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Essays in Quantitative Economics and International Finance

Serhiy Stepanchuk
University of Pennsylvania, serhiy@sas.upenn.edu

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Abstract
This dissertation uses quantitative dynamic models to study two separate questions in international finance and international labor markets.

In Chapter 2, we build a two-country two-good incomplete markets general equilibrium model of international portfolio choice, and use it to study global imbalances, in particular the large negative net foreign asset position, and international portfolio composition of the United States. We show that the ``exorbitant privilege'' of the US (ability to borrow in domestic currency) and lower volatility of macroeconomic shocks that the US has experienced since mid-80s can account for a large part of the negative net foreign position of the US, and also generate a realistic portfolio structure.

In Chapter 3, we study the differences in labor supply between the US and Europe. Using micro data from the US and eight European countries, we document that the difference between the US and Europe is mainly driven by the labor supply of women.European women work less than American women, whether it is single women, married women, or women with and without children. Using a larger number of countries, we also document that there is a strong correlation between divorce rates and female employment rates across countries and across time. A recent literature, including Prescott (2004) and Rogerson (2005), argues that differences in labor supply between the US and Europe can largely be explained by differences in tax rates. We use tax data from the OECD to develop tax schedules for a sample of 17 countries. The empirical correlation between hours worked and different measures of tax levels and progressivity is negative, however weak. Motivated by these observations, we develop a life-cycle model with heterogeneous agents, marriage and divorce and use it to study the impact of two mechanisms: 1) differences in marriage stability and 2) differences in tax systems on labor supply. There are three types of households; single males, single females and married households. Divorces and marriages occur stochastically. The main channel through which individual divorce and singlehood rates impact labor supply is by reducing the implicit insurance of marriage, and thereby providing incentives for individuals to invest in experience. We calibrate our model to US data and study how labor supply in the US changes as we introduce European tax systems, and as we replace the US divorce and marriage rates with their European equivalents. We find that the divorce and tax mechanisms combined on average explain 28% of the difference between the US and 11 European countries.

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Serhiy Stepanchuk

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Supervisor of Dissertation

Dirk Krueger, Professor of Economics

Graduate Group Chairperson

Dirk Krueger, Professor of Economics

Dissertation Committee

Harold Cole, Professor of Economics

Urban Jermann, Safra Professor of International Finance and Capital Markets

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ABSTRACT

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Serhiy Stepanchuk

Supervisor: Dirk Krueger, Professor of Economics

This dissertation uses quantitative dynamic models to study two separate questions in international finance and international labor markets.

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Chapter 1

Introduction

This dissertation uses quantitative dynamic models to study two separate questions in international finance and international labor markets.

In Chapter 2 (which is co-authored with Viktor Tsyrennikov), we build a two-country two-good incomplete markets general equilibrium model of international portfolio choice, and use it to study global imbalances, in particular the large negative net foreign asset position, and international portfolio composition of the United States. We assume that there are three assets that are traded internationally: claims to domestic and foreign equity, and a bond. We depart from a symmetric two-country setting, and evaluate the impact of the following two features of the economic environment on the international asset portfolio of the U.S.: a) the ability of the U.S. to issue bonds in domestic currency ("exorbitant privilege"), and b) lower volatility of output in the U.S. compared to the rest of the world. We model the "exorbitant privilege" by assuming that the bond's payoffs are denominated in domestic goods. We calibrate the model so that it is consistent with the data with respect to the behavior of the relative goods' prices, home equity bias and consumption-real exchange rate disconnect. We find that in equilibrium, the U.S. accumulates a negative position in the bond, and has a large negative overall net foreign asset position.

In Chapter 3 (co-authored with Hans Holter and Indraneel Chakraborty), we study the differences in labor supply between the US and Europe. Aggregate labor supply is higher in America than in Europe, and there is also substantial variation within Europe. Using micro data from the US and eight European countries, we document that the difference between the US and Europe is mainly driven by the labor supply of women. European women work less than American women, whether it is single women, married women, or women with and without children. Using a larger number of countries, we also document that there is a strong correlation between divorce rates and female employment rates across countries and across time. A recent literature, including Prescott (2004) and Rogerson (2005), argues that differences in labor supply between the US and Europe can largely be explained by differences in tax rates. We use tax data from the OECD to develop tax schedules for a sample of 17 countries. The empirical correlation between hours worked and different measures of tax levels and progressivity is negative, however, weak. Motivated by these observations, we develop a life-cycle model with heterogeneous agents, marriage and divorce and use it to study the impact of two mechanisms: 1) differences in marriage stability and 2) differences in tax systems on labor supply. There are three types of households; single males, single females and married households. Divorces and marriages occur stochastically. The main channel through which individual divorce and singlehood rates impact labor supply is by reducing the implicit insurance of marriage, and thereby providing incentives for individuals to invest in experience. We calibrate our model to US data and study how labor supply in the US changes as we introduce European tax systems, and as we replace the US divorce and marriage rates with their European equivalents. We find that the divorce and tax mechanisms combined on average explain 28% of the difference between the US and 11 European countries.

This finding is sensitive to the use of tax revenues.

Chapter 2

International Portfolios: An Incomplete Markets General Equilibrium Approach

2.1 Introduction

A recent wave of financial integration that started in mid-80's has led to a surge in international asset trade, and a build-up of large cross-border gross asset positions[1]. At the same time, there has been an emergence of the so-called "global imbalances". One of the most well-known aspects of the "global imbalances" that has recently came to the forefront of policy discussion has been a large and persistent negative net international asset position of the largest world economy, the United States. This has revived the interest in the analysis of the countries' international portfolios. For example, Obstfeld (2004) argues that understanding the structure of the countries' portfolios is of first-order importance for the analysis of external adjustment, claiming

[1] See Gourinchas & Rey (2005), Lane & Milesi-Ferretti (2005) and Lane & Milesi-Ferretti (2007) for a detailed account of these developments.

that "appropriate concepts of external balance adjustment cannot be defined without reference to the structure of national portfolios".

Figure 2.1, constructed using the data from the Lane & Milesi-Ferretti (2007) dataset, shows the sharp increase in the U.S. gross positions in two major asset categories – debt and equity, since mid-80's. It also shows the country's net position in these two asset categories [2].

Figure 2.1: US external positions in debt and portfolio equity

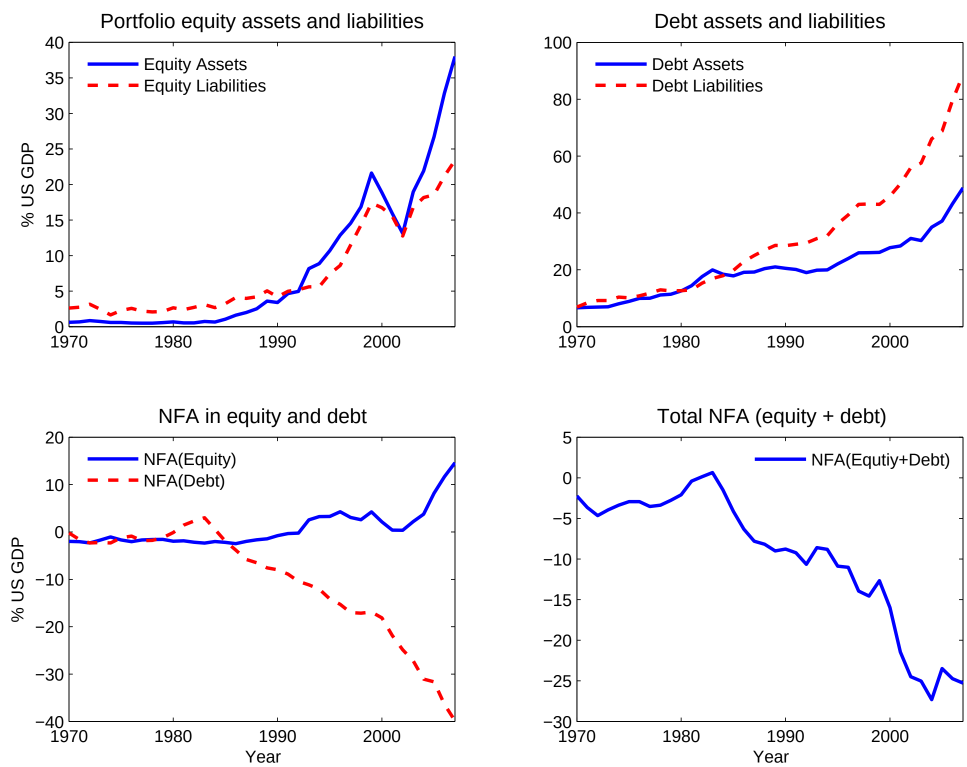


Figure 2.1 shows that the overall U.S. net foreign asset position has been driven by a growth in debt obligations, while the net position in equity has actually improved. Gourinchas & Rey (2005) conclude that "as financial globalization accelerated its

[2]The dataset is available on-line at `http://www.philiplane.org/EWN.html`. It contains the data on the net foreign asset positions (total and in several asset categories) for 178 countries for the 1970-2007 period.

pace, the U.S. transformed itself from a world banker into a world venture capitalist, investing greater amounts into high yield assets such as equity and FDI", while "its liabilities have remained dominated by bank loans, trade credit and debt, i.e. low yield safe assets". A similar observation is made by Obstfeld (2004), who states that for the United States, "the striking change since the early 1980s is the sharp growth in foreign portfolio equity holdings", while on the liabilities' side, "the most dramatic percentage increase has been in the share of U.S. bonds held by foreigners"[3].

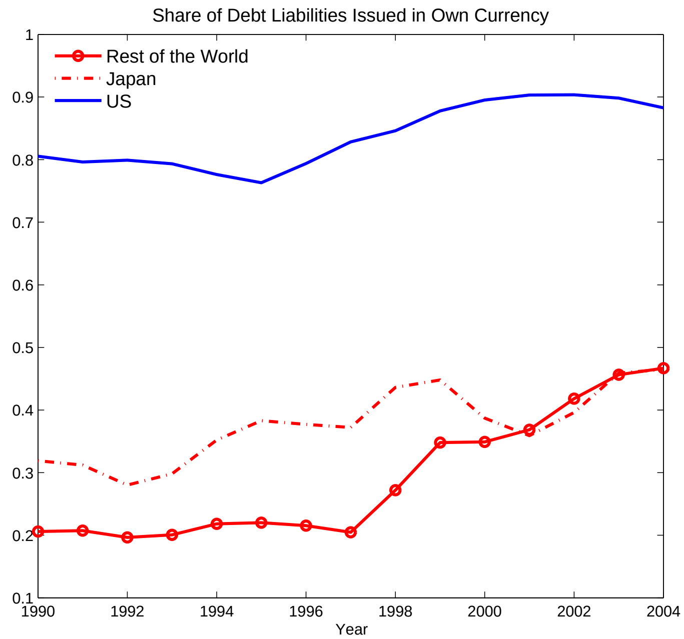
To explain the structure of the U.S. international portfolio, and the U.S. negative net foreign asset position, we develop a two-country general equilibrium model with many assets, incomplete financial markets and portfolio choice. We use the model to evaluate the effect of the following two features of the economic environment on the portfolio structure of the domestic economy:

1. *Exorbitant privilege.* Figure 2.2, constructed using the data from the Lane & Milesi-Ferretti (2007) and Lane & Shambaugh (2009) datasets, shows that the U.S., the issuer of the main global currency, has also been able to issue most of its debt liabilities in its own currency. Until the introduction of the Euro, only about 20% of the internationally-traded debt issued by all other countries has been denominated in the local currency of the issuing country. Even after the introduction of the Euro, this share remains under 50%. For comparison, the figure also shows the share of debt liabilities issued in local currency (Japanese yen) by Japan. Interestingly, over the 1990-2004 period (the period over which the data are available), on average the share of debt liabilities issued by Japan in the U.S. dollars (41.2%) exceeded the share of debt issued in yen (37.6%).

Eichengreen, Hausmann & Panizza (2002) analyze the determinants of the coun-

[3]See also Higgins, Tille & Klitgaard (2007), Tille (2005), Mendoza, Quadrini & Rios-Rull (2008) and Obstfeld & Rogoff (2005).

Figure 2.2: Share of debt liabilities issued in own currency: the U.S., Japan and the Rest of the World



try's ability to issue international debt in its own currency. They find that country size is the only robust determinant of the country's ability to issue debt in own currency. They also find that the effects of various measures of economic and financial development, the soundness of the country's monetary and fiscal policy, the degree of openness are statistically and economically insignificant. They conclude that the internationally traded debt "is concentrated in a very few currencies for reasons largely beyond the control of the excluded countries", and call this finding "the original sin"[4].

In our model, we will assume that the structure of the international debt market is exogenously fixed. We will model the exorbitant privilege by assuming that there is only one internationally-traded bond, whose payoff is denominated in terms of the

[4]Hasan (2010) develops a model where he shows that the debt issued in the currency of a larger country will have lower equilibrium interest rate. If this effect is large enough, one can imagine that no one would be willing to borrow in high-interest debt instruments issued in the currencies of smaller countries

domestic consumption good.

2. *Relative volatility of output shocks.* Fogli & Perri (2006) document that after 1985, the U.S. has experienced a larger fall in business cycle volatility compared to other countries, and propose this as an explanation for the deterioration in the U.S. net foreign asset position. Since their model has only one internationally-traded asset – a risk-less bond, they don't obtain any implications for the portfolio structure. We find that during the 1985-2007 period, the volatility of output in the U.S. has been about 1.5 times less volatile than in the rest of the OECD countries. We use our model to evaluate the impact of this difference in relative output volatilities on the international portfolio structure and the NFA position of the domestic economy.

We find that in our model, both the exorbitant privilege and the difference in output volatilities contribute to generating a large negative net foreign asset position for the home country, and also produce the portfolio structure that resembles the one observed in the U.S., with a short position in debt and a large share of wealth invested in risky assets like equity. We will show that the exorbitant privilege changes the correlation between the bond's payoff and home country non-traded income, and makes the bond whose payoff is denominated in home country goods an undesirable hedging instrument for domestic investors. Lower volatility of output in home country works through the precautionary savings channel, making domestic investors endogenously less risk-averse than their foreign counterparts. This causes foreign investors to accumulate precautionary savings in a relatively less risky bond.

Methodologically, we build on the numerical method developed in Kubler & Schmedders (2003). This allows us to obtain a numerical solution to our model that is globally accurate, while the perturbation methods for solving the international portfolio choice model recently developed in Tille & Van Wincoop (2010) and Devereux & Sutherland (2006) are designed to offer an accurate solution only locally around some fixed wealth distribution. The large magnitude of the negative NFA position that the U.S. has accumulated since 1985 imply large changes in the global financial wealth distribution, making the numerical method that we use a better suited tool for the analysis of international portfolio choice in the environment with global imbalances.

The structure of our paper is as follows. In section 2.2, we set up the model and reformulate it recursively, so that it can be analyzed using our numerical algorithm. In section 2.3, we consider a simplified version of our model to get some economic intuition for the mechanisms that drive our results. In section 2.4, we describe the algorithm, and test its performance in a special case of our model, where we can describe solution analytically. In section 2.6, we obtain and discuss the results from our calibrated model. Section 2.7 concludes.

2.2 Model

2.2.1 Economic Environment

We assume that there are only two countries, "Home" (representing the U.S.) and "Foreign" (representing the rest of the world, or RoW). Both economies are populated by a representative consumer/investor. Time is discrete and is indexed by $t = 0, \ldots, \infty$. Each period, both countries are endowed with some quantity of a local perishable good. The consumers in both countries enjoy both consumption goods, but have a relative preference towards the consumption of their local good ("home bias in consumption"). To differentiate between the domestic and foreign consumers, we will denote all foreign consumers' choice variables by an asterisk.

Each period t, one of finitely many possible states of the world, $z_t \in Z = \{z_1, \ldots, z_n\}$, realizes. The state of the world characterizes all relevant risks in our model. In particular, the endowment of both consumption goods in each period is a time-invariant function of the state of the world, $e_h : Z \to \mathbb{R}_{++}$ and $e_f : Z \to \mathbb{R}_{++}$. We will assume that only some part of the output in each country is "capitalized" – that is, it comes in the form of the dividends from the two Lucas trees that represent the stock indices in the two countries. The rest of the output is non-traded. We will refer to it as "wages", but as will be clear from our calibration, we intend it to also include the profits of the companies that are not publicly traded through the stock markets. The division of output between the dividends and the wages is also determined by the realization of the state of the world: $e_h(z_t) = d_h(z_t) + w_h(z_t)$ and $e_f(z_t) = d_f(z_t) + w_f(z_t)$[5].

We assume that the exogenous shock that determines the state of the world follows a first-order Markov process, with the probability transition matrix Π. A partial history of state realizations $(z_0, \ldots, z_t)$ is denoted by z^t. Given the partial history z^t, the conditional probability of state z_{t+1} next period depends only on the current realization of state z_t, and is denoted by $\pi(z_{t+1}|z_t)$.

Preferences. Consumers in both countries trade in financial and goods markets to achieve maximum expected lifetime utility. For domestic consumers, it is given by:

$$U_h(c) = E\Big[\sum_{t=0}^{\infty} \beta^t u(g_h(c_{ht}, c_{ft}))\Big| z_0\Big], \quad \beta \in (0,1). \tag{2.1}$$

Function g_h is the CES consumption aggregator, which takes the form:

$$g_h(c_h, c_f) = (s_h c_h^{\rho} + (1 - s_h) c_f^{\rho})^{1/\rho}, \quad \rho < 1,\ s_h > 0.5.$$

Parameter ρ controls the elasticity of substitution between the two consumption

[5]In particular, we will not assume, as is often done in the literature, that dividends are a constant fraction of the output. This will allow us to account for the fact that dividends are significantly more volatile than output.

goods. The assumption that $s_h > 0.5$ introduces the home bias in consumption.

We will assume that the instantaneous utility function u takes a standard CRRA utility function form:

$$u(g) = \frac{g^{1-\gamma}}{1-\gamma}, \quad \gamma > 0.$$

Foreign consumers maximize a similar lifetime expected utility function, with c^*_{ht}, c^*_{ft} and s_f replacing c_{ht}, c_{ft} and s_h.

Financial Markets. In a model with endowment economies and consumers with standard expected utility preferences, assuming complete financial markets has a number of unrealistic implications. First, Judd, Kubler & Schmedders (2000) demonstrate that this typically implies that consumers choose constant financial portfolios that do not change over time, with no trade in assets in any period beyond the initial period. This means that the only source of the changes in a country's net foreign asset position would be the valuation effects (changes in assets' prices), while the traditional measure of the current account which excludes the asset valuation changes is always zero. Second, Backus, Kehoe & Kydland (1992) show that it also has counterfactual implications for the cross-country correlation of consumption, while Backus & Smith (1993) show that it has counterfactual implications for the cross-country correlation between relative consumption and real exchange rate. We assume in our model that financial markets are incomplete, so that the number of internationally-traded assets with linearly independent payoffs is smaller than the number of the exogenous shock realizations. In particular, we assume that only the following 3 assets are traded internationally – two Lucas trees representing the stock indices in the two countries, and a single one-period internationally traded bond. The stocks represent claims to the future stream of dividends paid in the two countries' consumption goods, and are traded at the ex-dividend prices q_h and q_f. We assume that the bond's payoff is denominated in the consumption good of the home country. The price of the bond is given by q_b. With p_h and p_f denoting the prices of the two consumption goods, the payoffs to the two stocks and the bond are:

$$r_h \equiv q_h + p_h d_h,$$

$$r_f \equiv q_f + p_f d_f,$$

$$r_b \equiv p_h.$$

Taste shocks. We will show in section 2.3 that the position that domestic investors take in the bond in our model depends crucially on the behavior of the relative goods' prices [6]. To account for the behavior of the relative goods' prices in the data, we follow Stockman & Tesar (1995) and introduce taste shocks to our model, as a simple reduced-form way to model the demand-side shocks[7]. We model these taste shocks by allowing the preference weights, s_h and s_f, to vary with z_t. We assume that the taste shocks are i.i.d. and independent of the output shocks.

Government Debt. We abstract from the government sector and physical investments in our model. In public opinion, foreign debt in the United States is often related to the government debt. Figure 2.2.1 shows the evolution of the government debt and NFA position in the US (as a share of GDP) from 1970 to 2007[8]. It demonstrates that while the two closely tracked one another roughly till 1990, there appears to be little connection between the two since then. Figure 2.7 in appendix shows

[6] The behavior of relative prices also heavily influences the U.S. NFA position in the data. According to the data from the BEA, the average size of the annual changes in the U.S. NFA position that can be contributed to the exchange rate fluctuations over 1989-2009 period was 32% of the changes caused by the financial flows.

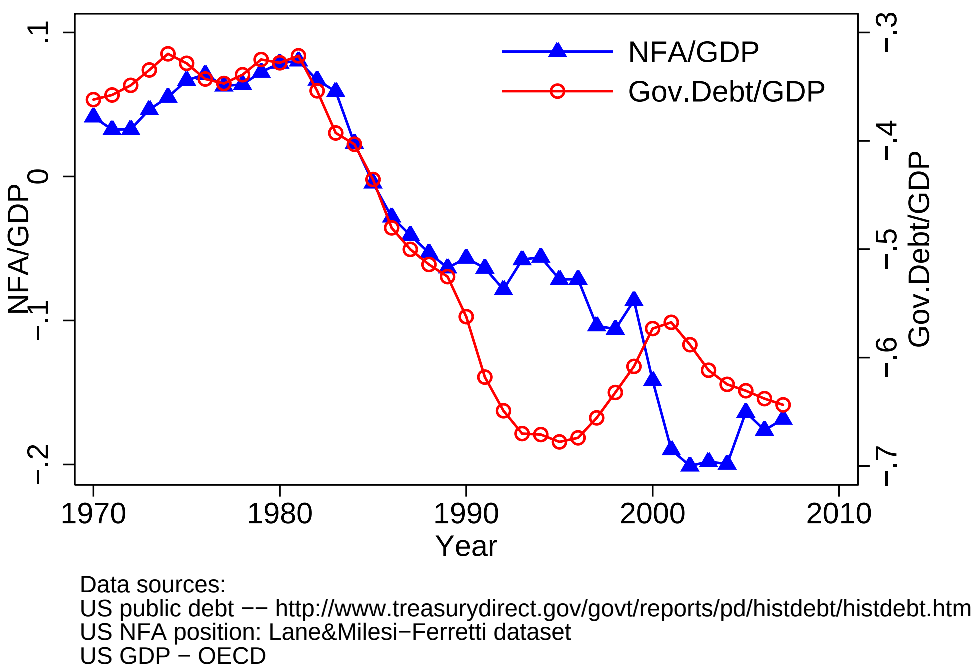
[7] Pavlova & Rigobon (2007) and Heathcote & Perri (2009) make a similar assumption.

[8] Government debt is shown with a negative sign, which corresponds to the negative bond position in our model.

the annual changes in the government debt/GDP and NFA/GDP positions over the same time. It confirms that the two variables followed each other till about 1970 (the correlation between them over the 1970-1989 period is 0.654), but have moved much further apart since then (the correlation between the two variables over the whole 1970-2007 period is only 0.013). Figures 2.8 and 2.9 in appendix show that qualitatively a similar picture arises when one considers the US NFA position in debt instruments ("bonds").

Figure 2.3: US Government Debt and NFA position

Scale for government debt/GDP is on the right, for NFA/GDP on the left



Budget constraints. Let (θ_h, θ_f, b) denote the domestic consumer's portfolio that consists of his position in the domestic and the foreign stock, and the bond. For each history of shocks z^t, domestic consumer faces a budget constraint that takes the following form:

$$p_h(z^t)c_h(z^t) + p_f(z^t)c_f(z^t) + q_h(z^t)\theta_h(z^t) + q_f(z^t)\theta_f(z^t) + q_b(z^t)b(z^t) = I_h(z^t), \quad (2.2)$$

where I_h is domestic consumer's "cash-in-hand" – the market value of their non-traded income and their financial portfolio (including the dividends):

$$I_h(z^t) \equiv p_h(z^t)w_h(z^t) + r_b(z^t)b(z^{t-1}) + r_h(z^t)\theta_h(z^{t-1}) + r_f(z^t)\theta_f(z^{t-1}).$$

Foreign consumers face similar budget constraints, with $(\theta_h^*, \theta_f^*, b^*)$, (c_h^*, c_f^*) and I_f replacing (θ_h, θ_f, b), (c_h, c_f) and I_h.

In addition to the budget constraints, we assume that investors face short-selling constraints on stocks, $\theta_h \geqslant 0$, $\theta_f \geqslant 0$, $\theta_h^* \geqslant 0$ and $\theta_f^* \geqslant 0$[9], and a borrowing limit on the bond that we describe next.

Borrowing limit. We assume that the amount that the investors in both countries can borrow using the internationally-traded bond is proportional to the lowest possible realization of the value of their wages next period. In particular, we require that:

$$\min_{z^{t+1}} \Big(kw_h(z^{t+1})p_h(z^{t+1}) + b(z^t)r_b(z^{t+1}) \Big) \geq 0, \quad \forall z^t \quad (2.3)$$

for some $k > 0$, and similarly for the foreign active traders. The parameter k controls the tightness of the borrowing limit.

Intuitively, if $k = 1$, this borrowing limit requires the consumer to be able to repay his debt for all possible realizations of the exogenous shock next period, using his non-traded income only [10]. We get additional flexibility by allowing k to be different from

[9] The short-selling constraints can be easily generalized to $\theta_h \geqslant -\bar{t}$, $\theta_f \geqslant -\bar{t}$, $\theta_h^* \geqslant -\bar{t}$ and $\theta_f^* \geqslant -\bar{t}$ for some $\bar{t} \in \mathbb{R}_+$.

[10] In this case, our formulation of the borrowing limit is similar to Kiyotaki & Moore (1997), who

1. In our baseline model in section 2.6.1, our strategy is to set k generous enough so that our results are not driven by potentially binding borrowing constraints (we set $k = 3$). One can also allow k to differ between the countries[11], however we do not use it in this paper.

Competitive equilibrium. For any initial realization of the exogenous state z_0 and the initial distribution of asset holdings $(\theta_h(z_0), \theta_f(z_0), b(z_0))$, we can define a competitive equilibrium in a standard manner, as:

- a prices system $\mathcal{P} = \{p_h(z^t), p_f(z^t), q_h(z^t), q_f(z^t), q_b(z^t), \ \forall z^t\}$,
- a consumption allocation $\mathcal{C} = \{c_h(z^t), c_f(z^t), c_h^*(z^t), c^*(z^t), \ \forall z^t\}$,
- portfolio positions $\mathcal{A} = \{\theta_h(z^t), \theta_f(z^t), \theta_h^*(z^t), \theta_f^*(z^t), \ \forall z^t\}$,

such that:

a) Given the prices, the consumption allocation and portfolio choices solve the optimization problem for every consumer.

b) For all z^t, markets clear:

$$
\begin{aligned}
c_h^*(z^t) + c_h(z^t) &= e_h(z^t), \\
c_f^*(z^t) + c_f(z^t) &= e_f(z^t), \\
\theta_h(z^t) + \theta_h^*(z^t) &= 1, \\
\theta_f(z^t) + \theta_f^*(z^t) &= 1, \\
b(z^t) + b^*(z^t) &= 0.
\end{aligned}
$$

require that the amount the borrower has to repay should not exceed the market value of his land holdings next period.

[11] This could potentially be useful to model the finding in Reinhart, Rogoff & Savastano (2003), who argue that many emerging economies can sustain substantially lower levels of debt compared to more advanced economies.

We add the following price normalization: $p_h(z^t) + p_f(z^t) = 1$ for all z^t.

