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Modeling the dynamics of network technology adoption and the role of converters

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Keywords
Externality, converters, dynamics, equilibrium

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***Abstract*—New network technologies constantly seek to displace incumbents. Their success depends on technological superiority, the size of the incumbent's installed base, users' adoption behaviors, and various other factors. The goal of this paper is to develop an understanding of competition between network technologies, and identify the extent to which different factors, in particular converters (a.k.a. gateways), affect the outcome. Converters can help entrants overcome the influence of the incumbent's installed base by enabling cross-technology interoperability. However, they have development, deployment, and operations costs, and can introduce performance degradations and functionality limitations, so that if, when, why, and how they help is often unclear. To this end, the paper proposes and solves a model for adoption of competing network technologies by individual users. The model incorporates a simple utility function that captures key aspects of users' adoption decisions. Its solution reveals a number of interesting and at times unexpected behaviors, including the possibility for converters to reduce overall market penetration of the technologies and to prevent convergence to a stable state; something that never arises in their absence. The findings were tested for robustness, *e.g.,* different utility functions and adoption models, and found to remain valid across a broad range of scenarios.**

***Index Terms*—Externality, converters, dynamics, equilibrium.**

I. Introduction

Advances in technology often see newer and better solutions replacing older ones. Networking is no exception. For example, the Internet competed against alternative packet data technologies before finally displacing the phone network as the de facto communication infrastructure. Recently, there have been calls for new architectures to succeed it, and these will face a formidable incumbent in the Internet. Their eventual success in replacing it will likely depend not just on technical superiority, but also on economic factors, and on their ability to win over the Internet's installed base.

A large installed base can give an incumbent an edge even if a new (entrant) technology is technically superior. The traditional networking approach to this problem has been converters (a.k.a. gateways) to ease migration from one technology to another. This is not unique to networks, but converters are particularly important in network settings where "communication" is the primary function, and its benefits grow with the number of users that can be reached, *e.g.,* as in Metcalfe's Law. Since converters allow users of one technology to connect with users of another, they are an important tool in the adoption of network technologies. However, developing, deploying, and operating converters comes at a cost, one that often grows as a function of the converter's quality. Further, converters can play a directionally ambiguous role. On one hand, a converter can help the entrant overcome the advantage of the incumbent's large installed base by allowing connectivity to it. On the other hand, the converter also helps the incumbent technology by mitigating the impact of its users migrating to the newer technology. Understanding the impact of converters on (network) technology adoption is, therefore, a topic that deserves further scrutiny.

In this paper, we develop a modeling framework to study adoption dynamics of entrant and incumbent technologies in the presence of network externalities. Specifically, we introduce a model for the utility derived by an individual user from a communication network, and use it to build an aggregate model for technology adoption that is consistent with individual rational decision-making. We apply the model to study the role that converters can play in the adoption of network technologies. Our main findings are:

- The adoption process can exhibit multiple steady state outcomes (equilibria); each associated with a specific region of initial adoption levels for the two technologies.
- Converters can help a technology improve its own standing, *i.e.,* market share, and even ensure its dominance while it would have entirely disappeared in the absence of converters.
- Improving converters efficiency can at times be harmful. They can result in lower market shares for an individual technology or for *both*.
- Converters can disrupt technology adoption and prevent both technologies from converging to a stable market share, *i.e.,* users switch back and forth between technologies. In contrast, this never arises without converters, with technology adoption always converging to a stable equilibrium.

The rest of this paper is organized as follows: Section II introduces our model and problem formulation. Section III characterizes technology adoption trajectories and equilibrium adoption levels. Section IV explores the role of converters in influencing adoption outcomes. Section V reviews prior work and positions the paper in the literature. We discuss the limitations of this study and conclude the paper with remarks

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on future work in Section VI.

II. Technology Adoption Model

A. Technology Valuation

As in most competitive situations, the choice of one technology over another depends on the "value" they provide. Value is a somewhat elusive concept that depends in part on the quality and functionality of the technology and its cost. In the context of network technologies whose main purpose is to enable communication among users, the *number* of users[1] accessible through it is another important component, often termed network effect or externality. As commonly done, we account for these factors and their effect on technology adoption through a *utility function.* For two competing network technologies, 1 and 2, the respective utility functions are given by eqs. (1) and (2).

$$U_1 = \theta q_1 + (x_1 + \alpha_1 \beta x_2) - p_1 \tag{1}$$

$$U_2 = \theta q_2 + (\beta x_2 + \alpha_2 x_1) - p_2 \tag{2}$$

Eqs. (1) and (2) consist of three distinct terms. Focusing on, say, Technology 1, the first term, θq_1 represents the stand-alone benefits from the technology, with q_1 representing the intrinsic quality of the technology, and θ a random variable accounting for heterogeneity in how users value technology. The quantity q_1 incorporates aspects of functionality, reliability, performance, etc., for the technology. In the rest of the paper, we assume $q_2 > q_1$, *i.e.,* Technology 2 is superior to Technology 1 and correspondingly can be viewed as the entrant with Technology 1 playing the role of the incumbent. The model, however, does not mandate such an assignment of roles, *e.g.,* it can be used to study settings where Technology 1 is the entrant and offers, say, a lesser quality but cheaper alternative than the incumbent Technology 2. The random variable $\theta \in [0, 1]$ determines the relative weight a user places on the intrinsic quality of a technology. It is private information, but we assume that the distribution of θ across users is known. We make the common assumption [2] that θ is uniformly distributed in the interval $[0, 1]$. This choice affects the magnitude of equilibrium adoption levels, but does not qualitatively affect findings regarding technology adoption dynamics and outcomes as demonstrated in Appendix F.

The second component of the user's utility is the network externality (or network effect), which refers to benefits derived from the ability to connect with other users. Network externalities are chosen to be proportional to the number of users each technology gives access to. This linear dependency of network benefits on the number of adopters is consistent with Metcalfe's Law and commonly used in the literature [7]. In Appendix F, we investigate other possible models and demonstrate the robustness of our findings across different functional forms for network externality, including non-linear ones. Denoting as x_1 and x_2 the fractions of adopters of each technology out of a large population of size N, the externality benefits for Technology 1 consist of x_1, the fraction of Technology 1 users, plus $\alpha_1 \beta x_2$, a term that includes the fraction of Technology 2 users weighed by two additional factors. The first, $0 \leq \alpha_1 \leq 1$, captures the availability of converters offering connectivity *from* Technology 1 *to* Technology 2 ($\alpha_1 = 0$ corresponds to no converter and $\alpha_1 = 1$ to "perfect" converters). The second parameter, β, allows different externality benefits for the two technologies[2]. We note that converters, once deployed, are available to *all* users of the technology. This corresponds to what we term "technology-level" converters, *i.e.,* their development and deployment are decisions made by the providers of network technologies.

Converters can be characterized as either *duplex* or *simplex*, *symmetric* or *asymmetric*, and *constrained* or *unconstrained*. Duplex converters provide bi-directional connectivity between technologies, while simplex converters are present in only one direction (most network technologies involve duplex converters, but the model does not mandate them). Asymmetric converters simply refer to the fact that converter efficiency can be different in each direction *i.e.,* $\alpha_1 \neq \alpha_2$. The notion of constrained vs. unconstrained converters arises in the presence of technologies that exhibit different externalities, *i.e.,* $\beta \neq 1$. For example, when $\beta > 1$, it captures whether converters allow users of Technology 1 access to the greater externality benefits of Technology 2 when connecting to its users. A converter is unconstrained if this is permitted, *i.e.,* $\alpha_1 \beta > 1$. We discuss an example where this can arise at the end of the section.

The last element of eqs. (1) and (2) is the price, $p_i, i \in \{1, 2\}$. Because of our focus on networks and connectivity that is typically offered as a *service* rather than a good or product, price is recurrent. In other words, maintaining connectivity through a particular network technology incurs new charges at regular intervals. As a result, users continuously reevaluate their technology choices, and can switch from one technology to another and possibly back. This assumes that switching costs are negligible. This assumption is for analytical tractability of the model, but the framework can be extended to incorporate switching costs, albeit at the cost of increased complexity. In Appendix G we discuss how switching cost can be included, its implications, and provide numerical and simulation based evidence to demonstrate that the qualitative results presented in this work remain valid even in the presence of switching costs.

We note that the model parameters, *i.e.,* q_i, p_i, α_i, β, are static and exogenously specified. An obvious extension is to make them time-varying, *e.g.,* technology gets better and/or cheaper as time goes by, and the outcome of strategic decision-making. Incorporating such effects is clearly of interest, especially in the context of competitive scenarios where firms may offer introductory pricing or seed the market to gain an initial foothold. This, however, requires that we first understand the basic tenets of technology adoption and dynamics in the simpler setting considered in this paper.

Another important question is how to assign actual values to the model's parameters. This is a topic that goes well

[1] Users can be individuals or organizations, and include resources and content.

[2] Eqs. (1) and (2) implicitly express utility in units of Technology 1 externality benefits, *i.e.,* Technology 1 externality benefits are equal to 1 when its penetration level is 100% ($x_1 = 1$).

beyond this paper, and we only point to a possible approach. A common method to estimate utility weights is conjoint analysis, a technique that has been widely adopted by marketing researchers and practitioners (see [9] for a detailed review). It relies on surveys offering users different combinations of functionality and attributes to extract a relative ordering among them, and ultimately produce individual weight assignments.

B. User Decisions

Given current adoption levels, x_1 and x_2, the utility functions of eqs. (1) and (2) identify how a user values each technology, which in turn determines her technology selection decisions. (Recall that the recurring nature of price makes this an ongoing decision process). Specifically, a user chooses Technology i whenever it provides a surplus that is both positive (Individual Rationality constraint) and higher than that of the other technology (Incentive Compatibility constraint). In other words, a user chooses

$$\begin{cases} \text{no technology} & \text{if} \quad U_i < 0 \text{ for all } i, \\ \text{Technology 1} & \text{if} \quad U_1 > 0 \text{ and } U_1 > U_2, \\ \text{Technology 2} & \text{if} \quad U_2 > 0 \text{ and } U_2 > U_1. \end{cases}$$

Note that the model assumes an exclusive choice of technology by users, *i.e.,* they select Technology 1, or 2, or neither, but not both. This translates into the constraint $0 \leq x_1 + x_2 \leq 1$. The dynamics of technology adoption arise from the dependency of the U_i's on the x_i's that change with users' adoption decisions. Capturing these dynamics, therefore, calls for specifying when users become aware of changes in the x_i's and update their adoption decisions. Knowledge of changes in adoption levels is likely to diffuse through the user population and users' reactions are often heterogeneous, *i.e.,* some switch quickly, while others defer. An approach, commonly used in individual-level diffusion models [10] and that captures these aspects is a continuous time approximation.

Specifically, assume that at time t the "current" technology adoption levels, $\underline{x}(t) = (x_1(t), x_2(t))$, are known to all users. With this information, users can compute their utility for each technology and make adoption decisions. Let $H_i(\underline{x}(t)),\ i \in \{1, 2\}$ denote the fraction of users for whom Technology i provides the highest (and positive) utility[3]. The quantity $H_i(\underline{x}(t)) - x_i(t)$ corresponds to the fraction of users that would normally proceed to adopt (disadopt) Technology i at time t. To capture a progressive adoption process, we assume that the rate of change in users' technology choices is proportional to this quantity, namely,

$$\frac{dx_i(t)}{dt} = \gamma\big(H_i(\underline{x}(t)) - x_i(t)\big),\ i \in \{1, 2\}, \tag{3}$$

The quantity $\gamma < 1$ is analogous to the hazard rate in diffusion models, and can be viewed as the expected conditional probability that an individual who has not yet adopted technology i will do so at time t. In our analysis, we assume that the propensity of individuals to adopt does not change with time, *i.e.,* γ is constant.

[3]We discuss the derivation of $H_i(\underline{x}(t))$ in Section III-A.

Two aspects of this diffusion process need further clarification. First, users are myopic. At any instant, the adoption decisions are driven by the number of adopters at that time $(x_i(t))$ and users are not able to anticipate likely adoption levels in the future. Second, the model identifies the rate of technology adoption across users, but not *which* users are making the change. To preserve consistency with user preferences, θ, we assume that the first users to adopt Technology i are those that stand to benefit most from the action. This ensures that at all times the sets of users having adopted either technology correspond to blocks of users with contiguous technology preferences.

The diffusion dynamics governed by eq. (3) can converge to an equilibrium $\underline{x}^*$ characterized by:

$$\left.\frac{dx_i(t)}{dt}\right|_{x_i(t)=x_i^*} = 0 \quad \Leftrightarrow x_i^* = H_i(\underline{x}^*) \text{ for } i \in \{1, 2\}. \tag{4}$$

In other words, at equilibrium, the fraction of users for whom it is individually rational and incentive compatible to choose Technology i equals the current fraction of adopters of Technology i. Based on this formulation, our goal is to characterize, as a function of the exogenous system parameters $\beta, p_i, q_i, \alpha_i$ for $i \in \{1, 2\}$, the equilibrium adoption levels, *i.e.,* the fixed points of eq. (4), and the dynamics leading to them.

Before exploring the dynamics and equilibria of technology adoption that the model gives rise to, we pause to briefly introduce a couple of examples that illustrate the model's parameters and applicability.

IPv4 vs. IPv6: The impending exhaustion of IPv4 addresses, *e.g.,* http:/www.potaroo.net/tools/ipv4 for a daily countdown, implies that service providers signing up new Internet customers will have to start using IPv6 addresses or charge more users who insist on an IPv4 address, *i.e.,* $p_{\text{IPv4}} = p_1 > p_2 = p_{\text{IPv6}}$. As technologies, although IPv4 and IPv6 are *incompatible,* they are largely similar so that for the purpose of our model one can reasonably assume $q_1 \lesssim q_2$ and $\beta = 1$. Because of their incompatibility, converters (gateways), *e.g.,* see [5] for a representative recent proposal, are needed for IPv6 users to access the IPv4 content that is the bulk of today's Internet content and unlikely to become natively accessible over IPv6 any time soon[4]. Conversely, those converters also enable the reverse flow from IPv4 to IPv6, *i.e.,* they are duplex converters, albeit not necessarily delivering the same performance in both directions, *i.e.,* they can be asymmetric, so that both α_1 and α_2 are non-zero but not always equal.

Users then decide between subscribing to an IPv4 or IPv6 service on the basis of price (p_i), the level of content they are able to access (x_i), and the quality of that access (α_i).

Low Def. vs. High Def. Video: The previous example illustrated a common adoption scenario with two mostly equivalent technologies and duplex, asymmetric converters. Because of the similarity of the two technologies $(\beta = 1)$, converters were by default constrained $(\alpha_1\beta \leq 1)$. However, when technologies exhibit significant differences in externality benefits,

[4]Although the servers hosting most web sites can typically get an IPv6 address, very few have bothered registering one with DNS, *e.g.,* see http://bgp.he.net/ipv6-progress-report.cgi.

e.g., $\beta > 1$, converters can be unconstrained ($\alpha_1\beta > 1$) and we present next a possible example.

