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Viktor Dotsenko

A. Brooks Harris
University of Pennsylvania, harris@sas.upenn.edu

David Sherrington

Robin B. Stinchcombe

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Disciplines
Physics

Replica Symmetry Breaking in the Critical Behaviour of the Random Ferromagnet

Viktor Dotsenko,[1,2] A. B. Harris,[1,3] David Sherrington,[1] and R. B. Stinchcombe[1]

1. Theoretical Physics, Oxford University,
1 Keble Rd., Oxford OX1 3NP, UK

2. Landau Institute for Theoretical Physics,
Russian Academy of Sciences, Moscow, Russia

3. Department of Physics, University of Pennsylvania,
Philadelphia, PA 19104-6396, USA

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Abstract

We study the critical properties of the weakly disordered p-component random Heisenberg ferromagnet. It is shown that if the specific heat critical exponent of the pure system is positive, the traditional renormalization group (RG) flows at dimensions $D = 4-\epsilon$, which are usually considered as describing the disorder-induced universal critical behavior, are *unstable* with respect to replica symmetry breaking (RSB) potentials as found in spin glasses. It is demonstrated that the RG flows involving RSB potentials lead to fixed points which have the structure known as the 1 step RSB, and there exists a whole spectrum of such fixed points. It is argued that spontaneous RSB can occur due to the interactions of the fluctuating fields with the local non-perturbative degrees of freedom coming from the multiple local minima solutions of the mean-field equations. However, it is not clear whether or not RSB occurs for infinitesimally weak disorder. Physical consequences of these conclusions are discussed.

1 INTRODUCTION

It has been many years since the effects produced by weak quenched disorder on the critical phenomena were considered to be qualitatively understood. In the most general terms the traditional point of view could be summarized as follows.

If the disorder is weak (e.g. the concentration of impurities is small), its effect on the critical behavior in the vicinity of the phase transition point T_c remains negligible so long as the correlation length R_c is not too large, i.e. for temperatures T not too close to T_c. In this regime the critical behavior will be essentially the same as in the pure system. However, as $\tau \equiv (T - T_c)/T_c \to 0$ and $R_c(\tau)$ becomes larger than the average distance between impurities, their influence can become crucial.

As T_c is approached the following change of length scale takes place. First, the correlation length of the fluctuations becomes much larger than the lattice spacing (which we take to be unity), and the system "forgets" about the lattice. The only relevant scale that remains in the system in this regime is the correlation length $R_c(\tau)$. Then, in the close vicinity of the critical point, R_c grows and becomes larger than the average distance between the impurities, so that the effective concentration of impurities, measured with respect to the correlation length, becomes large. Such a situation is reached for an arbitrary small concentration u of impurities. The strength of disorder, as scaled by u, affects only the width of the temperature region near T_c in which the effective concentration gets large. If $uR_c^D \gg 1$, where D is the spatial dimensionality, one has no grounds, in general, for believing that the effect of impurities will be small.

A very simple general criterion has been discovered, the so-called Harris criterion [1], which makes it possible to predict the effect of impurities qualitatively from only the critical exponents of the pure system. According to this criterion the impurities change the critical behavior only if α, the specific heat exponent of the pure system, is greater than zero (i. e. the specific heat of the pure system is divergent at the critical point); more properly, this criterion should be stated as $D\nu > 2$, where ν is the correlation length exponent.) According to the traditional point of view, when this criterion is satisfied, a new universal critical behavior, with new critical exponents, is established sufficiently close to the phase transition point [2, 3]. These new exponents were found to satisfy the opposite criterion (i. e. α was negative) and were therefore apparently stable. In contrast, when $\alpha < 0$ (the specific heat is finite), the impurities appear to be irrelevant, i.e. their presence does not affect the critical behavior.

We now consider this point in more detail. Near the phase transition point the D-dimensional Ising-like systems can be described in terms of the scalar field Ginsburg-Landau Hamiltonian with a double-well potential:

$$H = \int d^D x \left[\frac{1}{2} (\nabla \phi(x))^2 + \frac{1}{2} [\tau - \delta\tau(x)] \phi^2(x) + \frac{1}{4} g \phi^4(x) \right] . \qquad (1.1)$$

Here the quenched disorder is described by random fluctuations of the effective transition temperature $\delta\tau(x)$ whose probability distribution is taken to be symmetric and Gaussian:

$$P[\delta\tau] = p_0 \exp\left(-\frac{1}{4u} \int d^D x (\delta\tau(x))^2 \right) , \qquad (1.2)$$

where $u \ll 1$ is the small parameter which describes the disorder, and p_0 is the normalization constant. In Eq. (1.1) $\tau \sim (T - T_c)$ and for notational simplicity, we define the sign of $\delta\tau(x)$ so that positive fluctuations lead to locally ordered regions, whose effects are the object of our study.

Configurations of the fields $\phi(x)$ which correspond to local minima in H satisfy the saddle-point equation:

$$-\Delta\phi(x) + \tau\phi(x) + g\phi^3(x) = \delta\tau(x)\phi(x) . \qquad (1.3)$$

Such localized solutions exist in regions of space where $\tau - \delta\tau(x)$ assumes negative values. Clearly, the solutions of Eq. (1.3) depend on a particular configuration of the function $\delta\tau(x)$ being inhomogeneous. Let us estimate under which conditions the quenched fluctuations of the effective transition temperature are the dominant factor for the local minima field configurations.

