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Abstract

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In Chapter 2, the interest lays in the determinants of different life-cycle fertility outcomes across educational groups. The chapter presents a model where individuals deal with idiosyncratic shocks in the form of innovations to their market wages and to the efficacy with which they can control fertility outcomes. The model is estimated using data for the US and tested to see how it fares replicating facts of aggregate fertility under different counterfactual scenarios.

Chapter 3 (co-authored with Jose-Victor Rios-Rull), studies the cyclical behavior of the aggregate labor's share in total income, taking as a starting point models of business cycles driven by economy-wide technological shocks. The chapter looks at the co-movement of labor share and technological innovations in post war US history and assesses how well existing models can explain the facts.

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ESSAYS ON MACROECONOMICS AND LABOR
MARKETS: UNDERSTANDING IDIOSYNCRATIC AND
AGGREGATE SHOCKS

Se Kyu Choi

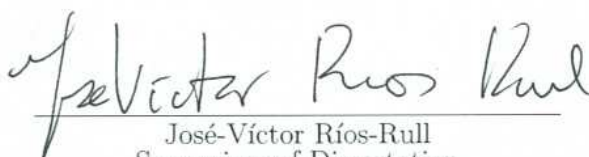
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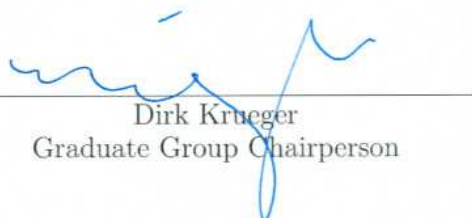
Economics

Presented to the Faculties of the University of Pennsylvania in Partial
Fulfillment of the Requirements for the Degree of Doctor of Philosophy

2009



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Supervisor of Dissertation



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Graduate Group Chairperson

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Se Kyu Choi

2009

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ABSTRACT

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UNDERSTANDING IDIOSYNCRATIC AND AGGREGATE SHOCKS

Se Kyu Choi
José-Víctor Ríos-Rull

This dissertation studies the importance of shocks in understanding economic outcomes, both at the aggregate and at the individual levels. The research in this document is separated into chapters that deal with somewhat dissimilar questions which are linked by the necessity to acknowledge and understand how unforeseeable shocks determine how agents make economic decisions. These shocks or innovations are a potential explanation for why, often similar economic actors face very different paths.

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Chapter 1

Introduction

This dissertation studies the importance of shocks in understanding economic outcomes, both at the aggregate and at the individual levels. The research in this document is separated into chapters that deal with somewhat dissimilar questions which are linked by the necessity to acknowledge and understand how unforeseeable shocks determine how agents make economic decisions. These shocks or innovations are a potential explanation for why, often similar economic actors face very different paths.

In Chapter 2, I present a life-cycle model with dynamic and imperfect fertility decisions in an environment where agents are subject to idiosyncratic and uninsurable labor income shocks and capital markets are imperfect. The model relies both on the time-allocation of mothers hypothesis (i.e., higher opportunity costs of children for higher wage earners) and on differential effectiveness in the use of contraceptive technologies (what one might consider idiosyncratic shocks to birth control efforts) to explain differences in fertility outcomes by education groups. I estimate the model using data on pregnancies and abortions from the National Survey of Family Growth and then I assess the effects on fertility rates of two quantitative experiments: a ban on abortions and equalization of job market opportunities across genders.

In Chapter 3 (which is co-authored with **José-Víctor Ríos-Rull**) we explore the

dynamics of the aggregate labor share for the US economy. We identify three interesting facts: Labor share is not constant (as it is usually assumed), it is counter cyclical and reacts strongly to technology shocks. We then explore the extent to which a family of real business cycles models, where wages are not set competitively (tailored to replicate cyclical facts about the labor market), is capable of generating the observed dynamics of labor share as described in [45]. We build upon [39], [3], [10], among others, who analyze models where wages are determined via Nash bargaining, employment lags productivity, and labor share falls with productivity innovations. While these models account for various business cycles properties, they fail in replicating the dynamic empirical response of the labor share to technological shocks; this occurs even after we change preferences and technology. However, changing the aggregate production function (from cobb-douglas to CES) delivers the best results, hinting the direction for future research.

Chapter 2

Idiosyncratic Shocks: Life-Cycle Fertility

2.1 Introduction

In this chapter I study life-cycle fertility patterns and the cross sectional distribution of births in the U.S. during the mid 1990s. Using data from the National Survey of Family Growth for the year 1995 (NSFG95), I document the following differences in fertility profiles by educational groups during their life-cycle: more educated individuals have fewer children, start their childbearing later and get less abortions than their less educated counterparts.

My approach to understand the facts is as follows. I embed a standard economic model of fertility (the allocation of mother's time variety¹) into a rich life-cycle, consumption-savings framework² where fertility is an endogenous choice subject to idiosyncratic shocks (undesired outcomes). Using this model, I revisit the question of why there exist a negative skill-fertility relationship and ask whether this model can accommodate the cross-sectional and life-cycle variation in the data.

¹This theory was first developed by [40] and [6] and used recently by [20] and [17], among others

²The basis of this structure is in [27] and in [26]. The main difference with those papers is that they don't have endogenous fertility. On the other hand, I depart from general equilibrium due to computational burden considerations

Results from the quantitative exercise show that differential fertility risk is important when trying to account for differences in the timing and the number of births across educational groups. Thus, relying on substitution effects alone, as is done in standard fertility theories, is not sufficient to produce a negative skill-fertility relationship while trying to match life-cycle facts: under imperfect capital markets, non-labor income (savings) is endogenous and higher for more skilled/lucky earners. This creates an income effect that can overpower the opportunity cost of having children. In other words, higher educated individuals with higher wages, can insure better against the costs of child rearing. This effect is reinforced if we account for marriage and positive assortative matching in the analysis: females with more education tend to marry more educated males, who have higher wages. Just like own savings, male wages act as an insurance against child expenses and the consumption cost of not working. Using my model, I can assess quantitatively the importance of the relevant margins for fertility decisions and show how fertility risk plays a role in generating different outcomes by educational groups.

Throughout this chapter, fertility risk has a dual meaning: earlier in life, it represents the fact that there exist failure when using contraceptive technologies and pregnancies may occur sooner than expected or when they were not wanted at all; later in life, fertility risk acquires a different connotation, since females who postpone childbearing find themselves dealing with biological constraints to conceive. I measure fertility risk by estimating my model using individual data from the NSFG95 on pregnancies, abortions and intentions on conceptions (whether pregnancies were planned or not).

After estimating my model, I perform two quantitative experiments and assess the impact on fertility rates: (i) banning abortion and (ii) equalizing job market opportunities across males and females. The first experiment raises total fertility rates by 7%, a small number considering that abortion rates in the data are close to 15%. This result comes from the fact that rational agents substitute abortion (a costly

but effective form of contraception) with more preemptive efforts against pregnancies when the abortion possibility is not available. On the other hand, equating job market opportunities across genders (by eliminating gender wage gaps in life-cycle profiles of mean income) raises the total fertility rate by almost 20%. This last exercise shows that the model estimates imply a strong income effect of wage profiles in the demand for children, so higher wages for females have a strong positive effect on births, even if the opportunity cost of children rises.

My approach borrows insights from the empirical microeconomic literature that studies life-cycle fertility³ using structural and dynamic models of fertility choice. From that literature, this chapter relates to [53] and [30] who acknowledge the importance of the stochastic nature of fertility. Wolpin analyzes how child mortality risk shapes fertility choices using Malaysian data; Hotz and Miller estimate birth control method choices by females in a life-cycle framework. However, my approach differs starkly in terms of assumptions regarding capital markets and preference heterogeneity (this is true for the whole literature and not just the specific papers mentioned above): I assume imperfect capital markets in the sense that agents can save but not borrow against their future earnings; also, I impose the same preferences for all agents, downplaying the role of unobserved heterogeneity in utility.

This chapter relates the most to [48] and [11]. The first paper provides evidence that more educated individuals are more efficient using different birth control methods, which is the main mechanism through which I obtain a negative skill-fertility relationship. However, [48] restrict their attention to all-white couples in intact first marriages, which might produce sample bias in their regression estimates. Also, they analyze a time period (late 1960s and early 1970s) when policies regarding birth control were different to the ones in the period I analyze: the pill was still not massively adopted by single females and abortion was not readily available to everyone. On the other hand, [11] studies fertility and educational attainment in the U.S. and develops a general equilibrium overlapping generations model in which agents choose

³See [29] for a survey

whether to conceive period by period and how much to consume and save. The main difference between this chapter and [11] is that he assumes fertility risk when couples are seeking a birth (in the form of a constant probability of getting pregnant if one chooses to) but perfect control when they don't want a pregnancy.

The present exercise also contributes to the literature that investigates the sources of lifetime inequality during using models with heterogenous households. As pointed out by [24], family composition shocks⁴ are an important source of both uninsurable economic risks (e.g. births) but also insurance (e.g. between couples in the same household), making the understanding of fertility decisions an integral part of this research agenda. This chapter provides a tractable fertility model that addresses questions regarding motives and timing of fertility decisions.

The structure of the chapter is as follows: In the next section I describe my data sources and the main stylized facts I want to explain. In section 2.3, I pose a simple static model in which I show why standard theories of fertility need to be expanded when moved to a life-cycle setting. In section 3.3, I describe my quantitative model. Sections 2.5 and 2.6 describe the functional forms used in the model and the specific estimation method to obtain model parameters. I show the estimation results and some quantitative experiments in section 2.7. The final section concludes.

2.2 The Facts

I use information from the National Survey of Family Growth (NSFG) to put forward a set of facts on U.S. fertility. The NSFG is compiled by the National Center for Health Statistics (NCHS) and gathers information on family life, fertility, use of birth control and other health related questions. I use the survey for the year 1995, which comprises around ten thousand women between the ages of 15 and 44.

For every survey participant, the NSFG collects retrospective information on usage of birth control methods, on a monthly basis for up to 5 years. Participants also

⁴See for example, [13]

answer questions on wantedness and timing of births and pregnancy outcomes for all pregnancies conceived during that 5 year period. The survey also contains information on educational attainment, marital status and other background information.

I present age specific fertility rates in figure 2.1 and age specific abortion rates in figure 2.2.

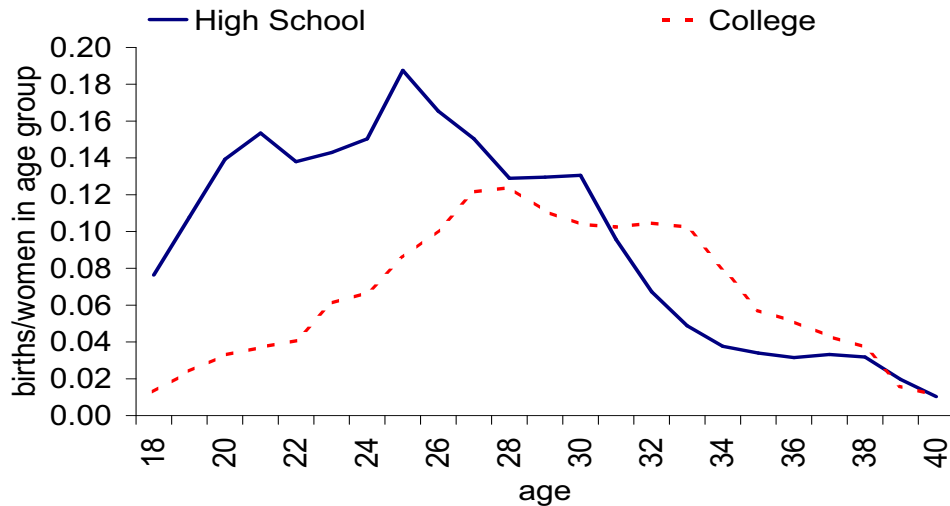


Figure 2.1: Age-specific fertility rates by education of the mother (NSFG 1995)

Both graphs present information on pregnancies occurring between 1994 and 1995, for each particular education-age group, i.e., I'm focusing on the cross-sectional dimension of the data⁵ Age specific fertility rates are defined by the ratio between the number of pregnancies in the specific education-age group and the total number of women in that group. Abortion rates are the number of abortions divided by the total number of women in each group. I divide groups according to educational attainment as follows: High School (those without any post-secondary education) and College (those with at least some post-secondary education). In both figures, I show smoothed statistics (moving averages of 3 years).

The following is a list of stylized facts from the data:

⁵Hence, I'm assuming NO cohort effects in fertility rates. A more complete discussion is in the Appendix.

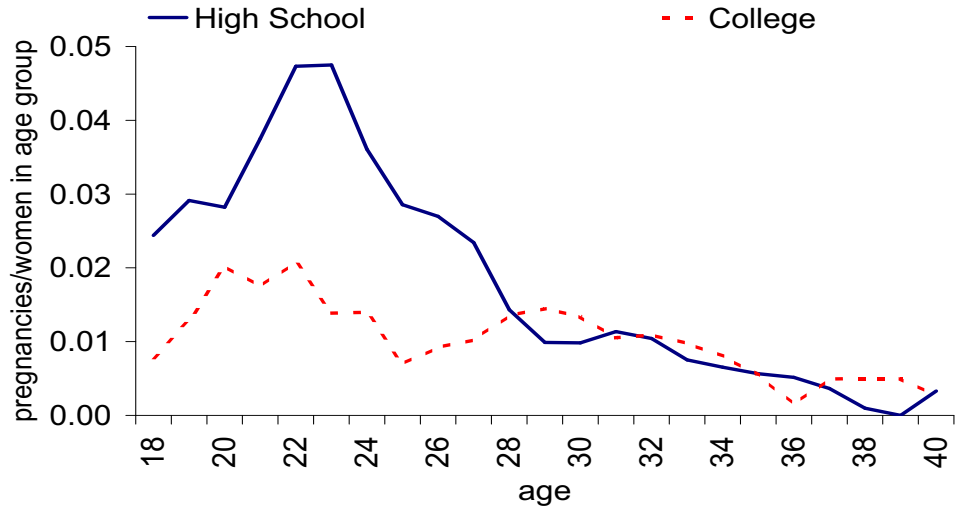


Figure 2.2: Age-specific abortion rates by education of the mother (NSFG 1995)

1. The education fertility gap: the high school group has a total fertility rate (TFR)⁶ of 2.2, while for the college group, it is 1.5 .
2. Timing of births: high school females start having children earlier than their college counterparts. According to figure 2.1, the age with the highest fertility rate is 25 for the High school group and 28 for the college one.⁷
3. Failure in fertility plans: the number of aborted pregnancies is higher for the high school group. The abortion rate for high school educated females is approximately 18 per 1000 women, while the number for college educated females is 11

All but the last fact have been well documented in recent economic literature. Since income of more educated individuals is higher, fact 1 above can be restated as the well known negative income-fertility relationship.⁸ The differential timing of

⁶Total fertility rates are the sum of of the age specific fertility rates and represent a cross-sectional measure of aggregate fertility

⁷A similar statistic is average age of first birth (22.7 and 26.2 for high school and college respectively)

⁸This observation goes back to [5]. [32] study Census data and find that this negative relationship is robust across time and different definitions of income

births is documented and studied by [9], who argue that returns to experience as well as marriage markets play an important role in explaining delay in childbirth.

The last fact shows that failure rates are more acute for the high school group. In terms of accuracy of this data, [15] claim that the introduction of computer assisted interviews in the NSFG for the year 1995 helped in reducing underreporting of abortions and unplanned pregnancies. Nevertheless, their study shows (by comparing implied abortion rates from the NSFG to data from abortion providers in the U.S.) that non reported abortion cases are still present and are higher for lower income groups (the high school group). Hence, differences in failure rates by educational groups are likely to be more pronounced if the data did not present any missed observations in the form of misreported abortions.

2.3 Example: A Static Model

The example below is useful to understand the basic features of standard economic theories of fertility. I also show how a static model of fertility can be modified to account for stochastic fertility outcomes and a dynamic setting (period by period choices). This example then shows where the standard time allocation theory might fail when faced with life-cycle considerations and why fertility risk is a natural solution.

Suppose that individuals derive utility from consumption c and the number of children k in the household. I assume separability in the utility from both elements and log-preferences. Agents have one unit of time which can be sold in the market at rate w and also have access to some non-labor income a . If there are children in the household, agents must spend a fraction $b(k) \in (0, 1)$ of their time taking care of them. This function is increasing in k . I ignore good-costs of children to keep the analysis simple. I model fertility choices in a two stage setting. In the first stage agents choose whether to increase the size of their household. During the second stage, agents choose optimal consumption.

In the second stage, agents solve the following problem, given the stock of children k (chosen during the previous stage)

$$\begin{aligned}
V(a, w, k) &= \max_c \log(c) + \gamma \log(k) \\
&\text{s.t.} \\
c + wb(k) &= a + w \\
\Rightarrow V(a, w, k) &= \log[a + w(1 - b(k))] + \gamma \log(k)
\end{aligned}$$

Besides being increasing, I assume that for any $k_1 > k_0$, $b(k)$ (time cost of children) satisfies the following

1. $V(0, w, k_0) > V(0, w, k_1)$
2. $\frac{1-b(k_0)}{1-b(k_1)} > \frac{a+w(1-b(k_0))}{a+w(1-b(k_1))}$

Assumption 1 states that if non-labor income is zero, the status quo in terms of family size is always preferred. The second assumption is a restriction on the way $b(k)$ affects the budget constraint of the household in terms of resources and time. Both assumptions are restrictive but provide unambiguous results in the examples below. Once we depart from these assumptions, however, answers must come from a quantitative exercise.

Deterministic Fertility Choice: In the first stage, and given a startup number of kids k_0 , the fertility problem is simply

$$v^f = \max\{V(a, w, k_0), V(a, w, k_1)\}$$

with $k_1 > k_0$. The two lemmas below show that the optimal policy function for kids is a step function, that jumps from k_0 (low fertility) to k_1 (high fertility) depending on both wages and non-labor income.

Lemma 2.3.1 *There exists a unique $w^*(a, k_0)$ such that $V(a, w^*, k_0) = V(a, w^*, k_1)$*

Proof In the Appendix. ■

Lemma 2.3.2 *There exists a unique $a^*(w, k_0)$ such that $V(a^*, w, k_0) = V(a^*, w, k_1)$*

Proof In the Appendix. ■

The first lemma says that below some threshold wage w^* , the optimal choice is to have high fertility k_1 . This is the standard negative income-fertility result. On the other hand, lemma 2.3.2 shows an opposing, "nesting effect": above some threshold a^* of non-labor income, individuals would choose higher fertility. Note that in a life-cycle setting, non-labor income can be thought of as savings from previous periods. Hence, the final income-fertility relationship cannot be derived as straightforward as in the static case. This is true in general, when non-labor income and labor earnings are positively correlated.

I will use a similar structure for the full quantitative exercise below. However, this basic framework is not suited to account for heterogeneity in fertility across individuals with the same wage or level of non-labor income. Below I introduce stochastic fertility and imperfect control and show how this extension provides a natural framework to understand the facts.