Consider a provider that offers its customers a video-conferencing service with the associated equipment. The service comes in two versions, high-definition (HD) and standard quality (SQ), *i.e.,* HD equipment generates a high-definition (q_2) video signal while SQ equipment produces a lower resolution ($q_1 < q_2$). Users derive value from video-conferencing with one another, with $\beta > 1$ reflecting the higher quality of an HD signal. The two services are priced accordingly ($p_2 > p_1$). However, because video is a highly asymmetric technology (encoding is hard but decoding is comparatively easy), it is possible for the provider to enable the *decoding* of HD signal on SQ equipment (and obviously conversely). This conversion can introduce quality degradations ($\alpha_1 < 1$), but more importantly it allows SQ users access to the externality benefits associated with receiving HD signals. Assuming HD↔SQ conversion is available in both direction, this is an instance of a duplex, possibly asymmetric ($\alpha_1 \neq \alpha_2$), and unconstrained ($\alpha_1\beta > 1$) converter.

Many users may then opt for the SQ service because of its lower price and the ability to still enjoy the higher benefits of viewing HD signals. On the other hand, if all users were to select the SQ service, those externality benefits would disappear. In general, users with high technology valuation (θ close to 1) may still opt for the HD service, but the decision depends on choices made by others.

III. Trajectories and Equilibria

Solving the evolution of technology adoption decisions over time described in eq. (3) calls for first computing expressions for $H_i(\underline{x}(t)),\ i = \{1,2\}$ as functions of known parameters.

A. Characterizing $H_i(\underline{x})$

For notational convenience we omit dependency on time and write $\underline{x}(t)$ simply as $\underline{x}$. Recall that $H_i(\underline{x}),\ i \in \{1,2\}$, corresponds to the fraction of users for whom it is rational to adopt Technology i, *given* the current adoption levels, $\underline{x}$. To determine the fraction of adopters of each technology, we introduce the notion of *indifference points,* which identify thresholds in users technology valuation (θ) corresponding to qualitatively significant changes in technology preference. Specifically, $\theta_i^0(\underline{x}), i \in \{1,2\}$ identify the θ value separating users with a negative utility for Technology i from those with a positive utility. In other words, for technology penetration levels $\underline{x}$, $\theta_i^0(\underline{x})$ is such that $U_i(\theta_i^0, \underline{x}) = 0$, and $U_i(\theta, \underline{x})$ is positive (negative) for θ values larger (smaller) than θ_i^0. Similarly, $\theta_2^1(\underline{x})$ corresponds to the θ value separating users preferring Technology 1 from those preferring Technology 2, *i.e.,* $U_1(\theta_2^1, \underline{x}) = U_2(\theta_2^1, \underline{x})$ and users with $\theta > \theta_2^1(\underline{x})$ derive greater utility from Technology 2 than Technology 1 (recall that $q_2 > q_1$).

From eqs. (1) and (2), $U_i(\theta_i^0, \underline{x}) = 0$ gives

$$\theta_1^0(\underline{x}) = \frac{p_1 - (x_1 + \alpha_1\beta x_2)}{q_1} \tag{5}$$

$$\theta_2^0(\underline{x}) = \frac{p_2 - (\beta x_2 + \alpha_2 x_1)}{q_2} \tag{6}$$

Similarly, $U_1(\theta_2^1, \underline{x}) = U_2(\theta_2^1, \underline{x})$ gives

$$\theta_2^1(\underline{x}) = \frac{(1-\alpha_2)x_1 - \beta(1-\alpha_1)x_2 + p_2 - p_1}{q_2 - q_1} \tag{7}$$

Combining eqs. (5)-(7) gives

$$\theta_2^1(\underline{x}) - \theta_1^0(\underline{x}) = \frac{q_2}{q_2 - q_1}(\theta_2^0(\underline{x}) - \theta_1^0(\underline{x})), \tag{8}$$

$$\theta_2^1(\underline{x}) - \theta_2^0(\underline{x}) = \frac{q_1}{q_2 - q_1}(\theta_2^0(\underline{x}) - \theta_1^0(\underline{x})), \tag{9}$$

from which the following Proposition can be derived.

Proposition 1:
If $\theta_1^0(\underline{x}) < \theta_2^0(\underline{x})$, then $\theta_2^1(\underline{x}) > \theta_2^0(\underline{x}) > \theta_1^0(\underline{x})$.
If $\theta_1^0(\underline{x}) \geq \theta_2^0(\underline{x})$, then $\theta_2^1(\underline{x}) \leq \theta_2^0(\underline{x}) \leq \theta_1^0(\underline{x})$.

Proposition 1 constrains the possible orderings of the indifference points given by eqs. (5)-(7), so that $H_i(\underline{x}), i \in \{1,2\}$ can be characterized in a compact manner.

$$H_1(\underline{x}) = \begin{cases} [\theta_2^1(\underline{x})]_{[0,1]} - [\theta_1^0(\underline{x})]_{[0,1]} & \text{if } \theta_1^0(\underline{x}) < \theta_2^0(\underline{x}) \\ 0 & \text{otherwise} \end{cases}$$

$$H_2(\underline{x}) = \begin{cases} 1 - [\theta_2^1(\underline{x})]_{[0,1]} & \text{if } \theta_1^0(\underline{x}) < \theta_2^0(\underline{x}) \\ 1 - [\theta_2^0(\underline{x})]_{[0,1]} & \text{otherwise} \end{cases} \tag{10}$$

where $y_{[a,b]}$ is the 'projection[5]' of y into $[a,b]$.

Based on the ordering of the indifference points θ_1^0, θ_2^0 and θ_2^1, and the outcome of their projections on $[0,1]$, the exact expressions for $H_1(.)$ and $H_2(.)$ can be computed. In Appendix A, we show that the entire (x_1, x_2) plane can be partitioned into nine distinct regions, R_1 to R_9 (see Figure 7); each associated with unique expressions for the $H_i(\underline{x})$ pair [6]. Furthermore, although dependent on the system parameters, the relative positions of all regions in the feasible solution space, $S = \{(x_1, x_2)\, s.t.\, 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1, x_1 + x_2 \leq 1\}$, remain fixed. This facilitates the derivation of a general solution, as we outline next.

B. Characterizing Adoption Trajectories

By combining eqs. (5) to (7) with eq. (10), explicit expressions can be obtained for $H_i(\underline{x})$ in each of the nine regions. These are listed in Table I. Using these expressions, it is now possible to solve eq. (3) and characterize the trajectory of technology adoption in each region. The trajectories have the following general form:

$$x_i(t) = a_i + b_i e^{\lambda_1 t} + c_i e^{\lambda_2 t}, \quad i \in \{1,2\} \tag{11}$$

where λ_1 and λ_2 can be positive, negative, or complex depending on the region. Individual solutions for each region are listed in Table II.

The full trajectory of technology adoption starting at some initial adoption levels $\underline{x}(0)$ within a given region, can then be obtained by "stitching" together trajectories in individual regions as region boundaries are crossed. The next question is to determine whether and where these trajectories may eventually converge as $t \to \infty$. We tackle this issue next.

[5] *i.e.,* its value is y for $y \in [a,b]$, a for $y < a$, and b for $y > b$.

[6] This method of partitioning the (x_1, x_2)-plane into nine regions remains applicable even for more generic network externality functional forms as explained in AppendixB

TABLE I
EXPRESSIONS FOR $H_i(\underline{x})$

R_1		$H_2(\underline{x}) = 1$
R_2	$H_1(\underline{x}) = 0$	$H_2(\underline{x}) = 1 - \frac{p_2-(\beta x_2+\alpha_2 x_1)}{q_2}$
R_3		$H_2(\underline{x}) = 0$
R_4	$H_1(\underline{x}) = 0$	$H_2(\underline{x}) = 1$
R_5	$H_1(\underline{x}) = \frac{(1-\alpha_2)x_1-\beta(1-\alpha_1)x_2+p_2-p_1}{q_2-q_1}$	$H_2(\underline{x}) = 1 - \frac{(1-\alpha_2)x_1-\beta(1-\alpha_1)x_2+p_2-p_1}{q_2-q_1}$
R_6	$H_1(\underline{x}) = \frac{(1-\alpha_2)x_1-\beta(1-\alpha_1)x_2+p_2-p_1}{q_2-q_1} - \frac{p_1-(x_1+\beta\alpha_1 x_2)}{q_1}$	$H_2(\underline{x}) = 1 - \frac{(1-\alpha_2)x_1-\beta(1-\alpha_1)x_2+p_2-p_1}{q_2-q_1}$
R_7	$H_1(\underline{x}) = 1$	
R_8	$H_1(\underline{x}) = 1 - \frac{p_1-(x_1+\beta\alpha_1 x_2)}{q_1}$	$H_2(\underline{x}) = 0$
R_9	$H_1(\underline{x}) = 0$	

TABLE II
TECHNOLOGY ADOPTION TRAJECTORIES

	$x_1(t)$	$x_2(t)$
R_1	$x_1(t_0)e^{-\gamma(t-t_0)}$	$\frac{(x_2(t_0)-1)}{x_1(t_0)}e^{-\gamma t}+1$
R_2	$c_1 e^{-\gamma(t-t_0)}$	$\frac{p_2-q_2}{\beta-q_2} + c_2 e^{-\gamma(1-\beta/q_2)(t-t_0)} - c_1\frac{\alpha_2}{\beta}e^{-\gamma t}$
	$c_1 = x_1(t_0)$	$c_2 = [x_2(t_0) + \frac{\alpha_2 x_1(t_0)}{\beta} - \frac{p_2-q_2}{\beta-q_2}]$
R_3	$x_1(t) = x_1(t_0)e^{-\gamma(t-t_0)}$	$x_2(t) = x_2(t_0)e^{-\gamma(t-t_0)}$
R_4	same as R_1	same as R_1
R_5	$x^*_{1\,R_5} + c_2 e^{-\gamma(t-t_0)} + c_1 e^{(-1+\frac{(1-\alpha_2)+\beta(1-\alpha_1)}{q_2-q_1})\gamma(t-t_0)}$	$x^*_{2\,R_5} + c_2\frac{(1-\alpha_2)}{\beta(1-\alpha_1)}e^{-\gamma(t-t_0)} - c_1 e^{(-1+\frac{(1-\alpha_2)+\beta(1-\alpha_1)}{q_2-q_1})\gamma(t-t_0)}$
	$c_1 = \frac{\beta(1-\alpha_1)}{1-\alpha_2+\beta(1-\alpha_1)}(x^*_{2\,R_5} - x_2(t_0))$ $-\frac{1-\alpha_2}{1-\alpha_2+\beta(1-\alpha_1)}(x^*_{1\,R_5} - x_1(t_0))$	$c_2 = (\frac{\beta(1-\alpha_1)}{1-\alpha_2+\beta(1-\alpha_1)})[x_1(t_0)+x_2(t_0)-1]$
R_6	$x^*_{1\,R_6} + c_1 K_1 e^{\frac{A+\sqrt{A^2-4B}}{2}\gamma(t-t_0)} + c_2 K_2 e^{\frac{A-\sqrt{A^2-4B}}{2}\gamma(t-t_0)}$	$x^*_{2\,R_6} + c_1 e^{\frac{A+\sqrt{A^2-4B}}{2}\gamma(t-t_0)} + c_2 e^{\frac{A-\sqrt{A^2-4B}}{2}\gamma(t-t_0)}$
	$c_1 = \frac{(1-\alpha_2)\{x^*_{1\,R_6}-x_1(t_0)-K_2(x^*_{2\,R_6}-x_2(t_0))\}}{(q_2-q_1)\sqrt{A^2-4B}}$	$c_2 = \frac{(1-\alpha_2)\{-(x^*_{1\,R_6}-x_1(t_0))+K_1(x^*_{2\,R_6}-x_2(t_0))\}}{(q_2-q_1)\sqrt{A^2-4B}}$
	$K_1 = \frac{\alpha_2+\beta(1-\alpha_1)-q_2/q_1-(q_2-q_1)\sqrt{A^2-4B}}{2(1-\alpha_2)}$	$A = \frac{1-\alpha_2+\beta(1-\alpha_1)}{q_2-q_1} + \frac{1}{q_1} - 2$
	$K_2 = \frac{\alpha_2+\beta(1-\alpha_1)-q_2/q_1+(q_2-q_1)\sqrt{A^2-4B}}{2(1-\alpha_2)}$	$B = (\frac{1}{q_1}-1)(\frac{\beta(1-\alpha_1)}{q_2-q_1}-1) + \frac{1-\alpha_2}{q_2-q_1}(\frac{\beta\alpha_1}{q_1}-1)$
R_7	$(x_1(t_0)-1)e^{-\gamma(t-t_0)}+1$	$x_2(t_0)e^{-\gamma(t-t_0)}$
R_8	$\frac{p_1-q_1}{1-q_1} + c_1 e^{-\gamma(1-\frac{1}{q_1})(t-t_0)} - c_2\beta\alpha_1 e^{-\gamma(t-t_0)}$	$c_2 e^{-\gamma(t-t_0)}$
	$c_1 = [x_1(t_0) + \beta\alpha_1 x_2(t_0) - \frac{p_1-q_1}{1-q_1}]$	$c_2 = x_2(t_0)$
R_9	$x_1(t) = x_1(t_0)e^{-\gamma(t-t_0)}$	$x_2(t) = x_2(t_0)e^{-\gamma(t-t_0)}$

C. Computing Steady-state Equilibria

From eq. (11), we see that a technology adoption trajectory in, say, region R_k, converges to a stable equilibrium $x_i(\infty) = a_i, i \in \{1,2\}$, if λ_1 and λ_2 are both negative (equilibrium is locally stable), *and* $(a_1, a_2) \in S \cap R_k$ (the equilibrium is valid, *i.e.,* in the region associated with the trajectory). In other words, solutions to eq. (4) $(H_i(\underline{x}^*) = x_i^*, i \in \{1,2\})$, must satisfy *stability* and *validity* conditions to be valid steady-state outcomes of the technology adoption process[7]. The simple nature of eq. (4) makes characterizing valid and stable solutions relatively straightforward, albeit tedious. The results are listed in Tables III and IV, where for readability, expressions for equilibria in R_5 and R_6, $\underline{x}^*_{R_5}$ and $\underline{x}^*_{R_6}$, are given separately in eqs. (12) and (13), respectively. Table III gives the stability conditions associated with each equilibrium, along with the joint validity and stability conditions (they are inter-dependent) in the last column.

