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# Binding energy of $^{22}\text{C}$

## Abstract

The sensitivity of the calculated matter radius to the binding energy is exploited to estimate the  $2n$  binding energy of  $^{22}\text{C}$ , using a recent experimental value of  $R_m=5.4(9)$  fm. The result is  $B_{2n} < 220$  keV, significantly smaller than another recent estimate.

## Disciplines

Physical Sciences and Mathematics | Physics

## Comments

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## Binding energy of $^{22}\text{C}$

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The sensitivity of the calculated matter radius to the binding energy is exploited to estimate the  $2n$  binding energy of  $^{22}\text{C}$ , using a recent experimental value of  $R_m = 5.4(9)$  fm. The result is  $B_{2n} < 220$  keV, significantly smaller than another recent estimate.

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The binding energy of  $^{22}\text{C}$  is poorly known (see Table I). The most recent mass evaluation [1] reports a mass excess of 53.28(90) MeV, corresponding to a  $2n$  binding energy of 420(940) keV. Because  $^{22}\text{C}$  is known to be bound, perhaps this value is better stated as  $420_{-420}^{+940}$ . A recent shell-model calculation [2] with shifted single-particle energies (shifted by  $-427$  keV to fit  $^{15}\text{C}$ ) reproduces reasonably well the ground-state (gs) binding energies of  $^{16,18,20}\text{C}$  and predicts  $B_{2n} = 601$  keV for  $^{22}\text{C}$ . A three-body theoretical calculation [3] finds  $B_{2n} = 388$  to  $573$  keV and an  $s^2$  fraction of  $0.915$ – $0.968$  for the last two neutrons. An early three-body Faddeev calculation [4] suggested  $B_{2n} = 1120$  keV. Abu-Ibrahim *et al.* [5] computed reaction cross sections and matter radii for an assumed range of  $B_{2n} = 122$ – $489$  keV. The connection between the radius and binding energy is well known. Reference [6] related the radii of stable light nuclei to the average binding energy per nucleon. For neutron-rich light nuclei, Refs. [7,8] explored the dependence of the matter radius on the binding energy of the valence neutron(s). For small binding energies, the reaction cross section  $\sigma_R$  and the calculated radius  $R_m$  are quite sensitive to the binding energy. This dependence arises naturally from a potential-model description of the wave function of the valence neutron(s), and we exploit that feature here.

A recent paper [9] reported results of a  $^{22}\text{C} + p$  reaction cross-section measurement at  $40A$  MeV. The value reported was 1338(274) mb. Using the Glauber model for the  $R_m$  dependence of  $\sigma_R$ , they deduced the rms matter radius to be  $5.4(9)$  fm, a rather large value, suggesting a neutron halo. Those authors found that they needed a very small binding energy to explain their results. In order to obtain a reaction cross section of  $\sim 1050$  mb (about  $1\sigma$  below the central value), they needed a  $2n$  binding energy of 10 keV. They also required a pure  $s^2$  configuration for the last two neutrons. (The radius is significantly larger for  $s^2$  than for  $d^2$ , especially at small binding.) This configuration is consistent with the conclusion of Horiuchi and Suzuki [3], who found that “ $^{22}\text{C}$  is an ideal  $s$ -wave two-neutron halo nucleus”. Before the experiment, for their assumed  $B_{2n}$  range of 122–489 keV, Ref. [5] computed the  $^{22}\text{C} + p$  reaction cross section at  $40A$  MeV to be in the range 957–1005 mb, and the matter radius range to be 3.6–4.1 fm. For their calculated range of  $B_{2n} = 390$ – $470$  keV, Ref. [3] computed  $R_m = 3.93$ – $4.12$  fm. These illustrate how unexpectedly large the values of Ref. [9] are.

We have used a simple model and the results of Ref. [9] to put a limit on the  $2n$  binding energy of  $^{22}\text{C}$ . Because our conclusion is significantly different from another recent estimate, [10] we thought it worthwhile to report our result.

Some time ago, one of us [11] used a simple expression to relate the square of the matter radius  $R_m^2$  of a neutron-rich nucleus to that of a core  $R_c^2$  and  $R_v^2$ , the expectation value of  $r^2$  computed with the wave function of the last neutron, assumed to be a single-particle neutron radial wave function calculated with a Woods-Saxon potential well having  $r_0, a = 1.25, 0.65$  fm. The well depth was adjusted to reproduce the separation energy. The formula was

$$R_m^2 = [(A - 1)/A](R_c^2 + R_v^2/A).$$

This formula is similar to ones that had been used previously [7,12–14] in other contexts. And it has been used subsequently by others. It is identical to the equation in Refs. [7,12].

