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High School Math Curriculum, Student's Course Selection and Education Outcomes

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Twenty-one states are increasing the requirements for a high school diploma so that all students graduate college-ready. The new graduation requirements include completion of Algebra, Geometry and Algebra II. Before this recent set of reforms, states had graduation requirements related to the number of math credits, irrespective of math course content. To quantify the potential impact of requiring Algebra, Geometry and Algebra II for high school graduation on educational attainment and math knowledge, I develop a dynamic, discrete choice model of high school attendance, math course selection and educational attainment. I estimate the parameters of the model using data from NELS:88/2000 under the old policy and simulate behavior under the new graduation requirement. Model simulations show that educational attainment at age 18 is very responsive to the policy change, but college completion by age 25 is less so. The on-time high school graduation rate falls from 84 to 59 percent, and the proportion of students opting for a GED during the four years of high school increases from 2 to 20 percent. The overall proportion of individuals who earn an advanced degree remains roughly constant, moving from 37 to 36 percent.

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HIGH SCHOOL MATH CURRICULUM, STUDENTS' COURSE SELECTION AND EDUCATION OUTCOMES

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HIGH SCHOOL MATH CURRICULUM, STUDENTS' COURSE SELECTION AND EDUCATION OUTCOMES

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ABSTRACT

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Kenneth I. Wolpin, Professor of Economics

Twenty-one states are increasing the requirements for a high school diploma so that all students graduate college-ready. The new graduation requirements include completion of Algebra, Geometry and Algebra II. Before this recent set of reforms, states had graduation requirements related to the number of math credits, irrespective of math course content. To quantify the potential impact of requiring Algebra, Geometry and Algebra II for high school graduation on educational attainment and math knowledge, I develop a dynamic, discrete choice model of high school attendance, math course selection and educational attainment. I estimate the parameters of the model using data from NELS:88/2000 under the old policy and simulate behavior under the new graduation requirement. Model simulations show that educational attainment at age 18 is very responsive to the policy change, but college completion by age 25 is less so. The on-time high school graduation rate falls from 84 to 59 percent, and the proportion of students opting for a GED during the four years of high school increases from 2 to 20 percent. The overall proportion of individuals who earn an advanced degree remains roughly constant, moving from 37 to 36 percent.

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Chapter 1

Introduction

In the 2006 Program for International Student Assessment (PISA), which tests the mathematics literacy of 15 year olds, American students ranked 34th overall, scoring lower on average than students in all other OECD countries other than Portugal, Italy, Greece, Turkey and Mexico. Within the US, highest level math completed in high school is correlated with college completion Adelman (2006). Twenty-one states are increasing the requirements for a high school diploma so that all students graduate college-ready.[1]

To design the new requirements, the American Diploma Project commissioned a study by Carnevale and Desrochers (2002) that identified jobs paying more than \$ 40,000 per year using Bureau of Labor Statistics and Consumer Price Survey data. Carnevale and Desrochers (2002) then identified the high school courses that distinguished individuals in those jobs at age 25 from others using the National Education Longitudinal Study of 1988. Carnevale and Desrochers (2002) identified completion of Algebra II as the benchmark course for individuals who go on to earn more than \$ 40,000 per year.[2] The American Diploma Project also surveyed faculty at 2-year and 4-year colleges regarding the content knowledge and skills

[1]See Achieve, Inc. (2010a) for details of the state-by-state breakdown of the reforms.
[2]Adelman (2006) also identifies completion of Algebra II as a key correlate of college completion.

necessary for success in the first year of college and worked with states to develop new standards, assessments and high school graduation requirements. The new graduation requirements include completion of Algebra, Geometry and Algebra II.

Before this recent set of reforms, states had graduation requirements related to the number of math credits, irrespective of math course content. Most high schools offered two kinds of math courses: basic math–which is non-academic and has titles like consumer math or general math–and academic math–which is typically taken in sequence of Pre-algebra, Algebra, Geometry, Algebra II, Pre-calculus, Calculus. Under the old policy, students could graduate by taking only basic math. The new policy requires students who would previously have taken two or three years of basic math to take and pass academic math to graduate from high school.

Requiring Algebra II for graduation has the potential to increase or decrease math achievement and educational attainment. As the policy intends, students might graduate with more academic math. However, students may also fail to graduate after attending high school for four years, drop out early in anticipation of failure, or opt for a GED instead of a high school diploma given the more stringent academic math requirements.[3] The effect of the policy will depend on how high school students respond to this change in the incentive structure.

To quantify the impact of requiring Algebra, Geometry and Algebra II for high school graduation on educational attainment and math knowledge, I develop a dynamic, discrete choice model of math course selection, credit accumulation and educational attainment. I estimate the parameters of the model under the old policy using data from the National Educational Longitudinal Study of 1988 (NELS:88/2000) and simulate behavior under the new graduation requirements. This approach allows me to perform an ex-ante policy evaluation. Because the first graduating class required to complete up to Algebra II was the New York state class of 2010, any evaluation of the effect of changing graduation requirements on educational attainment

[3]A GED is a high school equivalent certificate that an individual earns by taking a General Educational Development test or similar exam.

must necessarily be an ex-ante evaluation.

There is a large number of papers that estimate dynamic, discrete choice models. The two most closely related to this work are Eckstein and Wolpin (1999) and Arcidiacono (2004). Eckstein and Wolpin (1999) present a dynamic model of high school attendance that models credit accumulation, treating all units of high school credit equally, and includes working as an option that affects the probability of passing courses. This paper focuses instead on differentiating between kinds of high school courses and does not model the decision to work. Arcidiacono (2004) models the choice of college and college major, allowing for learning about ability. I focus on choices made during high school and model math knowledge as evolving over time based on those choices.

This paper also relates to the reduced form literature investigating the impact of high school course taking on student outcomes. Altonji (1995) uses data from the National Longitudinal Survey of 1972 to estimate the effect of high school course work on earnings and educational attainment, instrumenting for course work with school level averages. These instrumental variables estimates show little to no effect of additional units of high school math, science, English and social studies courses on wages or educational attainment, but find a large positive effect of the academic track in high school on educational attainment. Altonji (1995) speculates that this positive effect is due to the advanced content of the course work in the academic track.

Rose and Betts (2004) use the sophomore cohort of the High School and Beyond survey of 1980 to investigate the impact of different kinds of math credit on earnings. To directly address the speculation that advanced content of course work affects earnings, Rose and Betts (2004) construct their instrument to match that of Altonji (1995). These instrumental variable estimates of the impact of advanced mathematics course-taking find that credits earned in Algebra/Geometry increase earnings by 8-9% (Rose and Betts, 2004). These two papers, taken together, underscore the importance of differentiating between academic and basic math courses when accounting for the payoff to course-taking.

In the model, in each year of high school, students choose whether or not to attend school or get a GED and whether to take academic math, basic math or no math. At the end of the year, math knowledge accumulates, and students pass or fail the courses they chose to take. The payoff to each choice includes an immediate payoff and a future payoff. The future payoff includes the expected value of educational attainment at age 25, which depends on final math knowledge, the highest level academic math completed and on the mode of exit from high school–diploma, GED or no diploma.

I estimate the parameters of the model using data from the National Educational Longitudinal Study of 1988 (NELS:88/2000), which provides rich panel data on high school choices and educational outcomes. Measures of educational attainment at age 25 are available. Standardized tests were administered as part of data collection in 1988, 1990 and 1992. I take the math test scores to be direct measures of math knowledge. In the restricted access version of the data, I observe high school transcripts and use them to construct measures of students' progression through the academic math sequence. I estimate the model on a sub-sample of white males.

Model simulations show that educational attainment at age 18 is very responsive to the policy change, but college completion by age 25 is less so. When Algebra II is required for graduation, the on-time high school graduation rate falls from 84 to 59 percent, and the proportion of students opting for a GED during the four years of high school increases from 2 to 20 percent. The proportion of on-time high school graduates who earn an advanced degree (Associates or higher) increases from 44 to 60 percent. However, the overall proportion of individuals who earn an advanced degree remains roughly constant, moving from 37 to 36 percent. Thus, the new policy is not effective in increasing the proportion of students who complete 2-year or 4-year college degrees.

Given that school districts are held accountable for their graduation rates, a 15 percentage point reduction in the graduation rate is quite large. The policy makers who formulated the new graduation requirements see increasing standards as part of a program to increase the graduation rate:

> Although dropout rates are alarmingly high, particularly in our inner cities, there is no evidence that higher expectations for students increases their chances of dropping out. In fact, the opposite may be true: When students are challenged and supported, they rise to the occasion (Achieve, Inc., 2010b).[4]

To support this claim, Achieve, Inc. (2008) describes the experience of San Jose Unified School District in California (SJUSD) which increased graduation standards in 1998 to require completion of a rigorous college preparatory curriculum, including Algebra, Geometry and Algebra II.[5] In addition to increasing the requirements, SJUSD increased the support available to students, offering Saturday academies, summer school, tutoring, mentoring, after-school programs, summer bridge programs and alternative education programs. Given these supports, the graduation rate remained high and the proportion of students who graduated having completed the rigorous college preparatory curriculum increased from 37 to 66 percent (Achieve, Inc., 2008).[6]

The roll of the safety net appears to be quite important. In the pre-reform data used to estimate the model, 14 percent of high school graduates had not mastered whole number operations by the end of eighth grade, and 32 percent of high school

[4]Achieve, Inc. is the organization that operates the American Diploma Project, which guided the states in developing the new graduation requirements.

[5]The new graduation requirements matched the entrance requirements for the California State University system. These new requirements increased the requirements for English, social studies, math and science. The math requirements, however, were expected to be the binding constraint.

[6]Given that only 66 percent of graduates had completed the graduation requirements, the remainder presumably completed the alternative education programs offered as a safety net.

graduates had mastered whole number but not rational number operations by the end of eighth grade. The new policy specifies that by the end of high school students should "have intuitive understanding of an infinite series [and] know how to sum a finite or infinite geometric series" (Carnevale and Desrochers, 2002, 33). This project quantifies the effect of the policy on the dropout rate holding the rest of the educational environment constant, introducing no additional student support. The estimated 15 percentage point reduction in the graduation rate provides a measure of the challenge facing high schools implementing the new requirements and seeking to provide safety nets to students at risk of dropping out.

The remainder of the paper is organized as follows. Section 2 describes the intuition of the model, presents the full specification of the model and solves the model. Section 3 describes the construction of the data set, including the sample selection and variable creation, and derives the likelihood used to estimate model parameters. Two kinds of results are presented in section 4: a description of model estimates and the results of the counterfactual analysis in which Algebra II is required for graduation. Section 5 concludes.