2.2.2 Wealth-recursive equilibria

Since we are interested in finding the equilibrium in our model economy numerically, we will concentrate on equilibria that can be represented in a recursive form, as a map from some state space into all current endogenous variables (the policy functions), and a transition function that describes the evolution of the state variable(s) over time[12].

The choice of the state space has both theoretical and practical consequences. On the one hand, the current state must be a sufficient statistic for the future evolution of the system. On the other hand, a high-dimensional state-space can lead to insurmountable computational difficulties – the so-called "curse of dimensionality". The description of the budget sets of the investors in the previous section suggests that the distribution of "cash-in-hand", or wealth between the investors is a natural candidate for being the only endogenous state variable (in addition to exogenous shock) in our model. Following Kubler & Schmedders (2002), we will call this type of equilibrium a "wealth-recursive" equilibrium. Using wealth shares (as opposed to the beginning-of-period portfolios) as the single state variable offers the practical advantage of reducing the dimensionality of the numerical problem that we need to solve.

Wealth share as a state variable. For our numerical algorithm, it will be important to have a compact state space. We assume that the exogenous shocks come from some finite set. We define our endogenous continuous state variable, the wealth share, so

[12]Duffie, Geanakoplos, Mas-Colell & McLennan (1994) call this type of equilibria "dynamically simple". They argue that it is reasonable to concentrate on these equilibria, since equilibria that do not display some minimal regularity through time will require implausibly high degree of coordination between the agents. Krueger & Kubler (2008) provide an overview of this type of equilibria and their applications in macroeconomics.

that given our portfolio constraints, it will lie in the unit interval. To achieve this result, we first need to redefine the investors' "cash-in-hand" as:

$$\tilde{I}_h(z^t) = kw_h(z_t)p_h(z^t) + r_b(z^t)b(z^{t-1}) + r_h(z^t)\theta_h(z^{t-1}) + r_f(z^t)\theta_f(z^{t-1}),$$
$$\tilde{I}_f(z^t) = kw_f(z_t)p_f(z^t) + r_b(z^t)b^*(z^{t-1}) + r_h(z^t)\theta_h^*(z^{t-1}) + r_f(z^t)\theta_f^*(z^{t-1}).$$

The total wealth of the investors worldwide is:

$$\tilde{I}(z^t) = \tilde{I}_h(z^t) + \tilde{I}_f(z^t) = k(w_h(z_t)p_h(z^t) + w_f(z_t)p_f(z^t)) + r_h(z^t) + r_f(z^t),$$

which follows from the asset market clearing conditions, $b + b^* = 0$, $\theta_h + \theta_h^* = 1$ and $\theta_f + \theta_f^* = 1$. Note that with strictly positive prices, the total wealth is always strictly positive.

As the next step, let us define the wealth shares of domestic and foreign investors as:

$$\omega_h(z^t) = \frac{\tilde{I}_h(z^t)}{\tilde{I}(z^t)}, \quad \omega_f(z^t) = \frac{\tilde{I}_f(z^t)}{\tilde{I}(z^t)}. \tag{2.5}$$

Note that $\omega_h(z^t) + \omega_f(z^t) = 1$ for all z^t by construction. The following lemma is very useful for our computational algorithm.

Lemma 2.2.1. *Given the short-sale constraints on equity positions and the borrowing limit defined by 2.3, the wealth shares of domestic and foreign active traders remain in the unit interval,* $\omega_h(z^t) \in [0, 1]$, $\omega_f(z^t) \in [0, 1]$ *for all* z^t.

Proof. If the short-sale constraints on equity positions, and the borrowing limit 2.3 is always satisfied for both domestic and foreign investors, we obtain that $\omega_h(z^t) \geqslant 0$ and $\omega_f(z^t) \geqslant 0$ for all z^t. Since $\omega_h(z^t) + \omega_f(z^t) = 1$, the desired result follows immediately. □

We can now rewrite the budget constraints for the domestic investors as:

$$p_h(z^t)c_h + p_f(z^t)c_f + q_h(z^t)\theta_h' + q_f(z^t)\theta_f' + q_b(z^t)b' =$$
$$\omega_h(z^t)\tilde{I}(z^t) + (1-k)w_h(z_t)p_h(z^t)$$

Intuitively, the right-hand side of this equation, which determines the resources available to the consumer in node z^t, depends only on the current realization of the exogenous shock z_t, equilibrium prices (which together determine the total wealth, $\tilde{I}(z^t)$) and the investor's wealth share, $\omega_h(z^t)$, but not on the consumer's positions in each of the three assets separately. We thus can expect that $(\omega_h, z) \in [0,1] \times Z$ can serve as a sufficient statistic for for the whole past history z^t, so that we can use them as state variables.

2.3 Inspecting the mechanism

To gain some insight into the investors' incentives to hold the bond in our model, we consider a simplified version of our model where the bond is the only asset that is traded internationally, and investors hold all of the corresponding local stocks. To understand the investors' asset positions, we will use the indirect approach proposed in Svensson (1988). The idea of this approach is to look at the value that the investors attach to the asset before they participate in the financial markets[13]. Intuitively, the investor that values the asset more before the trade will end up holding the asset. Svensson (1988) shows that this intuition holds exactly in a 2-period model with only 1 asset, and holds as a tendency in a model with more assets.

[13] As a result of the trade in financial markets, the investors will choose their portfolios so that the marginal values that they attach to each asset will be equalized. However, before the trade, they will typically have different marginal valuations for the asset.

The value of the asset to the investor depends both on the asset's future stream of payoffs, and the investor's pricing kernel, which summarizes the investor's needs for income in different states of the world. In the appendix, we show that, after going through the so-called "two-stage budgeting" procedure, the pricing kernel of the domestic investor before the investors choose their asset positions for the next-period can be expressed as:

$$M_h(z^{t+1}) = \left(\pi_h(z^{t+1})^{\gamma-1}\left(p_h(z^{t+1})e_h(z^{z+t})\right)^{-\gamma}\right), \tag{2.6}$$

where $\pi_h = \left(s_h^{1/(1-\rho)}p_h^{\rho/(\rho-1)} + s_f^{1/(1-\rho)}p_f^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$ is the consumption-based domestic pricing index (home CPI), and $p_h e_h$ is the market value of the domestic investor's labor income. Similarly, foreign investor's pricing kernel (before the trade in financial markets) is:

$$M_f(z^{t+1}) = \left(\pi_f(z^{t+1})^{\gamma-1}\left(p_f(z^{t+1})e_f(z^{t+1})\right)^{-\gamma}\right). \tag{2.7}$$

The value of the bond to the domestic investor is:

$$q_b^h(z^t) = \frac{\beta}{M_h(z^t)}\left[E\left(M_h(z^{t+1})|z^t\right)E\left(r_b(z^{t+1})|z^t\right) + Cov\left(M_h(z^{t+1}), r_b(z^{t+1})|z^t\right)\right],$$

with a similar expression for the foreign investor, q_b^f.

The first term inside the square brackets can be interpreted as the present discounted value of the bond's payoffs to the investor. The second term shows the covariance, or "hedging" value of the bond's payoffs. It depends on how well-adapted the stream of bond's payoffs is to the investor's needs (expressed by the pricing kernel). For developing the intuition, we begin by considering the case where both investors start the period with equal wealth, and where the output in both countries has the same volatility. In this case, the first term in the above asset-pricing equation will be the same for both investors, and the investors will differ only in their covariance (or "hedging") value of the bond's payoffs.

Equation 2.6 shows that domestic investor's pricing kernel has two parts – one that contains the domestic CPI, and the other one that contains the domestic investor's labor income. With home bias in consumption ($s_h > 0.5$), domestic bonds have a positive hedging value against the domestic CPI shocks (assuming $\gamma > 1$), and its magnitude depends on the variance of the goods' prices. At the same time, since the bond's payoffs are denominated in terms of the same good as the domestic investor's next-period income, the bond will have a negative hedging value for the domestic investor against the fluctuations in his income. As we show next, the overall hedging value of the bond depends on both the variance of the relative goods' prices, and their covariance with the domestic output.

Using a linear approximation of the pricing kernel, $M_h = \pi_h^{\gamma-1} (p_h e_h)^{-\gamma}$, around $\bar{p}_h = \bar{p}_f$ and $\bar{e}_h = E(e_h)$, we get:

$$Cov(M_h, r_b) \approx C_1 \left((\gamma - 1)(2x - 1) - \gamma\right) Var(p_h) - C_2 Cov(e_h, p_h), \qquad (2.8)$$

where:

$$x = \frac{s_h^{1/(1-\rho)}}{s_h^{1/(1-\rho)} + (1 - s_h)^{1/(1-\rho)}} \in (1/2, 1), \quad \text{given } 1/2 < s_h < 1,$$

$$C_1 = \frac{\bar{\pi}_h^{\gamma-1}(\bar{w}_h \bar{p}_h)^{-\gamma}}{\bar{p}_h} > 0, \qquad \text{and } C_2 = C_1 \frac{\bar{p}_h}{\bar{e}_h} > 0.$$

With the home bias in consumption, the term $((\gamma - 1)(2x - 1) - \gamma)$ is negative for all parameter values, making the bond undesirable for domestic investors. At the same time, $Cov(e_h, p_h) < 0$ since higher relative output of the domestic good in our model leads to a decrease in domestic good's price, making the bond more desirable for the domestic investor. As a result, the investors' position in the bond will depend crucially on 3 things: the volatility of relative goods' prices (or terms of trade), their covariance with the output shocks, and the size of the home consumption bias. We will calibrate our model so that it is consistent with the data along these 3 dimensions, and will show that in such an environment, the domestic investor borrows using the bond.

In addition to this, when the volatility of output shocks is higher abroad, the volatility of foreign investor's labor income will also be higher. Because of the precautionary savings motive, this gives the foreign investor an additional incentive to save using the relatively safer asset, the bond.

2.4 Computing the equilibrium

One popular approach to solving the dynamic stochastic general equilibrium models is to use the so-called local perturbations approach[14], when one uses the information about the solution to the system of equations that characterize the equilibrium at some particular given point and their first, second and potentially higher-order derivatives at that point to obtain an approximation to the equilibrium policy and pricing functions in the form of a Taylor series.

Judd & Jinn (2002) discuss two cases when the application of the perturbation approach is problematic: 1) models with portfolio choice, 2) models where the distribution of wealth may matter. The model that we use contains both of these complications. In both cases, the problem is created by the fact that the steady state solution of the deterministic version of the model (which is typically used as a point of approximation) is not locally unique. The intuition for why this problem arises in

[14] Judd (1998) and Judd & Jinn (2002) provide an excellent introduction to this method.

the first case, with portfolio choice, is straightforward – in a deterministic version of the model without shocks, all assets become perfect substitutes, and the agents are indifferent between any portfolio shares. To understand the problem in the second case, one can consider a simplified version of our model with only one consumption good and one asset that is traded between the countries. Equilibrium in this case is characterized by the following 6 equations (the two Euler equations, two budget constraints and two laws of motion for exogenous output shocks):

$$q_{b,t}u'_h(c_{h,t}) = \beta E_t \left(u'_h(c_{h,t+1})\right),$$
$$q_{b,t}u'_f(c_{f,t}) = \beta E_t \left(u'_f(c_{f,t+1})\right),$$
$$q_{b,t}b_{h,t+1} + c_{h,t} = e_{h,t} + b_{h,t},$$
$$-q_{b,t}b_{h,t+1} + c_{f,t} = e_{f,t} - b_{f,t},$$
$$\log(e_{h,t+1}) = \rho \log(e_{h,t}) + \epsilon_{h,t},$$
$$\log(e_{f,t+1}) = \rho \log(e_{f,t}) + \epsilon_{f,t}.$$

In a stationary deterministic version of this model, the two Euler equations become identical (since $c_{h,t+1} = c_{h,t}$ and $c_{f,t+1} = c_{f,t}$), and the steady state solution for four equilibrium variables, $\left(b_h^{ss}, c_h^{ss}, c_f^{ss}, q_b^{ss}\right)$, is characterized by only 3 independent equations:

$$q_b^{ss} = \beta,$$
$$q_b^{ss}b_h^{ss} + c_h^{ss} = 1 + b_h^{ss},$$
$$-q_b^{ss}b_h^{ss} + c_f^{ss} = 1 - b_h^{ss},$$

As a result, there will be a continuum of possible steady state solutions.

The existence of the continuum of steady state solution makes it difficult to perform the first step of the algorithm, which fixes the constant terms of the policy and pricing functions. In addition, we have also found that even if one arbitrarily chooses one of the steady states as the approximation point, the algorithm often runs into difficulties performing the second step – finding the linear approximation to the solution of the deterministic version of the model around this steady state, as the number of stable eigenvalues in the linearized version of the model is smaller than the number of control variables[15].

Tille & Van Wincoop (2010) and Devereux & Sutherland (2006) address the first problem related to the portfolio choice. They develop algorithm that can choose one of the continuum of steady state portfolios by using different orders of approximation for different equations that describe the equilibrium (a higher order for Euler equations compared to the rest of the equilibrium system).

Schmitt-Grohe & Uribe (2003) suggest several ways to deal with the second problem – existence of continuum of steady state wealth distributions in a small open economy model, which can be applied in our model as well. However, these suggested solutions essentially arbitrarily choose one of the continuum of possible steady state wealth distributions, and penalize the agents for deviating from them. In this case, it is hard to say whether the long-run predictions of the model are driven by the features of the underlying model, or the fix itself. To illustrate this problem, we use the simplified one-good one-asset version of our model described above. We also assume that domestic agents are less risk-averse than foreigners (we set the parameter γ that controls the relative risk aversion at 2 for domestic consumers, and at 3 for foreigners). One could expect that precautionary motives would lead to domestic agents

[15]To find the local perturbation solution, we used the code described in Schmitt-Grohe & Uribe (2004). Klein (2000) describes the conditions that guarantee existence and determinacy of the solution to the linear rational expectations models.

accumulating negative bond holdings over the long run in this case. To induce the uniqueness of the of the steady state distribution of wealth, we introduce convex costs of holding a non-zero bond position (Schmitt-Grohe & Uribe (2003) call it "portfolio adjustment costs"), specified as:

$$\text{Cost}(b) = \frac{\kappa}{2} b^2.$$

Table 2.1 shows the long-run simulated means implied by the solution to the model using the projection approach, and using perturbations with different values of κ (column 2). These means are obtained using one long simulation with several million draws of the vector of exogenous shocks (figure 2.4 shows the simulated distributions). It also reports the simulated autocorrelation and standard deviation of bond position of country h, obtained by averaging over several thousand shorter simulations (with the length of each simulation equal to $T = 100$ periods) in columns 3 and 4. As is clear from the table, the long-run predictions obtained using the perturbation approach depends crucially on the magnitude of the portfolio adjustment costs[16]. Compared to the projection solution, they underpredict the magnitude of the negative bond position for high values of κ, and overpredict it for low values of κ. For values of κ smaller than 3^{-5}, the simulations based on the model solution eventually diverge to infinity, failing to produce a long-run distribution. Columns 3 and 4 show that the short-run behavior is less affected by the magnitude of κ, but it differs from that predicted by the projection solution.

We avoid these complications by using the projection method[17]. In addition, the solution obtained by using the projection method is designed to be globally accurate

[16]In this toy model, the steady state value of output in both countries is equal to 1. Thus, the average values of bond holdings reported in column 2 can be though of as the long-run debt-to-GDP ratio.

[17]See Judd (1998), chapter 11 for an excellent introduction to these methods.

Table 2.1: Simulated average bond holdings

Solution Method	Mean(b_h)	Corr$(b_{h,t}, b_{h,t+1})$	$\sigma(b_h)$
Projection	-0.178	0.924	0.080
Perturbation, $\kappa = 1^{-3}$	-0.011	0.982	0.0123
Perturbation, $\kappa = 5^{-4}$	-0.021	0.984	0.0121
Perturbation, $\kappa = 4^{-5}$	-0.355	0.985	0.0117
Perturbation, $\kappa = 3^{-5}$	-0.553	0.985	0.0117

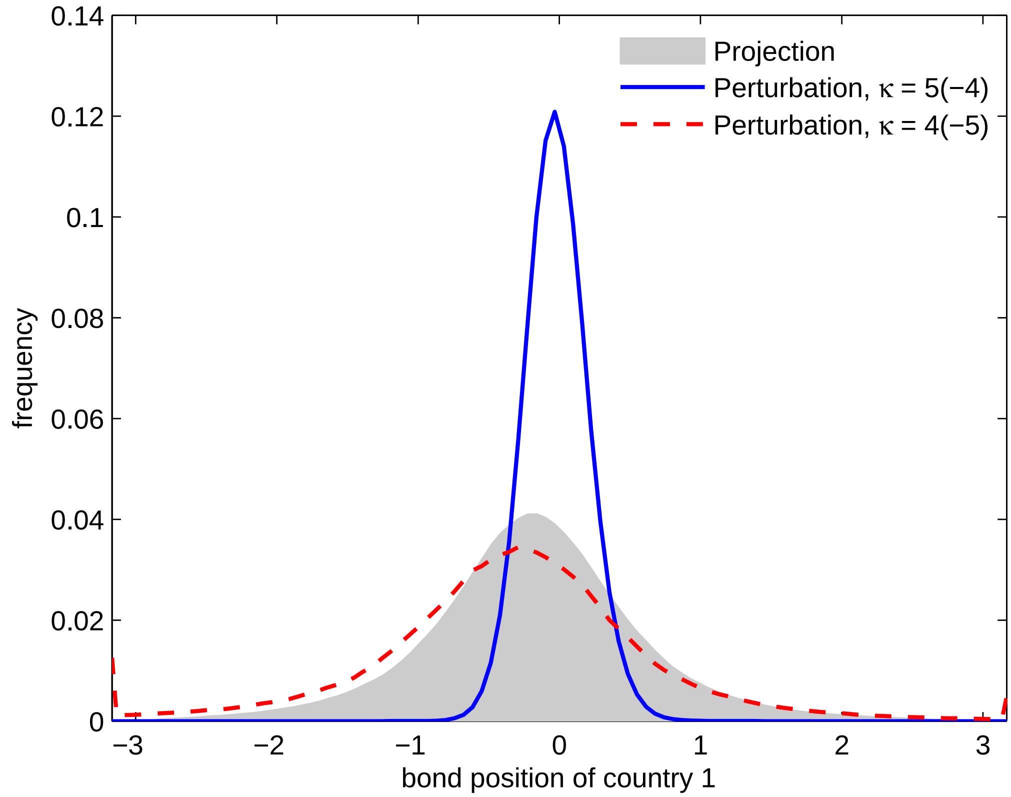


Figure 2.4: Simulated distributions of bond holdings

for all values of the state space, not only in some neighborhood of the point of approximation, as is in the case of local perturbations. We provide a more detailed description of the algorithm in appendix 2.8.3, while here we only outline its main features. We project the policy and pricing functions into the space of piecewise polynomials (splines). We start with some initial guess, and update the polynomial coefficients by solving the system of non-linear equations that describe the first-order optimality and market clearing conditions (described in details in the appendix) at each point on some predetermined grid over the state space, $[0, 1] \times Z$, and iterate until convergence.

Portfolio constraints can lead to non-differentiable policy functions, and the location of the kinks are not known a priori. To deal with this complication, we use several hundred grid points over $[0, 1]$. Another complication is that the portfolio constraints introduce inequalities (through Kuhn-Tucker complementarity conditions) into the system of temporal equilibrium conditions. We deal with this by using the "Garcia-Zangwill" trick (described in detail in Garcia & Zangwill (1981)). It essentially transforms the Kuhn-Tucker inequalities into equalities by an appropriate change of variables. We provide a brief explanation of how it works in the appendix.

The algorithm was implemented in Fortran 90. The code is available upon request [18].

We test the performance of our numerical algorithm in a special case of our model, where trade in two stocks is enough to achieve an efficient allocation, and we can describe the solution analytically. The results, which are provided in appendix 2.8.4, suggest that the numerical algorithm that we use produces a very accurate approximation to both the policy functions, and the ergodic distribution of the model (at

[18] To solve the non-linear system of equilibrium equations, we used two non-linear solvers: HYBRD and KNITRO (more robust).

least in this special case).

2.5 Calibration

In section 2.3, we show that the investors' position in the bond depends crucially on the volatility of the relative goods' prices (or terms of trade), their correlation with the output shocks, and the degree of the home bias. We calibrate our model so that it is consistent with the data along these three dimensions. To implement this calibration strategy, we need the data for the terms of trade. Because of the data availability limitations, we concentrate on the OECD economies. Thus, in our calibrated model, country 2 ("Foreign") stands for all OECD economies except the U.S. We are interested in the period since 1985, because it coincides with the period of increased international capital flows.

Data sources. We use the national income accounts data from the OECD.Stat database. Since we abstract from the government expenditures and the investment in the physical capital, we use the data for the final consumption expenditures of the households (series P31S14) plus net export (series B11) as our measure of output. Similar to Backus, Kehoe & Kydland (1994) and Stockman & Tesar (1995), we measure terms of trade as the ratio of the implicit price deflator for imports to the implicit price deflator for exports, where the deflators are computed as the ratios of current-price imports and exports to base-year-price imports and exports.

We also use Robert Shiller's dataset for the U.S. stock market dividends, and the dataset compiled by Lane and Milesi-Ferretti for the countries' net foreign asset positions and portfolio composition.

Output processes. We use HP-filtered series for consumption and net export (C+NX) for the U.S. and the rest of the OECD economies during the 1985-2007 period to construct the process for the output shocks in our two model economies. To obtain a measure of output volatility in our country 2 ("Foreign"), we compute a weighted average of standard deviations of our measure of output in each of the OECD country (except the U.S.), using the corresponding country's average share in total output as the weight. We find that the output abroad was about 1.5 times more volatile than in the U.S. during this period. Table 2.2 provides the details. It also shows that we find a similar difference in output volatilities if we use the growth rates of output instead of the HP-filtered series. Table 2.7 in the appendix provides more details for each of the country in our sample.

	HP-filtered series	**Growth rate series**
$std(y_t^{us}), \%$	0.984	0.881
$std(y_t^{row}), \%$	1.506	1.536
$corr(y_t^{us}, y_t^{row})$	0.127	0.131
$corr(y_t^{us}, y_{t-1}^{us})$	0.659	0.369
$corr(y_t^{row}, y_{t-1}^{row})$	0.619	0.364

Table 2.2: Summary statistics for the output processes in the U.S. and the rest of the OECD countries (RoW)

We assume that the output in our two model economies follows a bivariate VAR process, and approximate it with a discrete first-order Markov process with 9 states (3 for each country), using the approach described in Knotek & Terry (2008)[19].

An alternative way to obtain a measure of the volatility of output for our second model economy would be to first construct the total output for the rest of the world as the sum of the outputs of all individual RoW countries in our sample, and then compute its volatility. However, such total output for RoW turns out to be *less* volatile than the output in the U.S. (with $std(y_{hp}^{tot\,row}) = 0.56\%$), despite the fact that for all individual countries in our sample except one (France), output is more volatile

[19]This approach implements a bivariate version of the Tauchen discretization procedure using Monte Carlo simulation to compute the cell probabilities.

than in the U.S. (see table 2.7 in the appendix for more details)[20].

Dividends. Since we lack the data for the stock dividends for many countries in our sample, we will use the U.S. data to calibrate the dividend process. In particular, we use the data on the dividends accruing to the S&P Composite Stock Market Index from Robert Shiller's dataset. Figure 2.5 shows the joint behavior of dividends and our measure of output in the U.S. over the last four decades.

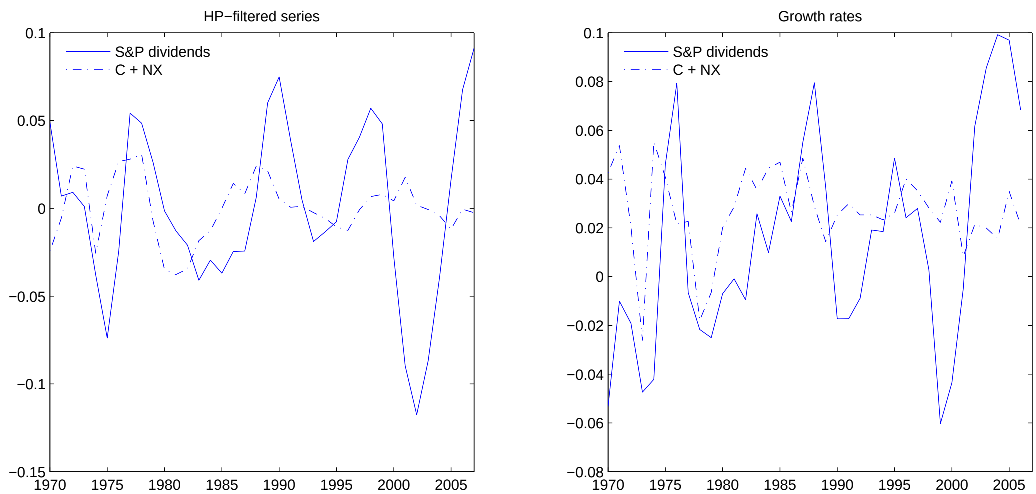


Figure 2.5: Dividends and output in the U.S.

The figure shows that since the mid-80s, there has been a reduction in the volatility of output in the U.S., which was not accompanied by a similar reduction in the volatility of many financial variables. In particular, stock dividends remained as (or even more) volatile as before 1985. As a result, during the 1985-2007 period, the dividends were significantly more volatile than our measure of output, and the correlation between the two was close to zero. Table 2.3 provides the details.

We choose the dividends in both countries to match the following 3 moments: (1)

[20]This second approach for measuring output volatility for the rest of the world implicitly assumes that individual output shocks are perfectly hedged between the RoW countries. We make the other extreme assumption that the RoW countries cannot hedge their shocks bileterally among themselves at all.

	HP-filtered series	**Growth rate series**
$std(d_t)/std(y_t)$	6.25	4.35
$corr(d_t, y_t)$	-0.02	-0.03

Table 2.3: Dividends and output in the U.S.

the relative volatility of dividends compared to volatility of output, $\sigma(d_t)/\sigma(y_t) = 6.25$, (2) the correlation between dividends and output, $\rho(d_t, y_t) = -0.02$ and (3) the average dividend share, $E\left(\frac{d_t}{y_t}\right) = 0.05$ (to match the average ratio of corporate dividends to our measure of output in the U.S.).

Taste shocks and other parameters. As we mentioned in section 2.2.1, to account for the behavior of relative goods' prices, we follow Stockman & Tesar (1995) and introduce taste shocks, as a reduced-form way to model the demand-side shocks. We assume that the preference weights in both countries, s_h and s_f, are stochastic, and take on one of the two possible values, $\bar{s}_i + \epsilon$ and $\bar{s}_i - \epsilon$. We assume that taste shocks are i.i.d., independent of the output shocks in both countries. We choose the variance of the taste shocks (by choosing ϵ), the average degree of the home bias in consumption (by choosing $\bar{s}_h$ and $\bar{s}_f$) and ρ to match the relative volatility of terms of trade to output, the correlation between the terms of trade and output, and the share of trade in output (as measured by $\frac{0.5(Im+X)}{C+X-Im}$) in our OECD sample[21]

We set the value of the discount factor β to 0.96, which gives us the annual interest rate of 4%, and set the value of γ (the parameter in the CRRA instantaneous utility function) to 2.

Table 2.4 summarizes our choice of all the parameters.

[21] We choose to match the trade share for the U.S. (16.3%). The average weighted trade share for all countries in our sample is equal to 29.9%. Matching this higher trade share increases the negative position of the domestic country in the bond.