This equation is basically equivalent, numerically, to modeling the density as the sum of a core density and the density of a valence neutron, a procedure which is very common in this field. In some cases (for example, in Refs. [9,15]), a cutoff radius is introduced to separate interior and exterior regions. Varying this cutoff radius (treated as a free parameter) can be used to artificially enhance the contribution of the neutron tail. We have no such parameter. Once the geometrical parameters of the Woods-Saxon well are chosen (and we always use the same ones), our method is parameter free.

In cases for which the  $A - 1$  member is unbound [and hence  $R_c(A - 1)$  is undefined], Ref. [11] assumed that the two neutrons share the  $B_{2n}$  energy so that  $R_v$  was computed for  $B_n = B_{2n}/2$ , and the core was assumed to be  $A - 2$ . The equation then becomes

$$R_m^2 = [(A - 2)/A](R_c^2 + 2R_v^2/A).$$

This  $2n$  procedure was applied to nuclei  $^{6,8}\text{He}$ ,  $^9\text{Be}$ ,  $^{11}\text{Li}$ ,  $^{14}\text{Be}$ ,  $^{17}\text{B}$ , and  $^{22}\text{C}$  in Ref. [11]. It was later proposed by Bhagwat *et al.* [16] for use in a much more sophisticated model. The  $2n$  procedure and the assumption of  $B_n = B_{2n}/2$  has become a common feature of work in this field [3,5,10,15–17].

Some have questioned the validity of the approximation that the radius of the  $A - 2$  core in nucleus  $A$  is the same as for the free nucleus  $A - 2$ . We are not alone in using this approximation. Its use is widespread in this field, even

TABLE I. Binding energy of  $^{22}\text{C}$ .

Method	$B_{2n}$ (keV)	Reference
Three-body Faddeev	1120	[4]
Masses	420(940)	[1]
Glauber	122–489 <sup>a</sup>	[5]
Shell model, shifted	601	[2]
Three-body	388–573	[3]
To fit $\sigma_R - \Delta\sigma_R$	10	[9]
To fit $R_m - \Delta R_m$	<400	[10]
To fit $R_m - \Delta R_m$	<220	Present

<sup>a</sup>Assumed (see text).

in models that are much more sophisticated than ours. For charge radii, a recent report [18] noted that the charge radius of  $^{11}\text{Be}$  was 0.106(24) fm larger than in  $^{10}\text{Be}$ . Those authors stated that the “increase from  $^{10}\text{Be}$  to  $^{11}\text{Be}$  can be attributed to motion of the  $^{10}\text{Be}$  core relative to the center of mass”. The different treatment of the center of mass is partially responsible for the fact that two different equations (as, e.g., here and in Ref. [10]) are widely used to relate the matter radius of a nucleus to those of its constituents. For  $^{11}\text{Li}$  and  $^9\text{Li}$  [19], the authors state “there is no change to the  $^9\text{Li}$  core caused by the halo neutrons”. In a very recent paper [20] concerning  $^{22,23}\text{O}$ , the authors assume the  $^{22}\text{O}$  core in  $^{23}\text{O}$  is the same as a free  $^{22}\text{O}$ .

In order to make a  $0^+$  state, the last two neutrons must be identical. So, having them share the binding energy equally is reasonable. The  $2n$  equation is identical to that of Ref. [16], but slightly different from that of Ref. [10] (see below). It is a special case of the generalized expression in Ref. [12]. Because  $^{21}\text{C}$  is unbound, we must use the  $2n$  procedure for  $^{22}\text{C}$ . Our approach is similar to that of Ref. [10], except that their equation has a larger coefficient of the second term than does ours. In our opinion, our form has a more solid foundation.

It is true that this expression contains no correlations between the two neutrons—other than the assumption that they are in the same orbital (to make a  $0^+$  state). The assumption of equal sharing of the  $2n$  binding energy must be true on average, because the two neutrons are identical. So, one way to think of the relation  $B_n = B_{2n}/2$  is that we are using the average rather than separately specifying each  $B_n$  and then averaging. We are ignoring the possibility of any small internal energy of the  $2n$  pair. We repeat, this assumption is in common use.