Stochastic Fertility Choice: Now, assume that individuals must exert contraceptive effort x in order to influence the probability of no-conception $\pi : \mathbb{R} \rightarrow (0, 1)$, which is an increasing and concave function. They also face some utility cost $c(x)$ of exerting effort, which is always positive, increasing and convex. Then, the problem during the fertility stage is

$$v_s^f = \max_x \pi(x)V(a, w, k_0) + [1 - \pi(x)]V(a, w, k_1) - c(x)$$

Using the first order condition from this problem as well as assumptions 1 and 2

from before, it can be shown that $\partial x/\partial w > 0$ and $\partial x/\partial a < 0$. Hence, if we define the expected fertility outcome given optimal effort x^* as

$$k_s^*(a, w) = \pi(x^*)k_0 + [1 - \pi(x^*)]k_1$$

we get that

$$\begin{aligned} \frac{\partial k_s^*}{\partial w} &< 0 \\ \frac{\partial k_s^*}{\partial a} &> 0 \end{aligned}$$

The optimal policy functions for fertility choice and their relation to both wages and non-labor income are depicted in figures 2.3 and 2.4 respectively. In both figures, the deterministic case is shown as a step policy function while the stochastic case is a smooth one.

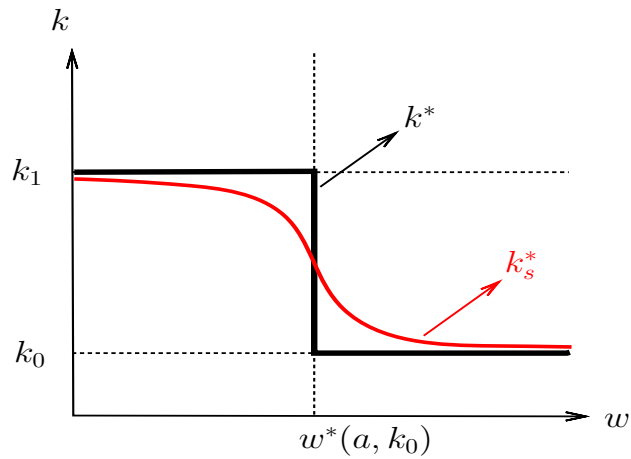


Figure 2.3: Optimal fertility choices with respect to wages

If we consider an economy populated by a continuum of individuals facing the same problem, stochastic fertility and imperfect control produce a non-degenerate

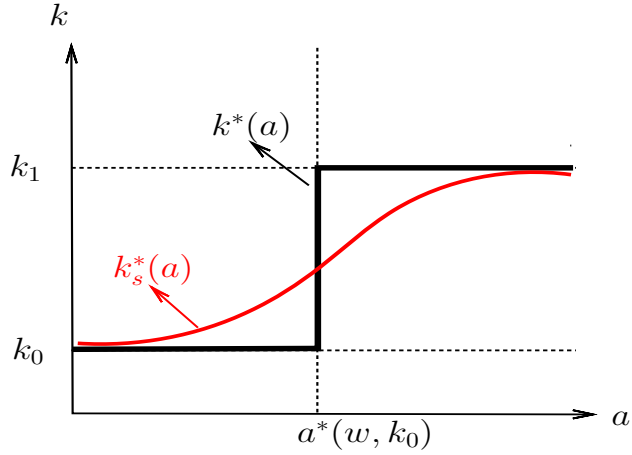


Figure 2.4: Optimal fertility choices with respect to non-labor income

fertility rate (unlike the deterministic case, where the fertility rate is a fixed number): it is an endogenous distribution that depends on incentives and the shape of both $\pi(x)$ and $c(x)$.

As I showed in the previous section, the sign of the wage-fertility relationship is ambiguous if we let labor and non-labor income to be positively correlated (as it is the case within educational groups), since they act as two forces influencing fertility in opposite directions. In the stochastic setting, these forces act in the same way on the optimal contraceptive effort, thus the level of contraceptive failures by skill group cannot be assessed either.

One way of rationalizing the educational fertility gap then, would be to implement a model with heterogeneous preferences for children.⁹ However, heterogeneous preferences can not explain higher levels of error in fertility plans by educational group (abortions and unplanned pregnancies). The alternative I propose is to allow for differential effectiveness of birth control effort on fertility outcomes. This approach has been proposed before by [48] and in the setting below, it also helps in matching the facts on abortion.

⁹See for example, [31]

2.4 A Quantitative Model

The model environment is an economy populated by agents of different gender (males and females) and education level (high and low) who live finite lives and face three types of exogenous and idiosyncratic shocks: to their life (survival shocks), to their household type (marital transition shocks) and to their earnings (shocks to the value of their market rewards). All agents derive utility from consumption and from the presence of children in the household. Agents supply labor inelastically to the market before retirement and every period they decide how much to consume and save for the future; they cannot borrow.

During the first part of their life-cycle, female agents are fertile (can conceive children) and decide on contraceptive efforts period by period. This effort influences imperfectly the probability of conception. Unwanted pregnancies can be aborted; both contraceptive effort and abortions come at a utility cost. After a birth, female agents must spend some time at home rearing their children. Male agents are not affected by this requirement.

State space. Let z be the state space that defines an agent in this economy. Throughout the discussion, I focus on the female's point of view:

$$z = \{e, e^*, i, k, m, \epsilon, \epsilon^*, i^*, a\} \quad (2.1)$$

Asterisks represent values for spouses (when applicable). Age is indexed by $i = \{i_0, \dots, I\}$, $k = \{1, 2, \dots, K\}$ represents the number of children living in the household (not the same as parity), $m = \{1, 2, 3\}$ is the type of household (1 = single, 2 = married, 3 = widowed/divorced¹⁰), $e \in \{\underline{e}, \bar{e}\}$ represents the education type of the agent (low, high), ϵ is the value of the multiplicative shock to labor earnings and a is the amount of real assets in the household.

¹⁰Features of widowed vs. divorced households are unified in a single state, since their distinctions in the data are not significant

For ease of exposition, in some sections of the chapter I use the following partition of the state space $\tilde{z} = \{m, e, e^*, \epsilon, \epsilon^*, i^*\}$ so that $z = \{i, a, k\} \times \tilde{z}$.

The Life-cycle proper. All agents start life at age i_0 (first year of adulthood) being one of two educational types: low (\underline{e}) or high (\bar{e}). This type doesn't change and can be considered as a decision taken before the events in the model. Agents can also start life as married or single and with or without children.

The maximum lifespan for all agents is of I years. Survival from age i to $i + 1$ is subject to state dependent mortality risk, i.e., the probability of surviving an additional year depends on the gender and the educational type of the agent. I denote this probability as $\delta_{i,e}$. The probability for males is δ_{i^*,e^*}^* .

With regard to labor markets, agents work until they reach age i_r . The retirement age is common for males and females. Female agents also make fertility decisions from i_0 to i_f , the last fertile age. This cut-off for the fertile period is common and known to all female agents.

Fertility and children. During their fertile years, females choose effort to determine the probability of a pregnancy. I denote this effort as $x \in \mathbb{R}$, which translates into a probability $\pi(x|i, m, e) \in (0, 1)$ of *no* conception. This stochastic production function of no pregnancies depends on the age of the female agent (to capture biological constraints on women's reproductive systems), her marital status (since conception opportunities might differ if a mate is present or not) and her education. Evidence of this last point is in [48], who estimate differential effectiveness rates of contraceptive use by educational attainment. The exertion of this effort comes at a utility cost of $C(x)$.

With complementary probability $(1 - \pi)$, a pregnancy occurs. If the pregnancy falls into the category of "unplanned/unwanted" (i.e., a positive amount of contraceptive effort was exerted), agents have the opportunity of getting an abortion at a utility cost κ_e . This cost depends on the educational level of the agent. If the pregnancy is

intended (i.e., $x < 0$) the agent keeps the child and the household increases its size by one.¹¹

I make the assumption that children are attached to female agents. I don't keep track of the age nor the sex of children in the household for reasons of computational burden. Instead, households face a constant hazard rate for the permanence of children in the household. I denote this hazard by s_k , which means that on average, children spend $1/s_k$ periods attached to their mothers.¹² Finally, no children can stay in the household after retirement of the mother.

Marital states. The transition through different marital status is stochastic and exogenous. The probability of going from m to m' (conditional on both spouses being alive, in case of agents being married) is given by $\Gamma_{i,e}(m'|m)$. I assume that mortality shocks hit the household before marital transition shocks.¹³

Markets. Agents sell their time to a spot market for labor, receiving a fixed price of w . They can also save positive amounts of resources, i.e., they can rent assets at the market rate r .

Labor endowments. Agents are endowed with state dependent efficiency profiles, $\varepsilon_{i,m,e}$ for females and ε_{i^*,m^*}^* for males. They also face idiosyncratic and persistent multiplicative income shocks (ϵ and ϵ^*). The processes generating these shocks are also state dependent. Hence, for males of age i^* , marital status m^* and education level e^* , labor income is given by

$$w\epsilon^*\varepsilon_{i^*,m^*,e^*}^*$$

Note that w is the market rental rate for efficiency units of labor. On the other hand, if children are present in the household, females need to devote some time

¹¹There is no child mortality risk nor multiple births

¹²This hazard rate is independent for each child in the household (if $k > 1$).

¹³This is important to calculate expectations over future states

taking care of them. These time requirements are reflected in $b(m, k) \in (0, 1)$, so that labor income of females/mothers is given by

$$w \in \varepsilon_{i,m,e} (1 - b(m, k))$$

Since I don't keep track of ages of children in the household, $b(m, k)$ is not time dependent. This simplifying assumption is in contrast of evidence that children require more time and money as they grow old.¹⁴

Preferences. Agents in the economy derive utility from per period consumption and the number of kids in the household. Hence, children are treated as durable goods in terms of utility and their characteristics (such as age and sex) are not qualities that enter agents utility function. In this chapter I restrict attention to preferences that are separable in consumption and number of children of the form

$$u(c|z) + \gamma g(k)$$

Preferences for consumption depend on the characteristics of the household (z), namely, the number of members living under the same roof. This is to capture economies of scale in consumption and the idea that marriage might create consumption habits.¹⁵

Since the focus of the chapter is on females and fertility, utility of married households is taken to be that of the female member. This could be the result of using unitary theories of the household or theories that allow for intra-household bargaining and the female having all the bargaining power. This assumption is restrictive, but necessary to keep this a feasible exercise.

Agents in this economy don't have the ability/desire of leaving bequests upon death and don't receive utility from their children once they leave the household.

¹⁴For example, see [4]

¹⁵See [27].

The Dynamic problem when fertile. There are three distinct stages in the life-cycle of a female agent: (1) work-fertile stage, (2) work - infertile stage and (3) Retirement. Figure 2.5 presents the timing of events during the first stage, when females make both fertility and consumption/savings choices.

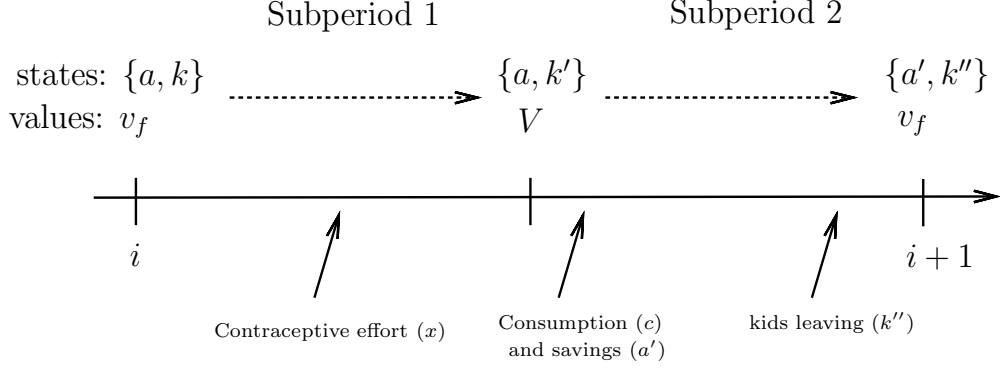


Figure 2.5: timeline of events

As seen in the figure, agents in this stage transit between subperiod 1, where they make fertility decisions and subperiod 2, where they choose consumption and savings for the future. Before transiting to subperiod 1 again, households face an updating in their stock of children (due to kids leaving their mothers).

The following bellman equation represents the problem of agents during sub-period 2 (once they have made contraceptive effort choices):

$$V(i, a, k, \tilde{z}) = \max_{c, y} u(c|z) + \gamma g(k) + \delta_{i,e} \beta E [v_f(i+1, a', k', \tilde{z}')|z] \quad (2.2)$$

st :

$$c + y = (1+r)a + w\epsilon_i \epsilon_{i,m,e} (1 - b(m, k)) \quad \text{if } m = \{1, 3\}$$

$$c + y = (1+r)a + w\epsilon_i \epsilon_{i,2,e} (1 - b(2, k)) + w\epsilon_{i^*}^* \epsilon_{i^*,2,e}^* \quad \text{if } m = 2, i^* < i_r$$

$$a' = \Phi(y, z'|z)$$

where m represents current marital status ($m = 1, 2, 3$ stands for single, married

and widowed/divorced respectively). The budget constraint accounts for different states, since married agents receive extra income from their spouses' labor, but only if the spouse is not retired ($i^* < i_r$). The Φ operator translates the amount of savings into next period assets given marital transitions and future states.¹⁶

Given optimal policies in subperiod 2, females make contraceptive effort choices in subperiod 1. The problem faced by them is:

$$\begin{aligned}
v_f(i, a, k, \tilde{z}) = & \max_x \pi(x|i, m, e)V(i, a, k, \tilde{z}) \\
& + [1 - \pi(x|i, m, e)] \max \left\{ \begin{array}{l} V(i, a, k + 1, \tilde{z}), \\ V(i, a, k, \tilde{z}) - \kappa_e \end{array} \right\} \\
& - C(x)
\end{aligned} \tag{2.4}$$

The value function at this stage is a convex combination of the continuation values with and without a new pregnancy. In the case of pregnancy (which occurs with probability $(1 - \pi(\cdot))$), agents have the chance of having an abortion at utility cost κ_e . Note that even though there are discrete outcomes following this stage (number of children in the household), the effort function convexifies the problem maintaining smoothness of the value function, which proves useful for solving (2.3) using standard continuous methods.¹⁷

¹⁶The particular form of Φ is given by:

$$\Phi(y, z'|z) = \begin{cases} y & if & (m' = 2|m = 2) \\ y & if & (m' = 1, 3|m = 1, 3) \\ y & if & (m' = 3|m = 2) \text{ (widowhood)} \\ \chi y & if & (m' = 3|m = 2) \text{ (divorce)} \\ y + a^* & if & (m' = 2|m = 1) \end{cases} \tag{2.3}$$

where (m', m) refers to a transition from m to m' next period. For example, when going from $m = 2$ (married) to $m = 3$ (through divorce), assets next period are a fraction χ of what is saved today, where $\chi \in (0, 1)$ reflects the partition of assets after a divorce. Note that when going from $m = 1$ (single) to $m = 2$ (married), assets next period are given by current savings plus what the prospective spouse brings to the household. This last variable (a^*) is a random variable that depends on the distribution of single agents of the opposite sex in the economy.

¹⁷Details of the numerical solution procedure are in the Appendix.

My approach to model fertility choices differs from those who try to understand choices of specific birth control methods by women.¹⁸ The setup above doesn't distinguish between different contraceptive methods nor their efficacy, but is general and its implementation straightforward.

Moreover, I allow the probability of no conception to be flexible enough so that overall fertility is not only due to failed birth control but also as the result of conscious efforts of females to start a family. Specifically, this means that the domain of π is the entire real line (contraceptive effort can be negative, in order to maximize the probability of conception) and the cost function is always positive, increasing away from zero. This general specification allows me to capture biological constraint on human fertility, which play a role in determining the optimal timing of births later in life.

The dynamic problem after fertile years. Once agents are past the fertile stage (cannot produce more children), they keep choosing optimal paths for consumption and savings until death. This stage in the life-cycle can also be divided into two: before and after retirement.

Before retirement ($i \leq i_r$), the problem of the agent is:

$$\begin{aligned}
V(i, a, k, \tilde{z}) &= \max_{c, y} u(c|z) + \gamma g(k) + \delta_{i,e} \beta E [V(i+1, a', k', \tilde{z}')|z] & (2.5) \\
st : & \\
c + y &= (1+r)a + w\epsilon_i \varepsilon_{i,m,e} (1 - b(m, k)) & \text{if } m = \{1, 3\} \\
c + y &= (1+r)a + w\epsilon_i \varepsilon_{i,2,e} (1 - b(2, k)) + w\epsilon_{i^*}^* \varepsilon_{i^*,2,e}^* & \text{if } m = 2, i^* < i_r \\
a' &= \Phi(y, z'|z)
\end{aligned}$$

The main difference between this Bellman equation and the one in (2.6) is that the the stock of children can only decrease from period to period.

¹⁸See for example [30] and [47].

After retirement, the problem reduces to

$$\begin{aligned}
V(i, a, 0, \tilde{z}) &= \max_{c,y} u(c|z) + \gamma g(k=0) + \delta_{i,e} \beta E [V(i+1, a', 0, \tilde{z}')|z] & (2.6) \\
st : & \\
c + y &= (1+r)a & \text{if } m = \{1, 3\} \\
c + y &= (1+r)a + w\epsilon_{i^*}^* \epsilon_{i^*,2,e^*}^* & \text{if } m = 2, i^* < i_r \\
a' &= \Phi(y, z'|z)
\end{aligned}$$

at this stage no children are present in the household ($k = 0 \forall i \geq i_r$) and the only resources available for non-married agents are past savings. On the other hand, if agents are married to working age individuals, they enjoy the extra labor income $w\epsilon_{i^*}^* \epsilon_{i^*,2,e^*}^*$.

2.5 Taking the Model to the Data

The solution of this model is a set of policy functions $x^{opt}(z|\Theta)$, $y^{opt}(z|\Theta)$ for contraceptive effort and savings respectively, given the current state z and other parameters, Θ (including prices). As it's usual, analytical expressions for the optimal policies are unfeasible, so I approximate them using numerical solutions to an empirical model with the following quantitative features.

Demographics and life-cycle. All agents start life at age 18 and cannot live longer than 95 years. Retirement is at 65 and the last fertile age is 40. A model period is one year when $i \in \{18, \dots, 40\}$, 5 years when $i \in \{40+1, \dots, 65\}$ and 10 years when $i \in \{65+1, \dots, 95\}$.¹⁹ Age specific mortality rates are taken from the National Center for Health Statistics and adjusted for educational attainment, as in [26].²⁰

¹⁹I do this to reduce the state space of the model. For details, see the Appendix.

²⁰Given the mapping from model periods to actual years, all age specific variables used in the computation are recalculated, depending on the stage of model life-cycle. Details in the Appendix.