Chapter 2

Model

The model describes an individual's high school attendance and math course selection in a dynamic, discrete choice framework. Each period of the model $t = 1, \ldots, 4$ corresponds to a year of high school, which consists of grades 9 through 12 and corresponds roughly to ages 14 to 18. At the start of each academic year, individuals choose among attending school, getting a GED and not attending school. If attending, individuals choose whether to take academic math, basic math or no math. In the spring of the academic year, math knowledge is realized based on this choice. At the end of the year, students pass or fail the courses they chose to take.

Choices are based on both an immediate and a future payoff. The future payoff includes the immediate payoffs in later years of high school and an expected payoff to educational attainment at age 25. The payoff to educational attainment at age 25 depends on highest math completed, math knowledge and whether or not students have graduated on-time or earned their GED during the four years of high school.

Individuals enter the model having successfully completed 8th grade having accrued math knowledge. Some students have already completed high school level academic math courses like Algebra. Eighth grade highest math completed and math knowledge are included as initial conditions in the model. Students also vary in their motivation, study habits and level of engagement with school. This individual level heterogeneity is modeled as a discrete, permanent characteristic called type.

All individuals have the option to take an academic math course, but differ in the courses they are eligible to take. This structure excludes the classic notion of tracking, wherein students are either placed in an academic track or a non-academic track and cannot switch.[1] Academic math courses are taken in a fixed sequence.[2] Highest math completed in eighth grade determines which course the student is eligible to take in ninth grade, the first decision period of the model. Advanced students who complete two courses in the sequence before high school can complete the entire math sequence through Calculus in high school taking a single academic math class a year.

The evolution of math knowledge and probability of passing for students who take academic math are defined in the model to capture the following scenario. A student takes an academic math course and, by sitting in class every day, is exposed to math content. This exposure probabilistically increases his math knowledge, which is measured by a test in the spring semester. After the test is given, the teacher passes or fails the student based on his math knowledge and homework completion. In the model, the distribution of math knowledge depends on previous knowledge and on whether the individual takes academic math. The probability that an individual passes the academic math courses he attempts depends on realized current math knowledge and type.

The distribution of math knowledge is not affected by the individuals choice to take basic math or attend school. Furthermore, the probability that the individual passes basic math courses does not depend on realized math knowledge. These assumptions reflect the fact that a student who takes basic math or does not take

[1] In the data, switching between academic and basic math is relatively common.

[2] The most common sequence is Pre-algebra, Algebra, Geometry, Algebra II, Pre-calculus, Calculus. Sequences are set by state or local standards.

math is not systematically exposed to math content at a level that would increase measured math knowledge. Basic math courses rarely cover content beyond rational number operations.

In addition to the immediate payoff of the choices, individuals choosing between taking academic and basic math consider the following incentives. Both academic and basic math credit count towards high school graduation which affects the expected payoff to educational attainment at age 25. Taking academic math also increases educational attainment at age 25 conditional on graduation. For high school graduates, final math knowledge and highest academic math completed affect the expected payoff. Taking academic math tends to increase both of these outcomes. However, some individuals may be more likely to pass basic math than academic math.

Depending on parameter values, the model is capable of generating a range of policy effects on educational attainment at age 25. For reasonable parameter values, however, I expect graduation rates to fall. The new policy does not introduce additional incentives to graduate: expected payoff to educational attainment at age 25 given final math knowledge and highest math completed is assumed to be policy invariant, and this payoff is the incentive motivating individuals to graduate. By requiring Algebra, Geometry and Algebra II for graduation, the policy does increase the cost of graduation. The magnitude of the decrease in the graduation rate will be determined by estimated parameter values.

There are reasonable parameter values for which the college graduation rate would increase. I expect that individuals who graduate from high school with higher math knowledge and higher levels of academic math completed are more likely to earn a 2-year or 4-year degree, and I expect that individuals who graduate under the new policy will have completed more academic math courses and therefore have higher math knowledge. This suggests a positive effect of the policy on 2-year and 4-year college completion for high school graduates. The overall effect of the policy on 2-year and 4-year college completion is determined by the relative magnitude of this positive effect on college completion of high school graduates and the negative effect on the graduation rate.

2.1 Model Specification

In each period, $t = 1, \dots, 4$, an individual makes his choice, d_t, which is a vector of mutually exclusive indicator variables corresponding to the following options: not attend school d_t^N, get a GED d_t^G, attend school without taking math d_t^S, take basic math d_t^B, take two basic math classes $d_t^{B\times 2}$, take academic math d_t^A, take two academic math classes $d_t^{A\times 2}$ and take a basic and an academic math class $d_t^{B\&A}$. The immediate payoff of each choice varies by type and previous choice. The flow payoff to attending school without taking math is normalized to zero: $u^A(\text{type}, d_{t-1}) = 0$. The flow payoff to other choices includes a type-specific payoff to the choice, a switching cost and a cost for taking multiple math courses.

The type-specific payoff is defined for the broad choice categories: not attending school ω_φ^N, getting a GED ω_φ^G, taking basic math ω_φ^B and taking academic math ω_φ^A. An individual pays a switching cost if he does not attend school and attended the previous year ω_{switch}^N, if he takes a basic math course and did not take at least one basic math course the previous year ω_{switch}^B or if he takes an academic math course and did not take at least one academic math course the previous year ω_{switch}^A. When a student takes multiple math courses in a year, he receives the type specific payoff to the kind of courses he takes, pays a switching cost if necessary and pays the cost of taking multiple math courses in a year, parameterized by $\omega_{\mathbf{double}}^B, \omega_{\mathbf{double}}^A, \omega_{\mathbf{double}}^{B\&A}$. See appendix A for a full specification of the flow utility of each choice.

Math knowledge $K_t \in \{0, \ldots, 4\}$, a discrete measure of proficiency, is then realized based on choice and previous knowledge. For all possible choices, math knowledge evolves according to the following ordered logit. The latent can be interpreted as the test score and the cut-points as the scores denoting a particular level of proficiency. Taking academic math increases the value of the latent and hence the probability of higher levels of knowledge. The interpretation of this assumption is that the individual learns math by being in an academic math class. Because basic math classes primarily address content corresponding to the lowest levels of math knowledge, I do not allow taking basic math to affect the latent. The latent is given by

$$K_t^* \quad = \quad \beta_\kappa^K \mathbb{1}\{K_{t-1} = \kappa\} + \beta_{\text{academic}}^K \mathbb{1}\left\{d_t^A + d_t^{A\times 2} + d_t^{B\&A} = 1\right\} + \epsilon_t^K,$$

where ϵ_t^K is independently and identically distributed according to the logistic distribution.[3] The coefficient of the indicator variable representing the lowest knowledge category is set to zero: $\beta_{\kappa=0}^K = 0$. The cut-points dividing categories are denoted by $\alpha_1^K, \ldots, \alpha_4^K$.

In an ordered logit framework, a particular category is observed when the latent falls into the relevant bin defined by the cut-points. In this case, the probability distribution of K_t is given by:

$$\mathbb{P}\{K_t = \kappa\} \quad = \quad \begin{cases} \mathbb{P}\{K_t^* < \alpha_1^K\} & \text{for } \kappa = 0 \\ \mathbb{P}\{\alpha_\kappa^K \leq K_t^* < \alpha_{\kappa+1}^K\} & \text{for } 0 < \kappa < 4 \\ \mathbb{P}\{\alpha_4^K \leq K_t^*\} & \text{for } \kappa = 4 \end{cases} .$$

An ordered logit structure is also used to model credit accumulation.

The model considers three kinds of credit: academic math credit A_t, basic math credit B_t and non-math credit C_t. Highest math completed is the sum of eighth grade highest math completed and units of academic math accumulated: $H_t = H_0 + A_t$.

[3]The logistic distribution has cumulative distribution function $F(z) = \frac{1}{1+\exp z}$. This function is of similar shape to the standard normal with slightly fatter tails.

Academic math credit and basic math credit both count towards high school graduation. Academic math credit directly enters the long-run payoff through highest math completed. Non-math credit summarizes all other high school graduation requirements. Non-math credit is accumulated only if the individual chooses to attend school; math credit is accumulated only if the individual chooses to take that kind of math course. Individuals enter high school having completed no credit $A_0 = B_0 = C_0 = 0$. As it is possible to fail courses, credit does not evolve deterministically. Students may earn up to two units of math credit, by earning two units of academic credit, two units of basic credit or one of each. Students can earn a single unit of non-math credit. Each kind of credit evolves according to an ordered logit specification.

The probability of passing academic math courses attempted is determined by type, current math knowledge, the course attempted and the number of courses attempted. An individual earns a_t units of academic math in year t and has completed a total of $A_t = A_{t-1} + a_t$ units at the end of the year. The realization of a_t is generated by an ordered logit with latent

$$a_t^* = \beta_\varphi^A \mathbb{1}\{\text{type} = \varphi\} + \beta_\kappa^A \mathbb{1}\{K_t = \kappa\} + \beta_m^A \mathbb{1}\{H_{t-1} + 1 = m\} + \epsilon_t^A,$$

where ϵ_t^A is independently and identically distributed according to the logistic distribution. As above, the coefficient of one of each set of indicator variables is set to zero: $\beta_{\varphi=\overline{\varphi}}^A = 0, \beta_{\kappa=0}^A = 0, \beta_{m=1}^A = 0$. This latent is the same whether the individual takes one or two academic math courses. However, the probability of passing differs in these cases. If the individual takes a single academic math course, the probability of earning one unit of credit is given by the probability that the latent is greater than the cut-point α^A. Individuals who attempt two academic math courses may earn zero, one or two units of credit. For these individuals, the probability of passing is determined the cut-points $\alpha_2^{A\times 2}$ and $\alpha_1^{A\times 2}$.[4]

[4] If an individual attempts two academic math courses and passes one, it is assumed that he passes the lower level academic math course.

The probability of passing basic math courses is determined by type and the number of courses attempted.[5] An individual earns b_t units of basic math credit in year t and has completed a total of $B_t = B_{t-1} + b_t$ by the end of the year. The realization of b_t is generated by an ordered logit with latent

$$b_t^* = \beta_\varphi^B \mathbb{1}\{\text{type} = \varphi\} + \epsilon_t^B,$$

where ϵ_t^B is identically and independently distributed according to the logistic distribution. The coefficient of the lowest type category is fixed at zero: $\beta^B_{\varphi=\overline{\varphi}} = 0$. If the individual attempts one basic math course, the cut-point is α^B. If the individual takes two basic math courses, the cut-points are $\alpha_2^{B\times 2}$ and $\alpha_1^{B\times 2}$.

An individual earns c_t units of non-math credit in year t and has completed a total of $C_t = C_{t-1} + c_t$ units at the end of year t. For individuals attending school, non-math credit c_t evolves according to an ordered logit with latent

$$c_t^* = \beta_\varphi^C \mathbb{1}\{\text{type} = \varphi\} + \epsilon_t^C,$$

where ϵ_t^C is identically and independently distributed according to the logistic distribution, and cut-point α^C. The coefficient of the lowest type category is fixed at zero: $\beta^C_{\varphi=\overline{\varphi}} = 0$. Type is the only covariate in the specification and captures the individual's motivation, study habits and choices related to non-math courses.

