_{j \in [0,1]}} \bar{p}_t \tilde{y}_t - \int p_{j,t} \tilde{y}_{j,t} dj$$

$$\tilde{y}_t \leq \left[\int y_{j,t}^{\frac{\nu-1}{\nu}} dj \right]^{\frac{\nu}{\nu-1}}$$

and

$$(\hat{y}_{j,t})_{j \in [0,1]}$$

are consistent with pointwise maximization of the final good producer given any chosen price $p \in \mathbb{R}_t$ by intermediate producer j given $\bar{p}_t, (p_{j,t})_{j \in [0,1] \setminus \{j\}}$

- markets clear: $\forall t, h$

$$\int l_{j,t} dj = 1$$

$$\int \omega_{j,t} dj = 1$$

$$\omega_t = 1$$

$$\hat{y}_{j,t}(p_{j,t}) = y_{j,t}$$

$$c_t + \int i_{j,t} dj = y_t$$

$$k_t = \int k_{j,t} dj$$

- The discount factor of the firm fulfills $\Lambda_{0,t} = \Pi_{s=0}^{t-1} \Lambda_{s,s+1}$ where $\Lambda_{t,t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\psi}} \frac{V_{t+1}^{(1-\gamma)(1-\frac{1}{\eta})}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]^{1-\frac{1}{\eta}}}$.⁹

We will focus on a symmetric equilibrium in the following. It then implies that prices and capital service choices are the same for all intermediate good producers.

C.2.2 Partial Characterization and Auxiliary Planner's Problem

It is well known in the literature and easy to check that the final good producer problem results in the following demand for intermediate good i in equilibrium: $\frac{p_{j,t}}{\bar{p}_t} \left(\frac{p_{j,t}}{\bar{p}_t} \right)^{-\nu} \bar{Y}_t$, where \bar{Y}_t is demand and equal to output in equilibrium and $\bar{p}_t = \left[\int_0^1 p_{j,t}^{1-\nu} dj \right]^{\frac{1}{1-\nu}}$. Imposing this in any intermediate good producing firm's problem and combining it with the problem of a capital good producer,¹⁰ we get as a representative firm's problem

$$\begin{aligned} \max_{k_{j,t}, i_{j,t}, l_{j,t}, P_{j,t}} \mathbb{E}_0 \sum_{t=0} \Lambda_{0,t} \left[\frac{p_{j,t}}{\bar{p}_t} \left(\frac{p_{j,t}}{\bar{p}_t} \right)^{-\nu} \bar{y} - i_{j,t} - w_t l_{j,t} \right] \\ \text{s.t.} \quad \left(\frac{p_{j,t}}{\bar{p}_t} \right)^{-\nu} \bar{y}_t = F(k_{j,t}, Z_t l_{j,t}) \\ k_{j,t+1} = (1 - \delta)k_{j,t} + G\left(\frac{i_{j,t}}{k_{j,t}}\right)k_{j,t}. \end{aligned}$$

We continue by taking first-order conditions (We drop the j index for now.). Let λ_t be the multiplier on the first constraint and μ_t on the second.

⁹This condition could be easily derived from assuming there is a full set of Arrow securities in zero net supply. In order not to further expand the notation, we directly impose the condition on the discount factor.

¹⁰A intermediate good producer's problem is static and the market for capital services is competitive. In addition, we assume a symmetric equilibrium so that the capital stock and capital and labor services are the same across firms. Therefore, we can combine the two problems without changing equilibrium allocations.

FOC:

$$\begin{aligned}
(p_t) \quad \mathbb{E}_t[\Lambda_{0,t}(1-\nu) \left(\frac{p_t}{\bar{p}_t}\right)^{-\nu} \frac{\bar{y}_t}{\bar{p}_t} - \lambda_t(-\nu p_t \left(\frac{p_t}{\bar{p}_t}\right)^{-\nu} \bar{y}_t)] &= 0 \\
(i_t) \quad \mathbb{E}_t[-\Lambda_{0,t} + \mu_t G'(\frac{i_t}{k_t})] &= 0 \\
(k_t) \quad \mathbb{E}_{t-1}[-\{\mu_{t-1}\}] + E_{t-1}[\lambda_t F_{K,t} + \mu_t(1-\delta + G(\frac{i_t}{k_t}) - G'(\frac{i_t}{k_t})) \frac{i_t}{k_t}] &= 0 \\
(l_t) \quad \mathbb{E}_t[\Lambda_{0,t}\{-w_t\} + \lambda_t F_{L,t}] &= 0
\end{aligned}$$

$F_{K,t}$ and $F_{L,t}$ denote the derivatives of F with respect to K and L given period t inputs. Imposing a symmetric equilibrium (all intermediate prices are the same) and using $q_t = \frac{1}{G'_t}$:

$$\begin{aligned}
(p_t) \quad \mathbb{E}_t[-\frac{(1-\nu)}{\nu} \Lambda_{0,t}] &= \mathbb{E}_t[\lambda_t] \\
(i_t) \quad \mathbb{E}_t[-\Lambda_{0,t} + \frac{\mu_t}{q_t}] &= 0 \\
(k_t) \quad \mathbb{E}_{t-1}[-\{\mu_{t-1}\}] + E_{t-1}[\lambda_t F_{K,t} + \mu_t(1-\delta + G(\frac{i_t}{k_t}) - \frac{i_t}{q_t})] &= 0 \\
(l_t) \quad \mathbb{E}_t[\{-w_t\} - \frac{(1-\nu)}{\nu} F_{L,t}] &= 0
\end{aligned}$$

Combining further and dropping the expectation operators where not necessary:

$$\begin{aligned}
w_t &= \frac{(\nu-1)}{\nu} F_{L,t} \\
\Lambda_{0,t} q_t &= \mu_t \\
-\frac{(1-\nu)}{\nu} \Lambda_{0,t} &= \lambda_t \\
q_{t-1} &= \mathbb{E}_{t-1}[\Lambda_{t-1,t} [\frac{\nu-1}{\nu} F_{K,t} + q_t(1-\delta + G(\frac{i_t}{k_t})) - \frac{i_t}{k_t}]]
\end{aligned}$$

The last equation is the Euler equation for capital accumulation which we will target in the computation. The reader should notice the distortion terms $\frac{\nu-1}{\nu}$ for

capital and labor compared to the "standard" Euler equation in a competitive model. The price of the final good in each period is set to one for simplicity. The total profits of the two types of firms combined can be seen to be (assuming a Cobb-Douglas production function) $F(k_t, Z_t l_t) * (\alpha + \frac{1-\alpha}{\nu}) - I_t$. It is easy to see that $\frac{1-\alpha}{\nu} F(k_t, Z_t l_t) - I_t$ is the share of the profit collected by the capital good producers while the remaining part of the $\frac{1}{\nu} F(k_t, Z_t l_t)$ is the profit of each intermediate good producer.