I divide educational or skill types into those with at most a high school diploma or GED, and those with some post secondary education (college, community college, vocational school, etc.). To calculate the proportion of these types, I use the Current Population Survey (CPS) between 1990 and 1995. The proportion of high school individuals is around 40%. The majority of agents start life as single and childless, but I allow some of them to be married and have children. The proportion of never married 18 year old females in the CPS is around 93% and females with kids is around 9%. When performing simulations of the model, I distribute women uniformly according to these statistics to determine their initial state.

Since non-married females can always find a (new) partner in the model, I need information on who they'd marry. Also from the CPS, I compute the proportion of couples by age and educational attainment of the partners, the age distribution of male partners for married females and the relative asset position of both non-married males and non-married females.²¹ Given this information, I construct education-specific grids with probabilities of marrying someone of characteristics given by $\{e^*, i^*, a^*\}$ (education, age and assets of prospective husbands). Since I'm not computing equilibrium, this procedure doesn't check for internal consistency of measures of agents (as in [27], where all these probabilities are endogenous objects).

Transitions between marital states come from the Panel Study of Income Dynamics (PSID) for the years 1990-1995. I follow all heads of household older than 18 years old (inclusive) and compute annual age and education specific transition probabilities between three states: single, married and divorced/widowed. Given variable specification in the PSID, married couples include cohabitating couples.

Preferences. I use an additively separable specification for instantaneous, per period utility: $u(c|z) + \gamma g(k)$. The marginal utility from consumption depends on the size of a household:

²¹My proxy for individual assets is the sum of interest, dividend and rent income as defined in the March supplements of the CPS

$$u(c|z) \equiv \tilde{u} \left(\frac{c}{1 + \mathbf{1}_{\{m=2\}}\phi_m + \mathbf{1}_{\{k>0\}}k\phi_k} \right) \quad (2.7)$$

Where $\mathbf{1}_{\{cond\}}$ is the indicator function that takes a value of one when "cond" is true and zero otherwise; ϕ_m and ϕ_k are equivalence scales which discount consumption in married households and households with children respectively. If $\phi_m, \phi_k < 1$, economies of scale in consumption exist in the household: expenditures to maintain the level of per capita utility constant, grow proportionally less than the number of household members.

The specific functional forms for \tilde{u} and g are given by

$$\tilde{u}(\mathbf{c}) = \frac{\mathbf{c}^{1-\eta_c} - 1}{1 - \eta_c} \quad g(k) = \frac{(1+k)^{1-\eta_k} - 1}{1 - \eta_k} \quad (2.8)$$

Fertility. I use the following function for π (the probability of NO conception, given effort x):

$$\pi(x|i, m, e) = \begin{cases} \pi^+(x|i, m, e) & \text{if } x > 0 \\ \pi^-(x|i, m, e) & \text{if } x < 0 \end{cases} \quad (2.9)$$

where

$$\pi^+(x|i, m, e) = \frac{\exp\{x\}}{\exp\{x\} + \varphi_{i,m,e}^+ \exp\{-x\}}$$

and

$$\pi^-(x|i, m, e) = \frac{\exp\{x\}}{\exp\{x\} + \varphi_i^- \exp\{-x\}}$$

In general, π is a modified logistic function with φ as a shift parameter. [44] use a similar framework to study fertility choice in an equilibrium model. Note that the higher $\varphi_{i,m,e}^+$, the higher the probability of a pregnancy when effort (x) is positive (females trying to avoid fertility), which means that I can parameterize higher

difficulty in controlling fertility by increasing $\varphi_{i,m,e}^+$. However, if women are trying to get pregnant (negative x), parameterizing π through the same $\varphi_{i,m,e}^+$ would not be realistic, since it would mean that ability in using contraceptive methods is negatively correlated with the ability of procuring a conception when it is desired. Hence, I use a different shifter, φ_i^- for this case. Note that contraceptive ability $\varphi_{i,m,e}^+$ depends on age, marital status and education and its parameterization is given by

$$\varphi_{i,m,e}^+ = \tilde{\varphi}_{i,m}^+ + \mathbf{1}_{\{e=\underline{e}\}}\bar{\varphi}_i$$

While ability in using contraceptive technology might depend on marital status and education, I assume that the technology of procuring a pregnancy depends mostly on biological constraints. This is represented by φ_i^- depending only on age.

I parameterize these age profiles using polynomial approximations on age: given the order for the polynomial (p , the same for all profiles) the number of parameters to be determined is then $p + 1$ (the number of polynomial coefficients) times 5 profiles: for singles ($\tilde{\varphi}_{i,1}^+$), for married ($\tilde{\varphi}_{i,2}^+$), for divorced/widowed ($\tilde{\varphi}_{i,3}^+$), for the extra risk faced by the high school group ($\bar{\varphi}_i$) and for the biological fertility profile φ_i^- . For example, the profile for married individuals is given by

$$\tilde{\varphi}_{i,2}^+ = \alpha_0^2 + \alpha_1^2 i + \alpha_2^2 i^2 + \dots + \alpha_p^2 i^p$$

while the profile for the excess fertility risk faced by the high school group is

$$\bar{\varphi}_i = \bar{\alpha}_0 + \bar{\alpha}_1 i + \bar{\alpha}_2 i^2 + \dots + \bar{\alpha}_p i^p$$

I reduce the number of parameters by assuming that the fertility control technology for singles is the same than for divorced/widowed agents ($\tilde{\varphi}_{i,1}^+ = \tilde{\varphi}_{i,3}^+$).

On the other hand, I parameterize the utility cost of exerting contraceptive effort as

$$C(x) = \frac{x^2}{2}\xi$$

This cost function is symmetric around zero, so I use it for both sides of the fertility problem: when females want to prevent or are seeking a pregnancy. This is not restrictive, given the asymmetric structure of π .

Earnings and Labor Supply. Endowments of labor efficiency profiles come from the CPS (years 1990-1995). I calculate annual labor earnings for the two educational groups (high school and college), by age and marital status. As in [27] and [26], I use annual earnings since they capture differences in the intensive margin of earnings by sex and marital status better than hourly earnings. To account for inflation, I adjust nominal values by the GDP deflator for the year 2000.

I restrict attention to childless females throughout the sample period. For males, I don't make that distinction, since the change in income due to the presence of own children in the household is not significant.

I attribute the time cost of child-rearing $b(m, k)$ to annual labor income differentials of females in fertile age (18 to 40) by number of children. This is different than accounting for hours worked by number of children in the household; it stands alternatively for different ways in which a child might change earnings ability of the mother (e.g., getting a job with more flexible schedule but lower pay, getting a job with lower pay but closer to home, not getting tenured at an academic job or not being made partner at the firm, etc.) other than through hours worked per week. The computed values are in table 2.1.

As seen from the table, time cost of children (or time away from the best paid market alternative) is increasing in the number of children present in the household. Note also that the cost increases faster in the number of kids for married women than for single ones.

For earnings shocks, I use an AR(1) specification

Children	$b(m = \{1, 3\}, k)$	$b(m = 2, k)$
0	0%	0%
1	5.9%	26.5%
2	16.9%	37.5%
3	41.0%	52.6%
4	61.3%	63.3%
5+	81.2%	72.8%

Table 2.1: Time cost of Children (in terms of full time work), CPS 1990-1995.

$$\epsilon'_e = \rho_e \epsilon_e + \mu'_e \quad (2.10)$$

where $\mu_e \sim N(0, \sigma_e)$. These shocks are gender and education specific. I take values of ρ_e, σ_e (for $e = \{\underline{e}, \bar{e}\}$) from [26], who uses the PSID between 1986-1992 to compute maximum likelihood estimates. As is common, I discretize both continuous processes using the method proposed by [52].

2.6 Estimation

Given the partial equilibrium nature of the exercise, I set several model parameters exogenously. First, the rental price of efficiency units of labor w is normalized to 1. I set the interest rate to equal the average of the 1-year Treasury Bill Rate (monthly auction averages).²² I let the discount factor β to be $1/(1+r)$. For equivalence scales, I use $\phi_m = 0.7$ and $\phi_k = 0.5$ (i.e., the OECD values).

The rest of the model parameters are determined jointly, by minimizing the square difference between data and model moments. The procedure is standard in the literature: (i) select which data targets to use (ii) guess values for model parameters (iii) solve the model and calculate optimal policies (iv) simulate life-cycles for a large

²²Series id TBIYA, on the St. Louis Fed Economic Data webpage.

number of individuals and compute model equivalents to the data targets (vi) calculate the error of the iteration (the sum of square values of the difference between every data and model moment) (vii) if the error is less than a pre-specified tolerance, exit; if not, update parameters according to some predefined rule and repeat from step (iii) until convergence. This is a simplified simulated method of moments estimation procedure, where the weighting matrix for moments is the identity matrix.

The list of moments is as follows:

- Age profile of pregnancy rates for non-married females by education²³: 46 moments = 23 ages \times 2 education levels
- Age profile of pregnancy rates for married females by education: 46 moments = 23 ages \times 2 education levels
- Age profile of abortion rates by education: 46 moments = 23 ages \times 2 education levels
- Age profile of unplanned pregnancy rates by education: 46 moments = 23 ages \times 2 education levels

In total, there are 184 moments to match. On the other hand, the number of model parameters depends on the choice of order for the polynomials that define age profiles for fertility parameters. I chose $p = 6$, so each age profile is defined by 7 coefficients. Thus, the model has 34 parameters to be determined jointly:

- curvature in the utility of consumption η_c (1)
- curvature in the utility of children η_k (1)
- multiplicative parameter in utility of children γ (1)
- utility cost of an abortion κ_e (2)

²³Note that I merge the statistics for both single and widowed/divorced females.

- utility cost of contraceptive effort ξ (1)
- contraceptive ability parameter $\tilde{\varphi}_{i,m}$ (14 parameters = 7 coefficients \times 2 marital states)
- conception ability parameter ϕ_i^- (7)
- contraceptive ability shifter for low skill/education group $\bar{\varphi}_i$ (7)

Solution to the model is by backwards recursion. In the last period of life there is no continuation value (I assume no bequests motives nor life insurance) hence optimal policies and value functions can be calculated recursively from the next to last period. Details of the procedure are in the Appendix.

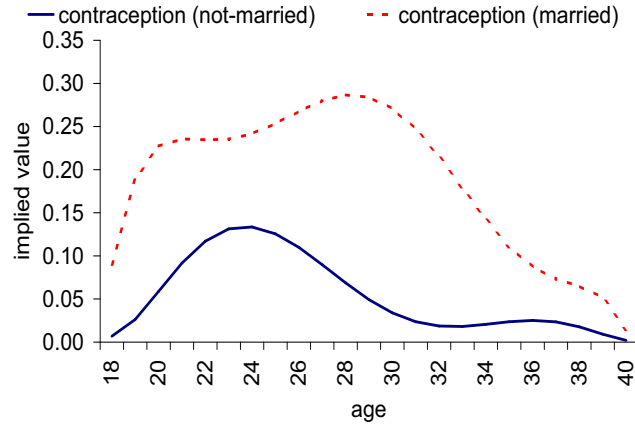
2.7 Results and Experiments

The estimated parameters are in table 2.2. The full list of coefficients for the polynomial functions used in parameterizing conception probabilities is in the Appendix. In figure 2.6, I present instead the implied age profiles given by the estimated polynomials.

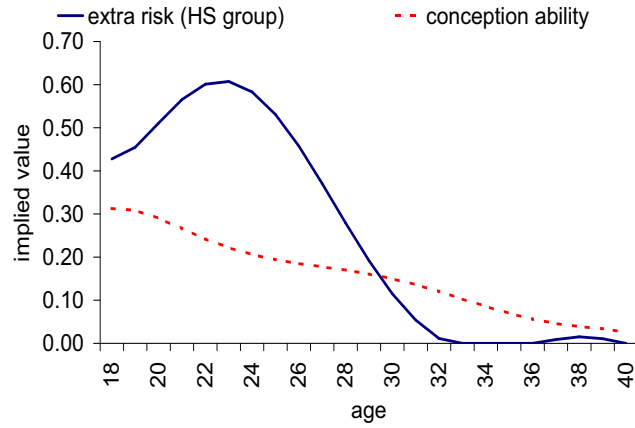
Parameter	value
η_c	1.79
η_k	1.45
γ	0.77
κ_{HC}	3.95
$\kappa_{College}$	1.42
ξ	0.95

Table 2.2: Model Parameters

The estimated curvature in the utility of consumption (η_c) equals 1.78, which is in line with the rest of the literature (the usual number lays between 1.5 and 2). Preferences are close to being homothetic: the value of η_k (1.45) is close to the one



(a) $\tilde{\varphi}_{i,\{1,3\}}$ and $\tilde{\varphi}_{i,2}$



(b) $\bar{\varphi}_i$ and ϕ_i^-

Figure 2.6: Estimated parameters for contraceptive technology and conception ability

for η_c . Overall, the value of both parameters indicate that consumption and children enter as complements in the utility function, so females enjoy the presence of children more when consumption levels are higher. Given increasing wages during the earlier stages of the life-cycle, this means that females would like to postpone childbearing as much as possible.

The utility cost of an abortion is around two and a half times higher for high school individuals than for college individuals (3.95 vs. 1.42 respectively).

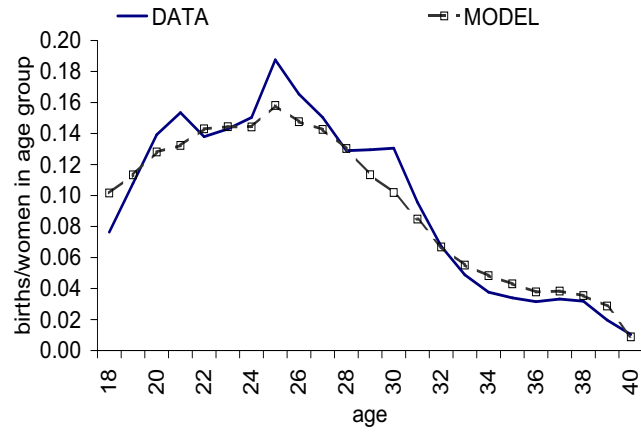
Figure 2.6 shows the age-profiles that describe fertility technology and restrictions during the life-cycle. All profiles are decreasing in age which is a reflection of decreasing chances of conception late in the fertile stage of life. Contraceptive parameters for married individuals are higher than for single ones, which means that birth control is easier when there is no steady partner of the opposite sex in the household. With respect to the risk faced by the high school group, results imply that females in that group have 17% more chances (on average in their lifetime) of having an unwanted pregnancy than their college counterparts. The calculation of this percentage comes from using the age profile $\bar{\varphi}_i$: the approximated probability of an unwanted pregnancy (when exerting a very low amount of effort) for every age is given by $\bar{\varphi}_i/1 + \bar{\varphi}_i$ (The average for all ages is 17%).

Figures 2.7 to 2.9 show the goodness of fit of the model.

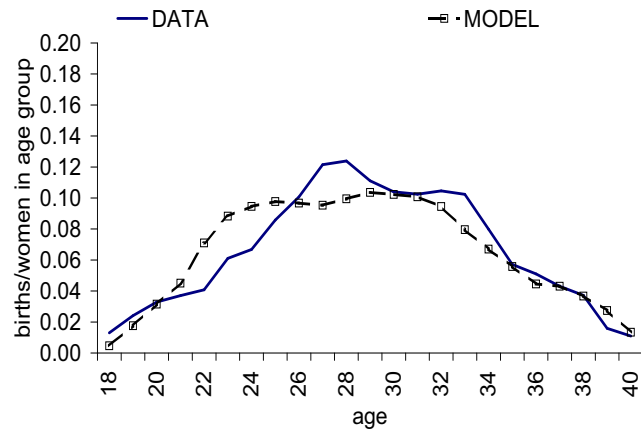
Overall, the model does a good job in replicating the stylized facts with respect to the number and timing of births across educational groups. Both simulated abortion and fertility rates follow closely their data counterparts; on the other hand, the rate of unwanted pregnancies is overpredicted for the high school group but the overall qualitative features of the data are preserved (differences among educational groups).

Relevant Margins Below I present exercises that show the importance of the two main ingredients of my theory, namely differential fertility risk across educational groups and self-insurance (the ability to save for the future).