Let us consider a large region Ω_L of a linear size $L >> 1$. The spatially average value of the function $\delta\tau(x)$ in this region could be defined as follows:

$$\delta\tau(\Omega_L) = \frac{1}{L^D} \int_{x \in \Omega_L} d^D x \delta\tau(x) . \qquad (1.4)$$

Correspondingly, for the characteristic value of the temperature fluctuations (averaged over realizations) in this region one gets:

$$\delta\tau_L = \sqrt{\overline{\delta\tau^2(\Omega_L)}} = \sqrt{2u} L^{-D/2} . \qquad (1.5)$$

Then, the average value of the order parameter $\phi(\Omega_L)$ in this region can be estimated from the equation:

$$-\tau + g\phi^2 = \delta\tau(\Omega_L) . \qquad (1.6)$$

One can easily see that if the value of τ is sufficiently small, i. e. if

$$\delta\tau(\Omega_L) >> \tau \qquad (1.7)$$

then the solutions of Eq.(1.6) are defined only by the value of the random temperature:

$$\phi(\Omega_L) \simeq \pm \left(\frac{\delta\tau(\Omega_L)}{g} \right)^{1/2} . \qquad (1.8)$$

Now let us estimate up to which sizes of locally ordered regions this may occur. According to Eq.(1.5) the condition $\delta\tau(\Omega_L) >> \tau$ yields:

$$L << \frac{u^{1/D}}{\tau^{2/D}} \ . \tag{1.9}$$

On the other hand, the estimation of the order parameter in terms of the saddle-point equation (1.6) could be correct only at scales much larger than the correlation length $R_c \sim \tau^{-\nu}$. Thus, one has the lower bound for L:

$$L >> \tau^{-\nu} \ . \tag{1.10}$$

Therefore, quenched temperature fluctuations are relevant when

$$\tau^{-\nu} << \frac{u^{1/D}}{\tau^{2/D}} \tag{1.11}$$

or

$$\tau^{2-\nu D} << u \ . \tag{1.12}$$

According to the scaling relations, one has $2 - \nu D = \alpha$. Thus one recovers the Harris criterion: if the specific heat of the pure system is positive, then in the temperature interval,

$$\tau < \tau_* \equiv u^{1/\alpha} \tag{1.13}$$

the disorder becomes relevant. This argument identifies $1/\alpha$ as the cross-over exponent associated with randomness.[4]

The above considerations also demonstrate another very important point: in the disorder dominated region one finds a *macroscopic* number of local minimum solutions of the saddle-point equation (1.3). Indeed, for a given realization of the random function $\delta\tau(x)$ there exists a macroscopic number of spatial "islands" where $\tau - \delta\tau(x)$ is negative (so that the local effective temperature is below T_c), and in each of these "islands" one finds two local minimum configurations of the field: one which is "up", and another which is "down", as indicated in Eq.(1.8). These local minimal energy configurations are separated by finite energy barriers, whose heights become larger as the size of the "islands" are increased.

Now, if one is interested in the critical properties of the system, one has to integrate over all local field configurations up to the scale of the correlation length. This type of calculation is usually performed using a Renormalization Group (RG) scheme, which self-consistently takes into account all the fluctuations of the field on length scales up to R_c.

The point, however, is that the traditional RG approach is only a perturbative theory (albeit a powerful one) in which one treats the deviations of the field around the ground state configuration, and it can not take into account other local minimum configurations which are "beyond barriers". This problem does

not arise in the pure systems, where the solution of the saddle-point equation is unique. However, in a situation like that discussed above, when one gets numerous local minimum configurations separated by finite barriers, the direct application of the traditional RG scheme may be questioned.

In a systematic approach one would like to integrate in an RG way over fluctuations around the local minima configurations. Furthermore, one also has to sum over all these local minima up to the scale of the correlation length. In view of the fact that the local minima configurations are defined by the random quenched function $\delta\tau(x)$ in an essentially non-local way, the possibility to implement successfully such a systematic approach seems rather hopeless.

On the other hand there exists another technique which has been developed specifically for dealing with systems which exhibit numerous local minima states. It is the Parisi Replica Symmetry Breaking (RSB) scheme which has proved to be essential in the mean-field theory of spin-glasses (see e.g. [5]). Recent studies show that in certain cases the RSB approach can also be generalized for situations where one has to deal with fluctuations as well [6],[7], [8].

In this paper we are going to study the critical properties of weakly disordered systems in terms of the RG approach generalized to take into account the possibility of the RSB phenomena. The model we will treat is the ferromagnetic $O(n)$ or Heisenberg model with weak random interactions. The idea is that hopefully, like in spin-glasses, this type of generalized RG scheme self-consistently takes into account relevant degrees of freedom coming from the numerous local minima. In particular, the instability of the traditional Replica Symmetric (RS) fixed points with respect to RSB indicates that the multiplicity of the local minima can be relevant for the critical properties in the fluctuation region.

It will be shown in the next Section that, whenever the disorder appears to be relevant for the critical behavior (in accordance with the Harris criterion), the usual RS fixed points (which used to be considered as providing new universal disorder-induced critical exponents) are unstable with respect to "turning on" an RSB potential. In the presence of such a potential the RG flows actually go to another type of fixed point having a structure known as 1-step RSB. The 1-step RSB structure is described by one parameter x_0 ($0 < x_0 < 1$), which is the coordinate of the RSB step, and this parameter remains arbitrary within the framework of the RG scheme. Therefore, within the framework of the formal RG consideration one finds a whole line of the fixed points (instead of the unique fixed point in the RS subspace), and correspondingly one obtains a whole spectrum of critical exponents.

Formally, the value of the parameter x_0 at the fixed point is defined by the "initial conditions" for the RG equations. This situation is qualitatively

different from the traditional RS one, where the fixed point appears to be universal, and does not depend on the the details of the starting values of the parameters of the Hamiltonian. Here, the existence of such RSB fixed points indicates that the actual values of the critical exponents could be *non-universal*, being dependent on the concrete characteristics of the disorder involved.