At the end of the last decision period, the individual has math knowledge K_T, accumulated credit A_T, B_T, C_T and highest math completed H_T. If an individual chooses to get a GED in year t, he exits the model with Exit $= GED$ after his math knowledge evolves; his terminal state variables are given by this last value of math knowledge $K_T = K_t$, credit accumulated when he chose to get a GED $A_T = A_{t-1}, B_T = B_{t-1}, C_T = C_{t-1}$, and highest math completed when he made his choice $H_T = H_{t-1}$. At the end of year four, the individual receives a high school

[5] I exclude math knowledge from the probability of passing basic math for the same reason I exclude taking math from the evolution of math knowledge: the content of basic math courses corresponds to the lowest categories of math knowledge.

diploma Exit $= HS$ if he completed four years worth of non-math credit $C_T = 4$ and two years of math credit $A_T + B_T \geq 2$. If the individual did not get a GED or a diploma, he receives no diploma Exit $= ND$.

Terminal state variables and mode of exit determine the final payoff the individual receives at the end of the model, that is, the expected payoff to educational attainment at age 25

$$V_T(\text{type}, K_T, A_T, B_T, C_T, H_T) \;=\; \sum_{Ed} \mathbb{P}\{Ed|\varphi, H_T, K_T, \text{Exit}\} V^{Ed}.$$

Educational attainment at age 25 Ed is measured by the highest degree or degree equivalent received and can take on the following values: less than high school DO, GED or high school equivalent GED, on-time high school diploma HS, 2-year college degree AA or 4-year degree or higher BA. Each of these levels of education is associated with a scalar-valued payoff, which are assumed to be weakly increasing: $V^{DO} = 0 \leq V^{GED} \leq V^{HS} \leq V^{AA} \leq V^{BA}$. The probability of each level of education depends on the mode of exit from high school. For high school graduates, highest level math completed, math knowledge and type also affect these probabilities.

If an individual graduates from high school on time, his educational attainment is at least HS. The probability of educational attainment for high school graduates is an ordered logit with latent

$$Ed^*_{HS} \;=\; \beta^{Ed}_{\varphi} \mathbb{1}\{\text{type} = \varphi\} + \beta^{Ed}_{\kappa} \mathbb{1}\{K_T = \kappa\} + \beta^{Ed}_{m} \mathbb{1}\{H_T = m\} + \epsilon^{Ed}_{HS},$$

where ϵ^{Ed}_{HS} is identically and independently distributed according to the logistic distribution. The following coefficients are fixed at zero: $\beta^{Ed}_{\varphi=\overline{\varphi}} = 0$, $\beta^{Ed}_{\kappa=0} = 0$ and $\beta^{Ed}_{m=1} = 0$. The cut-points are α^{HS}_{AA} and α^{HS}_{BA}.

Individuals who do not graduate from high school in four years can go on to earn a high school equivalent or GED, a 2-year college degree or a 4-year college degree or higher. I do not allow these individuals to earn a standard high school diploma in more than four years.[6] For individuals who did not graduate from high school

[6]This assumption is made for two reasons: individuals who graduate late have educational

or earn a GED (Exit $= ND$), educational attainment is distributed according to a ordered logit with latent

$$Ed^*_{NG} \quad = \quad \beta^{Ed}_{GED} \mathbb{1}\{\text{Exit} = GED\} + \epsilon^{Ed}_{NG}, \tag{2.1}$$

where ϵ^{Ed}_{NG} is identically and independently distributed according to the logistic distribution, and cut-points α^{NG}_{GED}, α^{NG}_{AA} and α^{NG}_{BA}. If Exit $= GED$, the probability of each level of educational attainment is given by a modified version of this ordered logit. To account for the fact that these individuals have already realized a level of attainment beyond the lowest category, the probability of Ed is given by the probability that the latent falls into the relevant bin conditional on the observation that the latent is above the cut-point for a GED.

2.2 Model solution

For $t = 1, \ldots, 4$ each individual chooses d_t to solve

$$V_t(S_{t-1}, d_{t-1}) \quad = \quad \max_{d_t} \sum_j V^j_t(S_{t-1}, d_{t-1}) d^j_t,$$

where $j \in \{N, G, S, B, B \times 2, A, A \times 2, B\&A\}$ indexes options and $V^j_t(S_{t-1}, d_{t-1})$ is the alternative-specific value function. The alternative specific value function is given by

$$\begin{aligned} V^j_t(S_{t-1}, d_{t-1}) \quad &= \quad u^j(\text{type}, d_{t-1}) + \eta^j_t \\ &\quad + \delta \mathbb{E}\{V_{t+1}(S_t, d_t) | S_{t-1}, d^j_t = 1\}, \end{aligned}$$

where $S_t = (\text{type}, H_0, K_t, A_t, B_t, C_t, \eta_t)$ is the vector of state variables and utility shocks. I assume that η^j_t is identically and independently distributed according to the standard extreme value type one distribution, which has cumulative distribution function $F(z) = e^{-e^{-z}}$.

attainment patterns that more closely resemble non-graduates than on-time graduates, and only four years of high school transcript data are reliably available.

I solve the model using backwards induction. First, I evaluate the final payoff $V_T(S_T)$ for all possible values of relevant individual characteristics at the end of the model $S_T = (\text{type}, K_T, H_T, \text{Exit})$. I take the expectation of this payoff conditional on the information available when the individual makes his final choice, which is either in the last period $t = 4$ or when he chooses to get a GED earlier ($d_t^G = 1$). The expected continuation value is given by:

$$\begin{aligned}
\mathbb{E}\{V_{t+1}(S_t, d_t)|S_{t-1}, d_t^j = 1\} &= \sum_{S_T} \mathbb{P}\{S_T|S_{t-1}, d_t^j = 1\} V_T(S_T), \\
\text{where} \quad & t = 4 \text{ and } j \text{ is unrestricted} \\
\text{or} \quad & t < 4 \text{ and } j = G.
\end{aligned}$$

The state transitions are constructed from the processes described above. Because ordered logits produce closed form expressions for probabilities, the expected final payoff has a closed form.

Next, taking $t = 4, 3, 2, 1$ successively, I evaluate the value function given the expected continuation value. Given the assumption on the distribution of the utility shock, the expectation of the value function with respect to the utility shock and the choice probabilities have the following closed form expressions:

$$\begin{aligned}
\mathbb{E}_\eta\{V_t(S_{t-1}, d_{t-1})\} &= \log\left(\sum_j \exp\{\nu^j\}\right) + \gamma \text{ and} \\
\mathbb{P}\{d_t^j = 1\} &= \frac{\exp\{\nu^j\}}{\sum_{\hat{j}} \exp\{\nu^{\hat{j}}\}}, \\
\text{where } \nu^j &= u^j(\text{type}, d_{t-1}) + \delta \mathbb{E}\{V_{t+1}(S_t, d_t)|S_{t-1}, d_t^j = 1\}
\end{aligned}$$

and γ is Euler's constant.[7] To complete the induction step, I take the expectation of the value function at time t given the information available when the individual makes his choice in the previous period:

$$\mathbb{E}\{V_t(S_{t-1}, d_{t-1})|S_{t-2}, d_{t-1}^j = 1\} = \sum_{S_{t-1}} \mathbb{P}\{S_{t-1}|S_{t-2}, d_{t-1}\} \mathbb{E}_\eta\{V_t(S_{t-1}, d_{t-1})\}.$$

[7]For a discussion of the the properties of the extreme value type one distribution, see Bierlaire, Bolduc, and McFadden (2003).

This yields the expected continuation value in the individual's earlier decision.

Chapter 3

Data and Estimation

The National Education Longitudinal Study of 1988 (NELS:88/2000) collected data on a nationally representative sample of 25,851 eighth grade students in the spring of 1988 and followed these students until 2000, when most were 25 or 26 years old. The data collection focused on schooling experiences and outcomes, administering standardized achievement measures and surveying youths, teachers and school administrators in 1988, 1990, 1992 when students were expected to be in eighth, tenth and twelfth grades, respectively.[1] Youths were subsequently interviewed in 1994, two years after their expected date of high school graduation, and in 2000. High school and post-secondary transcript data are available in the restricted access version of the data set.

For the analysis, I restrict attention to white males in states that require two years of math for high school graduation for whom I observe key measures. I choose to focus on a single demographic group in order to limit variation in outcomes likely to be driven by factors outside the model, for example, unequal opportunities across different demographic groups. To maximize sample size, I focus on the largest ethnic group: white, non-Hispanics. Within white non-Hispanics, I focus on males, as their dropout rates are higher than females and they are likely to be more sensitive to the

[1] Parents were also surveyed in 1988 and 1990.

Table 3.1: **Sample Selection** The sample size for each stage of sample selection and the proportion of the sample lost due to each new requirement are presented.

n	**(%) Attrition**	**Data requirement**
25,851	-	Original sample
17,274	33.2	High school transcript data available
5,414	68.7	White males
4,587	15.3	Public school students
4,107	10.5	8th grade math test scores available
4,102	0.1	Met math graduation requirements if graduated
3,806	7.2	Math progression measure available
2,535	33.4	Two years of math required for graduation

See appendix B for details of the construction of math progression and description of cases for which this measure is not available.

change in graduation requirements.

To estimate the model, I must also observe a high school transcript though it need not be complete: indeed, missing years from transcripts are how I identify the decision to not attend school. To avoid the problem of missing initial conditions, I require an eighth grade math test score. As it is public school students who are subject to the state graduation requirements, I further restrict attention to public school students. I restrict attention to individuals in states that require two years of math for graduation to reduce computational time.

Table 3.1 presents the stages of sample selection, describing the effect of each restriction on sample size.

3.1 Variable construction

The primary measures used to estimate the model were constructed from math achievement, high school transcript, survey and post-secondary educational attainment data. To construct measures of high school graduation status Exit_i, I used the graduation measure from the high school transcript and survey reports of GED receipt.[2] Individuals who graduated from high school by September of 1992 are high school graduates $\text{Exit}_i = HS$. Individuals who received a GED by September of 1992 are GED recipients $\text{Exit}_i = GED$ and their choice is noted $d_{it}^G = 1$ for the academic year they received the GED. The remaining students receive no diploma $\text{Exit}_i = ND$.

Measures of educational attainment Ed_i are constructed from post-secondary transcripts, survey reports and high school graduation status Ex_i. Post-secondary transcript is used to determine if the individual received a bachelor's degree or higher $Ed_i = BA$ or an associate's degree $Ed_i = AA$.[3] If an individual completes high school or gets a GED after September 1992 and does not go on to complete college, educational attainment is defined to be a GED or high school equivalent $Ed_i = GED$. Educational attainment is defined by high school graduation status for individuals who do not report any additional schooling after September of 1992.