Figure 2.10 shows the comparison between the baseline model and the case when

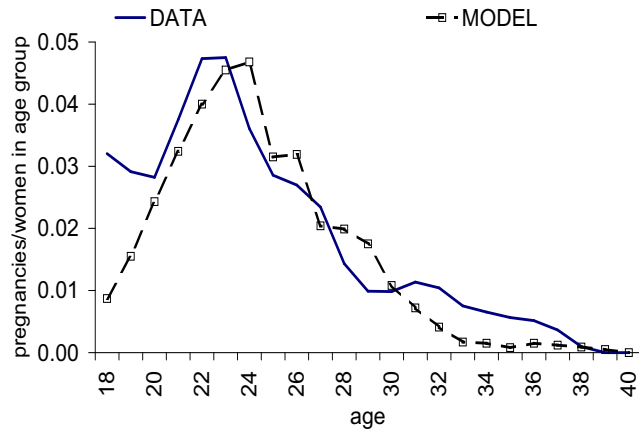


(a) High School

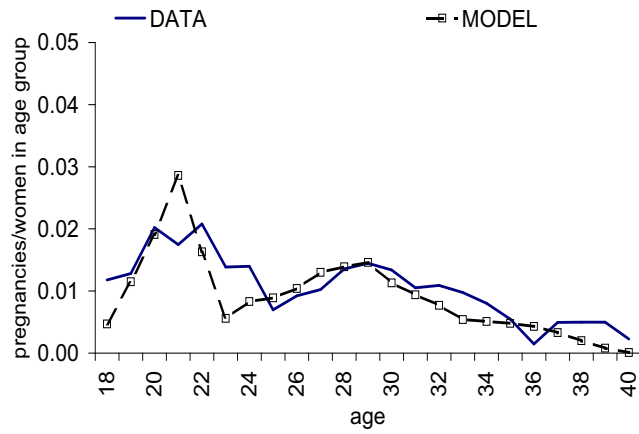


(b) College

Figure 2.7: Age specific fertility rates: Data and Model

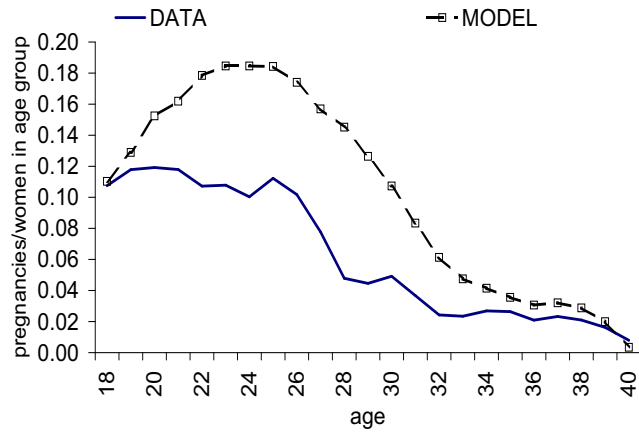


(a) High School

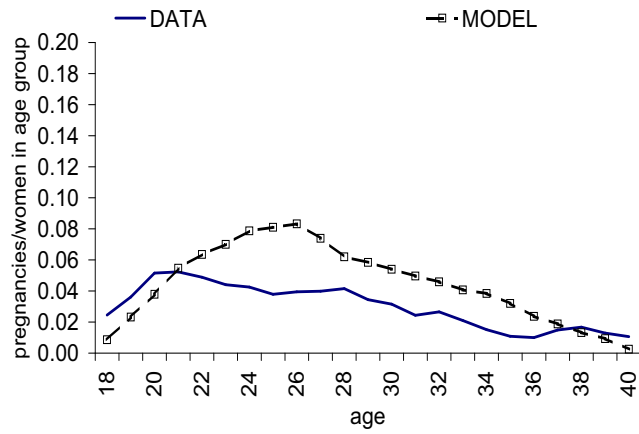


(b) College

Figure 2.8: Abortion rates: Data and Model

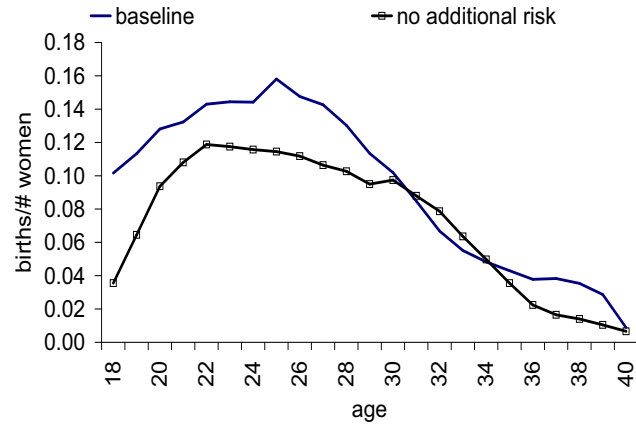


(a) High School

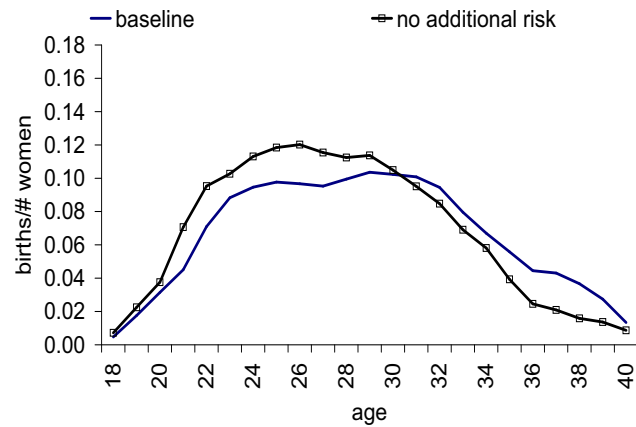


(b) College

Figure 2.9: Unwanted rates: Data and Model



(a) High School



(b) College

Figure 2.10: Age-specific fertility rates, different models

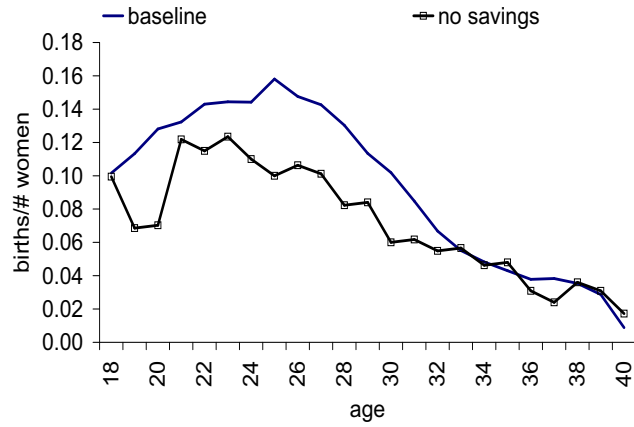
the model is calibrated to match the facts without extra risk for the high school group ($\bar{\varphi}_i = 0, \forall i$). As seen from the figure, the latter model is unable to match the higher fertility rates of high school educated individuals, while the rates for the college group are slightly overpredicted. In terms of timing of births, this alternative model correctly accounts for the delay in childbearing by females in the college group as opposed to those in the high school group; however, this difference is less pronounced than in the baseline model.

In the model where the two educational groups face the same contraceptive technology, the ratio between their TFRs is 0.94 (1.56 and 1.67 for high school and college educated individuals respectively) while the ratio in the data (and the baseline) is closer to 0.7. Hence, differential fertility risk earlier in life accounts for most of the differences in the number of births across educational groups during the life-cycle, leaving a small role for differences in wages. This result hints that differences in wage profiles help mostly in predicting the different timing of births: the flatter profiles of life-cycle wages of high school educated females makes them choose early childbirth (at the margin) given the complementarity between consumption and children.²⁴ On the other hand, college individuals face rapidly growing wage profiles, which induce them to delay fertility.

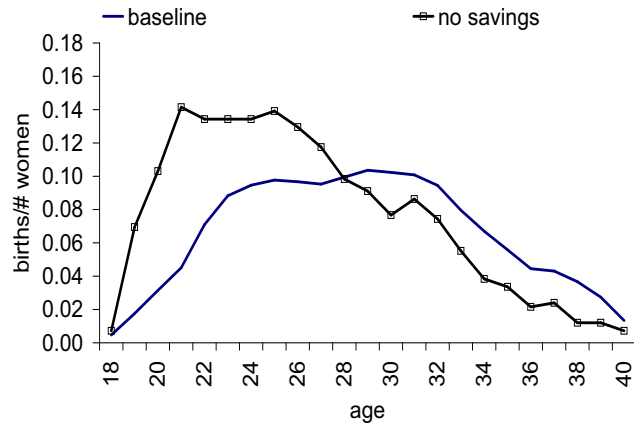
In the next simulation, I take the baseline model, shut down the ability to save and recalibrate the economy. Since I assume that after retirement agents don't receive any income other than past savings, I set the last period in the no-savings simulation to be i_r and discard the retirement stage (setting the value functions after age i_r equal to zero for any point in the state space).

The predicted fertility profiles of the no-savings case are in figure 2.11. Results from simulating this version of the model (no savings but with differential fertility risks) shows that the choice of assumption regarding capital markets changes the predictions of fertility models in non-trivial directions. It also helps in understand

²⁴After re-estimating the parameters, $\eta_c = 1.17$ and $\eta_k = 1.11$, so consumption and children are still complements, but the extent of this complementarity is smaller than in the baseline



(a) High School



(b) College

Figure 2.11: Age-specific fertility rates, different models

the intertemporal margins faced by individuals in the life-cycle.

Given the inability to save, all income has to be consumed at the end of each period. For individuals in both educational groups, consumption in this setting is, on average, higher than what they would have chosen had they been able to save. For individuals in the high school group, this leads to marginal lower fertility rates than in the baseline. The college group, however, faces increasing wage profiles, thus increasing utility. Given complementarity between consumption and children in the utility function, this 'forced' higher consumption for the group leads them to conceive more children to increase utility when wages and consumption are high. Since fertility choice is dynamic and females are restricted to one child per period, the timing of first child birth is shifted towards younger ages, which is a major counterfactual prediction of this variation of the model.

Fertility Shocks as Source of Lifetime Inequality From the quantitative exercise, 'fertility risk' (understood as the inability to control fertility outcomes in a perfect manner) arises as a type of uninsurable shock that has potential distributional consequences for wealth inequality. These consequences are underscored by the fact that 'fertility risk' has a different magnitude depending on educational attainment. In figure (2.12) I show simulated asset accumulation paths, by educational level of the female and total number of children during their lifetime. To analyze inequality, the figure shows the simple difference between percentiles 90 and 10 of the simulated wealth distribution.

A clear trend that arises from the figure is a negative relationship between wealth dispersion and family size, specially for the college educated group. This result comes from the assumption of kids 'costing' a fixed amount of time from mothers while they live at home. Bigger families imply more time away from market work and less exposure to idiosyncratic income shocks. Hence, children act as buffer from income shocks to differences in wealth accumulation. As seen in figure (2.12), this effect is stronger for college educated individuals, for whom income profiles are higher and

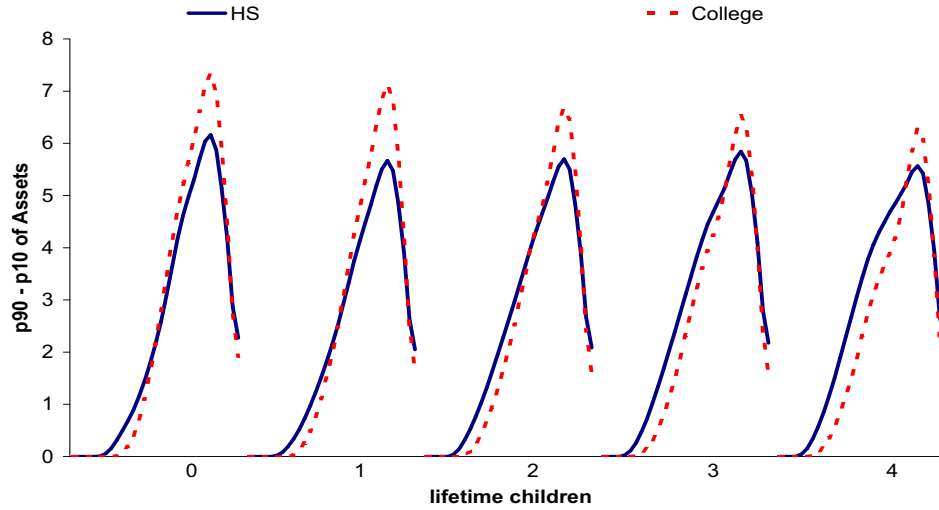


Figure 2.12: Difference in percentiles 90 versus 10 of lifetime assets, by education and family size

steeper than those of their less educated counterparts.

Quantitative Experiments: In this section, I discuss the effect of banning abortions and equating labor market opportunities across genders on total fertility rates, separated by educational group. Table 2.3 below shows the rates after performing each experiment.

	Fertility Rates		
	HS	College	Total
Data	2.21	1.52	1.94
Benchmark	2.15	1.51	1.89
No abortions	2.25	1.66	2.02
No gender gap	2.43	1.94	2.26

Table 2.3: Results from quantitative experiments

In the case of banning abortion (by increasing κ_e to a level in which abortions are prohibitive), the total fertility rate increases 6.87% with respect to the benchmark

(1.89 versus 2.02).²⁵ Given the sequential nature of the fertility process, rational agents react to this environment by exerting higher levels of contraceptive effort in order to avoid pregnancies that would have ended up in abortions otherwise. This is why the total fertility rate increases by only a fraction of the original abortion rate (around 15% of pregnancies).

In the second experiment, I endow all females with the job market prospects and risks of male workers, in terms of age-specific wage profiles (by marital status and education) as well as idiosyncratic wage shocks. This simulation entails an increase in the total fertility rate of 19.58% (from 1.89 in the baseline to 2.26). Although in this specification of the model females have higher wage profiles than in the baseline, thus higher opportunity costs of their time to rearing children (hence less incentives to give birth), fertility rates increase. The main reason for this result is that the parameterization of the model implies strong income effects of higher wage profiles (or "permanent" increases in wages) during the life-cycle; this is due to the complementarity between the utility of consumption and the utility of children in the household, which creates incentives for higher fertility when the whole wage profile increases (which is the case when we increase female profiles to eliminate the wage gap) given that individuals enjoy more kids when consumption levels are higher.

²⁵[36], uses difference in difference methods and exploits variation in Medicaid funding across U.S. states to conclude that making abortion illegal would increase total fertility rates between 3 and 5%.

2.8 Conclusion

In this chapter I study life-cycle fertility in the U.S., focusing on birth profile differences across educational groups (high school and college). To understand the facts on timing and number of births during the life-cycle, I develop a structural model where agents transit through different marital states, face idiosyncratic survival and earnings risk and capital markets are incomplete (individuals cannot borrow against their future earnings). In this setting, I embed a standard fertility model (the "time allocation of mothers" variety) and add the assumption of imperfect control of individuals over fertility outcomes. From the analysis, I conclude that differential fertility risk (in the form of ability to control fertility plans) across education groups is the main determinant of differences in timing and levels of fertility, while differences in marriage/labor markets play minor roles. This shows that standard fertility theories, which rely solely on substitution effects to produce negative skill-fertility relationships, cannot account for life-cycle nor cross sectional facts.

2.9 Appendix

2.9.1 Proofs

Proof of Lemma 2.3.1. Define the value of not increasing family size

$$\Delta V(a, w, k_0) \equiv V(a, w, k_0) - V(a, w, k_1)$$

existence of w^* comes from continuity of the log function and the use of the intermediate value theorem. First,

$$\begin{aligned} \lim_{w \rightarrow 0} \Delta V &= \log \left\{ \frac{a + w(1 - b(k_0))}{a + w(1 - b(k_1))} \right\} + \gamma \log \left\{ \frac{k_0}{k_1} \right\} \\ &= 0 + \gamma \log \left\{ \frac{k_0}{k_1} \right\} \\ &< 0 \end{aligned}$$

On the other hand, $\lim_{w \rightarrow \infty} \Delta V > 0$, by assumption 1. Hence, there must exist at least one wage such that $\Delta V = 0$. For uniqueness, we require $\frac{\partial \Delta V}{\partial w} \geq 0$, which comes from using assumption 2 ■

Proof of Lemma 2.3.2. This proof is analogous to the previous one. First, note that $\lim_{a \rightarrow 0} \Delta V = \log(k_0/k_1) < 0$. On the other hand, $\lim_{a \rightarrow \infty} \Delta V > 0$ by assumption 1, so applying the same logic as above, a^* exists. For uniqueness, we have that

$$\frac{\partial \Delta V}{\partial a} = \frac{1}{a + w(1 - b(k_0))} - \frac{1}{a + w(1 - b(k_1))}$$

which is strictly negative, because $b(k)$ is increasing. ■

2.9.2 Data

Figure 2.13 shows the profiles for labor endowments, computed from march supplements of the Current Population Survey (years 1990 to 1995). In the figure I show annual earnings for females, between ages 18 to 65, corrected for inflation using the GDP deflator for the year 2000. These profiles are smoothed using a 5th order polynomial.

To characterize the labor market, I also use gender and education specific idiosyncratic labor shocks. These shocks come from estimates from [26], who uses labor earnings data from the PSID to calculate the unobserved component of annual labor earnings. I use a standard discretization of the continuous AR(1) described in the paper. I choose to discretize the four processes (2 education groups and 2 genders) by a 3 state markov system. The standard in the literature is to use at least 5 states, but computational burden prevents me from using a more detailed shock structure. However, results in the chapter don't rely in the dimensionality of these shocks.

Also from the CPS, I calculate the proportion of females (by education) married to college educated males (irrespective of presence of children in the household), in order to measure positive assortative matching in the marriage market. As seen in figure 2.14, marriage indeed shows the positive assortative matching property.

I compute yearly survival probabilities by educational group using the information in [26]. I interpolate his 5 year values and smooth the resulting series with a second order polynomial. The resulting probabilities for female individuals are in figure 2.15.

To calculate transitions through marital states, I use the Panel Study of Income Dynamics (PSID) for the years 1990 through 1995. I use heads of household and wives (as defined in the PSID) to compute the following probabilities, by education and age: probability of remaining single, the probability of remaining married and the probability of getting married conditional on being divorced/widowed. Given these three probabilities, I can span all transitions (e.g., some probabilities are zero by definition and others are just complements). I extrapolate these probabilities

when necessary since the PSID doesn't have many observations for young/old heads of household. Given the short span of my chosen sample, individuals contribute at most 5 observations/years, making these probabilities a cross-section description of marital transitions during the mid 1990s in the U.S. Figures 2.16 and 2.17 show these transitions.

I assume simple age and asset distribution of prospective male partners. For ages I consider only 3 possible alternatives: same age, one year older and two years older ($i^* = \{i, i + 1, i + 2\}$), each occurring with probabilities $P(i^* = i) = 0.4$, $P(i^* = i + 1) = 0.41$ and $P(i^* = i + 2) = 0.19$, which come from CPS data. Age of partners is important since they determine the extra income for the household in terms of partner's labor earnings and the probability of death (hence, transitioning to widowhood status). Since the profiles for both characteristics are smoothed, the tradeoff between accuracy and simplicity of the solution by assuming such a narrow age distribution is lessened.

For assets, I calculate from CPS data the average annual non-labor income (dividends, interests and rents) for both single males and females. Single males have on average 20% higher non-labor income than single females. Hence, I create a simple three point distribution for assets of prospective partners $a^* = \{1.1a, 1.2a, 1.3a\}$, centered around the fact that on average $a^*/a = 1.2$. This simple distribution is uniform (equal probabilities for each point). Changing this distribution doesn't alter any of the qualitative results from the exercise.

A note on Total Fertility Rates and cohort effects: throughout the chapter, I assume no cohort effects in fertility rates. Although fertility has experienced significant changes during the 20th century, fertility rates for the cohorts considered in my analysis are quite stable. Figure 2.18 shows age specific fertility rates computed from the internet release of Vital Statistics of the United States for the year 1995 (tables 1-7).

The figure shows both total fertility rates for the cross section in 1995 and for cohorts (denoted by year of birth) across multiple survey years. The differences

between the cross-sectional profile and actual cohort profiles is minimal. This comes from the fact that I am considering a small window in the life-cycle of cohorts that are close together (at most 20 years between births).

2.9.3 Computation and Estimation details

To solve the model, I use a Chebyshev regression (as described in [33] and [25]) to approximate the optimal policies for savings and contraceptive effort and the value function along the asset space (the only continuous state variable in the model). My approximation is described by 7 collocation points and the use of a Chebyshev polynomial of degree 5. Increasing both the number of collocation points and/or the order of the polynomial doesn't improve the quality of the approximation significantly.

A note on the non-standard mapping between model periods and years: I adopt this procedure to save on computational time. A similar feature is present in [35]. In my model, $i_f = 40$ (last fertile age), $i_r = 45$ (stands for a retirement age of 65 years) and $I = 48$ (represents the age of 95, the last period of life).

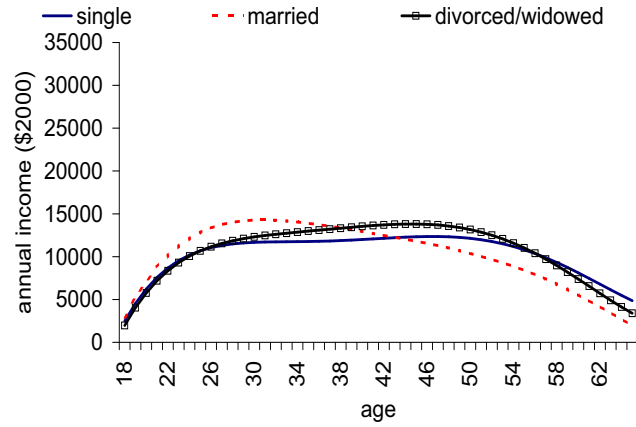
I assume that an individual aged $i \in \{i_f + 1, \dots, i_r\}$ experiences one model period as the average of 5 real years; when $i \in \{i_r + 1, \dots, I\}$, the experience is that of 10 averaged years. The external data used (and described in the previous section) is treated accordingly depending on the age of the individual: earnings, survival and transition probabilities, etc., are averaged in groups of 5 or 10 years accordingly.

