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**Abstract**
High temperature series for the magnetic susceptibility, $\chi$, of random anisotropy axis models in the limit of infinite anisotropy are presented, for two choices of the number of spin components, *m*. For *m*=2, we find $T_c$ =1.78 *J* on the simple cubic lattice, and on the face-centered cubic lattice we find $T_c$ =4.29 *J*. There is no divergence of $\chi$ at finite temperature for *m*=3 on either lattice. For the four-dimensional hypercubic lattice, we find finite temperature divergences of $\chi$ for both *m*=2 and *m*=3.

**Disciplines**
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# Long range order in random anisotropy magnets

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High temperature series for the magnetic susceptibility, $\chi$, of random anisotropy axis models in the limit of infinite anisotropy are presented, for two choices of the number of spin components, $m$. For $m = 2$, we find $T_c = 1.78J$ on the simple cubic lattice, and on the face-centered cubic lattice we find $T_c = 4.29J$. There is no divergence of $\chi$ at finite temperature for $m = 3$ on either lattice. For the four-dimensional hypercubic lattice, we find finite temperature divergences of $\chi$ for both $m = 2$ and $m = 3$.

There continues to be a great deal of controversy associated with the term "spin glass." Much of this controversy results from the lack of general agreement on what a spin glass is or is supposed to be. For example, Mukamel and Grinstein[1] and Chudnovsky[2] have argued that the "correct" model of a spin glass is a random anisotropy model, rather than a random exchange model.

Motivated by the results of a recent simulated annealing study,[3] we have calculated high temperature perturbation series for the magnetic susceptibility, $\chi$, of random anisotropy models on square, simple cubic, face-centered cubic, and hypercubic lattices. We will interpret the results of these calculations in the light of various other information, including the work of Pelcovits, Pytte, and Rudnick[4,5] (PPR). Using a spin-wave analysis, PPR found that ferromagnetism is unstable in random anisotropy models when the number of spatial dimensions, $d$, is less than or equal to 4. In contrast, our results indicate that for strong anisotropy the lower critical dimension is 3.

The simplest reasonable model for spin glass behavior is the Edwards–Anderson[6] (EA) Hamiltonian:

$$H_{\rm EA} = -\sum_{\langle ij\rangle} J_{ij} S_i S_j, \tag{1}$$

where $\langle ij\rangle$ is a sum over nearest-neighbor pairs on some lattice, the $J_{ij}$ are independent random variables whose probability distribution has the property that $2(\overline{J_{ij}})^2 < \overline{J_{ij}^2}$, and $S_i = \pm 1$. This model is a useful starting point, but it does not describe all of the behavior which is found in the experimental systems.[7-10]

An alternative model for strongly disordered magnetic systems was proposed by Harris, Plischke, and Zuckermann[11] (HPZ):

$$H_{\rm HPZ} = -J\sum_{\langle ij\rangle}\sum_{\alpha=1}^{m} S_i^\alpha S_j^\alpha - D\sum_i\left(\sum_{\alpha=1}^{m}(\hat n_i^\alpha S_i^\alpha)^2 - 1\right), \tag{2}$$

where $S_i$ is now an $m$-component spin and the $\hat n_i$ are uncorrelated random $m$-component unit vectors. This Hamiltonian may give rise to spin glass behavior under certain conditions, as was made clear by later work.[12-14]

When we go to the strong anisotropy limit, $D/J\to\infty$, each spin is constrained to be parallel to its local anisotropy axis. Equation (2) then reduces to

$$H_\infty = -J\sum_{\langle ij\rangle}(\hat n_i\cdot\hat n_j)S_i S_j \tag{3}$$

in the absence of an external magnetic field. Each $S_i$ is now an Ising variable, which takes on only the values $\pm 1$. This Hamiltonian was solved in the infinite range case by Derrida and Vannimenus,[15] and it is convenient for both computer modeling[16] and high temperature series expansions.[17,18]

