4-1-1990

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Suggested Citation:
Kumar, V. and J.F. Gardner. (1990). "Kinematics of Redundantly Actuated Closed Chains." *IEEE Transactions on Robotics and Automation*, Vol. 6(2)2. pp. 269 - 274.

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Disciplines
Engineering | Mechanical Engineering

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Kinematics of Redundantly Actuated Closed Chains

VIJAY KUMAR, MEMBER, IEEE, AND JOHN F. GARDNER

Abstract—**The instantaneous kinematics of a hybrid manipulation system, which combines the traditional serial chain geometry with parallelism in actuation, and the problem of coordination is discussed. The indeterminacy and singularities in the inverse kinematics and statics equations and measures of kinematic performance are analyzed. Finally, coordination algorithms that maintain an optimal force distribution between the actuators while avoiding or exploiting singularities are presented.**

I. Introduction

Broadly speaking, there are two types of geometries for robot manipulators: serial chain and parallel chain linkages. However, robotic systems such as two cooperating arms, walking vehicles, and multifingered grippers consist of several actively controlled articulations (serial linkages), which act in parallel on an object/end-effector/ground. Unlike serial manipulators, they include one or more closed kinematic chains in their structure, and in addition, unlike completely parallel manipulators, there is more than one actuator in a particular chain, and the number of actuators typically exceeds the mobility typically. In this paper, such devices are called *hybrid* manipulators.

Examples of robots with completely in-parallel actuation and the kinematic and dynamic analysis of such parallel systems have been presented in [1], [2], [7], and [12]. The dualities that exist between serial and parallel geometries and between kinematics and statics have been pointed out in [17]. The control problem for such dynamically constrained systems has also been analyzed (see, e.g., [4], [5], [16], [20]).

Hybrid systems are characterized by redundantly-actuated closed chains. The actuator rates are uniquely determined by the specified trajectory, but the actuator forces are underdetermined. The redundancy in such systems is dual to the kinematic redundancy in serial chain manipulators in which the number of actuators exceeds the dimension of the task space [11]. The presence of redundancy engenders a need for techniques that will resolve (or even exploit) the redundancy in the system.

Redundancy in manipulation systems with parallelism has been studied with reference to multifingered grippers [8], [15] and walking vehicles [3], [10], [14]. Since all these systems involve interaction between several actively controlled arms with a passive object [9], most reports have described attempts to optimize contact conditions [3], [8]–[10], [14]. These optimization efforts have largely ignored the performance of actuators at the joint level and have treated each articulation (leg/finger/arm) independently rather than considering the entire system. In research on multiple arm systems (see, e.g., [13], [16], [18], [20]), the force distribution problem has not been addressed directly. Instead, in most proposed control schemes, the force distribution is automatically determined by trajectory errors, which results in unacceptably large interaction forces, thus affecting the system performance adversely. Even in studies in which this problem has been addressed, the load distribution has been *a priori* specified [13], [16].

This paper addresses the issue of coordination in robotic systems with redundantly actuated closed chains. First, an instantaneous kinematic analysis of *hybrid* devices is presented. The singularities in the kinematics and statics equations, which are characteristic of systems with closed chains, are analyzed. We next discuss the kinematic characterization of the actuators to improve the understanding of the nature of redundancy in hybrid systems. Coordination schemes, which avoid singularities and compute optimal force distributions, are derived from this characterization. Simulations are used to demonstrate the efficacy of the proposed schemes.

Manuscript received October 18, 1988; revised October 31, 1989.
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IEEE Log Number 9035792.

II. Instantaneous Kinematic Analysis

The analysis in this paper is restricted to symmetric configurations (same number of links, joints, and actuators on each chain). It is assumed that the joints are frictionless and that inertial forces are negligible since the main objective of the paper is to gain an insight into the problem rather than work with a perfect model. Let the manipulation system be comprised of n parallel chains, each consisting of m serially connected links (and possessing m degrees of freedom), and let the task space of d-dimensional (where $d \leq m$). Let a ($\leq m$) be the number of actuators in each of the n chains. There are $r = (n \times a)$ actively controlled joints. The special case $a = 1$ ($r = n$) corresponds to a scheme of actuation that is completely parallel; an extensive survey of such mechanisms can be found in [7], whereas $n = 1$ (a must equal m in this case) denotes the standard serial arm.

