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Active Color Image Analysis for Recognizing Shadows

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Abstract

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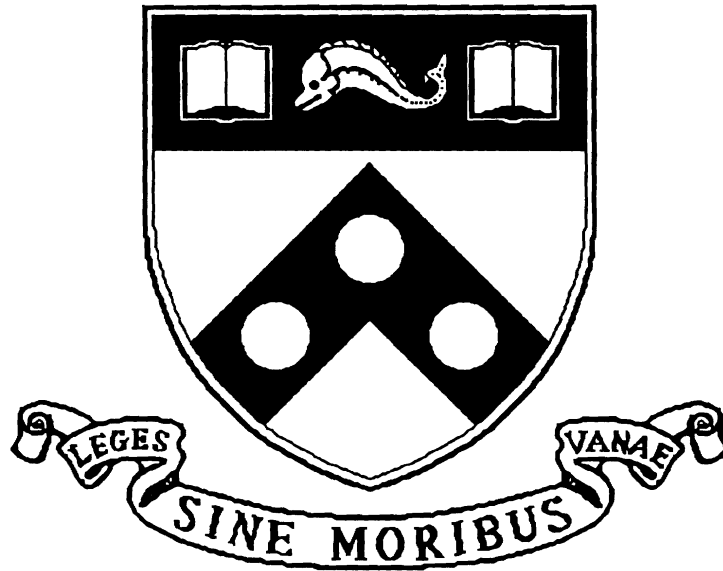
Comments

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Active Color Image Analysis for Recognizing Shadows

MS-CIS-92-82
GRASP LAB 336

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November 1992

Active Color Image Analysis for Recognizing Shadows¹

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Abstract

Many existing computer vision modules assume that shadows in an image have been accounted for prior to their application. In spite of this, relatively little work has been done on recognizing shadows or on recognizing a single surface material when directly lit and in shadow. This is in part because shadows cannot be infallibly recognized until a scene's lighting and geometry are known. However, color is a strong cue to the presence of shadows. We present a general color image segmentation algorithm whose output is amenable to the recovery of shadows as determined by an analysis of the physics of shadow radiance. Then, we show how an observer that can cast its own shadows can infer enough information about a scene's illumination to refine the segmentation results to determine where the shadows in the scene are with reasonable confidence. Having an observer that can actively cast shadows frees us from restrictive assumptions about the scene illumination or the reliance on high level scene knowledge. We present results of our methods on images of complex indoor and outdoor scenes.

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1 Shadows and Image Understanding

Many existing computer vision modules assume that shadows in an image have been accounted for prior to their application. For instance, object recognition systems assume that one can recover intrinsic properties of an object despite irradiance changes such as shadows. Similarly, shape from shading algorithms assume that surface radiance does not include shadow effects. In addition, it has long been recognized that identifying shadows in an image constrains the geometric interpretation of the pictured scene [14], [11]. However, despite the acknowledged importance of identify shadows for image understanding, attempts to date have been overly simplistic. In aerial image interpretation, for instance, shadows are considered to be those parts of an image whose values are below a threshold and are adjacent to a rectangular shape (a building) on the side away from the direction of illumination [9], [3], [4], [8].

In this work, we present a color image segmentation algorithm that recovers image regions amenable to the recovery of shadows. In addition, we sketch how this technique fits in with a larger system to enable an active observer to recognize shadows in a images of a scene.

We begin with a physical model for color image formation for a single surface material directly lit and in shadow. We discuss the limitations of previous color image work for shadows and then present the assumptions we need to make and the techniques we have for addressing the problem. These includes the use of an observer that can actively cast a shadow into the scene in order to examine the illumination conditions. Finally, we present results and a discussion of our work.