If we now take the limit $m\to\infty$ while holding $J^2/m$ fixed, it is easy to show that Eq. (3) reduces to Eq. (1), with a Gaussian probability distribution for the bond strengths. Therefore, rather than "Which is the real spin glass Hamiltonian, $H_{\rm EA}$ or $H_{\rm HPZ}$?," one should ask "What is the correct value of $m$ which describes my experimental system?." Beck has shown[9] that AuFe belongs to the class $m = 3$. Following Ioffe and Feigel'man,[19] we will claim that CuMn belongs to $m = 2$. But there also exist systems, such as $Eu_xSr_{1-x}S$, which are believed to belong to $m = \infty$.

The usual situation for nonrandom three-dimensional magnets is that each value of $m$ constitutes a different universality class. This means that the behavior near the critical point, $T_c$, depends in a very well-defined fashion on the parameter $m$. The behavior at low temperatures is then well described by some kind of a mean field theory. For random anisotropy and spin glass models, however, we have good reasons for suspecting that such a scenario may not work. PPR have given widely accepted (although nonrigorous) arguments which show that a ferromagnetic mean field theory does not provide a good description of the low temperature behavior of random anisotropy magnets in the absence of an external field, when $d\leqslant 4$. This conclusion was later confirmed for $m = 3$ on simple cubic lattices by numerical calculations.[3,13,16]

For the infinite anisotropy Hamiltonian, Eq. (3), it is straightforward to calculate the mean field transition temperature as a function of $m$ and the number of nearest neighbors of each spin, $z$. The ferromagnetic transition temperature is $T_c/J = z/m$, where we have set Boltzmann's constant to 1. The spin glass transition temperature is $T_{sg}/J = \sqrt{z}/m$. These results are obtained from a diagrammatic expansion for the free energy high temperature perturbation series.[18] Thus, we see that, in mean field theory, we will have a phase transition from the paramagnetic phase to a ferromagnetic phase as we lower the temperature, as long as $m < z$. If $m > z$

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the transition from the paramagnetic phase is into the spin glass phase.

We have calculated high temperature series expansions for the free energy, $F$, and magnetic susceptibility, $\chi$, of $H_\infty$ on various lattices, for several values of $m$. Details of the calculations, which are somewhat complicated by the fact that the bonds are not independent random variables, will be reported in a future publication.[20]

From the analysis of the ring diagrams,[18] it should be anticipated that $T_c(m) \approx T_I/m$ would be a good estimate of the critical temperature of $H_\infty(m)$ on some lattice with $m < z$, where $T_I$ is the critical temperature of the standard Ising model on the same lattice. If the PPR analysis[4] were applicable in the strong anisotropy limit, however, this would break down for $d \leqslant 4$. For $d < 4$, it is not unlikely that the phase transition, if any, will be first order, as is claimed for the random field Ising model.[21] Past experience suggests that results which are independent of the details of the lattice structure are probably reliable.

The susceptibility series for $m = 2$ and $m = 3$ on the simple cubic lattice are shown in Table I, and the series for the face-centered cubic lattice are shown in Table II. The extrapolation of the fcc $m = 2$ series is quite well described by a divergence of the form $\exp[A/(T - T_c)^\zeta]$, with $T_c/J = 4.29 \pm 0.01$ and $\zeta = 0.45 \pm 0.03$. The simple cubic $m = 2$ series is somewhat more difficult to extrapolate, because of interference by the antiferromagnetic singularity. After making a transformation to allow for this, we find $T_c/J = 1.78 \pm 0.01$ and $\zeta = 0.69 \pm 0.05$ for the simple cubic lattice. It is encouraging to note that $2T_c(2)/T_I$ is similar for the two lattices, as one would expect: 0.789 for simple cubic and 0.875 for fcc. The Ising model critical temperatures for these lattices were obtained from Ref. 22. Our analysis of the $m = 3$ series on these lattices indicates that $T_c = 0$ in both cases. The behavior of these series coefficients is fairly regular. The major source of uncertainty in our analysis is