	Value	Moment/Source
$std(e_t^{us})$	0.01	Volatility of output in US
$std(e_t^{row})$	0.015	Volatility of output in rest of OECD
$corr(e_t, e_{t-1})$	0.64	Persistence of output, OECD
$corr(e_t^{us}, e_t^{row})$	0.12	Cross-country correlation of output
$E(d_t/e_t)$	0.05	Corporate dividends/Output Ratio, US
β	0.96	Return on bond = 4%
σ	2.00	Common benchmark value
$\bar{s}$	0.616	Trade/Output in US = $0.5(X+M)/(C+NX) = 16.3\%$
ρ	0.71	std(terms of trade)/std(output), OECD
$std(\epsilon)/std(e_h)$	0.74	$corr$(terms of trade,output), OECD

Table 2.4: Parameter values

2.6 Results

2.6.1 Numerical results

In this section, we report our numerical results. We consider 2 versions of our model:

- *Model M1.* A model with the exorbitant privilege in the bond market, but the same volatility of output in the two countries.
- *Model M2.* A model with both the exorbitant privilege and smaller volatility of output in home country.

In model M1, we assume that the bond pays a unit of domestic good, but the two output processes in both countries are equally volatile (we set the standard deviation of output in both countries to 0.0125, which is the weighted average for all countries in our sample, including the U.S.). In other words, we switch off the different volatilities channel, and study the impact of the exorbitant privilege channel separately on our results. In model M2, we assume that both of our channels are operative, and we set all the model parameters to the values reported in table 2.4.

Figure 2.6 plots the simulated ergodic distributions from models M1 and M2, and compares them to the distribution generated by the fully symmetric model SYM (in which both the exorbitant privilege and different volatility channels are turned off). We use these simulated ergodic distributions to compute all the models' summary statistics that we report below.

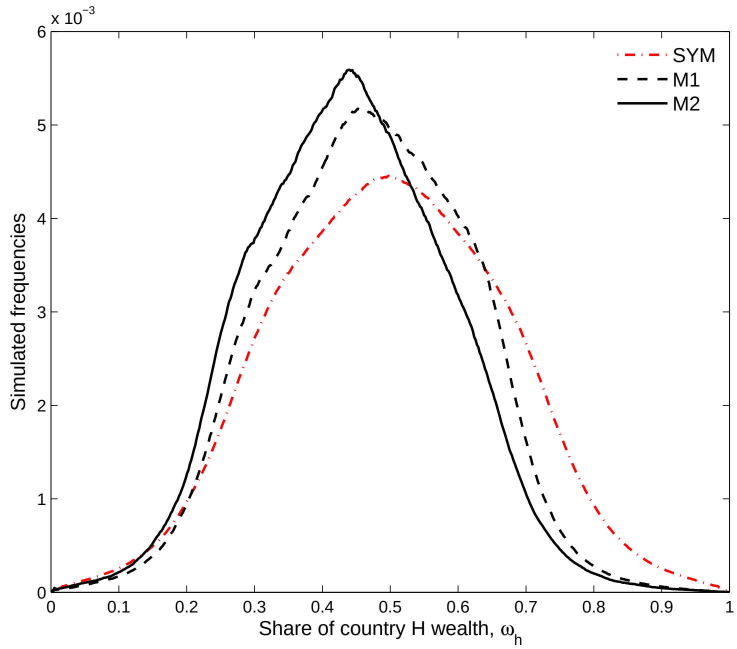


Figure 2.6: Simulated ergodic distributions

In both models M1 and M2, the ergodic distribution is skewed to the left, with domestic economy having a lower financial wealth in the long-run. Table 2.5 shows that this is caused by the large negative NFA position of the domestic economy. Table 2.5 compares the moments generated by our model to those in the data. The NFA(h) position reported in the "Data" column corresponds to the U.S. NFA position in two asset categories – equity and debt, as a percentage of $C + NX$ (our measure of output), in 2007, computed using the data in Lane & Milesi-Ferretti (2007). The table shows that the exorbitant privilege alone generates a large negative NFA position for the domestic country, close to a half of the negative NFA position observed for the U.S. Adding smaller output volatility in the home country amplifies this result, and generates a negative NFA position slightly higher than the one observed in the U.S. in 2007.

Table 2.5: Simulated and data moments

	Data	M1	M2
NFA(h)/output(h)	−38%	−23%	−43%
Wealth share, country H ($\omega(h)$)	n.a.	0.47	0.44
Domestic portfolio shares:			
Domestic stocks, $\mu(\theta_h)$	1.01	0.77	0.93
Foreign stocks, $\mu(\theta_f)$	0.32	0.54	0.58
Bond, $\mu(b)$	−0.33	−0.31	−0.50
Calibrated moments:			
std(terms of trade)/std(output)	1.962	1.994	1.901
corr(terms of trade,output)	0.187	0.191	0.187
Trade/output	0.163	0.167	0.164
Backus-Smith	0.075	0.966	0.969
Home Bias	0.761	0.588	0.616

Table 2.5 also reports portfolio shares for the domestic investor. The share of domestic stocks, for example, is defined as:

$$\mu(\theta_h) = \frac{q_h\theta_h}{q_h\theta_h + q_f\theta_f + q_b b},$$

and similarly for foreign stocks and the bond. To compute these shares in the data, we combine the Lane & Milesi-Ferretti (2007) dataset with the data on countries' stock market capitalizations from the World Bank dataset[22]. To compute portfolio

[22] This data is available on-line at `http://data.worldbank.org/indicator/CM.MKT.LCAP.CD`.

shares in the data, we calculate:

$$q_h\theta_h = \text{U.S. Stock Market Capitalization} - \text{U.S. Portfolio Equity Liabilities},$$
$$q_f\theta_f = \text{U.S. Portfolio Equity Assets},$$
$$q_b b = \text{U.S. Debt Assets} - \text{U.S. Debt Liabilities},$$

using the 2007 data.

Table 2.5 also shows that in all versions of our model, negative NFA of the domestic economy is associated with a large negative position in the bond, as is the case in the data for the U.S.

To summarize our main findings, we obtain that:

Result 1. A country that can issue bonds in own currency accumulates on average a sizable negative net foreign asset position in the long run. Similarly to what we find in the data for the U.S., this is driven by a large negative position in the bond, which is caused by the strong positive correlation between the bond's payoff and domestic labor income.

Result 2. A country with less volatile output shocks also accumulates a large negative NFA position in the long run. Similarly to the previous result, this is driven by a large negative position in the bond. In this case, the main cause is the larger precautionary motive of foreign investors.

Table 2.5 also reveals that the predictions of our model in terms of the Backus-Smith correlation (the correlation between the relative consumption growth rate, and the real exchange rate changes in the two countries) are at odds with what we observe in the data. Similarly, the share of stock holdings invested in domestic stocks ("Home bias") in our model is lower than in the data. We address these two issues in the following section.

2.6.2 Backus-Smith puzzle and home equity bias

To improve the ability of our model to match the data along the following two dimensions – low correlation between the growth rates of relative consumption and the real exchange rate (the Backus-Smith puzzle), and a high share of equity holdings invested into local stocks (home equity puzzle), we introduce two new features into our model. To address the Backus-Smith puzzle, we follow Heathcote & Perri (2009) and introduce discount factor shocks. In this new formulation, the discount factor in country i becomes $\tilde{\beta} = \beta * \delta_i$, where $\delta_i \in \{e^{\sigma_\delta}, e^{-\sigma_\delta}\}$. We assume that the discount factor shocks are i.i.d., and we choose σ_δ which regulates the size of the shock to match the Backus-Smith correlation. These discount factor shocks are observationally equivalent to a model with heterogeneous beliefs – i.i.d. surges in optimism and pessimism.

To obtain a measure of the Backus-Smith correlation in the data, we compute the weighted average correlation between the growth rates in relative aggregate consumption, and the ratio of the CPIs for each country in our OECD sample with the U.S. during the 1985-2007 period, again using the countries' relative output shares as weights. The Backus-Smith correlation that we compute in this way is equal to 0.075, which we report in table 2.5.

To improve our results in terms of the home equity bias, we assume that investing in the stock of another country entails a cost proportional to τ for the investor. We assume that these costs are paid in terms of the local goods of the country issuing the stock, and are rebated back to the local country's investors as a lump-sum. This means, for example, that domestic investors' payoff from holding a foreign stock is $r_f(1 - \tau p_f e_f)$. If domestic investor chooses to hold θ_f in foreign stocks, the amount equal to $\tau p_f e_f \theta_f$ is paid by domestic investors to foreign investors next period. The assumption that the investment costs are rebated to local investors allow us to compute a wealth-recursive equilibrium similar to the one defined in section 2.2.2, with ω_h now denoting the share of the worldwide investors' wealth owned by the domestic investors.

Table 2.6 reports our results from this modified model, where we keep both the exorbitant privilege, and the lower volatility of output in domestic country assumptions (we call this version of the model M3). To match the Backus-Smith correlation in the data, we need to set σ_δ, the parameter that controls the magnitude of the discount factor shocks, to 0.0073. To generate realistic home bias in equity, we need to introduce only very small investment costs proportional to $\tau = 16^{-6}$.

Table 2.6: Simulated and data moments

	Data	M3
NFA(h)/output(h)	−38%	−42%
Wealth share, country H ($\omega(h)$)	n.a.	0.44
Domestic portfolio shares:		
Domestic stocks, $\mu(\theta_h)$	1.01	1.18
Foreign stocks, $\mu(\theta_f)$	0.32	0.39
Bond, $\mu(b)$	−0.33	−0.57
Calibrated moments:		
std(terms of trade)/std(output)	1.962	1.914
corr(terms of trade,output)	0.187	0.185
Trade/output	0.163	0.164
Backus-Smith	0.075	0.070
Home Bias	0.761	0.752

Table 2.6 shows that model M3 closely matches the data in terms of the home equity bias and Backus-Smith correlation. At the same time, the results in terms of the domestic country's NFA position is similar to those obtained in model M2. Domestic country borrows using the bond, which leads to a sizable negative NFA position in the long run.

2.7 Conclusions

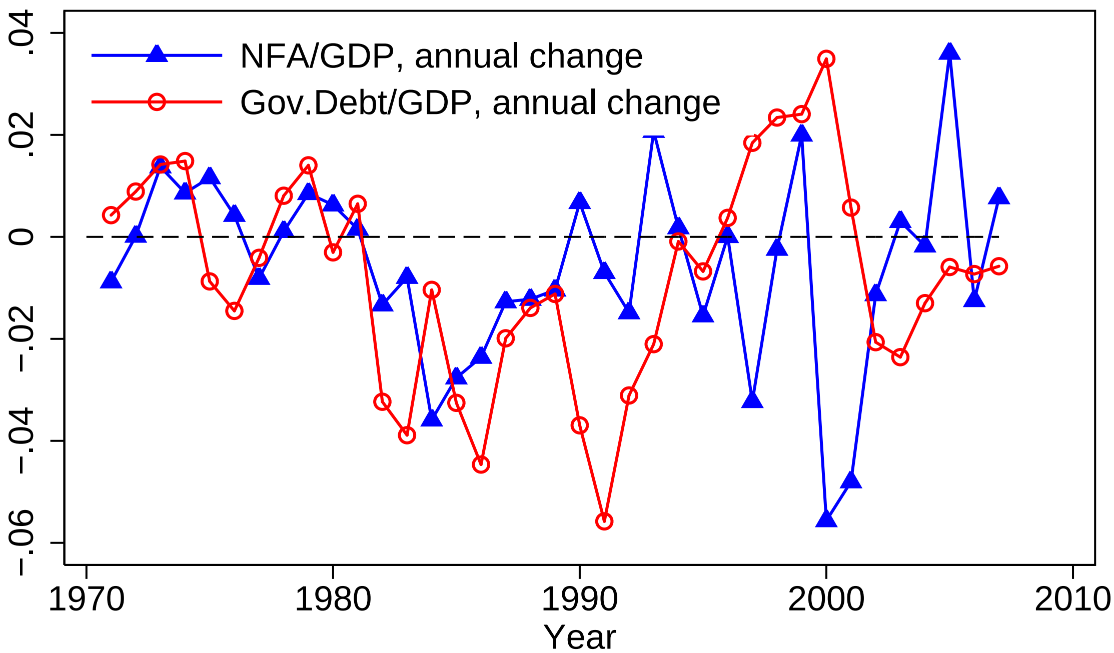
In this paper, we develop and solve an incomplete-markets general equilibrium model of international portfolio choice. We calibrate our model so that it is consistent with the behavior of the relative goods' prices, home equity bias and consumption-real exchange rate disconnect (Backus-Smith puzzle), and use our calibrated model to study the net foreign asset position and the structure of the international portfolio in the U.S. We adapt the algorithm developed in Kubler & Schmedders (2003) to obtain a globally accurate numerical approximation to our model's equilibrium. We use our model to study the impact of two features of economic environment on the international portfolio structure and NFA position of the U.S.: 1) exorbitant privilege – the ability of the U.S. to borrow internationally in its own domestic goods, and 2) lower volatility of output shocks in the U.S. compared to the rest of the world. We find that in our model, both of these two features contribute to generating a realistic portfolio structure for the U.S., with a large negative position in the bond and a sizable negative NFA position similar to what we find in the data.

The model and the global numerical solution method that we use are well-suited for the analysis of the international portfolios in the presence of large shocks, such as the 1998 Asian crisis or the slowdown of the output growth in Japan during the 90s. We expect that the introduction of these features into our model can improve our model's predictions in terms of the equity premium puzzle, and generate sizable positive net factor payments to the U.S. that we see in the data. We plan to incorporate these features into the model in our future research.

2.8 Appendix

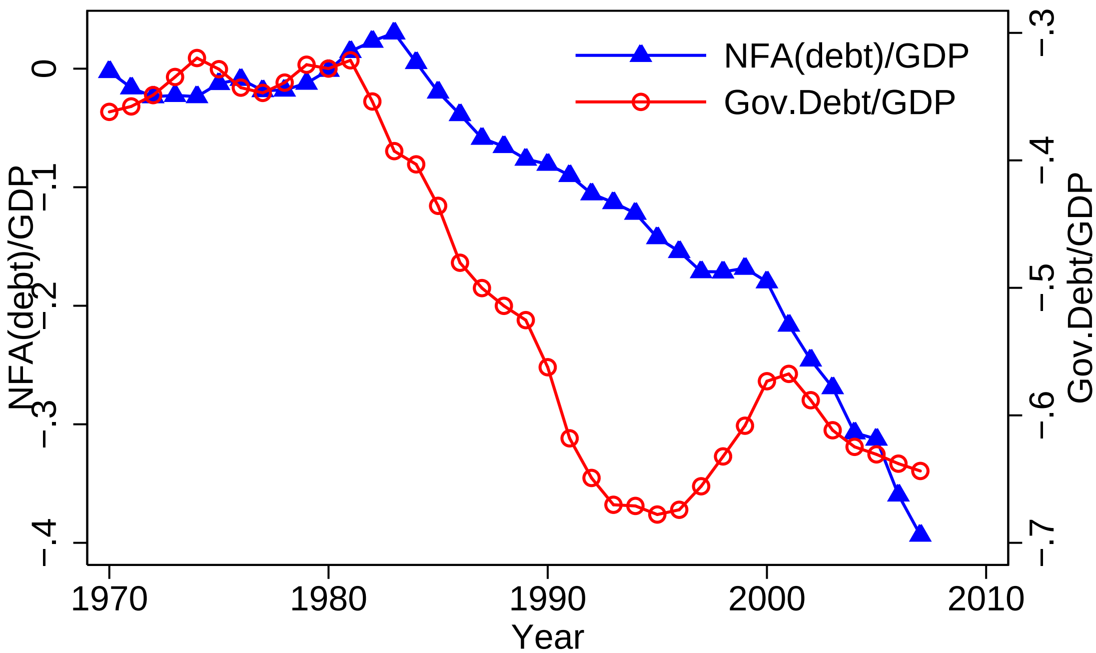
2.8.1 Data

Figure 2.7: Annual Changes in Government Debt/GDP and NFA/GDP



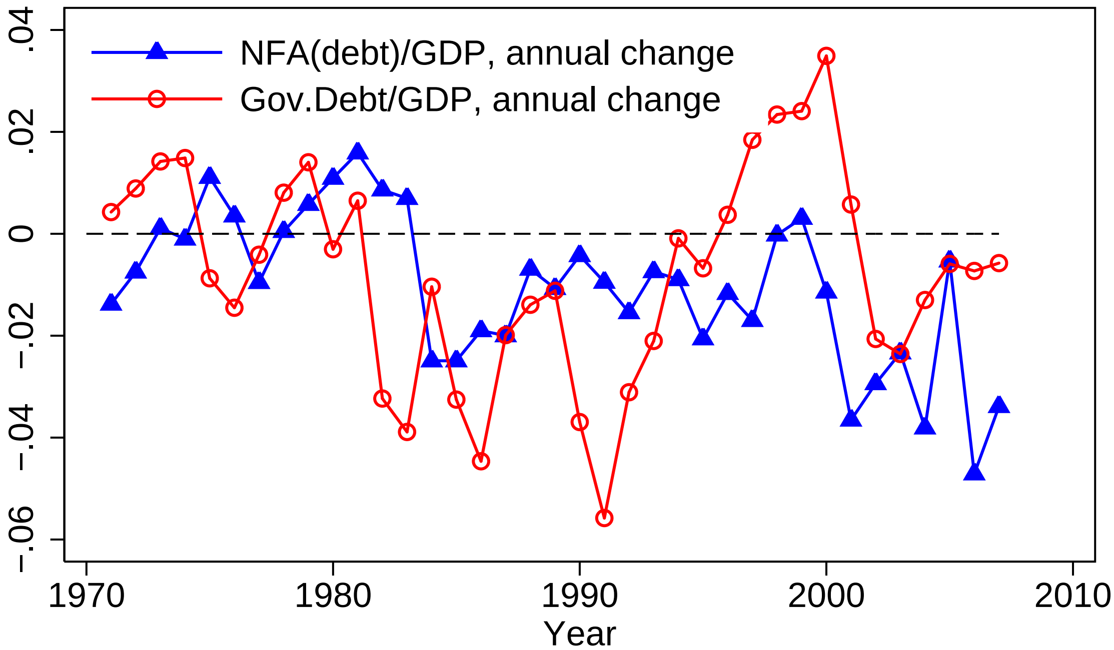
Data sources:
US public debt −− http://www.treasurydirect.gov/govt/reports/pd/histdebt/histdebt.htm
US NFA position: Lane&Milesi−Ferretti dataset
US GDP − OECD

Figure 2.8: US Government Debt and NFA in Debt Instruments



Data sources:
US public debt −− http://www.treasurydirect.gov/govt/reports/pd/histdebt/histdebt.htm
US NFA position: Lane&Milesi−Ferretti dataset
US GDP − OECD

Figure 2.9: Annual Changes in Government Debt/GDP and NFA(debt)/GDP



Data sources:
US public debt −− http://www.treasurydirect.gov/govt/reports/pd/histdebt/histdebt.htm
US NFA position: Lane&Milesi−Ferretti dataset
US GDP − OECD

Table 2.7: Output volatilities, HP-filtered and growth rate series

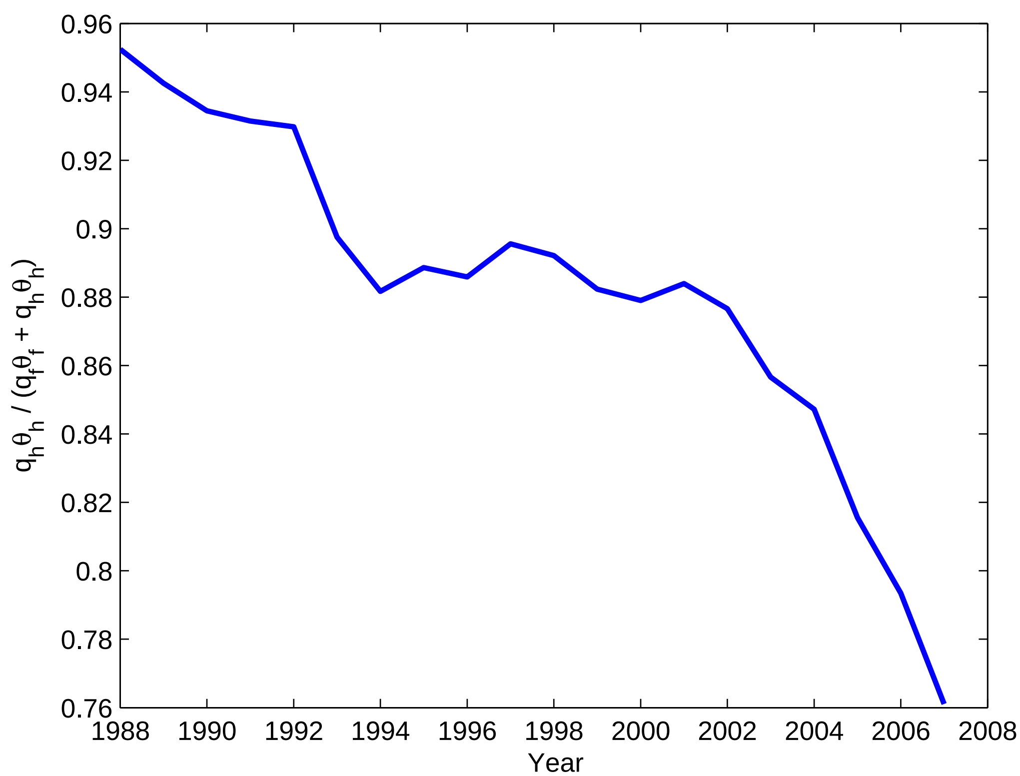
Country	$std(y^i_{hp}), \%$	$std(y^i_{\Delta}), \%$	**Weight**
AUS	2.096	2.375	0.025
AUT	1.444	1.633	0.011
BEL	1.133	1.257	0.014
CAN	2.453	2.108	0.045
DEN	1.218	1.718	0.009
FIN	2.428	2.316	0.007
FRA	0.905	0.751	0.082
GER	1.346	1.422	0.122
GRC	2.169	2.494	0.009
ICL	4.756	6.711	0.001
ITA	1.021	1.359	0.073
JAP	1.477	1.362	0.302
KOR	1.817	2.339	0.028
LUX	3.296	3.918	0.001
MEX	1.572	1.959	0.043
NRL	1.522	1.753	0.022
NZL	2.142	2.808	0.003
NOR	2.199	2.705	0.010
POR	1.699	1.604	0.007
SPN	1.027	0.951	0.036

Continued on next page

Table 2.7 – continued from previous page

Country	$std(y^i_{hp}), \%$	$std(y^i_{\Delta}), \%$	**Weight**
SWE	1.799	2.071	0.014
CHE	1.942	2.050	0.018
TUR	2.401	2.944	0.019
GBR	1.645	1.460	0.098
USA	0.984	0.881	n.a.

Figure 2.10: Share of domestic stocks in U.S. equity portfolio



2.8.2 Two-stage budgeting

The two-stage budgeting procedure separates the overall optimization problem of the investor into the static and the dynamic parts. The dynamic part deals with the reallocation of the investor's income across time and states of the world, through participation in the financial markets. In the static part, the investor decides on how to spend the income available to him in each date-event node on the consumption goods, by participating in the spot markets.

Let's start with the static problem. Suppose that the domestic consumer has some amount of income $\tilde{c}_h$ that he can spend on consumption in some date-event node. To decide how to spend this income on the two consumption goods, he solves:

$$\max \left(s_h c_h^{\rho} + s_f c_f^{\rho}\right)^{1/\rho}$$

$$\text{s.t. } p_h c_h + p_f c_f = \tilde{c}_h.$$

The solution to this problem is:

$$\left(c_h^{\max}, c_f^{\max}\right) = \left(\frac{\tilde{c}_h}{p_h + p_f\left((s_f p_h)/(s_h p_f)\right)^{1/(1-\rho)}}, \frac{\tilde{c}_h}{p_f + p_h\left((s_h p_f)/(s_f p_h)\right)^{1/(1-\rho)}}\right).$$

This produces the indirect utility from income:

$$v_h(\tilde{c}_h | p_h, p_f) = g(c_h^{\max}, c_f^{\max}) = \tilde{c}_h / \pi_h(p_h, p_f),$$

where $\pi(p_h, p_f) = \left(s_h^{1/(1-\rho)} p_h^{\rho/(\rho-1)} + s_f^{1/(1-\rho)} p_f^{\rho/(\rho-1)}\right)^{1-1/\rho}$ is the domestic consumption-based price aggregator (domestic CPI). We can substitute it into 2.1, and rewrite the dynamic problem as:

$$\max E\Big[\sum_{t=0}^{\infty}\beta^t\frac{(\tilde{c}_h/\pi_h)^{1-\gamma}}{1-\gamma}\Big|\mathcal{I}_0\Big], \tag{2.9}$$

subject to the sequence of budget constraints:

$$\tilde{c}_h(z^t)+q_h(z^t)\theta_h(z^t)+q_f(z^t)\theta_f(z^t)+q_b(z^t)b(z^t)=I_h(z^t)$$

We get that the marginal utility of income, or the pricing kernel of this consumer, is $M_h=(\pi_h)^{\gamma-1}\tilde{c}_h^{-\gamma}$. Before the investor chooses his portfolio for the next period, his income next period consists of only his labor income, so that $\tilde{c}_h=w_hp_h$. In the case when the bond is the only asset that is traded internationally, and investors hold the entire supply of local stocks, their next-period income is equal to the market value of the total domestic good's output, $\tilde{c}_h=e_hp_h$. This gives us the expression in 2.6.

2.8.3 Numerical algorithm

We approximate the policy and price functions with splines. We experimented with both cubic and piecewise linear splines, which produced practically identical solutions, and we ultimately chose piecewise linear functions for speed considerations. Since our model has only 2 countries, our endogenous state variable, *wealth share*, can be represented with a single number, $\omega\in[0,1]$. We form a grid M over $[0,1]$, and start with an initial guess for the policy and price functions, f^0. Good choice of f^0 is important for both convergence and speed of the algorithm. We choose either a solution to the static ("last-period") problem, or a solution for a "nearby" economy as our guess.