2 Spectral Model of Shadows

2.1 Shadows Without Other Reflectance Effects

Let $D(\lambda, p)$ be the amount of energy emitted at each wavelength λ by a source of illumination as measured at the back-projection of pixel p onto a given surface. We will refer to D as the direct light source. In addition to the light $D(\lambda, p)$, the light $A'(\lambda, p)$ that has been reflected or scattered in the environment, and any other direct sources of illumination $L_1(\lambda, p), \dots, L_n(\lambda, p)$ also strike the surface. Therefore, the total illumination striking the surface is

$$D(\lambda, p) + A'(\lambda, p) + \sum_{i=1}^n L_i(\lambda, p).$$

Assume now that an object is brought between the light source D and the surface. The reflected light in the scene changes due to reflections off the obstructing object, call it now $A(\lambda, p)$. So, the illumination striking the surface is now

$$\alpha D(\lambda, p) + A(\lambda, p) + \sum_{i=1}^n L_i(\lambda, p)$$

where $\alpha \in [0 \dots 1]$ indicates that the light source D will be only partially obstructed at some locations on the surface if D is not a point light source. The partial obstruction of an extended light source results in the penumbra of a shadow (sometimes referred to as the “soft edge” of a shadow).

Let $S(\lambda, p)$ be the surface reflectance (albedo). For the moment, we assume that $S(\lambda, p)$ is independent of the direction of illumination for this scene and hence that there are no specularities (high-lights) nor shading across the surface. Let $Q_j(\lambda)$ be the weighting function of the observer’s camera system for the j th filter ($j \in [1, \dots, m]$). Then, the light measured by the camera from the surface directly lit and in shadow for one filter is

$$I_j(p) = \int_{\Lambda} \left(\alpha D(\lambda, p) + A(\lambda, p) + \sum_{i=1}^n L_i(\lambda, p) \right) S(\lambda, p) Q_j(\lambda) d\lambda. \quad (1)$$

where Λ is the range in which $Q_j(\lambda)$ is non-zero.

We will use the following notation:

$$\mathbf{D}(p) = \begin{bmatrix} \int_{\Lambda} D(\lambda, p) S(\lambda, p) Q_1(\lambda) d\lambda \\ \vdots \\ \int_{\Lambda} D(\lambda, p) S(\lambda, p) Q_m(\lambda) d\lambda \end{bmatrix}$$

$$\mathbf{E}(p) = \begin{bmatrix} \int_{\Lambda} (A(\lambda, p) + \sum_{i=1}^n L_i(\lambda, p)) S(\lambda, p) Q_1(\lambda) d\lambda \\ \vdots \\ \int_{\Lambda} (A(\lambda, p) + \sum_{i=1}^n L_i(\lambda, p)) S(\lambda, p) Q_m(\lambda) d\lambda \end{bmatrix}$$

where $\mathbf{D}(p)$ and $\mathbf{E}(p)$ are m element vectors. From the above equation it follows that the image of the surface directly lit and in shadow is

$$\mathbf{I}(p) = \alpha \mathbf{D}(p) + \mathbf{E}(p). \quad (2)$$

The m color filters span a sub-space of color space and Equation 2 is the parametric form of a line in this color sub-space with parameter α . The end-point of the line at $\alpha = 0$ corresponds to the umbra of the shadow. The end-point where $\alpha = 1$ corresponds to the surface directly lit. The open interval of the line (where $0 < \alpha < 1$) corresponds to the penumbra of the shadow.

The images we work with were all taken with three color filters corresponding to red, green, and blue. When we use the terms *Red*, *Green*, and *Blue*, we will mean the image value at a pixel taken with the corresponding filter. Some images were converted to the form

$$(S_0, S_1, S_2) = (\text{brightness}, \sin \lambda, \cos \lambda)$$

where λ ranges over the visible wavelengths [6]. This space is convenient

for describing hue and color saturation. Hue is defined as $\tan^{-1}(\frac{S_2}{S_1})$. Color saturation is defined as $\frac{\sqrt{S_1^2+S_2^2}}{S_0}$. Equation 2 holds in both color spaces.

2.2 Shading, Inter-Reflections, and Specularities

Shading, or variations in the amount of light striking a surface due to a change in geometry, can complicate the model we have described above. For a scene with a single light source, no inter-reflections, and a uniformly colored Lambertian surface which receives varying amounts of illumination due to surface curvature or varying distance from the the light source, the reflection from the surface will describe a linear cluster in color space [12], [6]. This linear cluster will lie along the linear cluster resulting from a shadow penumbra under the same illumination conditions.

