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Series Study of a Spin-Glass Model in Continuous Dimensionality

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Abstract

A high-temperature series expansion for the Edwards and Anderson spin-glass order-parameter susceptibility is computed for Ising spins on hypercubic lattices with nearest-neighbor interactions. The series is analyzed by Padé approximants with Rudnick-Nelson-type corrections to scaling. The results agree with the first-order ε expansion of Harris, Lubensky, and Chen. The critical exponent γ_Q increases monotonically with decreasing dimension, d, for d<6, and apparently tends to infinity at d=4; however, the critical temperature does not appear to go to zero at d=4.

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[14]See, in particular, Eq. (3.17) on p. 81 of Mott and Davis (Ref. 13) for the definition of S_F.

Series Study of a Spin-Glass Model in Continuous Dimensionality*

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A high-temperature series expansion for the Edwards and Anderson spin-glass order-parameter susceptibility is computed for Ising spins on hypercubic lattices with nearest-neighbor interactions. The series is analyzed by Padé approximants with Rudnick-Nelson–type corrections to scaling. The results agree with the first-order ϵ expansion of Harris, Lubensky, and Chen. The critical exponent γ_Q increases monotonically with decreasing dimension, d, for $d<6$, and apparently tends to infinity at $d=4$; however, the critical temperature does not appear to go to zero at $d=4$.

Recently, there has been great theoretical interest in various models for spin-glasses.[1-4] The Hamiltonian is usually of the form

$$\mathcal{H}=\sum_{i,j}J_{ij}\vec{S}_i\cdot\vec{S}_j, \tag{1}$$

where $\{\vec{S}_i\}$ are n-component vectors and the $\{J_{ij}\}$ are randomly distributed over some probability distribution which is assumed to be translationally invariant. Edwards and Anderson[1] (EA) have given a mean-field analysis for a model in which the probability distribution for the $\{J_{ij}\}$ is symmetric, so that $[J_{ij}]_{\rm av}=0$, where $[\;]_{\rm av}$ denotes a configurational average over the probability distribution of the $\{J_{ij}\}$. The state of spin-glass order is characterized by the conditions

$$M(\vec{q})=\left[\lim_{N\to\infty}\frac{1}{N}\left|\sum_{i=1}^{N}\exp(i\vec{q}\cdot\vec{r}_i)\langle\vec{S}_i\rangle\right|\right]_{\rm av}=0 \tag{2}$$

for all $\vec{q}$, and

$$Q^2=\left[\lim_{N\to\infty}\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}Q_{ij}^2\right]_{\rm av}\neq 0, \tag{3}$$

where Q_{ij} is defined by

$$Q_{ij}^2=\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\langle S_i^{\alpha}S_j^{\beta}\rangle^2, \tag{4}$$

where α and β label spin components. The first condition, Eq. (2), indicates that there is no long-range ferromagnetic or antiferromagnetic order, but the second, Eq. (3), implies that the spins are "frozen" into a state with long-range (but nonperiodic) correlations. In this Letter, we report the results of a calculation of the order-parameter susceptibility, χ_Q, given by[3]

$$\chi_Q = \left[\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} (Q_{ij}^2 - Q^2) \right]_{\rm av}. \qquad (5)$$

The mean-field values of the critical exponents α associated with the specific heat and β defined by $Q \sim (T - T_F)^\beta$, where T_F is the freezing temperature, were found by EA to be

$$\alpha = -1, \quad \beta = 1. \qquad (6)$$

Consideration of the model on a Cayley tree enables one to determine the mean-field values of the exponents γ_Q and ν associated with the divergence in χ_Q and the correlation length, respectively, as[3]

$$\gamma_Q = 1, \quad \nu = \tfrac{1}{2}. \qquad (7)$$

Hyperscaling ($2\beta + \gamma_Q = d\nu$) then indicates that these exponents are correct for $d \geq d_c = 6$. For $d < d_c$, Harris *et al.*[3] have used the renormalization group (RG) to derive expansions for these exponents in powers of $\epsilon \equiv 6 - d$. To first order in ϵ, the results for the Ising ($n = 1$) spin-glass are

$$\alpha = -1 - 2\epsilon, \quad \beta = 1 + \tfrac{1}{2}\epsilon, \qquad \gamma_Q = 1 + \epsilon, \quad \nu = \tfrac{1}{2} + \tfrac{5}{12}\epsilon. \qquad (8)$$

One purpose of our work is to check the validity of this approach by comparing these results with those obtained below using high-temperature expansions.

We consider the Hamiltonian

$$\mathcal{H} = J \sum_{\langle ij \rangle} \epsilon_{ij} S_i S_j, \qquad (9)$$

where S_i is an Ising variable (± 1), each ϵ_{ij} assumes the values ± 1 randomly, with equal probability, and $\langle ij \rangle$ denotes a sum over nearest-neighbor pairs on a d-dimensional hypercubic lattice. This model is a particularly convenient choice for developing power-series expansions, as can be seen as follows. Let $w = \tanh^2(J/kT)$. Then

$$[\tanh^n(\epsilon_{ij} J/kT)]_{\rm av} = \begin{cases} w^{n/2}, & \text{if } n \text{ is even;} \\ 0, & \text{if } n \text{ is odd;} \end{cases} \qquad (10)$$

so that we can develop power series for thermodynamic functions by expanding in the variable w. In particular, we report here the results of a study of the order-parameter susceptibility χ_Q, defined in Eq. (5). Details of the calculation and a series for the free energy will be published elsewhere.