Given the function f^{n-1} that determines the behavior of the economy next period, we obtain a new function f^n by solving the following system of static and short-term dynamic optimality conditions ("expectations correspondence") + the "law of motion for wealth share" equation for each gridpoint $(\omega, z) \in M \times Z$:

$$p_f/p_h = (s_f/s_h)(c_h/c_f)^{1-\rho} = (s_f^*/s_h^*)(c_h^*/c_f^*)^{1-\rho}, \tag{2.10}$$

$$q_h = \beta E_{z^+}\left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_h^{n-1}(\omega^+, z^+)\right) + \phi_h, \tag{2.11}$$

$$q_h = \beta E_{z^+}\left(\frac{\lambda_f^{n-1}(\omega^+, z^+)}{\lambda_f} r_h^{n-1}(\omega^+, z^+)\right) + \phi_h^*, \tag{2.12}$$

$$q_f = \beta E_{z^+}\left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_f^{n-1}(\omega^+, z^+)\right) + \phi_f, \tag{2.13}$$

$$q_f = \beta E_{z^+}\left(\frac{\lambda_f^{n-1}(\omega^+, z^+)}{\lambda_f} r_f^{n-1}(\omega^+, z^+)\right) + \phi_f^*, \tag{2.14}$$

$$q_b = \beta E_{z^+}\left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_b^{n-1}(\omega^+, z^+)\right) + \nu \frac{\partial B_h(b)}{\partial b}, \tag{2.15}$$

$$q_b = \beta E_{z^+}\left(\frac{\lambda_f^{n-1}(\omega^+, z^+)}{\lambda_f} r_b^{n-1}(\omega^+, z^+)\right) + \nu^* \frac{\partial B_f(b^*)}{\partial b^*}, \tag{2.16}$$

$$\phi_h \geq 0,\ \theta_h \geq 0,\ \phi_h \theta_h = 0, \tag{2.17}$$

$$\phi_f \geq 0,\ \theta_f \geq 0,\ \phi_f \theta_f = 0, \tag{2.18}$$

$$\phi_h^* \geq 0,\ \theta_h^* \geq 0,\ \phi_h^* \theta_h^* = 0, \tag{2.19}$$

$$\phi_f^* \geq 0,\ \theta_f^* \geq 0,\ \phi_f^* \theta_f^* = 0, \tag{2.20}$$

$$\nu \geq 0,\ B_h(b) \geq 0,\ \nu B_h(b) = 0, \tag{2.21}$$

$$\nu^* \geq 0,\ B_f(b^*) \geq 0,\ \nu^* B_f(b^*) = 0, \tag{2.22}$$

$$\tilde{I} = k(w_h p_h + w_f p_f) + r_h + r_f, \tag{2.23}$$

$$p_h c_h + p_f c_f + q_h \theta_h + q_f \theta_f + q_b b = \omega \tilde{I} + (1-k) w_h p_h, \tag{2.24}$$

$$p_h c_h^* + p_f c_f^* + q_h \theta_h^* + q_f \theta_f^* + q_b b^* = (1-\omega)\tilde{I} + (1-k) w_f p_f, \tag{2.25}$$

$$\lambda_h = g_h^{1-\rho-\gamma} \left(s_h c_h^{\rho-1} + s_f c_f^{\rho-1} \right) \tag{2.26}$$

$$\lambda_f = g_f^{1-\rho-\gamma} \left(s_h^* (c_h^*)^{\rho-1} + s_f^* (c_f^*)^{\rho-1} \right) \tag{2.27}$$

$$p_h + p_f = 1 \tag{2.28}$$

$$c_h + c_h^* = e_h \tag{2.29}$$

$$\theta_h + \theta_h^* = 1, \quad \theta_f + \theta_f^* = 1, \quad b + b^* = 0 \tag{2.30}$$

$$\omega^+ = \frac{r_h^{n-1}(\omega^+, z^+)\theta_h + r_f^{n-1}(\omega^+, z^+)\theta_h + r_b^{n-1}(\omega^+, z^+) b + k w_h(z^+) p_h^{n-1}(\omega^+, z^+)}{r_h^{n-1}(\omega^+, z^+) + r_f^{n-1}(\omega^+, z^+) + k \left(w_h(z^+) p_h^{n-1}(\omega^+, z^+) + w_f(z^+) p_f^{n-1}(\omega^+, z^+) \right)} \tag{2.31}$$

Equation 2.10 is the static optimality condition. Equations 2.11 – 2.16 are short-term dynamic optimality (Euler) equations. Equations 2.17 – 2.22 are Kuhn-Tucker complementarity conditions. Equation 2.23 defines total financial wealth. Equations 2.24, 2.25 are the budget constraints. Equations 2.26 and 2.27 define marginal utility of income for domestic and foreign investors. Equation 2.28 is a price normalization. Equations 2.29 and 2.30 are market-clearing conditions. Finally, equation 2.31 implicitly defines next-period wealth share ω^+ as a function of today's portfolio choices.

Note that λ_h^{n-1}, λ_f^{n-1}, $r_h^{n-1} = q_h^{n-1} + p_h^{n-1} d_h$, $r_f^{n-1} = q_f^{n-1} + p_f^{n-1} d_f$, $r_b^{n-1} = \alpha p_h^{n-1} + (1-\alpha) p_f^{n-1}$, p_h^{n-1} and p_f^{n-1} are components of f^{n-1}. Also, note that $B_h(b)$ and $B_f(b^*)$ are the borrowing limits defined in 2.3.

We keep updating the policy and pricing functions until $||f^n - f^{n-1}|| < \epsilon$ for some small $\epsilon > 0$.

Garcia-Zangwill trick. Kuhn-Tucker complementarity conditions that involve inequalities can be transformed into equalities using the so-called Garcia-Zangwill trick. It is basically a change of variables. We will demonstrate how it works using the borrowing limit and the optimality condition for the bond holdings. Fix some positive integer $k \geq 2$ and define

$$\alpha^{+} = (\max[0, \alpha])^{k}, \alpha^{-} = (\max[0, -\alpha])^{k}$$

Note that $\alpha^{+} \geq 0$, $\alpha^{-} \geq 0$ and $\alpha^{+}\alpha^{-} = 0$. We can now reformulate the Euler equation and Kuhn-Tucker conditions of domestic consumer for bonds as follows:

$$q_b = \beta E_{z^+} \left(\frac{\lambda_h^{n-1}(\omega^+, z^+)}{\lambda_h} r_b^{n-1}(\omega^+, z^+) \right) + \alpha^{+} \frac{\partial B_h(b)}{\partial b}$$

$$\alpha^{-} - B_h(b) = 0$$

Similarly, we can transform all other Kuhn-Tucker conditions into equalities.

2.8.4 Testing the algorithm: a special case with analytic solution

To check the performance of our algorithm, we consider a special case of our model where we can characterize the solution analytically. We show that our numerical solution produces a very good approximation to the analytic solution in this case.

Consider the model that we described in section 2.2, with the following modifications (*Model LSR*[23].):

1. Consumers in both countries have identical preferences towards the two consumption goods. In particular, we will assume that the preference weights for both consumers are fixed at $s_h(z) = s_h^*(z) = 1/2$ for all z[24].

[23]The consumption allocation in the one-good version of this model is known in the general equilibrium literature as the "linear sharing rule (LSR)" (see Magill & Quinzii (1996), p. 173)

[24]Intuitively, with this assumption the model behaves as a model with only one consumption good.

2. The "wages" and the "dividends" are some fixed fractions of the output in both countries, so that $w_h(z) = \nu e_h(z)$ and $w_f(z) = \nu e_f(z)$ for some $\nu \in [0, 1)$ for all z (which implies that $d_h(z) = (1 - \nu)e_h(z)$ and $d_f(z) = (1 - \nu)e_f(z)$).

3. The initial portfolio distribution (or, alternatively, the initial wealth share of the home country, $\omega_h(z_0)$) is such that the short-selling constraints are not be binding at the solution that we present next (the precise meaning of this assumption will become clear when we present the solution).

The exact values for all other parameters in the model (ρ, σ, β and the specification of the two output processes) are not important for this example, but we will set them equal to the values from our calibrated model in section 2.5. We will assume that $\nu = 0.1$. On one hand, assuming a low share of labor income in total output will ensure that the short-selling constraints are not binding for a large portion of our state space. On the other hand, given our borrowing limit, assuming that labor share is zero would trivially imply a zero bond position, which we would like to avoid for the purpose of this exercise.

Proposition 2.8.1. *The model that satisfies assumptions* "Model LSR" *has an equilibrium with the following consumption allocation (which is Pareto optimal):*

$$c_h^{PO}(z) = ke_h(z), \quad c_f^{PO}(z) = ke_f(z),$$

$$c_h^{*PO}(z) = (1-k)e_h(z), \quad c_f^{*PO} = (1-k)e_f(z)$$

where $k = 1/\left(1 + \left[\frac{1-\mu^{po}}{\mu^{po}}\right]^{1/\sigma}\right)$ *and* μ^{po} *is the weight that the planner assigns to the domestic consumer (which depends on* $\omega_h(z_0)$*), and the following time-invariant portfolio allocation:*

$$\theta_h^{PO} = \frac{k-\nu}{1-\nu}, \quad \theta_f^{PO} = \frac{k}{1-\nu}, \quad b^{PO} = 0,$$

$$\theta_h^{PO*} = \frac{k}{1-\nu}, \quad \theta_f^{PO*} = \frac{k-\nu}{1-\nu}, \quad b^{PO*} = 0,$$

It is easy to check that $(c_h^{PO}, c_f^{PO}, c_h^{*PO}, c_f^{*PO})$ is indeed Pareto optimal, by verifying that it satisfies the first-order conditions in the planner's problem. It is also easy to check that the suggested consumption and portfolio allocations satisfy the budget constraints for both consumers at each note z^t. Finally, one can set the relative prices to be equal to the appropriate ratios of marginal utilities of either of the consumers, and check that the first-order conditions in the consumer's optimization problem are satisfied. Note that the Pareto weight μ^{po} depends on the initial distribution of wealth between the consumers (it must be chosen so that the budget constraints are satisfied). We need to assume that the initial wealth distribution is such that the predicted equity positions are non-negative ($\theta_h^{PO} \geqslant 0, \theta_f^{PO} \geqslant 0, \theta_h^{PO*} \geqslant 0, \theta_f^{PO*} \geqslant 0$), so that the short-selling constraints are satisfied. The predicted bond positions are identically zero, so that the borrowing limits are satisfied.

Note that our portfolio allocation in this case is the same as in Baxter & Jermann (1997), who consider the model with one consumption good and Cobb-Douglas production functions (which produce the result that wages and dividends are constant fractions of output). If $\nu = 0$ (no non-traded income), we get $\theta_h^{PO} = \theta_f^{PO} = k$ and $\theta_h^{PO*} = \theta_f^{PO*} = (1-k)$, so that both consumers should hold only a fixed share in a mutual fund that fully owns both equities. If $\nu > 0$, domestic consumers should reduce their exposure to domestic equity and increase their position in foreign equity.

Proposition 2.8.1 describes portfolio positions for a given Pareto weight μ^{po}. For every initial exogenous state z_0 and every initial wealth share $w_h(z_0)$, we can find μ^{po} numerically using the Negishi algorithm, as described in Judd et al. (2000).

Figure 2.11 compares the equity and bond positions obtained by our numerical solution with the ones from proposition 2.8.1. The upper part of the figure shows

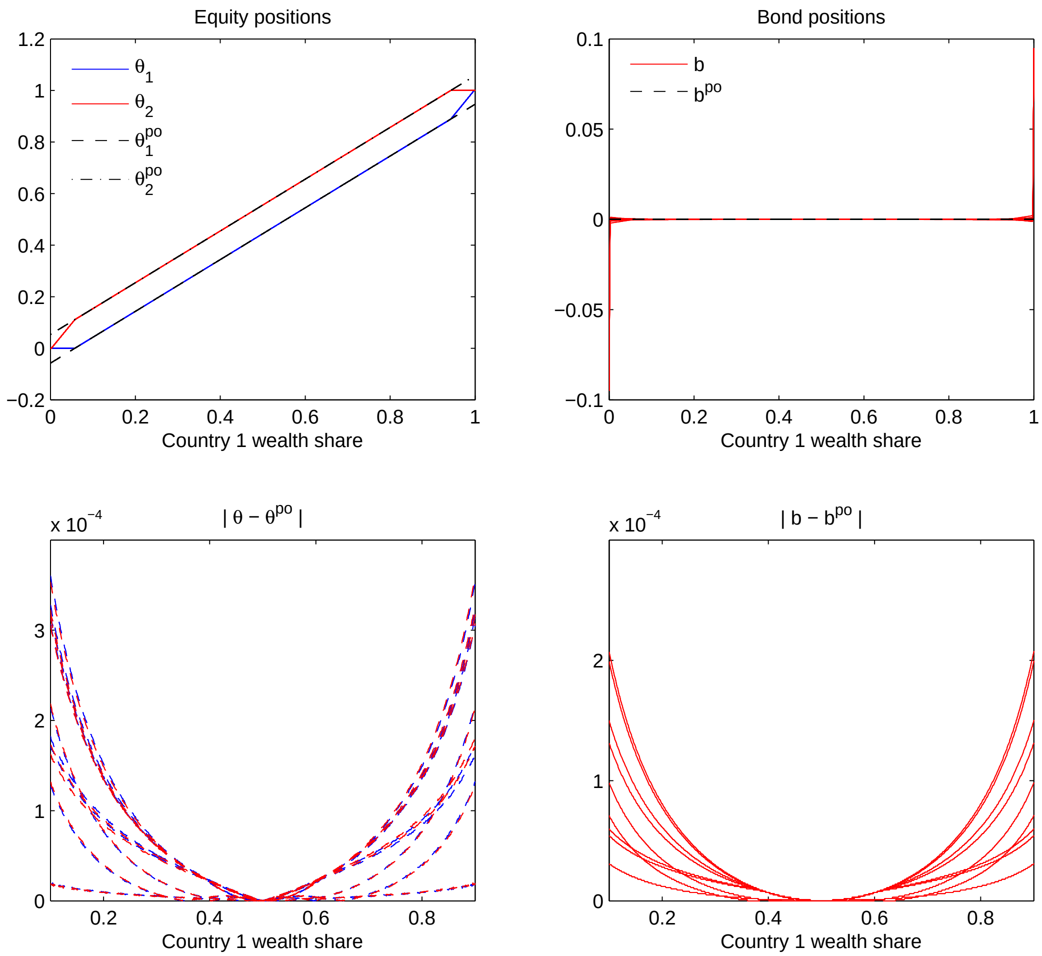


Figure 2.11: Numerical and analytic portfolios for Model LS

the two portfolio solutions over the whole state space, and the lower part shows the absolute difference between the two solutions (only the part of the state space where the short-sale constraints do not bind is shown). This figure demonstrates that our numerical solution is very close to the analytic solution.

Next, we compare the simulated stationary distributions of our endogenous state variable, ω_h, and simulated portfolio positions (with several million draws of the exogenous shock). We simulate the model assuming that $\omega_h(z_0) = 0.5$. We can expect that ω_h should stay close to 0.5. In fact, from proposition 2.8.1 it follows that there should be finitely many realized values for ω_h, equal to the number of exogenous states[25].

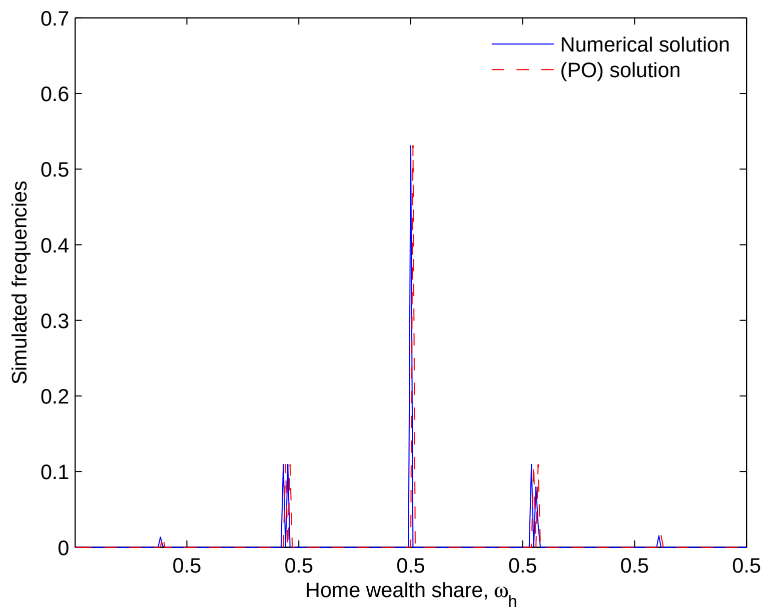


Figure 2.12: Stationary distributions of endogenous state variable

Figure 2.12 confirms that the stationary distribution obtained from our numerical solution is very close to the distribution implied by the analytic solution. Table 2.8 shows that the predicted portfolio positions are also very close to the ones from proposition 2.8.1.

We conclude that our numerical algorithm provides a very good approximation to both the policy functions, and the implied stationary distribution over ω_h.

[25]From the definition in 2.5, it is clear that ω_h depends on the selected portfolio positions, goods' and assets' prices and wage and dividend realizations. Proposition 2.8.1 shows that portfolio positions should not change over time. It also shows that the consumption allocations, and thus all the prices will be "strongly stationary" (will depend only on the realization of the exogenous state). It follows that ω_h will also depend only on the realization of the exogenous state.

	Numerical solution	(PO) solution
mean(ω_h)	0.5000	0.5000
mean(θ_h)	0.4444	0.4444
mean(θ_f)	0.5556	0.5556
mean(b)	0.0000	0.0000

Table 2.8: Simulated portfolio allocation

Chapter 3

Marriage Stability, Taxation and Aggregate Labor Supply in the US vs. Europe

3.1 Introduction

It is a well-known empirical finding that aggregate labor supply is higher in the United States than in Europe and that there is also substantial variation among European countries, see for instance Prescott (2004) and Rogerson (2006). Rogerson (2006) notes that these differences are an order of magnitude larger than the fluctuations at business cycle frequencies in post-WWII US data, and thus deserve serious attention. Are the differences in hours worked due to public policies or are they due to other fundamental differences between societies?

In this paper, we start by using micro level data to document the contribution of various demographic groups to the aggregate differences between the US and 8 European countries. We find that among the demographic groups that we consider, the largest contribution comes from women – in most European countries, women work substantially less than in the United States, while the difference in hours worked between European and American men is smaller, and in some cases practically non-existent. This is especially true for married women, but also holds for single women, and for women with and without children. We also document a negative cross-country correlation between tax level and labor supply, and a positive correlation between divorce rates and labor supply across countries and across time. Divorce rates are, however, in particular correlated with female labor supply. Motivated by these observations, we consider the following two potential driving forces for cross-country differences in labor supply: 1) cross-country differences in taxation; 2) cross-country differences in marriage stability.

To quantitatively assess the impact of taxes and marriage stability on labor supply we develop a life-cycle, overlapping-generations model with heterogeneous agents, marriage, and divorce. There are three types of households; single males, single females and married households. Divorces and marriages occur stochastically. The main channel through which individual divorce and singlehood rates impact labor supply is by reducing the implicit insurance of marriage, and thereby providing incentives for individuals to invest in experience accumulation. We calibrate our model to US data and study how labor supply in the US changes as we introduce divorce and marriage probabilities and tax systems from other countries. We find that the effect of making marriages more stable is a reduction in labor supply. This effect is particularly strong for female labor supply, because the woman is usually the lower earner in a married couple. Changing the US probabilities of marriage and divorce to their European equivalents accounts on average for 22% of the difference in hours worked between the US and 11 European countries. When we introduce European taxes and redistribute the increase in taxes evenly to all households, we can account for 19% of the difference in hours worked between the US and the average of the European countries. If the increased tax revenues from European taxation is not redistributed the average effect is an increase in labor supply. When using both the divorce and marriage probabilities and tax systems from the European countries, the model can on average account for 28% of the difference in hour worked between the US and Europe.

Cross-Country Differences in Labor Supply: Possible Explanations and Previous Literature

The economic literature has proposed several potential explanations for the observed cross-country differences in aggregate labor supply. Taxes have been suggested as a major contributor to the differences in labor supply by Prescott (2004) and Rogerson (2006), who used an infinite horizon, representative agent model to evaluate the impact of differences in average tax rates. We extend this argument, and use a life-cycle model with heterogeneous agents, who accumulate labor market experience, and reside in one- and two person households. This allows us to capture several dimensions of tax systems that cannot be captured in a representative agent model. We fit nonlinear income tax schedules that can capture the impact of both tax levels and tax progressivity on aggregate labor supply, as well as one the labor supply of various demographic groups. We are also able to capture the impact of joint versus separate taxation of married couples. As pointed out by Guner, Kaygusuz & Ventura (2008), separate taxation of married couples leads to a lower marginal tax rate on the secondary earner in a couple, and therefore encourages female labor supply. In Section 3.7, we find this to be an important effect in our model.

To the best of our knowledge, the role of differences in marriage stability in accounting for cross-country differences in labor supply has not been analyzed in the literature. Yet, our finding in Section II below that the biggest contribution to the cross-cross country differences in average hours worked comes from women, and in particular from prime-aged married women, suggests that one may need to pay attention to the cross-country differences in family dynamics. There is ample anecdotal evidence that compared to the US, marriages are more stable in Europe, especially in "catholic" European countries such as Italy, Spain, Ireland, and Greece where divorces have traditionally carried more social stigma with them. Our hypothesis is that more stable marriages provide implicit income and consumption insurance to the spouse who is not the main income earner in the family (the role that for various reasons is traditionally played by the wife), thus giving her/him less incentive to accumulate market experience.

One may argue that divorce and marriage decisions are also affected by economic conditions and that therefore we should make them endogenous choices. However, then we would need a systematic cross-country pattern in economic conditions that could account for both the pattern in divorce rates and in labor supply at the same time. This type of condition could be for instance cross-country differences in the gender wage gap, in the female return to labor market experience, or in the cost of having children. These explanations have been proposed in the literature trying to explain changes in female labor supply over time, see for instance Olivetti (2006) and Attanasio, Low & Sanchez-Marcos (2008). However, we have not been able to document a cross-country pattern in the gender wage gap or in the female return to labor market experience that would help us explain the observed patterns in aggregate labor supply and divorce rates. In Section 3.2, we argue that children are unlikely to be an important explanation, as the cross country differences in labor supply is not more pronounced for women with children. We therefore choose to study the economic implications of exogenous differences in marriage and divorce rates caused by "cultural" and/or legal factors. Crouch & Beaulieu (2006) documents a correlation between different types of divorce laws and divorce rates in the US and 22 European countries. Generally divorce laws are stricter in Europe. For instance, they require a longer waiting period before a divorce can be obtained. Johnson & Skinner (1986) provides empirical support to our theory about the impact of exogenous changes in the probability of divorces on female labor supply. They estimate a simultaneous model of future divorce probability and current labor supply using US data, and conclude that their results support the hypothesis that higher divorce probabilities increase labor supply, while the reverse effect appears insignificant. Stevenson (n.d.) documents that the US states who adopted unilateral divorce in the 1970s experienced a spike in female labor supply compared to states who did not.

One pronounced difference between the US labor market and those in many European countries is the more rigid regulations and laws in Europe, often referred to in the literature as labor market frictions. These are possible contributors to the higher observed unemployment rates and lower labor force participation rates in Europe. Unions are also much more common in Europe. Alesina, Glaeser & Sacerdote (2005) argues that regulations and unionization are more like explanations than taxes. We believe that they could also be contributing factors and that we should not hope for taxes and divorce rates to explain all of the cross country variation in labor supply. Out of all the above proposed explanations, however, differences in divorce rates stand out as a promising candidate for explaining why cross country differences is mainly driven by female labor supply.

The remainder of the paper is organized as follows: In Section 3.2, we study the contributions of different demographic groups to aggregate differences in labor supply between the US and 8 European countries. In section 3.3, we document a correlation between aggregate labor supply and taxation across countries and a correlation between aggregate labor supply and divorce rates across time and place. Section 3.4

studies the impact of divorce rates on labor supply in a simple model. Section 3.5 develops the quantitative model. Section 2.5 discusses data and calibration. In Section 3.7, we study the quantitative implications from changing the US divorce and marriage probabilities to their European counterparts and from introducing European tax schemes in the US. Section 3.8 concludes.

3.2 Which Demographics Groups Contribute to Differences in Aggregate Labor Supply: US vs. Europe

In this section, we use data from the Luxembourg Income Study (LIS) and the OECD Employment Database to analyze the contribution of various demographic groups to cross-country differences in aggregate labor supply. We find that women is the biggest contributor to the cross-country differences in labor supply. American women work more than European women, whether it is single women, married women, women with and without children. The contribution of women is the largest in Spain, Italy, Greece and Ireland – the countries where, as we document in the next section, marriages tend to be more stable.

Next, we analyze the importance of the intensive and extensive margins in accounting for the cross-country differences in labor supply, and find that they are both important. However, the extensive margin is particularly important for Spain, Italy, Greece and Ireland (coincidentally, these are the countries where the contribution of women is also particularly large), while the intensive margin is particularly important in Germany and Netherlands.

Data Description

The LIS database that we use contains micro-level data from the United States and a large number of European countries. The advantage of using this database is that the LIS team harmonizes and standardizes the micro data from the different countries' surveys in order to facilitate comparative research.

The LIS database provides information about individual hours worked per week and weeks worked per year[1]. We construct annual hours worked as the product of these two variables. To make our data comparable to the OECD aggregate-level estimates used by Rogerson (2006) and Prescott (2004), we include in our sample all individuals between 15 and 64 years of age. We make two adjustments to the LIS data. First, for several European countries the LIS database does not provide information about the labor market outcomes for 15 and/or 16 year-olds[2]. In these instances, we replace the missing values with the appropriate group averages from the US sample.

Table 3.1 reports the average annual hours worked by individuals who are from 15 to 64 years old in the US and a number of European countries, computed using the LIS data for year 2000. For comparison, the last two columns of the table also show the corresponding averages computed using the OECD data.

Unfortunately, for several European countries the average annual hours worked computed from the LIS data differ substantially from those reported by the OECD. Further research is needed to understand what causes this discrepancy. One possible explanation is that the LIS data does not capture the differences between the countries in the number of holidays and paid vacations[3].

[1] Variables *phoursu* and *pweektl*.

[2] For instance, German data does not have labor market information for both 15 and 16 year-olds, while for Spain and Ireland, this information is missing only for 15 year-olds

[3] A vast majority of individuals in all countries in the LIS data report either 0 or 52 weeks worked per year. At the same time, Jorgensen (2002) documents that individuals in most European

Table 3.1: Annual hours worked, all persons 15-64 years of age, 2000

Country	Annual Hours, LIS	% of the US, LIS	Number of obs, LIS	Annual Hours, OECD	% of the US, OECD
US	1375.46	100.0	84286	1360.69	100.0
Germany	1273.10	92.6	19845	965.91	70.99
Italy	1104.15	80.3	15354	1002.85	73.70
Spain	1127.14	81.9	9560	993.40	73.01
Ireland	1219.18	88.6	5992	1117.82	82.15
Austria	1375.30	100.0	4580	1132.39	83.22
Belgium	1344.42	97.7	4488	941.14	69.17
Netherlands	1240.23	90.2	8346	1117.82	72.76
Greece	1238.89	90.1	7309	1184.56	87.06

Since most of the previous research on the cross-country differences in labor supply has relied on the OECD data, we use the OECD data to determine the average country-level annual hours worked, and use the LIS data mainly to compute the contributions of various demographic groups to the cross-country differences. To account for the discrepancy between the OECD and LIS data, we uniformly scale all individual observations in each country in the LIS data so that the aggregate country-level averages that we obtain from the LIS data are equal to those reported by the OECD. Such adjustment makes the contributions of various demographic groups to the cross-country differences in aggregate-level average hours worked more uniform (in other words, we obtain a conservative estimate of the contribution of women to the cross-country differences, since this adjustment makes the contribution of separate demographic groups less pronounced)[4].

Table 3.2 shows the average annual hours worked for men and women separately, computed using the LIS 2000 data (adjusted as explained above). The table shows that the difference between the hours worked by European women and American

countries on average enjoy several more weeks of holidays compared to Americans.

[4]Our current adjustment is appropriate, for example, if the duration of vocations and holidays for each individual is a certain percent of his/her workdays. If, on the other hand, one assumes that the duration of vocations is the same for each individual, the differences in the contribution of various demographic groups would become more emphasized.

Table 3.2: Annual Hours Worked, Men and Women, 15-64 yrs. old, LIS 2000

Country	Men		Women	
	Annual Hours	% of US	Annual Hours	% of US
US	1596.82	100.0	1164.64	100.0
Germany	1225.33	76.7	716.87	61.6
Italy	1351.31	84.6	658.78	56.6
Spain	1355.47	84.9	633.17	54.4
Ireland	1517.71	95.0	718.02	61.7
Austria	1425.27	89.3	844.41	72.5
Belgium	1192.77	74.7	711.24	61.1
Netherlands	1319.30	82.6	675.91	58.0
Greece	1671.21	104.7	738.49	63.4

women is larger than the corresponding difference for men, both in percentage and in absolute terms. This difference between genders is more pronounced in Italy, Spain, Ireland and Greece, and less pronounced in Germany, Belgium and Austria.

Table 3.14 in the appendix shows the average annual hours worked of individuals in 3 different age groups: 1) "young" (15-20 year-olds), 2) "prime-aged" (21-55 year-olds) and 3) "old" (56-64 year-olds). There is substantial heterogeneity in hours worked by the "young" across the countries in our data (part of this could reflect poorer quality of the data for this age group). The hours worked by the "prime-aged" and "old" individuals in Europe are uniformly lower compared to the US.