However, shading can effect the term $\mathbf{E}(p)$ in Equation 2. In the case where a uniformly colored Lambertian surface is illuminated by light in varying amounts from two distinctly colored lights, the reflection from the surface will describe a planar cluster in color space. If the surface is illuminated by light in varying amounts from multiple, distinctly colored lights, the reflection from the surface will describe a volume in color space [6]. For multiple, differently colored light sources, the reflection distortion in color space due to a shadow being cast on a surface will be super-imposed on the volumetric cluster due to shading.

The light reflected from one surface onto a second surface serves as a source of irradiance for the second surface. As such, inter-reflections complicate our shadow model in the same way that multiple light source do.

If a surface is not perfectly Lambertian, then the image of that surface may include specularities. Specularities from the obstructed light source cannot occur in the umbra of a shadow. Specularities from the obstructed light source

that fall within the penumbra are visible but their shape will be truncated at the boundary of the umbra. Diffuse specularities due to a rough, microfaceted surface [13] can, however, become dimmer in a penumbra because the specular irradiance at a point on the image can be due to microfacets in the umbra as well as in the penumbra of a shadow.

3 Assumptions for Shadow Recognition

Many researchers assume that the light illuminating a shadow and the direct, obstructed light are spectrally proportional. Hence that $\mathbf{E}(p) = \beta\mathbf{D}(p)$ [10]. For this case, the color cluster for a single material directly lit and in shadow is a linear cluster that would pass through the origin of the color space if the line were extended. Gershon et al. [2] refer to shadows whose illumination satisfies this conditions as *ideal shadows*. They also considered the somewhat more general case of $\mathbf{E}(p) = \beta\mathbf{D}(p) + \mathbf{K}(p)$ where $\mathbf{K}(p) = [k_1, \dots, k_m]^T$ is constant. This case is referred to as that of *non-ideal shadows*.

Rather than require a strong, global constraint on the scene illumination as in [10] and [2], we prefer to use a weaker, local constraint during our initial color image segmentation. What we want to insure is that we can segment an image into regions such that if shadows are present, a uniformly colored surface directly lit and in shadow will be represented by a single region when the scene is lit by single compact light source or a cross section of the penumbra of a shadow will be represented as a single region when multiple compact light sources are present. Only those segmented regions with a linear model will be shadow candidate regions. The shadow candidate regions will be further analyzed for evidence to support or refute the hypothesis that a shadow is present. In general this will require bringing both geometric and spectral information to bear on the problem. However, here we concentrate on spectral

information.

In order to meet our goal for image segmentation, we will make the following assumption:

The Linear Color Cluster Assumption for Penumbrae:

We assume that the light irradiating a penumbra, with the exception of the the partially obstructed light, does not vary or varies insignificantly.

Consequently, the variation in reflection in a penumbra on a uniformly colored surface is due entirely to the obstruction of a direct light source. And, in order to find regions of an image that could represent the same surface lit and in shadow across a penumbra we present a scheme in which an image is segmented into line-like or uniform color clusters.

However under our assumption, line-like color clusters can still originate from physical phenomenon other than shadows. Shading, inter-reflections, highlights, or material changes may also produce line-like color clusters [5], [6]. We will examine later how to distinguish the case of shadows.

The analysis discussed so far was done strictly in color space and ignores image or scene locality. Because all the pixels in an image of a complicated scene taken together may result in many line-like color clusters, we introduce local image continuity as a constraint in our color image segmentation. So, we will only be looking for contiguous sets of pixels in an image that form line-like color clusters. This restricts our image interpretation to those shadows for which the same surface can be seen directly lit and adjacently in shadow. Note also that the boundary of the shadow must include a penumbra for our system to work. This means that we cannot handle scenes with point light sources. Fortunately, true point light sources are rare in non-laboratory environments.