In Section 3 we discuss the problem of the "initial conditions" for the RG calculations in more detail. The crucial problem for the present approach is to understand whether RSB is inherent in the random bond model or not. If, for instance, RSB does NOT occur spontaneously for the weakly random bond Heisenberg model, then one remains in the replica symmetric subspace of potentials and the traditional approach[2, 3] is probably valid. In contrast, it might be that disorder is always accompanied by a small amount of RSB. In that case, critical exponents, amplitude ratios and the like, would be determined by the RSB fixed point we find here. Based on general physical arguments we propose a mechanism whereby the multiple local minimum solutions discussed above can provide RSB interactions between the fluctuating fields. It is argued that to sum over all the local degrees of freedom, one has to sum over discrete local minima solutions first, and then one could initiate the RG integrations over the fluctuating fields. The point is that the fluctuating fields themselves are the deviations from the local minimum configurations, and that is why the summation over these quenched discrete degrees of freedom (together with the replica averaging over the quenched disorder) could provide additional non-trivial interactions among the fluctuating fields. According to the arguments presented in this Section, the partition function which describes these interactions is analogous to that of the random energy model (REM) of Derrida.[12] Using the known results[12] for the REM, it then follows that if the interaction, $V_{\mathrm{L-P}}$ between local solutions and perturbative fluctuations exceeds a critical strength, replica symmetry will be broken. Thus, we identify a scenario for obtaining spontaneous RSB from a random spin model. However, it remains an open question whether, when the bond randomness is arbitrarily weak, the interaction $V_{\mathrm{L-P}}$ exceeds the critical strength necessary for RSB. If so, the analogous REM has the 1-step RSB structure and the value of the coordinate of the step x_0 is defined by the concrete statistical characteristics of the disorder involved.

The remaining problems as well as future perspectives are discussed in the Conclusions.

2 THE RENORMALIZATION GROUP AND REPLICA SYMMETRY BREAKING

We consider the p-component ϕ^4 theory with quenched random effective temperature fluctuations, which near the transition point can be described by the usual Ginzburg-Landau Hamiltonian:

$$H[\delta\tau,\phi] = \int d^D x \Bigg[\tfrac{1}{2}\sum_{i=1}^{p} (\nabla\phi_i(x))^2 + \tfrac{1}{2}(\tau - \delta\tau(x))\sum_{i=1}^{p}\phi_i^2(x) + \tfrac{1}{4}g\sum_{i,j=1}^{p}\phi_i^2(x)\phi_j^2(x)\Bigg] , \tag{2.1}$$

where the quenched potential $\delta\tau(x)$ is distributed according to Eq. (1.2). In terms of the standard replica approach one has to calculate the following replica partition function:

$$\begin{aligned} Z_n &= \overline{(\int D\phi_i(x)\exp\{-H[\delta\tau,\phi]\})^n} \equiv \int D\delta\tau P[\delta\tau](\int D\phi_i(x)\exp\{-H[\delta\tau,\phi]\})^n \\ &= \int D\delta\tau(x)\int D\phi_i^a(x)\exp\Bigg[-\tfrac{1}{4u}\int d^D x[\delta\tau(x)]^2 \\ &- \int d^D x[\tfrac{1}{2}\sum_{i=1}^{p}\sum_{a=1}^{n}[\nabla\phi_i^a(x)]^2 + \tfrac{1}{2}[\tau-\delta\tau(x)]\sum_{i=1}^{p}\sum_{a=1}^{n}[\phi_i^a(x)]^2 \\ &+ \tfrac{1}{4}g\sum_{i,j=1}^{p}\sum_{a=1}^{n}[\phi_i^a(x)]^2[\phi_j^a(x)]^2]\Bigg] , \end{aligned} \tag{2.2}$$

where the superscript a labels the replicas. (Here and in what follows all irrelevant pre-exponential factors are omitted.) After Gaussian integration over $\delta\tau(x)$ one gets:

$$\begin{aligned} Z_n = \int D\phi_i^a(x)\exp\Bigg[-\int d^D x\Bigg(&\tfrac{1}{2}\sum_{i=1}^{p}\sum_{a=1}^{n}[\nabla\phi_i^a(x)]^2 + \tfrac{1}{2}\tau\sum_{i=1}^{p}\sum_{a=1}^{n}[\phi_i^a(x)]^2 \\ &+ \tfrac{1}{4}\sum_{i,j=1}^{p}\sum_{a,b=1}^{n} g_{ab}[\phi_i^a(x)]^2[\phi_j^b(x)]^2\Bigg)\Bigg] , \end{aligned} \tag{2.3}$$

where

$$g_{ab} = g\delta_{ab} - u \ . \tag{2.4}$$

To study the critical properties of this system we are going to apply the standard RG procedure developed for dimensions $D = 4 - \epsilon$, where $\epsilon \ll 1$.

Along the lines of the usual rescaling scheme (see e.g. [9]) one gets the following RG equations for the interaction parameters g_{ab}:

$$\frac{dg_{ab}}{d\xi} = \epsilon g_{ab} - \frac{1}{8\pi^2}(4g_{ab}^2 + 4g_{aa}g_{ab} + p\sum_{c=1}^{n} g_{ac}g_{cb}) \ , \tag{2.5}$$

where ξ is the standard rescaling parameter.

If one takes the matrix g_{ab} to be replica symmetric, as in the starting form of Eq. (2.4), then one would recover the usual RG equations for the parameters g and u, and eventually one would obtain the well known results for the fixed points and the critical exponents [2, 3]. Here, however, we are going to study the stability of the RG flows with respect to RSB in the matrix g_{ab}. We leave the question as to how perturbations out of the RS subspace could arise until the next Section, and formally consider the RG eqs.(2.5) assuming that the matrix g_{ab} has a Parisi RSB structure (which also include the RS structure as a special case).