After these derivations it is easy to see now that the auxiliary planner's problem defined in the main text leads to the same Euler equation as the one of the firm if we impose symmetry. Given that also the resource constraints are the same we see that the symmetric equilibrium and the planner's problem result in the same allocations. If we solve the latter we can as usual use first order conditions to obtain prices.

C.2.3 State Space Selection for the Computation

In order to solve the model numerically using the planner's problem, we normalize all quantities that grow over time - consumption, investment and capital - and the value function in period t by Z_t . To be more specific, if X_t is the value of $X \in \{c, i, \bar{k}, k\}$ in period t , we define $\tilde{X}_t = \frac{X_t}{Z_t}$. Normalizing this way we get the following first-order conditions for the planner's problem, where we also normalize the multipliers accordingly.¹¹

$$w_t = \frac{(\nu - 1)}{\nu} F_L(\tilde{K}_t, L_t)$$

¹¹Assuming F is Cobb-Douglas and of degree 1, F_L, F_K are of degree 0. It is also the case that $\frac{\tilde{i}_t}{\tilde{k}_t} = \frac{i_t}{k_t}$, which takes care of normalizing q .

$$\begin{aligned}
\Lambda_{0,t}q_t &= \mu_t \\
-\frac{(1-\nu)}{\nu}\tilde{\Lambda}_{0,t} &= \tilde{\lambda}_t \\
q_{t-1} &= E_{t-1}\tilde{\Lambda}_{t-1,t}\left[\frac{\nu-1}{\nu}F_K + q_t(1-\delta + G(\frac{\tilde{i}_t}{\tilde{k}_t})) - \frac{\tilde{i}_t}{\tilde{k}_t}\right] \\
\tilde{\Lambda}_{t,t+1} &= \beta\left(\frac{\frac{Z_{t+1}\tilde{c}_{t+1}}{Z_t}}{\tilde{c}_t}\right)^{-\frac{1}{\psi}}\frac{\tilde{V}_{t+1}^{(1-\gamma)(1-\frac{1}{\eta})}}{\mathbb{E}_t[\tilde{V}_{t+1}^{1-\gamma}]^{1-\frac{1}{\eta}}} \\
\tilde{k}_{t+1}\frac{Z_{t+1}}{Z_t} &= (1-\delta)\tilde{k}_t + G(\frac{\tilde{i}_t}{\tilde{k}_t}) \\
\tilde{y}_t &= \frac{1}{\nu}\tilde{k}_t^\alpha + \frac{\nu-1}{\nu}\tilde{k}_t^\alpha \\
\tilde{V}_t &= [(1-\beta)\tilde{c}_t^{\frac{1-\gamma}{\eta}} + \beta\mathbb{E}_t[\frac{Z_{t+1}}{Z_t}\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{\eta}}]^{1-\frac{\eta}{1-\gamma}}.
\end{aligned}$$

It should be noted that Z_t drops out everywhere beside in the ratio $\Delta Z_{t+1} = \frac{Z_{t+1}}{Z_t}$. Furthermore, ΔZ_{t+1} factors out in the discount factor and the value function. It follows that Z_t does not influence the value of the normalized variables solving the equations beyond the effect it might have on the expectation for ΔZ_{t+1} . We therefore use the beliefs after updating using Z_t , g_t and $s_{2,t}$ as a state variable. This allows us to drop Z_t and g_t as states when solving the model in its normalized form. We have to incorporate the possible realizations of the shocks and beliefs into the transition probabilities only when taking expectations. We are left with both capital stocks $\tilde{\tilde{k}}_t, \tilde{k}_t$, updated beliefs $\hat{\mu}_t$ and the state of the volatility shock $s_{2,t}$. But once the Euler equation for capital was derived while correctly distinguishing the two capital stocks, we can impose $\tilde{K}_t = \tilde{k}_t$ without changing the results. This allows us to drop one capital stock in computing the solution, leaving us with two continuous and one discrete state variable for the purpose of numerically solving the normalized problem of the planner.

C.3 Computation - Projection

We follow [Krueger and Kuebler \(2003\)](#) by applying a Smolyak collocation method. Our system of state variables contain the capital stock in the beginning of a period and the belief about the state of the long run component of technology growth after observing the shock and the signal. In addition, we have a state variable taking two values for the variance, and we therefore use two sets of polynomials, one for the high and one for the low volatility regime, to approximate policy and value functions.¹² We outline the applied procedure in the following with references to more detailed descriptions. To solve for the coefficients of the polynomials, we use a time iteration procedure for the investment decision and the value function.

- Step 0: Define a tolerance (we used 10^{-5} but checked for the benchmark calibration that the results do not change significantly, if we use 10^{-6} instead). Select an upper and lower bound for capital and the belief to define the intervals the approximation should focus on. Compute the collocation points as described in [Krueger and Kuebler \(2003\)](#). For the baseline problem, we need to fit the investment policy function and the value function. As an initial guess, we use constant functions at the deterministic steady-state levels of these variables (we later used the approximations obtained from previous solutions as an initial guess). Denote this initial guess by $V_{p,i}^0$ and $I_{p,i}^0$ for $i = 1, 2$, where $i = 1$ is the low variance state and $i = 2$ the high variance state. Set $n=1$, Norm=10. Go to Step 1.
- Step 1: Enter iteration n . Calculate $I_{p,i}^n(x)$ by using the normalized Euler equation for capital of the planner imposing equilibrium conditions (espe-

¹²We refer the reader to the model appendix for the arguments on why these states suffice.

cially $k = K$) in each point x of the grid and using $I_{p,i}^{n-1}$ to determine policy choices in the next period. For the computation of the expectation, see below. Go to step 2.

- Step 2: Use $I_{p,i}^n(x)$ to obtain $V_{p,i}^n(x)$ using the budget constraint and the definition of V and $V_{p,i}^{n-1}$ for the values in the next period. Go to step 3.
- Step 3: Use the interpolation rule as described in [Krueger and Kuebler \(2003\)](#) to obtain the approximation polynomials $V_{p,i}^n$ and $I_{p,i}^n$ for $i = 1, 2$. Go to step 4.
- Step 4: Compute Norm as the maximum norm of the difference of the values in the collocation for this and the previous approximation normalizing by the previous approximation. If the Norm is smaller than the tolerance end, otherwise set $n=n+1$ and go to Step 1.

After solving for the investment policy function, the value function and the law of motion for K , we compute Euler equation errors on a grid different from the one used to solve the model to check the quality of the approximation. For taking the expectation, we used that conditional on the beliefs and the realization of the volatility state the distribution of productivity growth shocks is normal and applied a five point Gauss Hermite Quadrature formula ([\(Judd, 1998\)](#) pages 261-263). We also checked with more points resulting only in very mild changes in the results. To obtain the full approximation, we use the transition matrix for the stochastic volatility and Bayes' rule to update beliefs given technology growth shocks and signals. In practice we use an accelerator method as described in [Judd \(1998\)](#) to speed up the computation.