[7]Our model is well-behaved and instances of boundary fixed points do not arise

$$
\begin{aligned}
x^*_{1\;R_5} &= \frac{(p_2-p_1)-\beta(1-\alpha_1)}{(q_2-q_1)-[(1-\alpha_2)+\beta(1-\alpha_1)]} \\
x^*_{2\;R_5} &= 1 - x^*_{1\;R_5} = \frac{(q_2-q_1)-(p_2-p_1)-(1-\alpha_2)}{(q_2-q_1)-[(1-\alpha_2)+\beta(1-\alpha_1)]}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
x^*_{1\;R6} &= \frac{p_1q_2-p_2q_1+\beta\alpha_1(p_2-q_2)-\beta(p_1-q_1)}{(q_1-1)(\beta-q_2)+(q_1-\alpha_1\beta)(q_1-\alpha_2)} \\
x^*_{2\;R6} &= \frac{p_2q_1-p_1q_1-p_2+p_1\alpha_2+{q_1}^2-q_1q_2+q_2-q_1\alpha_2}{(q_1-1)(\beta-q_2)+(q_1-\alpha_1\beta)(q_1-\alpha_2)}
\end{aligned} \tag{13}
$$

The derivations are mechanical in nature, but we review the implications and properties of their solutions.

First, possible equilibria include instances where one technology wipes out the other while achieving either full $(x_i^* = 1)$ or partial $(0 \le x_i^* < 1)$ market penetration, and instances where both technologies coexist, again at either full $(x_1^*+x_2^* = 1)$ or partial market penetration $(0 \le x_1^* + x_2^* < 1)$. Instances where both technologies die-out, *i.e.,* $\underline{x}^* = (0,0)$, while possible (the equilibrium lies in regions R_3 or R_9), are absent from Table III, as we restrict our focus to scenarios where Technology 1 survives in the absence of the Technology 2's introduction. This precludes a $(0,0)$ outcome.

Second, although not explicitly indicated in Table III, configurations can be found for which the validity and stability conditions of multiple equilibria are simultaneously satisfied. In other words, depending on the initial conditions $\underline{x}(0)$, technology adoption converges to different outcomes. The following proposition identifies the configurations of multiple equilibria that can simultaneously arise for a given set of parameter values.

Proposition 2: The only combination of multiple valid and stable equilibria that can coexist are:

1. $(1,0)$ and $(0,1)$
2. $(x^*_{1_{R_8}},0)$ and $(0,1)$
3. $(x^*_{1_{R_8}},0)$ and $(0,x^*_{2_{R_2}})$
4. $(1,0)$ and $(0,x^*_{2_{R_2}})$
5. $(x^*_{1_{R_5}},1-x^*_{1_{R_5}})$ and $(0,x^*_{2_{R_2}})$
6. $(x^*_{1_{R_6}},x^*_{2_{R_6}})$ and $(0,1)$
7. $(x^*_{1_{R_6}},x^*_{2_{R_6}})$ and $(1,0)$

Additionally, no combination of three or more equilibria can coexist as valid and stable equilibria.

The proof of the above proposition is available in Appendix D. When multiple equilibria arise, the initial market penetration determines the equilibrium to which the adoption process converges. Therefore it is useful to identify the set of all initial market levels, $\underline{x}(0)$, for which the adoption trajectory converges to a particular stable equilibrium. This set is known as the 'Basin of Attraction' of that stable equilibrium. If the stable equilibrium is the only stable equilibrium in the system *i.e.,* globally stable, then the entire region S is its basin of attraction. That is, all starting points lead to the equilibrium. But whenever a pair of stable equilibria coexist, a 'separatrix', demarcating the basins of attraction of the two stable equilibria can be computed. The expressions for the separatrices are provided in Table VI of Appendix E.

Figure 1 provides an illustrative example. The figure, called a phase diagram, shows the path of the diffusion process in the (t,x_1,x_2) space projected onto the (x_1,x_2) plane. In other words, it plots $x_1(t)$ versus $x_2(t)$ and is what one would see if one stood high on the time axis and looked down into the (x_1,x_2) plane, sometimes referred to as the phase plane. We observe that there are two stable steady-state equilibria (of the form $(0,x_2^*)$ and $(x_1^*,0)$) and an unstable equilibrium in R_6. A separatrix passes through this unstable equilibrium, separating the basins of attraction of the stable equilibria.

The framework developed here can be used in a wide range of situations to model the dynamics of adoption. As an illustration of the useful insights that such a model can offer, we apply our model to studying the role of converters in the adoption of incompatible technologies. We see from Tables III and IV that converters can influence (through the parameters α_i) both the validity and the stability of equilibria. In other words, converters may lead technology adoption to an entirely different equilibrium. A rapid inspection of Table II shows that a similar conclusion holds for trajectories. In particular, converters can affect the values of λ_1 and λ_2 of eq. (11). Investigating if and when such changes can happen, is the topic of Section IV.

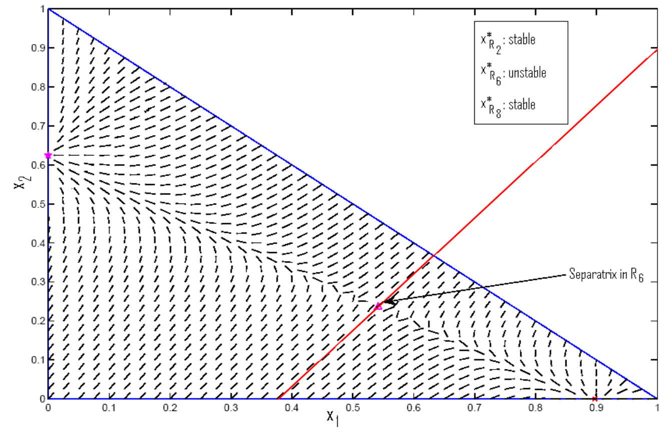

Fig. 1. Separatix and the Basins of Attraction
$(p_1=1.2,q_1=2.95,p_2=2.54,q_2=5.1,\alpha_2=\alpha_1=0.01,\beta=1)$

IV. The Impact of Converters

As we shall see, converters are capable not just of shifting equilibria around; they can also *eliminate* or *create* equilibria. An exhaustive investigation of the full influence of converters, while possible, results in a situation where it is difficult to "see the forest for the trees." As a result, we focus on what we believe are some of the more revealing and significant effects of converters. We identify the reasons behind these effects, and provide conditions under which they can arise.

The investigation proceeds along the following thrusts: (i) Can converters help a network technology improve its market standing and in particular avoid elimination? (ii) Can improving the efficiency of one's converter hurt a technology? (iii) Can improving the efficiency of one's converter hurt the overall market? and (iv) Can the introduction of converters affect overall market stability? Note that when referring to converters of a particular technology, we mean converters developed by that technology provider to let its users communicate with users of the other technology. This distinction is moot when using symmetric converters, but worth highlighting as the model allows it.

A. Impact on Adoption Levels

We begin our investigation with a simple numerical example that illustrates how converters can induce drastic changes in the adoption of network technologies. Specifically, consider the scenario of Figure 2 that shows two adoption outcomes for the same two network technologies $(p_1=1.01,q_1=0.7,p_2=2.5,q_2=2.51,\beta=3)$, with and without converters.

The plot on the left corresponds to a scenario without converters $(\alpha_1=\alpha_2=0)$ and in which Technology 2 eventually eliminates Technology 1 and achieves full market penetration[8]. This corresponds to a single, stable equilibrium $(0,1)$. The right hand plot shows how the use of perfect converters results in the elimination of the original $(0,1)$ equilibrium, so that the only possible outcome of technology adoption is now one where both technologies co-exist.

[8]Note that this is a scenario in which Technology 1 is marginally competitive, *i.e.,* if left alone it would achieve a relatively low market penetration.

TABLE III
CONDITIONS FOR STABLE, VALID EQUILIBRIA

Region	Equilibria	Stability Conditions	Validity and Stability Conditions
R_1	$(0,1)$	always locally stable	$p_2 \leq \beta,\ \alpha_1 \leq \frac{p_1}{\beta} + \frac{q_1}{q_2}(1-\frac{p_2}{\beta})$
R_2	$(0, \frac{p_2-q_2}{\beta-q_2})$	$\beta < q_2$	$\beta < p_2 < q_2$ $\alpha_1\beta(q_2-p_2) \leq \beta(q_1-p_1) + p_1q_2 - p_2q_1$
R_4	$(0,1)$	always locally stable	$p_1 < \alpha_1\beta,\ \ \frac{p_1}{\beta} + \frac{q_1}{q_2}(1-\frac{p_2}{\beta}) \leq \alpha_1 \leq 1 + \frac{p_1-p_2}{\beta}$
R_5	$(x^*_{1R_5}, 1-x^*_{1R_5})$ (See Eq. (12))	$q_2 - q_1 > 1 - \alpha_2 + \beta(1-\alpha_1)$	$p_2 - p_1 > \beta(1-\alpha_1)$ $q_2 - q_1 - (p_2 - p_1) \geq 1 - \alpha_2$ $q_2 - q_1 > \beta(1-\alpha_1) + 1 - \alpha_2$ $\alpha_1\beta(\alpha_2 + q_2 - q_1 - p_2) \geq \beta - p_2 - p_1(\beta - \alpha_2 - (q_2 - q_1))$
R_6	$(x^*_{1R_6}, x^*_{2R_6})$ (See Eq. (13))	See Table IV	$0 < x^*_{1R_6},\ 0 < x^*_{2R_6},\ 0 < x^*_{1R_6} + x^*_{2R_6} < 1$
R_7	$(1,0)$	always locally stable	$p_1 \leq 1,\ \alpha_2 \leq 1 + p_2 - p_1 - (q_2 - q_1)$
R_8	$(\frac{p_1-q_1}{1-q_1}, 0)$	$1 < q_1$	$1 < p_1 < q_1$ $\alpha_2(q_1-p_1) \leq (1-q_1)(q_2-p_2) + q_1(q_1-p_1)$

TABLE IV
STABILITY CONDITIONS FOR $\underline{x}^*_{R_6}$

Case	Conditions
$A^2 - 4B \geq 0$ (Ref. Table II for exp. of A and B)	$A < 0 \Leftrightarrow \beta(1-\alpha_1) - \alpha_2 < 2(q_2-q_1) - \frac{q_2}{q_1}$ $B > 0 \Leftrightarrow (q_1-1)(\beta-q_2) + (q_1-\alpha_1\beta)(q_1-\alpha_2) < 0$
$A^2 - 4B < 0$	$A < 0 \Leftrightarrow \beta(1-\alpha_1) - \alpha_2 < 2(q_2-q_1) - \frac{q_2}{q_1}$

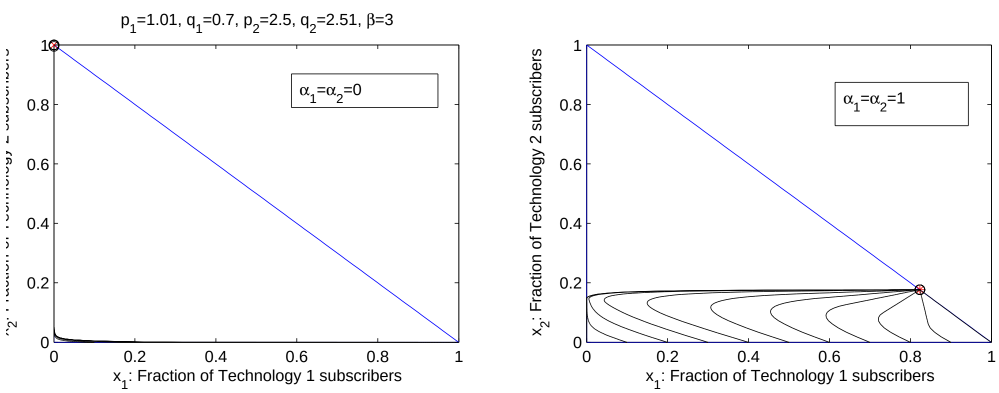

Fig. 2. On the effect of converters on the existence of equilibria.$(p_1 = 1.01, q_1 = 0.7, p_2 = 2.5, q_2 = 2.51, \beta = 3)$

Figure 2 answers our question regarding a technology's ability to avoid elimination through the introduction of converters, and thus leading to a new equilibrium adoption outcome. We now state it more formally in the following proposition.

Proposition 3: Converters can help a technology alter market equilibrium from a scenario where it has been eliminated to one where it coexists with the other technology, or even succeeds in nearly eliminating it.

The proofs of Proposition 3 and subsequent propositions can all be found in Appendix D.

As discussed above, Figure 2 provides a sample configuration illustrating Proposition 3, *i.e.,* Technology 1 goes from elimination to dominating Technology 2 simply by introducing an efficient converter. Table III identifies that the equilibrium $(0,1)$ becomes invalid when the converter efficiency of Technology 1 verifies $\alpha_1 > 1 + \frac{p_1-p_2}{\beta}$. Note that since $0 \leq \alpha_1 \leq 1$, this requires $p_1 < p_2$. Assuming this is the case, the difference between the maximum intra-network benefits of Technology 2 and the maximum cross-networks (through the converter) benefits that the users of Technology 1 derive, becomes at this point equal to the price differential between the two technologies. As a result, low-end users (with small θ values) become indifferent to choosing either technology *i.e.,* $\theta^1_2 = 0$, and any further increase in α_1 leads them to switching to Technology 1. Depending on the values of the other system parameters, it is possible that further increases in α_1 can allow it to nearly eliminate Technology 2. Note that while Technology 1 may succeed in nearly eliminating Technology 2, a small number of users of Technology 2 must remain present to contribute externality benefits to the users of Technology 1, and those scenarios typically require large β values. Note also that as illustrated in Figure 2 that considers symmetric converters, the outcome is not one that can be changed by the other technology deploying its own converters. This is a general phenomenon, and outcomes induced by introducing unidirectional converters can typically not be fully reversed through the deployment of converters operating in the other direction. In other words, most if not all of the results in this section also hold under the constraint of symmetric converters (we will explicitly highlight those that don't).

A similar set of results hold for Technology 2 that, under some conditions, can enjoy the same benefits from converters. The symmetric condition that allows Technology 2 to overcome elimination ($(1,0)$ is now the initial equilibrium), is to introduce a converter whose efficiency α_2 exceeds $\alpha_2 \geq 1 + (p_2 - p_1) - (q_2 - q_1)$. In other words, Technology 2 needs to develop a converter whose efficiency compensates for both the maximum intra-network benefits of Technology 1 and the difference between the price and quality differentials of the two technologies[9]. At that point, $\theta_2^1 = 1$ so that with any further improvement in its converter efficiency, Technology 2 will start attracting some high-end users (large θ values) and eventually re-emerge. As with Technology 1, further improvements in its converter efficiency can in some cases allow Technology 2 to nearly wipe out Technology 1, although again not entirely.

Similar results can also be obtained from Table III for $(x_1^*, 0)$ and $(0, x_2^*)$, *i.e.,* instances when the elimination of a technology does not coincide with full market penetration for the other.

Proposition 3 focused on a scenario where converters allows a technology to avoid elimination. Next, we explore whether it is possible for an increase in converter efficiency to actually harm a technology, *i.e.,* lower its market penetration.

Proposition 4: Technology 1 can hurt its market penetration by introducing a converter and/or improving its efficiency if Technology 2 offers higher externality benefits ($\beta > 1$) and the users of Technology 1 are able to access these benefits ($\alpha_1\beta > 1$). Furthermore, whenever Technology 1 hurts its own market penetration, it also reduces the overall market penetration. Conversely, Technology 2 can never hurt itself while improving its own converter efficiency.