Figure 3.1 plots the age profiles, using more detailed data (5-year age groups), separately for men and women for the US and European countries. This figure illustrates that there is a larger difference in hours worked between the US and Europe for women than for men. It also suggests that while the age profiles for men appear to have similar shapre in the US and Europe (with hours worked peaking in the middle age group, 35-44 year-olds), in most European countries (with the exception of Germany and Austria) the age profiles for women look markedly different, with hours worked peaking earlier than in the US.

Figure 3.1: Average hours worked by gender and age group

Averages are adjusted so that the total average across all subgroups is equal to the one reported by the OECD.

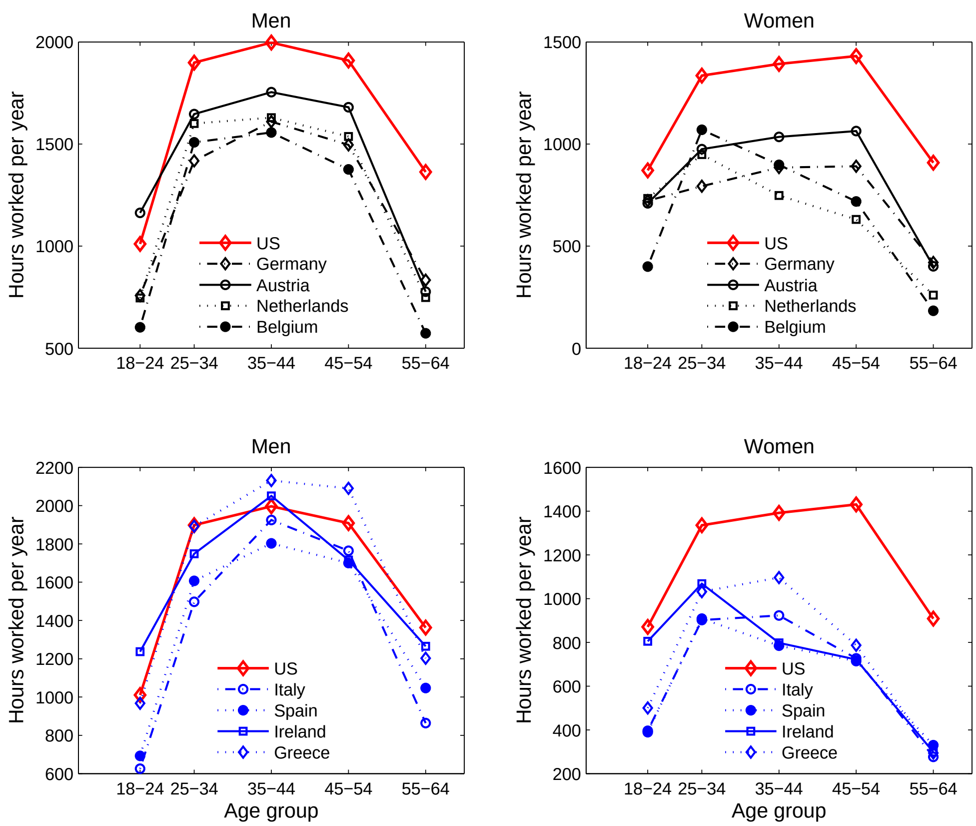


Table 3.3 compares the average annual hours worked by marital status and gender. It shows that in percentage terms married women in Europe display a bigger difference (work less) relative to their American counterparts than do single women. For men, the pattern is much less clear.

Given that we find that the difference in hours worked between the US and Europe is larger for women than for men, it is natural to ask whether this is related to women reducing their labor supply as a result of having children. Figure 3.12 in the appendix shows that in most of the countries where women worked the least compared to the US (Italy, Spain and Greece, but not in Ireland), women in fact tended to have fewer children than in the US.

Table 3.16 shows the hours worked by men and women split into three groups: 1) "child 3", which includes the individuals who have a child under 3 years of age, 2) "child 6", which includes the individuals who have a child under 6 years of age, 3) "no child", which includes individuals with no small children. According to the table, it is only in Germany and Austria that mothers with small children reduce their labor supply further compared to the US. In the countries where women worked the least (Italy, Spain, Greece and Ireland), the percentage difference with the US in hours worked for mothers with small children is smaller than for women without small children.

These two observations: 1) that fertility in the US is relatively high; 2) women with small children in Europe do not reduce their labor supply relative to their American counterparts, suggest that having small children is not a major reason for the difference in women's labor supply between the US and Europe.

Group Contribution Decomposition

To analyze the contribution of various demographic groups to the difference between aggregate labor supply in the US and the European countries in our sample, we

Table 3.3: Annual hours worked, by Gender and Marital Status, LIS 2000

Country	Men				Women			
	Married		Single		Married		Single	
	Annual Hours	% of US	Annual Hours	% of US	Annual Hours	% of US	Annual Hours	% of US
US	1965.87	100.0	1183.67	100.0	1207.27	100.0	1114.78	100.0
Germany	1398.72	71.2	1022.14	86.4	631.36	52.3	826.44	74.1
Italy	1620.99	82.5	982.29	83.0	651.98	54.0	669.62	60.1
Spain	1675.59	85.2	945.39	79.9	616.55	51.1	656.87	58.9
Ireland	1916.06	97.5	1107.74	93.6	692.64	57.4	747.04	67.0
Austria	1508.93	76.8	1324.17	111.9	807.33	66.9	891.72	80.0
Belgium	1328.43	67.6	971.42	82.1	713.24	59.1	708.17	63.5
Netherlands	1461.07	74.3	1134.34	95.8	553.21	45.8	856.81	76.9
Greece	1896.69	96.5	1276.75	107.9	748.15	62.0	719.60	64.6

perform the following decomposition. Suppose we divide each country's sample into n different groups. Then the difference between the aggregate average annual hours worked in the US, H^{us}, and in country j, H^j, can be written as:

$$\begin{aligned} H^{us} - H^j &= \sum_{i=1}^{n} \omega_i^{us} h_i^{us} - \sum_{i=1}^{n} \omega_i^j h_i^j \\ &= \underbrace{\sum_{i=1}^{n} (h_i^{us} - h_i^j)\omega_i^{us}}_{\text{behavioral effect}} + \underbrace{\sum_{i=1}^{n} (\omega_i^{us} - \omega_i^j) h_i^j}_{\text{composition effect}} \end{aligned}$$

where ω_i^j is the share of observations that come from group i in country j's sample, while h_i^j is the average annual hours worked by individuals in this group.

We divide the data into 12 demographic groups, according to gender, marital status and age (using 3 age groups). We are interested in analyzing the first summand in the expression above, which we call the behavioral effect, after removing the sample composition effect (which amounts to looking at a hypothetical case where the composition of the samples in different countries would be identical). Tables 3.20 and 3.21 in the appendix show the sample compositions in all our countries. It is worth noting that the total contribution of the compositional effects is quite small – in most cases, it is smaller than 5% of the total difference in average hours, except for Belgium (-8.717%), Greece (-7.176%) and Netherlands (-6.174%). Tables 3.17 and 3.18 show the contribution of different demographic groups to the aggregate difference in hours worked, weighted by the size of the appropriate group in the US sample, $\frac{h_i^{us}-h_i^j}{H^{us}-H^j}\omega_i^{us}$.

These tables show that women in general contribute more to the differences in labor supply than men. We find that in all countries, the contribution of women is larger than 50%. This difference between the contribution of the two genders is especially large in the four "catholic" countries – Spain, Italy, Ireland and Greece, where it ranges from 66% in Italy to 101% in Greece. In all countries except Belgium, married prime-aged women are the biggest contributing group. In Spain, Italy, Ireland and Greece single prime-aged women are the second-largest contributing group.

Intensive vs Extensive Margin

Table 3.2 shows the contribution of intensive and extensive margins to the overall cross-country differences in labor supply, using the following decomposition formula:

$$\begin{aligned} H^{US} - H^i &= H_{\text{empl}}^{US} \cdot \text{Share}_{\text{empl}}^{US} - H_{\text{empl}}^i \cdot \text{Share}_{\text{empl}}^i \\ &= \underbrace{\left(H_{\text{empl}}^{US} - H_{\text{empl}}^i\right) \text{Share}_{\text{empl}}^{US}}_{\text{Intensive Margin}} + \underbrace{\left(\text{Share}_{\text{empl}}^{US} - \text{Share}_{\text{empl}}^i\right) H_{\text{empl}}^i}_{\text{Extensive Margin}} \end{aligned}$$

From the OECD data, one can compute the total average hours worked in country i, H^i, as the product of the hours worked by employed persons, H_{empl}^i, and the share of the population which is employed, $\text{Share}_{\text{empl}}^i$. Table 3.2 reports the contributions of intensive and extensive margins as a percentage of the total difference in hours worked between the US and country i, $H^{US} - H^i$. As can be seen from the table, both margins appear to be important. The contribution of the extensive margin is particularly large in Greece, Italy, Spain and Ireland. The intensive margin is more important in the Netherlands and Germany.

Table 3.4: Contribution of Intensive and Extensive Margins to Cross-Country Differences in Labor Supply

Country	Intensive Margin, %	Extensive Margin, %
Germany	68.21	31.79
Italy	-5.16	105.16
Spain	21.34	78.66
Ireland	35.82	64.18
Austria	57.87	42.13
Belgium	51.46	48.54
Netherlands	92.44	7.56
Greece	-119.62	219.62

3.3 Possible Determinants of Labor Supply: Taxes and Marriage Stability

In this section, we analyze the empirical relationship between hours worked in the US and Europe, and the following two candidate explanations for cross-country differences in labor supply: 1) differences in taxes; 2) differences in marriage stability. Taxes have been suggested as a major contributor to cross country differences in labor supply in the literature (see Prescott (2004) and Rogerson (2006)). Marriage stability is a new explanation in this context, motivated by our finding in section 3.2 that women are the biggest contributor to the cross-country differences in labor supply. Our hypothesis is that more stable marriages provide consumption insurance, thereby reducing the incentives to accumulate labor market experience, in particularly for women (who usually are secondary earners). Conversely, a higher probability of divorce can increase the value of market experience for the woman who has a higher probability of ending up as a single earner.

We first compare and discuss some features of the tax systems in the US and

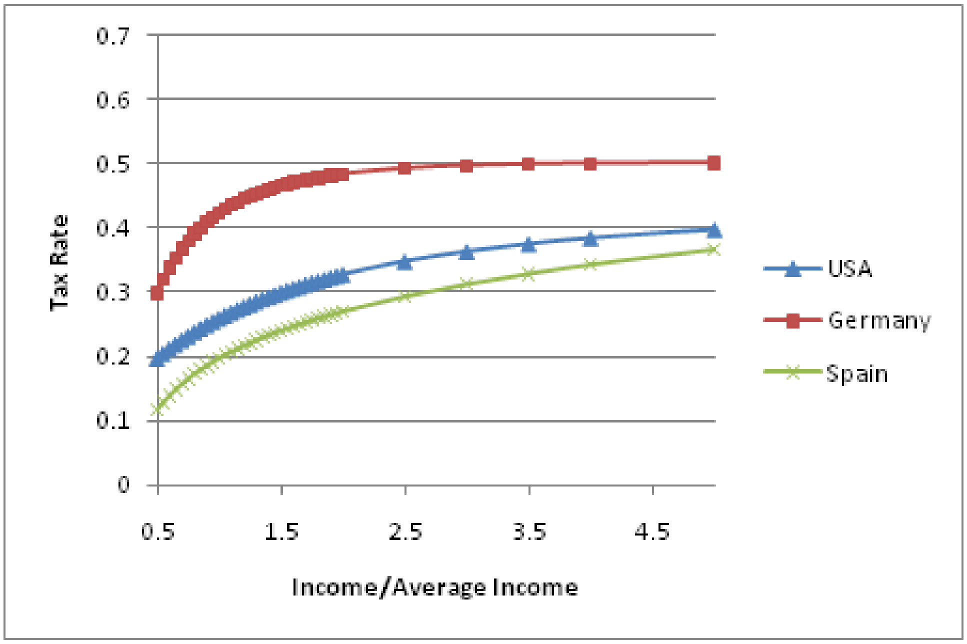
Europe with particular focus on the 9 countries in Table 1: the US, Germany, Italy, Spain, Ireland, Austria, Belgium, the Netherlands, and Greece. We then study the correlation between labor supply and various measures of tax levels, tax progressivity, and marriage stability in a larger sample of countries. We find that there is positive correlation between taxes and aggregate labor supply, and negative correlation between marriage stability and aggregate labor supply, but in both cases, the correlation is not very strong. In addition, when we regress average annual hours worked in each country on different measures of taxation and marriage stability separately, the regression coefficients have the expected sign, but are only marginally statistically significant (at 10% significance level), and the R^2 of the regressions are very low. However, when we combine a measure of tax levels and divorce rates in the same regression, both regression coefficients become highly statistically significant, and the adjusted R^2 increases considerably (to 49.4%). We conjecture that the importance of these two mechanisms is different for different groups of countries within Europe. Finally, we document strong correlation between female employment rates and divorce rates[5]. These observations motivate us to more carefully study the impact of taxes and marriage stability on labor supply in a structural model.

Labor Income Taxes in the US and Europe

There are many issues to consider when comparing labor income taxes across countries. (i) Firstly, both the levels and progressivity of taxes may be of interest, when studying the impact of taxation on labor supply. (ii) Secondly, taxes differ with respect to marital status. In the US, Germany, Spain, and Ireland married couples are taxed jointly, while in Italy, Austria, Belgium, the Netherlands, and Greece they are taxed separately. In the whole OECD there are 19 countries practicing separate

[5]Unfortunately, we are restricted to using the employment rates when we look at the labor supply by gender, since the OECD does not provide information for hours worked separately for men and women.

Figure 3.2: Country Labor Income Tax Functions (singles)



taxation of married couples and 11 countries practicing joint taxation. There may also be slightly different schemes for married households with 1 and 2 earners. (iii) Finally, taxes vary with the number of children in the household. In this section, we will focus on the taxes paid by single households without children[6].

For each country in Table 3.19, we fit a polynomial tax function, based on tax data from the OECD[7]: Among our countries, labor income taxes are the lowest in Spain and Greece, moderate to low in the US, and highest in Germany and Belgium. In figure 3.2 we plot fitted labor income tax schedules for single individuals in Spain, the US, and Germany.

Columns 1 and 2 of Table 3.19 display the top marginal tax rates and the income level where they become effective for single households in the US and many Western

[6]Essentially, we abstract in this section from points (ii) and (iii) above. We do it here because taxes paid by an average single household without children is the measure that is most easily comparable between the countries. In sections 3.5-3.7, we differentiate between the taxes paid by single and married households within the structural model of labor supply.

[7]See Appendix

European countries. There are not always large differences in the maximum tax rates but the income level where they become effective also vary greatly. In Germany, for instance, the top tax rate becomes effective already at 1.5 times average earnings, while in the US the top marginal rate first becomes effective at 9 times average earnings. Column 4 of Table 1 displays the labor income tax paid by singles with average earnings across countries.

A person making labor supply decisions will care about his marginal tax rate in addition to his tax level. It is possible that tax progressivity, and not only the level of taxes are important for the cross country pattern in labor supply. A commonly used measure for tax progressivity is so-called progressivity wedges, see for instance Guvenen, Kuruscu & Ozkan (2009):

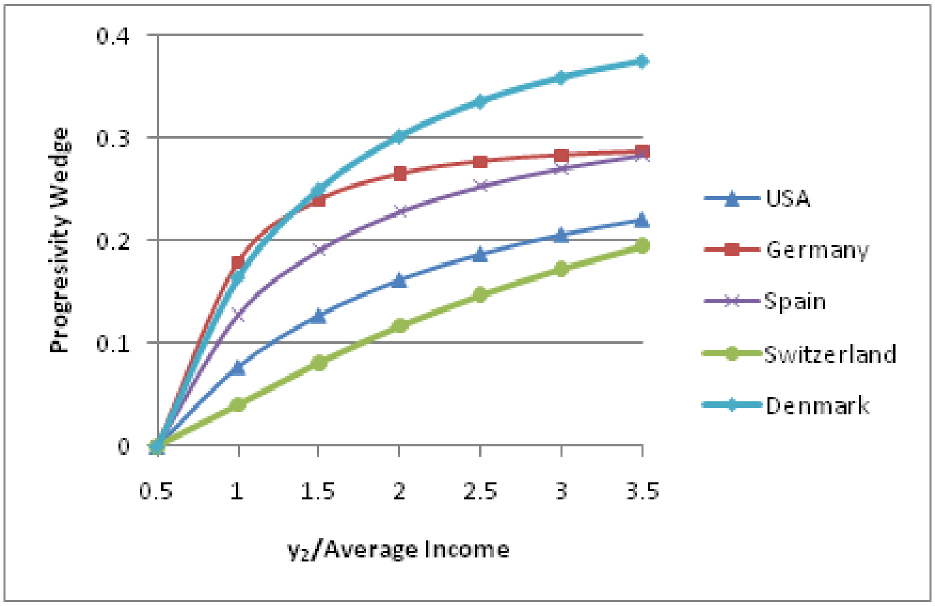
$$PW(y_1, y_2) = 1 - \frac{1 - \tau(y_2)}{1 - \tau(y_1)}$$

This measure says something about how fast the tax rate increases as earnings increase from y1 to y2. If there is a flat tax, then the progressivity wedge would be zero for all levels of y_1 and y_2. Figure 3.3 plots progressivity wedges for $y_1 = 0.5AE$ for the US, Germany, Spain, Denmark, and Switzerland. Among the 17 countries in Table 3.19, Denmark has the most progressive taxes and Switzerland the least progressive. The US is among the countries with the least progressive taxes, while Germany are among the countries with the most progressive taxes.

Consumption Taxes

Consumption taxes also have an impact on labor supply decisions. The second column of Table 2 reports these flat taxes in (2001). The consumption tax varies from 7.6% in Switzerland on the low end to 25% in Denmark and Sweden on the high end. Among our 9 countries, the US stands out with low consumption taxes.

Figure 3.3: Country Tax Functions (Singles)



Correlation between Labor Supply and Taxes and Labor Supply and Divorce Rates

In Figure 3.4, we plot the correlation between labor supply and four tax-related measures. They are: the average labor income tax rate at average earnings, the average effective tax rate on labor income at average earnings, the top marginal tax rate, and the tax progressivity wedge at $y_1 = 0.5AE$, $y_2 = 2AE$. The effective tax rate on labor income, τ, as defined in Prescott (2004) is:

$$\tau = 1 - \frac{1 - \tau_l}{1 + \tau_c}$$

It is the fraction of labor income that is taken in the form of taxes, holding investment fixed. In other words a measure that combines labor income tax and consumption tax into a single tax rate.

As can be seen from Figure 3.4, there is generally a negative but weak correlation between different measures of taxes and aggregate hours worked. The strongest correlation, -0.45, is with the effective tax rate at average earnings. There is a negative

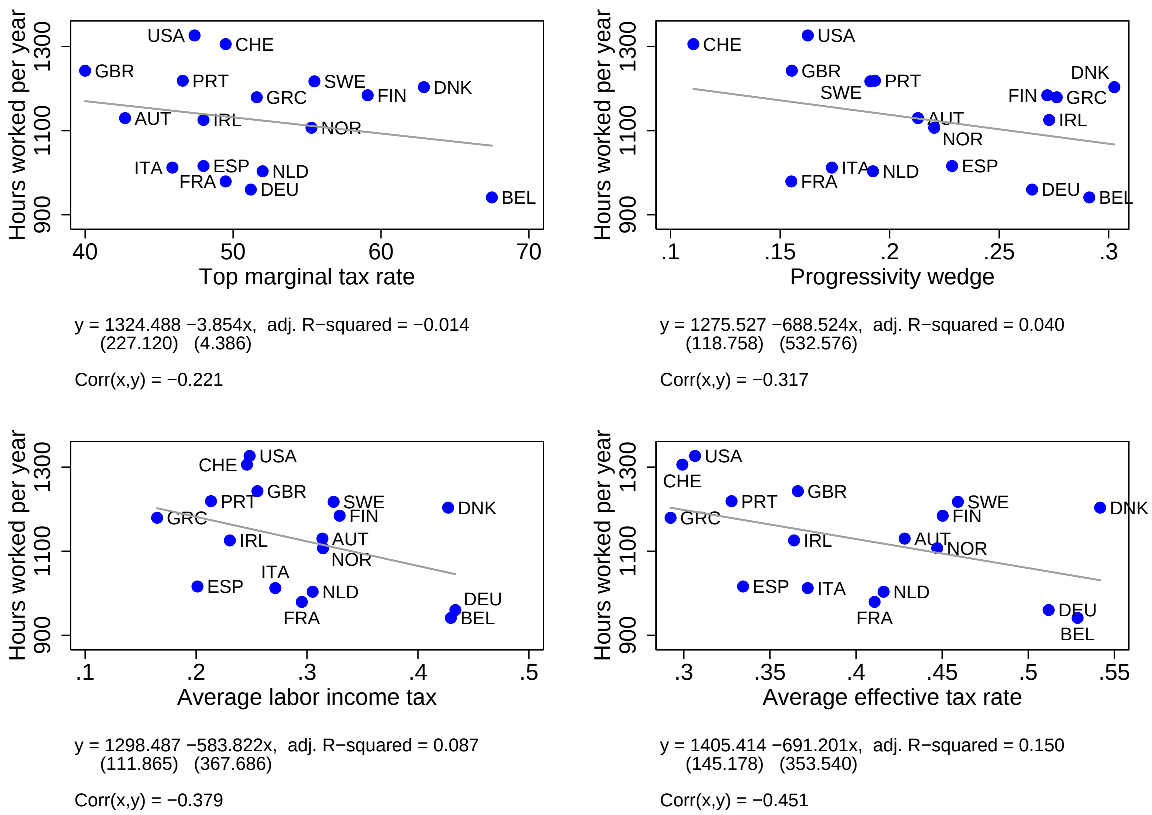


Figure 3.4: Relationship between annual hours and tax measures by country

relationship between labor supply and all our tax measures, but only the regression coefficient for the effective tax rate at average earnings is marginally statistically significant at the 10% level. In addition, the largest adjusted R^2 in the regressions is 15%, so taxes alone do not explain much of the cross- country variation in labor supply.

In figure 3.5, we plot the correlation between divorce rates and aggregate labor supply. The data for divorce rates in European countries is constructed using Eurostat data, while for the US we use the National Vital Statistics data provided by the Centers for Decease Control and Prevention, and the US Census data. As can be seen from Figure 3.5, there is a positive relationship between average annual hours worked and divorce rates. The regression coefficient is almost statistically significant at the 5% level, and the adjusted R^2 is only 13.7%.

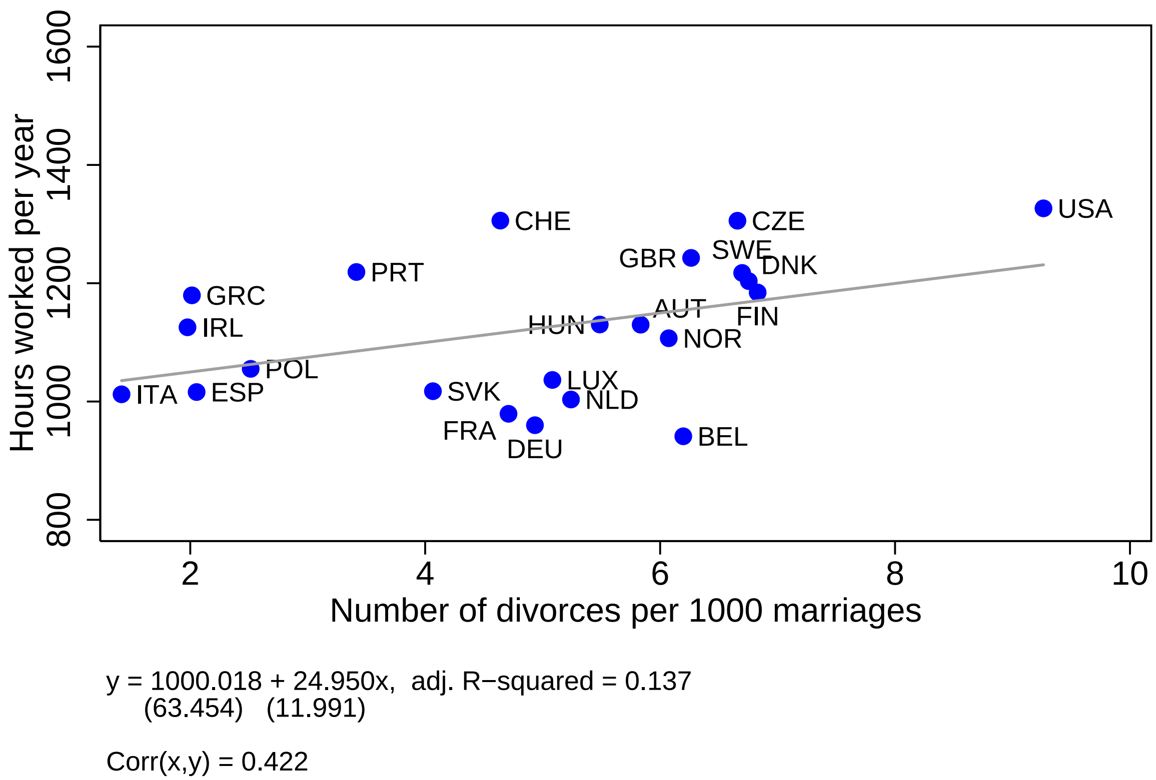


Figure 3.5: Relationship between annual hours and divorce rate by country

In Table 3.5 we present the results from a regression of labor supply on divorce rate and each of the different tax measures. In two cases (when using the average labor income tax and average effective tax rate), the coefficients for both the divorce rates and the tax measure that we use are statistically significant at any conventional significance level, and the adjusted R^2 improves substantially to 49.4%. Using both taxes and divorce rates together explains a significant share of the cross- country variation in labor supply.

Guner et al. (2008) argue that one of the features of the tax system that can

Table 3.5: Regressing Average Hours Worked on Divorce rate and Tax Measures

	(I)	(II)	(III)	(IV)
Const	1321.283***	1166.408***	1258.655***	1383.385***
	(207.819)	(137.197)	(83.996)	(112.1656)
Divorce rate	27.101*	19.428	42.036***	36.733***
	(13.694)	(13.418)	(11.627)	(10.968)
Top marginal tax rate	−6.409	–	–	–
	(4.215)			
Progressivity wedge	–	−629.513	–	–
		(515.734)		
Average labor income tax	–	–	−1156.867***	–
			(316.286)	
Average effective tax rate	–	–	–	−1088.327***
				(297.347)
adjusted R^2	0.151	0.106	0.494	0.494

Standard errors are in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

be particularly important for the labor supply of the married couples is whether the labor income of the couple is taxed jointly or separately. Table 3.25 in the appendix reports the regression results when we add a dummy variable equal to 1 for countries in our sample that practice separate taxation. Table 3.25 shows that the coefficient for separate taxation in 3 out of 4 regressions reported in the table has the expected positive sign, but is not statistically significant.

Unfortunately, the OECD dataset does not provide data for hours worked separately for men and women, but it does provide data on employment rates by gender. Figure 3.6 shows the relationship between the divorce rates and employment ratios by country for men and women separately. It shows that for both men and women, this relationship is positive, but the magnitude of the coefficient is about three times as large for women as it is for men. In addition, the coefficient is statistically significant for women and not statistically significant for men.