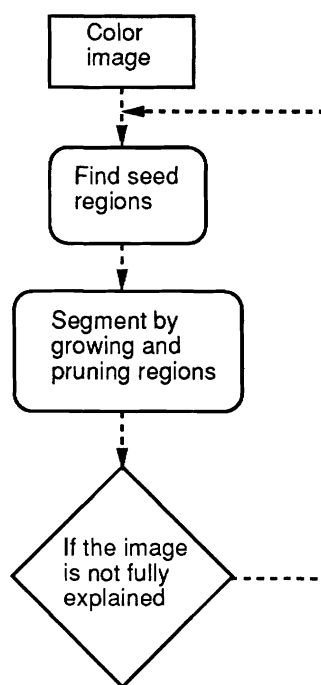


Figure 1: Color Image Segmentation Outline

4 Color Image Segmentation

Our color image segmentation is founded on three ideas. First, to use line-like color models in color space which are our shadow candidate regions. Second, to dove-tail the processing between color-space and image-space in order to take into account aspects of each. And finally, the realization that segmentation should be the search for the best description of an image in terms of primitive models [7]. An outline of the control structure of the algorithm is given in Figure 1.

The bulk of the work is accomplished through a segmentation by region growing and pruning algorithm. This algorithm is based on [7] and is outlined below:

WHILE change **DO LOOP**

Grow each region based on the region model.

Update each region to fit the new and old data.

Consider a higher order model.

Prune away regions based on overlap, size, goodness of fit and model order.

END LOOP

Unlike [7] our region models are not bivariate polynomials functions of pixel location. Instead, our region models are uniform or linear functions in color space. The tolerance criteria for region growing is determined when the seed regions are found or is set prior to segmentation based on a model of camera noise. The tolerance is allowed to vary uniformly with pixel intensity because greater color variation is possible for brighter image pixels.

Prior to region growing, it is necessary to find seed regions with which to begin the growing. We find seed regions based on strong color samples found in a color histogram of the image or based on the image area within a grid which is overlain on the image. The square elements of the grid are 7x7 pixels. Histogramming results in larger seed regions if large parts of the image correspond to a single surface material under fairly constant illumination. Consequently, we begin the image processing by looking for seed regions through histogramming. The algorithm switches to the grid method when histogramming is no longer successful at finding compact seed region with low color variance.

We have used a two dimensional color space for the histogramming. The 2D color spaces used were $(\frac{Green}{Red}, \frac{Blue}{Red})$ and $(\frac{S_1}{S_0}, \frac{S_2}{S_0})$. We believe that any one or two dimensional color space that de-emphasizes color brightness is a suitable choice for the initial histogramming. The 2D color spaces we used were convenient for us to implement. (See Figure 2 for a sample 2D color histogram.)

In the 2D color histogram we look for strong peaks. (See [1] for the algorithm used.) Strong peaks in the histogram correspond to dominant colors in the original image. Each peak is used to label pixels in the original image with the peak color. The tolerance criteria for region growing is determined based on the variance of the peak found in the histogram.

The initial 3D color models for seed regions are either uniformly colored or correspond to lines in 3D color space that pass through the origin. If a uniform region does not grow during an iteration then a linear model is tried. The linear model is accepted if the error is relatively small and the region can be grown by a considerable amount. Through the process of choosing a new model during updating, linear models need no longer pass through the origin. Consequently, we can recover shadows that do not fit the criteria of being *ideal shadows*. See [7] for details of how the models are updated and the regions are pruned.

Our color image segmentation decouples the region models from the individual pixels. The models apply to region pixels en masse. The decoupling insures that a a single material illuminated by one compact light source on which numerous distinct shadows are cast is still recognized as one image region. Our post-segmentation processing will concentrate on analyzing individual regions to see if they contain shadows and will not need to compare regions that are not adjacent.

5 Employing an Active Observer

Because line-like color clusters can result from physical events besides shadow, our observer must examine the shadow candidate regions found by our color segmentation procedure to determine if a shadow is present. Here we present further tests of the color clusters to support or refute their origin from shadows.

The simplest shadow criteria for the linear color clusters follows from the fact that shadows are darker than the same surface directly lit. Consequently, for a color space whose bases are band-limited functions (such as red, green, blue), the linear color cluster for a shadow must not get brighter along any of the bases in one direction along the line.

In addition, if all the shadows in a scene are illuminated by similarly colored light, we expect the shadows to show a similar relative change in brightness, hue and color saturation with respect to a single surface material. We propose to judge any trends in the ambient light illuminating the shadows based on experiments the observer does by actively casting its own shadows.