Measures of credit accumulation $\{A_{it}, B_{it}, C_{it}\}_{t=1}^4$, and highest math completed $\{H_{it}\}_{t=0}^4$ and choice $\{d_{it}\}_{t=1}^4$ were constructed from high school transcript data given the measure of Exit_i defined above. Appendix B describes how academic math course progression A_{it}, H_i, basic math credit accumulation B_{it} and the choices to take academic and basic math courses d_{it}^B, $d_{it}^{B\times 2}$, d_{it}^A, $d_{it}^{A\times 2}$, $d_{it}^{B\&A}$ are constructed. To define C_{it}, individuals who graduated were assumed to accumulate one unit of non-math credit each year. For individuals who did not graduate, the number of courses passed in a year was used to determine non-math credit accumulation. Individuals attended without taking math d_{it}^S if they attempted more than 1.25 Carnegie credits, each of which corresponds to a year long course.

To measure math knowledge K_{it}, I use criterion-referenced proficiency scores provided by NELS:88/2000.[4] The test scores measure proficiency at each of the

[2] In the very few cases where the high school transcript outcome was missing and the transcript was observed, I used survey reports of graduation.

[3] To avoid biasing the measure downwards, Ed_i is defined as missing when the individual reports receiving a post-secondary degree and post-secondary transcripts were not received by NELS.

[4] Psychometric properties of the test and information on construction of the proficiency scores

following levels:

> Math Level 1: Simple arithmetical operations on whole numbers: essentially single step operations which rely on rote memory.
> Math Level 2: Simple operations with decimals, fractions, powers and roots.
> Math Level 3: Simple problem solving, requiring the understanding of low level mathematical concepts.
> Math Level 4: Understanding of intermediate level mathematical concepts and/or having the ability to formulate multi-step solutions to word problems.
> Math Level 5: Proficiency in solving complex multi-step word problems and/or the ability to demonstrate knowledge of mathematics material found in advanced mathematics courses =(Rock et al., 1995, 61-62).

A test score of zero indicates that the individual is not proficient at any of the following levels, and a test score of three indicates that the individual is proficient at levels one through three but not at levels four or five. Test items measuring proficiency at level five were only administered in the final year. To insure that math knowledge is measured consistently throughout high school, I group levels four and five together, so that $K_{it} \in \{0, \ldots, 4\}$ for $t = 1, \ldots, 4$.[5] In this measure, the top category is interpreted as proficiency at level four or five. Tests were administered in 1988, 1990 and 1992, which corresponds to $t = 0, 2, 4$ in the model. In section 3.2, I describe how I account for missing years of data in detail.

Years of math required for graduation are taken from the state graduation requirements for 1992 (IES National Center for Education Statistics, 1995). Connecticut, Iowa, Massachusetts and Nebraska did not have state level mathematics graduation

may be found in Rock, Pollack, and Quinn (1995).

[5]The eighth grade measure only allows for proficiency up to level three: $K_{i0} \in \{0, \ldots, 3\}$. Because the sample is selected so that there are no missing observations of K_{i0}, this does not present a problem.

requirements, requiring local school boards to set their own standards. For students in these states, I looked at school administrators' reports of graduation requirements and the courses completed by high school graduations to impute graduation requirements.

3.2 Likelihood

In the empirical model specification, individuals are indexed by i. Given initial conditions $\text{type}_i, K_{i0}, H_{i0}$, the probability of observing a sequence of choices d_{it}, state variables $S_{it} = (H_{i0}, K_{it}, A_{it}, B_{it}, C_{it})$, final state $S_{iT} = (H_{iT}, K_{iT}, \text{Exit})$ and final educational attainment Ed_i is given by

$$\left[\prod_{t=1}^{4} \mathbb{P}\{d_{it}|\varphi, S_{i,t-1}, d_{i,t-1}\}\mathbb{P}\{S_{it}|\varphi, S_{i,t-1}, d_{it}\}\right]\mathbb{P}\{Ed_i|\varphi, S_{iT}\}. \tag{3.1}$$

If individual level heterogeneity type were observed and no data were missing, the expression above would be the individual likelihood.

However, individual level heterogeneity is unobserved, capturing individual characteristics like motivation and study skills. These characteristics are assumed to co-vary with 8th grade math knowledge and 8th grade highest math according to a multinomial logit:

$$\begin{aligned} \mathbb{P}\{\text{type} = \varphi\} &= \frac{\exp x^{\varphi}}{\sum_{\hat{\varphi}} \exp x^{\hat{\varphi}}}, \\ \text{where } x^{\overline{\varphi}} &= 0 \\ \text{and } x^{\varphi} &= \beta^{\varphi}_{\kappa}\mathbb{1}\{K_{i0} = \kappa\} + \beta^{\varphi}_{m}\mathbb{1}\{H_{i0} = m\} + \alpha^{\varphi}, \text{ otherwise.} \end{aligned}$$

The parameter of the lowest knowledge category is set to zero $\beta^{\varphi}_{\kappa=0} = 0$. The parameters of advanced levels of 8th grade highest math completed are set to zero $\beta^{\varphi}_{m\geq 2} = 0$. The dependence of type on eighth grade math knowledge and highest math completed is not assumed to represent a causal relationship.

Observations of math knowledge and educational attainment are missing in some cases, but observations of variables constructed from high school transcripts are never missing. Because tests were not administered in 1989 or 1991, K_{i1} and K_{i3} are missing for all individuals. Scores are missing for some individuals years when the tests were administered as well: 19 percent of individuals are missing K_{i2} and 33 percent of individuals are missing K_{i4}. Educational attainment is missing for 42 percent of the sample. Missing observations of knowledge K_{it} and educational attainment Ed_i are assumed to be missing at random conditional on observed data and type.

To calculate the individual likelihood, I take the expectation of expression 3.1 with respect to type and missing observations of Ed_i and K_{it}. Because the expression 3.1 already includes probability statements for Ed_i and K_{it} given observed data and type, taking the expectation with respect to missing observations only requires summing over all possible values of the missing variable. To take the expectation with respect to Ed_i and K_{it}, I introduce κ_t as the hypothetical value of K_{it} and ed as the hypothetical value of Ed_i.

$$
\begin{aligned}
&\mathcal{L}_i\left(\theta|\{d_{it}\}_{t=1}^{4}, \{S_{it}\}_{t=1}^{T}, Ed_i\right)\\
&= \sum_{\varphi} \mathbb{P}\{\varphi|K_{i0}, H_{i0}\} \cdot\\
&\cdot \left(\sum_{\kappa_1} \cdots \sum_{\kappa_T} \left[\prod_{t=1}^{T} \mathbb{1}\{K_{it} = \kappa_t \text{ or } K_{it} \text{ missing}\}\right] \cdot\right.\\
&\cdot \left(\sum_{ed} \mathbb{1}\{Ed_i = ed \text{ or } Ed_i \text{ missing}\} \cdot\right.\\
&\cdot \left.\left.\left[\prod_{t=1}^{4} \mathbb{P}\{d_{it}|\varphi, \tilde{S}_{i,t-1}, d_{i,t-1}\}\mathbb{P}\{\tilde{S}_{it}|\varphi, \tilde{S}_{i,t-1}, d_{it}\}\right] \mathbb{P}\{ed|\varphi, \tilde{S}_{iT}\}\right)\right)
\end{aligned}
$$

where $\tilde{S}_{it}$ and $\tilde{S}_{iT}$ are vectors of state variables with the hypothetical value κ_t substituted for K_{it} and θ is the vector of all parameters. If K_{it} or Ed_i is observed, the indicator functions only include the case where the hypothetical value κ_t or ed are equal to the observed value, ruling out all other hypothetical values.

To estimate θ, I maximize the log-likelihood of the parameters given observed data:

$$\hat{\theta} = \text{argmax}_{\theta} \sum_{i} \log \mathcal{L}_i \left(\theta | \{d_{it}\}_{t=1}^{4}, \{S_{it}\}_{t=1}^{T}, Ed_i \right).$$

I use DUMPOL, a simplex routine based on Nelder-Mead (1965) available in the IMSL library, to maximize the function.

Chapter 4

Results

This section presents the estimation results and the results of the counterfactual policy analysis in which I impose the new graduation requirement.

The estimation results are discussed in sections 4.1 and 4.2 First I illustrate significant patterns in the data and discuss the extent to which estimated model parameters reproduce these patterns. Then I describe individual choices and outcomes by type to provide detail about the mechanisms operating in the model. Estimated values and standard errors are available for each of the 83 model parameters in appendix C.1.

To evaluate the effect of requiring Algebra, Geometry and Algebra II for graduation, I change the model to reflect the new policy and simulate behavior holding all other aspects of the model constant. The results of this analysis are discussed in section 4.3

4.1 Model fit

Final highest math completed H_{iT}, math knowledge K_{iT}, high school graduation status Exit_i and educational attainment at age 25 Ed_i are the primary outcomes of interest. Each of these outcomes is generated by the interaction of the 83 parameters in the model. As there is no specific parameter of interest, I focus on the extent to which simulations generated by the estimated model parameter values are able to reproduce patterns in the data.

Table 4.1 presents the choice proportions observed in the data and those produced in simulations. Note that the taking a single academic math course accounts for 61 percent of all choices. The options to not attend, attend without taking math, take one basic math course or take one academic math courses represent 95 percent of all choices. Simulated data from the estimated parameter values very closely fit the choice patterns in the data.

Table 4.1: **Model Fit: Choice (%)** All choices averaged over all years are presented.

Choice	Data	Sim.
Not attend	4.0	4.3
GED	0.4	0.4
Attend, no math	16.1	15.8
Basic	13.8	13.8
Basic x 2	0.6	0.7
Academic math	61.0	60.8
Academic math x 2	2.5	2.6
Basic & academic	1.7	1.8
Total	100.0	100.0

More than 70 percent of individuals take academic math in the first two years. This proportion falls in the third and fourth years, ending at 38 percent. As expected, the proportion of individuals attending without taking math increases as the proportion of individuals taking academic math decreases. Model simulations capture these dynamic patterns. Table 4.2 presents choice proportions each year for the most common choices.

The proportion of individuals who take basic math decreases each year, moving from 19 percent in the first year to 10 percent in the fourth. This pattern does not appear in the model simulations, which hold the proportion of individuals choosing

Table 4.2: **Model Fit: Choice by Year (%)** Only choices representing 4 percent or more are included. Columns do not sum to 100 percent.

	Year			
Choice	1	2	3	4
Data				
Not attend	1.3	2.0	4.8	7.9
Attend, no math	3.4	5.0	17.7	38.5
Basic	19.3	15.3	10.7	9.7
Academic math	70.8	73.2	61.7	38.0
Simulations				
Not attend	3.4	3.4	4.9	5.4
Attend, no math	6.6	9.8	16.2	30.5
Basic	14.5	13.3	13.5	13.9
Academic math	67.9	71.4	60.2	43.6

basic math roughly constant. Switching costs provide incentive to persist in taking basic math until graduation requirements are completed. However, individuals are generally more likely to pass basic than academic math and therefore have an incentive to switch into basic math courses if they are in danger of not completing the math requirement. In the data, transitions between basic and academic math are common in both directions.