For the ith serial chain, the end-effector rate ${}^i\dot{\theta}$ is given by

$$ {}^i\dot{x} = {}^iJ\,{}^i\dot{\theta} \tag{1}$$

where the leading superscript i denotes the ith serial chain, ${}^i\dot{q}$ is the $(m \times 1)$ vector of joint rates, and ${}^iJ$ is the $(d \times m)$ Jacobian matrix for the ith chain. Similarly, using the principle of virtual work

$$ {}^i\tau = {}^iJ^{T\,i}\mathbf{f} \tag{2}$$

where ${}^i\tau$ is the $m \times 1$ vector of joint torques (or forces), and ${}^if$ is the vector of forces and moments exerted by the ith chain on the end effector (platform) or object. Obviously, m-a joint torques in ${}^i\tau$ must equal zero. Equations (1) and (2) can be written for any of the n serial chains in the system.

Let us assume for the moment that the ith chain is in a nonsingular configuration, that is, $m = d$, and the m joint freedoms in the ith chain are linearly independent. We can invert ${}^iJ$ in (1) to obtain the joint rates required to effect a desired velocity of the end effector in the task space. If this is done for all the n chains, we can write the inverse equations for the mn rates compactly:

$$\begin{bmatrix} {}^1\dot{\theta} \\ {}^2\dot{\theta} \\ \vdots \\ {}^n\dot{\theta} \end{bmatrix} = \begin{bmatrix} {}^1J^{-1} \\ {}^2J^{-1} \\ \vdots \\ {}^nJ^{-1} \end{bmatrix} \dot{x}. \tag{3}$$

The rates for the $r(= na)$ actively controlled joints can be extracted from (3)

$$\dot{\Theta}_a = \Gamma \dot{x} \tag{4}$$

where $\dot{\Theta}_a$ is a $r \times 1$ vector of joint rates, where the subscript 'a' indicates that only the rates of the actively controlled joints are included in the vector. Each row of Γ (a $r \times d$ Jacobian matrix) is a row in the $mn \times d$ matrix of inverses in (3). Equation (4) represents the inverse rate kinematics equations for the system. Clearly, if $r = d = na$ and Γ is nonsingular, the direct kinematics equations may be obtained by inverting (4):

$$\dot{x} = \Gamma^{-1}\dot{\Theta}_a. \tag{5}$$

Notice that in systems with parallelism, the inverse kinematics (3), (4) are simpler than the direct kinematics [12], [17]. Further, the inverse kinematics require that each Jacobian matrix (${}^iJ$) be nonsingular, but the direct kinematics, in addition to that, require that Γ be nonsingular.

Similar transformations from joint space to Cartesian (end-effector) space are possible in statics in the absence of friction and gravity. By using the principle of virtual work, from (4), we obtain (τ_a is the vector of joint torques for all the actively controlled joints)

$$\tau_a^T \dot{\Theta}_a = \tau_a^T \Gamma \dot{x} = \mathbf{w}^T \dot{x} \tag{6}$$

or

$$w = \Gamma^T \tau_a. \tag{7}$$

Once more, if $r = na = d$ and Γ is invertible, the inverse problem can be solved.

A. Singularities

There are two types of special configurations that are encountered in control algorithms for such hybrid systems. The first type of singularity is caused by the joint freedoms in a single chain becoming linearly dependent, which results in *that* chain loosing one or more degrees of freedom, thus constraining the end effector along one or more directions. This is the well-known kinematic singularity, which is characteristic of serial chain arms. However, even if none of the n serial chains are singular (that is, there is no kinematic singularity), it is possible for the d rows in the $d \times d$ Γ matrix to become linearly dependent. In this case, the direct kinematics transformation (5) is not possible. In such a situation, a desired unique velocity of the end effector cannot always be specified. Alternatively, the manipulator is underconstrained, and there exist one or more wrenches that cannot be resisted. Therefore, alternatively, we can describe this singularity as a singularity in the "inverse statics" since it also prevents the determination of τ_a for a given w in (7). In other words, this type of singularity is dual to the first kind and may even be called a *static* singularity as opposed to a *kinematic* singularity. We note that since we are only concerned about *geometric* singularities, computational singularities that arise from a particular mathematical modeling technique are not an issue here.

B. Redundancies

If any of the n chains has more than d joints, i.e., $m \geq d$, the inverse kinematics problem is underdetermined. It is possible to find more than one set of joint rates for a desired end-effector velocity, and this situation is called kinematic redundancy. Optimization techniques to resolve the kinematic redundancy can be found in the literature [6] and is beyond the scope of this study. In this paper, we will concentrate on systems in which $m = d$. A situation that is dual to this occurs in hybrid systems when the problem of determination of the forces and moments exerted by each of the n chains of the end effector is underspecified. In this case, the problem is statically indeterminate—the situation of *static* redundancy is dual to the concept of *kinematic* redundancy. This problem of distributing the load between the n chains is called the force distribution problem [2], [9]–[11], [14].