We employ an observer that can extend a probe into the environment in order to cast shadows. If we assume the environment is unchanging during the extension of the shadow probe then any shadows that are cast and are visible can be easily detected by taking the difference of images. We assume the observer is mobile and that in time the observer will image a shadow that it has cast if the scene illumination is such that shadows are possible.

With the probe shadow detected, the observer has one sample of what the environment looks like directly lit and in shadow. This information is used in two ways. First, to partially segment the image and second to estimate the general color effects of shadows in the environment.

We segment the image area of the probe shadow using our color segmentation algorithm. Then, we allow the results to grow into the surrounding parts of the image. Thus, we assume that the shadow we cast on a material will look like other shadows cast on the same material. (See Figure 2).

In analyzing the probe shadows, the observer infers information about the illumination situation and characteristics.

We begin by examining the situational information that can be gained from the probe shadow. If no shadow is cast, we assume that there is no compact

light source. If the shadow cast has no detectable penumbra, then we assume that there is a point light source and that our shadow recognition system is not applicable. If we find disjoint shadows cast by the probe, we know that there are multiple compact light sources present. If only one compact light source is indicated, then shadows are expected strictly within image regions. However, if there are multiple compact light sources, then a shadow cast on a uniformly colored surface may be found across several contiguous regions due to complex shading. In future work we will address the problem of recognizing that a shadow that is split over several regions is a single shadow.

Note that the observer can test the validity of the Linear Color Cluster Assumption for Penumbrae for those shadows that it actively cast. The penumbra of these shadows can be found independently of our segmentation algorithm (see [1]) and the penumbra color values can be compared against a linear color model to test the assumption. If the assumption holds for the probe shadow, we will consider our segmentation for shadows applicable to the scene.

We now examine the spectral characteristics of the scene illumination that can be inferred from the probe shadow. We assume that the color trends we see in the probe shadow will hold for all the shadows in the scene. For instance, if all the probe shadows show a bias towards blue along a measure of hue then we will expect all shadows to follow this rule under the present lighting conditions. We look for trends along the criteria of hue among the probe shadows. (This is similar in spirit to the *pull factor* for *non-ideal shadows* of Gershon et al. [2].) We also examine if the ratio of a surface in the probe umbra to the same surface directly lit is constant (The case of an *ideal shadow*). If the ratio is bounded for a variety of surfaces in shadow, then the observer can use this ratio as a color criteria for detecting shadows. At present, we test for the *ideal shadow* case and then if that does not hold, whether or not there is a bias for a particular hue direction.

6 Results

In Figure 2 we show the complete processing of one image. The coordinates of the pictured 2D color histogram are $(\frac{S_1}{S_0}, \frac{S_2}{S_0})$. Unsaturated colors are near the origin and saturated colors are at the periphery. Red is to the right, green at the bottom, and blue is at the upper left corner. Strong responses can be seen for the white background (the spot near the center of the histogram) and for the red block (the spot near the right of the histogram). There is a weaker response for the green block not in shadow (the spot at the bottom center of the histogram). The green block in shadow is a line from green to red in the histogram. In the scene, the red block is at the right and the green block is in the center of the image towards the back of the scene.

The middle left picture in the figure shows the segmented region found from the probe shadow colored white and super-imposed on the original image. The pictures at the bottom of the figure shows the gray-scale coded regions from the color image segmentation. Black indicates that no region was fit to that location on the image. In the final results of the segmentation at the bottom right of the figure, some parts of the image remain unexplained. In areas where no seed region is fit and no region is grown, the image is better explained by texture, edges, or higher order color models than those we recover. In the area where the two blocks meet, there are strong inter-reflections from the shiny plastic of the blocks. This results locally in a non-line-like color cluster which our system was not designed to recover. (See [5] or [6] for work on recovering inter-reflections). The shadows in the scene are recognized as compatible with the probe results of a shadow that is nearly an *ideal shadow* with a slight bias to red.

In the next two figures no probe was used and only the success of the color image segmentation algorithm is tested.