Another type of correlation involves configuration mixing. But because we are seeking an upper limit, and because the  $s^2$  configuration gives the largest radius, any configuration mixing would make our  $B_{2n}$  limit even smaller. It is now clear that  $^{22}\text{C}$  should be  $s^2$  [3,9]. Any admixture of  $d^2$  will decrease the calculated  $R_m$  and hence decrease the limit on  $B_{2n}$  even further. This can easily be seen in Fig. 1 by comparing our calculations (with 100%  $s^2$ ) to those of Ref. [3] (with 91–97%  $s^2$ ). The experimental value of  $R_m$  for  $^{20}\text{C}$  is [21] 2.98(5) fm. Here, we use 3.0 fm as the core radius in the  $^{22}\text{C}$  calculations. While it is possible that the  $^{20}\text{C}$  core in  $^{22}\text{C}$  may not be identical to a free  $^{20}\text{C}$ , such a difference is very small compared to the large effect being considered here.

We have performed several calculations for  $^{22}\text{C}$  for different  $2n$  binding energies. Figure 1 displays a representative sample of the calculated values of  $R_m$  vs the assumed  $2n$  separation energy (diamonds and a curved line). The sensitivity of the calculated value of  $R_m$  to the value of  $B_{2n}$  is apparent. Horizontal lines are plotted at the upper and lower  $1\sigma$  limits on the experimental value [9] (squares). The variation in the calculated value caused by varying the input value of  $B_{2n}$  is large, but is significantly less than the uncertainty in the experimental value. For any given binding energy and configuration, the uncertainty in the calculated matter radius is smaller than the size of the points in the figure. The triangle represents the predicted range of  $B_{2n}$  and  $R_m$  from Ref. [3]. Their matter radius range is 3.93–4.12 fm, for their predicted  $B_{2n}$  range of 388–573 keV. Their calculations found an  $s^2$  fraction of 0.915–0.968. Increasing that to 1.0, as used here and in Refs. [9] and [10], would increase their  $R_m$  values slightly.

We are not concerned here with the procedure of extracting a matter radius from a measured cross section. We assume the groups doing that extraction have correctly assigned the uncertainties. The large uncertainty in the reported value of  $R_m$  contains a contribution from the uncertainty in the measured cross section and the uncertainty in the model extraction of a matter radius from that cross section. There is no systematic uncertainty in our treatment. For the given binding energy and configuration, our calculated  $R_m$  is fully determined.

We see that at the  $1\sigma$  lower limit on  $R_m$ , we have a  $1\sigma$  upper limit of  $B_{2n} < 220$  keV. Of course, if the  $1\sigma$  lower limit on  $R_m$  should become smaller as the uncertainties are decreased in future experiments, our estimated upper limit on  $B_{2n}$  would increase. We encourage additional measurements of the reaction cross section for  $^{22}\text{C}$ . Reference [10] reported an estimate of  $B_{2n} < 400$  keV. They get a large radius at larger  $B_{2n}$  partly because (for  $^{22}\text{C}$ ) their coefficient of  $R_v^2$  is 10% larger than ours, but that cannot explain all of the difference. We await an experimental determination of this important binding energy.

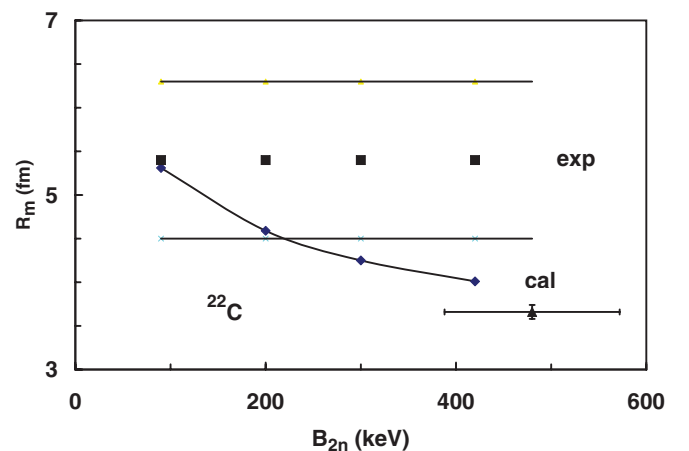


FIG. 1. (Color online) Squares and horizontal lines labeled “exp” represent the central value and the  $\pm 1\sigma$  uncertainties for  $R_m = 5.4(9)$  fm, extracted from the reaction cross section in Ref. [9]. The plot of the calculated  $R_m$  vs an assumed  $2n$  separation energy is shown as diamonds and a curved line. The triangle displays the range of  $R_m$  and  $B_{2n}$  predicted by Ref. [3].

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