Some notes on the estimation: The parameterization of age profiles for contraceptive and conception ability are in table 2.4

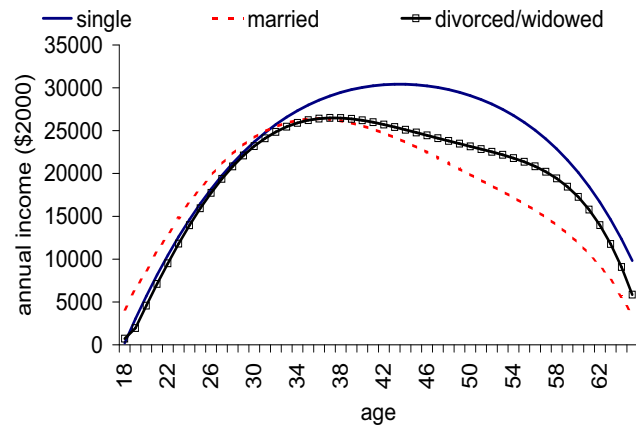
	$\widehat{\varphi}_{i,m=\{1,3\}}^+$ (not-married)	$\widehat{\varphi}_{i,2}^+$ (married)	$\overline{\varphi}_i$ (High School)	φ_i^- (conception)
constant	2.05E-02	-1.17E-01	4.74E-01	2.89E-01
i	-3.86E-02	2.81E-01	-1.02E-01	4.34E-02
i^2	2.99E-02	-8.57E-02	6.68E-02	-2.30E-02
i^3	-5.34E-03	1.25E-02	-1.19E-02	3.58E-03
i^4	3.98E-04	-9.12E-04	8.47E-04	-2.59E-04
i^5	-1.35E-05	3.18E-05	-2.69E-05	8.85E-06
i^6	1.71E-07	-4.24E-07	3.17E-07	-1.15E-07

Table 2.4: polynomial coefficients for parameterization of fertility profiles

To accelerate the estimation algorithm, I use a Beowulf cluster with 20 processors. I parallelize at the parameter level, using the APPSPACK software available free on the web. See [19] and [37] for details.



(a) High School



(b) College

Figure 2.13: Labor Endowments by educational group

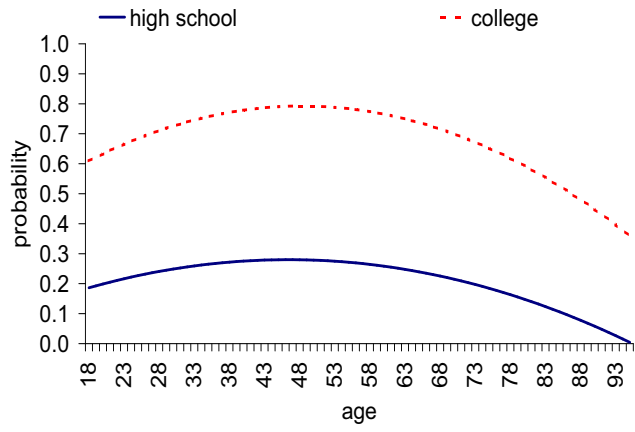


Figure 2.14: Probability of being married to college male, by education of female

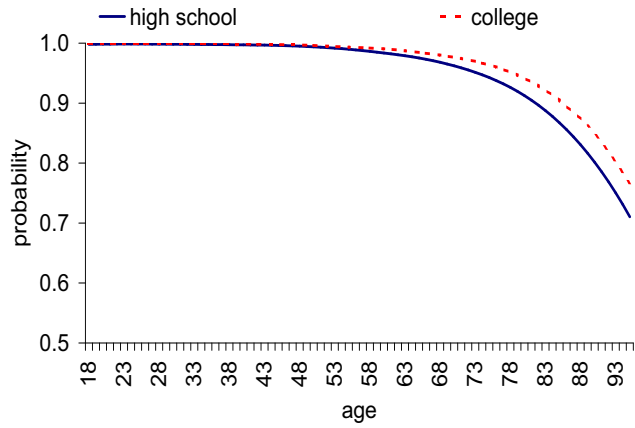
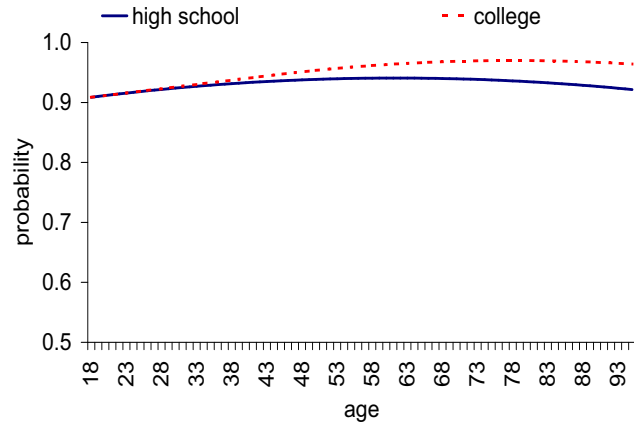
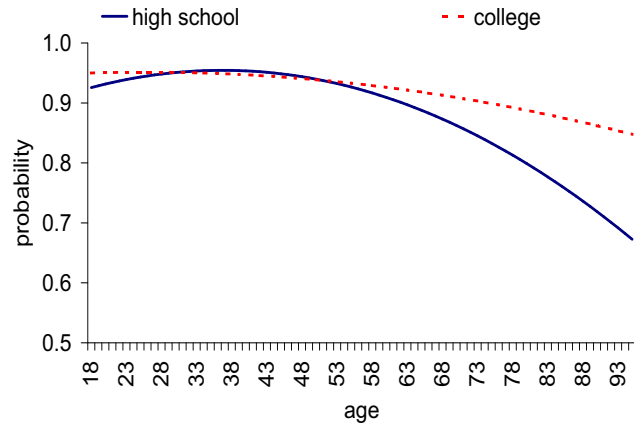


Figure 2.15: Survival probability by education

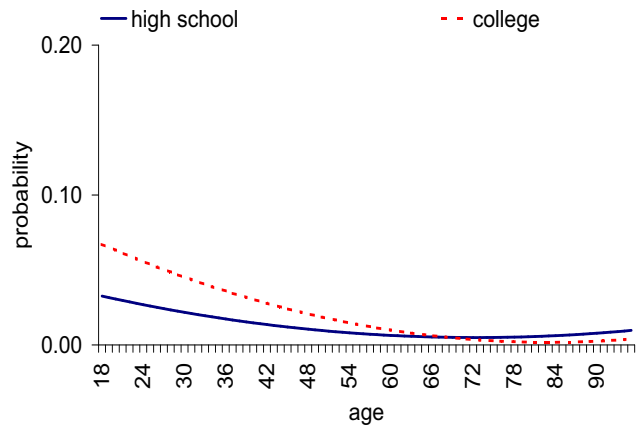


(a) single to single



(b) married to married

Figure 2.16: Transition probabilities for marital states



(a) div/wid to married

Figure 2.17: Transition probabilities for marital states

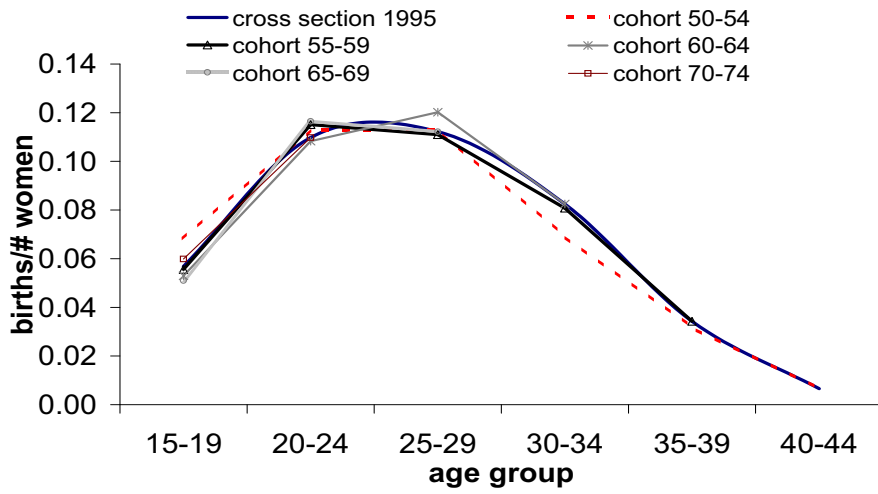


Figure 2.18: Age Specific Fertility Rates by birth cohort

Chapter 3

Aggregate Shocks: Labor Markets

3.1 Introduction

Most business cycle research is performed using the notion that factor shares of income (capital and labor) are constant. This is usually achieved by assuming a Cobb-Douglas production function and marginal productivities as factor prices. We can trace this assumption choice from a generalized belief in [34]’s stylized facts about growth. However, these facts represent (at best) gross generalizations of reality and fail to hold under rigorous statistical scrutiny. As noted by [45], factor shares of income are not constant. They identify an *overshooting* property: the impulse response of bivariate vector autoregressions between the Solow residual and the labor share displays the latter variable falling after a contemporaneous positive shock in technology, but then labor share experiences a long-lasting and persistent increase in its value, peaking five years later at a level larger (in absolute terms) than the initial drop. When looked in detail the dynamic behavior of labor share is strongly influenced by the lagged and strong behavior of employment and by the persistent response of output.

In this chapter (co-authored with José-Víctor Ríos-Rull) we explore whether breaking one of the two assumptions that lead to constant factor shares, namely competitive factor pricing induces dynamics of the labor share like those observed. Specifically,

we explore the extent to which a family of real business cycles models where wages are not set competitively tailored to replicate cyclical facts about the labor market, is capable of generating the observed dynamics of labor share as described in [45]. We build upon [39], [3], [10], among others, who analyze models where factor prices are not set competitively, employment lags productivity, and labor share falls with productivity innovations.

In these models, frictions exist in the labor market and wages are non-competitive, in the tradition of the Mortensen-Pissarides search and matching model.¹ More specifically, these models are characterized by labor markets where search frictions prevent the seamless allocation of workers to jobs; also, the non-competitive wage setting (usually, Nash Bargaining) introduces some rigidity in real compensations which in turn creates a wedge between wages and average labor productivity.

We see this model as a natural extension of standard business cycle models and a place where we can depart from constant factor shares of income by assuming non competitive wages in a transparent and intuitive way. Moreover, and as we will show below, the response of labor share to innovations in technology is hump shaped, driven in a big part by the hump shaped response of employment and total hours. This is not possible to replicate in models where labor inputs to production adjust instantaneously to the cycle, hence the need for some kind of frictions in the determination of aggregate employment. An additional benefit of taking the search and matching framework as a baseline is that it naturally replicates the fact that the immediate response of the labor share to productivity innovations is negative.² Since labor share is comprised of the wage bill (real wages times labor input) over total output, the negative instantaneous response of the simulated labor share to positive innovations in technology is reproduced in the model because labor inputs are fixed (given the search frictions) and wages react sluggishly to the cycle (given Nash bargaining).

¹See [43]

²This fact has been well documented in [39], [3], [10], [45], among others.

Our results show that this class of models fails in replicating the overshooting property of the labor share: unlike the data, responses in the model economy are short-lived. Even though the model presented here is tailored to mimic the behavior of labor share 5 to 10 periods after a shock in technology, it moves very little afterwards. In other words, the model reverses almost immediately to one that looks more like competitive pricing.

We calibrate our baseline model to average properties of the data, such as employment rates, fraction of GDP devoted to creating vacancies and the like. We also calibrate our model so that it matches the observed response of employment and the immediate response of labor share to productivity innovations. We do so not so much to ask how good a model is this of labor fluctuations but to see whether a version of this model capable of generating employment fluctuations generates the observed dynamics of labor share. As we have said, it does not.

There has been a recent controversy on the ability of the Mortensen-Pissarides search and matching type of models to replicate volatility of vacancies and unemployment as seen in US data. This discussion relates closely to the work presented here, since it addresses the same margins we study.

The main point was raised by [50], who performs a standard calibration of the Mortensen-Pissarides model and finds that it cannot replicate the volatility of vacancies and unemployment as seen in US data. [50] blames Nash bargaining, arguing that equilibrium wages in that model are not "rigid" enough: productivity increases are followed closely by real wages, eroding profits and firms' incentives to post vacancies during the cycle. This point is underscored by [22] who studies a model where rigid wages (in fact, wages in his chapter are static) generates high volatility of vacancies and unemployment.

An alternative view is presented by [21] who show that a different calibration of the model can deliver the facts: They point out that Shimer's calibration (high bargaining power of workers and low utility from leisure) effectively kills incentives

for vacancy posting: the surplus of the match³ is high (given low value of leisure), so increases in labor productivity increase the surplus by a proportional small amount.

Moreover, much of these increases are taken by workers with high bargaining power. Hence, by setting a high value of leisure and a low bargaining power, [21]’s version of the model produces accurate cyclical responses of vacancies and unemployment.

Our findings shed some light on the controversy, since our calibration strategy imposes the response of employment observed in the data while freeing the value of home production when not working. Our results confirm (in the context of the present model) that in order to generate large employment responses, the outside option value for the worker has to be quite large.⁴

We do explore some alternative calibration strategies to see whether the additional discipline implied by the dynamics of labor share sheds some lights on the properties of calibration strategies. We find that changing the baseline calibration to an alternative in the spirit of [21] where we target an extremely low value of the Nash weight of the worker increases the cyclical response of vacancies and employment; nevertheless, increased responses of employment are matched with a lower-than-the-baseline response in real wages, leaving the response of the labor share close to what is seen in the baseline. The use of this alternative calibration strategy could be seen as a way of affecting the hiring margin of the model.

We also explore a variant of the baseline, designed to increase real wages after a positive technology shock and that we refer to as the "Garrison" preferences where we pose curvature in bodies making it increasing costly in utility terms to increase the

³in the Mortensen-Pissarides model, the static surplus of a match is labor productivity minus utility from not working for the workers. In steady state, the surplus also depends on the average duration of a match.

⁴An important alternative finding is that of [41] who poses a slight variation of the model wherein workers who separate from a match, go to the pool of prospective hires immediately, and not on the next period as is traditional in the literature. When the model is posed like this, the value of home production implied solely by the utility of leisure is supplemented by a higher continuation value for the workers, which in turn decreases the value of each match. By the logic presented in [21], this helps in creating the incentives for high vacancy posting.

fraction of the population working. This is an attempt to vary the wage margin in the model. However, again we find that we cannot generate accurate dynamic responses of labor share to productivity innovations, the reason being that while wages move more employment move less.

We take the inability of the model to generate the observed dynamics of labor share (under very distinct calibration strategies) as evidence that in non competitive wage setting the hold up problems that create movements in the wedge between real wages and labor productivity fades too quickly: in the model, firms have rational expectations on what will happen with the cost of employment after a productivity shock and they can act upon that by modifying vacancy posting; hence, the model has embedded strong forces that nullify the effects of non competitive wages.

In addition, the failure of these models shows that the dynamics of the cyclical behavior of factor shares in income poses a strict discipline that may be used to discriminate between alternative classes of models. In this context, we think that a next step is to explore models with technologies that are not Cobb-Douglas, and that are susceptible to induce interesting dynamics: we present a variation from our benchmark model, where we use a CES production function. This modification in the baseline economy looks the most promising and points out to new directions for future research.

This chapter is not the first one in modeling an endogenous non-constant labor share⁵, nor the first one pointing out the necessity of understanding its cyclical behavior.⁶ However, it is (to the best of our knowledge) the first one trying to explain the overshooting property, or in other words, the medium-run frequency movements of endogenous labor market variables, identified as impulse response functions from

⁵See for example [18], [8], [14], [23], [28] and [2] among others.

⁶For example, [16] and [49] argue that the labor share is better suited than the output gap to estimate inflation dynamics in environments where staggered contracts and rigidities in wages are present, hence, the need to understand labor share dynamics in order to understand/predict trends in inflation.

data. In doing this, we take into account both relative correlations and levels of endogenous variables influenced by technological shocks; most of the existing literature focuses only on correlations (and only in the short-run) and dismisses information contained on impulse response functions.

The structure of the chapter is as follows: the next section discusses the data, estimation procedures and facts about the labor share at quarterly frequency. Section 3.3 describes our baseline model. Section 3.4 discusses our calibration strategy while section 3.5 shows properties of the model economy under the proposed calibration. In section 3.6 we consider different calibration strategies in order to check for robustness; we then propose a simple deviation from the baseline economy, which clarifies the link between non-competitive wage setting through Nash-bargaining and the dynamics of employment, hours and the labor share. We conclude in section last section.

3.2 Cyclical Behavior of the Labor Share: The Facts

Here we briefly summarize the findings in [45]. The facts we are interested in can be summarized in the following figures and tables that we have calculated from US data, from the first quarter of 1964:I to the last quarter of 2004.⁷ Figure 3.1 shows the labor share; Table 3.1 presents the standard business cycle statistics; the series are in logs and then hp filtered.⁸

The facts can be summarized by

1. Labor share is quite volatile: its standard deviation is 42% that of output.
2. It is countercyclical: its correlation with contemporaneous output is -0.13.
3. It is highly persistent: Autocorrelation of 0.78

⁷We use the labor share as constructed by [45]. We use data from NIPA and BLS. Standard data and standard construction of variables...

⁸Taking logs allows the interpretation of its volatility as percentages.

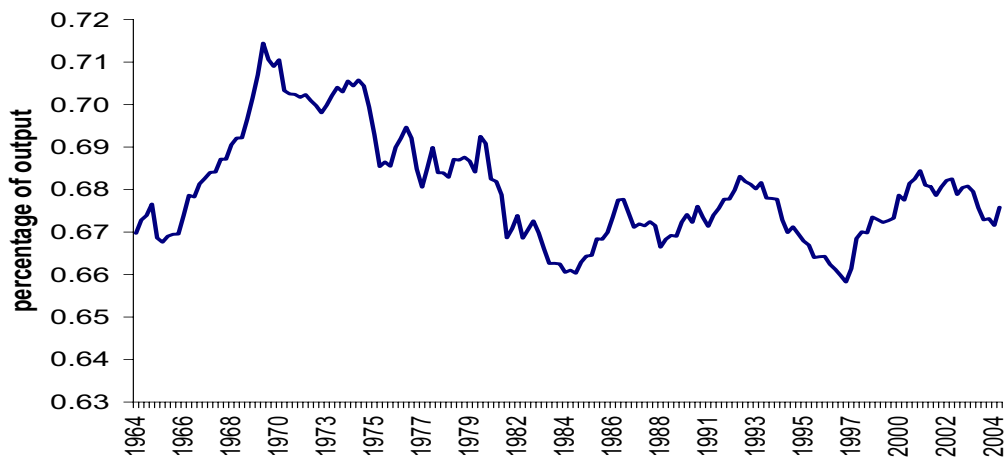


Figure 3.1: The Labor Share, U.S. 1964.I-2004.IV

	σ_x	σ_x/σ_{GNP}	$\rho(x, GNP)$	$\rho(x_t, x_{t-1})$
<i>GNP</i>	1.56	1.00	1.00	.86
log Labor Share	.66	.42	-.13	.78

Cross-correlation of GNP_t with Labor Share $_{t+i}$

$i =$										
-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
-0.27	-0.29	-0.29	-0.25	-0.20	-0.13	0.10	0.27	0.39	0.45	0.45

Table 3.1: Business Cycle statistics of the labor share

4. It lags output by a year
5. It overshoots. When we estimate a vector autoregression (VAR) of order 1 (without constant) between the Solow residual and the labor share and then plot the impulse response (IR) function for the labor share, we get figure 3.2. The term "overshooting" is due to the fact that, while the instantaneous response of labor share to a technology shock is negative, the overall effect is positive and long lasting.