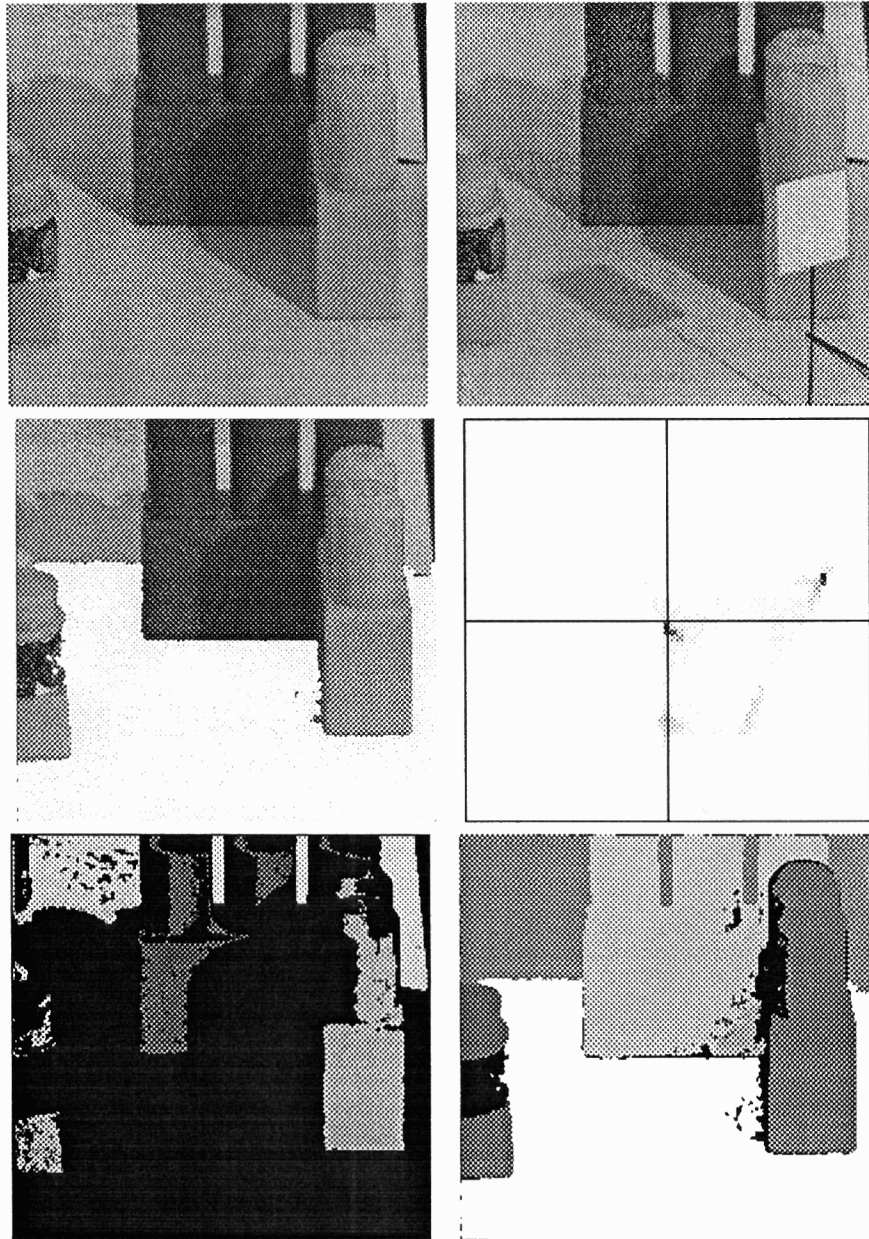


Figure 2: Color Image Segmentation: Scene 1. Top Left: The original image. Top Right: The same scene after introducing the shadow probe. Middle Left: The partial segmentation based on the probe. Middle Right: The 2D color histogram of the remaining parts of the image. Bottom Left: The seed regions from the color histogram. Bottom Right: The results of segmentation. (See the text for a complete explanation.)