Note that the proposition implicitly assumes asymmetric converters, *i.e.,* explores the effect of unidirectional converter introduction or improvement.

The following discussion tries to shed light on when and why the outcome of Proposition 4 arises. Intuitively, the original impetus for Technology 1 to improve the efficiency of its converters, is to make itself more attractive to potential users by allowing them to better tap into the (higher) externality benefits of Technology 2. It may then attract new users, either from among those that had not previously adopted any technology or among users of Technology 2 who decide to switch to Technology 1. It is the acquisition of the latter type of users that can prove harmful to Technology 1. Specifically, because $\alpha_1\beta > 1$, the switching of users from Technology 2 to Technology 1 negatively affects the externality benefits of all Technology 1 users. When β is high, the decrease in externality benefit can be significant. As illustrated in Figure 4, the result of this decrease can be that some low-end (small θ)

[9]The price differential must be lower than the quality differential, *i.e.,* $p_2 - p_1 < q_2 - q_1$, for this to be possible.

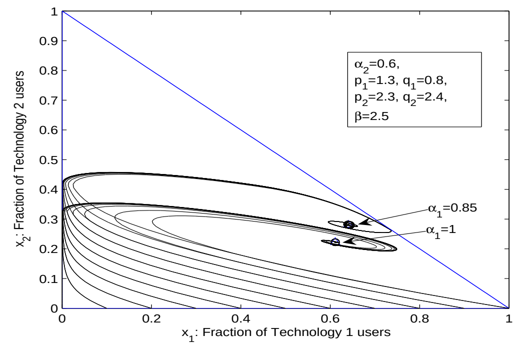

Fig. 3. Better converters harm Technology 1 and the overall market when α_1 is increased from 0.85 to 1.
$(p_1 = 1.3, q_1 = 0.8, p_2 = 2.3, q_2 = 2.4, \alpha_2 = 0.6, \beta = 2.5)$

users decide to leave Technology 1 and exit the market. When the influx of new users is less than the outflow, the overall penetration of Technology 1 decreases. Figure 3 shows an instance of such a decrease. Additionally, the same reasoning shows that this also results in a decrease in overall market penetration (both x_1 and x_2 decrease).

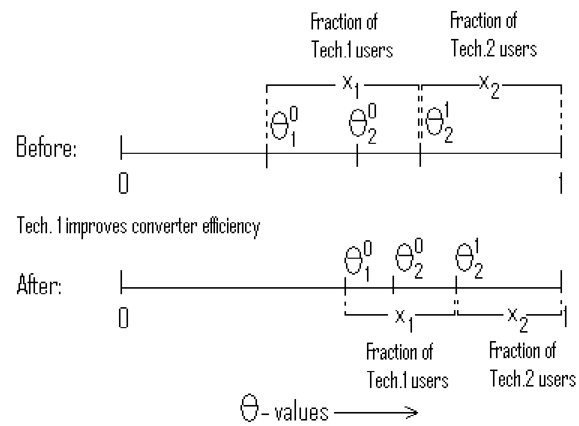

Fig. 4. Technology 1 hurts itself as well as the overall market penetration.

When $\beta \geq 1$, it is easy to see that the above argument does not hold for Technology 2, so that it cannot experience such a reversal when improving its own converter. Appendix D provides a proof that this property actually holds for all values of β, *i.e.,* even when $\beta \leq 1$.

Proposition 4 indicated that Technology 1 could not only hurt itself through better converters, but also the overall market penetration. The next proposition states that this can actually happen because of either Technology 1 or Technology 2, and formally identifies conditions under which this takes place.

Proposition 5: Both technologies can hurt overall market penetration through better converters. Technology 2 can have such an effect only when $\alpha_1\beta < 1$, *i.e.,* Technology 1 users derive lesser externality benefits from connecting to Technology 2 users than to their peers. Conversely, Technology 1 demonstrates this behavior only when $\alpha_1\beta > 1$, *i.e.,* its users derive greater externality benefits from connecting to Technology 2 users than to their peers.

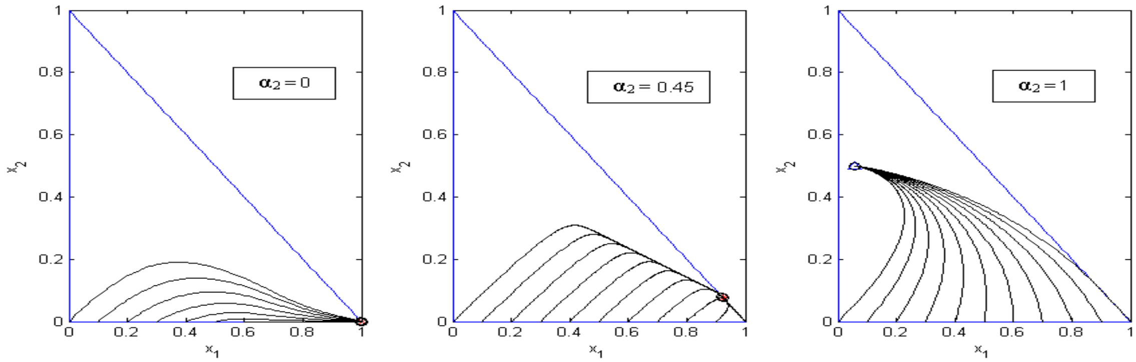

Fig. 5. Greedy Technology 2 harms overall market penetration. ($p_1 = 0.9, q_1 = 1.9, p_2 = 2.7, q_2 = 4.3, \alpha_1 = 0, \beta = 1.2$)

As the discussion of Proposition 4 highlighted how this occurred with Technology 1, we focus instead on Technology 2. The motivation for better converters remains the same, namely, allow users of Technology 2 to derive higher externality benefits by connecting to users of Technology 1. This improvement in the externality benefits of Technology 2 leads some users (those close to the θ_2^1 boundary) to switch. When $\alpha_1\beta < 1$, the migration of those users from Technology 1 to Technology 2 translates into a net drop in the overall utility Technology 1 offers its remaining users (the externality benefits contributed by every user that migrates goes down from a relative weight of 1 to one of $\alpha_1\beta < 1$). This decrease in Technology 1 value then leads some low valuation users (small θ) to drop out altogether, which brings the overall market penetration down.

Figure 5 provides a representative example. In this configuration, in the absence of converters, Technology 1 had reached full market penetration. When Technology 2 introduces a converter of efficiency $\alpha_2 = 0.45$, it emerges and both technologies coexist at equilibrium, while still achieving full market penetration. If the efficiency of Technology 2 converter further improves, it still sees a rise in its own market penetration, but the overall market penetration now decreases to $\approx 55\%$, as low valuation users drop out.

B. Impact on Adoption Dynamics

The previous sub-section explored the effect that converters can have on equilibria. In this sub-section we extend the investigation to both trajectories and equilibria. In particular, we concentrate on an unexpected effect of converters, one that can be shown not to be possible in their absence, namely, the possibility that the introduction of converters can render the process of technology adoption unstable. In the next proposition, we quantify this potential for instability and the conditions under which it can arise.

Proposition 6: The introduction of converters can create "boom and bust" cycles in the technology adoption process. This behavior arises only when Technology 2 exhibits higher externality benefits ($\beta > 1$) than Technology 1 and the users of Technology 1 are unconstrained in their ability to access these benefits ($\alpha_1\beta > 1$).

Conversely, the next corollary establishes that this never occurs in the absence of converters. The proofs of both are again in Appendix D.

Corollary 4.1: In the absence of converters, technology adoption trajectories always converge to a stable equilibrium.

Before trying to offer some insight into the emergence of instabilities when converters are introduced, we offer an example to illustrate the type of outcomes that can arise.

Figure 6 provides a sample scenario of converters affecting the stability of technology adoption, and in particular introducing cycles in the adoption trajectories. The left-hand-side of the figure shows how in the absence of converters, Technology 2 displaces Technology 1 and achieves full market penetration. The introduction of a reasonably efficient converter ($\alpha_1 \approx 0.623$) by Technology 1, however, drastically changes the situation by introducing two new equilibria; both of them unstable (middle diagram). As a result, while the original equilibrium of $(0, 1)$ remains valid, its basin of attraction has now shrunk considerably . Instead, under most initial conditions, a cyclical pattern of adoption decisions emerges. In other words, users repeatedly switch back and forth between the two network technologies. Matters only become worse if the efficiency of the converter of Technology 1 continues improving[10], and with a perfect converter the original equilibrium of $(0, 1)$ has all but disappeared and only one, unstable equilibrium remains around which adoption decisions keep circling.

The intuition behind the emergence of such a situation is somewhat similar to that of a technology harming itself and/or the overall market through the introduction of better converters. Specifically, consider an instance where Technology 2 offers higher externality benefits that users of Technology 1 can tap into if a converter is available. When converters are absent, users that value the higher quality of Technology 2 adopt it (when it offers a higher overall utility), eventually leading to full adoption as shown on the left most part of Figure 6. However, once a converter is introduced, users have the option to remain with Technology 1 (and enjoy its lower price) without forfeiting all the benefits of Technology 2, and in particular its externality benefits. As a result, while Technology 2 will initially still gain market share by attracting high technology valuation users away from Technology 1, this now happens with Technology 1 also gaining new customers

[10] As mentioned before, similar situations arise under symmetric converters.

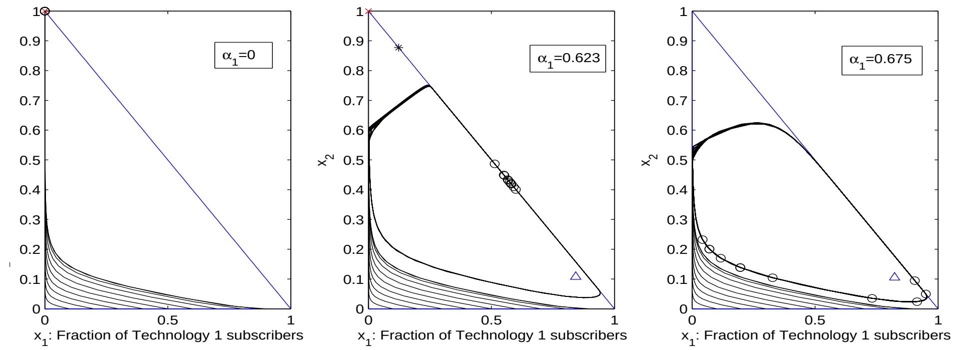

Fig. 6. Effect of converters on adoption stability $(p_1 = 1.05, q_1 = 0.4, p_2 = 2.1, q_2 = 2.11, \alpha_2 = 0.3, \alpha_1 = 0.675, \beta = 2.8)$

(low technology valuation customers are now adopting because of the externality benefits accessible through the new users who joined Technology 2). This combined effects results in a steady increase in overall market share until a limit is reached. This limit corresponds to a point where Technology 2 has tapped out all the high technology valuation users it could attract. As Technology 2 growth tapers off, Technology 1 continues growing as it still attracts new low technology valuation customers. Continued growth in Technology 1 customer base eventually makes it attractive to some mid-range technology valuation customers (the latest ones to have joined Technology 2) that start switching back to it. This fuels an accelerated growth in the user base of Technology 1 that now acquires customers from both Technology 2 and non-adopters. This continues until the user base of Technology 2 becomes so small that it starts affecting the ability of Technology 1 to grow. At this point, both technologies start loosing customers. This ends when the customer base of Technology 1 is small enough to allow Technology 2 to again start attracting customers (its own customer base had by then all but disappeared), and the process repeats anew.

Quantitative support for the above intuition can be extracted from Table II. Consider trajectories in the region R_6. Their expressions include a term of the form $\sqrt{A^2 - 4B}$, where both A and B depend on the converter efficiency terms α_1 and α_2. Changing α_1 and/or α_2 can affect the sign of $A^2 - 4B$, and in particular allow it to become negative. This introduces complex exponents, or rather $\cos(.)$ and $\sin(.)$ terms in the adoption trajectories. It is these terms that allow the "changes of direction" needed for cycles in the adoption trajectories. When converters are absent $(\alpha_1 = \alpha_2 = 0)$, $A^2 - 4B$ is always positive and trajectories cannot change direction (their slope has a constant sign), which precludes the cycles. This is essentially what is behind Corollary 4.1.

V. Related work

Modeling the diffusion of new products and technologies has a long tradition in marketing. Fourt and Woodlock [8] first proposed a product diffusion model in which a fixed fraction of consumers who have not yet bought the product do so at every period; this is known as the constant hazard rate model. Bass [1] proposed an extension that additionally incorporates word-of-mouth communication between current adopters and potential adopters. A large body of work has since built on these earlier models (see [13] for an overview of this literature). Although most of the literature deals with single-product settings, Norton and Bass [15] study the joint diffusion of successive generations of technologies. Their model belongs to a class of substitution models that assume that the newer generation eventually replaces the earlier generation and thus their interest is only in the time it takes for this to occur. Significantly, both single-product and multiple-generation diffusion models focus on aggregate adoption dynamics without explicitly modeling individual decision-making processes. The advantage of such an approach is that it results in relatively simple diffusion models that can, in turn, be used to study dynamic policies (*e.g.,* dynamic pricing). Unfortunately, these aggregate models do not shed sufficient light on the decision processes that lead to the emergent system dynamics or the exact mechanism through which various decision variables (pricing, quality, advertising, etc.) impact adoption decisions.

A few models have focused on individual-level adoption (*e.g.,* [10]). These models provide far greater insight into the mechanism through which rational individual decision-making results in aggregate system dynamics. Given the complexity of these models, much of the progress to date has been in settings with a single technology. In contrast, the adoption of new network technologies is often influenced by incumbents. Moreover, all of the above models and indeed much of the literature refers to generic durables, *e.g.,* washing machines. Such models do not account for the unique features of network technologies, including network externalities and the role of converters.

A recent stream of work in economics has studied the role of network externalities on equilibrium adoption of standards and technologies. Cabral [2] develops a model of individual decision-making in the presence of network externalities and characterizes the aggregate adoption dynamics. He shows that network externalities are potential drivers of S-shaped diffusion curves. We build on Cabral's model but differ in our focus by considering a two-technology setting. Put another way, we are interested in the diffusion of a new network technology in the presence of an incumbent. A related paper by Farrell and Saloner [6] evaluates the impact of an installed base on the transition to a new standard. They show that the installed base can cause "excess inertia" which prevents the transition to the new standard. At the same time, the adoption of the new standard by a few users can create "excess momentum" as well. In their model, users are homogeneous except for the time of their arrival into the system. As a result, they observe a bandwagon effect in which the adoption of a standard by one set of users makes the same choice more attractive to all other users. Thus, one standard always wins and coexistence is not feasible. Choi [3] extends the model by Farrell and Saloner [6] to include converters and shows that converters can in some instances blockade the transition by weakening the threat of being stranded for users of the incumbent technology. In a more recent study, Joseph et al. [11] also show that increase in efficiency of a converter can hinder the adoption of a new network architecture.

