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Abstract

We have computed energies of ^{16}Ne levels in a core plus two-nucleon space, using known ^{16}C energies and existing wave functions. We have then used these energies to compute properties of the first three levels of ^{17}Na. Significant differences are found with results of a recent microscopic-cluster-model formulation.

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Coulomb energies in ^{16}Ne and low-lying levels of ^{17}Na

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We have computed energies of ^{16}Ne levels in a core plus two-nucleon space, using known ^{16}C energies and existing wave functions. We have then used these energies to compute properties of the first three levels of ^{17}Na. Significant differences are found with results of a recent microscopic-cluster-model formulation.

Little is known about the nucleus ^{17}Na. In 1966, Kelson-Garvey [1] estimated its mass excess to be 35.61 MeV, a value that corresponds to $E_p = 4.3$ MeV (unbound) with respect to ^{16}Ne $+ p$. Recently, a microscopic-cluster-model (MCM) calculation [2] (hereinafter referred to as TD) estimated the ground state (gs) to have $E_p = 2.4$ MeV. Here, we give results of our calculations for this nucleus.

The levels of ^{17}Na are perhaps best discussed in terms of those of its mirror ^{17}C. In ^{17}C, a low-lying triplet contains $3/2^+$ gs, $1/2^+$ at 210(6) keV, and $5/2^+$ at 331(6) keV [3]. As in ^{19}O, the $3/2^+$ state contains virtually no $d3/2$ single-particle (sp) strength, but rather is predominantly $(d5/2)^3$. On the other hand, the wave function of the $1/2^+$ state is dominated by a $2s1/2$ sp amplitude. Because of the so-called Thomas-Ehrman shift, this $1/2^+$ state is expected to be the gs of ^{17}Na, as indeed found by TD. Presumably it is the $3/2^+$ state whose energy was estimated by Kelson-Garvey [1].

In their work, TD allowed the last two neutrons in ^{16}C to occupy the $0d5/2$, $1s1/2$, and $0d3/2$ orbitals. For ^{16}C $+ n$ relative orbital angular momentum, they allowed $\ell = 0$, 2, and 4. Their last neutron was coupled to the gs and the $2^+_{1,2}$, 3^+_1, and 4^+_1 states of ^{16}C. They fine-tuned their parameters to fit the properties of these ^{17}C levels and then computed ^{17}Na. Our calculations considered only s or d for the last neutron. We assume the wave-function amplitudes in ^{17}Na are equal to those in ^{17}C. In our model, the wave function of a given state in ^{17}C is written as ^{17}C$_i = \Sigma\ a_{ijk}\ ^{16}C_j\ n_k$, where the subscript j runs over the low-lying states of ^{16}C and n_k is an s- or d-shell neutron. We equate the squares of the a's with the spectroscopic factors listed in Table II (taken from TD). In TD, the S's are calculated from the $(^{17}$C/^{16}C) overlaps. For each component of each wave function, a Woods-Saxon potential with radius and diffusivity parameters r_0, $a = 1.25$, 0.65 fm, was used for the ^{16}C $+ n$ relative motion. (No spin-orbit term was included. Experience has shown that it has negligible effect on Coulomb energies and on widths at given energies.) The well depth was adjusted to fit the physical binding energy of the ^{17}C state. This potential was then used, with the addition of the Coulomb potential of a uniformly charged sphere to compute energies for each component of ^{17}Na $= ^{16}$Ne $+ p$. The energy E_i of each component was computed in this way,

and the predicted energy of each ^{17}Na state obtained from the expression $E = \Sigma\ S_i E_i / \Sigma\ S_i$. Here E_i is the energy of the ith component of the wave function, and S_i is the spectroscopic factor of that component in the total wave function. This procedure assumes the expansion coefficients for n and p wave functions are the same, and the mirror states differ only in the radial form of the wave function. This technique is relatively simple, but the model has worked well for a variety of light mirror pairs, giving agreement at the 40–70-keV level.

Recently, we predicted the gs energy of ^{19}Mg, for which earlier calculations had covered a wide range. Our calculated value [4] was $E_{2p} = 0.87(7)$ MeV. A later experiment [5] found $E_{2p} = 0.75(5)$ MeV.

To compute energies in ^{17}Na, we need energies of excited states in ^{16}Ne. The 2^+_1 energy is known [6] to be 1.69 MeV, but the 2^+_2, 3^+, and 4^+ states are unknown. So, first we must calculate those energies, which we do in a basis of two sd-shell nucleons outside ^{14}C (for ^{16}C) and outside ^{14}O (for ^{16}Ne). We also need energies of the $1/2^+$ and $5/2^+$ states of ^{15}F, which we take to be $E_p = 1.356(40)$ and 2.785(46) MeV, respectively, from Ref. [7]. We use two-particle wave functions from Ref. [8]. The energy is fixed in the neutron-rich nucleus ^{16}C, and then computed in the proton-rich mirror, keeping the structure amplitudes unchanged as outlined previously. The energy is $E = \Sigma\ S_i E_i / \Sigma\ S_i$ as before. For 2^+_1, we get $E_x = 1.683$ MeV, to be compared with the known value of 1.69 [6]. From past experience, we expect the uncertainties in our energies to be $\leqslant 70$ keV, including the uncertainties in the $1/2^+$ and $5/2^+$ energies in ^{15}F. Calculated energies for ^{16}Ne are listed in Table I.