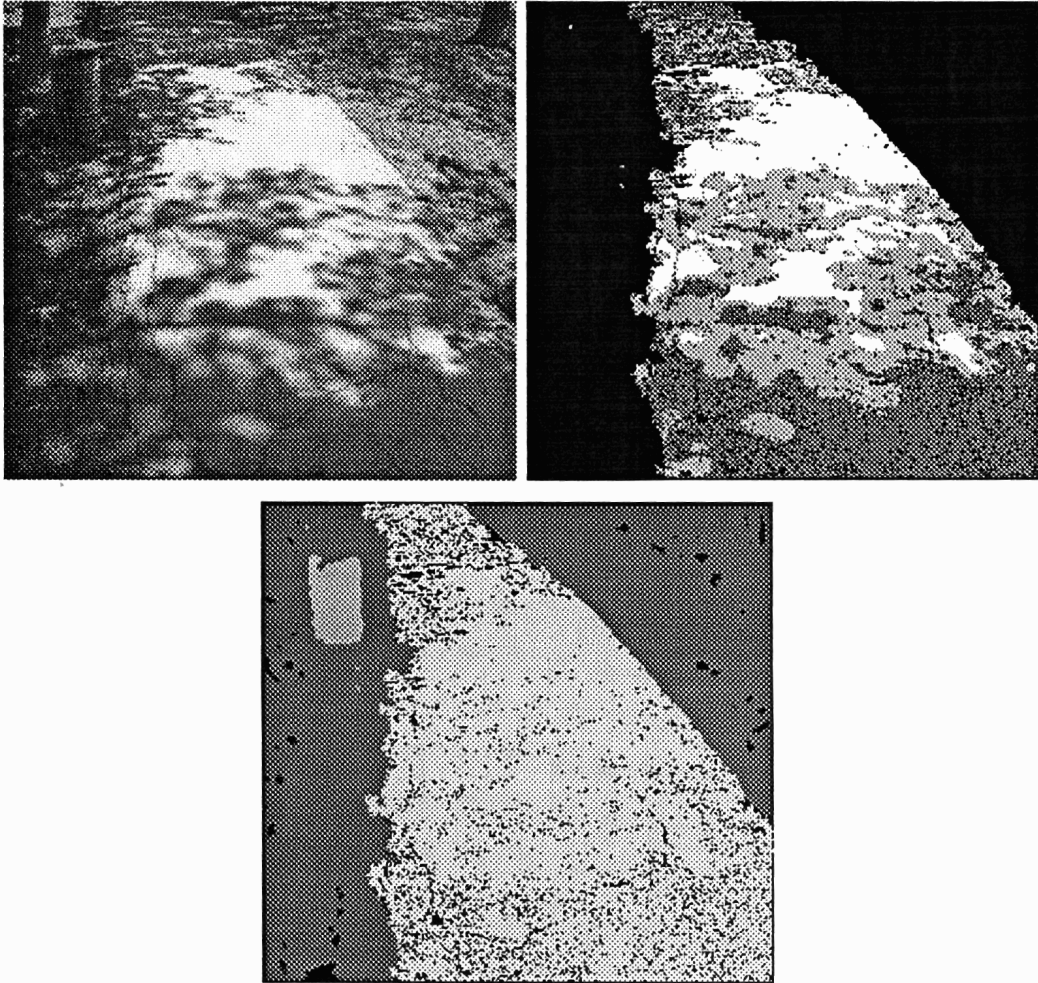


Figure 3: Color Image Segmentation: Scene 2. Top Left: An image of a road courtesy of the Carnegie Mellon Navlab project. Top Right: Where the recovered umbra and penumbra of the shadow on the road are. The umbra is dark gray, the penumbra light gray, and the road directly lit is white. Bottom: The full segmentation of the original image. The different regions are indicated by changes in the gray-scale. Black indicates no region was found.