According to the standard technique of the Parisi RSB algebra (see e.g. [5],[10]), in the limit $n \to 0$ the matrix g_{ab} is parametrized in terms of its diagonal element $\tilde{g}$ and the off-diagonal *function* $g(x)$ defined in the interval $0 < x < 1$:

$$g_{ab} \to (\tilde{g}; g(x)) \tag{2.6}$$

The RS situation corresponds to the case $g(x) = const$ independent of x. All the operations with the matrices in this algebra can be performed according to the following simple rules:

$$g_{ab}^k \to (\tilde{g}^k; g^k(x)) \tag{2.7}$$

and

$$(\hat{g}^2)_{ab} \equiv \sum_{c=1}^{n} g_{ac}g_{cb} \to (\tilde{c}; c(x)) \tag{2.8}$$

where

$$\begin{aligned} &\tilde{c} = \tilde{g}^2 - \textstyle\int_0^1 dx g^2(x) \\ &c(x) = 2(\tilde{g} - \textstyle\int_0^1 dy g(y))g(x) - \textstyle\int_0^x dy[g(x) - g(y)]^2 \end{aligned} \tag{2.9}$$

Using the above rules, and redefining (just to simplify formulae): $g_{ab} \to 8\pi^2\epsilon g_{ab}$ and $\xi \to \frac{1}{\epsilon}\xi$, from the eqs.(2.5) one gets:

$$\frac{d}{d\xi}g(x;\xi) = g(x;\xi) - 4g^2(x;\xi) - (4+2p)g(x;\xi)\tilde{g}(\xi) + \\ +2pg(x;\xi)\int_0^1 dy g(y;\xi) + p\int_0^x dy[g(x;\xi) - g(y;\xi)]^2 \tag{2.10}$$

$$\frac{d}{d\xi}\tilde{g}(\xi) = \tilde{g}(\xi) - (8+p)\tilde{g}^2(\xi) + p\int_0^1 dy g^2(y;\xi) \tag{2.11}$$

The usual RS equations are recovered if one takes $g(x;\xi) \equiv g(\xi)$ independent of x.

Then, for the fixed point, $\frac{d}{d\xi}g(x) = 0$, $\frac{d}{d\xi}\tilde{g} = 0$ one obtains the following equations:

$$g(x) - 4g^2(x) - (4+2p)g(x)\tilde{g} + \\ +2pg(x)\int_0^1 dy g(y) + p\int_0^x dy[g(x) - g(y)]^2 = 0 \tag{2.12}$$

$$\tilde{g} - (8+p)\tilde{g}^2 + p\int_0^1 dy g^2(y) = 0 \tag{2.13}$$

It is interesting to note that the structure of Eq. (2.12) is similar to that for the Parisi function $q(x)$ near T_c in the SK model of spin-glasses (SG). Although the cubic terms in the "order parameter" function $g(x)$ are not included here, they could be calculated in the next order of the loop expansion of the RG. The most essential difference from the situation in the SK spin-glasses is the presence of the term $g^2(x)$ in Eq. (2.12) (there is no such term in the SK model). This is the typical term which is known to produce 1step RSB in other SG-like systems [11], and in this case the higher order terms produce no qualitative change in the results.

From the Eqs. (2.12) and (2.13) one can easily find out what should be the structure of the function $g(x)$ at the fixed point. Taking the derivative over x twice, one gets, from Eq. (2.12): $g'(x) = 0$. This means that either the function $g(x)$ is constant (which is the RS situation), or it has the step-like structure.

Let us consider the simplest ansatz, which is the 1step RSB (actually, a bit later we are going to argue, that there are no many-step fixed points). Thus we assume that

$$g(x) = \begin{cases} g_0 & \text{for } 0 \le x < x_0 \\ g_1 & \text{for } x_0 < x \le 1 \end{cases} \tag{2.14}$$

where $0 \le x_0 \le 1$ is the coordinate of the step.

Then, from Eqs. (2.12) and (2.13) one gets three equations for three parameters $\tilde{g}, g_0$ and g_1:

$$
\begin{aligned}
&(4-2px_0)g_0^2 - 2p(1-x_0)g_1g_0 + (4+2p)\tilde{g}g_0 = g_0 \\
&-px_0g_0^2 + (4-2p+px_0)g_1^2 + (4+2p)\tilde{g}g_1 = g_1 \\
&-px_0g_0^2 - p(1-x_0)g_1^2 + (8+p)\tilde{g}^2 = \tilde{g}
\end{aligned}
\tag{2.15}
$$

Note again, that for $g_0 \equiv g_1$ one obtains the usual RS fixed point equations, and the parameter x_0 drops out from the equations.

Equations (2.15) have several solutions. Among them there are two fixed points which are the usual RS ones [2, 3]:

$$
g_0 = g_1 = 0; \quad \tilde{g} = \frac{1}{8+p} \tag{2.16}
$$

and

$$
g_0 = g_1 = -\frac{4-p}{16(p-1)}; \quad \tilde{g} = \frac{p}{16(p-1)} \tag{2.17}
$$

The first of these corresponds to the pure system, and is stable when the disorder is irrelevant according to the Harris criterion. The disorder-induced fixed point (2.17) is usually considered to be the one which describes the new universal critical behaviour in systems with impurities. This fixed point has been shown to be stable (with respect to the RS deviations!) for $p < 4$, which is consistent with the Harris criterion since the specific heat critical exponent associated with this fixed point is negative and that associated with the pure system fixed point is positive. (For $p = 1$ this fixed point involves an expansion in powers of $(\epsilon)^{1/2}$ [3]. This structure is only revealed within a two-loop approximation and was therefore not located in the early work [2].)