Finally, we also need to compute asset pricing functions. In order to do this, we follow the same procedure as before for investment and value functions. We fix those solutions and then iterate on the price using the Euler equations for the specific asset we want to value until convergence.

Appendix D

Appendix to “Identifying Neutral Technology Shocks”

D.1 Proofs and Derivations

D.1.1 Proof of Lemma 1

Consider a variable $X_t = \sum_{i=1}^M X_{it}$, with long-run response g_X for variable X and g_{X_i} for the components $X_i \neq 0$. By the definition of the impulse response

$$IR_t^X(x) = (e^{g_X x} - 1)E_0[X_t] = \sum_{h=1}^H IR_t^{X_i}(x) = \sum_{h=1}^H (e^{g_{X_h} x} - 1)E_0[X_{h,t}]$$

and therefore after canceling terms

$$e^{g_X x} E_t[X_t] = \sum_{h=1}^H e^{g_{X_h} x} E_0[X_{h,t}]$$

Taking the l 'th derivative w.r.t. x yields

$$g_X^l e^{g_X x} E_0[X_t] = \sum_{h=1}^H g_{X_h}^l e^{g_{X_h} x} E_0[X_{h,t}]$$

and dividing by g_X^l ,

$$e^{g_X x} E_0[X_t] = \sum_{h=1}^H \left(\frac{g_{X_h}}{g_X} \right)^l e^{g_{X_h} x} E_0[X_{h,t}]$$

This implies that $g_{X_h} \leq g_X$ since otherwise the RHS converges to ∞ for $l \rightarrow \infty$.

Then, since

$$\begin{aligned} \exp(g_X x) E_0(X_t) &= \sum_{h=1}^H \exp(g_{X_h} x) E_0(X_{ht}), \\ g_X &= g_{X_1} = \cdots = g_{X_h} = \cdots = g_{X_H}. \end{aligned}$$

D.1.2 Proof of Theorem 1

The argument has two parts. The first part is to show that a neutral technological shock has the properties stated in the theorem and the second part is to show that any other shock with these properties is a neutral shock. In order to prove the first part, note that the resource constraint implies

$$IR_t^Y(x) = IR_t^I(x) + IR_t^C(x),$$

where I is total investment and C is total consumption of output Y . Equivalently

$$e^{g_Y x} E_0[Y_t] = e^{g_I x} E_0[I_t] + e^{g_C x} E_0[C_t].$$

Dividing by $e^{g_Y x}$ yields

$$E_0[Y_t] = e^{(g_I - g_Y)x} E_0[I_t] + e^{(g_C - g_Y)x} E_0[C_t].$$

Taking derivatives w.r.t. x :

$$0 = (g_I - g_Y)e^{(g_I - g_Y)x} E_0[I_t] + (g_C - g_Y)e^{(g_C - g_Y)x} E_0[C_t].$$

Since this holds for all x , it must be the case that $g_Y = g_I = g_C$. Capital accumulation implies that

$$IR_{t+1}^{K_j}(x) = IR_t^{K_j}(x)(1 - \delta_j) + IR_t^{I_j}(x)$$

and equivalently

$$e^{g_{K_j}x} E_0[K_{j,t+1}] = e^{g_{K_j}x} E_0[K_{j,t}] + e^{g_{I_j}x} E_0[I_{j,t}].$$

This yields

$$e^{g_{K_j}x} \{E_0[K_{j,t+1}] - (1 - \delta_j)E_0[K_{j,t}]\} = e^{g_{K_j}x} E_0[I_{j,t}] = e^{g_{I_j}x} E_0[I_{j,t}],$$

and thus $g_{K_j} = g_{I_j}$. By Lemma 1, it must be the case that $g_{I_j} = g_I$, and thus $g_{K_j} = g_I \forall j$. This implies $g_Y = g_C = g_{I_j} = g_{K_j} = g$. By Assumption 1, this also means that $g_{L_j} = 0 \forall j$. Since the production function features constant return to scale, we have $g = 1$. Constant returns to scale also implies that the marginal products of capital is not influenced by the shock in the long run

$$\begin{aligned} & F_{K_j}(e^x K_{1,t}, \dots, e^x K_{J,t}, e^x Z_t L_{1,t}, \dots, e^x Z_t L_{N,t}; \theta_t) & (D.1) \\ & = F_{K_j}(K_{1,t}, \dots, K_{J,t}, Z_t L_{1,t}, \dots, Z_t L_{N,t}; \theta_t), \end{aligned}$$

and that the marginal product of labor increases by x percent,

$$\begin{aligned} & F_{L_n}(e^x K_{1,t}, \dots, e^x K_{J,t}, e^x Z_t L_{1,t}, \dots, e^x Z_t L_{N,t}; \theta_t) \\ &= e^x F_{L_n}(K_{1,t}, \dots, K_{J,t}, Z_t L_{1,t}, \dots, Z_t L_{N,t}; \theta_t). \end{aligned} \quad (\text{D.2})$$

For the impulse responses we thus get

$$IR_t^{F_{K_j}}(x) = 0$$

$$IR_t^{F_{L_j}}(x) = (e^x - 1)E_0(F_{L_j}).$$

This proves the first part of the characterization theorem.

The second part of the proof shows that any other shock with these properties is the neutral technology shock, which establishes that no other shock has these properties. To this aim, consider an innovation to a non-neutral permanent shock θ_i of x percent at time 0 and consider how this changes a variable X_t . The impulse response compares variables in two scenarios: one where the shock happens and one where it does not. Denote variables X_t with a $\tilde{\cdot}$ (i.e. \tilde{X}_t is conditional on the shock) in the first scenario and without a $\tilde{\cdot}$, X_t in the second scenario. The impulse response of a variable X_t therefore equals

$$E_0(\tilde{X}_t) - E_0(X_t),$$

and for $t \geq T$:

$$\tilde{Y}_t = \exp(x)Y_t, \tilde{K}_{jt} = \exp(x)K_{jt}, \tilde{L}_{jt} = L_{jt}, \tilde{\theta}_i(t) = \exp(x)\theta_i(t)$$

Using this notation we get on the one hand

$$\begin{aligned}\tilde{Y}_t &= F(\tilde{K}_{1,t}, \dots, \tilde{K}_{J,t}, Z_t \tilde{L}_{1,t}, \dots, Z_t \tilde{L}_{N,t}; (\theta_1(\mathbf{t}), \dots, \tilde{\theta}_i(\mathbf{t}), \dots)) \\ &= F(\exp(x)K_{1,t}, \dots, \exp(x)K_{J,t}, Z_t L_{1,t}, \dots, Z_t L_{N,t}; (\theta_1(\mathbf{t}), \dots, \mathbf{exp}(\mathbf{x})\theta_i(\mathbf{t})))\end{aligned}\quad (\text{D.3})$$

and on the other hand that

$$Y_t = F(K_{1,t}, \dots, K_{J,t}, Z_t L_{1,t}, \dots, Z_0 L_{N,0}; (\theta_1(\mathbf{t}), \dots, \theta_i(\mathbf{0}), \dots)). \quad (\text{D.4})$$