Table 4.3: **Model Fit: Highest math completed (%)** Final values computed after the fourth year of high school or at the end of the last year attended

Highest math completed	Data	Sim.
None	12.8	14.8
Pre-algebra	8.2	9.8
Algebra	17.1	15.3
Geometry	15.6	18.0
Algebra II	25.5	22.5
Pre-calculus	11.3	11.1
Calculus	9.5	8.6
Total	100.0	100.0

Math knowledge and highest math completed co-evolve as individuals choose

Table 4.4: **Model Fit: Math knowledge (%)** Highest level of proficiency computed after the fourth year of high school for those attending that year

Math Knowledge	Data	Sim.
0-None	5.3	5.6
1-Whole Number Operations	16.6	19.5
2-Rational Number Operations	13.1	12.7
3-Simple Problem Solving	23.5	23.5
4-Intermediate or Advanced Problem Solving & Concepts	41.6	38.7
Total	100.0	100.0

to take academic math. Overall, 13 percent of individuals complete no academic math, and 22 percent have not mastered rational number operations, which is the primary content of the first course in the academic math sequence. The proportion of individuals who complete Algebra II or higher is 46 percent, while 42 percent demonstrate mastery of the intermediate or advanced level problem solving and concepts taught in these courses. Table 4.3 presents final highest math completed, and table 4.4 presents final math knowledge. Model simulations slightly under-predict the proportion of individuals who pass Algebra II or higher and the proportion of individuals who demonstrate mastery of algebra II content. Simulations over-predict the number of individuals who take no academic math and those who score in the lowest knowledge category.

Table 4.5: **Model Fit: High school graduation status (%)**

Exit	Data	Sim.
No diploma	13.3	15.4
GED	1.6	1.5
Diploma	85.1	83.1
Total	100.0	100.0

Individual choices and credit accumulation determine high school graduation status. In the data, 85 percent of individuals graduate, 2 percent receive a GED before they are expected to graduation and the remaining 13 percent receive no diploma.

Table 4.5 presents high school graduation status. Model simulations slightly under-predict graduation.

Table 4.6: **Model Fit: Educational attainment at 25 (%)**

	High School Graduation Status					
	HS Grad		**GED**		**No Diploma**	
Attainment at 25	Data	Sim.	Data	Sim.	Data	Sim.
Less than High School	-	-	-	-	51.3	52.6
GED/HS equivalent	-	-	96.0	97.4	47.5	46.2
On-time HS Diploma	55.1	56.4	-	-	-	-
AA degree	8.2	8.3	0.0	2.6	0.6	0.0
BA/BS or higher	36.7	35.3	4.0	0.0	0.6	1.3
Total	100.0	100.0	100.0	100.0	100.0	100.0

Very few individuals who do not receive a standard on-time high school diploma go on to finish either a two-year or 4-year college degree: only 4 percent of GED recipients and 1 percent of those with no diploma. In contrast, 45 percent of on-time high school graduates go on to earn a post-secondary degree. Table 4.6 presents educational attainment at age 25 by high school graduation status.

4.2 Model intuition

To understand the incentives driving the patterns described above, it is important to understand the role of unobserved heterogeneity, called type in the model section. Type affects utility from choices, probability of earning credits attempted and educational attainment of high school graduates. The distribution of type depends on observed initial conditions: eighth grade math knowledge and highest math completed. In estimation, I fixed the number of types at three.

Type one has the lowest levels of initial conditions and accounts for 14 percent of simulations. Only 2 percent of type one individuals complete Algebra II or higher levels of academic math. Of the three types, type one is least likely to graduate with

Table 4.7: **Type Probability** The probability of type given 8th grade highest math completed and math knowledge is presented in percentage terms.

	Highest math completed		
Math knowledge	0	1	2+
Probability type = 1			
0	34.3	16.5	11.3
1	25.9	11.7	7.9
2	19.3	6.4	2.4
3	10.0	1.8	0.4
13.9 % of simulations			
Probability type = 2			
0	65.7	83.5	88.7
1	74.1	88.3	92.1
2	73.1	64.6	37.2
3	55.5	26.6	9.3
61.2 % of simulations			
Probability type = 3			
0	0.0	0.0	0.0
1	0.0	0.0	0.0
2	7.6	29.0	60.3
3	34.5	71.5	90.3
25.0 % of simulations			

Table C.10 presents the distribution of initial conditions in the sample.

87 percent of type one individuals receiving no diploma and only 3 percent graduating. Type one high school graduates have a near zero probability of completing a 2-year or 4-year college degree.

Type two is the most common type, representing 61 percent of simulations. Type two is more academically oriented than type one, but less academic than type three. Only 33 percent of type two individuals complete Algebra II or higher, but 96 percent of type two individuals graduate. Thus this is the type most likely to be affected by requiring Algebra, Geometry and Algebra II for graduation. Type two is less likely to complete a 4-year college degree than type three. This is due to the direct effect of type on educational attainment of high school graduates and the indirect effect operating through highest math completed and math knowledge, both of which are lower on average for type two individuals.

The academic type, type three, accounts for 25 percent of simulations and enters the model with high math knowledge and several academic math courses completed. This type tends to take academic math all four years and graduates 98 percent of the time. Given that 93 percent of this type complete Algebra II or higher levels of academic math, this type is not expected to be affected by the new graduation requirements.

Table 4.7 presents the probability of each type given eighth grade math knowledge and highest math completed. Choices made by each type, averaged over all years, are presented in table 4.8. Table 4.9 describes graduation status by type and table 4.10, educational attainment. Tables presenting final math knowledge, highest math completed and educational attainment by type are available in appendix C.2.

The observed differences in choices across types are generated by the following incentives. Expected payoff to educational attainment varies by type, as does flow utility. The probability of passing courses varies by type and further increases the variation in expected payoff to attempting academic math, basic math and non-math courses.

Table 4.8: **Choice by Type (%)** All choices averaged over all years are presented.

	Type		
Choice	1	2	3
Not attend	27.9	0.1	0.0
GED	2.3	0.1	0.1
Attend, no math	12.8	21.0	6.7
Basic	26.1	16.7	1.7
Basic x 2	1.1	0.8	0.7
Academic math	26.3	57.3	86.9
Academic math x 2	0.4	2.8	3.3
Basic & academic	3.1	1.5	1.3
Total	100.0	100.0	100.0

Table 4.9: **High School Graduation Status by Type** The proportions of individuals who receive a high school diploma, get a GED and receive no diploma at age 18 are presented by type.

	Type		
Exit	1	2	3
No Diploma	87.4	3.4	1.4
GED	9.3	0.2	0.5
Diploma	3.3	96.4	98.1
Total	100.0	100.0	100.0

Table 4.10: **Educational Attainment at 25 by Type (%)**

	Type		
Educational Attainment	1	2	3
Less than High School	45.1	1.8	0.8
GED/HS equivalent	50.3	1.7	1.1
On-time HS Diploma	3.3	66.0	24.4
AA degree	0.5	7.7	9.2
BA/BS or higher	0.9	22.7	64.6
Total	100.0	100.0	100.0

The expected payoff to educational attainment at age 25 is constructed from the utility payoff associated with each possible level of attainment and the probability of each level of attainment. The estimated values of utility for each level of attainment are presented in table 4.11. Table 4.12 presents the probability of each level of attainment and the expected payoff for non-graduates. Type, final math knowledge and highest math completed affect the probability of attainment for high school graduates, but do not affect the distribution of educational attainment for those who get a GED during the four years of high school or exit with no diploma. Table 4.13 presents expected payoff to educational attainment for high school graduates.

Table 4.11: **Payoff to Educational Attainment at 25**

Parametrization			**Value**
V^{ND}	$=$	0	0.00
V^{GED}	$=$	$\exp\{\omega^{GED}\}$	2.15
V^{HS}	$=$	$\exp\{\omega^{GED}\} + \exp\{\omega^{HS}\}$	5.03
V^{AA}	$=$	$\exp\{\omega^{GED}\} + \exp\{\omega^{HS}\} + \exp\{\omega^{AA}\}$	5.03
V^{BA}	$=$	$\exp\{\omega^{GED}\} + \exp\{\omega^{HS}\} + \exp\{\omega^{AA}\} + \exp\{\omega^{BA}\}$	37.57

Table 4.12: **Expected Payoff for Non-Graduates** At the end of high school, the expected long-run payoff is calculated by weighting the payoff for each possible level of educational attainment at age 25 by the probability of that level of attainment and summing. For non-graduates, no covariates affect the probability distribution of educational attainment

	Prob. of Attainment at 25(%)					**Expected Payoff**
HS Exit	BA	AA	HS	GED	ND	$V_T(S_T)$
ND	0.8	0.4	0.0	47.4	51.3	1.35
GED	2.7	1.4	0.0	95.9	0.0	3.15

Type one is less likely to pass academic, basic or non-math credits attempted than the other types. The proportions of individuals who accumulate one unit of academic math credit, basic math credit and non-math credit, given that they attempt to complete a unit of credit, are presented by type in table 4.14. The estimated parameter values determining the probability of passing academic math are available in table C.3. In these estimates, type three is the reference category. The coefficient

Table 4.13: **Expected Payoff for HS Graduates** At the end of high school, the expected long-run payoff is calculated by weighting the payoff for each possible level of educational attainment at age 25 by the probability of that level of attainment and summing. The probability distribution of educational attainment for high school graduates varies by type, final math knowledge and highest math completed

Final State			**Prob. of Attainment at 25(%)**					**Expected Payoff**
Type	$\mathbf{K_T}$	$\mathbf{H_T}$	*BA*	*AA*	*HS*	*GED*	*ND*	$V_T(S_T)$
1	1	0	0.0	0.0	100.0	0.0	0.0	5.03
1	3	3	0.0	0.0	100.0	0.0	0.0	5.03
1	4	5	0.0	0.0	100.0	0.0	0.0	5.03
2	1	0	5.1	2.8	92.1	0.0	0.0	6.69
2	3	3	27.0	10.1	63.0	0.0	0.0	13.81
2	4	5	45.7	11.6	42.7	0.0	0.0	19.91
3	1	0	12.4	6.0	81.6	0.0	0.0	9.06
3	3	3	49.3	11.5	39.3	0.0	0.0	21.07
3	4	5	68.9	9.2	21.9	0.0	0.0	27.45

The levels of H_T and K_T presented represent typical patterns for each type, with the lowest levels typical of type 1 and highest levels of type 3.

of type two is negative and significantly different from zero. The coefficient of type one is smaller, i.e. more negative, than that of type two and the different between the two coefficients is statistically significant. Higher levels of math knowledge increase the probability of passing academic math. For both basic and non-math credits, differences in coefficients between types two and three are not statistically significant and simulated proportions are virtually identical. Tables C.4 and C.5 present estimates for basic and non-math credit accumulation parameters.