TABLE I. High-temperature series coefficients for the magnetic susceptibility, $\chi$, of random anisotropy axis models in the infinite anisotropy limit, Eq. (3), for the simple cubic lattice. The numbers displayed are $c_n$, defined by $\chi = (1/mT)(1 + \Sigma_n c_n [J/mT]^n)$, where $m$ is the number of spin components.

| $n$ \ $m$ | 2 | 3 |
|---|---|---|
| 1 | 6.0 | 6.0 |
| 2 | 30.0 | 30.0 |
| 3 | 144.0 | 139.2 |
| 4 | 666.0 | 618.0 |
| 5 | 3020.0 | 2622.171 428 571 43 |
| 6 | 13 436.0 | 10 751.794 285 714 3 |
| 7 | 58 918.666 666 666 7 | 42 217.536 |
| 8 | 255 460.666 666 667 | 160 460.605 714 286 |
| 9 | 1 095 867.2 | 583 308.554 805 194 |
| 10 | 4 662 697.333 333 33 | 2 027 333.898 745 82 |
| 11 | 19 674 854.186 666 7 | 6 637 797.310 305 46 |
| 12 | 82 500 121.333 333 3 | 20 264 446.693 317 0 |
| 13 | 343 685 731.923 808 | 56 161 109.833 998 2 |
| 14 | 1 424 431 147.907 72 | 130 827 918.366 620 |
| 15 | 5 872 789 753.31103 | 206 252 296.859 672 |

TABLE II. Series coefficients for the face-centered cubic lattice. The notation is the same as in Table I.

| $n$ \ $m$ | 2 | 3 |
|---|---|---|
| 1 | 12.0 | 12.0 |
| 2 | 132.0 | 132.0 |
| 3 | 1392.0 | 1382.4 |
| 4 | 14 292.0 | 13 965.6 |
| 5 | 143 992.0 | 137 048.502 857 143 |
| 6 | 1 430 256.0 | 1 312 032.0 |
| 7 | 14 048 493.333 333 3 | 12 286 661.869 714 3 |
| 8 | 136 736 137.333 333 | 112 746 484.355 265 |
| 9 | 1 320 751 369.066 67 | 1 014 963 605.083 62 |

our lack of knowledge about the nature of the transition, which is difficult to quantify. The fact that the divergence of $\chi$ does not appear to be a power law is consistent with the lack of a magnetization[3] for $T < T_c$ in three-dimensional random axis models.

The $\chi$ series for $m = 2$ and $m = 3$ on the four-dimensional simple hypercubic lattice are given in Table III. For $d = 4$ we fit our results for $\chi$ with a simple power-law divergence, $(T - T_c)^{-\gamma}$. We find $T_c/J = 3.215 \pm 0.005$ and $\gamma = 1.192 \pm 0.008$ for $m = 2$, and $T_c/J = 2.005 \pm 0.005$ and $\gamma = 1.46 \pm 0.04$ for $m = 3$. The values of $mT_c(m)/T_I$ are higher than in $d = 3$: 0.962 for $m = 2$, and 0.900 for $m = 3$, where we use $T_I = 6.6817J$.[23] The assumption of a power-law form is reasonable, at least for large $D/J$, if there is a nonzero magnetization in four dimensions for $T < T_c$.

Our series results for $d = 4$ do not agree with the results of PPR[4,5] for the small $D/J$ limit, since we find $T_c(3) > 0$. Our results for $d = 3$, however, are similar to what they claimed would occur in $d = 4$. PPR predicted a special behavior, related to that of the nonrandom $m = 2$, $d = 2$ ferromagnet, for the $m = 2$ case when $d = 4$, with no divergence of $\chi$ for $m > 2$. Since there are no spin waves in the limit $D/J \to \infty$, it is not simple to relate the results of PPR to our work.