III. Kinematic Characterization of Actuators

A. Partitioning of Actuators

If the rows of Γ are denoted by $R_1, R_2, \cdots, R_r$, (7) can be rewritten as

$$w = \Gamma^T \tau = [R_1^T R_2^T \cdots R_r^T][\tau_1 \tau_2 \cdots \tau_r]. \tag{8}$$

This represents an underdetermined set of equations when $r > d$. In such a situation, it is possible to designate any d of the r available actuators as *primary* (the other $d - r$ are called *secondary*) similar to the approach followed in [4] and [5]. In this framework, the primary actuator set would control the motion of the manipulated object, whereas the *secondary* actuators would cater to secondary objectives—one alternative is the control of interaction or constraint forces on the object [20]. In any event, the control inputs corresponding to the primary actuator set, in general, span the task space. From this point on, we ignore such secondary considerations and assume that joints with secondary actuators are free.

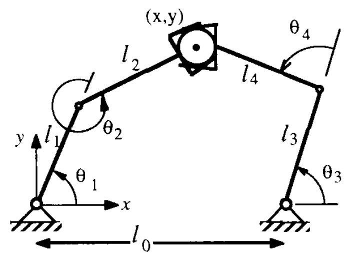

Fig. 1. Manipulation with two planar arms.

Clearly, there are ${}^rC_d$ choices for the primary actuator set. The input vector of torques corresponding to a particular set that consists of actuators $i_1, i_2, \cdots, i_d$ is denoted here as $u_{i_1, i_2, \cdots, i_d}$. Each such $r \times 1$ vector has d nonzero torques $(\tau_{i_1}, \tau_{i_2}, \cdots, \tau_{i_d})$; the other $r - d$ torques are zero. These ${}^rC_d$ vectors span the r-dimensional joint space, and any vector of inputs (torques) can be expressed as a linear combination of these vectors. Any of the ${}^rC_d$ primary actuator sets can be used to control the motion and the input vector $u_{i_1, i_2, \cdots, i_d}$ can be found by solving for the d unknowns $\tau_{i_1}, \tau_{i_2}, \cdots, \tau_{i_d}$

$$\mathbf{w} = \Gamma^T_{i_1, i_2, \cdots, i_d} [\tau_{i_1} \tau_{i_2} \cdots \tau_{i_d}]^T, \quad \text{where}$$
$$\Gamma_{i_1, i_2, \cdots, i_d} = [R_{i_1}^T R_{i_2}^T, \cdots, R_{i_d}^T]^T. \tag{9}$$

B. Measures of Kinematic Performance

Several measures of kinematic performance have been sought for serial robot manipulators. The *manipulability* [19] and the condition number [15] are two well-known indices. Both measures of kinematic optimality may be adapted to meet our requirements.

For a given primary actuator set consisting of actuators $i_1, i_2, \cdots, i_d$ (input vector $u_{i_1, i_2, \cdots, i_d}$), we can define

$$\mu_{i_1 i_2 \cdots i_d} = \sqrt{\det(\Gamma_{i_1 i_2 \cdots i_d} \Gamma^T_{i_1 i_2 \cdots i_d})} \quad \text{and}$$
$$r_{i_1 i_2 \cdots i_d} = \frac{1}{c(\Gamma_{i_1 i_2 \cdots i_d})} \tag{10}$$

as measures of optimality of the primary actuator sets, where $c(\cdot)$ is the condition number. Clearly, $\mu_{i_1 i_2 \cdots i_d}$ and $r_{i_1 i_2 \cdots i_d}$ are functions of the position of the object in task space as well as the choice of actuators $i_1, i_2, \cdots, i_d$. It is important to note, however, that $\mu_{i_1 i_2 \cdots i_d}$ is the volume (except for a multiplicative constant) of the *force ellipsoid* given by

$$\|u_{i_1, i_2, \cdots, i_d}\|^2 = w^T (\Gamma^T_{i_1 i_2 \cdots i_d} \Gamma_{i_1 i_2 \cdots i_d})^{-1} w = 1. \tag{11}$$

A larger value of $\mu_{i_1 i_2 \cdots i_d}$ implies that for a given load, the input torques are smaller. Similarly, $r_{i_1 i_2 \cdots i_d}$ is a measure of the isotropy of the force ellipsoid.

C. A Planar Dual Arm Manipulation System—An Example

As an example, we consider two planar robot manipulators, where each has two revolute joints. We model the gripped object as a small cylinder (the radius is small compared with the link lengths), and the interaction between the two arms is modeled as a revolute joint. In these circumstances, the system may be modeled as a closed five-bar chain (with five revolute joints) and four actuators, as is shown in Fig. 1. The mobility of the linkage is equal to two, and the task space is the 2D translational space. Since the number of control inputs (actuators) is four, the system is redundant.