Table 4.14: **Credit Accumulation by Type** The proportions of individuals who accumulate one unit of academic math credit, basic math credit and non-math credit, given that they attempt to complete a unit of credit, are presented by type given.

	Type		
Credit	1	2	3
Academic math $a_t = 1$	55.2	81.0	90.1
Basic math $b_t = 1$	63.4	95.5	95.6
Non-math $c_t = 1$	55.5	99.7	99.6
Total	100.0	100.0	100.0

4.3 Counterfactual policy analysis

To evaluate the effect of requiring Algebra, Geometry and Algebra II for graduation, I change the model to reflect the new policy and simulate behavior holding all other aspects of the model constant. In addition to requiring individuals to complete Algebra, Geometry and Algebra II for graduation, I remove the option to take basic math courses. If an individual has not completed the math graduation requirements, I remove the option of attending school without taking math. These changes are reflect the expected response of schools to the new policy.

Counterfactual simulations show that the proportion of individuals who graduate from high school on-time falls from 84 percent to 59 percent. The decrease in the graduation rate corresponds to a 18 percentage point increase in the proportion of individuals receiving a GED and a 5 percentage point increase in the proportion of individuals receiving no diploma at their expected date of high school graduation. Table 4.15 describes high school graduation rates under the old policy and the new.

Table 4.15: **Policy Impact on High School Graduation Status (%)**

	Policy Simulation	
High School Graduation Status	Old	New
No diploma	14.5	20.3
GED	1.5	20.4
Diploma	83.9	59.3
Total	100.0	100.0

The policy increases educational attainment of high school graduates. Under the original graduation requirements, 44 percent of high school graduates completed a 2-year or 4-year college degree by age 25. This proportion increased to 60 percent when Algebra, Geometry and Algebra II are required for graduation. Table 4.16 shows the educational attainment of high school graduates under both policy scenarios.

The overall effect of the policy on college completion at age 25 depends on the relative magnitude of the positive effect on college completion of high school graduates

Table 4.16: **Policy Impact on Educational Attainment at Age 25 of High School Graduates (%)**

	Policy Simulation	
Educational Attainment of High School Graduates	Old	New
On-time HS Diploma	55.9	40.5
AA degree	8.3	10.1
BA/BS or higher	35.7	49.4
Total	100.0	100.0

Table 4.17: **Policy impact on Educational Attainment at Age 25 (%)**

	Policy Simulation	
Educational Attainment	Old	New
Less than High School	7.5	10.3
GED/HS equivalent	8.3	29.2
On-time HS Diploma	46.9	24.0
AA degree	7.1	6.3
BA/BS or higher	30.1	30.1
Total	100.0	100.0

and the negative effect of the policy on high school graduation rate. The proportion of individuals receiving a 2-year or 4-year college degree remains roughly constant, moving from 37 percent to 36 percent. The policy is not effective at increasing college completion.

Moreover, the proportion of individuals who have received an on-time high school diploma and completed no additional schooling decreases from 47 percent to 24 percent, while the proportion of individuals who complete a GED or other high school equivalent increases from 8 percent to 10 percent. Table 4.17 compares the distribution of educational attainment at age 25 under the old policy to that under the new policy.

The policy increases the proportion of individuals who complete Algebra II and increases the proportion of individuals who complete no academic math, but does not shift the overall distribution of math knowledge. Under the old policy 44 percent of the sample completed Algebra II or a higher level course. The new policy increased this proportion to 50 percent. Table 4.18 presents highest level academic math completed for each simulation, and table 4.19 presents final math knowledge by simulation.

Table 4.18: **Policy impact on Highest Level Academic Math Completed (%)**

	Policy Simulation	
Highest level math completed	Old	New
None	14.1	18.4
Pre-algebra	8.7	6.9
Algebra	15.9	4.8
Geometry	17.7	9.5
Algebra II	24.2	34.2
Pre-calculus	11.2	17.3
Calculus	8.2	8.9
Total	100.0	100.0

Table 4.19: **Policy impact on Math Knowledge (%)**

	Policy Simulation	
Math Knowledge	Old	New
0	6.9	7.9
1	19.7	19.0
2	13.4	12.6
3	23.0	21.6
4	37.0	38.9
Total	100.0	100.0

The tables presented above describe the effect of the policy on the distribution of outcomes of interest. They do not, however, describe the effect of the policy on an individual. One of the strengths of a counterfactual analysis using simulations is that it is possible to compare a particular individual's choices and outcomes under one scenario to his choices and outcomes under another scenario.

To simulate an individual's actions, I use observations of eighth grade math knowledge K_0 and highest math completed H_0 and draw type from the estimated distribution. I then draw a sequence of utility shocks η_{it} that determine the choice given flow utility and expected future payoffs, a sequence of error terms $\epsilon_{it}^K, \epsilon_{it}^A, \epsilon_{it}^B, \epsilon_{it}^C$ that determine the realization of math knowledge and credit accumulation, and a final error term ϵ_i^{Ed} that determines educational attainment. The policy change does not affect these random variables–the randomness comes from occurrences in the individual's life. Holding initial conditions, utility shocks and error terms constant and comparing individual outcomes across the two policy simulations allows me to answer the question: does this individual benefit from the policy.

Table 4.20: **Policy Impact on Final Math Knowledge by 8th Grade Math Knowledge (%)**

	8th Grade Math Knowledge				
Math Knowledge	0	1	2	3	Total
Decreased	25.6	16.8	4.9	1.4	11.5
Unchanged	54.1	60.8	80.5	94.2	72.9
Increased	20.3	22.4	14.5	4.4	15.6
Total	100.0	100.0	100.0	100.0	100.0

Though the overall distribution of math knowledge does not change systematically, the policy increased the final math knowledge of 16 percent of individuals and decreased the math knowledge of 12 percent of individuals. Table 4.20 shows individual impacts by eighth grade math knowledge. Among individuals with eighth grade math knowledge in category one or higher, that is among individuals who had mastered at least whole number operations by the end of eighth grade, more individuals benefit from the policy than are negatively affected. However, of those individuals who are in the lowest math knowledge category and have not mastered whole number operations, 26 percent have a lower final math knowledge under the new policy and 20 percent higher.

Requiring Algebra II for graduation increases highest math completed for 32 percent of the sample and decreases it for 12 percent. A larger proportion of individuals has increased highest math completed than has decreased highest math completed

Table 4.21: **Policy Impact on Highest Math Completed by 8th Grade Math Knowledge (%)**

	8th Grade Math Knowledge				
Highest Math Completed	0	1	2	3	Total
Decreased	23.2	18.0	6.5	3.4	12.4
Unchanged	44.9	44.5	57.9	74.6	55.4
Increased	31.9	37.5	35.6	22.0	32.3
Total	100.0	100.0	100.0	100.0	100.0

for all levels of starting math knowledge. As expected individuals who enter high school with math knowledge in category three, which roughly corresponds to mastery of Algebra I content, are the least affected by the policy in terms of highest math completed. Table 4.21 presents the effect of the policy on individual highest math completed by initial math knowledge. Given the constraint in the counterfactual simulations that individuals who attend school must take academic math until they fulfill the graduation requirement, highest math completed will decrease only if individuals choose to not attend school, to get a GED or to stop taking math after completing the requirement.

Table 4.22: **Policy Impact on Years Attended by 8th Grade Math Knowledge (%)**

	8th Grade Math Knowledge				
Years Attended	0	1	2	3	Total
4 fewer	32.6	23.0	5.7	0.9	14.8
3 fewer	5.7	4.9	1.8	0.3	3.1
2 fewer	3.9	2.8	1.3	0.3	2.0
1 fewer	8.5	6.3	3.2	0.8	4.5
Unchanged	48.9	62.6	87.8	97.7	75.3
1 more	0.3	0.3	0.1	0.0	0.2
2 more	0.1	0.0	0.0	0.0	0.0
3 more	0.0	0.0	0.0	0.0	0.0
Total	100.0	100.0	100.0	100.0	100.0

The new graduation requirements reduce the number of years of high school attended for 24 percent of the sample. Indeed, 15 percent of the sample attended four fewer years of high school under the new policy than they would have under the old policy. This difference is primarily driven by individuals who previously attended four years of high school choosing to get a GED instead of attending their first year of high school. Table 4.22 provides details of the effect of the new policy on the years of high school attended.

Table 4.23 compares individual high school graduation status under the old and new graduation requirements. Overall 17 percent of the sample consists of individuals who graduate under the old policy and get a GED under the new. Individuals who graduated under the old policy and receive no diploma under the new account for another 8 percent of the sample. In total, 26 percent of the sample has reduced high school graduation outcomes under the new policy. A few individuals benefit: 3 percent of the sample receive no diploma under the old policy and opt for a GED under the new policy. Individuals with low entering math knowledge are more negatively affected: 47 percent of those who enter with math knowledge in the lowest category have reduced graduation status under the new policy, and 7 have increased graduation status.

Table 4.23: **Policy Impact on High School Graduation Status by 8th Grade Math Knowledge (%)**

HS Graduation Status		**8th Grade Math Knowledge**				
Old Policy	New Policy	0	1	2	3	Total
No Diploma	No Diploma	21.2	14.8	8.6	2.7	11.3
No Diploma	GED	6.3	4.1	1.8	0.5	3.0
No Diploma	Diploma	0.2	0.3	0.2	0.1	0.2
GED	No Diploma	2.3	1.7	0.7	0.2	1.2
GED	GED	0.5	0.3	0.3	0.4	0.4
GED	Diploma	0.0	0.0	0.0	0.0	0.0
Diploma	No Diploma	8.4	10.4	8.1	3.3	7.7
Diploma	GED	36.0	26.9	7.1	1.0	17.1
Diploma	Diploma	25.0	41.5	73.3	91.8	59.1
Total		100.0	100.0	100.0	100.0	100.0

Educational attainment decreases due to the new graduation requirements for 25 percent of the sample and increases for only 7 percent. This pattern holds for all levels of eighth grade math knowledge. Educational attainment decreases for more individuals than it increases due to the policy change. Table 4.24 presents the individual level analysis of the effect of the policy on educational attainment.

Table 4.24: **Policy Impact on Educational Attainment by 8th Grade Math Knowledge (%)**

	8th Grade Math Knowledge				
Attainment	0	1	2	3	Total
Decreased	44.5	37.6	15.6	4.8	25.1
Unchanged	47.1	53.6	76.9	91.0	67.6
Increased	8.4	8.8	7.5	4.2	7.3
Total	100.0	100.0	100.0	100.0	100.0

Chapter 5

Discussion

To quantify the impact of requiring Algebra, Geometry and Algebra II for high school graduation on educational attainment and math knowledge, I develop a dynamic, discrete choice model of math course selection, credit accumulation and educational attainment. I estimate the parameters of the model under the old policy using data from the National Educational Longitudinal Study of 1988 (NELS:88/2000) and simulate behavior under the new graduation requirements.