An important distinction of our work relative to these papers is that we incorporate heterogeneity in user preferences. We show that this gives rise to equilibria in which the technologies may coexist, *i.e.,* neither network technology fully captures the market. Further, very little attention is paid to the adoption path in these papers because all users make the same decision. In contrast, we show that the heterogeneity across users can result in interesting adoption dynamics including non-monotonic evolution of the market shares of the technologies. Additionally, these papers focus on environments in which users make the decisions to adopt the converters. This is meaningful in environments in which the converter functionality and its deployment resides with individual users, *e.g.,* converters for two incompatible software applications that a user decides to download. In contrast, our interest is in environments in which converters are usually deployed by the technology providers upon incurring high fixed costs, and in the process made available to all its users.

VI. Conclusion and Extensions

The paper provides a framework to study the adoption and diffusion of a new network technology in the presence of an incumbent and offers insight into the role of converters. Our model accounts for both externalities and user heterogeneity, and helps reveal several unexpected behaviors. Of note are that the presence of converters can hurt overall market penetration, and that under certain conditions they can preclude the adoption process from ever converging. As shown in Appendix F, those behaviors remain present across a wide range of utility models that differ from the one used for analytical tractability in this paper.

As the first step of our investigation in the dynamics of technology adoption in the presence of converters, the paper and its model clearly have limitations that we plan to address in the future. As mentioned earlier, allowing some of the system parameters to be time-varying is of obvious interest. Similarly, letting prices be endogenous variables, *e.g.,* to optimize revenue or as a result of dynamic pricing policies, is another direction we have started investigating.

In addition, while the paper concentrated on a setting in which switching costs are negligible, there are obviously many environments where such an assumption is inadequate. Exploring the extent to which the results are robust when switching costs are present is, therefore of interest. For example, very high switching costs may make it infeasible for users to switch back and forth between technologies, thereby affecting the likelihood of observing boom-and-bust cycles.

Our work represents an initial step towards understanding adoption dynamics of network technologies. Further work building on this paper would likely provide further insight.

VII. Acknowledgments

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Appendix

A. Solution Regions

This Appendix shows why eq. (10) gives rise to a partition of the (x_1, x_2) plane that, irrespective of the choice of system parameters, consists of only nine regions with fixed relative positions, and each associated with a unique expression for $H_i(\underline{x}),\ i = \{1, 2\}$.

The line $\theta_1^0(\underline{x}) = \theta_2^0(\underline{x})$ separates regions that require different expressions of $H_i(\underline{x}), i = \{1, 2\}$. For points above that line $(\theta_1^0(\underline{x}) > \theta_2^0(\underline{x}))$, the expression of $H_2(\underline{x})$ depends on the projection of $\theta_2^0(\underline{x})$ on $[0, 1]$. Therefore the lines $\theta_2^0(\underline{x}) = 0$ and $\theta_2^0(\underline{x}) = 1$ delineate regions associated with different $H_2(\underline{x})$. Similarly, for points below that line, the lines $\theta_1^0(\underline{x}) = 0$, $\theta_1^0(\underline{x}) = 1$, $\theta_2^1(\underline{x}) = 0$ and $\theta_2^1(\underline{x}) = 1$ introduce additional region boundaries for $H_i(\underline{x}),\ i = \{1, 2\}$. Expressions for all the lines can be obtained from eqs. (5) to (7).

Arguably, so many different lines with slopes varying with system parameters could give rise to arbitrary intersections, and therefore patterns of regions. Fortunately, this is not so. It can be easily shown[11] that irrespective of the choice of system parameters, the lines $\theta_2^0(\underline{x}) = 0, \theta_1^0(\underline{x}) = 0$ and $\theta_2^1(\underline{x}) = 0$ always intersect at a point P, and the lines $\theta_2^0(\underline{x}) = 1, \theta_1^0(\underline{x}) = 1$ and $\theta_2^1(\underline{x}) = 1$ always intersect at a point Q, with both P and Q lying on the line $\theta_1^0 = \theta_2^0$. The points P and Q act as "anchors" of the partition of the solution space. A representative configuration is shown in Figure 7, where reach region has been labeled as in Table I.

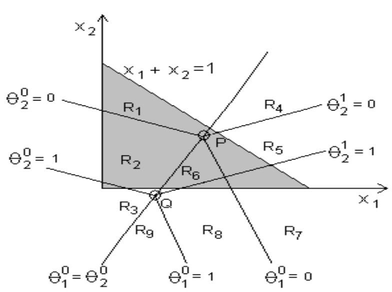

Fig. 7. Region Partitions

B. Generalized Region Partition

The analysis of solution regions in the above subsection was based on the fact that for linear externality functions, the (x_1, x_2)-plane can be partitioned into nine regions, each representing a unique ordering of the indifference points, and therefore different expressions for $H_i(\underline{x})$ and diffusion trajectory. The unique points P and Q on the line $\theta_1^0(\underline{x}) = \theta_2^0(\underline{x})$ acted as 'pivots' for the partition of the plane into nine regions. Although one may expect that for arbitrary externality functions, the lines denoting the region boundaries will intersect in arbitrary ways, we show here that even for more generic monotonically increasing network externality functions, the two 'pivot' points P and Q remain unique. This uniqueness of P and Q, along with constraints on how the boundary lines can intersect as imposed by the monotonic property of externality functions, result in the partitioning of the plane into "nine" regions.

Let the network externality of the two technologies, Technology 1 and 2 be given by positive increasing externality functions $g_1(x_1)$ and $g_2(x_2)$ for the respective adoption levels of x_1 and x_2 (*i.e.,* $g_i(x_i) \geq 0,\ g_i'(x_i) > 0,\ i = \{1, 2\}$).

The end user's utility from using Technologies 1 and 2 is given by:

$$U_1 = \theta q_1 + (g_1(x_1) + \alpha_1 \beta g_2(x_2)) - p_1 \quad (14)$$

$$U_2 = \theta q_2 + (\beta g_2(x_2) + \alpha_2 g_1(x_1)) - p_2 \quad (15)$$

Setting $U_i(\theta, \underline{x}) = 0$, we get

$$\theta_1^0(\underline{x}) = \frac{p_1 - (g_1(x_1) + \alpha_1 \beta g_2(x_2))}{q_1} \quad (16)$$

$$\theta_2^0(\underline{x}) = \frac{p_2 - (\beta g_2(x_2) + \alpha_2 g_1(x_1))}{q_2} \quad (17)$$

Similarly, setting $U_1(\theta, \underline{x}) = U_2(\theta, \underline{x})$ gives

$$\theta_2^1(\underline{x}) = \frac{(1-\alpha_2) g_1(x_1) - \beta(1-\alpha_1) g_2(x_2) + p_2 - p_1}{q_2 - q_1} \quad (18)$$

To simplify notation, we use from now on θ_i^0 and θ_2^1 instead of $\theta_i^0(\underline{x})$ and $\theta_2^1(\underline{x})$. After simple manipulations, we get

$$\theta_2^1 - \theta_1^0 = \frac{q_2}{q_2 - q_1}(\theta_2^0 - \theta_1^0), \quad (19)$$

$$\theta_2^1 - \theta_2^0 = \frac{q_1}{q_2 - q_1}(\theta_2^0 - \theta_1^0) \quad (20)$$

Given that Technology 2, the entrant, is technically superior (*i.e.,* $q_2 > q_1$), from the above relation we establish the following Proposition.

Proposition 7: If $\theta_1^0 < \theta_2^0$, then $\theta_2^1 > \theta_2^0 > \theta_1^0$. If $\theta_1^0 \geq \theta_2^0$, then $\theta_2^1 \leq \theta_2^0 \leq \theta_1^0$.

$$H_1(\underline{x}) = \begin{cases} [\theta_2^1]_{[0,1]} - [\theta_1^0]_{[0,1]} & \text{if } \theta_1^0 < \theta_2^0 \\ 0 & \text{otherwise} \end{cases}$$

$$H_2(\underline{x}) = \begin{cases} 1 - [\theta_2^1]_{[0,1]} & \text{if } \theta_1^0 < \theta_2^0 \\ 1 - [\theta_2^0]_{[0,1]} & \text{otherwise} \end{cases} \quad (21)$$

where $x_{[a,b]}$ is the projection of x into the interval $[a, b]$, *i.e.,* is equal to x for $x \in [a, b]$, a for $x < a$, and b for $x > b$.

As the preference levels θ of all users lie in $[0, 1]$, Equation (21) fully determine $H_i(\underline{x})$, albeit with possibly different expressions depending on the outcome of the projections of the indifference thresholds on $[0, 1]$. Hence, we partition the (x_1, x_2) plane into regions where $H_i(\underline{x})$ has a unique expression. This can be achieved by combining Equation (16) to (18) with Equation (21).

The line $\theta_1^0(\underline{x}) = \theta_2^0(\underline{x})$ separates regions that require different expressions of $H_i(\underline{x}), i = \{1, 2\}$. For points above that line $(\theta_1^0(\underline{x}) > \theta_2^0(\underline{x}))$, the expression of $H_2(\underline{x})$ depends on the projection of $\theta_2^0(\underline{x})$ on $[0, 1]$. Therefore the lines $\theta_2^0(\underline{x}) = 0$ and $\theta_2^0(\underline{x}) = 1$ delineate regions associated with

[11] By solving simple systems of linear equations.

different $H_2(\underline{x})$. Similarly, for points below that line, the lines $\theta_1^0(\underline{x}) = 0$, $\theta_1^0(\underline{x}) = 1$, $\theta_2^1(\underline{x}) = 0$ and $\theta_2^1(\underline{x}) = 1$ introduce additional region boundaries for $H_i(\underline{x}),\ i = \{1, 2\}$. Expressions for all the lines can be obtained from eqs. (16) to (18).

Next we show that irrespective of the choice of system parameters, the lines $\theta_2^0(\underline{x}) = 0, \theta_1^0(\underline{x}) = 0$ and $\theta_2^1(\underline{x}) = 0$ always intersect at a point P, and the lines $\theta_2^0(\underline{x}) = 1, \theta_1^0(\underline{x}) = 1$ and $\theta_2^1(\underline{x}) = 1$ always intersect at a point Q, with both P and Q lying on the line $\theta_1^0(\underline{x}) = \theta_2^0(\underline{x})$. The points P and Q act as "anchors" of the partition of the solution space.

We denote the lines $\theta_1^0(\underline{x}) = 0$ and $\theta_2^0(\underline{x}) = 0$ by functions $f_1(x_1, x_2) = 0$ and $f_2(x_1, x_2) = 0$. Let (x_1^*, x_2^*) denote the co-ordinates in (x_1, x_2)-plane where these lines intersect (*i.e.,* $f_i(x_1^*, x_2^*) = 0, i = \{1, 2\}$). Note that the lines $\theta_2^1(\underline{x}) = 0$ and $\theta_1^0(\underline{x}) = \theta_2^0(\underline{x})$, which can then be represented as $f_2(x_1, x_2) - f_1(x_1, x_2) = 0$ and $(1/q_2)f_2(x_1, x_2) - (1/q_1)f_1(x_1, x_2) = 0$ respectively, also must pass through (x_1^*, x_2^*). This point of intersection of all these lines can be labeled as P. Additionally, it can be seen that if any two of these lines intersect at some point, all the other curves must also pass through that point. Similarly, we obtain the other 'pivot' point, Q, at which the lines $\theta_1^0(\underline{x}) = \theta_2^0(\underline{x}), \theta_2^0(\underline{x}) = 1, \theta_1^0(\underline{x}) = 1$ and $\theta_2^1(\underline{x}) = 1$ must intersect.

Proof of Uniqueness of P and Q

Let us consider the intersection of the lines $\theta_1^0(\underline{x}) = 0$, $\theta_2^0(\underline{x}) = 0$, $\theta_2^1(\underline{x}) = 0$ and $\theta_1^0(\underline{x}) = \theta_2^0(\underline{x})$ at some point P as shown in Figure 8. Assume that there exist another such point P' where all the lines again intersect (because we showed that whenever any two of these lines intersect, the other lines should also intersect). However, using eqn.(5-7) and $g_i'(x_i) \geq 0,\ i = \{1, 2\}$, we see that the line $\theta_2^1 = 0$ is always increasing in x_1 and x_2, while the line $\theta_0^1 = 0$ is decreasing in x_2 for increase in x_1. Therefore in the entire region $\theta_1^0 < \theta_2^0$ the lines can only intersect once, and therefore the point P must be unique. A similar argument holds for point Q as well.

Thus the (x_1, x_2)-plane can only be partitioned into the nine regions shown in Figure 8, and the relative positions of these regions in the plane remain fixed. Moreover, each of these regions is a connected set. As shown in the figure, each region corresponds to a different arrangement of the indifference points with respect to the $0, 1$ boundary under the two feasible orderings (from Proposition 7). The classification of these nine regions based on the different orderings is provided in Table V. A brief explanation of the meaning of the regions is provided next.

Meaning of Regions

Each region correspond to a particular ordering of the indifference points, which in turn maps to unique expressions for $H_i(\underline{x})$ in eqn.(21). For example, consider the region R_8, which is the set of all (x_1, x_2) penetration levels for which $\theta_1^0 < \theta_2^0$, $1 \leq \theta_1^2$ and $0 \leq \theta_1^0 < 1$. In this region, because $\theta_1^2 > 1$ for any current (x_1, x_2) adoption levels, no user has a preference for choosing Technology 2 over 1. But since $0 < \theta_1^0 < 1$, some users whose preference $\theta_1^0 < \theta$ derive positive utility from Technology 1, will be willing to adopt it.

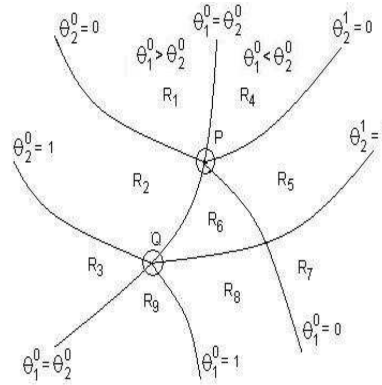

Fig. 8. Generalized Region Partitions

TABLE V
PARTITIONS CHARACTERIZING $H_i(\underline{x})$

$\theta_1^0 \geq \theta_2^0$		$\theta_1^0 < \theta_2^0$		
Region	condition	Region	condition	
R_1	$\theta_2^0 \leq 0$	R_4	$\theta_2^1 \leq 0,$	$0 \leq \theta_1^0$
R_2	$0 < \theta_2^0 < 1$	R_5	$0 < \theta_2^1 < 1,$	$\theta_1^0 \leq 0$
R_3	$1 \leq \theta_2^0$	R_6	$0 < \theta_2^1 < 1,$	$0 < \theta_1^0 < 1$
		R_7	$1 \leq \theta_2^1,$	$\theta_1^0 \leq 0$
		R_8	$1 \leq \theta_2^1,$	$0 < \theta_1^0 < 1$
		R_9	$1 \leq \theta_2^1,$	$1 \leq \theta_1^0$

Therefore if at any instant t, the system reaches adoption levels (x_1, x_2) in Region R_8, the diffusion from that point on in the inside of Region R_8 will proceed with a decrease in the value of x_2 *i.e.,* users leave Technology 2 as $\theta_2^1 > 1$. This fact is also reflected in the exponentially decreasing value of the $x_2(t)$ co-ordinate of the diffusion trajectory in R_8 (as given in Table II). Each region can be interpreted in a similar manner, and Table V essentially connects this abstract notion of each region to the corresponding ordering of the indifference thresholds that define it.