However, besides these two RS fixed points there exist the following two non-trivial 1step solutions with $g_0 \neq g_1$:

$$
g_0 = -\frac{4-px_0}{16(px_0-1)}; \quad g_1 = \tilde{g} = \frac{px_0}{16(px_0-1)} \tag{2.18}
$$

and

$$
\begin{aligned}
&g_0 = 0; \quad g_1 = -\tfrac{4-p}{16(p-1)-px_0(8+p)} \\
&\tilde{g} = \tfrac{p(1-x_0)}{16(p-1)-px_0(8+p)}
\end{aligned}
\tag{2.19}
$$

The stability analysis for the above fixed points with respect to general RSB deviations would appear to be technically a rather difficult problem, but one can easily check the stability within the considered 1step RSB subspace. The calculations, although a bit cumbersome, are straightforward. One has

to linearize the "dynamical" equations (2.10) near a given fixed point, and then one has to find the eigenvalues. The fixed point is stable only if all three eigenvalues are negative. We omit this purely technical analysis and report the results only. The pure system fixed point (2.16) appears to stable (in accordance with the Harris criterion) for $p > 4$, when the pure system specific heat critical exponent is negative.

The traditional RS fixed point (2.17) appears to be *always unstable*. The three eigenvalues of the linearized equations are: $\lambda_1 = -1/2, \quad \lambda_2 = -\frac{(4-p)}{8(p-1)}$ and $\lambda_3 = +\frac{(4-p)}{8(p-1)}$. The instability appears just due to the possibility of creating a "step".

The first 1step RSB fixed point (2.18) appears to be stable in the region of parameters: $1 < px_0 < 4$.

Finally, the second 1step RSB fixed point (2.19) is stable for:

$$\begin{aligned} & 1 < p < 4; \\ & 0 < x_0 < x_c(p) \equiv \tfrac{16(p-1)}{p(8+p)} \end{aligned} \tag{2.20}$$

In particular, $x_c(p = 2) = 4/5$ and $x_c(p = 3) = 32/33$ (while $x_c(p = 4) = 1$).

Note, that in addition to the fixed points listed above there exist several other 1step RSB solutions of the eqs.(2.15) which we do not reproduce here because they are always unstable.

Actually the "physical" fixed point is that in Eq. (2.19) (with the stability conditions (2.20)), and not that in the Eq. (2.18), which has to be considered as "unphysical". The point is that according to the arguments which are due to be presented in the next Section, whenever the RSB perturbation comes into play, it always requires (according to its physical origin) that $g_1 < g_0$. On the other hand, simple numerical solution of the evolution Eqs. (2.10) clearly demonstrates that if the initial values of the parameters are bounded such that $g_1 < g_0$, then, whatever the actual starting values of the parameters are, one always ends up at the fixed point (2.19) and not the one (2.18). In the opposite case, $g_1 > g_0$, the RG trajectories always go to the fixed point (2.18), but this situation must be considered as "unphysical".

As for the possibility to have many-step solutions of the fixed point Eqs. (2.12) and (2.13), the generalization of the above-considered approach is straightforward, although technically it is much more cumbersome. On the other hand, the direct numerical observations of the RG trajectories for the case of several steps in the initial function $g(x)$ is still rather simple. Such an analysis convincingly demonstrates that whatever the starting conditions are, one always ends up in the 1step fixed point (2.19), where the value of the parameter x_0 is the coordinate of the "right-most" step in the initial configuration of the function $g(x)$.

It should be stressed here that, within the present pure RG considerations, the value of the parameter x_0 which defines the fixed points, remains arbitrary. In this sense, one can say that in the problem under consideration there exists a whole line of fixed points instead of the unique universal one. Formally, the value of the parameter x_0 which characterizes a given fixed point is defined by the initial conditions of the RG equations, and in this sense one could argue that the critical behavior in systems with such disorder is not universal.

Another painful problem is to elucidate what is going on if the value of the parameter x_0 happens to be in the "instability region" $x_c(p) < x_0 < 1$. Formally, in this case the RG trajectories go to infinity, and it means that one has to take into account the next order RG terms, which hopefully could stabilize the situation, just as they do for the Ising ($p = 1$) case.[3] We leave these problems for future analysis.

Consider now what the consequences of the existence of the above 1step RSB fixed point are for the critical exponents. The RG equations for the mass term of the Hamiltonian in the Eq.(2.3) are:

$$\frac{d}{d\xi}\tau_{ab} = 2\tau_{ab} - \epsilon[2\tau_{ab}g_{ab} + p\delta_{ab}\sum_{c=1}^{n}\tau_{cc}g_{cb}] \tag{2.21}$$

This is a general (one-loop) RG equation for an arbitrary mass matrix τ_{ab}. In our case the initial matrix is diagonal: $\tau_{ab}(\xi = 0) = \tau_0\delta_{ab}$, and it remains diagonal (in the higher orders of the RG as well) whatever RSB takes place in the interaction matrix g_{ab}. This is just a general consequence of the absence of fields which would break the symmetry $\phi \to -\phi$. At the 1step RSB fixed point (2.19) for the rescaling of the (diagonal) mass term one gets:

$$\begin{aligned} \tfrac{d\ln\tau}{d\xi} &= 2 - \epsilon[(2+p)\tilde{g} - p(1-x_0)g_1 - px_0g_0] = \\ &= 2 - \epsilon\tfrac{6p(1-x_0)}{16(p-1)-px_0(p+8)} \equiv \Delta_\tau \end{aligned} \tag{2.22}$$

According to the standard scaling relations for the critical exponent of the correlation length one finds:

$$\nu(x_0) = \frac{1}{\Delta_\tau} = \frac{1}{2} + \frac{1}{2}\epsilon\frac{3p(1-x_0)}{16(p-1) - px_0(p+8)} \tag{2.23}$$

Correspondingly, for the critical exponent of the specific heat: $\alpha = 2-(4-\epsilon)\nu$, one obtains:

$$\alpha(x_0) = -\frac{1}{2}\epsilon\frac{(4-p)(4-px_0)}{16(p-1) - px_0(p+8)} \tag{2.24}$$

Thus, depending on the value of the parameter x_0 one finds a whole *spectrum* of the critical exponents. In particular, the possible values of the specific heat critical exponent appear to be in the following band:

$$-\infty < \alpha(x_0) < -\epsilon\frac{(4-p)}{8(p-1)} \tag{2.25}$$

The upper bound for $\alpha(x_0)$ is achieved in the RS limit $x_0 \to 0$, and it coincides with the usual RS result [2]. On the other hand, as x_0 tends to the "border of stability" $x_c(p)$ of the 1step RSB fixed point, formally the specific heat critical exponent tends to minus infinity.