Figures 3.13 and 3.14 in the appendix show the relationship between our tax measures and employment ratios for women and men respectively. None of the tax

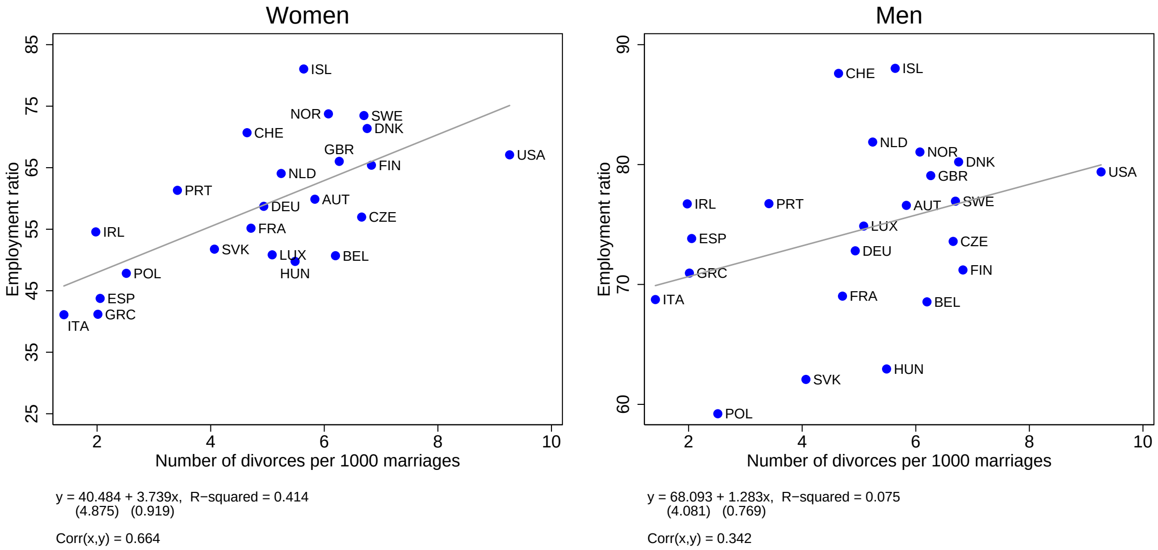


Figure 3.6: Relationship between divorce rates and employment ratios for men and women by country

measures is statistically significant for either of the sexes, and in many cases the relationship appears to be negative. We conclude that our macro level data suggest that while both tax measures and divorce rates appear to be related to annual hours worked, taxes appear to impact mostly the intensive margin (hours worked for those who are employed), while divorce rates appear to be related to the extensive margin – the employment ratios (see figure 3.15), and this relationship appears much stronger for women.

Finally, Table 3.6 shows the panel regression results, when regressing employment ratios on divorce rates for men and women separately, using the data from 1990 to 2009 (one obtains a qualitatively similar results when starting at an earlier date)[8].

[8]Since the Eurostat data on the number of divorces that we use to construct the divorce rate measure spans different time periods for different countries, we have an unbalanced panel. The US data start in 2000. Also, the data here lacks observations for some European countries, such as Spain and Greece, altogether. In our previous cross-sectional plots for 2001, we used the Eurostat Census 2001 data on the number of married people for these countries, but this data is available only for one year, 2001.

The panel regression results provide further support to our finding that divorce rates appear to affect mostly the labor supply of women.

Table 3.6: Relationship between employment ratios and divorce rates, panel regression results

Employment rate	Women	Men
Constant	51.809***	72.681***
	(2.795)	(2.076)
Divorce rates	1.685***	0.323
	(0.398)	(0.283)

Standard errors are in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In this section, we have documented an empirical relationship between aggregate labor supply and taxes and aggregate labor supply and divorce rates. This motivates our study in the next three sections of the impact of taxes, divorce- and marriage probabilities on labor supply in a structural model.

3.4 Gaining Intuition: Labor Supply and Divorce in a Simple Two-Period Model

In this section, we outline the intuition for the effect of divorce rates on women's labor supply using a simplified two-period version of our model[9]. We describe our full model in the next section.

Consider a family that consists of a husband (a "man") and a wife (a "woman") who live for 2 periods. Suppose that both members of the family have 1 unit of time at their disposal in each period. For simplicity, assume here that the husband always works full-time, while the wife has to decide how much time to spend working in period 1 and in period 2. Assume that the husband's wage in period 1 is $w_{1,m}$, while

[9] The intuition concerning the effect of taxation is described very well in Rogerson (2007), Guner et al. (2008) etc.

the wife's wage in the first period is $w_{1,f}$. Suppose that their wages in the second period increase linearly with the amount of time they spend working in period 1, with parameters k_m and k_f controlling the "returns to experience" for the husband and the wife. Thus, the husband's wage in period 2 is $w_{1,m} + k_m$ (since the husband always works full-time), while the wife's wage in period 2 is $w_{1,f} + k_f h_{1,f}$. Assume that with probability π_d, the couple divorces before the second period starts. Suppose that they cannot save or borrow in period 1.

At the start of period 1, the couple jointly solves:

$$\begin{aligned}
\max_{\substack{c_1,c_2,c^s_{2,m},c^s_{2,f},\\ h_{1,f},h_{2,f},h^s_{2,f}}} \quad & \alpha\log(c_1/e) + (1-\alpha)\log(1-h_{1,f}) + (1-\pi_d)(\alpha\log(c_2/e) \\
& + (1-\alpha)\log(1-h_{2,f})) + \pi_d\left(\log(c^s_{2,m}) + \alpha\log(c^s_{2,f}) + (1-\alpha)\log(1-h^s_{2,f})\right) \\
\text{s.t.:} \quad & c_1 = w_{1,m} + w_{1,f}h_{1,f}, \\
& c_2 = w_{1,m} + k_m + (w_{1,f} + k_f h_{1,f})h_{2,f}, \\
& c^s_{2,m} = w_{1,m} + k_m, \\
& c^s_{2,f} = (w_{1,f} + k_f h_{1,f})h^s_{2,f}.
\end{aligned}$$

where $h_{2,f}$ is the woman's choice of work in period 2 in case she stays married, $h^s_{2,f}$ is her choice of work if she gets divorced, and e is the adult equivalence scale.

The solution is characterized by the following 3 first-order conditions:

$$\frac{1-\alpha}{1-h_{2,f}} = \frac{\alpha}{c_2}(w_{1,f} + k_f h_{1,f}), \tag{3.1}$$

$$\frac{1-\alpha}{1-h^s_{2,f}} = \frac{\alpha}{c^s_{2,f}}(w_{1,f} + k_f h_{1,f}), \tag{3.2}$$

$$\frac{1-\alpha}{1-h_{1,f}} = \frac{\alpha}{c^s_{1,f}}w_{1,f} + (1-\pi_d)\frac{\alpha}{c_2}k_f h_{2,f} + \pi_d\frac{\alpha}{c^s_{2,f}}k_f h^s_{2,f}. \tag{3.3}$$

First, let us consider how a change in the probability of divorce, π_d, affects the woman's choice of labor supply in period 1, $h_{1,f}$. An increase in π_d will affect $h_{1,f}$ both directly through equation 3.3, and also indirectly through the effect of the change in $h_{1,f}$ on $h_{2,f}$ and $h^s_{2,f}$ in equations 3.1 and 3.2, which feeds back into c_2 and $c^s_{2,f}$ in equation 3.3. For simplicity, let us disregard the indirect effect, and concentrate on the direct effect in equation 3.3. On the right hand side of that equation, we have the marginal benefit of an increase in the wife's work in period 1, which includes both an immediate increase in consumption in period 1, and the increase in consumption in period 2 because of the accumulation of the woman's experience (and increased period 2 wages). An increase in π_d effectively decreases the weight put on the second period's marginal utility of consumption in case the couple stays married, and increases the weight on the second period's marginal utility of consumption of the divorced woman. Intuitively, because the income of the married couple also includes the income of the husband (which typically is larger than the income of the wife), we get $c_2 > c^s_{2,f}$. From equations 3.1 and 3.2, it also follows that $h^s_{2,f} > h_{2,f}$, so that $\frac{\alpha}{c^s_{2,f}} h^s_{2,f} > \frac{\alpha}{c_{2,f}} h_{2,f}$, and such re-weighting increases the marginal benefit from the woman's work in period 1. This increases the woman's incentive to work in period 1.

Given the utility function that we have assumed in this section, one can in fact show that[10]

Proposition 3.4.1. *The increase in divorce probability increases woman's labor supply in period 1:*

$$\frac{\partial h_{1,f}}{\partial \pi_d} > 0.$$

It is clear from equation 3.3 that for the change in divorce probability to have an impact on the woman's labor supply, we need $k_f > 0$ (returns to experience must be

[10] See Appendix for the proof.

positive). One can expect this impact to be larger, the bigger is the gender wage gap ($\frac{w_m}{w_f}$). One could also be tempted to conclude from equation 3.3 that the effect of the change in divorce probability is stronger, the bigger is the returns to experience. However, even though this is true for fixed c_2 and $c^s_{f,f}$, and we found it to be true for a variety of reasonable choices of parameters in this simple two-period model, this could be at least partially offset by the income effect of the increase in k_f, which could be larger for the single woman.

An increase in woman's labor supply in period 1 leads to accumulation of experience, and thus higher wages in period 2. On one hand, this gives both the married and the single woman an incentive to increase labor supply in period 2 through the substitution effect. However, there is also potentially an offsetting income effect. Intuitively, the income effect will be stronger for the divorced woman who does not have access to her spouse's income (and thus, its is more likely that the married woman will increase her labor supply in period 2). Given the utility function we have assumed in this section, we get $h^s_{2,f} = \alpha$ and $h_{2,f} = \frac{\alpha(w_{1,f}+k_f h_{1,f}+w_{1,m}+k_m)-(w_{1,m}+k_m)}{w_{1,f}+k_f h_{1,f}}$, so that $\frac{\partial h^s_{2,f}}{\partial \pi_d} = 0$ and $\frac{\partial h_{2,f}}{\partial \pi_d} = \frac{\partial h_{2,f}}{\partial h_{1,f}} \frac{\partial h_{1,f}}{\partial \pi_d} = \frac{k_f(w_{1,m}+k_m)(1-\alpha)}{(w_{1,f}+k_f h_{1,f})^2} \frac{\partial h_{1,f}}{\partial \pi_d} > 0$

To see that the increased probability of divorce can also increase labor supply of single women, imagine that there are 3 periods of active life, all women are single in period 0, but they are certain to get married in period 1 (and periods 1 and 2 are the same as the above), and that the the wages the woman receives in period 2 increase both in experience accumulated in period 0 and 1.

3.5 Quantitative Model

The stationary economy is populated by three types of households: single males, single females, and married couples. Individuals start their life at age 20. They live for at least 65 years, and at most 95 years, but enter retirement at age 65. A model period is 1 year, so there are a total of 45 model periods of active work life. Single households face an age-dependent probability of becoming married, while married couples face an age dependent probability of divorce. One is more likely to be married to someone with the same level of education. We assume that marriage will always happen to a partner of the same age, and that married couples die together. Households decide whether to participate in the labor market, how much to consume, and how much to save, and they accumulate labor market experience.

Labor Income. The wage, w, of an individual depends on his level of education, $j \in \{\text{hs}, \text{c}\}$ (where "hs" stands for high school and "c" stands for college), gender, $g \in \{\text{m}, \text{f}\}$, and years of labor market experience, x:

$$w(j, g, x) = e^{\gamma_{0jg} + \gamma_{1jg} x + \gamma_{2jg} x^2 + \gamma_{3jg} x^3} \tag{3.4}$$

Given this wage function, the beginning wage levels as well as the returns to experience are allowed to differ by level of education and gender.

Preferences. The momentary utility function of single individuals, U^S, depends on labor market participation, $n \in 0, 1$, consumption, c, and on gender:

$$U_g^S(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - F_g n \tag{3.5}$$

F_g is here a fixed, gender specific, disutility from working. Married couples have a joint utility function, U^M, with shared consumption, measured in adult equivalents:

$$U^M(c, n_m, n_f) = \frac{\left(\frac{c}{e}\right)^{1-\sigma}}{1-\sigma} - F_m n_m - F_f n_f \tag{3.6}$$

Household's Problem

Written recursively, a single household's problem can be formalized as follows:

$$\begin{aligned} V^S_{g,j}(k,x,t) &= \max_{c,n,k'} U^S_g(c,n) + \beta\Big((1-\pi_m(t))V^S_{g,j}(k',x',t+1) \\ &+ \pi_m(t)E_{j_p,k'_p,x'_p}\left[V^M_{j,j_p}(k'+k'_p,x',x'_p,t+1)\right]\Big) \\ \text{s.t.:}\quad c(1+\tau_c)+k' &= k(1+r)+nw(j,g,x)(1-\tau_{n,S}(w(j,g,x)n))+(1-n)T, \\ x' &= x+n, \quad n\in\{0,1\}, \quad k'\geq 0, \quad c>0 \end{aligned} \tag{3.7}$$

k here is the level of asset holdings, r is the risk-free interest rate, and β the time discount factor. τ_c is a constant consumption tax, while τ_n is a nonlinear labor income tax. In the US and some European countries, the tax schedule is dependent on whether a person is single or married. T is an individual's income if he chooses not to participate in the labor market. The sources of such income would be unemployment benefits, social aid, transfers from relatives and charities and so on. $\pi_m(t)$ is a age-dependent probability of becoming married in the next period. The subscript, p, stands for partner. In the case that an individual becomes married in the next period, the expectation of next period's utility must be taken with respect to the distribution over potential partners' education, experience, and asset holdings, $Q^{jgt}(j_p,x'_p,k'_p)$. An individual is more likely to find a partner of his own education group, and the distribution of partners naturally varies by gender and age. The distribution over x'_p and k'_p is derived from the individuals' optimal desicions.

Married couples maximize their joint utility and face a time-dependent probability, $\pi_d(t)$, of becoming divorced. When couples divorce, they split their assets evenly.

Their problem can be written as:

$$\begin{aligned}
V^M_{j_m,j_f}(k,x_m,x_f,t) &= \max_{c,k',n_m,n_f} U^M(c,n_m,n_f) \\
&+ \beta \Big((1-\pi_d(t))V^M_{j_m,j_f}(k',x'_m,x'_f,t+1) \\
&+ \pi_d(t) \Big(V^S_{m,j_m}(k'/2,x'_m,t+1) + V^S_{f,j_f}(k'/2,x'_f,t+1)\Big)\Big) \\
\text{s.t:} \quad c(1+\tau_c)+k' &= k(1+r) + (n_m w_m + n_f w_f)(1-\tau_{n,M}(n_m w_m + n_f w_f)) \\
&+(2-(n_m+n_f))T \\
x'_m = x_m + n_m, \quad x'_f &= x_f + n_f, \quad n_f, n_m \in \{0,1\}, \quad k' \geq 0, \quad c > 0 \qquad (3.8)
\end{aligned}$$

Retired households make no labor supply decisions but receive an amount of social security, $\Phi(g)$, depending on their gender. We assume that retired households do not marry or get divorced, and that husband and wife die at the same time. Their problem, if single, is simply:

$$\begin{aligned}
V^S_g(k,t) &= \max_{c>0,k'\geq 0} U^S_g(c) + \Omega(t)\beta V^S_g(k',t+1) \\
\text{s.t.:} \quad c(1+\tau_c)+k' &= k(1+r) + \Phi(g), \qquad (3.9)
\end{aligned}$$

where $\Omega(t)$ is the probability of survival until the next period. Married retirees solve:

$$\begin{aligned}
V^M(k,t) &= \max_{c>0,k'\geq 0} U^M(c) + \Omega(t)\beta V^M(k',t+1), \\
\text{s.t.:} \quad c(1+\tau_c)+k' &= k(1+r) + \Phi(m) + \Phi(f), \qquad (3.10)
\end{aligned}$$

3.6 Calibration

This section describes the calibration of the model parameters. We calibrate our model to match the appropriate moments from the US data. We use data from different sources. We try to use data from 2000 or the year closest to 2000 that we can obtain. Many parameters can be calibrated to direct empirical counterparts without solving the model. They are listed in Table3.7. The 7 parameters in Table 2 below are, however, calibrated using an exactly identified simulated method of moments approach. We use the data from the European countries in our sample only to obtain the estimates of tax polynomials and age-specific marriage and divorce probabilities, which we use in section 3.7 in our counterfactual experiments.

Preferences

The momentary utility function is a standard CRRA utility function in equations 3.5 and , with consumption measured in adult equivalents, $\frac{c}{e}$. We use the OECD adult equivalence scale and set $e = 1.7$ for married couples, and $e = 1.0$ for singles. Consistent with a survey of the empirical literature in Browning et. al. (1999), we set the coefficient of relative risk aversion, σ, equal to 2. The discount factor, β, and fixed costs of working, F_m and F_f, are among the estimated parameters. The corresponding data moments are the mean asset holdings of households with head aged $20 - 64$, taken from the PSID (99-05), and the male- and female employment rates, taken from OECD 2000.

Risk Free Interest Rate

Given the partial equilibrium nature of the model, we take the risk free rate as fixed and calibrate it using the data. We set the risk free rate equal to the average of 3-month t-bill rates minus inflation over the period from 1947-2008 based on data from the Federal Reserve Bank of St. Louis[11].

Wages

We calibrate the experience profile of wages exogenously, using the PSID from 1968-1997. After 1997 it is not possible to get years of actual labor market experience from the PSID. We regress earnings on a 3rd order polynomial in years of labor market experience and control for the year of birth. We estimate different returns to experience for each gender / education group. To get levels of earnings that are in line with the asset holdings, we include a parameter controlling the average earnings of each gender / education group in the structural estimation. The corresponding data moments are the average wage of each group in the PSID 99-05.

Taxes

The labor income tax schedule is a polynomial function of an individual's earnings relative to the average earnings, AE, equation 3.12 in the appendix. As described in more detail in the appendix, we fit this polynomial to labor income tax data from the OECD tax database (2001). This data is constructed by the OECD based on tax laws from different countries. It is well suited for cross country comparisons, see also see Guvenen et al. (2009). For those countries who practice joint taxation of married couples, we fit a different tax schedule for married and single individuals. Coming up with an accurate estimate of consumption taxes in the US is complicated by the fact that there are local county-level taxes in addition to state taxes. Vertex Inc. (a consulting company) estimated that the average consumption tax in the US was 8.4% in 2002. We use that number. For simplicity, we abstract from capital taxes. we do this because different types of capital is taxed differently, and this also differs across countries. Households do for instance have about half of their wealth in their homes

[11]Series TB3MS and GDPDEF.

Table 3.7: Parameters Calibrated Outside of the Model

Parameter	Value	Description	Target
r	0.011	Risk free interest rate (annual)	3-mnth T-bill minus inflation (1947-2008)
σ	2	$u(c,n) = \frac{(c/e)^{(1-\sigma)}}{(1-\sigma)}$	Browning et. al. (1999)
e	1.0 or 1.7		OECD equivalence scale.
$\gamma_{1hsm}, \gamma_{2hsm}, \gamma_{3hsm}$	0.066, -20(-4), 17(-6)	$w_{hsm} = e^{(\gamma_{0hsm}+\gamma_{1hsm}x+\gamma_{2hsm}x^2+\gamma_{3hsm}x^3)}$	PSID (1968-1997)
$\gamma_{1cm}, \gamma_{2cm}, \gamma_{3cm}$	0.109, -32(-4), 26(-6)	$w_{cm} = e^{(\gamma_{0cm}+\gamma_{1cm}x+\gamma_{2cm}x^2+\gamma_{3cm}x^3)}$	
$\gamma_{1hsf}, \gamma_{2hsf}, \gamma_{3hsf}$	0.069, -16(-4), 12(-6)	$w_{hsf} = e^{(\gamma_{0hsf}+\gamma_{1hsf}x+\gamma_{2hsf}x^2+\gamma_{3hsf}x^3)}$	
$\gamma_{1cf}, \gamma_{2cf}, \gamma_{3cf}$	0.064, -12(-4), 6(-6)	$w_{cf} = e^{(\gamma_{0cf}+\gamma_{1cf}x+\gamma_{2cf}x^2+\gamma_{3cf}x^3)}$	
τ_{s0}, τ_{s1}	1.727, -6.450	$\tau(y) = \tau_{s0} + \tau_{s1}(y/AE)^{0.2}$	OECD tax data (01)
τ_{s2}, τ_{s3}	8.995, -5.000	$+\tau_{s2}(y/AE)^{0.4} + \tau_{s3}(y/AE)^{0.6}$	
τ_{s4}	0.988	$+\tau_{s4}(y/AE)^{0.8}$	
τ_{m0}, τ_{m1}	2.162, -7.302	$\tau(y) = \tau_{m0} + \tau_{m1}(y/AE)^{0.2}$	OECD tax data (01)
τ_{m2}, τ_{m3}	9.222, -4.736	$+\tau_{m2}(y/AE)^{0.4} + \tau_{m3}(y/AE)^{0.6}$	
τ_{m4}	0.872	$+\tau_{m4}(y/AE)^{0.8}$	
τ_c	0.084	Consumption tax	Vertex Inc. (2002)
T	\$8440	income if not working	CEX 2000-2001
$\Phi(m), \Phi(f)$	\$12600, \$9680	Social security	S.S. Admin. (2000)
$\pi_m(t)$	Varies	Prob. of marriage	CPS (1999-2001)
$\pi_d(t)$	Varies	Prob of divorce	CPS (1999-2001)
$\Gamma(t)$	Varies	Death probabilities	NCHS (1991-2001)
Fraction w. some college.	0.533		CPS (1999-2001)
Prob. intra ed. marriage	0.737		CPS (1999-2001)
k_0	8260	Savings at age 20	NLSY97
M_0	0.126	Share of married 20 year-olds	CPS (1999-2001)

Table 3.8: Parameters Calibrated Endogenously

Parameter	Description	Data Moment	Value
γ_{0hsm}	$w_{hsm} = e^{(\gamma_{0hsm}+\gamma_{1hsm}x+\gamma_{2hsm}x^2+\gamma_{3hsm}x^3)}$	Mean male hs-wages	-1.438
γ_{0cm}	$w_{cm} = e^{(\gamma_{0cm}+\gamma_{1cm}x+\gamma_{2cm}x^2+\gamma_{3cm}x^3)}$	Mean male c-wages	-1.464
γ_{0hsf}	$w_{hsf} = e^{(\gamma_{0hsf}+\gamma_{1hsf}x+\gamma_{2hsf}x^2+\gamma_{3hsf}x^3)}$	Mean female hs-wages	-2.081
γ_{0cf}	$w_{cf} = e^{(\gamma_{0cf}+\gamma_{1cf}x+\gamma_{2cf}x^2+\gamma_{3cf}x^3)}$	Mean female c-wages	-1.692
β	Discount factor	Mean assets	1.001
F_m	Fixed cost of working	Male employment rate	2.092
F_f	Fixed cost of working	Female employment rate	2.265

which may or may not be taxed. In the US, interest income is taxed as labor income, while dividends and capital gains are subject to capital gains tax. The return on capital is, however, set very conservatively in our calibration. It is set equal to the returns on risk free bonds, which was 1.1% over the past 60 years.

Death Probabilities and Social Security

The probability that a retiree will survive to the next period, we obtain from the National Center for Health Statistics (1991-2001). We assume that all retirees receive the same constant Social Security benefit, only dependent on gender. We obtain the average benefit for males and females from the Annual Statistical Supplement to the Social Security Bulletin (2000).

Marriage and Divorce Probabilities

To compute the age-specific probabilities for marriage and divorce for the US, we use the data from the CPS March supplement from 1999-2001. For most European countries, we use the data from Eurostat on-line database[12]. For some European countries, we supplement it with the data from the IPUMS International.

We assume the stationary environment, where the probabilities of getting married and divorced don't change over time (we allow them to depend on the age of the

[12] Available at http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/search_database.

person, but not on his/her cohort)[13]. We also assume that the probability of getting married is the same for those who get married for the first time, and those who were previously divorced. This allows us to compute the probabilities using the following approach. Let M_t and D_t be the share of the married and divorced persons respectively at age t[14]. Then the probability of getting married at age t, $\pi_m(t)$, and the probability of getting divorced at age t, $\pi_d(t)$, is pinned down by:

$$M_{t+1} = (1 - M_t)\pi_m(t) + M_t(1 - \pi_d(t)),$$
$$D_{t+1} = D_t(1 - \pi_m(t)) + M_t\pi_d(t).$$

We smooth the resulting age-profiles for $\pi_m(t)$ and $\pi_d(t)$ by fitting a polynomial. Figure 3.7 shows the resulting probability profiles for the US, Germany and Italy[15].

Figure 3.7 shows that the probability of getting divorced is noticeably higher in the US than Italy, and somewhat higher than in Germany. At the same time, the probability of getting married reaches its peak in the US somewhat earlier compared to the two European countries[16].

Fixed Cost of Working and Income if Not Working

The data moments for the fixed cost of working for men and women are the male and

[13]Figure 3.16 in appendix shows the number of divorces per 1000 marriages for 3 countries – US, Italy and Netherlands over a span of 10 (in case of US) to 20 (in case of Netherlands and Italy) years. It shows that even though the number of divorces have been increasing in Italy and decreasing in the US, these changes over time were rather slow and small compared to the differences in levels.

[14]Figure 3.18 in appendix shows the share of married women in the countries in our sample.

[15]Countries like Spain, Ireland, Greece and Portugal have marriage and divorce probabilities similar to Italy, and countries like Netherlands and Belgium are similar in this respect to Germany.

[16]The computed probabilities use the data for women. We get a qualitatively similar picture when using the data for both men and women (with the exception that men in all countries tend to get married somewhat later than women).

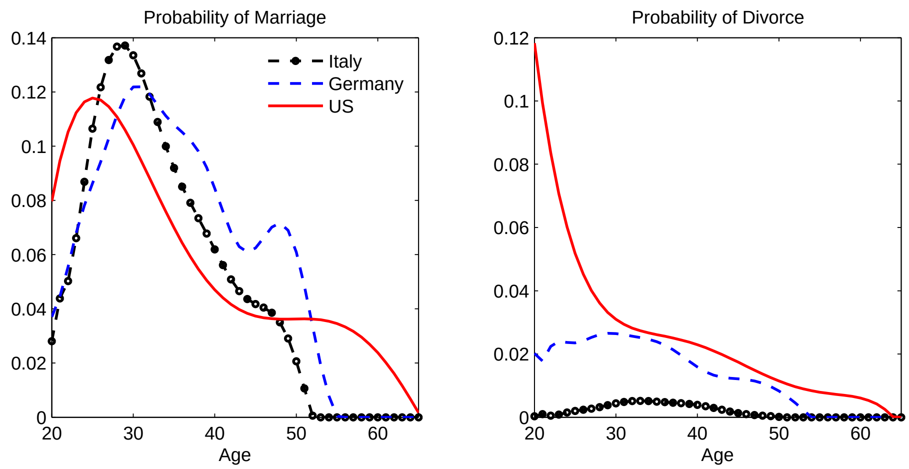


Figure 3.7: Age-dependent probabilities of marriage and divorce

female employment rates in 2000, taken from the OECD. As an approximation for income when not working, we take the value of non-housing consumption of households with income less than \$5000 per year from the 2000-2001 Consumer Expenditure Survey. The sources of such income would be unemployment benefits, social aid, gifts from relatives and charities etc.

Estimation Method

7 model parameters are calibrated using an exactly identified simulated method of moments approach. We minimize the squared percentage deviation of simulated model statistics from the 7 data moments in Table 3. Let $\Theta = \{\gamma_{0hsm}, \gamma_{0cm}, \gamma_{0hsf}, \gamma_{0cf}, \beta, F_m, F_f\}$ and let $V(\Theta) = (V_1(\Theta), \ldots, V_7(\Theta))'$ denote the vector where $V_i(\Theta) = (\bar{m} - \hat{m}(\Theta))/m$ is the percentage difference between empirical moments and simulated moments. Then:

$$\hat{V} = \min_{\Theta} V(\Theta)'V(\Theta) \tag{3.11}$$

Table3.8 summarizes the estimated parameter values. As can be seen from Table 3.9, we get close to match all the moments exactly.