Let (x, y) be the coordinates of the object and θ_i be the joint variables. Further, let c_i, s_i, c_{ij}, and s_{ij} denote $\cos\theta_i$, $\sin\theta_i$,

$\cos(\theta_i + \theta_j)$, and $\sin(\theta_i + \theta_j)$, respectively. The rate kinematics equations are

$$\dot{x} = [\dot{x}\ \dot{y}]^T = {}^1J[\dot{\theta}_1\ \dot{\theta}_2]^T = {}^2J[\dot{\theta}_3\ \dot{\theta}_4]^T \tag{12}$$

where

$${}^1J = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} \end{bmatrix}$$

and

$${}^2J = \begin{bmatrix} -(l_3 s_3 + l_4 s_{34}) & -l_4 s_{34} \\ (l_3 c_3 + l_4 c_{34}) & l_4 c_{34} \end{bmatrix}.$$

Inverting the Jacobians analytically, we get expressions similar to (4) and (7):

$$\dot{\Theta}_a = [\dot{\theta}_1\ \dot{\theta}_2\ \dot{\theta}_3\ \dot{\theta}_4]^T = \Gamma \dot{x}$$

$$w = [w_x\ w_y]^T = \Gamma^T \tau = [\mathbf{R}_1 \mathbf{R}_2 \mathbf{R}_3 \mathbf{R}_4][\tau_1 \tau_2 \tau_3 \tau_4]^T$$

where

$$\Gamma = \begin{bmatrix} \dfrac{c_{12}}{l_1 s_2} & \dfrac{s_{12}}{l_1 s_2} \\ -\dfrac{1}{s_2}\left(\dfrac{c_1}{l_2} + \dfrac{c_{12}}{l_1}\right) & -\dfrac{1}{s_2}\left(\dfrac{s_1}{l_2} + \dfrac{s_{12}}{l_1}\right) \\ \dfrac{c_{34}}{l_3 s_4} & \dfrac{s_{34}}{l_3 s_4} \\ -\dfrac{1}{s_4}\left(\dfrac{c_3}{l_4} + \dfrac{c_{34}}{l_3}\right) & -\dfrac{1}{s_4}\left(\dfrac{s_3}{l_4} + \dfrac{s_{34}}{l_3}\right) \end{bmatrix} \tag{13}$$

Clearly, there are ${}^4C_2$ (= 6) choices for the primary actuator set. The six input vectors, denoted by u_{12}, u_{13}, u_{14}, u_{23}, u_{24}, and u_{34} span the 4D joint space, and any vector of torques can be expressed as a linear combination of these vectors. The input vector u_{ij} for the primary actuator set consisting of actuators i and j that is required for a load w can be found by solving the above equations for the two unknowns τ_i and τ_j.

For example, for $u_{13} = [\tau_1 \quad 0 \quad \tau_3 \quad 0]^T$

$$w = \Gamma_{13}^T \begin{bmatrix} \tau_1 \\ \tau_3 \end{bmatrix}$$

where

$$\Gamma_{13} = \begin{bmatrix} \dfrac{c_{12}}{l_1 s_2} & \dfrac{s_{12}}{l_1 s_2} \\ \dfrac{c_{34}}{l_3 s_4} & \dfrac{s_{34}}{l_3 s_4} \end{bmatrix}.$$

Even for the simple geometry shown in Fig. 1, the workspace is replete with singularities. These can be classified into six categories (cases A–F), all of which are shown in Fig. 2 and described in Table I.

If the primary actuator set consists of actuators from one of the two arms only (that is, the inputs u_{12} or u_{34} are considered), a special configuration or singularity occurs when an arm is completely extended or retracted (cases A and F in Fig. 2 and Table I); the determinant of ${}^1J$ (or ${}^2J$) equals zero. If the system is considered to be a closed chain, that is, the dichotomy of two arms is abandoned, we are automatically faced with more special configurations. For example, for u_{13}, equating the determinant of Γ_{13} to zero yields upon simplification the condition $\sin(\theta_1 + \theta_2 - \theta_3 - \theta_4) = 0$. This corresponds to the distal links of each manipulator being aligned, or more precisely, to axes of joints 2, 4, and 5 being coplanar. In this situation, a force exerted perpendicular to the plane of the axes cannot be resisted, and the object, therefore, is unrestrained. Similarly, the singularities for other actuator sets can be found. Unless a simple leader-follower type of a scheme is employed for coordination, all these singularities can be expected to affect the control of the system. A good coordination algorithm must avoid (or exploit) these singularities appropriately, as is shown in the next section.