C. Conditions for valid and stable equilibria

The expressions in Table III and IV for the validity and stability conditions for each of the equilibrium are rearranged and presented below for clarity.

Region R_1 – Equilibrium: $(0, 1)$
Validity and Stability Conditions are:

$$p_2 \leq \beta \tag{22}$$
$$\alpha_1 \leq \frac{p_1}{\beta} + \frac{q_1}{q_2}\left(1 - \frac{p_2}{\beta}\right) \tag{23}$$

Region R_2 – Equilibrium: $(0, \frac{p_2 - q_2}{\beta - q_2})$
Validity and Stability Conditions:

$$\beta < p_2 < q_2 \tag{24}$$
$$\alpha_1 \leq \frac{\beta(q_1 - p_1) + p_1 q_2 - p_2 q_1}{\beta(q_2 - p_2)} \tag{25}$$

Region R_3 – Candidate equilibrium $(0,0)$

By assumption, this is not a feasible equilibrium.

Region R_4 – Equilibrium: $(0,1)$

Validity and Stability Conditions:

$$p_1 \leq \alpha_1\beta \tag{26}$$

$$\frac{p_1}{\beta} + \frac{q_1}{q_2}\left(1 - \frac{p_2}{\beta}\right) \leq \alpha_1 \leq \frac{p_1}{\beta} + 1 - \frac{p_2}{\beta} \tag{27}$$

Region R_5 – Equilibrium: $({x^*_1}_{R5}, {x^*_2}_{R5})$

$$\begin{aligned} {x^*_1}_{R5} &= \frac{(p_2 - p_1) - \beta(1-\alpha_1)}{(q_2 - q_1) - [\beta(1-\alpha_1) + (1-\alpha_2)]} \\ {x^*_2}_{R5} &= 1 - {x^*_1}_{R5} = \frac{(q_2 - q_1) - (p_2 - p_1) - (1-\alpha_2)}{(q_2 - q_1) - [\beta(1-\alpha_1) + (1-\alpha_2)]} \end{aligned}$$

Rewriting the Validity and Stability conditions given in Table III, we have:

$$p_2 - p_1 > \beta(1-\alpha_1) \tag{28}$$

$$q_2 - q_1 - (p_2 - p_1) \geq 1 - \alpha_2 \tag{29}$$

$$q_2 - q_1 > \beta(1-\alpha_1) + 1 - \alpha_2 \tag{30}$$

$$\alpha_1\beta(\alpha_2 + q_2 - q_1 - p_2) \geq \beta - p_2 - p_1(\beta - \alpha_2 - (q_2 - q_1)) \tag{31}$$

Region R_6 – Equilibrium: $({x^*_1}_{R6}, {x^*_2}_{R6})$

$$\begin{aligned} {x^*_1}_{R6} &= \frac{p_1q_2 - p_2q_1 + \beta\alpha_1(p_2 - q_2) - \beta(p_1 - q_1)}{(q_1 - 1)(\beta - q_2) + (q_1 - \alpha_1\beta)(q_1 - \alpha_2)} \\ {x^*_2}_{R6} &= \frac{p_2q_1 - p_1q_1 - p_2 + p_1\alpha_2 + q_1^2 - q_1q_2 + q_2 - q_1\alpha_2}{(q_1 - 1)(\beta - q_2) + (q_1 - \alpha_1\beta)(q_1 - \alpha_2)} \end{aligned}$$

The validity conditions for these equilibrium expression requires ${x^*_1}_{R6} \geq 0$, ${x^*_2}_{R6} \geq 0$ and ${x^*_1}_{R6} + {x^*_2}_{R6} < 1$. We will denote the numerators of ${x^*_1}_{R_6}$ and ${x^*_2}_{R_6}$ in eq. (13) as N_1 and N_2, respectively, and their common denominator as D. Table IV shows that the if $A^2 - 4B \geq 0$ the stability conditions require $A < 0$ and $B > 0$, while if $A^2 - 4B < 0$ then $A < 0$ is required (and $B > 0$ since $B > A^2/4 > 0$). Thus an equilibrium in R_6 can satisfy stability conditions only if $B > 0$ and $A < 0$. Additionally $B > 0$ implies that the denominator of the expressions for the equilibrium adoption levels (given in eqs. (13)) is negative (*i.e.,* $D < 0$).

Therefore a valid, stable equilibrium in R_6 must have:

$$\begin{aligned} &A < 0: \\ &\beta(1-\alpha_1) - \alpha_2 < 2(q_2 - q_1) - \frac{q_2}{q_1} \end{aligned} \tag{32}$$

$$\begin{aligned} &D < 0 \ (B > 0): \\ &(q_1 - 1)(\beta - q_2) + (q_1 - \alpha_1\beta)(q_1 - \alpha_2) < 0 \end{aligned} \tag{33}$$

$$\begin{aligned} &N_1 \leq 0: \\ &\alpha_1\beta(q_2 - p_2) \geq \beta(q_1 - p_1) + p_1q_2 - p_2q_1 \end{aligned} \tag{34}$$

$$\begin{aligned} &N_2 \leq 0: \\ &\alpha_2(q_1 - p_1) \geq (1 - q_1)(q_2 - p_2) + q_1(q_1 - p_1) \end{aligned} \tag{35}$$

$$\begin{aligned} &\frac{N_1 + N_2}{D} < 1: \\ &\alpha_1\beta(\alpha_2 + q_2 - q_1 - p_2) \\ &< \beta - p_2 - p_1(\beta - \alpha_2 - (q_2 - q_1)) \end{aligned} \tag{36}$$

Region R_7 – Equilibrium: $(1,0)$

Validity and Stability Conditions:

$$p_1 \leq 1 \tag{37}$$

$$\alpha_2 < 1 + (p_2 - p_1) - (q_2 - q_1) \tag{38}$$

Region R_8 – Equilibrium: $(\frac{p_1 - q_1}{1 - q_1}, 0)$

Validity and Stability Conditions:

$$1 < p_1 < q_1 \tag{39}$$

$$\alpha_2(q_1 - p_1) \leq (1 - q_1)(q_2 - p_2) + q_1(q_1 - p_1) \tag{40}$$

Region R_9 – Equilibrium: $(0,0)$

By assumption, $(0,0)$ is not a feasible equilibrium.

D. Proofs of Propositions

Proof of Proposition 2:

In this proof we will show that the following pairs of equilibria cannot coexist together as valid and stable equilibria. Consequently, it is easy to verify that the only combination of multiple equilibria that can coexist are the ones mentioned in Proposition 2.

1. $(1,0)$ and $\underline{x}^*_{R_8}$
2. $(1,0)$ and $\underline{x}^*_{R_5}$
3. $(0,1)$ and $\underline{x}^*_{R_2}$
4. $(0,1)$ and $\underline{x}^*_{R_5}$
5. $\underline{x}^*_{R_8}$ and $\underline{x}^*_{R_5}$
6. $\underline{x}^*_{R_8}$ and $\underline{x}^*_{R_6}$
7. $\underline{x}^*_{R_2}$ and $\underline{x}^*_{R_6}$
8. $\underline{x}^*_{R_5}$ and $\underline{x}^*_{R_6}$

The following analysis will use the expressions for validity and stability conditions for the different equilibria listed in the Subsection C of the Appendix.

1. $(1,0)$ and $\underline{x}^*_{R_8}$

This pair cannot coexist because the equilibrium $(1,0)$ in R_7 requires $p_1 \leq 1$ (eq. (37)) while the equilibrium $\underline{x}^*_{R_8}$ requires $p_1 > 1$ (eq. (39)).

2. $(1,0)$ and $\underline{x}^*_{R_5}$

Eq.(38) for equilibrium $(1,0)$ in R_7 and condition in eq.(29) for $\underline{x}^*_{R_5}$ have contradictory requirements, and therefore these pair cannot coexist.

3. $(0,1)$ and $\underline{x}^*_{R_2}$

The equilibrium $(0,1)$ can either exist in Region R_1 or R_4. In either it requires $\beta \geq p_2$ for being a valid, stable equilibrium. While this is explicit for Region R_1 (refer to eq.(22)), the relation is implicitly implied by the conditions of Region R_4. Note that eq.(26) and eq.(27) can be written together as $0 \leq \alpha_1\beta - p_1 \leq \beta - p_2$. Thus $(0,1)$ equilibria requires $\beta \geq p_2$ which contradicts with the requirement in eq.(24) for $\underline{x}^*_{R_2}$.

4. $(0,1)$ and $\underline{x}^*_{R_5}$

The equilibrium $(0,1)$ is valid in R_4 if $\alpha_1 \leq \frac{p_1}{\beta}+1-\frac{p_2}{\beta}$. It is valid in R_1 if the bound is stricter *i.e.,* $\alpha_1 \leq \frac{p_1}{\beta}+\frac{q_1}{q_2}\left(1-\frac{p_2}{\beta}\right)$ (since $\frac{q_1}{q_2}<1$ and $\beta \geq p_2$).

However equilibrium $\underline{x}^*_{R_5}$ requires $\alpha_1 > \frac{p_1}{\beta}+1-\frac{p_2}{\beta}$ from eq.(28). Therefore the two equilibria cannot coexist.

5. $\underline{x}^*_{R_8}$ and $\underline{x}^*_{R_5}$

Equilibria $\underline{x}^*_{R_8}$ requires $q_1 > p_1$ from eq.(39). When this relation holds, the condition in eq.(40) for $\underline{x}^*_{R_8}$ and eq.(29) for equilibria $\underline{x}^*_{R_5}$ can be written as:

$$\begin{aligned} &\frac{(1-q_1)(q_2-p_2)+q_1(q_1-p_1)}{q_1-p_1} \geq \alpha_2 \\ &\qquad > 1-(q_2-q_1)+p_2-p_1 \\ \Rightarrow \quad &(q_2-q_1-(p_2-p_1)) \\ &\qquad > p_1(q_2-q_1-(p_2-p_1)) \end{aligned} \tag{41}$$

Since $(q_2-q_1-(p_2-p_1)) \geq 1-\alpha_2 \geq 0$ from eq.(29), we must have $p_1 < 1$, which contradicts with the requirement in eq.(39) for the equilibrium $\underline{x}^*_{R_8}$. Therefore this pair cannot coexist.

6. $\underline{x}^*_{R_8}$ and $\underline{x}^*_{R_6}$

Condition in eqs.(39) and (40) when considered together contradicts the requirement of eq.(35). Therefore these equilibria cannot coexist as valid, stable equilibria.

7. $\underline{x}^*_{R_2}$ and $\underline{x}^*_{R_6}$

Eqs.(24) and (25) for $\underline{x}^*_{R_2}$ together contradict the requirement in eq.(34) for $\underline{x}^*_{R_6}$, and thus cannot coexist as valid, stable equilibria pair.

8. $\underline{x}^*_{R_5}$ and $\underline{x}^*_{R_6}$

The condition in eq.(31) for $\underline{x}^*_{R_5}$ and eq.(36) cannot hold together and therefore these equilibria never coexists as a pair of valid, stable equilibria.

Proof: No combination of three or more equilibria can coexist as valid, stable equilibria in the presence of converters

Given Proposition 2, all but one combination of three equilibria can be excluded from further consideration as at least a pair of equilibria in these combinations will not coexist as per the proposition. The only combination of three equilibria that can potentially coexist is $\{(0,1),(1,0),\underline{x}^*_{R_6}\}$. We will show that the validity and stability conditions of these three equilibria cannot be satisfied together. Thus since no pair of three equilibria may coexist, it will directly follow that no combination of four or more equilibria can therefore coexist, thus proving the present proposition.

The equilibrium $(0,1)$ to exist in R_4 can be shown to require $\beta \geq p_2$ and $\alpha_1\beta \leq \beta+p_1-p_2$ from eqs.(26) and (27). For $(0,1)$ to exist in R_1 the constraint imposed by eq.(23) is even more stringent than $\alpha_1\beta \leq \beta+p_1-p_2$. Therefore the validity and stability of $(0,1)$ requires at least $\alpha_1\beta \leq \beta+p_1-p_2$ and $\beta \geq p_2$. Using this and eq.(38) for $(1,0)$ in R_7, we have:

$$\begin{aligned} (\alpha_1-1)\beta \quad &\leq p_1-p_2 \leq \quad 1-\alpha_2-(q_2-q_1) \\ \Rightarrow (\alpha_1-1)\beta \quad &\leq \quad 1-\alpha_2-(q_2-q_1) \end{aligned}$$

Using the above inequality and eq.(32), we get:

$$\begin{aligned} \Rightarrow q_2-q_1-1 \quad &\leq \quad \beta(1-\alpha_1)-\alpha_2 \\ &< \quad 2(q_2-q_1)-q_2/q_1 \\ \Rightarrow (q_2-q_1)(q_1-1) \quad &> \quad 0 \\ \Rightarrow q_1 \quad &> \quad 1 \ \ (as \ \ q_2>q_1) \end{aligned}$$

Eq.(35) gives:

$$(\alpha_2-q_1)(q_1-p_1) \geq (1-q_1)(q_2-p_2)$$

Since we have $1 \geq p_1$ from eq.(37) as a condition for the $(1,0)$ equilibrium in R_7 and we established that $q_1>1$, we get $q_1>p_1$. This in addition to the relation $q_1 > 1 \geq \alpha_2$, enforces $q_2>p_2$ for the previous inequality expression.

Eq.(35) could also be rearranged as:

$$(\alpha_2-q_1+q_2-p_2)(q_1-p_1) \geq (q_2-p_2)(1-p_1)$$

The expression on the right hand side is positive since $q_2>p_2$ and $1 \geq p_1$ as discussed previously. Therefore the left hand side expression also needs to be positive. Using eq.(36) and $q_1>p_1$, we must have $\alpha_2-q_1+q_2-p_2>0$.

Now using eqs.(34) and (36), and the facts $q_2>p_2$ and $\alpha_2-q_1+q_2-p_2>0$ as established above, we can write:

$$\begin{aligned} \frac{\beta(q_1-p_1)+p_1q_2-p_2q_1}{q_2-p_2} \quad &\leq \quad \alpha_1\beta \\ &< \quad \frac{\beta-p_2-p_1(\beta-\alpha_2-(q_2-q_1))}{\alpha_2+q_2-q_1-p_2} \\ \Rightarrow \alpha_2(q_1-p_1) \quad &< \quad (1-q_1)(q_2-p_2)+q_1(q_1-p_1) \end{aligned}$$

It can be easily seen that the above inequality contradicts with the condition in eq.(35).

Hence all the validity and stability conditions for the three equilibria $\{(0,1),(1,0),\underline{x}^*_{R_6}\}$ cannot be satisfied together.