The susceptibility series was computed to order w^{10} as a function of lattice dimensionality; and the coefficients are given in Table I. The method of Fisher and Gaunt[5] was used to obtain d-dimensional lattice constants. Correlation functions were computed by standard counting techniques; and configurational averages were taken by retaining only those diagrams in which each bond (i.e., each ϵ_{ij}) occurs an even number of times. The series was analyzed by the method of Dlog Padé Approximants.[6] As shown by Van Dyke and Camp,[7] it is necessary to take corrections to scaling into account in order to obtain

TABLE I. Expansion coefficients defined by $\chi_Q = 1 + \sum_{n,m \geq 1} a_{nm} d^n w^m$.

n	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$	$m=9$	$m=10$
1	2	−2	2	26	−118	−326	4034	16 282	−209 408⅔	−1 523 914
2	0	4	−8	−16	216	104	−7576	−16 368	469 597⅓	2 498 769⅓
3	0	0	8	−24	−64	496	2936	−7032	−319 813⅓	−797 498⅔
4	0	0	0	16	−64	−176	1280	5232	44 426⅔	−265 621⅓
5	0	0	0	0	32	−160	−416	3424	9440	55 242⅔
6	0	0	0	0	0	64	−384	−896	9088	16 384
7	0	0	0	0	0	0	128	−896	−1792	23 552
8	0	0	0	0	0	0	0	256	−2048	−3328
9	0	0	0	0	0	0	0	0	512	−4608
10	0	0	0	0	0	0	0	0	0	1024

satisfactory results for d near d_c. We used the Rudnick-Nelson[8] (RN) form

$$\chi_Q = A[1+(1-B/\Delta_1)(t^{\Delta_1}-1)]^{\theta} t^{-\gamma}, \qquad (11)$$

where $t=(1-w/w_c)$, with $w_c = \tanh(J/kT_F)$, and A and B are nonuniversal constants. A RN-type analysis, modified to take into account the fact that $\eta = O(\epsilon)$ for the spin-glass (SG) model, yields $\Delta_1 = \frac{1}{2}\epsilon$ and $\theta_{SG} = 2(\partial\gamma_Q/\partial\epsilon)_{\epsilon=0} = 2$.

We tested the above result, Eq. (11), against our numerical work as follows. We first constructed the expansion in powers of w for the quantity $\hat{\chi}$ defined by

$$\hat{\chi} = \chi[1+(1-B/\Delta_1)(t^{\Delta_1}-1)]^{-\theta}. \qquad (12)$$

We then fixed $B=0.24$ by requiring $\gamma(d_c)=1$. A Dlog Padé analysis of $\hat{\chi}$ then gave γ_Q and w_c as functions of d (or ϵ) with no free parameters, since they are independent of the normalization constant A. The values of $\gamma_Q(d)$ and $w_c(d)$ obtained by this method are given in Table II. Values obtained neglecting corrections to scaling, i.e., taking $\hat{\chi}=\chi$, are shown for comparison. The shift in the apparent value of $\gamma(d_c)$ produced by the corrections to scaling is approximately linear in the product $B\theta$. Thus the large shift in γ seen in the spin-glass relative to the corresponding shift in the Ising ferromagnet[5,7] (FM) is to be expected, since $\theta_{SG}/\theta_{FM}=6$.

The results in Table II show that the critical point approaches the self-avoiding walk (SAW) limit for large d:

$$w_c - w_{SAW} = (2d-1)^{-1}, \qquad (13)$$

with $w_c > w_{SAW}$ as expected. Our results for γ_Q are in precise agreement with the RG prediction[3] of Eq. (8). Our analysis also indicates that γ_Q diverges to infinity at $d=4$. Scaling relations imply that other exponents must diverge simultaneously. This result lends support to the prediction[9] that the SG-FM-PM (paramagnet) phase diagram undergoes qualitative changes at $d=4$, because $\alpha_{FM}>0$, for $2<d<4$.

Below $d=4$, our analysis of the susceptibility series shows no indication of singularities on the positive real w axis. Instead, the physical singularity above $d>4$ splits into a complex-conjugate pair whose approximate locations are given by

$$w_c \approx 0.4 \pm 20\,(4-d)i, \qquad (14)$$

for d just below 4. This indicates that while there may very well be a phase transition at finite temperature for $d<4$, the EA order parameter is probably not relevant.[10]

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TABLE II. Padé analysis of $(\partial/\partial\omega)\ln\chi_Q(\omega)$.

d	Without RN[a] w_c	γ_Q	With RN[b] w_c	γ_Q
6.00	0.1023	1.32	0.1019	1.00
5.75	0.1091	1.40	0.1089	1.25
5.50	0.1171	1.52	0.1172	1.53
5.25	0.1271	1.69	0.1273	1.84
5.00	0.1400	1.95	0.1400	2.23
4.75	0.156	2.4	0.156	2.7
4.50	0.179	3.4	0.180	3.7
4.25	0.221	7	0.221	7.1
4.00	0.38	∞	0.39	∞

[a]I.e. assuming no correction to scaling, $\chi_Q \sim t^{-\gamma}$.

[b]Using Eq. (11) with $B=0.24$, $\theta=2$, and $\Delta_1 = \min(\frac{1}{2}\epsilon, \frac{1}{2})$.

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