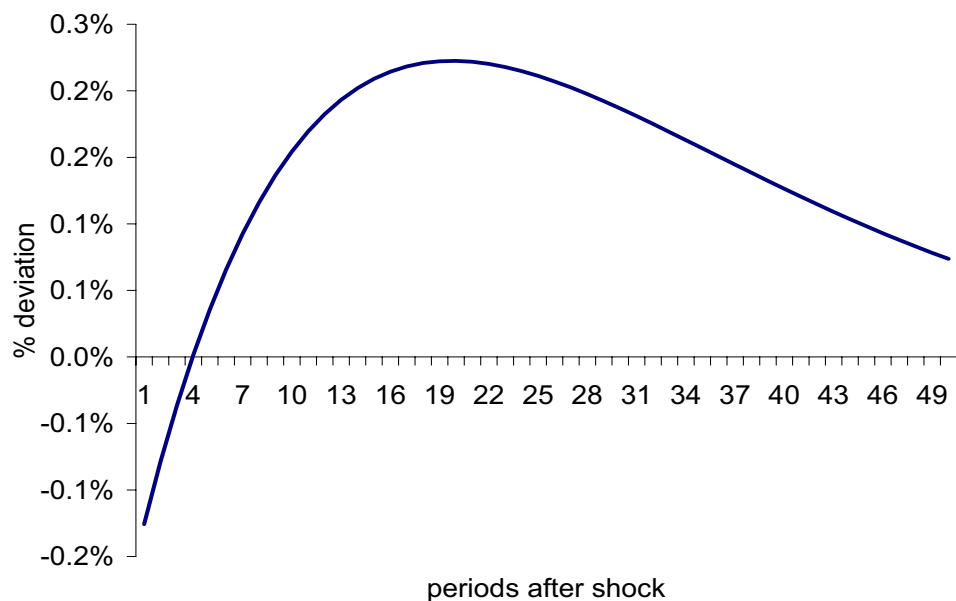


Figure 3.2: Response of Labor Share to a shock in the Solow residual

Basically, this "hump-shaped" response of the labor share is due to the hump-shaped response of employment during an expansion. Hours per worker and wages have weaker responses. This is shown in figure 3.3, where each response function is calculated from a bivariate VAR of order 1, between the variable of interest and the solow residual.

We take these VARs as statistics from the data. In the results section, we will take simulated data and construct these same figures and statistics and compare them.

3.3 The model

Here we describe the model that we take as a baseline. As pointed out in the introduction, this model is appealing for the problem at hand because it has three main ingredients: non competitive wages, a role for frictions in employment and forces that create a countercyclical labor share.⁹

⁹See [43], [39], [3] and [10], among others.

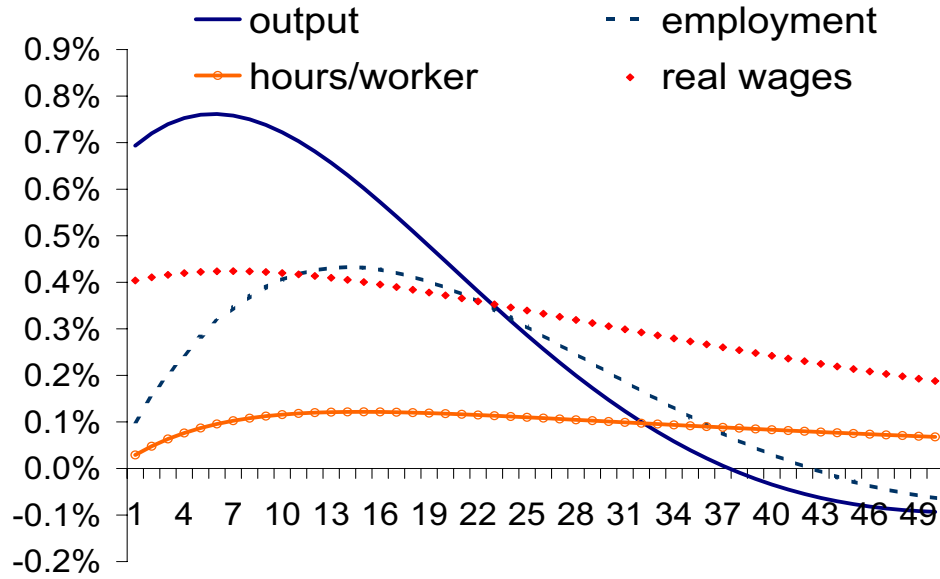


Figure 3.3: Response of Labor Share components to a shock in the Solow residual

The environment can be described as follows. Time is discrete and goes on forever. There is a continuum of identical and infinitely lived large families, or households, each of measure one. The household consumes (c), accumulates assets (a) which they rent to the firms and depreciates at rate δ (in equilibrium, assets and capital are the same) and provides labor: there is a fraction n of individuals in the family, or household members, that is matched with firms or employed, the rest, $1 - n$, is not employed but willing to be assigned to any open job. The amount of hours worked by those employed is determined in a decentralized way in the job.

Firms produce the unique good in the economy, using a constant returns to scale technology, subject to aggregate and persistent productivity shocks (z):

$$y = e^z F(k, nh) \tag{3.1}$$

Inputs to this technology are capital k and labor in the form of workers, n , times hours per worker, h . There are search frictions in the labor market: workers and

firms need to be matched. Over time there is attrition of matches and firms post vacancies, v , that result in additional future matches according to a standard matching function that depends on the aggregate level of vacancies and size of the non-working population, $M = M(V, 1 - N)$. Following the literature, we assume that this function is homogenous of degree one in both arguments (V and $1 - N$) and that there is an exogenous and constant separation rate χ between jobs and workers. The search component of the model makes employment a predetermined state variable at the beginning of each period; also, its value evolves according to the matching function.

When solving their problem, both the household and the firm take as given the interest rate (r), which is determined in a perfectly competitive capital market. On the other hand, the real wage (w) and hours per worker (h) are determined by a pairwise Nash bargaining game between firms and individuals.

The assumption of large families helps us distinguishing employment and hours per worker in equilibrium, as well as the employment status of individuals inside the family. The latter provides a simple framework to understand labor search and matching in this model, since we don't have to distinguish between employed or unemployed families, but rather employed or unemployed individuals.

The aggregate state in this economy is the aggregate shock, the amount of capital and the fraction of the population matched to firms, $S = \{z, K, N\}$. The household state is the aggregate state S and its own state variables which are its assets and the fraction of its members that have a job, i.e. that are attached to a firm, $s_H = \{a, n\}$. Assets in this economy are both physical capital and firms. The latter have value because they are matched to workers by posing costly vacancies in exchange for future profits. Without loss of generality we close the stock market in this economy and post the dividends (not the rental income of capital) as an endowment of households. This simplifies dramatically the definition of equilibrium and avoids cumbersome notation. Consequently, the household solves the following recursive problem:

$$V(S, s_H) = \max_{\{c, a'\}} u(c) + n\nu(1 - h) + (1 - n)\nu(1) + \beta E[V(S', s'_H)|z] \quad (3.2)$$

s.t.

$$c + a' = w(S, s_H)h(S, s_H)n + (1 + r - \delta)a + \pi(S) \quad (3.3)$$

$$n' = (1 - \chi)n + \Psi(S)(1 - n) \quad (3.4)$$

$$S' = G(S) \quad (3.5)$$

$$z' = \rho z + \epsilon', \quad \epsilon \sim iid(0, \sigma_\epsilon^2) \quad (3.6)$$

where $u(c)$ is utility of consumption, a are the assets of the household, $\nu(\cdot)$ is the utility of leisure, $\pi(S)$ are dividends from the firm to the households. We denote with primes next period's values of variables. $G(S)$ is the law of motion of aggregate variables, and $\Psi(S)$ is the job-finding rate,. This rate is derived from the usual matching function, i.e.,

$$\Psi = \frac{M(V(S), 1 - N)}{(1 - n)} \quad (3.7)$$

The household takes as given by functions $w(S, s_H)$ and $h(S, s_H)n$ the allocation of hours and the determination of wages. As we will see below they are set by decentralized bargaining.

Since we use separable utility between consumption and leisure, the intra-household consumption level doesn't depend on employment status, that is, the household perfectly insures its non-working members. In equilibrium, this means that unemployed individuals are better off than employed ones: they receive the same consumption stream c and enjoy all the leisure, while the employed agents spend h working for the firms. This issue was discussed by [10]¹⁰ but has no implications for our goal of replicating the cyclical properties of the factor shares.

Turning now to the problem of firms, they rent the capital from the households and post vacancies on the job market, which turn into matches with one period delay,

¹⁰In their paper, they introduce non-separable utility between consumption and leisure, a la [46]

an intrinsically dynamic problem so the firm considers the future paths of vacancies and unemployment in order to make its decisions today. The representative firm takes the aggregate and individual state variables, S and $s_F = \{n\}$ respectively, and solves the following dynamic problem:

$$\Omega(S, s_F) = \max_{\{v, k\}} y - w(S, s_F)h(S, s_F)n - rk - c_v v + E \left[\tilde{R}(S') \Omega(S', s'_F) | \mathcal{S} \right] \quad (3.8)$$

s.t.

$$y = e^z F(k, nh) \quad (3.9)$$

$$n' = (1 - \chi)n + \Phi(S) v \quad (3.10)$$

$$S' = G(S) \quad (3.11)$$

$$z' = \rho z + \epsilon', \quad \epsilon \sim iid(0, \sigma_\epsilon^2) \quad (3.12)$$

where c_v is the cost of posting a vacancy, Φ is the job-filling rate ($\Phi \equiv M/V$), F is the production function, and G is the law of motion of aggregate state variables that in equilibrium is determined by the actions of individual households. The firm also takes as given functions $w(S, s_F)$ and $h(S, s_F)n$ that determine the allocation of hours and the level of wages.

Note that, the discount factor for the firm is none other than the rate of return of the economy which in equilibrium is given by the standard FOC using aggregate variables

$$\tilde{R}' \equiv \beta \frac{u_c[C(S')]}{u_c[C(S)]}. \quad (3.13)$$

where $u_c(\cdot)$ is the marginal utility of consumption for the households.

From the problem of the firm, we can derive the first order condition for vacancies

$$c_v = \beta \Phi(S) E \left\{ \frac{u_c[C(S')]}{u_c[C(S)]} \Omega_n(S', n') | S \right\} \quad (3.14)$$

where Ω_n (and now we start omitting arguments to avoid exhausting notation) is the

value of an additional worker to the firm, i.e.

$$\Omega_n = \frac{\partial y}{\partial n} - w h + (1 - \chi) \beta E \left[\frac{u'_c}{u_c} \Omega'_n \right] \quad (3.15)$$

Equations (3.14) and (3.15) say that the firm posts vacancies until the (constant) marginal cost of the vacancy equates the probability of getting a new worker times the marginal benefit of that additional worker.

As stated before, in this model wages are not equal to the marginal productivity of labor, but are determined by the outcome of a bilateral Nash bargaining game between the firm and the individual.¹¹ As opposed to the standard [43] model where the outside option for the workers is a fixed parameter, in this model the value of not engaging in production for a worker is determined endogenously; the outside option for the worker is related to the extra leisure that unemployed individuals enjoy (remember that their consumption is insured by the family unit) and the option value of being matched to another job next period.

Specifically, the setup of the bargaining game has the following components. First, the value of an additional worker for the household ($\equiv \frac{\partial V}{\partial n}$) is given by

$$V_n = u_c w h - \nu(1) + \nu(1 - h) + (1 - \chi - \Psi) \beta E [V'_n] \quad (3.16)$$

and the value of an additional worker for the firm ($\equiv \frac{\partial \Omega}{\partial n}$) is given by

$$\Omega_n = y_{nh} h - w h + (1 - \chi) \beta E \left[\frac{u'_c}{u_c} \Omega'_n \right] \quad (3.17)$$

where $y_{nh} = \frac{\partial y}{\partial(nh)}$ denotes the output produced by someone who works h hours. This is an important assumption, that goes to the nature of the bargaining protocol.

With these elements, we can define the axiomatic Nash bargaining problem for

¹¹This is important, specially if the production function is Cobb-Douglas, since competitive pricing of the factors under that particular production function gives rise to constant factor shares.

which the outcome determines both the wage and the hours per worker:

$$(w, h) = \arg \max_{w, h} (V_n/u_c)^\mu (\Omega_n)^{1-\mu} \quad (3.18)$$

where μ is the bargaining power of the worker. Note that the value of the marginal worker for the household (V_n) is multiplied by u_c (the marginal utility from consumption) in order to transform everything into units of the consumption good.

Taking the derivatives with respect to real wages and hours, we get two conditions: the first one is the "sharing rule" of production surplus and the second is a static condition for determining the length of the workweek,

$$\mu u_c V_n = (1 - \mu) W_n \quad (3.19)$$

$$u_c y_{nh} = \nu_h(1 - h) \quad (3.20)$$

To solve for the equilibrium wage, we have to use the first order condition for vacancies (3.14) as well as the sharing rule (3.19) to get

$$w h = \mu \left[\frac{\partial y}{\partial n} + c_v \frac{v}{1-n} \right] + (1 - \mu) \left[\frac{\nu(1) - \nu(1-h)}{u_c} \right] \quad (3.21)$$

This is an analog to the wage equation derived by [43] in the simpler setup where productivity and the outside option of the worker are constants. In words, the wage bill is a weighted average of (i) the marginal productivity of the worker plus the average savings in vacancy postings per unemployed individuals and (ii) the outside option of the worker, which in this case is simply the forfeited leisure incurred by the individual who works h hours at the firm.

The bargaining protocol that we have posed implicitly assumes that in case that the negotiations break down between any specific worker and firm, the negotiations of the firm (and the worker) with other workers and (firms) are unaffected. This is an extreme assumption and an alternative has been posed nicely by [51] where firms and

workers internalize that any breakdown of negotiations affects negatively the position of the firm with other workers. Unfortunately, the simultaneous determination of hours with this alternative bargaining protocol seems intractable.

To finalize we should impose the equilibrium conditions that both households and firms are representative. We omit them since they are very well known.

3.4 Calibration of the Model

The calibration process consists of selecting functional forms, specifying parameter values and the targets that the model economy has to satisfy that restrict the values of those parameters. Of course, the number of targets has to be at least that of parameters.

3.4.1 Functional Forms and Parameters

A model period is taken to be one quarter. The production function is Cobb-Douglas with exponential depreciation. The utility function is separable in consumption (where we use log utility) and leisure, with

$$\nu(\ell) = \begin{cases} \gamma \frac{\ell^{1-\eta}}{1-\eta} & \text{if } \ell \in [0, 1) \\ \gamma_u & \text{if } \ell = 1 \end{cases} \quad (3.22)$$

We follow [3] and introduce a differentiated parameterization for the leisure in the household. where γ_u is a constant. The different values for leisure of the employed versus the non-employed can be interpreted as differential efficiency in home production given the labor force status¹², commuting time or search costs. In practical terms, this extra parameter allows for more flexibility in the calibration of the model since it can be set independently from the leisure of the employed. The additional flexibility becomes quite handy when noticing that it allows for accommodating the large lag

¹²See [3], page 115.

implied by the model for workers that have been just separated from a match, in joining the set of prospective hires (12 weeks) and the large group of non-workers in this economy. As stated before, [41] uses a disciplined value of home production in an environment where if a worker breaks up with a firm, it becomes immediately eligible to search for another job.

The matching function has constant returns to scale, and it is also assumed to be of the Cobb-Douglas form¹³.

$$M(V, 1 - N) = \omega V^\psi (1 - N)^{1-\psi} \quad (3.23)$$

Given these expressions, the equilibrium of the model is characterized by the following system of nonlinear equations:

$$\begin{aligned} Y &= e^z K^\theta (Nh)^{1-\theta} \\ N' &= (1 - \chi)N + \omega V^\psi (1 - N)^{1-\psi} \\ K' &= (1 - \delta)K + I \\ Y &= I + C + c_v V \\ \Phi &= \omega V^{\psi-1} (1 - N)^{1-\psi} \\ 1 &= \beta E \left[\frac{C}{C'} \left(1 - \delta + \theta \frac{Y'}{K'} \right) \right] \\ \frac{c_v}{\Phi} &= \beta E \left[\frac{C}{C'} \left((1 - \theta) \frac{Y'}{N'} - w'h' + (1 - \chi) \frac{c_v}{\Phi'} \right) \right] \\ (1 - \theta) \frac{Y}{Nh} &= C\gamma(1 - h)^{-\eta} \\ wh &= \mu \left[(1 - \theta) \frac{Y}{N} + c_v \frac{V}{1 - N} \right] + (1 - \mu)C \left[\gamma_u - \gamma \frac{(1 - h)^{1-\eta}}{1 - \eta} \right] \\ z' &= \rho z + \epsilon' \end{aligned}$$

Given any parametrization, we solve this model with a first order approximation of the system around the stochastic steady state.

¹³See [50] and [7]

There are 13 parameters in this model that we have to specify. There are preference, matching technology and bargaining parameters.

Preference parameters

1. β Discount factor.
2. γ Multiplicative coefficient of leisure.
3. γ_u Utility level of unemployment.
4. η Exponential Coefficient of leisure.

Production parameters

5. θ Coefficient of capital in the production function. Under competitive factor prices, $1 - \theta$ is labor share. This is not the case in this model.
6. δ Depreciation rate.
7. σ^2 Standard deviation of the innovation to the productivity shocks.
8. ρ Autocorrelation of the productivity shocks.

Matching Technology

9. c_v Cost of posing a vacancy.
10. ω General level of job creation.
11. ψ Coefficient of vacancies in the matching function.
12. χ Job destruction rate.

Bargaining protocol

13. μ The weight of the firm in the bargaining process.

3.4.2 Calibration Targets

We turn now to specify the targets that we chose to determine the values of these parameters. Unlike traditional work in business cycle research we pose, not only targets that are first moments, but also targets that are second moments. We defend this choice below.