Model simulations show that educational attainment at age 18 is very responsive to the policy change, but college completion by age 25 is less so. The on-time high school graduation rate falls from 84 to 59 percent, and the proportion of students opting for a GED during the four years of high school increases from 2 to 20 percent. The proportion of on-time high school graduates who earn an advanced degree (Associates or higher) increases from 44 to 60 percent. However, the overall proportion of individuals who earn an advanced degree remains roughly constant, moving from 37 to 36 percent. Thus, the new policy is not effective in increasing the proportion of students who complete 2-year or 4-year college degrees.

Appendix A

Model Specification

The flow utility for each choice varies by type and previous choice. The full-specification of flow utility for each choice is:

$$
\begin{aligned}
u^N(\text{type}, d_{t-1}) &= \omega_\varphi^N \mathbb{1}\{\text{type} = \varphi\} + \omega_{\text{switch}}^N \mathbb{1}\{d_{t-1}^N = 0\}, \\
u^G(\text{type}, d_{t-1}) &= \omega_\varphi^G \mathbb{1}\{\text{type} = \varphi\}, \\
u^S(\text{type}, d_{t-1}) &= 0, \\
u^B(\text{type}, d_{t-1}) &= \omega_\varphi^B \mathbb{1}\{\text{type} = \varphi\} + \omega_{\text{switch}}^B \mathbb{1}\{d_{t-1}^B + d_{t-1}^{B\times 2} + d_{t-1}^{B\&A} = 0\}, \\
u^{B\times 2}(\text{type}, d_{t-1}) &= \omega_\varphi^B \mathbb{1}\{\text{type} = \varphi\} + \omega_{\text{switch}}^B \mathbb{1}\{d_{t-1}^B + d_{t-1}^{B\times 2} + d_{t-1}^{B\&A} = 0\} + \omega_{\text{double}}^B, \\
u^A(\text{type}, d_{t-1}) &= \omega_\varphi^A \mathbb{1}\{\text{type} = \varphi\} + \omega_{\text{switch}}^A \mathbb{1}\{d_{t-1}^A + d_{t-1}^{A\times 2} + d_{t-1}^{B\&A} = 0\}, \\
u^{A\times 2}(\text{type}, d_{t-1}) &= \omega_\varphi^A \mathbb{1}\{\text{type} = \varphi\} + \omega_{\text{switch}}^A \mathbb{1}\{d_{t-1}^A + d_{t-1}^{A\times 2} + d_{t-1}^{B\&A} = 0\} + \omega_{\text{double}}^A \\
& \text{and} \\
u^{B\&A}(\text{type}, d_{t-1}) &= \omega_\varphi^B \mathbb{1}\{\text{type} = \varphi\} + \omega_{\text{switch}}^B \mathbb{1}\{d_{t-1}^B + d_{t-1}^{B\times 2} + d_{t-1}^{B\&A} = 0\} \\
&+ \omega_\varphi^A \mathbb{1}\{\text{type} = \varphi\} + \omega_{\text{switch}}^A \mathbb{1}\{d_{t-1}^A + d_{t-1}^{A\times 2} + d_{t-1}^{B\&A} = 0\} + \omega_{\text{double}}^{B\&A}.
\end{aligned}
$$

Appendix B

Data

I constructed the academic math course progression A_{it}, H_i, basic math credit accumulation B_{it} and the choices to take academic and basic math courses d_{it}^{B}, $d_{it}^{B\times 2}$, d_{it}^{A}, $d_{it}^{A\times 2}$, $d_{it}^{B\&A}$ through the following steps.

1. I identified academic math classes as Pre-Algebra, Algebra, Geometry, Algebra II-Trigonometry, Pre-Calculus and Calculus courses and categorized all other math classes as basic math.[1]

2. For each of these categories, I defined units attempted as the number of year-long courses attempted. If the course grade was listed as withdrew, incomplete, non-graded, blank, or missing, that course was not considered to be attempted.

3. For each of these categories, units completed was given by the total number of Carnegie credits–a standard measure of year long course credits–earned. Generally, the transcript reported credits completed if the student earned a grade of D or better.

4. If in one year an individual attempted more than two units of basic math and no academic math, I assumed he only attempted two units. All other

[1]For this process, I followed the National Assessment of Education Progress (NAEP) categories available in the NELS:88/2000 electronic code book entry for variable F2RMAT_C.

individuals who attempted more than two math courses were dropped from the sample.

5. Based on units attempted, I defined the choices to take academic and basic math courses: $d_{it}^{B}, d_{it}^{B\times 2}, d_{it}^{A}, d_{it}^{A\times 2}, d_{it}^{B\&A}$.

6. Based on units completed, I defined academic and basic math credit accumulation A_{it}, B_{it}.

7. I constructed 8th grade highest math completed H_0 based on A_t and the sequence of math courses attempted in the following manner.

   (a) Individuals who never attempted academic math are assumed to have completed no academic math before high school $H_0 = 0$.

   (b) Because schools vary in the order of math courses they offer, I defined highest math completed H_t for each of the following progressions:

      i. 1 Pre-Algebra, 2 Algebra, 3 Geometry, 4 Algebra II-Trigonometry, 5 Pre-Calculus, 6 Calculus

      ii. 1 Pre-Algebra, 2 Algebra, 3 Algebra II-Trigonometry, 4 Geometry, 5 Pre-Calculus, 6 Calculus

      iii. 1 Algebra 1/2, 2 Algebra 2/2, 3 Geometry, 4 Algebra II-Trigonometry, 5 Pre-Calculus, 6 Calculus

      iv. 2 Pre-Algebra, 3 Algebra, 4 Geometry, 5 Algebra II-Trigonometry, 6 Calculus

      v. 2 Pre-Algebra, 3 Algebra, 4 Geometry, 5 Pre-Calculus, 6 Calculus

   (c) One of these progressions is consistent with an individual's course sequence if H_0, calculated by $H_0 = H_t - A_t$ is constant over t.

   (d) I assigned each individual to the most common progression consistent with their data. Progression 1 was consistent with the largest number of individual's course patterns, followed by 3, 2, 4 and then 5.

(e) I corrected for the following sources of error:

   i. If an individual moved on to the following class without passing the previous one, I assumed they passed the earlier course;

   ii. If an individual retook a course they had previously passed, I assumed they had failed it the first time;

   iii. If an individual never passed an academic math course, I assumed they kept retaking the first course attempted.

(f) For individuals who still did not have a valid progression, I constructed H_t by counting backwards from Calculus or Pre-Calculus if the individual attempted one of these courses.

8. Individuals who remained without a constructed value of H_0 were dropped from the analysis.

Appendix C

Results

C.1 Estimates

Table C.1: **Estimates: Type Probability** The type distribution follows a multinomial logit with the following parameter values. The coefficients of $K_0 = 0, H_0 \geq 2$ are fixed at 0.

Estimate	Std. Err.	Param.	Variable
Type $\varphi = 1$			
-2.138185	15.741139	$\beta_{\kappa=1}^{\varphi=1}$	8th grade math knowledge $K_0 = 1$
-18.275000	1.525571	$\beta_{\kappa=2}^{\varphi=1}$	8th grade math knowledge $K_0 = 2$
-20.448023	1.487896	$\beta_{\kappa=3}^{\varphi=1}$	8th grade math knowledge $K_0 = 2$
4.151427	0.685511	$\beta_{m=0}^{\varphi=1}$	8th grade highest math completed $H_0 = 0$
1.711821	0.606727	$\beta_{m=1}^{\varphi=1}$	8th grade highest math completed $H_0 = 1$
15.058979	1.487447	$\alpha^{\varphi=1}$	Constant
Type $\varphi = 2$			
-1.737584	15.745478	$\beta_{\kappa=1}^{\varphi=2}$	8th grade math knowledge $K_0 = 1$
-17.592985	1.511491	$\beta_{\kappa=2}^{\varphi=2}$	8th grade math knowledge $K_0 = 2$
-19.382351	1.464144	$\beta_{\kappa=3}^{\varphi=2}$	8th grade math knowledge $K_0 = 2$
2.757808	0.518854	$\beta_{m=0}^{\varphi=2}$	8th grade highest math completed $H_0 = 0$
1.290417	0.383797	$\beta_{m=1}^{\varphi=2}$	8th grade highest math completed $H_0 = 1$
17.104284	1.477662	$\alpha^{\varphi=2}$	Constant

Table C.2: **Estimates: Knowledge Transition** The distribution of knowledge follows an ordered logit with the following parameter values. The coefficient of $K_{t-1} = 0$ is fixed at 0.

Estimate	**Std. Err.**	**Param.**	**Variable**
Latent K_t^*			
1.858338	0.141029	$\beta^K_{\kappa=1}$	Previous math knowledge $K_{t-1} = 1$
4.092834	0.156567	$\beta^K_{\kappa=2}$	Previous math knowledge $K_{t-1} = 2$
7.057578	0.169508	$\beta^K_{\kappa=3}$	Previous math knowledge $K_{t-1} = 3$
9.331041	0.192744	$\beta^K_{\kappa=4}$	Previous math knowledge $K_{t-1} = 6$
1.180017	0.068022	$\beta^K_{\text{academic}}$	Take academic math $d_t^A + d_t^{A\times 2} + d_t^{B\&A} = 1$
Cut-points			
0.289363	0.094316	α^K_1	Division between $K_t = 0$ and $K_t = 1$
3.248968	0.134669	α^K_2	Division between $K_t = 1$ and $K_t = 2$
5.244428	0.143544	α^K_3	Division between $K_t = 2$ and $K_t = 3$
8.229996	0.163633	α^K_4	Division between $K_t = 3$ and $K_t = 4$

Table C.3: **Estimates: Academic Math Credit Accumulation** The distribution of units of academic math credit earned follows an ordered logit with the following parameter values. The coefficients of type $\varphi = 3, K_t = 0, H_{t-1} + 1 = 1$ are fixed at 0.