Constant returns to scale and $\tilde{Y}_t = \exp(x)Y_t$ imply that

$$\begin{aligned}\tilde{Y}_t &= \exp(x)Y_t \\ &= F(\exp(x)K_{1,t}, \dots, \exp(x)K_{J,t}, \exp(x)Z_t L_{1,t}, \dots, \exp(x)Z_t L_{N,t}; (\theta_1(\mathbf{t}), \dots, \theta_i(\mathbf{t})))\end{aligned}\quad (\text{D.5})$$

Equating the last two expressions for \tilde{Y}_t gives for all x and $t \geq T$ that

$$\begin{aligned}&F(\exp(x)K_{1,t}, \dots, \exp(x)K_{J,t}, Z_t L_{1,t}, \dots, Z_t L_{N,t}; (\theta_1(\mathbf{t}), \dots, \mathbf{exp}(\mathbf{x})\theta_i(\mathbf{t}))) \\ &= F(\exp(x)K_{1,t}, \dots, \exp(x)K_{J,t}, \exp(x)Z_t L_{1,t}, \dots, \exp(x)Z_t L_{N,t}; (\theta_1(\mathbf{t}), \dots, \theta_i(\mathbf{t}))).\end{aligned}\quad (\text{D.6})$$

Thus the first line - the effect of a x percent shock to θ_i - is equivalent to the latter line which is the effect of a x percent shock to neutral technology ($\tilde{Z}_t = \exp(x)Z_t$). Since this identity holds for all x , θ_i is a neutral technology shock.

Note that the proof at no point uses that the shock θ directly enters the production function, i.e. it applies also to non-technology shocks, e.g. preference shocks, government expenditure shocks or wage mark-up shocks.

D.1.3 Proof of Lemma 2

In order to check that our state space is minimal, one needs to verify the observability and controllability conditions are satisfied in our state space model. The observability matrix is given by:

$$\mathcal{O}_{(n(n+k-1)) \times (n+k)}^n = \begin{bmatrix} \mathbf{B}_{(n+k-1) \times (n+k)} \\ \mathbf{B}\Phi_{(n+k) \times (n+k)} \\ \dots \\ \mathbf{B}\Phi^n_{(n+k) \times (n+k)} \end{bmatrix}.$$

The observability condition is satisfied if $\mathcal{O}_{(n(n+k-1)) \times (n+k)}^n$ is of full rank. First notice that \mathbf{B} is of rank $n + k - 1$. Now, suppose that the observability condition is violated. That would imply the existence of a $n + k$ dimensional vector $\xi \neq \mathbf{0}$ such that:¹

$$\mathbf{B}\xi = \mathbf{0} = \mathbf{B}\Phi\xi$$

Given our knowledge of the \mathbf{B} matrix, that would imply that the vector ξ is equal to

$$\xi = (\chi, \underbrace{-\chi, \dots, -\chi}_{n-1 \text{ elements}}, 0, \dots, 0)^{tr}$$

for some $\chi \neq 0$. The last k elements, corresponding to the ξ vector are equal to

¹The nullspace of \mathbf{B} is one-dimensional, that means it is generated by a non-zero vector \mathbf{x} . The nullspace of $\mathbf{B}\Phi$ is one-dimensional as well. If the observation matrix has rank $n + k - 1$ then the nullspace of these of two matrices are identical and generated by the same vector \mathbf{x} .

zero since these variables are observable. As a result we have

$$\mathbf{\Phi}\xi = \chi \left(\phi_{1,1} - \sum_{l=2}^n \phi_{1,l}, \dots, \phi_{j,1} - \sum_{l=2}^n \phi_{j,l}, \dots, \phi_{n,1} - \sum_{l=2}^n \phi_{n,l}, 0, \dots, 0 \right)^{tr},$$

which equals using that the off-diagonal elements in the first row ($\phi_{1,j} = 0$) are zero,

$$\mathbf{\Phi}\xi = \chi \left(\phi_{1,1}, \dots, \phi_{j,1} - \sum_{l=2}^n \phi_{j,l}, \dots, \phi_{n,1} - \sum_{l=2}^n \phi_{n,l}, 0, \dots, 0 \right)^{tr} \quad (\text{D.7})$$

Multiplying this vector with \mathbf{B} maps it to zero, so that we get the set of equations:

$$-\phi_{1,1} = \phi_{j,1} - \sum_{l=2}^n \phi_{j,l} \quad \forall 2 \leq j \leq n, \quad (\text{D.8})$$

contradicting Assumption 2 ii). Thus, by contradiction we must have that ξ is not in the nullspace of $\mathbf{B}\mathbf{\Phi}$. Thus, the observability matrix is of full rank and the system is observable.

The controllability matrix is given by:

$$\mathbf{C}_{n+1 \times (n)^2}^n = \left[\mathbf{R}_{(n+1) \times n} \mathbf{\Phi} \mathbf{R}_{(n+1) \times n} \dots \mathbf{\Phi}^n \mathbf{R}_{(n+1) \times n} \right].$$

That the controllability matrix in our state space system is of full rank follows from Assumption 2 i).

As a result, our state space realization is observable and controllable, hence minimal.

D.1.4 Proof of Theorem 2

Suppose the state space is described by the matrices $(\hat{\mathbf{B}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{R}})$ which are related to the original one as follows:

$$\begin{aligned}\hat{\mathbf{B}} &= \mathbf{B}\mathbf{T} \\ \hat{\mathbf{\Phi}} &= \mathbf{T}^{-1}\mathbf{\Phi}\mathbf{T} \\ \hat{\mathbf{R}} &= \mathbf{T}^{-1}\mathbf{R}\mathbf{U}\end{aligned}\tag{D.9}$$

We show now that \mathbf{T} is the identity matrix and that \mathbf{U} is as described in the theorem.