Household Targets

1. Annual rate of return of the economy, 4%.
2. Steady state hours per worker, 433 per quarter (1733 per year, about 1/3 of total time).
3. Frisch elasticity of hours for those that work, .5.
4. Steady state employment rate, 75%. Depending on the definition of employment and unemployment, different authors target different values for this variable. [3], [39] and [10], among others, take 0.57 as their target; [50], [21] use a much higher number (around 0.94). Our number is on the very high end of the historical employment rate. We follow in this regard [38] and [42]. Although the simulated volatility of vacancies and unemployment are sensitive to this target, the responses of labor share to technology are not, so we settle on an average employment rate.¹⁴

Production Targets

5. Consumption to output ratio, 75%.
6. Measured labor share of output, 67%.
7. Standard deviation of measured Solow residual, .64%.

¹⁴Alternatively we could have targeted the duration of unemployment or the transition of unemployment to employment (as [3] and [10] do).

8. Autocorrelation of measured Solow residual, .93.¹⁵

Employment Turnover Targets

For the targets below, we follow [3], [1] and [10].

9. Separation rate, 15% per quarter.
10. Vacancy expenditures to output ratio, .5%.
11. Job filling rate in the steady state of 0.9.

Business Cycle Targets

The targets listed so far are steady state targets (except for the implied process for the Solow residual). For the last two targets, we impose two business cycle targets:

13. The immediate response of labor share to a productivity shock, -0.1263% .
14. The size of the response in employment to a productivity shock, 0.4326% .

We compute these last two targets (both with real and model simulated data) by way of estimating vector autorregresions between the technological shock and log-detrended variables (construction procedures described in [45]). We identify the immediate response of labor share to a productivity shock as the first element of the impulse response function of labor share in the VAR; the size of the response in employment is taken as the maximum value of the corresponding impulse response function. We choose these targets since they provide information on the workers' bargaining weight and the elasticity of the matching function to the number of vacancies, respectively.

Our calibration strategy entails looking for parameter values in order to match all of the targets above. There is a number of parameters that we can set in advance:

¹⁵This targets arise from our estimation of a VAR(1) between labor share and productivity, as described in [45].

parameter	Description	Value
β	Discount Factor	0.990
δ	dep. rate	0.012
ρ	Autoregressive parameter of tech. shock	0.930
σ_ϵ	Std. dev. of tech. shock	0.0064
θ	Share of Capital in Cobb-Douglas	0.327
χ	Separation rate	0.150
ψ	Elasticity of matching fnc. wrt vacancies	0.669
ω	Scale parameter in matching function	0.718
η	Curvature parameter in utility of leisure	4.000
γ	Scale parameter in utility of leisure	0.663
γ_u	Leisure utility of unemployed	0.326
c_v	Cost of posting a vacancy	0.037
μ	Bargaining power of workers	0.410

Table 3.2: Parameter values

$\{\beta, \delta, \chi, \eta\}$ while for the rest, we solve a system of 11 non-linear equations (model targets) in 11 unknowns (model parameters). We hit all steady state targets almost perfectly, while we approximate the business cycle ones very closely.¹⁶

3.4.3 Properties of the Calibrated Economy

It is useful to comment on the parameters obtained in the calibration process which are $\rho \sigma_\epsilon \theta \psi \omega \gamma \gamma_u c_v \mu$. The properties of the shock while not identical are quite similar to the ones that result from a direct estimation of the Solow residual. The value of leisure is very high, it is so high in fact that there is no way to describe its value in terms of commuting costs (for those who work). We can however describe its value in terms of the consumption value of the discontinuity that it implies by solving the following equation for x

$$u(c^*) + \gamma \lim_{\ell \rightarrow 1} \frac{\ell^{1-\eta}}{1-\eta} = u(c^* + x) + \gamma_u. \quad (3.24)$$

¹⁶The immediate response of labor share is -0.106% (-0.1263% in the data), and the size of the response in employment is 0.4258% (0.4326% in the data).

where c^* is the steady state value of consumption. The implied value turns out to be about 90% of c^* .

Another important feature of the calibrated parameters is that the Nash weight of workers is .41, on the lower end of those in the literature. This value is right between those used in the [50] vs. [21] debate. The former pairs a high weight for workers (0.72) with a low value of home production or unemployment insurance; the latter pair a high value of home production with a very low value of workers' weight (0.052). Hence, our calibration lays in between these two opposites: we have a very high value for the utility of the non-employed and a bargaining weight for workers that is a midpoint of the values used by [50] and [21].

Compare the values used in our environment (where upon the breakdown of a match a worker has to stay out of the workforce for 3 months and afterwards joins the 25% of the population that is willing to be matched) with those in [41], who poses that when bargaining breaks, workers can join the rank of employable within the same period. He models utility in the household without any sort of discontinuities with respect to leisure¹⁷ and a very low value for the bargaining weight of the worker (around 0.05). This type of calibration might seem insufficient to generate high volatility of vacancies since the outside option of workers is low; but as noted earlier, the fact that workers don't have to sit-out one period after a match breaks, reduces the surplus of each match by increasing the continuation value of the worker.

¹⁷[41] has non-separable utility between consumption and leisure, so equation (3.24) looks differently in his setup:

$$\lim_{\ell \rightarrow 1} u(c^*, \ell) = u(c^* + x, 1)$$

Since there is no form of discontinuity between $u(\cdot, \ell)$ and $u(\cdot, 1)$, $x = 0$.

3.5 Business Cycle Behavior of the Baseline Model Economy

We solve the model numerically by local approximation of the non-linear system of equations presented above and simulate paths for all endogenous variables. We then calculate statistics in the same manner for both real and model simulated data. Table 3.3 compares the cyclical properties of the model against quarterly US data, while figure 3.5 compares impulse response functions.

Table 3.3 shows that the model replicates well the main features of the US business cycle: standard deviation of output is closely replicated, as well as the cross-correlation of output with all endogenous variables in the table. This is not surprising as we targeted the volatility of employment and as the model has built in the propagation mechanism of a delayed response in employment.

As in [39], [3] and [10], key cyclical facts about the labor market are captured in the baseline: total hours moves more than in a standard RBC model, but still less than in the data; employment and hours per worker are pro-cyclical, with employment lagging the cycle slightly by one quarter; labor share is countercyclical, again as targeted.

		Cross correlation of output with:												
		σ	$\sigma(x)/\sigma(y)$	$x(-5)$	$x(-4)$	$x(-3)$	$x(-2)$	$x(-1)$	x	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$
US Data														
output		1.56	1.00	0.03	0.24	0.46	0.68	0.86	1.00	0.86	0.68	0.46	0.24	0.03
tot. hours		1.52	0.97	-0.14	0.04	0.25	0.48	0.71	0.89	0.91	0.83	0.69	0.49	0.30
Employment		1.28	0.82	-0.22	-0.05	0.15	0.38	0.62	0.82	0.90	0.88	0.79	0.64	0.45
hours		0.43	0.27	0.14	0.29	0.42	0.57	0.68	0.70	0.55	0.29	0.08	-0.15	-0.29
wages		0.79	0.51	0.11	0.16	0.20	0.22	0.18	0.17	0.05	-0.01	-0.09	-0.09	-0.13
labor's share		0.66	0.42	-0.27	-0.29	-0.29	-0.25	-0.20	-0.13	0.10	0.27	0.39	0.45	0.45
consumption		1.24	0.79	0.24	0.45	0.62	0.77	0.86	0.85	0.67	0.48	0.27	0.06	-0.10
investment		7.09	4.53	0.12	0.28	0.45	0.62	0.78	0.90	0.77	0.56	0.32	0.08	-0.16
L. prod.		0.74	0.47	0.36	0.44	0.47	0.45	0.36	0.30	-0.04	-0.25	-0.44	-0.50	-0.54
Model														
output (y)		1.48	1.00	-0.01	0.14	0.34	0.58	0.85	1.00	0.85	0.58	0.34	0.14	-0.01
tot. hours		1.06	0.72	0.00	0.14	0.31	0.52	0.75	0.95	0.94	0.66	0.38	0.14	-0.04
Employment		1.02	0.69	-0.03	0.09	0.24	0.44	0.66	0.89	0.97	0.71	0.43	0.19	0.00
hours		0.19	0.13	0.18	0.29	0.43	0.57	0.68	0.56	0.05	-0.11	-0.17	-0.21	-0.22
wages		0.46	0.31	-0.11	0.05	0.26	0.52	0.79	0.89	0.59	0.43	0.31	0.21	0.12
labor's share		0.15	0.10	-0.18	-0.27	-0.38	-0.49	-0.57	-0.41	0.12	0.23	0.25	0.24	0.22
consumption		0.30	0.21	-0.29	-0.14	0.07	0.34	0.65	0.86	0.77	0.67	0.56	0.44	0.33
investment		7.26	4.92	0.04	0.19	0.38	0.60	0.85	0.99	0.84	0.56	0.30	0.09	-0.08
L. prod. (y/Nh)		0.57	0.39	-0.04	0.11	0.31	0.55	0.79	0.82	0.44	0.29	0.18	0.10	0.03

Table 3.3: Cyclical Properties of U.S. Economy (1964:I, 2004:IV) and Model

The main finding of this chapter is shown in Figure 3.4. Labor share is flat after four periods and most of the initial drop targeted in the calibration has disappeared after one period. Moreover, the response of labor share never changes sign let alone moves in the opposite direction in the amount that it does in the data.

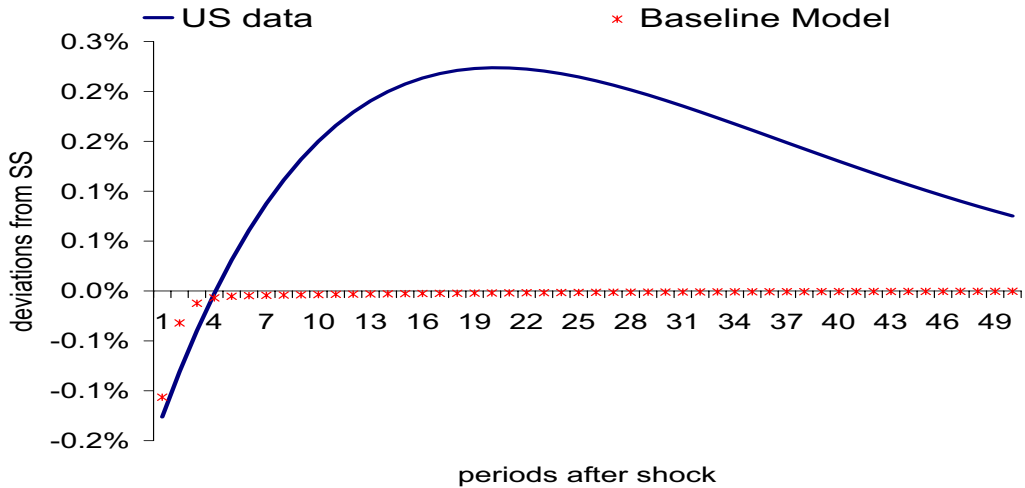


Figure 3.4: Dynamics of Labor Share

The four panels of figure 3.5 show the complete story. From 3.5(a), we see that we hit very closely our target of employment response to a technological shock, although the timing is not perfect. The model succeeds in hitting the high levels of employment fluctuations, with a calibration that yields a relatively low value of the worker’s Nash bargaining weight. This is partly due to a low target for employment, as pointed out by [12]: we use a target of 0.75, which is above the historical target of 0.57 used by [3] and [10], but below the one close to 0.95 used more recently by [50] and [21]. This is also partly due to the implied high value of home production yielded by parameter γ_u .

Even though employment creation might be sensitive to the choice of this target, the ability of the model to produce ”overshooting” of the labor share is not, as it will

be clear below.

In 3.5(c) and 3.5(d), we see that wages respond similarly to its value in the data but hours do not. In particular, hours in the model respond immediately because they substitute for bodies that cannot be increased. Once bodies are available, hours go back to the steady state level.

Finally, 3.5(b), shows the response of output. As in the data, the baseline shows a humped-shaped response, but with higher values at the beginning of the time period, due mainly to the difference between responses in the data and the baseline of hours per worker.

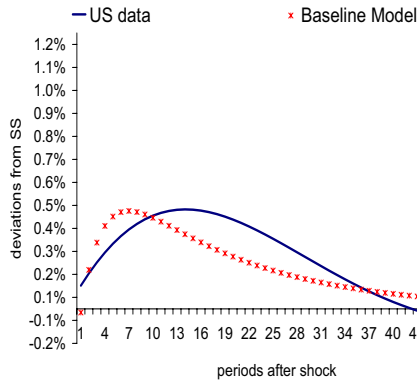
The failure of the baseline in replicating the overshooting of the labor share can be attributed to a mix of failed model responses: employment and wages respond less during the medium run (10 to 40 periods after the initial shock) but the main culprit seems to be hours per worker.

3.6 Robustness and Extensions

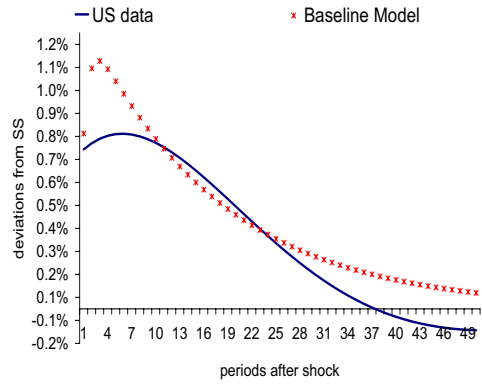
We study the extent to which our results are affected by our particular calibration. We recalibrate the model in the spirit of [21] and compare the implied model responses. Then, we take a small departure from the baseline economy, in order to clarify the possibility of increasing the response of labor share by varying the margins of the wage setting protocol.

3.6.1 A Different Calibration

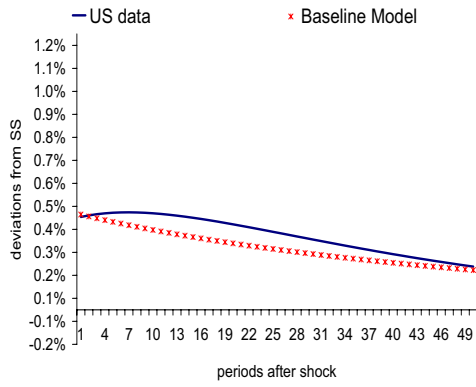
The main point argued by [21] with respect to the calibration of the model, is that in order to increase the ability of the search and matching framework to propagate technological shocks, incentives for the firm to post costly vacancies have to be significant. This implies a calibration where firms have small accountable profits but high bargaining position. Hence, when there is a positive technological innovation, the



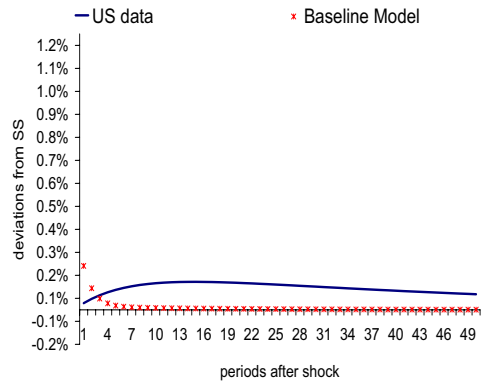
(a) Employment



(b) Output



(c) Wages



(d) Hours per Worker

Figure 3.5: Response to a Technological Shock: Data and Baseline

parameter	Baseline	Hagedorn Manovskii Calibration
θ	0.327	0.346
ψ	0.669	0.574
ω	0.718	0.693
γ	0.663	0.554
γ_u	0.326	0.389
c_v	0.037	0.048
μ	0.410	0.050

Table 3.4: Parameter values, alternative calibrations

percentage of increase in profits is large for the firms; moreover, firms want to post vacancies since they have the better bargaining position. By altering this margin in the model, we want to see if we can achieve higher responses of labor share due to increased responses in the extensive margin of employment.

We achieve this alternative parameterization of the model by setting the bargaining power of workers μ to 0.05 which is the value used by [21]. We then recalibrate the rest of the parameters by using the steady state targets described in section 3.4.2; in order to maintain the spirit of the suggested calibration, we replace the business cycle targets with a target for the firms profits of 0.03.¹⁸ The result of this calibration is presented in table 3.4. Notice that now the value of home production while working increases further although by a lot less than in the original discussion between those authors which may be due in part to the fact that we started with a relatively low bargaining power of workers.

Comparisons in model responses are shown in figures 3.6 and 3.7. As seen in figure 3.6, the use of this alternative calibration also fails in creating a positive response of the labor share to a technology shock. Notice that since the initial drop in labor share is not a target under the Hagedorn–Manovskii calibration, the model overreacts in this dimension (compared to the baseline), but again, it goes back to its long run

¹⁸This target is used by [50] and [21]

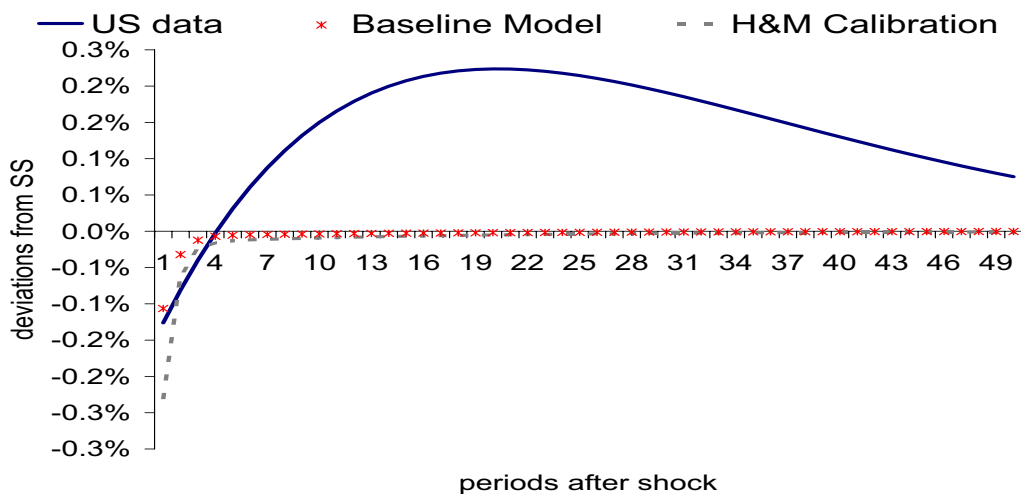


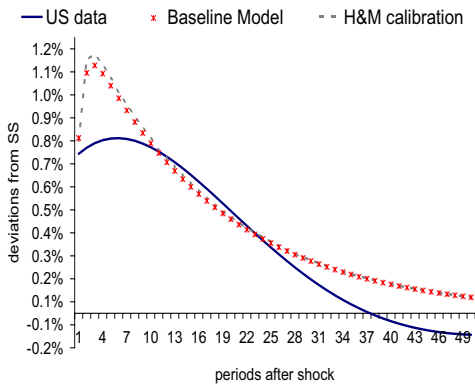
Figure 3.6: Dynamics of Labor Share

average pretty fast.