Note that, as usual, to obtain the leading fluctuation correction to the critical exponent of the correlation functions (usually called η) one has to study the RG fixed points in the next order ($\sim \epsilon^2$) approximation.

3 POSSIBLE SCENARIO FOR SPONTANEOUS RSB

In this Section we will present qualitative arguments showing how RSB perturbations could be spontaneously generated in the random bond model. For simplicity we set $p = 1$. We start by considering the partition function for a fixed configuration of $\delta\tau(x)$:

$$Z[\delta\tau] = \int D\phi(x) \exp\{-H[\phi;\delta\tau]\} \ , \tag{3.1}$$

where

$$H[\phi;\delta\tau] = \int d^D x \left(\frac{1}{2}[\nabla\phi(x)]^2 + \frac{1}{2}[\tau - \delta\tau(x)]\phi(x)^2 + \frac{1}{4}g\phi(x)^4\right) \ . \tag{3.2}$$

for a given realization $\delta\tau(x)$, the saddle-point equation,

$$-\Delta\phi(x) + (\tau - \delta\tau\ (x)\phi(x) + g\phi(x)^3 = 0 \tag{3.3}$$

has many local minima solutions. We denote such a local solution by $\psi^{(i)}(x)$ with $i = 1, 2, \ldots N_0$. If the size L_0 of an "island" where $\delta\tau(x) > 0$ is not too small, then the value of $\psi^{(i)}(x)$ in this "island" should be $\sim \pm\sqrt{\delta\tau(x)/g}$, where $\delta\tau(x)$ should now be interpreted as the value of $\delta\tau$ averaged over the region of size L_0. Such "islands" occur at a certain finite density per unit volume. Thus the number of such local solutions, N_0, is macroscopic: $N_0 = \kappa V$, where V is the volume of the system and κ is a constant. An approximate global extremal solution $\Phi(x)$ is constructed as the union of all these local solutions without

regard for interactions between "islands." Each local solution can occur with either sign, since we are dealing with the disordered phase:

$$\Phi^{(\alpha)}[x;\delta\tau(x)] = \sum_{i=1}^{\kappa V} \sigma_i \psi^{(i)}(x) \ , \tag{3.4}$$

where each $\sigma_i = \pm 1$. Accordingly, the total number of global solutions must be $2^{\kappa V}$. We denote these solutions by $\Phi^{(\alpha)}[x;\delta\tau(x)]$, where $\alpha = 1, 2, ..., K = 2^{\kappa V}$. (In this type of symbol we later write simply $\delta\tau$ for $\delta\tau(x)$.) As we mentioned, it seems unlikely that an integration over fluctuations around $\phi(x) = 0$ will include the contributions from the configurations of $\phi(x)$ which are near a $\Phi(x)$, since $\Phi(x)$ is "beyond a barrier," so to speak. Therefore, it seems appropriate to include separately the contributions from small fluctuations about each of the many $\Phi^{(\alpha)}[x;\delta\tau]$. Thus we have to sum over the K global minimum solutions (non-perturbative degrees of freedom) $\Phi^{(\alpha)}[x;\delta\tau]$ and also to integrate over "smooth" fluctuations $\varphi(x)$ around them:

$$\begin{aligned} Z[\delta\tau] &= \int D\varphi(x) \sum_\alpha^K \exp\left(-H\left[\Phi^{(\alpha)}[x;\delta\tau] + \varphi(x);\delta\tau\right]\right) \\ &= \int D\varphi(x) \exp\left(-H[\varphi;\delta\tau]\right) \times \tilde{Z}[\varphi;\delta\tau] \ , \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} \tilde{Z}[\varphi;\delta\tau] &= \sum_\alpha^K \exp\left(-H[\Phi^{(\alpha)}]\right) \\ &\times \exp\left(-\int d^D x \left[\tfrac{3}{2} g \Phi^{(\alpha)}[x;\delta\tau]^2 \varphi(x)^2 + g\Phi^{(\alpha)}[x,\delta\tau]\varphi(x)^3\right]\right) \ . \end{aligned} \tag{3.6}$$

Next we carry out the appropriate average over quenched disorder. To do this, we need to average the nth ($n \to 0$) power of the partition function. This is accomplished by introducing the replicated partition function, Z_n, as

$$Z_n = \int D\delta\tau \int D\varphi_a \exp\left(-\frac{1}{4u}\int d^D x [\delta\tau(x)]^2 - \sum_{a=1}^n H[\varphi_a;\delta\tau]\right) \times \tilde{Z}_n[\varphi_a;\delta\tau] \ , \tag{3.7}$$

where the subscript a is a replica index and

$$\tilde{Z}_n[\varphi_a;\delta\tau] = \sum_{\alpha_1\ldots\alpha_n}^{K}$$

$$\exp\left(-\sum_a H[\Phi^{(\alpha_a)}] - \int d^D x \sum_a \left[\tfrac{3}{2} g\Phi^{(\alpha_a)}(x)^2\varphi_a(x)^2 + g\Phi^{(\alpha_a)}(x)\varphi_a(x)^3\right]\right), \tag{3.8}$$

where $\Phi(x)$ stands for $\Phi[x,\delta\tau]$.