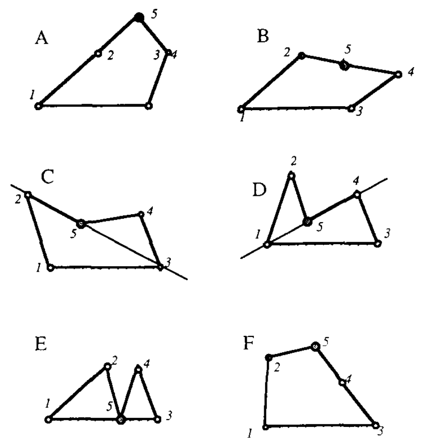

Fig. 2. Singular configurations in manipulation with two planar arms. In all cases, three of the five joint axes are coplanar (see Table I). Cases A and F correspond to the "usual" kinematic singularities, whereas cases B–E are the static singularities.

TABLE I
SINGULAR CONFIGURATIONS FOR TWO COOPERATING PLANAR ARMS

CASE	PRIMARY ACTUATOR SET	MATHEMATICAL CONDITION FOR SINGULARITY	GEOMETRIC DESCRIPTION (COPLANAR AXES)
A	1, 2	$\sin\theta_2 = 0$	1, 2, 5
B	1, 3	$\sin(\theta_1 + \theta_2 - \theta_3 - \theta_4) = 0$	2, 4, 5
C	1, 4	$l_4 \sin(\theta_1 + \theta_2 - \theta_3 - \theta_4) + l_3 \sin(\theta_1 + \theta_2 - \theta_3) = 0$	2, 3, 5
D	2, 3	$l_2 \sin(\theta_1 + \theta_2 - \theta_3 - \theta_4) + l_1 \sin(\theta_1 - \theta_3 - \theta_4) = 0$	1, 4, 5
E	2, 4	$l_2 l_4 \sin(\theta_1 + \theta_2 - \theta_3 - \theta_4) + l_1 l_4 \sin(\theta_1 - \theta_3 - \theta_4) + l_2 l_3 \sin(\theta_1 + \theta_2 - \theta_3) + l_1 l_3 \sin(\theta_1 - \theta_3) = 0$	1, 3, 5
F	3, 4	$\sin\theta_4 = 0$	3, 4, 5

IV. COORDINATION OF REDUNDANT ACTUATORS

We now address the problem of specifying torque set points for the controller and resolving the redundancy in (8) in an effective manner. It is clear from the discussion in the previous section that negotiating the myriad singularities is one of the key issues. However, if one considers the major task of the system to be that of resisting an applied load, it makes sense to take advantage of some of the singularities. In particular, the singularities that belong to the first class (kinematic singularities) are potentially attractive since they represent configurations in which externally applied loads are not reacted by the actuators at all. If we formulate a control scheme that actually favors those sets of actuators that are at or near kinematic singularities and avoid those that are at or near "static" singularities, we can achieve greater load (for example, lifting) capacity with less joint deflections, greater accuracy, and superior performance.

The simplest coordination scheme involves switching from one set of primary actuators to another in order to constantly command the optimal primary actuator set while the secondary actuator set is idle. However, this would result in a discontinuity in the torques at the instants when the actuator sets are switched. To circumvent this, we propose a coordination scheme based on different weighted averages