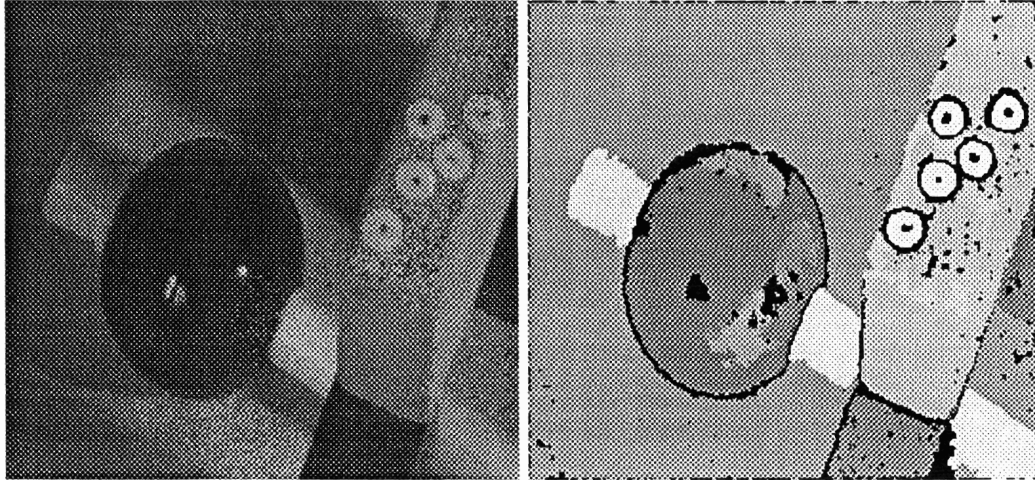


Figure 4: Color Image Segmentation: Scene 3. Left: A scene illuminated by two strong source of illumination. Right: Gray-scale coded regions from the color segmentation algorithm. Black indicates no region was found.

Figure 3 demonstrates that our color image segmentation can handle complex, natural shadowing. We also use this example to demonstrate that although one material directly lit and in shadow (in this case the road) is recovered as a single image region, we still have a representation of the umbra, penumbra, and directly lit parts of the surface. Some small, isolated parts of this image remain unexplained due to camera noise, cracks on the road, and variations in the ground cover. Note that the partially shadowed green garbage can is successfully recovered against the grass except for the dark interior of the can. However, the shadowed tree trunks and the interior of the garbage can have a similar color to the grass in shadow. All are of such low brightness that there is not enough hue information to distinguish them.

In Figure 4 the scene is illuminated by two strong, compact light sources. Evidence of this can be seen in the two specularities on the plastic ball in the center of the image. On the right side of the image are three colored wooden blocks arranged in an arch. The blocks cast a shadow across the ball and

much of the ground surface. Because of the illumination situation, there is shading within the shadow cast on the ball. Consequently, we cannot expect to recover the entire ball with one line-like color cluster model. However, if the Linear Color Cluster Assumption holds, we should be able to recover at least the penumbra of the shadow as one region. In fact, the penumbra and most of the ball is recovered as a single image region. This demonstrates how weak a constraint the Linear Color Cluster Assumption is on the scene illumination. Again, small parts of the image remain unexplained. These unexplained areas are due mainly to color variation on the wood blocks and the high image variance of the specularities. Both phenomenon fall below the image resolving abilities of our seed region generation. Without the ability to find small seed regions, the system cannot recover small regions.

7 Conclusion

We have presented a general algorithm for segmenting color images into regions that form uniform or linear color clusters in color space. Shadows will be represented as piece-wise linear color clusters under our Linear Color Cluster Assumption for Penumbrae. Under this assumption, the width of a penumbra will always be a single segmented region. Consequently, all linear color clusters become shadow candidate regions. Many of these regions can be discounted as shadows because they do not show a darkening simultaneously along each of red, green, and blue. Other regions can be discounted because they show color trends not compatible with the results from the probe shadow. For a complete shadow recognition system, geometric constraints would also need to be brought to bear on the remaining shadow candidate regions [1].

Rather than rely on restrictive or untestable assumptions, or high level knowledge about the scene illumination [2], we have made use of an observer

than can actively cast shadows. There are environments in which it may be unlikely that the observer will see the probe's shadow. However, for relatively uncluttered scenes, if shadows are present, the observer will be likely to cast a visible shadow. By employing an active observing paradigm and addressing the problem of shadow recognition prior to recovering high level scene knowledge, we have a system that meets the needs of a large number of visual modules for geometric recovery that assume that shadows have been accounted for. In addition, because of our need for only relatively weak a priori assumptions, our system can handle scenes with complex geometry and illumination, as is demonstrated by our results.

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