It is obviously hopeless to try to make a systematic evaluation of this replicated partition function. The global solutions $\Phi^{(\alpha)}$ are complicated implicit functions of $\delta\tau(x)$. These quantities have fluctuations of two different types. In the first instance, they depend on the stochastic variables $\delta\tau(x)$. But even when the $\delta\tau(x)$ are completely fixed, $\Phi^{(\alpha)}(x)$ will depend on α (which labels the possible ways of constructing the global minimum out of the choices for the signs $\{\sigma\}$ of the local minima). A crude way of treating this situation is to regard the local solutions $\psi^{(i)}(x)$ as if they were random variables, even though $\delta\tau(x)$ has been specified. This randomness, which one can see is not all that different from that which exists in a spin glass, is the crucial one which we claim may lead to RSB. Accordingly, we no longer bother to keep track explicitly of the fluctuations in $\Phi^{(\alpha)}(x)$ due to its dependence on $\delta\tau(x)$. Instead, we introduce a distribution function for $\psi(x)$. The simplest and the most natural distribution function is the Gaussian one:

$$P[\{\psi(x)\}] \equiv P[\Phi^{(\alpha)}[x;\delta\tau]] = \prod_i \exp\left(-\tfrac{1}{2\Delta}\int d^D x[\psi^{(i)}(x)^2 - \psi_0(x)^2]^2\right)$$

$$= \exp\left(-\tfrac{1}{2\Delta}\int d^D x[\Phi^{(\alpha)}(x)^2 - \Phi_0(x)^2]^2\right), \tag{3.9}$$

where $\psi_0(x) = \sqrt{\delta\tau(x)/g}$, Δ is a parameter, which in principle should be defined through the original parameters g and u, and $\Phi_0(x)$ is the value of Φ given by Eq. (3.4) when $\psi^{(i)}(x)$ is replaced by ψ_0. The above distribution function exhibits two Gaussian maxima (with a mean square width equal to Δ) around the values $\pm\sqrt{\delta\tau(x)/g}$ in the "islands", over which, on average, $\delta\tau(x) > 0$. The width Δ reflects the distribution in the magnitude of $\psi^{(i)}(x)$ which results from distant "islands" fluctuating between their "up" and "down" states.

Since the distribution (3.9) is symmetric with respect to the signs of the $\psi^{(i)}$, the term $\Phi^{\alpha_a}(x)\varphi_a(x)^3$ in the partition function (3.8) can produce only interactions of the order φ^6, which are irrelevant for the critical properties. Therefore, this term can be safely omitted. Using the equation (3.3) for the energy in a given minimum one easily gets:

$$H[\Phi^{(\alpha)}] = -\frac{1}{4}\int d^D x \Phi^{(\alpha)}(x)^4 \ . \tag{3.10}$$

Then, for the partition function $\tilde{Z}_n$ one obtains:

$$\tilde{Z}_n \simeq \prod_\alpha \left[\int D\Phi^{(\alpha)}(x) \exp\left(-\tfrac{1}{2\Delta} \int d^D x [\Phi^{(\alpha)}(x)^2 - \Phi_0(x)^2]^2 \right) \right]$$

$$\times \sum_{\alpha_1 \ldots \alpha_n}^K \exp\left(\tfrac{1}{4} g \int d^D x \sum_a \Phi^{\alpha_a}(x)^4 - \tfrac{3}{2} g \int d^D x \sum_a \Phi^{(\alpha_a)}(x)^2 \varphi_a(x)^2 \right) . \tag{3.11}$$

The non-trivial RSB effects we are looking for come from the integration in the vicinity of the points $\psi_i(x) = \pm\psi_0(x)$. Assuming that the parameter Δ is small enough, one can redefine $\Phi^\alpha(x)^2 = \Phi_0(x)^2 + z_\alpha(x)$.

Then, for the part of the partition function $\tilde{Z}_n$ which contains the integration over $z_\alpha(x)$ one obtains:

$$\tilde{Z}_n \simeq \prod_\alpha \left[\int D z_\alpha(x) \exp\left(-\tfrac{1}{2\Delta} \int d^D x z_\alpha^2(x) \right) \right]$$

$$\times \sum_{\alpha_1 \ldots \alpha_n}^K \exp\left[\tfrac{1}{4} g \int d^D x \sum_a z_{\alpha_a}^2(x) - \tfrac{1}{2} g \int d^D x \sum_a z_{\alpha_a}(x) [3\varphi_a(x)^2 - \Phi_0(x)^2] \right] . \tag{3.12}$$

At this stage the problem (3.12) seems to be similar to that of the Random Energy Model (REM) [12]. Leaving apart the details of the rigorous consideration, one can obtain the correct solution for this problem in the framework of the following simplified heuristic procedure. Each term in the exponent of the partition function (3.12) is the sum of n values $\{z_{\alpha_a}\}$'s chosen out of the total $K = 2^{\kappa V}$ ones. From the solution of the REM it is seen that the leading contribution to the partition function comes from the configurations in which in the summations $\sum_{a=1}^n z_{\alpha_a}$ one takes n/x_0 different α_a's, repeated x_0 times. Here x_0 is a parameter originally defined in the interval $1 \le x_0 \le n$, which (as usual in the replica formalism) turns into $1 \ge x_0 \ge 0$ in the limit $n \to 0$. This parameter has to be fixed by extremizing the free energy. If such an extremum is achieved for $x_0 = 1$, then one gets the RS solution, otherwise the system appears to be in the 1step RSB state.

According to the above ansatz for the partition function (3.12) one gets:

$$\tilde{Z}_n \simeq (2^{\kappa V})^{\frac{n}{x_0}} \prod_{c=1}^{n/x_0} \Bigg[\int Dz_{\alpha_c}(x) \exp\Bigg(-\tfrac{1}{2\Delta'} \int d^D x z^2_{\alpha_c}(x) \\ -\tfrac{1}{2} g \int d^D x z_{\alpha_c}(x) [3 \textstyle\sum_{b=1}^{x_0} \varphi^2_{cb}(x) - x_0 \Phi_0(x)] \Bigg) \Bigg] \ . \tag{3.13}$$

Here $(2^{\kappa V})^{\frac{n}{x_0}}$ is the combinatoric entropy factor, and $\frac{1}{\Delta'} = \frac{1}{\Delta} - \frac{1}{2} g$.