Estimate	**Std. Err.**	**Param.**	**Variable**
Latent a_t^*			
-1.928465	0.186635	$\beta^A_{\varphi=1}$	Type $\varphi = 1$
-0.519613	0.150679	$\beta^A_{\varphi=2}$	Type $\varphi = 2$
-0.118118	0.256765	$\beta^A_{\kappa=1}$	Current math knowledge $K_t = 1$
0.415210	0.245061	$\beta^A_{\kappa=2}$	Current math knowledge $K_t = 2$
1.155606	0.247390	$\beta^A_{\kappa=3}$	Current math knowledge $K_t = 3$
1.315831	0.253529	$\beta^A_{\kappa=4}$	Current math knowledge $K_t = 4$
-0.481956	0.121138	$\beta^A_{m=2}$	Math course attempted $H_{t-1} + 1 = 2$
-0.491711	0.141511	$\beta^A_{m=3}$	Math course attempted $H_{t-1} + 1 = 3$
-1.520246	0.148683	$\beta^A_{m=4}$	Math course attempted $H_{t-1} + 1 = 4$
-0.846905	0.204437	$\beta^A_{m=5}$	Math course attempted $H_{t-1} + 1 = 5$
-0.032490	0.327432	$\beta^A_{m=6}$	Math course attempted $H_{t-1} + 1 = 6$
Cut-points for one academic math course $d_t^A + d_t^{B\&A} = 1$			
-2.043576	0.280105	α^A	Division between $a_t = 0$ and $a_t = 1$
Cut-points for two academic math courses $d_t^{A\times 2} = 1$			
-3.166758	0.461685	$\alpha^{A\times 2}_1$	Division between $a_t = 0$ and $a_t = 1$
-0.417583	0.296626	$\alpha^{A\times 2}_2$	Division between $a_t = 1$ and $a_t = 2$

Table C.4: **Estimates: Basic Math Credit Accumulation** The distribution of units of basic math credit earned follows an ordered logit with the following parameter values. The coefficient of type $\varphi = 3$ is fixed at 0.

Estimate	**Std. Err.**	**Param.**	**Variable**
Latent b_t^*			
-2.445976	0.641391	$\beta^B_{\varphi=1}$	Type $\varphi = 1$
0.071092	0.664492	$\beta^B_{\varphi=2}$	Type $\varphi = 2$
Cut-points for one basic math course $d_t^B + d_t^{B\&A} = 1$			
-3.026421	0.631845	α^B	Division between $b_t = 0$ and $b_t = 1$
Cut-points for two basic math courses $d_t^{B\times 2} = 1$			
-4.849891	0.989488	$\alpha_1^{B\times 2}$	Division between $b_t = 0$ and $b_t = 1$
-0.642550	0.688503	$\alpha_2^{B\times 2}$	Division between $b_t = 1$ and $b_t = 2$

Table C.5: **Estimates: Non-math Credit Accumulation** The distribution of units of non-math credit earned follows an ordered logit with the following parameter values. The coefficient of type $\varphi = 3$ is fixed at 0.

Estimate	**Std. Err.**	**Param.**	**Variable**
Latent c_t^*			
-5.238035	0.383815	$\beta^C_{\varphi=1}$	Type $\varphi = 1$
0.271570	0.523505	$\beta^C_{\varphi=2}$	Type $\varphi = 2$
Cut-point			
-5.466620	0.369108	α^C	Division between $c_t = 0$ and $c_t = 1$

Table C.6: **Estimates: Educational Attainment at Age 25 of High School Graduates** The distribution of educational attainment follows an ordered logit with the following parameter values for high school graduates. The coefficients of type $\varphi = 3, K_T = 0, H_T = 0$ are fixed at 0.

Estimate	Std. Err.	Param.	Variable
Latent Ed^*_{HS}			
-1303.422323	-	$\beta^{Ed}_{\varphi=1}$	Type $\varphi = 1$
-0.985307	0.191503	$\beta^{Ed}_{\varphi=2}$	Type $\varphi = 2$
2.003486	0.701986	$\beta^{Ed}_{\kappa=1}$	Final math knowledge $K_T = 1$
0.923275	0.695771	$\beta^{Ed}_{\kappa=2}$	Final math knowledge $K_T = 2$
2.179944	0.690085	$\beta^{Ed}_{\kappa=3}$	Final math knowledge $K_T = 3$
2.313671	0.682632	$\beta^{Ed}_{\kappa=4}$	Final math knowledge $K_T = 4$
0.797464	0.123689	$\beta^{Ed}_{m=1}$	Highest math completed $H_T = 1$
1.367145	0.146481	$\beta^{Ed}_{m=2}$	Highest math completed $H_T = 2$
1.673064	0.160094	$\beta^{Ed}_{m=3}$	Highest math completed $H_T = 3$
2.193663	0.175935	$\beta^{Ed}_{m=4}$	Highest math completed $H_T = 4$
2.343320	0.182951	$\beta^{Ed}_{m=5}$	Highest math completed $H_T = 5$
2.697626	0.196506	$\beta^{Ed}_{m=6}$	Highest math completed $H_T = 6$
Cut-points			
3.395959	0.683257	α^{HS}_{AA}	Division between $Ed = HS$ and $Ed = AA$
3.860262	0.638381	α^{HS}_{BA}	Division between $Ed = AA$ and $Ed = BA$

The coefficient of type $\varphi = 1$ is fixed at -1303.422323 and therefore a standard error cannot be estimated. The probability that a type one individual attains a BA or AA conditional on graduating from high school is essentially zero, which sets the parameter to an arbitrarily large negative number.

Table C.7: **Estimates: Educational Attainment at Age 25 of Non-Graduates** The distribution of educational attainment follows an ordered logit with the following parameter values for those who exit the model with a GED or with no diploma. The coefficient of Exit= ND is fixed at 0. See the discussion of equation 2.1 for details.

Estimate	Std. Err.	Param.	Variable
Latent Ed^*_{NG}			
0.826703	2.204270	β^{ED}_{GED}	Earn a GED in $t = 1, \ldots, 4$ Exit= GED
Cut-points			
0.050946	0.176204	α^{NG}_{GED}	Division between $Ed = ND$ and $Ed = GED$
4.363660	0.895908	α^{NG}_{AA}	Division between $Ed = GED$ and $Ed = AA$
4.771001	1.099354	α^{NG}_{BA}	Division between $Ed = AA$ and $Ed = BA$

Table C.8: **Estimates: Flow Utility** to a particular choice given previous choice and type is constructed from the following parameters. See appendix A for details.

Estimate	**Std. Err.**	**Param.**	**Variable**
Type specific payoff to choice			
0.794968	0.106918	$\omega^N_{\varphi=1}$	Type $\varphi = 1$ payoff to not attending $d^N_t = 1$
-2.606776	0.960912	$\omega^N_{\varphi=2}$	Type $\varphi = 2$ payoff to not attending $d^N_t = 1$
-2.207316	1.733938	$\omega^N_{\varphi=3}$	Type $\varphi = 3$ payoff to not attending $d^N_t = 1$
-2.734562	1.547899	$\omega^G_{\varphi=1}$	Type $\varphi = 1$ payoff to getting a GED $d^G_t = 1$
-4.985967	1.964070	$\omega^G_{\varphi=2}$	Type $\varphi = 2$ payoff to getting a GED $d^G_t = 1$
-1.953785	1.732581	$\omega^G_{\varphi=3}$	Type $\varphi = 3$ payoff to getting a GED $d^G_t = 1$
0.913128	0.096740	$\omega^B_{\varphi=1}$	Type $\varphi = 1$ payoff to taking basic math $d^B_t = 1$
-0.455545	0.060908	$\omega^B_{\varphi=2}$	Type $\varphi = 2$ payoff to taking basic math $d^B_t = 1$
-0.641922	0.171319	$\omega^B_{\varphi=3}$	Type $\varphi = 3$ payoff to taking basic math $d^B_t = 1$
0.952573	0.099373	$\omega^A_{\varphi=1}$	Type $\varphi = 1$ payoff to taking academic math $d^A_t = 1$
-2.109323	0.159598	$\omega^A_{\varphi=2}$	Type $\varphi = 2$ payoff to taking academic math $d^A_t = 1$
-0.388200	0.222499	$\omega^A_{\varphi=3}$	Type $\varphi = 3$ payoff to taking academic math $d^A_t = 1$
Double course cost			
-3.165308	0.128827	ω^B_{double}	Two basic math courses $d^{B\times 2}_t = 1$
-4.287743	0.110507	ω^A_{double}	Two basic math courses $d^{A\times 2}_t = 1$
-2.880074	0.098012	$\omega^{B\&A}_{\text{double}}$	One basic and one academic course $d^{B\&A}_t = 1$
Switching cost			
-0.113358	0.180583	ω^N_{switch}	Not attending given $d^N_{t-1} = 0$
-1.222773	0.063043	ω^B_{switch}	Taking basic math given $d^B_{t-1} + d^{B\times 2}_{t-1} + d^{B\&A}_{t-1} = 0$
-1.989764	0.066244	ω^A_{switch}	Taking academic given $d^A_{t-1} + d^{A\times 2}_{t-1} + d^{B\&A}_{t-1} = 0$

Table C.9: **Estimates: Payoff to Educational Attainment at 25** is constructed from the following parameters. Table 4.11 in the Results section lists the payoff associated with each level of education implied by these estimates.

Estimate	**Std. Err.**	**Param.**	**Variable**
0.763890	0.793507	ω^{GED}	Educational attainment at 25 $Ed = GED$
1.015576	0.472063	ω^{HS}	Educational attainment at 25 $Ed = HS$
-2073.646061	-	ω^{AA}	Educational attainment at 25 $Ed = AA$
3.504065	0.087283	ω^{BA}	Educational attainment at 25 $Ed = BA$

The coefficient ω^{AA} is fixed at -2073.646061 and therefore a standard error cannot be estimated. The requirement that the payoff to a 2-year degree is at least as large as the payoff to a high school diploma $V^{HS} \leq V^{AA}$, which sets the parameter value to an arbitrarily large negative number.

C.2 Results

Table C.10: **Initial Conditions:** K_0, H_0 The percentage of individuals with each combination of 8th grade highest math completed and math knowledge is presented.

	Highest math completed H_0			
Math knowledge K_0	0	1	2+	Total
0	10.5	6.1	0.2	16.8
1	16.8	15.9	0.7	33.3
2	6.6	16.3	2.4	25.3
3	3.0	11.8	9.7	24.5
Total	36.9	50.1	13.0	100.0

Table C.11: **Highest Level Math Completed by Type** (%)

		Type	
Highest level math completed H_T	1	2	3
None	49.1	11.9	0.0
Pre-algebra	26.1	8.2	0.1
Algebra	16.4	22.1	0.5
Geometry	7.1	24.8	6.2
Algebra II	1.1	26.2	32.1
Pre-calculus	0.1	5.3	32.0
Calculus	0.1	1.5	29.2
Total	100.0	100.0	100.0

Table C.12: **Math Knowledge by Type (%)** Highest level of proficiency computed after the fourth year of high school for those attending that year

	Type		
Math Knowledge K_T	1	2	3
0-None	14.6	7.7	0.4
1-Whole Number Operations	34.0	23.2	3.1
2-Rational Number Operations	18.0	15.7	5.3
3-Simple Problem Solving	19.5	24.2	22.2
4-Intermediate or Advanced Problem Solving & Concepts	13.8	29.3	69.0
Total	100.0	100.0	100.0

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