Additionally, since no pair of three valid, stable equilibria can coexist a set of given parameter values, it follows that no combinations of four or more equilibria can coexist either, thus completing the proof.

Proof of Proposition 3:

The proposition has two parts: converters can help a technology (i) alter market equilibrium from a scenario where it has been eliminated to one where it coexist with the other technology; (ii) and even succeed in nearly eliminating it.

Condition (i) is relatively easy to establish. Consider a scenario where one of the technologies has been eliminated, *i.e.,* an equilibrium of the form $(1,0)$, $(0,1)$, $(0,x_2^*)$ or $(x_1^*,0)$. The validity conditions from Table III identify the minimum converter efficiency required to invalidate that equilibrium. From the table, such an invalidation is easily seen to correspond to the re-emergence of the other technology (these are the only equilibria whose validity conditions are compatible with the invalidation of the previous equilibrium), and thus co-existence of the two technologies.

Turning to condition (ii), assume that for a given set of system parameters, $(0, 1)$ is the initial equilibrium in the absence of converters. Users with the lowest technology valuation ($\theta = 0$) must, therefore, derive greater utility from Technology 2 than Technology 1 *i.e.,* $U_1(\theta = 0) < U_2(\theta = 0)$. This implies

$$\beta > p_2 - p_1 \tag{42}$$

Next, using perfect, symmetric converters ($\alpha_1 = \alpha_2 = 1$), we show that is is possible to satisfy both eq. (42) and the validity conditions of an equilibrium of the form $(1 - x_2^*, x_2^*)$, with x_2^* arbitrarily small. This identifies a configuration satisfying condition (ii).

An equilibrium of the form $(1 - x_2^*, x_2^*)$ requires that users of preference $\theta = 0$ adopt Technology 1, *i.e.,* $U_1(\theta = 0) \geq 0$, thus

$$x_2^* \geq \frac{p_1 - 1}{\beta - 1} \tag{43}$$

and users with preference $\theta = 1 - x_2^*$ are to be indifferent to the two technologies *i.e.,* $U_1(1 - x_2^*, x_2^*) = U_2(1 - x_2^*, x_2^*)$. This gives

$$x_2^* = 1 - \frac{p_2 - p_1}{q_2 - q_1} \tag{44}$$

From eq. (44), for Technology 2 to nearly disappear, *i.e.,* $x_2^* \approx 0$, we need $p_2 - p_1 \lesssim q_2 - q_1$. We also need β large enough for eqs. (43) and (42) to continue holding. Combinations of system parameters that allow these conditions to be simultaneously satisfied are easily found, which establishes that the introduction of converters can take the system from an equilibrium of the form $(0, 1)$ to one of the form $(1 - \epsilon, \epsilon)$, where $\epsilon \approx 0$.

Consider now the reverse scenario, where the equilibrium in the absence of converters is $(1, 0)$ for $\alpha_1 = \alpha_2 = 0$. For this, we need $\theta_2^1 > 1$ and $\theta_0^1 < 0$, *i.e.,*

$$1 + p_2 - p_1 > q_2 - q_1 \tag{45}$$

$$p_1 < 1 \tag{46}$$

As before, we assume next perfect, symmetric converters, and establish that with them it is possible to achieve a new equilibrium of the form $(x_1^*, 1 - x_1^*)$, where $x_1^* \approx 0$. The new equilibrium requires that users with preference $\theta = 0$ derive positive utility from the Technology 1, *i.e.,*

$$(\beta - 1)x_1^* < \beta - p_1, \tag{47}$$

and that users with preference $\theta = x_1^*$ be indifferent to the two technologies *i.e.,* $U_1(x_1^*, 1 - x_1^*) = U_2(x_1^*, 1 - x_1^*)$.

$$x_1^* = \frac{p_2 - p_1}{q_2 - q_1} \tag{48}$$

It is again easy to find a combination of system parameters that simultaneously satisfy eqs. (45) to (48), while ensuring $x_1^* \approx 0$.

Proof of Proposition 4:

We first consider Technology 1 hurting itself by introducing or improving a converter. Converter efficiencies affect the expressions of the adoption levels only for the equilibria in R_5 and R_6 (eq.(12) and (13)). Region R_5 is easily eliminated from consideration as its validity conditions can be shown to force a positive derivative of x_1^* w.r.t. α_1. Therefore, the remainder of the proof focuses on a stable equilibrium in R_6.

As before, the numerators of ${x^*_1}_{R_6}$ and ${x^*_2}_{R_6}$ in eq. (13) are denoted as N_1 and N_2 respectively, and with D as their common denominator. The stability of the equilibrium can be shown to imply that $D < 0$. The requirement $D < 0$ implies $N_1 < 0$ and $N_2 < 0$, which has important consequences on the impact of converter efficiency.

Specifically, better converters hurt Technology 1 if

$$\frac{\partial {x^*_1}_{R_6}}{\partial \alpha_1} = \frac{(\beta - q_2)N_2}{D^2} < 0 \tag{49}$$

Since $N_2 < 0$, the derivative is negative only if $\beta > q_2$. As a result, for better converters to hurt the incumbent, the condition $\beta > q_2$ and one of the sets of stability conditions in Table IV must be simultaneously satisfied. We use the Mathematica symbolic manipulation software to establish that the intersection of parameter sets satisfying these combinations of conditions is non-empty. Figure 3 is an instance of one combination of parameters in that intersection.

To prove that $\alpha_1\beta > 1$ is a necessary condition for this behavior to arise, we will show that if $\alpha_1\beta \leq 1$ then the validity and stability conditions of R_6 and the condition $\beta > q_2$, required for this behavior, cannot hold together. The proof will proceed by considering several subcases depending on the relationships between the parameters.

(A) Case: $q_1 > 1$

From eq. (33) for $D < 0$ we have

$(q_1 - 1)(\beta - q_2) + (q_2 - \alpha_1\beta)(q_1 - \alpha_2) < 0$

Using the fact that $\beta > q_2$ and $q_1 > 1 > \alpha_2$, we find that the above inequality can only hold if $q_1 < \alpha_1\beta \Rightarrow \alpha_1\beta > 1$.

(B) Case: $q_1 \leq 1$

Here we will need to consider two subcases for $q_1 \geq \alpha_2$ and $q_1 < \alpha_2$.

(B.1) Subcase: $q_1 \geq \alpha_2$

Since $\beta > q_2$ and $q_1 \geq \alpha_2$, it implies $\beta > \alpha_2 + q_2 - q_1$. Note that this is also the condition for the converter of Technology 1 to hurt the overall market penetration. We show in the proof of Proposition 5 that this condition for the drop in overall penetration, can only be satisfied with the validity and stability conditions for ${\overline{x}^*}_{R_6}$ only if $\alpha_1\beta > 1$. Therefore this particular subcase will require $\alpha_1\beta > 1$ to hold.

(B.2) Subcase: $q_1 < \alpha_2$

For this subcase we will again need to consider two more subcases: (a) $q_2 \geq p_2$ and (b) $q_2 < p_2$.

(B.2.a) subcase: $q_2 \geq p_2$

If $q_2 \geq p_2$ and $q_1 < \alpha_2$ then $\alpha_2 + q_2 - q_1 - p_2 > 0$. From eqs. (34) and (36) and using the fact that $\beta > q_2 \geq p_2$, we have

$$\begin{aligned}\frac{\beta(q_1-p_1)+p_1q_2-p_2q_1}{q_2-p_2} &\le \alpha_1\beta \\ &< \frac{\beta-p_2-p_1(\beta-\alpha_2-(q_2-q_1))}{\alpha_2+q_2-q_1-p_2} \\ \Rightarrow \alpha_2(q_1-p_1) &< (1-q_1)(q_2-p_2)+q_1(q_1-p_1)\end{aligned}$$

It can be easily seen that the above inequality contradicts the condition in eq.(35). Therefore, the relationships considered in this subcase cannot hold together.

(B.2.b) subcase: $q_2 < p_2$

In this subcase we again need to consider two further subcases depending on the parameter relations: (i) $q_1 \ge p_1$ and (ii) $q_1 < p_1$.

When $q_1 \ge p_1$, using the facts that $\beta > q_2$, $q_1 \ge p_1$, $q_2 < p_2$ and eq. (34), we get $\alpha_1\beta \le q_1$. This also implies $\alpha_1 q_2 < q_1$ since $\beta > q_2$. However, the relation, $\alpha_1 q_2 < q_1 < \alpha_2 < 1$, and eq.(32) together imply $q_1 > q_2$ which contradicts the requirement $q_2 > q_1$ of the model. Therefore this subcase cannot arise.

The subcase $q_1 < p_1$ also cannot arise by our assumption that $(0,0)$ is an invalid equilibrium. When both $q_1 < p_1$ and $q_2 < p_2$ then both $\theta_1^0(x_1 = 0, x_2 = 0) > 0$ and $\theta_1^0(x_1 = 0, x_2 = 0) > 0$, which makes $(0,0)$ a valid equilibrium.

Technology 2 cannot hurt itself while improving its converter efficiency, α_2.

Proof: The only equilibrium outcomes where the adoption level of Technology 2 varies as a function of α_2 are those that arise in regions R_5 and R_6 (as given by Eqs. (12) and (13)). If the equilibrium is in R_5 (*i.e.,* the technologies coexist at full market penetration) the derivative of the adoption level x_2 w.r.t. α_2 is

$$\frac{\partial x^*_{2_{R_5}}}{\partial \alpha_2} = \frac{(p_2-p_1)-\beta(1-\alpha_2)}{[(q_2-q_1)-(1-\alpha_2)-\beta(1-\alpha_1)]^2} \tag{50}$$

This expression is always positive since $(p_2-p_1) > \beta(1-\alpha_2)$ is a required validity condition for the equilibrium in R_5. thus increasing α_2 cannot hurt Technology 2 for an equilibrium in R_5.

Next we consider the effect of α_2 on the equilibrium in R_6. In this region, the indifference points obey the relation $0 < \theta_1^0 \le \theta_2^0 \le \theta_2^1 < 1$. To show that Technology 2 cannot hurt itself by increasing α_2, we will consider the two cases $\alpha_1\beta \le 1$ and $\alpha_1\beta > 1$ separately.

For $\alpha_1\beta \le 1$, consider that $x^*_{1_{R_6}}(0)$ and $x^*_{2_{R_6}}(0)$ are the initial equilibrium adoption levels. Since

$$U_2 - U_1 = \theta(q_2-q_1)+\beta(1-\alpha_1)x_2-(1-\alpha_2)x_1-(p_2-p_1)$$

on increasing α_2, the difference of $U_2 - U_1$ is increased. Therefore a small fraction, say δ, of users of Technology 1 switch to Technology 2, thus making $x^*_{1_{R_6}}(1) = x^*_{1_{R_6}}(0) - \delta$ and $x^*_{2_{R_6}}(1) = x^*_{2_{R_6}}(0) + \delta$. The indifference point θ_2^1 shifts to the right to $\theta_2^1(1) = \theta_2^1(0) - \delta$. The second order effect of the switch-overs leads to changes in the adoption decisions of the lower-end users of Technology 1. The indifference point θ_1^0 shifts to $\theta_1^0(1) = \theta_1^0(0) - \frac{(\alpha_1\beta-1)\delta}{q_1}$. Since $\alpha_1\beta \le 1$, if θ_1^0 shifts, it will shift to the right; thus further decreasing x_1. The new adoption level of Technology 1, therefore, becomes $x^*_{1_{R_6}}(1) = x^*_{1_{R_6}}(0) - \delta + \frac{(\alpha_1\beta-1)\delta}{q_1}$. Given these new values for $x^*_{1_{R_6}}(1)$ and $x^*_{2_{R_6}}(1)$, a new value can be computed for θ_2^1:

$\theta_2^1(1) = \theta_2^1(0) - \delta - \frac{\delta}{q_2-q_1}[\beta + (1-\alpha_1\beta)(1+\frac{1}{q_1})]$.

Since $q_2 > q_1$ and $\alpha_1\beta \le 1$, the change in $\theta_2^1(1) - \theta_2^1(0)$ is again negative *i.e.,* θ_2^1 shifts further to the left, leading to more users switching from Technology 1 to 2. Thus, the compounding of the first and second order effects of a small increase in α_2 leads to decreases in x_1 decreases and increases in x_2. Both reinforce the initial increase in $U_2 - U_1$ after increasing α_2. As a result, as the process converges to a new equilibria after an increase in α_2, the final x_2 value exceeds the original one. Hence, improving its converter cannot hurt Technology 2.

We consider next the case $\alpha_1\beta > 1$. In this scenario, we know[12] that the overall market penetration must increase when increasing α_2. Therefore, if Technology 2's market share were to drop upon increasing α_2, then the market share of Technology 1 must increase so that the overall market share increases. We proceed to show that such a scenario is infeasible.

Assume that $\alpha_1\beta > 1$ and Technology 2 hurts itself by increasing α_2, *i.e.,* the indifference point θ_2^1 moves to the right to $\theta_2^1 + \epsilon_1$, $(\epsilon_1 \gtrsim 0)$, then a user with preference θ is in the range $\theta_2^1 < \theta < \theta_2^1 + \epsilon_1$ will switch from using Technology 2 to using Technology 1. The switch-over of each such users decreases the utility U_1 of Technology 1 users by an amount $(\alpha_1\beta - 1) > 0$. This affects the lower-end users of Technology 1, *i.e.,* users with preference in the range $\theta_1^0 < \theta < \theta_1^0 + \epsilon_2$, $(\epsilon_2 \gtrsim 0)$, whose utility then becomes negative. These users, therefore, disadopt Technology 1. These disadoptions imply that the overall market penetrations decreases, which contradicts the fact that the overall market cannot drop when $\alpha_1\beta > 1$. This establishes that it is not possible for Technology 2 to hurt itself by increasing α_2.

Proof of Proposition 5:

Using the same notation as in Proposition 4, decreasing the overall market penetration by increasing the converter efficiency of Technology 1 requires $\frac{\partial(x^*_{1R_6}+x^*_{2R_6})}{\partial\alpha_1} < 0$.

$$\frac{\partial(x^*_{1R_6}+x^*_{2R_6})}{\partial\alpha_1} = \frac{\beta(\beta-\alpha_2-(q_2-q_1))N_2}{D^2}$$

Using the same reasoning as in the proof of Proposition 4, a valid and stable equilibrium in R_6 implies $D < 0$, and consequently $N_2 < 0$. The above derivative is, therefore,

[12]Using the notation form the proof of Proposition 5, note that $N_1 \le 0$. Therefore, for $\alpha_1\beta > 1$, the derivative $\frac{\partial(x^*_{1R_6}+x^*_{2R_6})}{\partial\alpha_2} = \frac{(1-\alpha_1\beta)N_1}{D^2}$ is strictly positive for $N_1 < 0$ while it equals zero for $N_1 = 0$. However $N_1 = 0$ corresponds to the $x^*_{1_{R_6}} = 0$, *i.e.,* Technology 1 has no users, in which case an increase in α_2 can never hurt x_2. Therefore our present discussion only requires us to consider the case where the derivative is strictly increasing.

negative only if $\beta > (q_2 - q_1) + \alpha_2$. There are many combinations of parameters that simultaneously satisfy this condition and the validity and stability conditions of an equilibrium in R_6. Figure 3 is again one such combination. Furthermore, this condition $\beta > (q_2 - q_1) + \alpha_2$ can only hold along with the validity and stability conditions for the equilibrium in R_6 only if $\alpha_1\beta > 1$. We now provide the proof for this.