A recent simulated annealing calculation[3] by one of the authors has given solid evidence for the existence of an infinite susceptibility phase for $m = 2$, but not for $m = 3$, on the simple cubic lattice. This is in excellent agreement with our analysis of the $\chi$ series. It is also interesting to compare our results with the best existing Monte Carlo calculations.[16] The Monte Carlo results do not indicate a divergence in $\chi$ for $d < 4$. For $m = 2$ on a square lattice they show a specific heat peak centered at $T = 1.3J$, and for $m = 3$ on a simple cubic lattice they show a peak at $T = 1.4J$, with no indication of long-range order for these cases. Unfortunately, there do not seem to be any published Monte Carlo results for $m = 2$ in $d = 3$.

The work of Bray and Moore[24] has demonstrated that there is no finite temperature phase transition for $d = 2$, for any value of $m \geqslant 2$. Our series analysis agrees with this result, giving no indication of a divergence in $\chi$ for any $m \geqslant 2$ on the square lattice (not shown). Therefore, we conclude that for large $D/J$ the lower critical dimension for the existence of a finite temperature phase transition is 3. If our assumption

TABLE III. Series coefficients for the four-dimensional simple hypercubic lattice. The notation is the same as in Table I.

| n \ m | 2 | 3 |
|---|---|---|
| 1 | 8.0 | 8.0 |
| 2 | 56.0 | 56.0 |
| 3 | 384.0 | 377.6 |
| 4 | 2584.0 | 2494.4 |
| 5 | 17 274.666 666 666 7 | 16 245.028 571 428 6 |
| 6 | 114 613.333 333 333 | 104 768.0 |
| 7 | 757 768.888 888 889 | 670 029.494 857 143 |
| 8 | 4 989 673.777 777 78 | 4 260 084.745 142 85 |
| 9 | 32 783 035.377 777 8 | 26 933 817.114 597 4 |
| 10 | 214 851 732.622 222 | 169 584 501.785 457 |
| 11 | 1 405 984 012.397 04 | 1 063 439 719.414 12 |
| 12 | 9 185 249 515.306 67 | 6 648 582 831.526 47 |
| 13 | 59 942 779 289.8222 | 41 441 798 194.8441 |
| 14 | 390 714 537 058.417 | 257 712 485 935.393 |
| 15 | 2 544 687 649 225.37 | 1 598 946 446 345.96 |

about power-law behavior in $d = 4$ is correct, then the upper critical dimension, for which the transition becomes mean-field-like, is at least 5 for large $D/J$. The comparison between our results and those of PPR would then imply that these critical dimensions change at some intermediate value of $D/J$.

The case $m = 2$ in $d = 3$ deserves further investigation. As we have already pointed out, there do not yet seem to be any Monte Carlo results. It is our expectation that, when these calculations are done, they will show a real phase transition to a $\chi = \infty$ phase. We would not be surprised, however, if this transition turns out to be first order, but with a very small latent heat. Whether the transition is first or second order may depend on the value of $D/J$.

Finally, we discuss the interesting question of which experimental systems might be expected to exhibit the infinite $\chi$ behavior. Obvious candidates are Tb-rich amorphous TbFe alloys[25,26] and TbCo alloys.[27] A more intriguing possibility is CuMn, and the conceptually similar system YGd.[28] The active degree of freedom here is the phase of the spin density wave,[29] which is linearly polarized and couples quadratically to the alloy disorder. This idea has been discussed in some detail by Ioffe and Feigel'man,[19] and we encourage the interested reader to consult their work. The addition of Au or Pt to CuMn[30] destroys the linear polarization of the spin density wave, because of the spin-orbit coupling. This changes the nature of the phase transition, probably by inducing a crossover to $m = 3$ behavior. Similar behavior is seen in stressed and impure Cr.[31]

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