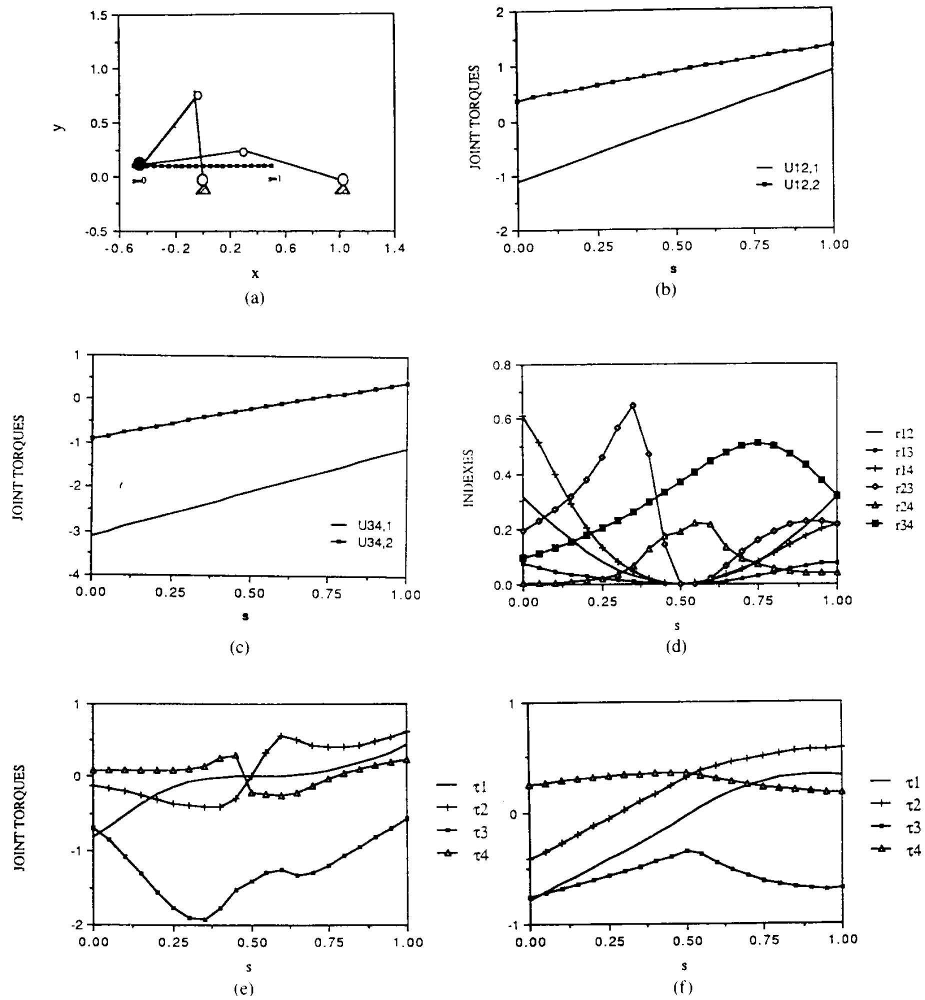

Fig. 3. Simulation results (Example 1). The trajectory is a horizontal path $(-0.5 \le x \le 0.5,\ y = 0.1)$ with a constant load of $-i - 2j$ applied on the object (including the weight). s is a parameter that varies from 0 to 1 along the trajectory, and $u_{ij}{}^1$ and $u_{ij}{}^2$ denote t_i and t_j $(i < j)$, respectively, for the primary actuator set consisting of actuators i and j. The force distributions in (d) and (e) are derived from (14) with r_{ij} and μ_{ij} as weighting factors, respectively.

of the ${}^rC_d$ possible input vectors that enables a continuous switching between different actuator sets.

For a given primary actuator set consisting of actuators $i_1, i_2, \cdots, i_d$, the input vector $u_{i_1, i_2, \cdots, i_d}$ can be computed unless $\Gamma_{i_1, i_2, \cdots, i_d}$ is singular. If $\rho_{i_1, i_2, \cdots, i_d}$ is an index that is used as a measure of performance of $u_{i_1, i_2, \cdots, i_d}$, an appropriate vector of torques can be computed quite simply:

$$\tau = \frac{\sum_{i_1 i_2 \cdots i_d} \rho_{i_1 i_2 \cdots i_d} u_{i_1 i_2 \cdots i_d}}{\sum_{i_1 i_2 \cdots i_d} \rho_{i_1 i_2 \cdots i_d}}. \tag{14}$$

The use of either $\mu_{i_1 i_2 \cdots i_d}$ or $r_{i_1 i_2 \cdots i_d}$ as a weighting factor makes it possible to accommodate singularities with ease since in either case, the weighting factor vanishes when the input vector corresponding to a singular $\Gamma_{i_1, i_2, \cdots, i_d}$ matrix. Using $\mu_{i_1 i_2 \cdots i_d}$ results in a larger force ellipsoid for the system and therefore smaller torques. This automatically favors kinematic singularities (see cases A and F in Fig. 2 and Table I) but avoids "static" singularities (cases B, C, D, and E in Fig. 2 and Table I). The use of $r_{i_1 i_2 \cdots i_d}$ on the other hand would increase the isotropy of the force ellipsoid but would also (indirectly) avoid "static" singularities that cause the force ellipsoid to flatten out along one of its principal axes.

A. Example—A Simulation of the Dual-Arm Manipulation System

We illustrate the application of (14) using the indexes $\mu_{i_1 i_2 \cdots i_d}$ and $r_{i_1 i_2 \cdots i_d}$ with simulations on a quasistatic model of the system in the example considered earlier (see Figs. 3 and 4). We consider the two trajectories shown in Figure 3(a) and 4(a) with a constant load, w, of $-i - 2j$ units as examples.

The horizontal trajectory of Fig. 3(a) passes through a kinematic singularity (case A) at $x = 0$. Fig. 3(b) and (c) shows the torques required to counteract the applied load in a master-slave type control scheme. Fig. 3(b), actuators 1 and 2 (input vector, u_{12}) and Fig. 3(c),