Let $\chi_i = (0, \dots, \underbrace{1}_i, \dots, 0)'$ be the unit vector with the i 'th entry equal to 1 and other entries equal to zero. Consider the long-run effect of χ_1 , that is the long-run effect of a neutral technology shock, which equals

$$\mathbf{B}\mathbf{T}^{-1}(\mathbf{I} - \mathbf{\Phi})^{-1}\mathbf{R}\mathbf{U}\chi_1 = \mathbf{B}(\mathbf{I} - \mathbf{\Phi})^{-1}\mathbf{R}\mathbf{U}\chi_1$$

since $\mathbf{B}\mathbf{T}^{-1} = \mathbf{B}$. Let

$$\mathbf{U}\chi_1 = \sum_{i=1}^{n+1} u_{i1}\chi_i,$$

where u_{i1} is the $(i, 1)$ entry of \mathbf{U} . Then the long-run effect of χ_1 equals

$$\sum_{i=1}^{n+1} u_{i1}v_i,$$

where v_i is the true long-run effect (i.e. for the state space described by the true matrices $(\mathbf{B}, \mathbf{\Phi}, \mathbf{R})$ of χ_i :

$$v_i = \mathbf{B}(\mathbf{I} - \mathbf{\Phi})^{-1}\mathbf{R}\chi_i.$$

We impose the balanced growth restriction which states that the long-run effect of χ_1 equals v_1 , so that

$$v_1 = \sum_{i=1}^{n+1} u_{i1} v_i,$$

The RHS is the long-run response to the shock $\mathbf{U}\chi_1$ which equals the long-run response of neutral technology (χ_1) on the LHS (v_1). Theorem 1 implies that only neutral technology has this property so that $\mathbf{U}\chi_1 = \chi_1$, i.e. first column of U is the vector $(1, 0, \dots, 0)'$. Since $\mathbf{U}\mathbf{U}' = I$ this implies that the first row of \mathbf{U} equals $(1, 0, \dots, 0)$. Finally we use that the first row of

$$\mathbf{T}^{-1}\mathbf{R}\mathbf{U}$$

is $(1, 0, \dots, 0)$. Using the properties of \mathbf{U} , we also know that the first row of

$$\mathbf{T}^{-1}\mathbf{R}$$

is $(1, 0, \dots, 0)$. Since $\tilde{\mathbf{R}}$ is invertible, we have that $\kappa_2 = \kappa_3 = \dots = \kappa_n = 0$. Furthermore since $r_{zz} = 1$ we also have $\kappa_1 = 0$, so that $\hat{\mathbf{R}} = \mathbf{R}\mathbf{U}$, what completes the proof since $\hat{\mathbf{R}}\hat{\mathbf{R}}' = \mathbf{R}\mathbf{U}\mathbf{U}'\mathbf{R}' = \mathbf{R}\mathbf{R}'$.

D.2 Standard Approaches to Controlling for Input Heterogeneity

D.2.1 Jorgenson's Correction

The fact that inputs heterogeneity complicates the measurement of technology is a well known problem in the growth accounting literature. Here we discuss the

most widely accepted procedure that was developed by [Jorgenson \(1966\)](#). An alternative but closely related procedure due to [Hansen \(1993\)](#) is discussed in [Appendix D.2.2](#). Central to these approaches is the approximation of the growth rate of L_t^e in terms of a weighted sum of the hours worked by different groups of individuals:

$$\Delta \log(L_t^e) \approx \sum_{j=1}^J a_{j,t} \Delta \log(L_{j,t}). \quad (\text{D.10})$$

The procedures differ in the way the weights $\{a_{j,t}\}$ are computed. Jorgenson uses the following Tornqvist aggregator:

$$a_{j,t} = \frac{\nu_{j,t} + \nu_{j,t-1}}{2}, \text{ where } \nu_{j,t} = \frac{w_{i,t} L_{i,t}}{\sum_j w_{j,t} L_{j,t}}. \quad (\text{D.11})$$

As shown in [Diewert \(1976\)](#), this would be the right correction to make in the case that L_t^e is a deterministic homogeneous translog function of the J groups considered, $\log(L_t^e) = f(\log(\mathbf{L}_t))$, where \mathbf{L}_t is the vector of hours worked by the J groups.² Using the properties of quadratic function (e.g., translog as defined in [footnote 2](#)), one obtains:

$$\begin{aligned} \Delta \log(L_t^e) &= f(\log(\mathbf{L}_t)) - f(\log(\mathbf{L}_{t-1})) \\ &= \frac{1}{2} [\nabla f(\log(\mathbf{L}_t)) + \nabla f(\log(\mathbf{L}_{t-1}))]' (\log(\mathbf{L}_t) - \log(\mathbf{L}_{t-1})), \end{aligned} \quad (\text{D.13})$$

where the matrix $\nabla f(\log(\mathbf{L}_t))$ collects the partial derivatives of $f(\cdot)$. Under the

² Defined by

$$\ln f(x) = \alpha_0 + \sum_{k=1}^K \alpha_k \ln x_k + \frac{1}{2} \sum_{m=1}^K \sum_{l=1}^K \gamma_{ml} \ln x_m \ln x_l, \quad (\text{D.12})$$

where $\sum_{k=1}^K \alpha_k = 1$, $\gamma_{ml} = \gamma_{lm}$ and $\sum_{l=1}^K \gamma_{ml} = 0$ for $j = 1, 2, \dots, K$.

additional assumption that prices equal marginal products at all points in time, the Jacobian $\nabla f(\log(\mathbf{L}_t))$ is equal to $\frac{w_{i,t}L_{i,t}}{\sum_j w_{j,t}L_{j,t}}$. Thus, equation D.10 is exact for a homogeneous translog aggregator when the weights are Tornqvist indexes of labor shares of different groups. All other functional forms, e.g., CES aggregator, will generate a bias.

A fundamental problem of this strategy arises when hours in efficiency units is not a deterministic aggregator of hours worked. An implicit assumption in this procedure is that the parameters of the aggregator have to be constant, making it for example difficult to explain movements in the skill premium. Thus even if the aggregator satisfies the functional form requirements at every point of time but parameters are changing over time, technology is measured with a bias. In order to make this point explicit, suppose that $\log(L_t^e) = f(\log(\mathbf{L}_t), \Theta_t)$, where Θ_t is a vector of time varying observable or unobservable factors and parameters. In this environment, one immediately verifies that equation D.14 is an incorrect expansion for L_t^e as it neglects changes in Θ_t .

D.2.2 Hansens' Correction

Hansen (1993) measures the efficiency units of labor as

$$\sum_i \alpha_i L_{i,t}, \tag{D.14}$$

where α_i is the constant weight of group i . The weights α_i are the average hourly earnings

$$\alpha_i = \frac{HE_i}{HE}, \tag{D.15}$$

where HE_i is average hourly earnings for group i and HE is average hourly earnings.