After simple algebra one obtains:

$$\tilde{Z}_n \simeq \exp\Bigg(\tfrac{9}{8} g^2 \Delta' \int d^D x \textstyle\sum_{c=1}^{n/x_0} \sum_{b,b'=1}^{x_0} \varphi^2_{cb} \varphi^2_{cb'} \\ -\tfrac{3}{4} g^2 \Delta' x_0 \int d^D x \Phi_0 \textstyle\sum_{a=1}^{n} \varphi^2_a + \tfrac{1}{8} g^2 \Delta' n x_0 \int d^D x \Phi_0^2 + \tfrac{n}{x_0} \kappa V \ln 2 \Bigg) \ . \tag{3.14}$$

Coming back to the initial problem of integration over the fluctuations $\varphi(x)$, Eq. (3.7), one finds that the second term in the exponent (3.14) gives an irrelevant shift of the mass term in the Hamiltonian $H[\varphi(x)]$, while the first term in (3.14) provides the RSB structure of the matrix g_{ab} in the interaction term:

$$\frac{1}{4} \sum_{a,b=1}^{n} g_{ab} \varphi^2_a \varphi^2_b \tag{3.15}$$

The matrix g_{ab} appears to have 1step RSB block structure described by the parameters (in notations of the Section 2):

$$\begin{aligned} \tilde{g} &= g - u - \tfrac{9}{2} g^2 \Delta' \\ g_1 &= -u - \tfrac{9}{2} g^2 \Delta' \\ g_0 &= -u \ . \end{aligned} \tag{3.16}$$

Since g, u, and Δ' are all positive by definition, one finds the following restrictions on the values of the interaction parameters: $g_0 < 0$, $g_1 < 0$, $g_1 \leq g_0$ and $\tilde{g} > g_1$.

In the original REM problem [12], after the partition function is calculated, the parameter x_0 is fixed by extremizing the resultant free energy with respect to x_0. Here the parameter x_0 enters into the further problem of integration over fluctuations which can be done in terms of the RG procedure. However, in terms of the RG technique one usually gets the results only for the singular part of the free energy, which is actually small (in τ) compared to the whole

free energy. All that makes the problem of fixing the parameter x_0 rather non-trivial.

It should be stressed, however, that aim of the considerations of this Section was to identify the physical mechanism which may, in principle, give rise to RSB perturbations whose treatment then follows from the usual RG calculations of the critical behaviour. It will be very difficult to make our arguments really precise. It is clear, in view of the somewhat slippery assumptions made above, that the actual contribution to the interactions of the fluctuating fields, coming from the non-perturbative degrees of freedom, could have an even more sophisticated RSB structure, than the simple 1step form obtained above. However, the main point of the present discussion was to demonstrate that, as far as the effective interactions of the fluctuating fields are concerned, the RSB perturbations could exist in the critical region. To what extend such RSB perturbations are relevant for the critical properties, can then be analysed (as we have done) in terms of the traditional RG approach. The results of Section 2 clearly demonstrate that whenever the weak disorder is relevant (i. e. if the specific heat of the pure system is positive), and if spontaneous RSB occurs, then the the critical behavior is modified in a dramatic way.

4 CONCLUSIONS

We may summarize our conclusions concerning the random bond p-component Heisenberg ferromagnet as follows.

1. The traditional fixed points[2, 3] in the weakly random ferromagnet for the case when randomness is relevant are only stable within the space of replica symmetric potentials. Therefore, the corresponding results for the critical exponents and other critical properties, are correct only as long a replica symmetry breaking does NOT occur spontaneously. When randomness is not relevant, our analysis reduces to the standard one.[2]
2. Spontaneous replica symmetry breaking has a dramatic effect on the renormalization group flows and therefore on the critical properties. At first order in ϵ and for p not close to 1 (the Ising limit), the stable fixed point corresponds to one step replica symmetry breaking, in close analogy with the random energy model.[12] Presumably a calculation at higher order in ϵ would lead to predictions for how the correlation functions would reflect a breaking of replica symmetry.
3. At first order in ϵ, there is an instability region near $p = 1$ where we find no stable fixed point in the presence of replica symmetry breaking. This result is in close analogy to the known result in the absence of

replica symmetry breaking, namely that only at two-loop order does one recover a stable fixed point[3] not present in first order.[2]

4. The replica symmetry broken fixed point is characterized by a parameter x_0 whose value is not fixed in the order to which we work here. Accordingly, two main possibilities exist. In the first case, the value of this parameter may be determined in higher order in ϵ, in which case one would have universal exponents, amplitude ratios, etc., as usual for a critical point. In the second case, one would have continuously variable exponents and a corresponding lack of universality. We have no idea what aspects of the randomness, when varied, would lead to variation in the critical exponents.

5. A key question, which remains unanswered, is whether or not in the case of *arbitrarily* weak randomness our model has spontaneous replica symmetry breaking. We have given a scenario by which replica symmetry can be spontaneously broken by interactions, $V_{\rm LP}$, between the perturbative fluctuations (usually treated within a renormalization group context) and fluctuations about local mean field solutions (ignored in previous treatments). In this scenario we relate the partition function due to fluctuations about local mean field solutions to the random energy model.[12] The crucial question, which we can not answer, is whether $V_{\rm LP}$ is large enough that the analogous random energy model is in its replica symmetry broken phase or not. So, it is possible that there is a critical strength in the randomness, below which replica symmetry breaking does not occur.

6. One may mention related work. For instance, the existence of local solutions to the mean field equations reminds one of the Griffith phase, in which field derivatives of the free energy are anomalous even outside the critical region [13]. While the present paper was in preparation we learned about similar RSB instability in the RG flows in the 2D random field XY model [14].

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