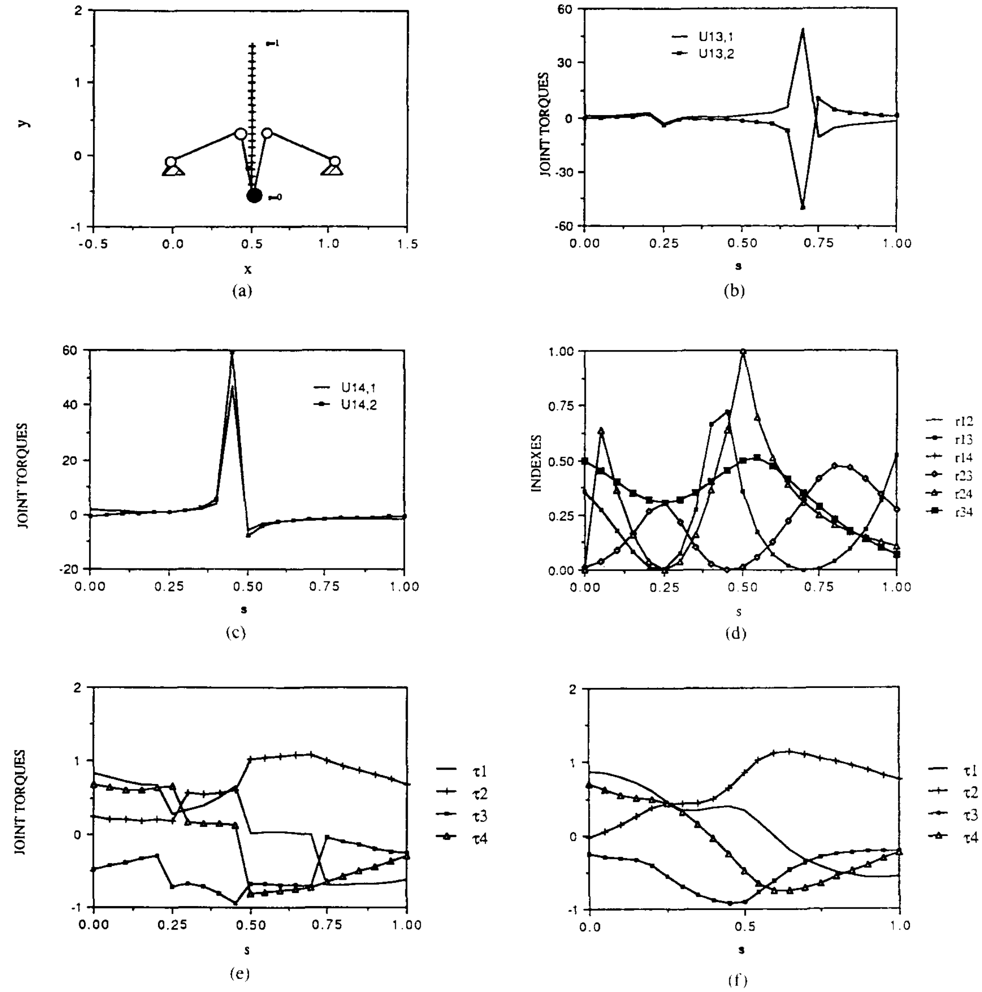

Fig. 4. Simulation results (Example 2). The trajectory is a vertical path ($x = 0.5$, $-0.5 \leq y \leq 1.5$) with a constant load of $-i - 2j$ applied on the object (including the weight). s is a parameter that varies from 0 to 1 along the trajectory, and $u_{ij},^1$ and $u_{ij},^2$ denote t_i and t_j ($i < j$), respectively, for the primary actuator set consisting of actuators i and j. The force distributions in (d) and (e) are derived from (14) with μ_{ij} and μ_{ij}^2 as weighting factors, respectively.

actuators 3 and 4 (input vector, u_{34}) are considered to be primary actuator sets. Since arm 1 (input u_{12}) is near a kinematic singularity, u_{12} is preferable to u_{34}. Other actuator sets have different characteristics. Fig. 3(d) shows values of r_{ij} (reciprocals of the six condition numbers) as functions of position. Recall that as r_{ij} approaches 1, the force ellipsoid approaches perfect isometry (a preferred condition). With a proper switching algorithm, it is therefore possible to solve the control problem in such a way that r_{ij} is maintained at a value above 0.3 (see Fig. 3(d)) through the trajectory. Fig. 3(e) shows the actuator torques that result from implementing a weighted average scheme with $\rho_{ij} = r_{ij}$, as is described in (14). On the other hand, using (14) with μ_{ij} as a weighting factor results in a much better force distribution from the point of view of torque requirements, as is shown in Fig. 3(f). The tradeoff is clear. By increasing the isotropy of the force ellipsoid, we also decrease the volume of the ellipsoid. We also obtain a smooth variation of torques in Fig. 3(f), which is important, especially for high-speed maneuvers, since this affects how well the actuators will be able to faithfully produce the desired torques.