Table 3.9: Calibration Fit

Moment	Data	Model
Mean wage of high school educated males	0.396	0.396
Mean wage of college educated males	0.594	0.594
Mean wage of high school educated females	0.255	0.255
Mean wage of college educated females	0.372	0.372
Mean assets	1.200	1.198
Male employment rate	0.841	0.841
Female employment rate	0.699	0.700

3.7 Counterfactual Experiments

In Section 3.3, we have documented a correlation between labor supply and tax levels and labor supply and divorce rates across countries and across time. This motivates the study, in this section, of the quantitative impact of cross country differences in tax schemes and divorce rates on labor supply. When we perform the policy experiments, we keep taxes, old age social security, and income when not working as functions of average earnings in the economy. In this way if the society becomes richer or poorer because of a counterfactual experiment, taxes and social security payments will adjust accordingly. Since there is no public good in the model, we do not keep a balanced government budget and excess tax revenues are assumed to finance bureaucracy.

The Effect of Marriage and Divorce Probabilities on Labor Supply

In this subsection, we use our model that we described in Section 3.5, and calibrated to match the US economy in Section 3.6, to study the impact of marriage and divorce probabilities on labor supply. We do this by imposing the marriage and divorce probabilities that we computed for each of the European countries in our sample on the model. Figure 3.8 shows how it affects hours worked[17]. We obtain a positive correlation between the model's predictions and the data (equal to 0.467). As we

[17]Since we do not have the intensive margin in our model, we compute the predicted annual hours worked for all European countries in our sample as a product of employment rates predicted by our model and hours worked by employment persons in the data in the US.

expect, higher marriage stability reduces labor supply both in the model and in the data.

Ideally, if the model matched the data perfectly, all observations would be located somewhere on the diagonal line. The distance from the diagonal shows the discrepancy between the data and the model prediction.

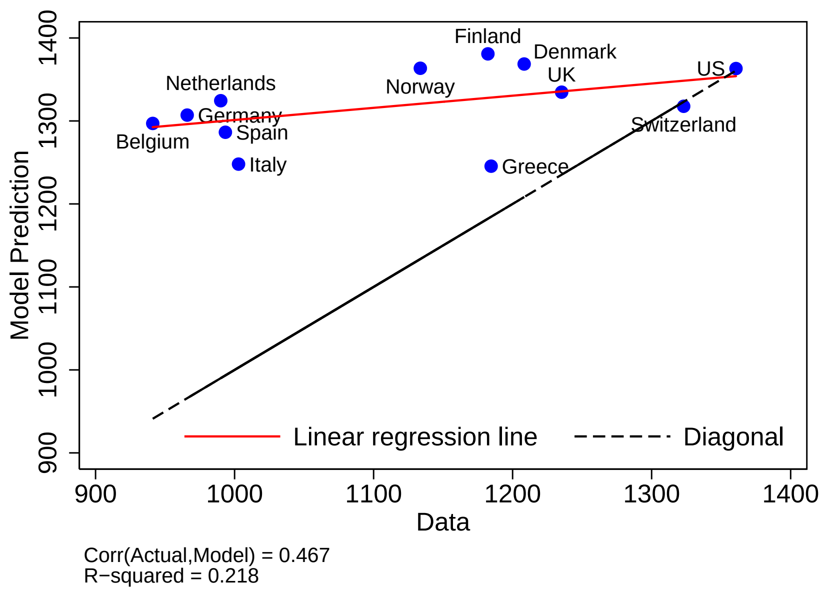


Figure 3.8: The Impact of Marriage and Divorce Probabilities on Hours Worked, Both Genders

Table 3.10 and figures 3.9 and 3.10 illustrate the impact on the employment rates for men and women. Unfortunately, we cannot perform the comparison by gender in terms of the hours worked because of the lack of the data. However, as we show in Section 3.3, marriage stability appears to affect mostly the extensive margin.

Figure 3.9 shows a rather high correlation (equal to 0.825) between our model's predictions and data for the individuals of both genders – higher marriage stability appears to reduce labor supply both in the model and in the data. Figure 3.10 shows that the correlation between the model predictions and the data is even higher for women (equal to 0.889). Figure 3.19 in appendix shows that the correlation between

Table 3.10: The Impact of Marriage and Divorce Probabilities on Employment Rates

Country	Aggregate Employment Rates		Female Employment Rates		Male Employment Rates	
	Actual	Model	Actual	Model	Actual	Model
US	0.770	0.770	0.699	0.699	0.841	0.841
Greece	0.610	0.704	0.450	0.608	0.781	0.800
Italy	0.574	0.706	0.421	0.616	0.728	0.795
Spain	0.610	0.727	0.445	0.646	0.774	0.809
Belgium	0.662	0.733	0.564	0.649	0.759	0.817
Switzerland	0.809	0.745	0.715	0.669	0.903	0.821
Germany	0.687	0.739	0.610	0.659	0.762	0.819
Netherlands	0.737	0.749	0.637	0.668	0.835	0.829
UK	0.737	0.755	0.665	0.685	0.810	0.825
Norway	0.805	0.771	0.763	0.702	0.847	0.840
Denmark	0.779	0.774	0.733	0.703	0.823	0.844
Finland	0.718	0.781	0.684	0.720	0.751	0.841

the model's predictions and the data is substantially worse for men (equal to 0.474). This is not surprising, as we expect the marriage stability mechanism to be able to better account for the behavior of women.

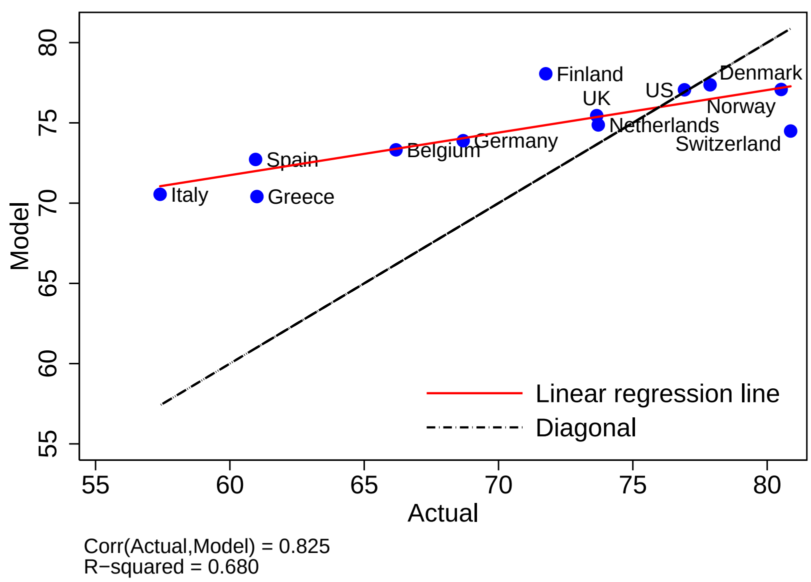


Figure 3.9: The Impact of Marriage and Divorce Probabilities on Employment Rates, Both Genders

We conclude that the marriage stability mechanism works in the right direction in our model by reducing the labor supply in the countries with more stable marriages.

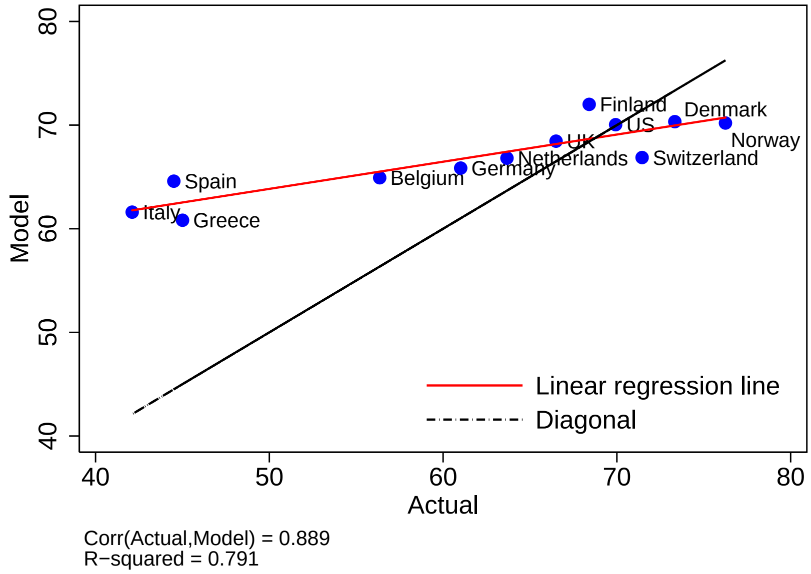


Figure 3.10: The Impact of Marriage and Divorce Probabilities on Employment Rates, Women

As one would expect, this mechanism appears to be able to account better for the labor supply of women.

The Impact of Differences in Taxation on Labor Supply

Figure 3.20 compares the predictions of our model to the data when we assume that the divorce and marriage probabilities in all countries are the same as in the US, but replace the tax system in the model by the one computed for each country using the OECD data (as described in section 3.6), and furthermore assume that all the difference in tax revenues that result from the change of the tax system go to waistful government spending. The figure shows that there is little impact on hours worked in our model in this case. We in fact obtain a negative correlation between our model's predictions and the data. Table 3.22 shows that one feature of the tax system that appears to be particularly important in our model is whether the married couples are taxed jointly or separately. In table 3.24, we see that our model predicts that labor supply is noticeably higher in the countries that practice separate taxation. Table 3.23 shows that this is primarily driven in the model by higher employment ratios of women.

Figure 3.21 shows that the predictions of our model improve when we assume that the additional tax revenues are redistributed to all the agents in the economy as a lump sum. This illustrates that the use of the tax revenues is crucial in our model for taxes to have a negative effect on labor supply.

The Combined Impact of Divorces and Taxation on Labor Supply

Figure 3.11 shows the impact of both the divorce and tax mechanisms combined in our model. When we include both mechanisms in the model, the correlation between the model's predictions and the data increase to 0.637, and we are able to explain 41% of the variation in hours worked in the data (as shown by the R^2).

On average, the experiment with changing only the divorce rates can account for 22% of the difference between the US and European countries in our sample, the experiment with changing only the tax system (and assuming redistribution of the additional tax revenues) can account for 19% of the difference, and in the experiment with both mechanisms included we account for 28% of the difference.

As can be seen from table 3.11, for Italy, Spain and Greece marriage stability appears to be a more important mechanism, while taxes is a relatively good predictor of labor supply in Germany, Belgium and Scandinavia. One interesting observation is that by a more careful modeling of the tax systems and introduction of the divorce mechanism we are able to resolve what Rogerson (2007) calls a puzzle, the fact that Scandinavian countries have among the highest taxes but still greater labor supply than a country like Germany. An important feature of the tax system in all Scandinavian countries (except Norway) is separate taxation of married couples. As was pointed out by Guner et al. (2008), this can help explain higher labor supply in these countries. For Denmark and Finland, the average tax level mechanism cannot account for the higher labor supply in these countries compared with Germany, as average tax level is higher in Denmark and about the same in Finland. However, both of these countries have separate taxation of married couples.

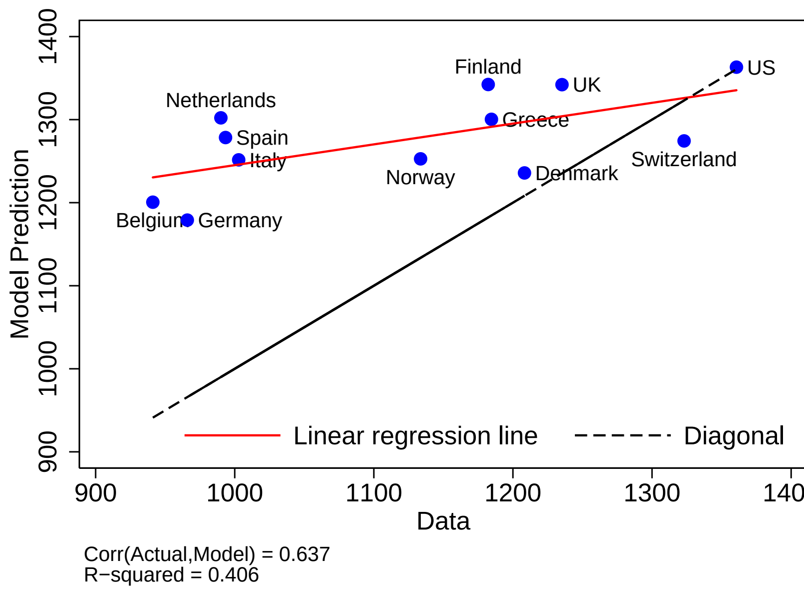


Figure 3.11: The Impact of Taxes and Marriage and Divorce Probabilities on Hours Worked, Both Genders

We conclude that our counterfactual experiments suggest that both the divorce and the tax mechanisms are important for accounting for the differences in labor supply between the US and Europe. The significance of these two mechanisms appear to vary for different European countries. When combined, they on average allow us to account for 28% of the difference.

3.8 Conclusion

In this paper we show that prime aged women is the largest contributor to differences in aggregate labor supply between the US and Europe. We document a negative cross-

Table 3.11: Labor Supply, Taxation and Marriage and Divorce Rates

The table shows the hours worked (predicted from the model and taken from the data) as a percent of hours worked in the US

Country	Divorces	Taxation	Divorces and Taxation	Data
US	100.000	100.000	100.000	100.000
Greece	91.536	104.381	95.566	87.056
Italy	91.718	99.362	91.978	73.701
Spain	94.539	100.871	93.954	73.007
Belgium	95.319	91.562	88.234	69.167
Switzerland	96.840	99.869	93.655	97.234
Germany	96.060	88.782	86.648	70.987
Netherlands	97.334	97.685	95.696	72.762
UK	98.088	99.765	98.634	90.783
Norway	100.208	92.056	92.069	83.313
Denmark	100.585	89.287	90.821	88.802
Finland	101.482	97.282	98.647	86.886

country correlation between tax levels and labor supply and a positive correlation between divorce rates and labor supply across time and place. The latter correlation is, however, driven by a strong correlation between female labor supply and divorce rates.

To quantify the impact of differences in tax schemes and divorce / marriage rates on labor supply, we develop a life-cycle, overlapping-generations model with heterogeneous agents, marriage, and divorce. We calibrate our model to US data and study how labor supply in the US change as we introduce European tax systems, and as we replace the US divorce and marriage rates with their European equivalents. Changing the US probabilities of marriage and divorce to their European counterparts on average accounts for 22% of the difference in hours worked between the US and the 11 European countries. When we also introduce European taxes and redistribute the increase in taxes evenly to all households, we can account for 28% of the difference in hours worked between the US and Europe.

3.9 Appendix

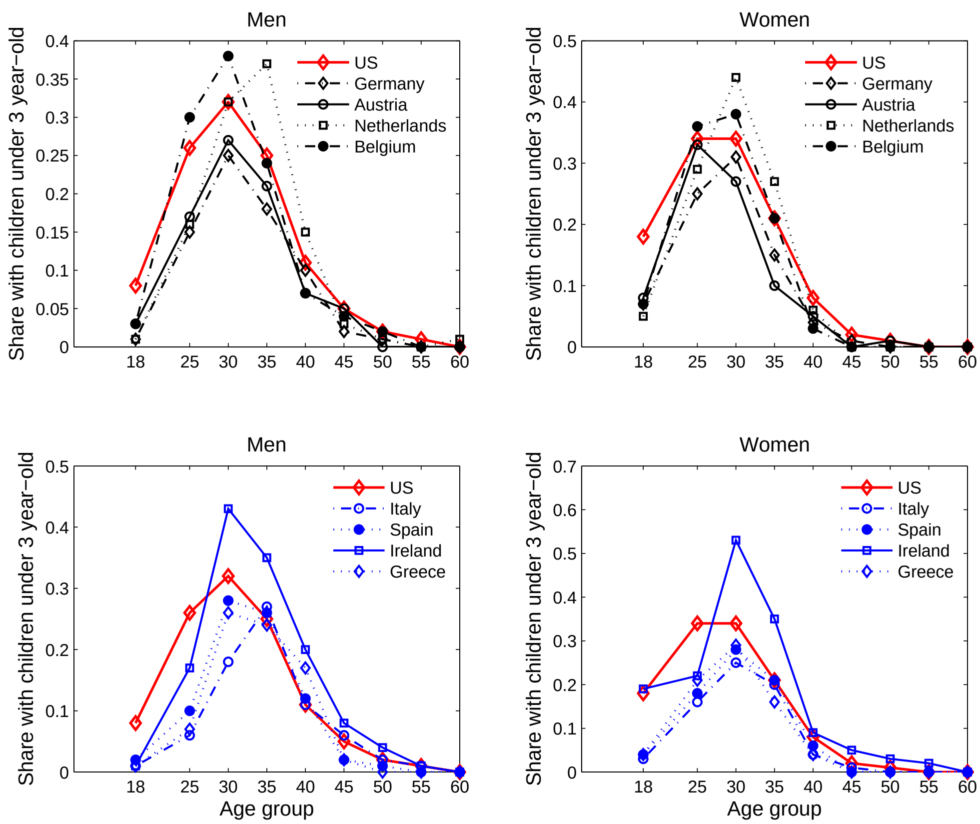


Figure 3.12: Share of persons with children younger than 3 years old, by age group

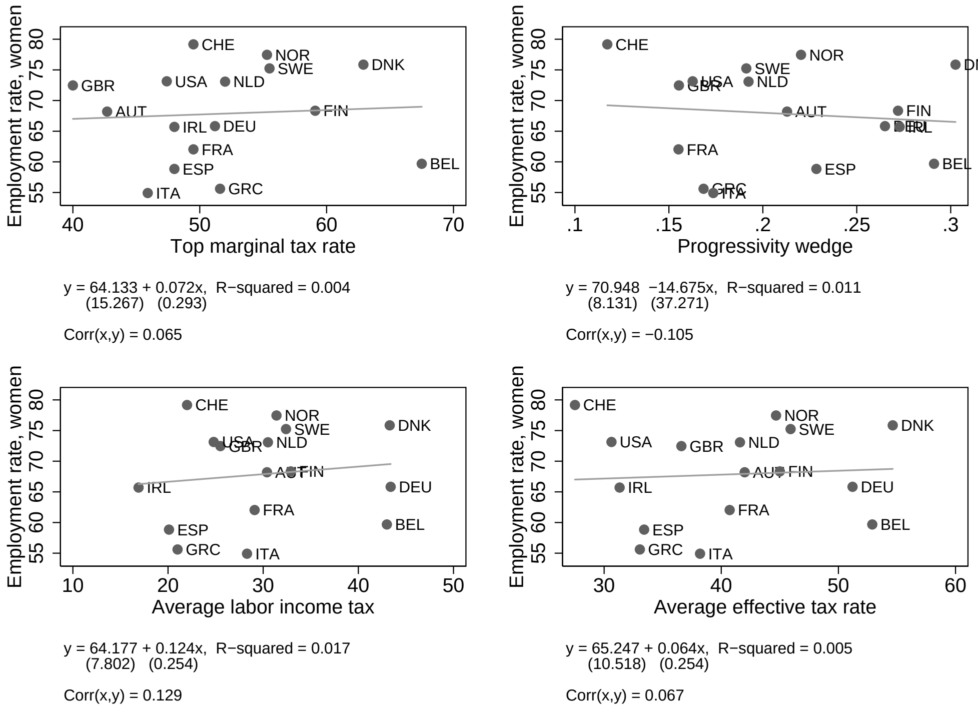
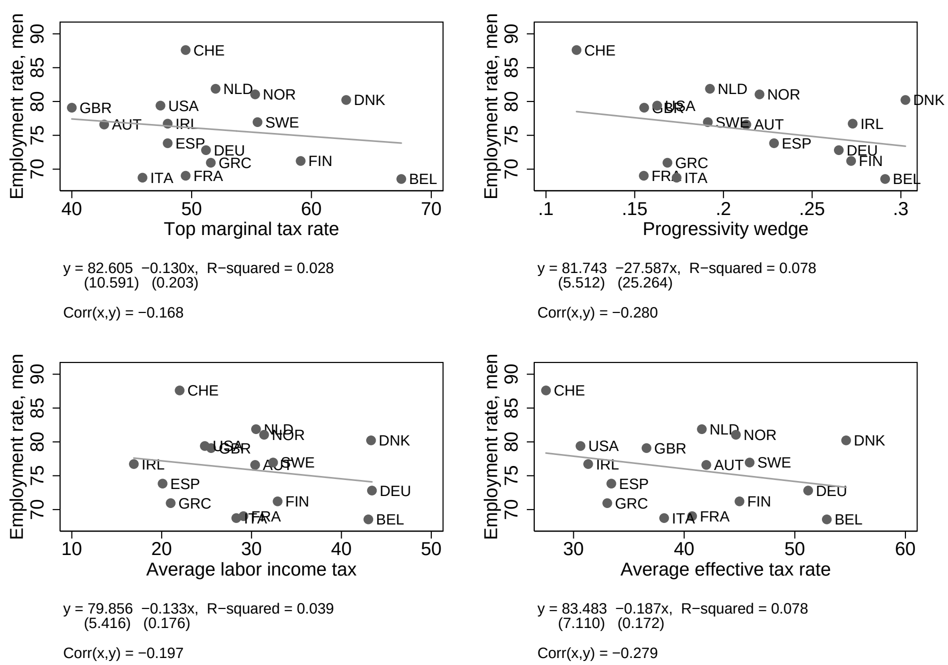
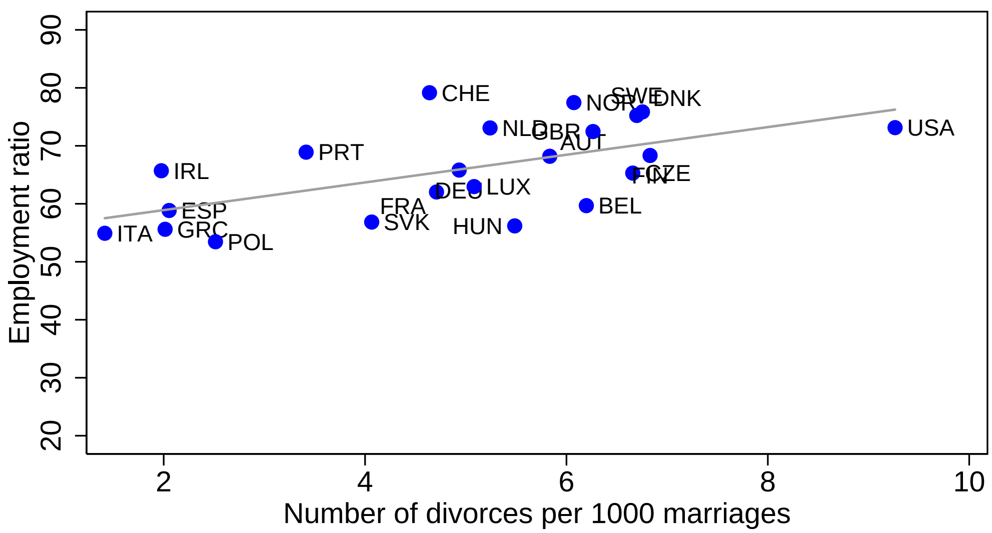


Figure 3.13: Relationship between tax measures and employment ratios for women

Figure 3.14: Relationship between tax measures and employment ratios for men





y = 54.144 + 2.387x, adj. R−squared = 0.330
(3.752) (0.709)

Corr(x,y) = 0.601

Figure 3.15: Relationship between divorce rates and employment ratios for both genders

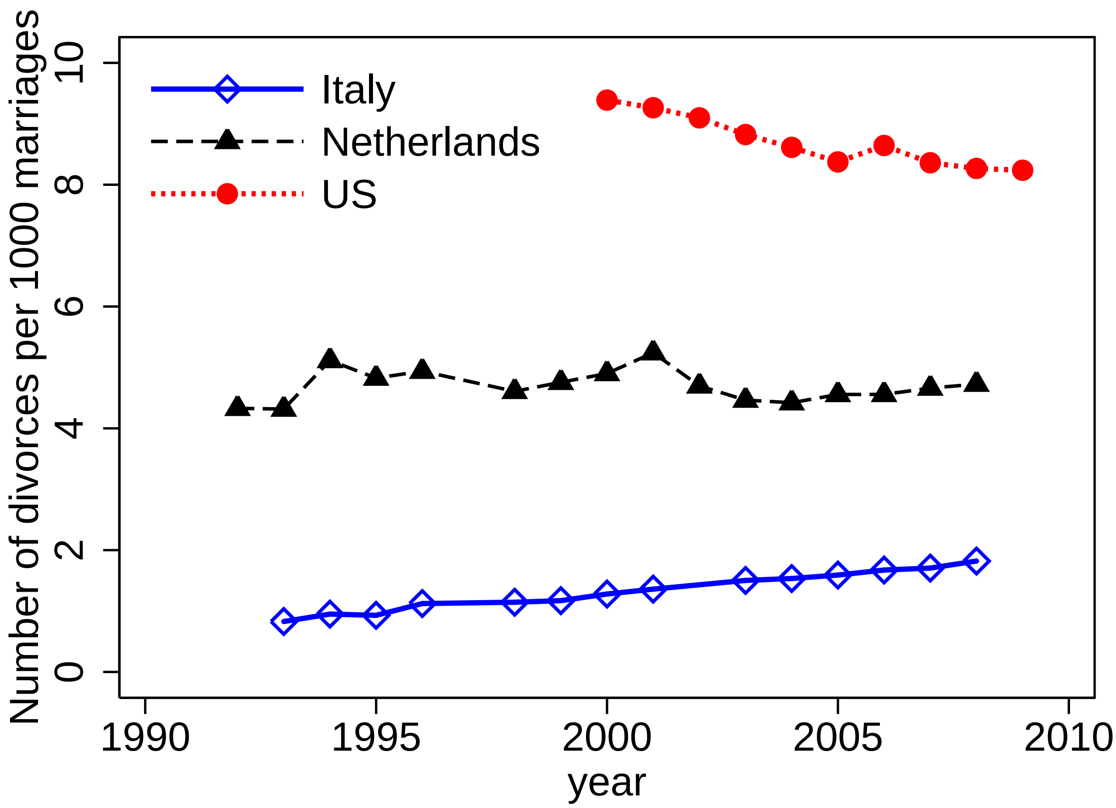


Figure 3.16: Trends in the number of divorces over time

3.9.1 Fitting Tax Functions Based on Data from the OECD

For every country in Figure 3, we fit the below polynomial where an individuals average tax rate is a function of his earnings relative to the average earnings in the economy:

$$\tau(y) = \tau_0 + \tau_1 \left(\frac{y}{AE}\right)^{0.2} + \tau_2 \left(\frac{y}{AE}\right)^{0.4} + \tau_3 \left(\frac{y}{AE}\right)^{0.6} + \tau_4 \left(\frac{y}{AE}\right)^{0.8} \tag{3.12}$$

We use this functional form because it generally gives us a very good fit, R2, and because we get functions that are strictly increasing and well behaved on a relatively wide range of labor income. We use labor income tax data from the OECD Tax-Benefit Calculator[18] and the OECD Tax Database[19]. This data is constructed by the OECD based on tax laws from different countries. The OECD Tax-Benefit Calculator gives the gross- and net-, after taxes and benefits, labor income, by family type in 2001. For single individuals we can get tese data for every percentile of average labor income for a range between 50% and 200% of average labor income. For married couples, one spouse's earnings have to be fixed at either 0%, 67%, 100% or 167% of average labor income, while the other spouse's earnings can take any whole percent value between 50% and 200% of average labor income. For countries that practice joint taxation of married couples, we fit different polynomials for married and single. We use the data for single and married individuals without children. For married individuals, we let the couples be as symmetric as possible. In the US this is inconsequential, since the tax system is completely symmetric, i.e. it does not matter who makes the income. The OECD Tax Database provides the top marginal tax rate in each country and the starting point for this tax rate for single individuals. To get

[18]Available at: www.oecd.org/document/18/0,3343,en_2649_34637_39717906_1_1_1_1,00.html.