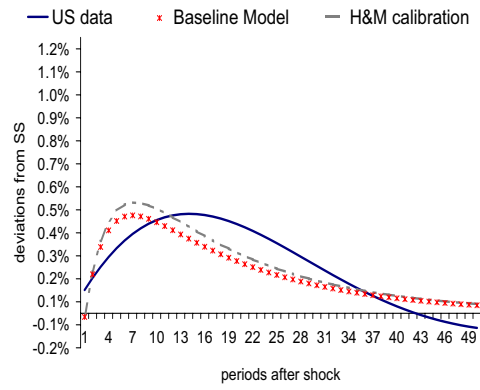
From figure 3.7 and its panels, we see that this alternative parameterization affects mostly employment and real wages, leaving hours per worker and output fairly untouched: Indeed, employment has a stronger response as opposed to the baseline, while real wages are more sluggish under the new calibration. Combined, these responses leave the labor share as in the baseline. Hence, although the insight provided by [21] (namely, that a different calibration alone is able to create the incentives for the firm to create more employment during the cycle) also works in this model, it doesn't affect our goal of matching the dynamics of the labor share.

3.6.2 The "Garrison" Effect

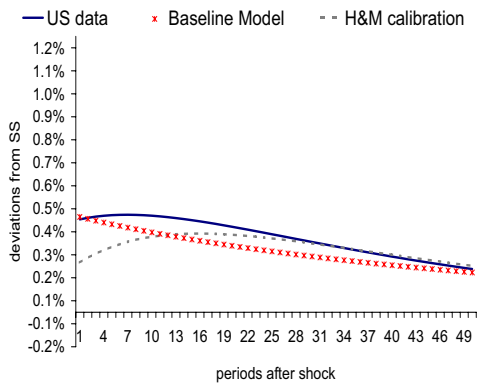
In this section we present a small deviation from the baseline model in order to understand better the reason for its inability in replicating the overshooting property of the labor share. In the previous section we analyzed whether a different calibration strategy would deliver. Here we go one step further and force some particular mechanisms on the model. Below we describe this mechanisms, explain how we recalibrate



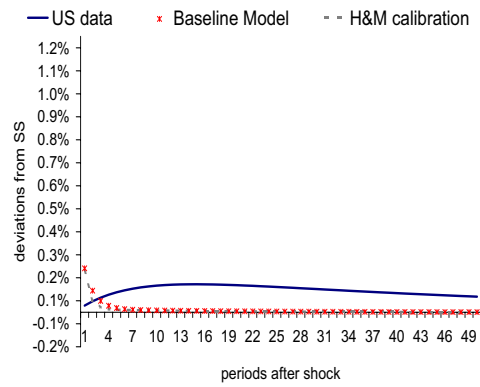
(a) Output



(b) Employment



(c) Wages



(d) Hours per Worker

Figure 3.7: Response to a Technological Shock: Data and Baseline with [21] Calibration

the model and show its performance in terms of impulse response functions.

Recall that the instantaneous utility of the household was given by

$$\mathcal{U} = U(c) + N\nu(1 - h) + (1 - N)\nu(1)$$

Consider the following alternative:

$$\mathcal{U}_g = U(c) + N\nu(1 - h) + (1 - N)^\kappa\nu(1)$$

where we denote $\kappa \in [0, 1)$ as the "Garrison" effect, since it captures the notion that having N going to 1 (most members of the household working), might be increasingly costly in terms of leisure, say, because of increasing returns to scale in household production.

The "Garrison" effect increases the in-household (non-working) option value for the workers, hence affecting the wage bill. Denote \tilde{b} as follows

$$\tilde{b} \left(\equiv -\frac{\partial \mathcal{U}}{\partial N} \right) = \nu(1) - \nu(1 - h)$$

With Garrison, we have that

$$\tilde{b}_g \left(\equiv -\frac{\partial \mathcal{U}_g}{\partial N} \right) = \kappa(1 - N)^{\kappa-1}\nu(1) - \nu(1 - h)$$

hence, with $\kappa < 1$ the reluctance to work by unemployed increases with N . Basically, through the "Garrison" effect, we are forcing the firm to pay higher wages during an expansion following a positive technology shock since the outside option value for the worker improves more than in the baseline in an expansion.

Given the exercise nature of this extension to the baseline, we solve the model for different values of κ instead of trying to calibrate it to some data target. We let $\kappa \in \{0.25, 0.50, 0.75\}$ and recalibrate the rest of the parameters in the model as described in section 3.4. The resulting parameter values are in table 3.5.

parameter	$\kappa = 0.25$	$\kappa = 0.50$	$\kappa = 0.75$
θ	0.323	0.313	0.333
ψ	0.877	0.954	1.021
ω	0.823	0.878	0.919
γ	0.762	0.673	0.664
γ_u	0.393	0.344	0.301
c_v	0.031	0.018	0.017
μ	0.619	0.671	0.644

Table 3.5: Parameter values, Baseline + Garrison

To get a better idea of the margins involved, in the next figures we plot model responses of the option values for the workers under the parameterization when $\kappa = 0.5$. We label these option values as *in-firm* and *in-household* because they represent the values of either working for the firm or staying at home enjoying leisure. They also represent the components of the wage bill, as calculated in equation (3.21).

In figure 3.8, panel (a) shows the response to technology of the *in-firm* value:

$$\frac{\partial y}{\partial n} + c_v \frac{v}{1-n}$$

while panel (b) shows the response of the *in-household* value:

$$\frac{\tilde{b}_g}{u_c} \equiv \frac{\kappa(1-N)^{\kappa-1}\nu(1) - \nu(1-h)}{u_c}$$

Figure 3.9 shows the impulse response function of the labor share to a technology shock while the four panels of figure 3.10 show the response of output, employment, wages and hours per worker.¹⁹ As seen in the figure, the introduction of this tweak in the model doesn't change the fact that the response of the labor share fades too quickly when compared to the data. The more telling story is in panel (b) of figure

¹⁹The calibration is not entirely successful: Maybe, a conflict of steady state targets given the presence of κ prevents us from hitting all rbc targets.

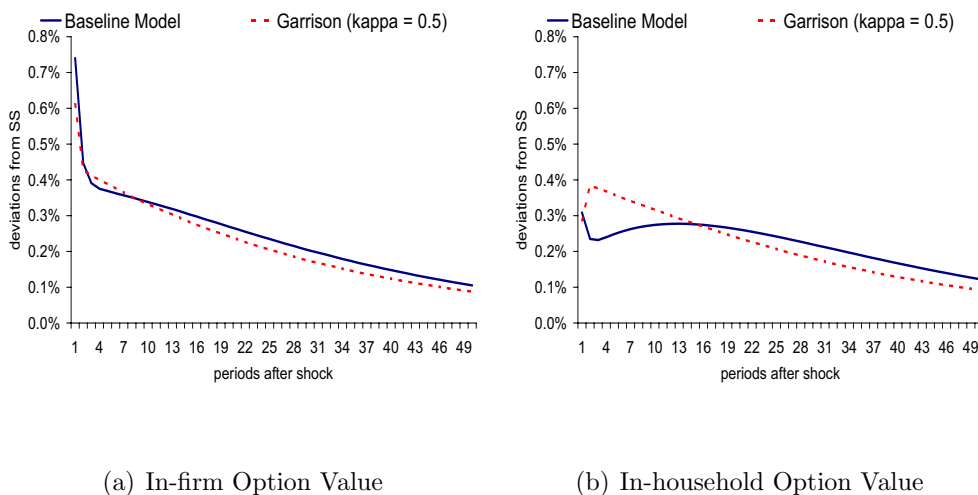


Figure 3.8: Response to a Technological Shock: Option Values for Workers

3.8, where the response of the *in-household* option value is depicted: through the Garrison preferences, we are forcing this value to be higher than the baseline throughout the transition back to the steady state. Nevertheless, we have two mechanisms that work against a positive response of labor share to technology. First, the same linkage between average labor productivity and real wages (through Nash bargaining) depresses employment in the face of higher wages. This is the same insight provided by [50]. In other words, although we force the model to produce real wages that are higher than the baseline, firms react by posting less vacancies and creating less employment.

Second, 3.8(b) shows that the effect of this artificial wedge is short-lived, given the convergence of the responses by the baseline and Garrison models just after 15 periods.

However, again we find that we cannot generate accurate dynamic responses of labor share to productivity innovations, the reason being that while wages move more employment move less.

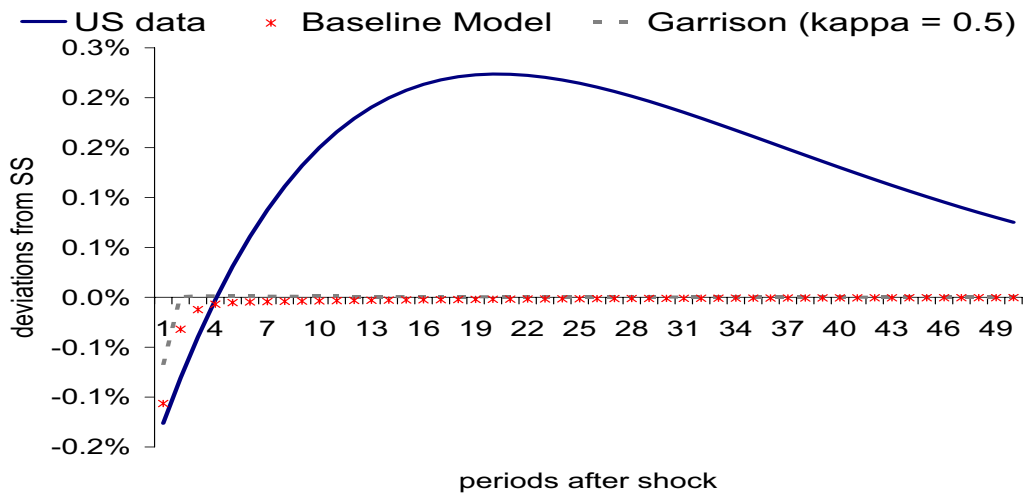


Figure 3.9: Dynamics of Labor Share

3.6.3 CES Technology

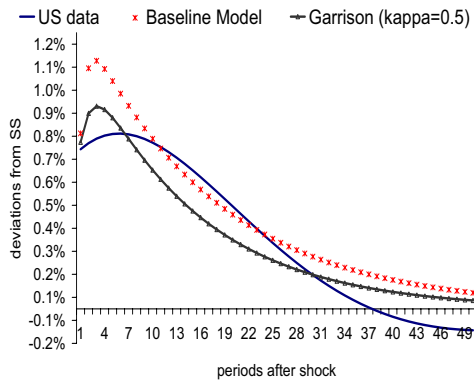
Finally, we pose the same benchmark model but change the aggregate technology. We want to ask whether an aggregate Cobb-Douglas production function imposes movements in shares that are too restrictive; hence, we feel that analyzing a CES (constant elasticity of substitution) production function might be a natural step in that direction. We use the exact same model, but change the production function to

$$Y = e^z [\theta K^{-\nu} + (1 - \theta)(Nh)^{-\nu}]^{-\frac{1}{\nu}}$$

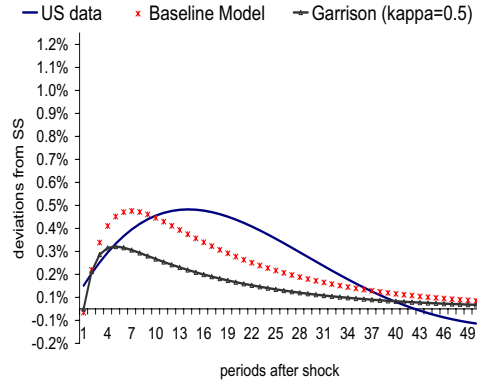
For the simulation of this economy, we use the same calibration as in the benchmark, but calibrate θ in order to obtain the same steady state factor shares. We set ν to 0.3 so to have an elasticity of substitution between capital and labor of 0.75 (in the Cobb-Douglas case, such elasticity is one).

Figures 3.11 and 3.12 show the impulse response functions of labor share and its components to a shock in technology when the aggregate production function is CES.

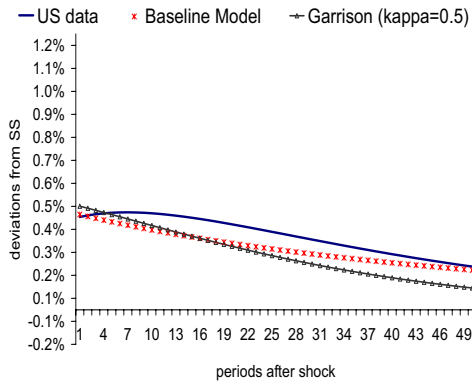
As seen in figure 3.12, the model with CES technology performs very similarly to the benchmark economy. However, the biggest departure comes from the reaction



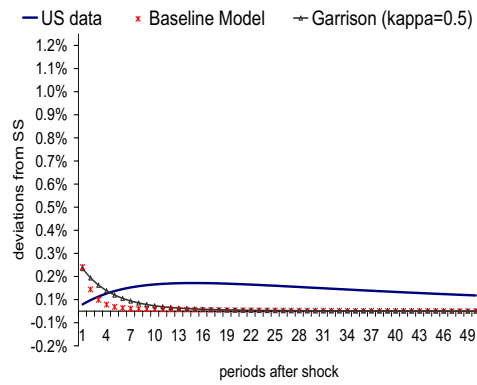
(a) Output



(b) Employment



(c) Wages



(d) Hours per Worker

Figure 3.10: Response to a Technological Shock: The Garrison Effect

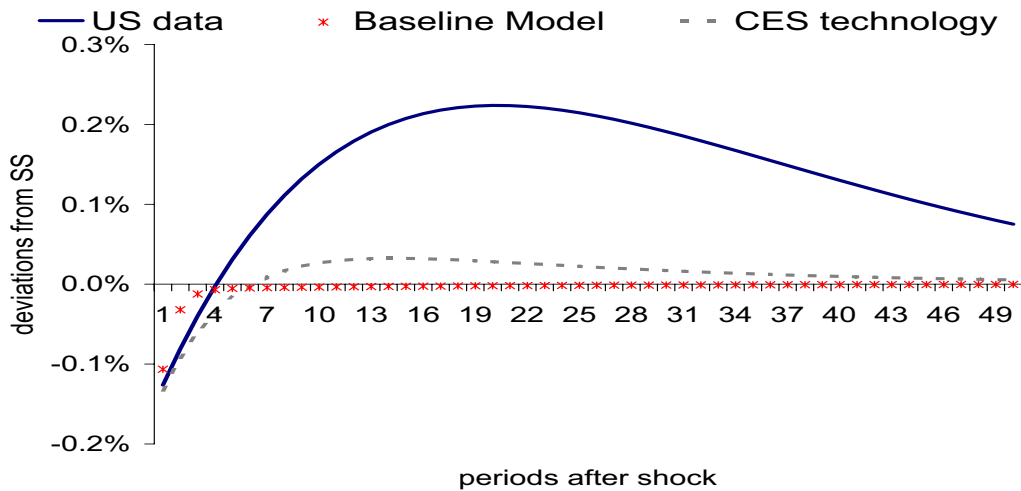


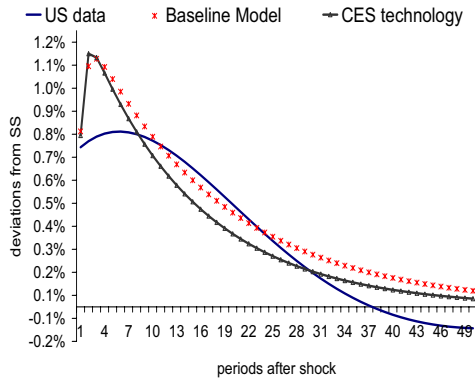
Figure 3.11: Dynamics of Labor Share

of the labor share (figure 3.11): it becomes positive after the 6th period and slowly reverts to zero. This is in contrast to every model presented so far and although the dynamics presented in the figure are not strong, they show that Cobb-Douglas technology indeed seems to pose a rigid share structure for the artificial economy.

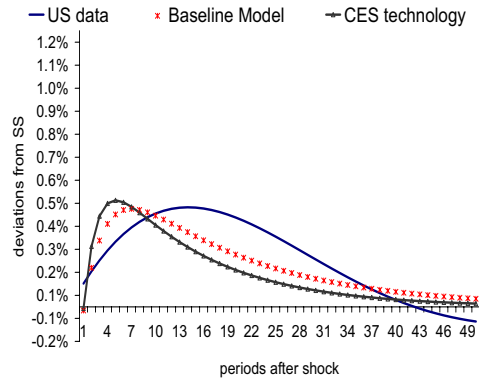
3.7 Conclusion

In this chapter we explore the extent to which models tailored to replicate cyclical facts about the labor market, are capable of generating the observed dynamics of labor share as described in [45]. We build upon [39], [3], [10], among others, who analyze models where factor prices are not set competitively, employment lags productivity, and labor share falls with productivity innovations.

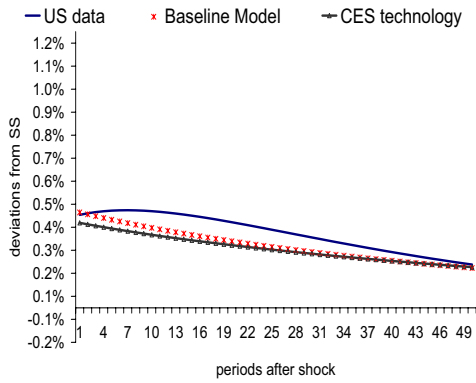
Our results show that the search and matching framework, along with non competitive wage setting fail in replicating empirical responses of the labor share to technological shocks: unlike in the data, responses by the model labor share are short-lived and fail to "overshoot". This negative result is robust to perturbations to the benchmark model (different preferences/technology) and is linked to the fact that



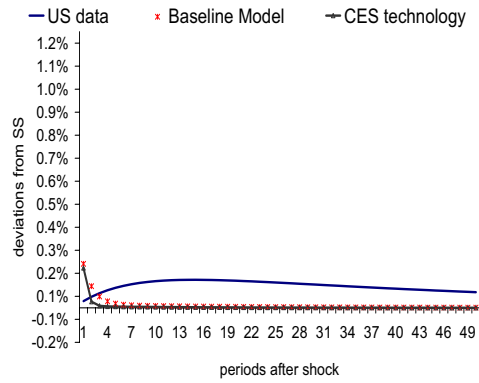
(a) Output



(b) Employment



(c) Wages



(d) Hours per Worker

Figure 3.12: Response to a Technological Shock: CES technology

the wage setting protocol used in these type of models (Nash bargaining) is not able to create a persistent wedge between real wages and labor productivity. Moreover, the failure of these models shows that the dynamics of the cyclical behavior of factor shares in income poses a strict discipline that may be used to discriminate between alternative classes of models.

In relative terms, our best results are given by economies where the aggregate production function is CES (instead of the standard Cobb-Douglas); we take this as evidence that exploring different technologies might be a good direction for further research.

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