In the second example, the trajectory is a vertical path (see Fig. 4(a)), which passes through three singularities (cases B, C, and D). The so-called "static" singularities can be clearly seen in Fig. 4(b) from the time histories of u_{13} and u_{14} (cases B and C, respectively). Fig. 4(c) once again suggests that a simple switching algorithm would ensure a condition number less than 3.0 ($r_{ij} > 0.33$) through the trajectory. The force distribution resulting from μ_{ij} as a weighting factor is shown in Fig. 4(d). This time, the torques are discontinuous in the slope (as opposed to the torques in Fig. 3(f)). This is a consequence of the singularities. However, if the weighting factor is made a higher power of μ_{ij}, for example, with $\rho_{ij} = \mu_{ij}^2$, we obtain a force distribution in which the maximum torque is approximately the same, but the torques are much smoother, as is shown in Fig. 4(f) (compare with Fig. 3(f)).

In conclusion, the use of weighting factors enables efficient utilization of all the actuators and allows a "closed chain" approach to the problem as opposed to a "master-slave" approach. The weighting factors circumvent singularities in the statics equations, whereas the kinematic singularities are exploited to minimize motor torques. This approach results in an even distribution of forces, as is evidenced by Fig. 3(e) and (f) and Fig. 4(e) and (f).

B. Remarks

1. Dynamic Loads and Inertial Forces: The assumptions regarding the quasi-static nature of the problem are realistic in some applications, as was mentioned earlier. However, if inertial forces become significant, they must be incorporated in the vector $\boldsymbol{w}$. Now, $\boldsymbol{w}$ varies with time, but this does not preclude the application of the ideas presented in this paper. The only difference is that the "static" singularities are not as important at high speeds since the inertia of the system will carry it through such singularities. It may be speculated that control problems will not be as severe. Nevertheless, at low speeds, even if the inertia of the system is significant, these singularities (belonging to the second kind) are an important consideration.

2. Computational Load: The computational complexity of the suggested scheme increases with the dimension of the task space. This is particularly so since $r_{i_1 i_2 \cdots i_d}$ requires the computation of eigenvalues of a matrix and is unsuitable for on-line computation. However, the use of $\mu_{i_1 i_2 \cdots i_d}$ is well suited to real-time operation, especially since the input vectors $\boldsymbol{u}_{i_1 i_2 \cdots i_d}$ as well as the weighting factors $\mu_{i_1 i_2 \cdots i_d}$ can be computed in parallel. Even in 3D geometry, the complexity in the computation of the weighting factors is similar to the complexity in the computation of the determinant of a Jacobian matrix in an industrial robot, which is not at all expensive when efficient analytical expressions for the determinant are used.

In fact, (14) can be further simplified if $\rho_{i_1 i_2 \cdots i_d} = \mu_{i_1 i_2 \cdots i_d}$

$$\tau = \frac{\sum_{i_1 i_2 \cdots i_d} \mu_{i_1 i_2 \cdots i_d} \boldsymbol{u}_{i_1 i_2 \cdots i_d}}{\sum_{i_1 i_2 \cdots i_d} \mu_{i_1 i_2 \cdots i_d}} = \frac{\sum_{i_1 i_2 \cdots i_d} \operatorname{adj}(\Gamma_{i_1 i_2 \cdots i_d}) \boldsymbol{w}}{\sum_{i_1 i_2 \cdots i_d} \mu_{i_1 i_2 \cdots i_d}} \qquad (15)$$

where adj $(\cdot)$ represents the adjoint of the matrix. Now, no inverses have to be computed to find $\boldsymbol{u}_{i_1 i_2 \cdots i_d}$, and there is never any scope for division by a small number.

V. Concluding Remarks

The kinematics of a hybrid manipulation system, which combines the traditional serial chain geometry with parallelism in actuation, is discussed. These systems have two key attributes: a closed chain structure and redundancy in actuation. In this paper, the structural characteristics and the kinematic performance of such systems are studied. The special configurations or singularities, which are classified into two dual categories, and are dual to one another, are described. A coordination algorithm that automatically avoids undesirable singularities while favoring preferred singularities is developed. Simulation results demonstrate the utility and practicality of the scheme.

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Modal State Position Controller for the Utah/MIT Dextrous Hand

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***Abstract*—Control of multifingered robotic hands is a natural example of *hybrid* control. Contact forces must be controlled without exceeding force thresholds while the fingers are moved to impart motion to an object. This paper describes a modal state position control algorithm suited for hybrid manipulation tasks. Position tracking is robust, allowing force control to be executed in parallel according to task requirements. Hybrid control is achieved by the summed actions of the primary position controller and one or more secondary force controllers. The position control algorithm is outlined for both joint space and Cartesian space applications. Implementation and performance issues on the Utah/MIT dextrous hand are discussed.**

Manuscript received March 28, 1989; revised November 27, 1989.

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IEEE Log Number 9034534.

1042-296X/90/0400-0274$01.00 © 1990 IEEE