We first compute a log-linear approximation of $\log(\sum_i \alpha_i L_{i,t})$ with respect to $\log(L_{i,t})$:

$$\log\left(\sum_i \alpha_i L_{i,t}\right) \approx \sum_i \frac{\alpha_i \bar{L}_i}{\sum_j \alpha_j \bar{L}_j} \log(L_{i,t}), \quad (\text{D.16})$$

where \bar{L}_i is the average labor supply of group i . In addition to this approximation, a second difference between Hansen and Jorgensen is that they use different coefficients. Jorgensen uses $\nu_{j,t}$, an average of two adjacent periods whereas Hansen uses

$$\frac{\alpha_i \bar{L}_i}{\sum_j \alpha_j \bar{L}_j}, \quad (\text{D.17})$$

a time average for the full sample. This means the second bias in the measurement due to differences in computing averages of wages equals

$$\nu_{j,t} - \frac{\alpha_i \bar{L}_i}{\sum_j \alpha_j \bar{L}_j}. \quad (\text{D.18})$$

After these approximations, Hansen measurement is equal to Jorgensen and thus is unbiased if and only if the aggregator is a homogeneous translog function (with constant coefficients).

D.2.3 Estimation of Solow Residual in Practice

The current the state-of-the-art measurement of Solow residual in the data is based on IV-regression methods described in [Basu et al. \(2006\)](#). As their methodology differs from the Solow residual construction we used in the main text, a few details

should be mentioned. First, it is well-known that if there might be increasing returns to scale, time-varying factor utilization, or if factors are not paid their marginal products, tfp measured as Solow residual will be biased. To overcome this limitation, [Basu et al. \(2006\)](#), following the insight in [Hall \(1988, 1990\)](#), treat Equation (4.2) as a regression. As input choices are likely endogenous to innovations in the technology estimated as the residual, the regression is estimated using instrumented variables. The instruments are required to affect the input choice but to be uncorrelated with innovation in technology. The authors use oil prices, growth in real government defense spending, and “monetary shocks” from a non-structural VAR. Their estimates are based on the data described in [Jorgenson et al. \(1987\)](#) that controls for changes in labor composition using the Jorgenson’s correction.

D.3 Calibration of the Simple RBC Model

The vector of structural parameters of our model is given by:

$$\theta = \underbrace{[\beta, \delta, \alpha, h_s^*, h_u^*, u, \phi^*, \bar{v}, \gamma]}_{\theta_1}, \underbrace{[\rho_\phi, \sigma_\phi, x_l, \rho_z, \sigma_z, \sigma_a, \rho_q, \sigma_q, \rho_g, \sigma_g]}_{\theta_2}.$$

Model period is one quarter. We use quarterly post-84 data on the US economy in order to calibrate the vector θ . The parameters in θ_1 are pinned down using long run average for selected time series. In particular, the parameters β , α and δ are chosen so that, in a deterministic steady state of the model, the real interest rate, the depreciation rate of capital and a labor income share are respectively 1%, 2.5% and 66%, values that are common in the business cycle literature. The growth rate of neutral technology shocks, γ , is chosen so to match an average

growth rate of GDP per capita equal to 2%. The parameters h_u^* , h_s^* , u and ϕ^* are chosen so that the model matches a fraction of 0.29 hours worked by low-skilled individual, 0.36 by high-skilled individuals, a fraction of low-skilled individuals over total population of 0.64 and a skill premium equal to 1.7. These numbers are calculated using CPS quarterly data (1979-2006) on wages and hours worked by education level.³ Finally, we fix the average Frisch elasticity $\bar{\nu}$ to 1.

The remaining parameters in θ_2 are calibrated via a Simulated Method of Moments (SMM) algorithm. In particular, let $\mathbf{m}_{\mathbf{T}}$ be a vector of sample moments for selected time series of length \mathbf{T} computed using US data. We denote by $\mathbf{m}_{\mathbf{T}}(\theta)$ their model counterpart when the vector of structural parameter is θ . θ is chosen to minimize a weighted distance between model and data moments:

$$\min_{\theta_2} [\mathbf{m}_{\mathbf{T}} - \hat{\mathbf{m}}(\theta)]' \mathbf{W}_{\mathbf{T}} [\mathbf{m}_{\mathbf{T}} - \hat{\mathbf{m}}(\theta)],$$

where $\mathbf{W}_{\mathbf{T}}$ is a diagonal matrix whose nonzero elements are the inverse of the variance of the corresponding moment. The empirical moments included in the vector $\mathbf{m}_{\mathbf{T}}$ are standard measures of cyclical variation and comovement for post 1984 quarterly US data. The time series used are the growth rate in GDP, private non-durable consumption, private nonresidential investment, total hours worked in the business sector, total hours of low and high skilled individuals in the business sector, nominal wages for these two demographic groups, labor productivity. For each of these time series, we compute the sample standard deviation, the first order autocorrelation and the cross-correlation with GDP growth. We collect these sample moments in the vector $\mathbf{m}_{\mathbf{T}}$. The associated model's moments are calculated via a Monte Carlo procedure. In particular, for each θ , we solve for the

³We define high-skilled individuals as those possessing college education and low-skilled individuals as those with no college education. See Appendix D.5.

policy functions using first order perturbation. We next simulate a realization of length \mathbf{T} for the model's counterparts of the above time series and calculate the vector $\hat{\mathbf{m}}_{\mathbf{T}}(\theta)$. We repeat this procedure $M = 300$ times, each time changing the seed used in the simulation. We then take the (component wise) median of $\hat{\mathbf{m}}(\theta)$ across the Monte Carlo replications.

Table A-1 summarizes the procedure used for the calibration of our model and reports numerical values for the structural parameters. Table A-2 reports the fit our model in terms of the calibration targets. We can verify that the calibrated model is consistent along many dimensions with the behavior of aggregate time series.

Table A-1: **Calibrated Parameter Values: RBC Model**

Parameter	Value	Source
α	0.33	Labor Income Share
δ	0.025	Depreciation of Capital Stock
β	0.99	Real Interest Rate
γ	1.004	Average GDP growth per capita
h_s^*	0.36	Weekly Hours per Individual (College)
h_u^*	0.29	Weekly Hours per Individual (no College)
u	0.64	% of Individuals without College
μ_ϕ	0.39	Skill Premium
$\bar{\nu}$	1.00	Fixed
x_s	0.85	Calibrated
ρ_ϕ	0.74	Calibrated
ρ_a	1.00	Calibrated
ρ_z	0.26	Calibrated
ρ_g	0.97	Calibrated
ρ_q	0.99	Calibrated
$\sigma_a \times 100$	1.14	Calibrated
$\sigma_\phi \times 100$	1.32	Calibrated
$\sigma_z \times 100$	0.74	Calibrated
$\sigma_g \times 100$	0.18	Calibrated
$\sigma_q \times 100$	0.12	Calibrated

