3-1-1970

Spin-Wave Damping and Hydrodynamics in the Heisenberg Antiferromagnet

A. Brooks Harris
University of Pennsylvania, harris@sas.upenn.edu

Deepak Kumar
University of Pennsylvania

Bertrand I. Halperin

Pierre C. Hohenberg

Recommended Citation

Harris, A., Kumar, D., Halperin, B. I., & Hohenberg, P. C. (1970). Spin-Wave Damping and Hydrodynamics in the Heisenberg Antiferromagnet. *Journal of Applied Physics, 41* (3), 1367-1362. http://dx.doi.org/10.1063/1.1658943

Spin-Wave Damping and Hydrodynamics in the Heisenberg Antiferromagnet

Abstract

The Dyson-Maleev formalism is used to calculate the decay rate of antiferromagnetic spin waves at low temperatures and long wavelengths. Various regimes must be distinguished depending on the relation between the wavevector k, the temperature T, and the anisotropy energy. For the isotropic system the relevant parameters are (a) the incident energy, (b) the thermal energy, (c) the deviation from linearity ("curvature energy") of thermal spin waves, and (d) the curvature energy of the incident spin wave. In the anisotropic case the damping of the k=0 mode has the same dependence on spin-wave energy as in the isotropic system. In all cases, the decay rate is small compared to the frequency, which implies that the spin waves are appropriate elementary excitations for small k and T, and that they interact weakly among themselves in this limit. For $k \to 0$ with T fixed, the decay rate is proportional to k^2 in the isotropic system. This agrees with an earlier hydrodynamic prediction and contradicts previous microscopic calculations. In this low-k limit the full spin-spin correlation function is calculated, and it agrees with the hydrodynamic form proposed earlier. The possibility of experimental verification of these predictions is briefly discussed.

Disciplines

Physics

JOURNAL OF APPLIED PHYSICS VOLUME 41, NUMBER 3 1 MARCH 1970

Spin Dynamics

L. R. WALKER, *Chairman*

Spin-Wave Damping and Hydrodynamics in the Heisenberg Antiferromagnet*

A. B. HARRIS AND D. KUMAR
University of Pennsylvania, Philadelphia, Pennsylvania 19104
AND
B. I. HALPERIN AND P. C. HOHENBERG
Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey 07974

The Dyson–Maleev formalism is used to calculate the decay rate of antiferromagnetic spin waves at low temperatures and long wavelengths. Various regimes must be distinguished depending on the relation between the wavevector k, the temperature T, and the anisotropy energy. For the isotropic system the relevant parameters are (a) the incident energy, (b) the thermal energy, (c) the deviation from linearity ("curvature energy") of thermal spin waves, and (d) the curvature energy of the incident spin wave. In the anisotropic case the damping of the $k=0$ mode has the same dependence on spin-wave energy as in the isotropic system. In all cases, the decay rate is small compared to the frequency, which implies that the spin waves are appropriate elementary excitations for small k and T, and that they interact weakly among themselves in this limit. For $k\to 0$ with T fixed, the decay rate is proportional to k^2 in the isotropic system. This agrees with an earlier hydrodynamic prediction and contradicts previous microscopic calculations. In this low-k limit the full spin–spin correlation function is calculated, and it agrees with the hydrodynamic form proposed earlier. The possibility of experimental verification of these predictions is briefly discussed.

Using the Dyson–Maleev representation for the spin Hamiltonian of an antiferromagnetic insulator,[1,2] we have calculated the decay rate of a long-wavelength spin wave, and the dynamic spin-correlation function, to lowest order in the temperature. The results obtained disagree with previous microscopic calculations,[3–5] but they agree in detail with the predictions of a recent "hydrodynamic" analysis,[6] when the long wavelength limit is taken at fixed temperature. Calculations have been carried out in both quantum-mechanical and classical low-temperature regimes, for isotropic and anisotropic models. In this note we shall summarize the principal results; the detailed calculations will be published elsewhere.[7]

I. LOWEST-ORDER CALCULATION: QUANTUM CASE

To lowest order in the spin-wave interaction the decay of a spin wave is due to scattering from thermally excited spin waves. The conservation laws of energy and momentum define a scattering surface which depends sensitively on the relations among the energy of the incoming spin wave[8] $H_E\epsilon_{\mathbf{k}}$, the thermal energy $k_BT\equiv\frac{1}{2}H_E\tau$, the anisotropy energy H_A, and the "curvature energy" for both incoming and thermal spin waves. The curvature energy represents the deviation from linearity of the spin-wave excitation spectrum, and is of order $H_E\epsilon_{\mathbf{k}}^3$ and $H_E\tau^3$, for the incoming and thermal spin waves, respectively. A number of different regimes must be distinguished for the decay rate $\Gamma_{\mathbf{k}}$, depending on the relative magnitudes of the various energies.