We now use those ^{16}Ne energies to compute energies in ^{17}Na for the lowest three states. We have done this using spectroscopic factors for two sets of wave functions—the MCM set labeled V2 in TD, and the shell-model (SM) values they quote. Spectroscopic factors for the ^{17}C states are listed in Table II. Many of the S's are similar for V2 and SM, but there are a few serious differences. For example, the SM results have large $3^+ \times d$ and $2^+ \times d$ parentage for the $1/2^+$ state. It is puzzling that those numbers are very small in V2, as pointed out by TD. By antisymmetry, the configuration $(d5/2)^2_{0^+}(s1/2)$, after recoupling, contains

TABLE I. Core excitation energies (MeV) in ^{16}C and ^{16}Ne.

	Excitation energy		
J^π	^{16}C (Expt, Ref. [6])	^{16}Ne (Present)	^{16}Ne (V2, Ref. [2])
2_2^+	3.986	3.670	4.2
3^+	4.088	3.258	6.6
4^+	4.142	4.162	3.3

both $(d5/2)(s1/2)_{2,3}(d5/2)$. We have also computed the decay widths for decay to the lowest 0^+ and 2^+ states, as did TD. These used the same potential parameters mentioned earlier.

Results for the expected ^{17}Na energies are listed in Table III, where they are compared with those of TD. Significant differences are noted, both in absolute energy of the gs and in relative spacing of the excited states. Our result with the V2 amplitudes is $E_p = 2.71$ MeV for the $1/2^+$ gs, compared to 2.40 MeV in TD. Our V2 excitation energies for $3/2^+$ and $5/2^+$ are 0.67 and 1.03 MeV, respectively, compared to 0.17 and 0.57 MeV in TD. Our proton separation energies for $3/2^+$ and $5/2^+$ are nearly identical for V2 and SM, but the $1/2^+$ is less unbound by 0.34 MeV with V2 than with SM.

Our calculated width (not listed) for decay of the $1/2^+$ state to 2_1^+ is 3.5 keV using SM S's. This value would result in a $2^+/0^+$ branching ratio of about 1.5×10^{-3} for the $1/2^+$ state. For the $5/2^+$ state, this ratio is about 0.37.

TABLE II. Spectroscopic factors for ^{16}C $+ n = {}^{17}$C (from Ref. [2]).

		$3/2^+$		$1/2^+$		$5/2^+$	
Core	sp	V2	SM	V2	SM	V2	SM
0^+	s	–	–	0.828	0.644	–	–
	d	0.010	0.035	–	–	0.558	0.701
2_1^+	s	0.328	0.163	–	–	0.037	0.096
	d	1.260	1.445	0.034	0.415	0.520	0.226
2_2^+	s	0.030	0.225	–	–	0.050	0.014
	d	0.127	0.090	0.366	0.372	0	0.631
4^+	d	0.372	0.381	–	–	0.969	0.916
3^+	s	–	–	–	–	0	0.301
	d	0.026	0.285	0.091	1.027	0.060	0.003

TABLE III. Proton energies (relative to ^{16}Ne $+ p$) and widths (both in MeV) for lowest three levels of ^{17}Na.

J^π		Present			TD (Ref. [2])		
		Energy	$\Gamma(0)$	$\Gamma(2)^a$	Energy	$\Gamma(0)$	$\Gamma(2)$
1/2+	V2	2.712	2.2	–	2.40	1.36	–
	SM	3.054	2.3	–			
3/2+	V2	3.384	–	0.23, 0.031	2.57	0.001	0.024
	SM	3.387	–	0.11, 0.035			
5/2+	V2	3.741	0.27	0.047, 0.030	2.97	0.123	0.021
	SM	3.723	0.35	0.12, 0.013			

^{a}Widths in this column are given for $\ell = 0$, followed by $\ell = 2$. TD gives only the sum.

Because several of our decay widths are different from those of TD, we have recalculated widths using V2 S's and energies from TD. These are listed in Table IV, in comparison with the widths from TD. Our width for $5/2^+ \rightarrow 2_1^+$ decay is only about 60% of the one given by TD. The other three are all similar to those of TD, leading to the conclusion that the differences in widths in Table III arise primarily from differences in energy. Because the widths vary greatly with energy, better width predictions await knowledge of the energies. It would be very interesting to find a reaction to populate these ^{17}Na levels.

TABLE IV. Widths for ^{17}Na $\rightarrow {}^{16}$Ne $+ p$, computed using energies from TD.

				$\Gamma_{\rm calc}$ (MeV)	
Decay	ℓ	E_p (MeV)		Present	TD
$1/2^+ \rightarrow$ gs	0	2.40		1.6	1.36
$1/2^+ \rightarrow 2_1^+$	2	0.71		5.4×10^{-6}	–
$3/2^+ \rightarrow 2_1^+$	0	0.88		0.019	
	2	0.88		0.0009	
			sum	0.020	0.024
$5/2^+ \rightarrow$ gs	2	2.97		0.13	0.123
$5/2^+ \rightarrow 2_1^+$	0	1.28		0.010	
	2	1.28		0.003	
			sum	0.013	0.021

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