[19]Available at: www.oecd.org/document/60/0,3343,en_2649_34533_1942460_1_1_1_1,00&&en-USS_01DBC.html.

the tax at earnings above 200% of average labor income, we use this information. For many countries the top marginal tax rate kicks in before 200% of average labor income but in the US, for instance, the top marginal tax rate starts at about 9 times average earnings. We then assume that the marginal tax rate increases linearly between 2 times average earnings and the point where the top marginal tax rate becomes effective. For countries that practice joint taxation of married couples, we assume that the top marginal tax rate for married starts at twice the level for singles.

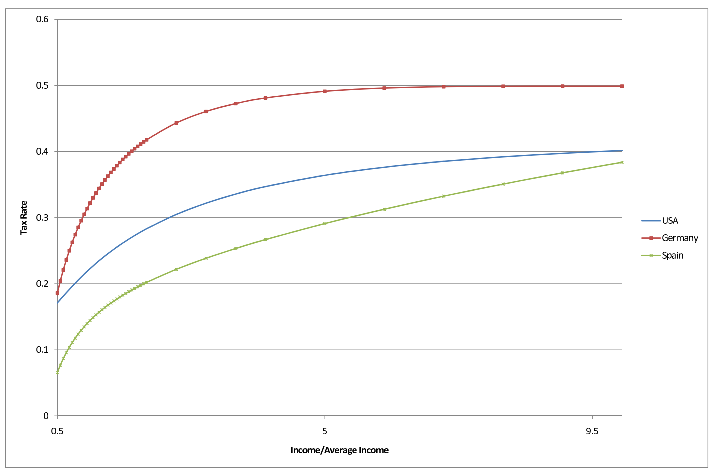
Country	τ_0	τ_1	τ_2	τ_3	τ_4	R^2
France	-0.4677592	2.062677	-2.743411	1.820481	-0.4305004	0.9989
Germany	-0.5409343	-0.9886915	4.474231	-3.421762	0.7909097	0.9962
Ireland	1.612143	-6.871639	9.391285	-4.898055	0.8901651	0.9940
Norway	-5.335858	14.96881	-15.43612	7.362051	-1.335945	0.9981
Portugal	3.907341	-12.23614	13.88106	-6.514196	1.101643	0.9995
Spain	-2.811092	8.034616	-8.401096	4.023208	-0.7058137	0.9959
Switzerland	-16.09581	48.2164	-53.35435	26.20165	-4.78368	0.9950
USA	2.16239	-7.301506	9.221961	-4.736035	0.8718943	0.9949

Table 3.12: Country Tax Functions for Married Couples

Table 3.13: Country Tax Functions for Singles

Country	τ_0	τ_1	τ_2	τ_3	τ_4	R^2
Austria	-5.626168	16.19854	-16.39948	7.397988	-1.250442	0.9937
Belgium	-4.587984	13.62661	-14.19084	6.823648	-1.24974	0.9959
Denmark	0.1422833	-2.357568	5.737164	-3.968169	0.8855884	0.9940
Finland	-1.387284	2.706099	-0.9767094	-0.0860593	0.0717587	0.9987
France	0.7157418	-2.514716	3.64648	-1.88936	0.3320441	0.9980
Germany	-6.582745	19.08046	-19.22463	8.580912	-1.430125	0.9964
Greece	-5.55185	14.76655	-14.7313	6.887032	-1.237959	0.9909
Ireland	-1.75284	2.625375	0.1463597	-1.13193	0.3456357	0.9983
Italy	-1.555522	2.965259	-0.9916236	-0.3076185	0.1599916	0.9992
Netherlands	1.126893	-4.322011	6.331867	-3.487033	0 .6651015	0.9899
Norway	2.335783	-8.6315	11.83152	-6.471281	1.25354	0.9988
Portugal	2.604929	-9.655736	12.78917	-6.821912	1.293703	0.9994
Spain	-2.640157	7.853874	-8.641411	4.527437	-0.9025463	0.9979
Sweden	5.645098	-18.75109	23.36599	-12.24517	2.322895	0.9968
Switzerland	-1.4185	5.181097	-6.488006	3.771889	-0.8035895	0.9985
UK	-0.3775787	0.2900424	1.07663	-0.9579886	0.2236049	0.9953
USA	1.727408	-6.44973	8.994808	-4.999817	0.9875019	0.9969

Figure 3.17: Country Tax Functions (Married)



3.9.2 Computational Details

Computation of Optimal Policies

We put boundaries on the capital space and pick a 16 point grid in $K = [k^m in, k^m ax]$. Capital is the only continuous state variable. Let $J = \{hs, c\}$ be the state space for whether an individual is high school or college educated, $X = \{0, , 44\}$ be the state space for the number of years of labor market experience, and $T = \{20, , 95\}$ be the state space for age. The state space for working age married individuals is then: $T \times J \times J \times X \times X \times K$, for working age single individuals it is: $T \times J \times X \times K$, and for retired individuals, both married and single it is: $T \times K$. We compute the household's optimal policies for each state by iterating backwards. We start from age 95, the last period of life. In that period, the next period's value function is 0, and the optimal policy is to consume as much as possible. Knowing the value function at age 95, we can compute optimal policies and value functions for age 94, and so on. The labor supply decisions are discrete, and so we compare the different options. For each choice of labor, we must solve for the optimal level of next period's capital. We find the optimal choice of capital by "golden search". To interpolate next period's value function outside of the grid, we use cubic splines.

Simulation

We simulate an over lapping generations economy with 100 000 men and 100 000 women in each identical generation. Knowing today's state, the policy functions, and next period's marital status, we can find the next period's state. To determine next period's marital status, we draw a random number, $\nu \in (0, 1)$, for every single individual and every married couple in each time period. We use the age dependent probabilities for divorce and marriage to determine whether a single individual is going to marry or a couple is going to split. We only let the random number drawn by the single men determine if they are going to get married. Then to find them a partner, we sort single men- and women by their random number and find a partner for each man that is going to change status. We also make sure that the right number of men marries someone with the same level of education.

Partial Equilibrium

When we calibrate the model we must have equilibrium in the marriage market, in the sense that single individuals must have rational expectations about their potential partners in the next period. This expectation must be taken with respect to education, experience, and asset holdings, $Q^{jgt}(j_p, x_p^{'}, k_p^{'})$. Given his own education, an individual knows the likelihood of marrying someone whit high school and college education in the next period. We keep track of the distribution of single individuals in each education group with respect to capital and experience at every age. We start out with an educated guess and then solve the model iteratively until we reach a fixed point.

When we perform the policy experiments we must also solve for a fixed point in terms of the average earnings in the economy because the tax functions, the social security payments, and the value of not working are kept as functions of average earnings. Finally when redistributing the increase in tax revenues, we must solve for a fixed point in terms of the lump sum redistribution.

Table 3.14: Annual Hours Worked, by Age Group, LIS 2000

Country	15-20 yr	% of US	21-55 yr	% of US	56-64 yr	% of US
US	363.70	100.0	1600.89	100.0	1077.54	100.0
Germany	310.46	85.4	1154.65	72.1	582.38	54.0
Italy	102.50	28.2	1232.94	77.0	505.38	46.9
Spain	167.36	46.0	1177.30	73.5	644.34	59.8
Ireland	336.59	92.5	1309.16	81.8	782.43	72.6
Austria	571.16	157.0	1325.48	82.8	507.15	47.1
Belgium	90.54	24.9	1132.67	70.8	320.16	29.7
Netherlands	352.51	96.9	1152.01	72.0	446.21	41.4
Greece	173.91	47.8	1422.52	88.9	698.62	64.8

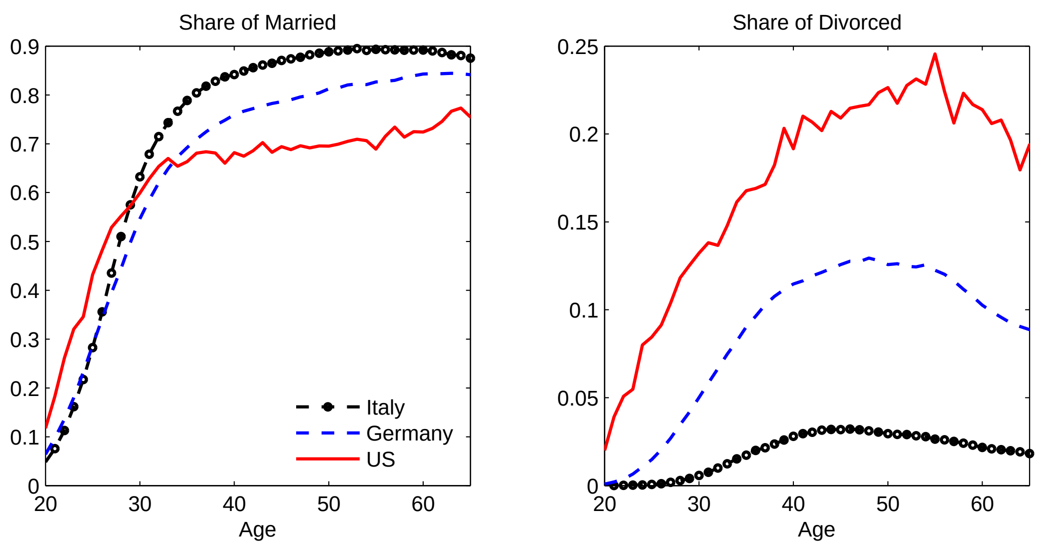


Figure 3.18: Share of Married and Divorced Women at Different Ages

Table 3.15: Annual hours worked, by age group and sex, LIS 2000

Country	Men						Women					
	15-20 yr	% of US	21-55 yr	% of US	56-64 yr	% of US	15-20 yr	% of US	21-55 yr	% of US	56-64 yr	% of US
US	380.80	100.0	1865.56	100.0	1309.24	100.0	345.88	100.0	1349.64	100.0	874.74	100.0
Germany	333.71	87.6	1464.09	78.5	779.80	59.6	287.22	83.0	857.98	63.6	395.93	45.3
Italy	130.80	34.3	1645.44	88.2	782.11	59.7	72.63	21.0	827.07	61.3	239.35	27.4
Spain	243.36	63.9	1587.55	85.1	992.96	75.8	85.96	24.9	768.40	56.9	321.42	36.7
Ireland	432.49	113.6	1761.80	94.4	1274.16	97.3	230.79	66.7	865.56	64.1	283.31	32.4
Austria	696.83	183.0	1649.07	88.4	725.37	55.4	452.59	130.9	1004.45	74.4	296.82	33.9
Belgium	155.56	40.9	1426.76	76.5	498.50	38.1	19.62	5.7	868.73	64.4	156.29	17.9
Netherlands	337.53	88.6	1530.10	82.0	679.06	51.9	366.06	105.8	788.86	58.4	225.45	25.8
Greece	261.82	68.8	1948.03	104.4	1169.27	89.3	101.26	29.3	931.88	69.0	277.77	31.8

Table 3.16: Annual hours worked, with and without children, LIS 2000

Country	Men						Women					
	child 3	% of US	child 6	% of US	no children	% of US	child 3	% of US	child 6	% of US	no children	% of US
US	2096.01	100.0	2093.84	100.0	1502.11	100.0	946.43	100.0	1021.13	100.0	1197.06	100.0
Germany	1604.33	76.5	1585.37	75.7	1170.99	78.0	196.58	20.8	304.35	29.8	786.22	65.7
Italy	2027.87	96.7	1976.34	94.4	1257.59	83.7	757.66	80.1	744.57	72.9	645.82	54.0
Spain	1883.10	89.8	1871.86	89.4	1273.15	84.8	676.93	71.5	642.64	62.9	631.69	52.8
Ireland	2045.85	97.6	2063.94	98.6	1390.88	92.6	680.39	71.9	639.05	62.6	740.95	61.9
Austria	1725.81	82.3	1751.53	83.7	1370.35	91.2	434.21	45.9	543.47	53.2	895.71	74.8
Belgium	1525.43	72.8	1540.27	73.6	1118.88	74.5	852.11	90.0	856.72	83.9	678.38	56.7
Netherlands	1668.32	79.6	1681.76	80.3	1232.26	82.0	583.29	61.6	568.38	55.7	702.31	58.7
Greece	2195.55	104.7	2218.34	105.9	1582.30	105.3	899.60	95.1	883.38	86.5	716.79	59.9

Table 3.17: Contribution of different demographic groups to the difference in average hours worked between the US and Europe

Germany

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.391	0.507	0.171	0.743
21-55:	25.379	10.538	33.299	14.594
56-64:	6.053	1.074	4.465	2.787
Total:	43.941		56.058	

Austria

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.422	-9.176	-0.720	-2.356
21-55:	27.700	3.199	34.182	20.023
56-64:	12.037	0.308	8.663	5.719
Total:	34.488		65.511	

Belgium

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	-0.103	3.025	0.362	4.172
21-55:	26.239	8.854	22.143	16.891
56-64:	7.473	1.789	5.763	3.393
Total:	47.276		52.723	

Netherlands

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.380	0.434	0.030	-0.379
21-55:	23.357	6.533	36.989	14.941
56-64:	6.649	1.690	6.215	3.160
Total:	39.043		60.956	

Table 3.18: Contribution of different demographic groups to the difference in average hours worked between the US and Europe, continued

Greece

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.760	3.320	0.698	7.129
21-55:	-6.865	-3.162	45.841	29.511
56-64:	4.366	0.271	10.115	8.016
Total:	-1.309		101.309	

Ireland

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.590	-1.744	0.648	2.452
21-55:	4.882	7.890	44.842	26.557
56-64:	-0.173	0.511	8.148	5.397
Total:	11.956		88.043	

Spain

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	-0.036	2.149	0.346	3.894
21-55:	12.859	12.912	33.838	21.756
56-64:	3.853	0.699	5.518	2.211
Total:	32.436		67.563	

Italy

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.407	4.092	0.395	4.285
21-55:	11.360	10.105	32.058	19.440
56-64:	6.371	1.376	5.980	4.132
Total:	33.710		66.289	

Table 3.19: Tax-Related Measures by Country (2001)

Country	Max marginal rate	Earnings level where the max marginal rate becomes effective	Consumption tax	Average labor income tax rate paid by the average worker
Austria	42.7%	2.2*AE	20.0	32.0%
Belgium	67.5%	1.2*AE	21.0	42.2%
Denmark	62.9%	1.0*AE	25.0	43.9%
Finland	59.1%	2.1*AE	22.0	32.8%
France	49.5%	1.8*AE	19.6	29.0%
Germany	51.2%	1.5*AE	16.0	42.4%
Greece	51.6%	3.8*AE	18.0	16.5%
Ireland	48.0%	1.1*AE	21.0	23.3%
Italy	45.9%	3.7*AE	20.0	27.0%
Netherlands	52.0%	1.4*AE	19.0	31.5%
Norway	55.3%	2.4*AE	24.0	31.8%
Portugal	46.6%	4.9*AE	17.0	21.3%
Spain	48.0%	4.2*AE	16.0	19.7%
Sweden	55.5%	1.5*AE	25.0	33.8%
Switzerland	49.5%	3.9*AE	7.6	23.8%
UK	40.0%	1.3*AE	17.5	25.5%
USA	47.4%	9.0*AE	8.4	26.0%

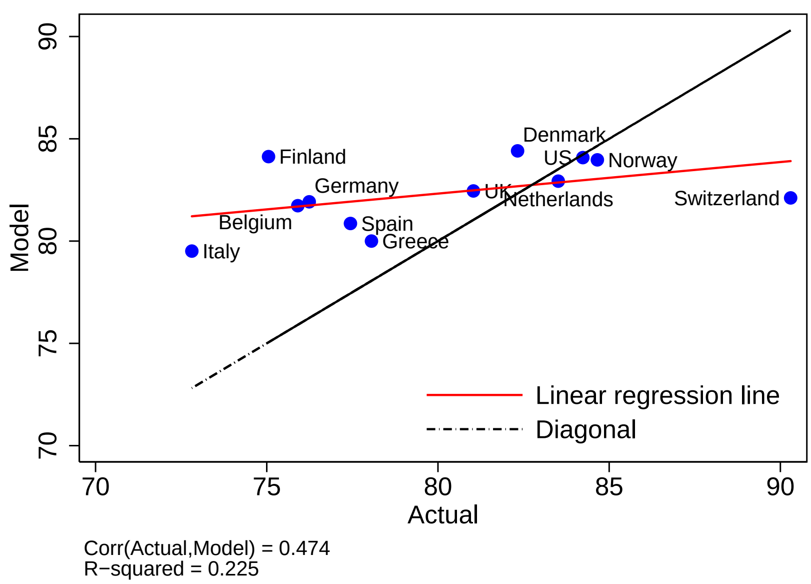


Figure 3.19: The Impact of Marriage and Divorce Probabilities on Employment Rates, Men

Table 3.20: Sample compositions

United States

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.001	0.067	0.003	0.063
21-55:	0.215	0.151	0.234	0.151
56-64:	0.041	0.012	0.039	0.022

Germany

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.000	0.049	0.000	0.049
21-55:	0.195	0.156	0.219	0.146
56-64:	0.070	0.020	0.067	0.029
Total contribution of compositional effects = 2.798%				

Spain

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.000	0.057	0.001	0.053
21-55:	0.224	0.153	0.239	0.139
56-64:	0.056	0.008	0.055	0.015
Total contribution of compositional effects = -4.692%				

Italy

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.000	0.052	0.001	0.049
21-55:	0.219	0.149	0.248	0.125
56-64:	0.068	0.009	0.061	0.019
Total contribution of compositional effects = -2.597%				

Austria

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.001	0.043	0.001	0.045
21-55:	0.208	0.169	0.231	0.149
56-64:	0.063	0.012	0.050	0.028
Total contribution of compositional effects = -1.030%				

Table 3.21: Sample compositions, continued

Belgium

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.000	0.039	0.000	0.036
21-55:	0.242	0.129	0.264	0.150
56-64:	0.054	0.014	0.053	0.020
Total contribution of compositional effects = -8.717%				

Greece

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.000	0.040	0.001	0.048
21-55:	0.227	0.129	0.269	0.112
56-64:	0.078	0.005	0.075	0.017
Total contribution of compositional effects = -7.176%				

Ireland

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.000	0.071	0.000	0.064
21-55:	0.211	0.160	0.223	0.156
56-64:	0.042	0.015	0.043	0.014
Total contribution of compositional effects = -2.310%				

Netherlands

	Men		Women	
Age:	Married	Single	Married	Single
15-20:	0.000	0.038	0.001	0.041
21-55:	0.223	0.160	0.251	0.147
56-64:	0.053	0.014	0.052	0.019
Total contribution of compositional effects = -6.174%				

Table 3.22: The Impact of Taxation (without Redistributiion) on Hours Worked

The table shows the hours worked (predicted from the model and taken from the data) as percent of the hours worked in the US

Country	Aggregate Employment Rates	
	Actual	Model
Countries with joint taxation of married couples:		
US	100.000	100.000
Germany	70.987	98.400
Norway	83.313	101.351
Spain	73.007	101.625
Switzerland	97.234	100.012
Countries with separate taxation of married couples:		
Greece	87.056	107.839
Italy	73.701	109.412
Belgium	69.167	110.231
Netherlands	72.762	109.555
UK	90.783	108.411
Denmark	88.802	111.193
Finland	86.886	110.959

Table 3.23: The Impact of Taxation (without Redistribution) on Employment Rates

Country	Aggregate Employment Rates		Female Employment Rates		Male Employment Rates	
	Actual	Model	Actual	Model	Actual	Model
Countries with joint taxation of married couples:						
US	0.771	0.771	0.699	0.700	0.841	0.841
Germany	0.687	0.757	0.610	0.693	0.762	0.821
Norway	0.805	0.780	0.763	0.710	0.847	0.850
Spain	0.610	0.782	0.445	0.716	0.774	0.857
Switzerland	0.809	0.769	0.715	0.699	0.903	0.840
Countries with separate taxation of married couples:						
Greece	0.610	0.830	0.450	0.779	0.781	0.880
Italy	0.574	0.842	0.421	0.800	0.728	0.883
Belgium	0.662	0.848	0.564	0.809	0.759	0.887
Netherlands	0.737	0.843	0.637	0.800	0.835	0.885
UK	0.737	0.834	0.665	0.783	0.810	0.885
Denmark	0.779	0.855	0.733	0.823	0.823	0.888
Finland	0.718	0.854	0.684	0.816	0.751	0.891

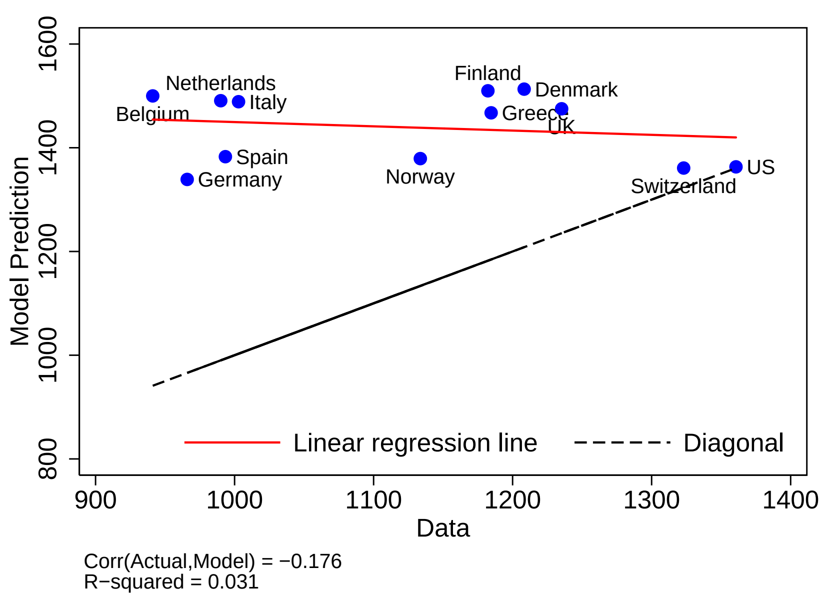


Figure 3.20: The Impact of Taxation without Redistribution on Employment Rates, Both Genders

Table 3.24: The Impact of Taxation (with Redistribution) on Hours Worked

The table shows the hours worked (predicted from the model and taken from the data) as percent of the hours worked in the US

Country	Aggregate Employment Rates	
	Actual	Model
Countries with joint taxation of married couples:		
US	100.00	100.00
Germany	88.782	70.987
Norway	92.056	83.313
Spain	100.871	73.007
Switzerland	99.869	97.234
Countries with separate taxation of married couples:		
Greece	104.381	87.056
Italy	99.362	73.701
Belgium	91.562	69.167
Netherlands	97.685	72.762
UK	99.765	90.783
Denmark	89.287	88.802
Finland	97.282	86.886

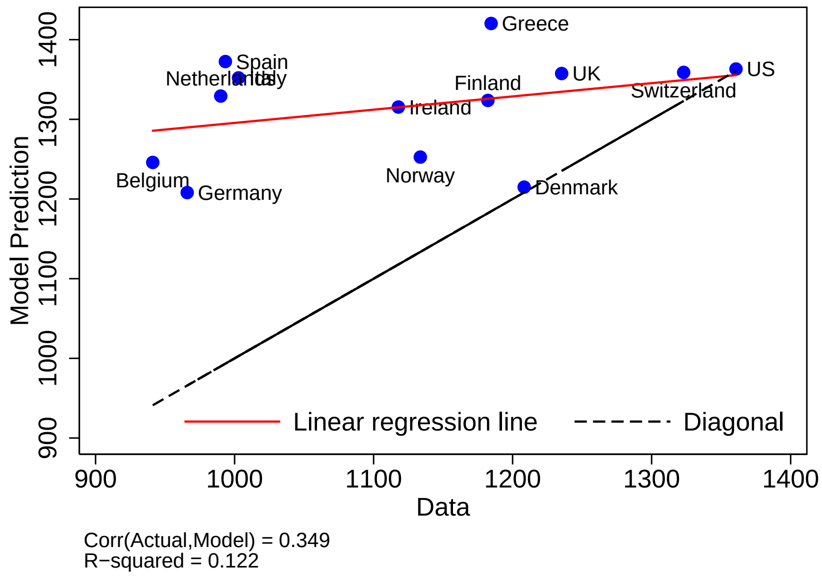


Figure 3.21: The Impact of Taxation with Redistribution on Hours Worked, Both Genders

Table 3.25: Regressing Average Hours Worked on Divorce rate and Tax Measures (including joint versus separate taxation of married couples)

	(I)	(II)	(III)	(IV)
Const	1321.374***	1166.408***	1258.269***	1395.375***
	(217.377)	(142.989)	(86.756)	(115.362)
Divorce rate	27.097*	19.428	41.959***	36.638***
	(14.248)	(14.142)	(12.010)	(11.163)
Top marginal tax rate	−6.413	–	–	–
	(4.497)			
Progressivity wedge	–	−629.507	–	–
		(557.163)		
Average labor income tax	–	–	−1183.122***	–
			(334.931)	
Average effective tax rate	–	–	–	−1160.108***
				(318.644)
Separate Taxation	0.197	-0.002	16.046	32.918
	(59.042)	(61.365)	(45.244)	(45.770)
adjusted R^2	0.085	0.037	0.460	0.476

Standard errors are in parentheses * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Proof of Proposition 3.4.1

Given the choice of the utility function, one can solve for $h_{2,f}$ and $h^s_{2,f}$ in terms of $h_{1,f}$ from equations 3.1 and 3.2, and after plugging these solutions into 3.3, obtain that the dependence of $h_{1,f}$ on π_d is implicitly defined by:

$$\begin{aligned}
G(h_{1,f}, \pi_d) = & \frac{\alpha w_{1,f}}{w_{1,m} + w_{1,f} h_{1,f}} \\
& + (1 - \pi_d) \left(\frac{k_f}{w_{1,f} + k_f h_{1,f} + w_{1,m} + k_m} \left(\alpha + (\alpha - 1) \left(\frac{w_{1,m} + k_m}{w_{1,f} + k_f h_{1,f}} \right) \right) \right) \\
& + \pi_d \left(\frac{\alpha k_f}{w_{1,f} + k_f h_{1,f}} \right) - \frac{1 - \alpha}{1 - h_{1,f}} = 0
\end{aligned}$$

Using the implicit function theorem, one can show that:

$$\begin{aligned}
& \text{sign}\left(\frac{\partial h_{1,f}}{\partial \pi_d} \right) = \text{sign}\left(\frac{\partial G}{\partial \pi_d} \right) \\
& = \text{sign}\left(\frac{\alpha}{w_{1,f} + k_f h_{1,f}} - \frac{1}{w_{1,f} + k_f h_{1,f} + w_{1,m} + k_m} \left(\alpha + (\alpha - 1) \left(\frac{w_{1,m} + k_m}{w_{1,f} + k_f h_{1,f}} \right) \right) \right)
\end{aligned}$$

Since $\frac{w_{1,f}+k_f h_{1,f}+w_{1,m}+k_m}{w_{1,f}+k_f h_{1,f}} > 1 > 1 + \frac{\alpha-1}{\alpha} \left(\frac{w_{1,m}+k_m}{w_{1,f}+k_f h_{1,f}} \right)$, we get $\frac{\partial h_{1,f}}{\partial \pi_d} > 0$.

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