In the quantum-mechanical low-temperature regime ($k_BT\ll H_E\equiv\hbar\omega_E=2JzS$, i.e., $\tau\ll 1$) we obtain the following results for the isotropic model[8] on a bcc lattice:

Regime A: $\epsilon_{\mathbf{k}}\ll\tau^3\ll 1$,

$$\Gamma_{\mathbf{k}}=2\omega_E S^{-2}\epsilon_{\mathbf{k}}^2\tau^3(2\pi)^{-3}[a\,|\ln\tau|+a']; \qquad (1)$$

Regime B: $\tau^3\ll\epsilon_{\mathbf{k}}\ll\tau\ll 1$,

$$\Gamma_{\mathbf{k}}=(8/3)\omega_E S^{-2}\epsilon_{\mathbf{k}}^2\tau^3(2\pi)^{-3}[b\ln(\tau/2\epsilon_{\mathbf{k}})+b']; \qquad (2)$$

Regime C: $\tau\ll\epsilon_{\mathbf{k}}\ll\tau^{1/3}\ll 1$,

$$\Gamma_{\mathbf{k}}=(\pi/108)\omega_E S^{-2}\epsilon_{\mathbf{k}}\tau^4; \qquad (3)$$

Regime D: $\tau^{1/3}\ll\epsilon_{\mathbf{k}}\ll 1$,

$$\Gamma_{\mathbf{k}}=\tfrac{1}{2}\omega_E S^{-2}\tau^5\zeta(5)(2\pi)^{-3}[g(k)\epsilon_{\mathbf{k}}^2]^{-1}, \qquad (4)$$

where a, a', b, and b' are constants of order unity, and $g(\hat{k})$ is a numerical function of the angle $\hat{k}$; the values of these constants are given in Ref. 7.

For the model with axial single-ion anisotropy[2] $H_A\ll H_E$, the decay rate of a $k=0$ spin wave may be expressed in terms of its energy $H_E\epsilon_0=(2H_AH_E+H_A^2)^{1/2}$. The results are then analogous to those for the isotropic model.

Regime A′: $\epsilon_0\ll\tau^3\ll 1$,

$$\Gamma_0=\tfrac{3}{2}\omega_E S^{-2}\epsilon_0^2\tau^3(2\pi)^{-3}[a\,|\ln\tau|+a'-(8/9)\pi^2\ln 2]; \qquad (5)$$

Regime B′: $\tau^3 \ll \epsilon_0 \ll \tau \ll 1$,

$$\Gamma_0 = 2\omega_E S^{-2}\epsilon_0^2\tau^3(2\pi)^{-3} \times[b\ln(\tau/2\epsilon_0)+b'-(4/3)\zeta(2)]; \quad (6)$$

Regime C′: $\tau \ll \epsilon_0 \ll 1$,

$$\Gamma_0 = 64\omega_E S^{-2}(2\pi)^{-3} \times[\exp(-H_E\epsilon_0/k_BT)](k_BT/H_E)^2\epsilon_0^3. \quad (7)$$

II. SELF-CONSISTENCY AND HIGHER-ORDER TERMS

The foregoing results may be shown to be unmodified to leading order in the temperature when the damping of the intermediate spin waves is taken into account in a self-consistent manner. The calculations may also be generalized to the case of an incident spin wave whose energy and momentum are not related by the resonance condition $\hbar\omega = H_E\epsilon_{\mathbf{k}}$. An analysis of the terms left out of the lowest-order calculation shows these to be of higher order in either (k_BT/H_E) or $(zS)^{-1}$, for long wavelengths. In particular it may be argued, although not proved rigorously, that terms of relative order $(k_BT/H_E\epsilon_{\mathbf{k}})$, for instance, do not appear in the perturbation series. We may thus conclude that the lowest-order diagrams already yield the correct long-wavelength (hydrodynamic) behavior for the spin-wave damping, in contrast to the case of phonons in a crystal.[9]

III. THE CLASSICAL LOW-TEMPERATURE DOMAIN

The classical limit is obtained by taking $S\rightarrow\infty$, $\hbar\rightarrow 0$, $J\rightarrow 0$ with $\hbar S \equiv \frac{1}{2}N_0$, and $2zJS^2 \equiv \frac{1}{2}N_0\omega_E \equiv k_BT_0$ remaining finite. In this case the decay rate may again be calculated, to lowest order in k and (T/T_0), yielding

$$\Gamma_{\mathbf{k}} = (4\eta/3\pi)\omega_E(T/T_0)^2\epsilon_{\mathbf{k}}^2, \quad (8)$$

where η is a numerical constant depending on the form of the magnetic lattice. The corrections to this formula are of relative order $\epsilon_{\mathbf{k}}$ and (T/T_0).