Table A-2: RBC Model Calibration Targets: Data and Model

Moment	Data	Model	Moment	Data	Model	Moment	Data	Model
St Dev(ΔY_t)	0.88	0.63	Acorr(ΔY_t)	0.15	0.47	Corr($\Delta Y_t, \Delta C_t$)	0.17	0.55
St Dev(ΔC_t)	0.47	0.56	Acorr(ΔC_t)	0.14	0.42	Corr($\Delta Y_t, \Delta I_t$)	0.13	0.33
St Dev(ΔI_t)	2.15	3.25	Acorr(ΔI_t)	0.30	0.36	Corr($\Delta Y_t, \Delta I_t$)	0.04	0.44
St Dev(ΔH_t)	0.77	0.75	Acorr(ΔH_t)	0.02	0.53	Corr($\Delta Y_t, \Delta \frac{Y_t}{H_t}$)	0.18	0.11
St Dev($\Delta \frac{Y_t}{H_t}$)	0.56	0.53	Acorr($\Delta \frac{Y_t}{H_t}$)	-0.11	-0.20	Corr($\Delta Y_t, \Delta W_{s,t}$)	0.30	0.06
St Dev($\Delta W_{s,t}$)	0.98	1.53	Acorr($\Delta W_{s,t}$)	-0.07	0.12	Corr($\Delta Y_t, \Delta W_{u,t}$)	0.05	0.06
St Dev($\Delta W_{u,t}$)	0.78	1.13	Acorr($\Delta W_{u,t}$)	-0.01	0.06	$\frac{\text{St Dev}(\Delta H_{s,t})}{\text{St Dev}(H_{u,t})}$	1.20	1.15

D.4 A New-Keynesian Model with Heterogeneous Labor

In this section we describe the New-Keynesian model which we use in the main text. The model is identical to [Christiano et al. \(2005b\)](#) and [Smets and Wouters \(2007b\)](#), except that we have two different type of labor (u)nskilled and (s)killed labor. There is a mass one of workers (on the unit interval), unskilled workers on the interval $[0, u]$ and skilled workers on $[1 - u, 1]$. We also have a richer specification of uncertainty. The sources of uncertainty in the model are shocks to TFP, investment, the disutility of labor, discount factor, the wage markup, the price markup, the skill premium, government spending and monetary policy.

D.4.1 Final-Good Firms

The final consumption good Y is a composite made of intermediate goods Y_j and is sold in a perfectly competitive market at price P_t and equals

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{1+\lambda_{f,t}}} dj \right]^{1+\lambda_{f,t}}, \quad (\text{D.19})$$

where $\lambda_{f,t}$ is an exogenous shock whose law of motion will be specified later, and Y_{jt} is intermediate good i . The inflation rate $\pi_t = P_t/P_{t-1}$. Bonds pay a return $e^b R$, where e^b is a risk-premium shock on the nominal return R .

D.4.2 Intermediate-Goods Firms

A monopolist produces intermediate good $j \in [0, 1]$ using the following technology:

$$Y_{jt} = \begin{cases} K_{jt}^\alpha [e^{Z_t} L_{jt}^e]^{1-\alpha} - Z_t F & \text{if } \geq F \\ 0 & \text{otherwise} \end{cases}, \quad (\text{D.20})$$

where $0 < \alpha < 1$,

$$L_{jt}^e = L_{s,jt}^\phi L_{u,jt}^{1-\phi}$$

and $L_{s,t} = sh_{s,t}$ is total hours worked by skilled individuals, and $L_t = L_{s,t} + L_{u,t}$ is total hours worked. Here, L_{jt}^e and k_{jt} denote the time t labor and capital services used to produce the j^{th} intermediate good. The fixed cost of production are denoted $F > 0$. Intermediate firms rent capital and labor in perfectly competitive factor markets. Profits are distributed to households at the end of each time period. Let R_t^k and W_t denote the nominal rental rate on capital t services and the wage rate, respectively. A firm's real marginal cost is $s_t = \delta S_t(Y)/\delta Y$, where

$$S_t(Y) = \min_{K, L_s, L_u} r_t^k K + w_t^u L_u + w_t^s L_s \quad (\text{D.21})$$

$$Y \text{ given by } (\text{D.20}) \quad (\text{D.22})$$

where $r_t^k = R_t^k/P_t$, $w_t^s = W_t^s/P_t$ and $w_t^u = W_t^u/P_t$. Given our functional forms, we have

$$s_t = \left(\frac{1}{\alpha}\right)^\alpha \left[\left(\frac{1}{\phi(1-\alpha)}\right)^\phi \left(\frac{1}{(1-\phi)(1-\alpha)}\right)^{1-\phi} \right]^{1-\alpha} (r_t^k)^\alpha [(w_t^s)^\phi (w_t^u)^{1-\phi}]^{1-\alpha} e^{Z_t(\alpha-1)} \quad (\text{D.23})$$

Price setting by firms is as in Calvo (1983) with a constant probability, $1 - \theta_p$, of being able to reoptimize its nominal price.

D.4.3 Households

There is a continuum of households, indexed by $j \in [0, 1]$. As in [Christiano et al. \(2005b\)](#) and [Smets and Wouters \(2007b\)](#) all households - skilled and unskilled - are homogeneous with respect to consumption and asset holdings but are heterogeneous with respect to the wage rate they earn and the hours they work. The utility function of the j^{th} household of type $T \in \{u, s\}$

$$E_{t-1}^j \sum_{l=0}^{\infty} \beta^{l-t} \left[u(c_{t+l} - hc_{t+l-1}) - \frac{e^{A_{t+l}}}{1 + \nu_T} h_{j,t+l}^{1+\nu_T} \right]. \quad (\text{D.24})$$

Here, E_{t-1}^j is the expectation operator, conditional on aggregate and household j 's idiosyncratic information up to, and including, time $t - 1$; c_t denotes time t consumption; h_{jt} denotes time t hours worked. The household's stock of physical capital, \bar{k}_t , evolves according to

$$\bar{k}_{t+1} = (1 - \delta)\bar{k}_t + e^{qt} \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \quad (\text{D.25})$$

The physical rate of depreciation is denoted δ , I_t denotes time t investment, and S is the adjustment cost function, with the following properties: $S(e^\gamma) = 0$, $S'(e^\gamma) = 0$ and $S''(e^\gamma) = \kappa$, where γ is mean growth rate of Z_t .

Capital services, k_t , are related to the physical stock of capital by $k_t = u_t \bar{k}_t$. Here, u_t denotes the utilization rate of capital, which at cost $a(u_t)\bar{k}_t$ (in consumption goods) is set by the household. We assume that $u_* = 1$ in steady state, that $a(1) = 0$ and we define $\gamma_u = a''(1)$.

D.4.4 The Wage Decision

Households are monopoly suppliers of a differentiated labor service, $h_{u,jt}$ for unskilled and $h_{s,jt}$ for skilled workers. They sell this service to a representative, competitive firm for skilled/unskilled workers that transforms it into an aggregate labor input, $L_{s,t}$ and $L_{u,t}$ respectively, using the following technologies:

$$L_{T,t} = \left[\int_0^1 h_{T,jt}^{\frac{1}{1+\lambda_{w,t}}} dj \right]^{1+\lambda_{w,t}}, \quad (\text{D.26})$$

for $T \in \{s, u\}$. In each period, a household faces a constant probability, $1 - \theta_w$, of being able to reoptimize its nominal wage.

D.4.5 Monetary Policy

We assume that monetary policy is described by an interest rate rule given by

$$\frac{R_t}{R_t^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi^*} \right)^{r_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{r_y} \right]^{1-\rho_R} \exp(\epsilon_t^r), \quad (\text{D.27})$$

where R^* is the steady state nominal gross interest rate, π^* is steady state inflation rate, and Y_t^* is the natural level of output, i.e. the output level in the flexible price and wage economy.

D.4.6 The aggregate resource constraint

The aggregate resource constraint is

$$c_t + g_t + u_t + a(u_t) \leq Y_t, \quad (\text{D.28})$$

where g_t is government expenditure.

D.4.7 Stochastic Structure

In addition to monetary policy there are eight additional sources of uncertainty.

The law of motion for these shocks are given by:

$$z_t - z_{t-1} = \gamma + \rho_z(z_{t-1} - z_{t-2}) + \varepsilon_{z,t} \quad (\text{D.29})$$

$$\epsilon_t^r = \varepsilon_{r,t} \quad (\text{D.30})$$

$$A_t = \rho_a A_{t-1} + \varepsilon_{a,t} \quad (\text{D.31})$$

$$\phi_t = \rho_\phi \phi_{t-1} + \varepsilon_{\phi,t} \quad (\text{D.32})$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \quad (\text{D.33})$$

$$q_t = \rho_q q_{t-1} + \varepsilon_{q,t} \quad (\text{D.34})$$

$$b_t = \rho_b b_{t-1} + \varepsilon_{b,t} \quad (\text{D.35})$$

$$\lambda_{wt} = \rho_w \lambda_{wt-1} + \varepsilon_{w,t} \quad (\text{D.36})$$

$$\lambda_{ft} = \rho_w \lambda_{ft-1} + \varepsilon_{f,t} \tag{D.37}$$

The innovations follow a standard normal random vector.

Table A-3: Calibrated Parameter Values: New Keynesian Model

Parameter	Value	Source	Parameter	Value	Source
α	0.20	SSK	x_s	0.85	SMM
h	0.66	SSK	ρ_ϕ	0.08	SMM
κ	2.5	SSK	ρ_a	0.99	SMM
γ_u	0.30	SSK	ρ_z	0.05	SMM
θ_p	0.66	SSK	ρ_g	0.90	SMM
θ_w	0.25	SSK	ρ_q	0.38	SMM
λ_p	0.15	SSK	ρ_w	0.10	SMM
λ_w	0.15	SSK	ρ_p	0.10	SMM
ρ_r	0.86	SSK	ρ_b	0.50	SMM
ψ_π	3.05	SSK	$\sigma_a \times 100$	1.71	SMM
ψ_g	0.06	SSK	$\sigma_\phi \times 100$	1.01	SMM
π^*	2.94	SSK	$\sigma_z \times 100$	0.60	SMM
g^*	0.28	SSK	$\sigma_g \times 100$	0.01	SMM
δ	0.025	Depreciation of Capital Stock	$\sigma_q \times 100$	0.82	SMM
γ	1.004	Average GDP growth per capita	σ_w	0.41	SMM
h_s^*	0.36	Weekly Hours per Individual (College)	$\sigma_p \times 100$	0.15	SMM
h_u^*	0.29	Weekly Hours per Individual (no College)	$\sigma_b \times 100$	0.36	SMM
u	0.64	% of Individuals without College	$\sigma_r \times 100$	0.02	SMM
μ_ϕ	0.39	Skill Premium			
\bar{v}	1.00	Fixed			

Table A-4: New Keynesian Model Estimation Targets: Data and Model

Moment	Data	Model	Moment	Data	Model	Moment	Data	Model
St Dev(ΔY_t)	0.75	0.63	Acorr(ΔY_t)	0.48	0.47	Corr($\Delta Y_t, \Delta C_t$)	0.82	0.55
St Dev(ΔC_t)	0.64	0.56	Acorr(ΔC_t)	0.42	0.42	Corr($\Delta Y_t, \Delta I_t$)	0.70	0.36
St Dev(ΔI_t)	2.38	3.25	Acorr(ΔI_t)	0.39	0.36	Corr($\Delta Y_t, \Delta \pi_t$)	-0.14	-0.05
St Dev(π_t)	1.58	1.03	Acorr(π_t)	0.40	0.57	Corr($\Delta Y_t, \Delta H_t$)	0.73	0.54
St Dev(ΔH_t)	1.03	0.75	Acorr(ΔH_t)	0.27	0.53	Corr($\Delta Y_t, \Delta \frac{Y_t}{H_t}$)	-0.02	0.03
St Dev($\Delta \frac{Y_t}{H_t}$)	0.70	0.53	Acorr($\Delta \frac{Y_t}{H_t}$)	-0.11	-0.18	Corr($\Delta Y_t, \Delta W_{s,t}$)	0.30	0.06
St Dev($\Delta W_{s,t}$)	0.83	1.53	Acorr($\Delta W_{s,t}$)	-0.07	0.07	Corr($\Delta Y_t, \Delta W_{u,t}$)	0.05	0.06
St Dev($\Delta W_{u,t}$)	1.25	1.13	Acorr($\Delta W_{u,t}$)	-0.01	-0.23	Corr($\Delta Y_t, R_t$)	-0.18	0.18
St Dev(R_t)	1.24	2.53	Acorr(R_t)	0.96	0.98			

D.5 Data Construction

GDP growth: Log difference of gross domestic product in per capita term (chained dollars). Data are quarterly, 1979:Q1-2012:Q4. The source is *Bureau of Economic Analysis*, National Income and Product Accounts Tables, Table 7.1.

Consumption growth: Log difference of personal consumption expenditures in per capita term (chained dollars). Data are quarterly, 1979:Q1-2012:Q4. The source is *Bureau of Economic Analysis*, National Income and Product Accounts Tables, Table 7.1.

Investment growth: Log difference of nonresidential gross private domestic investment (chained dollars). Data are quarterly, 1979:Q1-2012:Q4. The source is *Bureau of Economic Analysis*, National Income and Product Accounts Tables, Table 1.1.6.

Inflation: Log difference of GDP deflator. Data are quarterly, 1979:Q1-2012:Q4. The series is downloaded from the FRED database of the Federal Reserve Bank of St. Louis (GDPDEF).

Federal Funds Rate: Quarterly averages of monthly effective Federal Funds Rate. Data are quarterly, 1979:Q1-2012:Q4. The series is downloaded from the FRED database of the Federal Reserve Bank of St. Louis (FEDFUNDS).

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