IV. SPIN-CORRELATION FUNCTIONS

The Dyson–Maleev representation for spin operators may be used to calculate the dynamic spin-correlation functions at long wavelengths. Denoting by C_Q^{+-} and C_S^{+-} the correlation functions transverse to the direction of antiferromagnetic alignment, for staggered and total spins, respectively, we have for the isotropic model to lowest order in the temperature,

$$C_Q^{+-}(\mathbf{k},\omega) = (32k_BTS\hbar/ck^2) \times \frac{D'c^2k^4+\frac{3}{4}D'k^2(\omega^2-c^2k^2)}{[(\omega-ck)^2+(\frac{1}{2}D'k^2)^2][(\omega+ck)^2+(\frac{1}{2}D'k^2)^2]}, \quad (9)$$

$$C_S^{+-}(\mathbf{k},\omega) = (2k_BTS\hbar/c) \times \frac{D'c^2k^4+\frac{1}{4}D'k^2(\omega^2-c^2k^2)}{[(\omega-ck)^2+(\frac{1}{2}D'k^2)^2][(\omega+ck)^2+(\frac{1}{2}D'k^2)^2]}, \quad (10)$$

where $c=\frac{1}{2}\omega_E$ is the spin-wave velocity in frequency units, k is a dimensionless wavenumber, and D' is the temperature-dependent damping constant for spin waves, obtainable from Eqs. (1) and (8), for quantum and classisal cases, respectively, (note that $\epsilon_k \sim \frac{1}{2}k$). The form of Eqs. (9) and (10) is identical to the one predicted earlier from the hydrodynamic theory [see Eq. (6.11) of Ref. 6], and a comparison of the expressions yields the transport coefficients of the macroscopic theory.

V. CONCLUSION

In the isotropic model, the decay rate vanishes more rapidly than the spin-wave frequency as the wavenumber k goes to zero. This confirms that the antiferromagnetic spin waves interact weakly at long wavelengths. This result disagrees with conclusions drawn from previous calculations,[3,4] but is closely analogous to the situation in ferromagnets.[10]

Since the predicted decay rate is very small at low temperatures and long wavelengths, it cannot be easily observed by neutron scattering experiments. A method which seems more hopeful is the parallel pumping technique,[11] which has higher resolution. The damping of the uniform mode in the anisotropic model may be measured by antiferromagnetic resonance,[12] and the experimental results agree qualitatively with theory. One difficulty, however, is that our theoretical model for the anisotropy may not be a very accurate one.

Finally, we note that the calculations reported here have also been carried out[7] using the Holstein–Primakoff[13] formalism. The results for observable quantities agree in the two formalisms, except in the domain $\epsilon_{\mathbf{k}} \ll \tau^5$, where the Holstein–Primakoff modes are not self-consistent in lowest order.

* The portion of the work done at the University of Pennsylvania was partially supported by the U.S. Office of Naval Research and the National Science Foundation.

[1] A. B. Harris, Phys. Rev. Lett. **21**, 602 (1968).
[2] A. B. Harris, Phys. Rev. **183**, 486 (1969), and references therein.
[3] M. I. Kaganov, V. M. Tsukernik, and I. Y. Chupis, Fiz. Metal. Metaloved. **10**, 797 (1960) [Phys. Metal. Metallog. **10**, 154 (1960)].
[4] K. Tani, Progr. Theoret. Phys. (Kyoto) **30**, 580 (1963); **31**, 335 (1964).
[5] J. Solyom, Zh. Eksp. Teor. Fiz. **55**, 2355 (1968) [Sov. Phys.—JETP **28**, 1251 (1969)].
[6] B. I. Halperin and P. C. Hohenberg, Phys. Rev. (to be published).
[7] A. B. Harris, D. Kumar, B. I. Halperin, and P. C. Hohenberg (to be published).
[8] Unless otherwise stated, the notation is the same as in Refs. 1 and 2.
[9] See, for instance, L. J. Sham, Phys. Rev. **156**, 494 (1967).
[10] F. J. Dyson, Phys. Rev. **102**, 1217, 1230 (1956).
[11] L. W. Hinderks and P. M. Richards, Phys. Rev. **183**, 575 (1969).
[12] F. M. Johnson and A. H. Nethercot, Jr., Phys. Rev. **114**, 705 (1959).
[13] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).