Proof: $\alpha_1\beta > 1$ is a necessary condition for the incumbent to hurt the overall market

First consider the case when $q_1 \leq 1$. From eq. (32) and $\beta > (q_2 - q_1) + \alpha_2$, we have

$0 < \beta - \alpha_2 - (q_2 - q_1) < \alpha_1\beta + q_2 - q_1 - \frac{q_2}{q_1}$

$\Rightarrow \alpha_1\beta - q_1 > \frac{q_2}{q_1}(1 - q_1)$

$\Rightarrow \frac{\alpha_1\beta - q_1}{1-q_1} > \frac{q_2}{q_1} > 1$

Since $q_1 \leq 1$, we need $\alpha_1\beta > 1$.

Next consider the case when $q_1 > 1$. For this case, we will need to consider two subcases, corresponding to $\alpha_2 + q_2 - q_1 - p_2 > 0$ and $\alpha_2 + q_2 - q_1 - p_2 < 0$.

Subcase (1): Let $\alpha_2 + q_2 - q_1 - p_2 > 0$

When $q_1 > 1$, the above condition implies $q_2 - p_2 > q_1 - \alpha_2 > 0$ (*i.e.,* $q_2 > p_2$). However, when $\alpha_2 + q_2 - q_1 - p_2 > 0$ and $q_2 > p_2$, then eqs. (34) and (36) together result an inequality that contradicts the inequality in eq. (35). Since the conditions considered in this subcase cannot hold together, we do not need to consider it further.

Subcase (2): Let $\alpha_2 + q_2 - q_1 - p_2 < 0$

Let us assume that $\alpha_1\beta < 1$. We show here that the conditions for validity and stability of the R_6 equilibrium, and $\beta > (q_2 - q_1) + \alpha_2$ cannot hold together if $\alpha_1\beta < 1$. Using eq.(36), we have

$\alpha_2 + q_2 - q_1 - p_2 < \beta - p_2 - p_1(\beta - \alpha_2 - (q_2 - q_1))$

$\Rightarrow p_1 < 1 < q_1$

From eq.(33) we have

$(q_1 - 1)(\beta - q_2) < (q_1 - \alpha_2)(\alpha_1\beta - q_1) < 0$

which implies $\beta < q_2$ (since $q_1 > 1$).

Using the condition $\beta - \alpha_2 - (q_2 - q_1) > 0$ needed for hurting the overall market and eq.(35) and the condition $p_1 < 1 < q_1$ obtained previously, we get

$\beta - (q_2 - q_1) > \alpha_2 > \frac{(1-q_1)(q_2-p_2)+q_1(q_1-p_1)}{q_1-p_1}$

$\Rightarrow (\beta - q_2)(q_1 - p_1) > (1 - q_1)(q_2 - p_2)$

$\Rightarrow q_2 > p_2$ since $\beta < q_2$ and $q_1 > 1 > p_1$.

Now using the condition of this subcase *i.e.,* $\alpha_2 + q_2 - q_1 - p_2 < 0$ and eq. (35) we get $p_1 > 1$, which contradicts the previously obtained relation $p_1 < 1 < q_1$. Therefore when $\alpha_1\beta < 1$, all these conditions do not hold together and the so behavior will not arise for this case.

However, when $\alpha_1\beta > 1$, it can be shown using Mathematica that for this subcase there exists numerical values for the various parameters for which the overall market drops. The above analysis of all the different cases establishes that $\alpha_1\beta > 1$ is a necessary condition for this behavior to arise.

Similarly, when Technology 2 increases its converter efficiency, the overall market penetration will drop if

$$\frac{\partial(x^*_{1R_6} + x^*_{2R_6})}{\partial\alpha_2} = \frac{(1 - \alpha_1\beta)N_1}{D^2} < 0 \tag{51}$$

For a valid, stable equilibrium in R_6, we have $N_1 < 0$, and therefore the above expression is negative only if $\alpha_1\beta < 1$. This establishes the second part of Proposition 5.

Proof of Proposition 6:

In Figure 6 we identified a scenario where instabilities in adoption dynamics arose for $\alpha_1\beta > 1$, and the parameter values satisfied $A^2 - 4B < 0$.

To prove that $\alpha_1\beta > 1$ is necessary for the formation of cyclic instability, we proceed in two steps: (i) we show that if $A^2 - 4B \geq 0$ then cycles (closed orbits) cannot arise in the adoption trajectories, and (ii) if $\alpha_1\beta \leq 1$ then $A^2 - 4B \geq 0$ always holds, and therefore there cannot be any closed orbits.

It may be noted that if any such 'closed orbit' or cycle were to arise in the adoption process, then its locus must be the equilibrium in R_6. This follows from the Index Theory which states that if J is a closed orbit[13] of a system enclosing an open set D (*i.e.,* $A = D \cup J$ is a compact set), then the set D must include an equilibrium point. Thus, every closed orbit in the plane encloses an equilibrium point. In our adoption process, a cyclic trajectory can either lie entirely inside the S-plane or may touch its boundaries. Note that when the trajectory touches or includes a portion of the boundary, it is not possible to have an equilibrium on the boundary itself because then the system would have attained stability as soon as the trajectory reaches that equilibrium. Therefore every closed trajectory must enclose an equilibrium that lie exclusively in the interior of the S-plane. The equilibrium in R_6 is the only equilibrium that satisfies this requirement (as all the others lie on the boundaries *i.e.,* x_1, x_2-axes or the line $x_1 + x_2 = 1$ by their definition). So the equilibrium in R_6 will be the focus for the proof of the two steps mentioned earlier.

(i) Proof: If $A^2 - 4B \geq 0$ then cycles cannot arise in the adoption trajectories

To show that cycles cannot arise in the adoption trajectories when $A^2 - 4B \geq 0$, we have to consider the two possibilities that the equilibrium in R_6 is either stable or unstable.

(a) $A^2 - 4B \geq 0$ and $\underline{x}_{R_6}$ is stable.

We will prove that for this case, the entire R_6 region is the basin of attraction of the stable equilibrium $\underline{x}_{R_6}$ and therefore a closed trajectory cannot pass through (or be entirely located in) R_6 as it would have converged to that equilibrium. Furthermore, we show that it is not possible to realize any closed trajectory that has the R_6 equilibrium as its locus but does not ever pass through the Region R_6. These statements together eliminates the possibility of cycles in this case.

A stable equilibrium in R_6 requires $A < 0$ and $B > 0$. When $A^2 - 4B \geq 0$, we have $A - \sqrt{A^2 - 4B} \leq A +$

[13] It is a trace of the trajectory of a non-trivial (i.e. not a point) periodic solution [16]

$\sqrt{A^2-4B} < 0$. Therefore once a trajectory enters R_6, the exponential terms in its expression (Table II) decrease exponentially over time and converges to the equilibrium. In other words, the entire R_6 region is the basin of attraction of the stable equilibria located in it. Hence a closed trajectory cannot be realized if it were to pass through R_6.

Recall that in order to necessarily eliminate the possibility of $(0,0)$ being a valid equilibrium, the point P (in Figure 7) cannot lie in the positive quadrant of the (x_1,x_2) plane. As a result, the region R_6 will always touch either the boundary x_1 or x_2 axis. Recall that we previously established that if a closed orbit were to arise in this system, the equilibrium in R_6 must lie in its interior. However, since the region R_6 touches at least one of the axes, it is never possible to realize a closed orbit that encircles the equilibrium in R_6 as its locus but doesn't pass through this region. Therefore, cyclic instabilities cannot be realized for this case.

(b) $A^2-4B \geq 0$ and $\underline{x}_{R_6}$ is unstable.

In this proof we will need two more results from the Index Theory. First, the index of a closed orbit, J, is +1 [14]. All such closed orbits (trajectories) must encircle the equilibrium in R_6. Moreover, since the equilibrium $\underline{x}_{R_6}$ is the only equilibrium point enclosed by J, the index of this equilibrium is I($\underline{x}_{R_6}$)=+1 [15]. Therefore a closed contour, C, around $\underline{x}_{R_6}$ in its neighborhood set must also have the same index as I($\underline{x}_{R_6}$) *i.e.,* +1.

However, C can have an index of +1 only if either all the trajectories are pointing radially inward(outward) towards(from) the equilibrium. But we show below that when $A^2-4B \geq 0$ and the equilibrium is unstable, the trajectories in R_6 take the shape of hyperbolas, which implies that all the trajectories are not directed consistently either inward or outward from the equilibrium, and thus the index computed for the closed contour, C, will not be +1. This contradiction implies that a closed orbit J cannot be present, which will therefore eliminate the possibility of cyclic instability in this case.

Using the expressions for the trajectories in R_6 provided in Table II and the fact that $A^2-4B \geq 0$ and the equilibrium is unstable, the expressions for the trajectories around $\underline{x}_{R_6}$ can be rearranged in the following form to show their hyperbolic shapes:

$$(x_1 - x^*_{1R_6} - K_2(x_2 - x^*_{2R_6}))(x_1 - x^*_{1R_6} - K_1(x_2 - x^*_{2R_6})) = -c_1c_2(K_1-K_2)^2 e^{A\gamma(t-t_0)} \quad (52)$$

Substituting $\overline{x}_1 = x_1 - x^*_{1R_6}$, $\overline{x}_2 = x_2 - x^*_{2R_6}$, $a = \frac{\alpha_2+\beta(1-\alpha_1)-\frac{q_2}{q_1}}{(q_2-q_1)\sqrt{A^2-4B}}$, we have $K_1 = a-1,\ K_2 = a+1$.

Eqn. (52) can therefore be written as:

$$(\overline{x}_1 - a\overline{x}_2)^2 - \overline{x}_2{}^2 = -4b^2c_1c_2e^{A\gamma(t-t_0)} \quad (53)$$

The above expression clearly shows that at any time t, the trajectories around the equilibrium have hyperbolic shapes.

[14]ref. [12], pp. 299, lemma 7.1(c)

[15]ref. [16], pp.50

Thus, when $A^2-4B \geq 0$, irrespective of whether the equilibrium in R_6 is stable or not, there cannot be any cyclic trajectories in the adoption process.

(ii) Proof: If $\alpha_1\beta \leq 1$ then $A^2-4B \geq 0$

Note that $a^2+b^2 \geq 2ab$ for any real numbers a, b. Hence we have

$$\frac{\left(1-\alpha_2+\beta(1-\alpha_1)\right)^2}{(q_2-q_1)^2} + \frac{1}{q_1^2} \geq \frac{2(1-\alpha_2)+2\beta(1-\alpha_1)}{q_1(q_2-q_1)} > 0$$

since $0<\alpha_1<1, 0<\alpha_2<1, \beta>0$.

Using the expression for A^2-4B and the above inequality, we get:

$$A^2-4B \geq \frac{4(1-\alpha_1\beta)(1-\alpha_2)}{q_1(q_2-q_1)} \geq 0 \quad if\ \alpha_1\beta \leq 1$$

Since we can only have cyclic instability in the system when $A^2-4B<0$ and that this condition can only be satisfied when $\alpha_1\beta > 1$, it also becomes necessary that $\beta > 1$ (as $\alpha_1 < 1$).

E. Discussion on Separatrices

Table VI provides the characterization of the separatrices in each region when the unstable equilibrium that it passes through lies in that region. The regions R_1, R_3, R_7, R_9 are not included in the table as no separatrix can arise in them because by definition these equilibria are always stable whenever they are valid.

We briefly illustrate how the expressions for the separatrices may be derived. Consider the separatrix of Region R_2 passing through the unstable equilibrium $(0, x^*_{2_{R_2}})$. Note that the equilibrium is valid but unstable, and thus $q_2 < \beta$ from Table II. From the expressions of trajectory in R_2 it is clear that if $q_2 < \beta$, $x_2(t)$ increases if $c_2 > 0$ while it decreases if $c_2 < 0$. Depending on the sign of c_2, the trajectory diverges in opposite directions. Thus $c_2 = 0$ separates the trajectories with different convergence behaviors and is the candidate for the local separatrix in R_2, which gives

$$x_2 = \frac{p_2-q_2}{\beta-q_2} - \frac{\alpha_2}{\beta}x_1$$

as the expression for the boundary separating different basins of attraction.

TABLE VI
CANDIDATES FOR LOCAL SEPARATRIX

Region	Candidate of local separatrix
R_2	$x_2 = -\frac{\alpha_2}{\beta}x_1 + \frac{p_2-q_2}{\beta-q_2}$
R_5	$x_2 = \frac{1-\alpha_2}{\beta(1-\alpha_1)}(x_1 - x^*_{1R_6}) + x^*_{2R_5}$
R_6	$x_2 = \frac{2(1-\alpha_2)(x_1-x^*_{1R_6})}{\alpha_2+\beta(1-\alpha_1)-\frac{q_2}{q_1}+(q_2-q_1)\sqrt{A^2-4B}} + x^*_{2R_6}$
R_8	$x_2 = -\frac{1}{\beta\alpha_2}x_1 + \frac{p_1-q_1}{\beta\alpha_2(1-q_1)}$

F. Robustness to Alternative Models

In the paper we have identified several interesting behaviors that arise in presence of converters for a model where users are heterogeneous in their evaluation of the technology's quality and benefit from linear externality. However, many of the behaviors identified in this work will be present in a wide range of alternative models.

To show this robustness of our findings we first show that quantitatively similar behaviors arise for more generic distribution of heterogeneous user preferences. We do so by considering Beta-distribution of the user preference with positive and negative skewness. Following this, we also consider some different types of network externality benefits, namely, sub-linear, super-linear and logarithmic network benefits and provide examples that demonstrate similar behaviors as well. We also consider the case where user heterogeneity is extended to network benefits in addition to the technology's quality. Again for all these scenarios we present illustrative examples for qualitatively similar behaviors of interest.

1) Beta-distribution of User preference: We consider the same user utility functions as those in eqs.(1) and (2), but the heterogeneous user preferences are assumed to follow a Beta-distribution on $[0, 1]$ as opposed to an Uniform distribution. The density of Beta-distribution is given by $\frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}$ where $B(a, b)$ is the beta function with parameters a and b. Its skewness is given by $\frac{2(b-a)\sqrt{a+b+1}}{(a+b+2)\sqrt{ab}}$. We show qualitatively similar behaviors for both beta distributions with positive and negative skewness in the following cases.

(i) Positively skewed Beta-distributions

Figure 9 shows that instabilities can arise even with such alternative distributions where $a = 1.45, b = 2$ with a positive skewness of 0.25.

Figure 10 shows that Technology 1 can hurt itself as well as the overall market penetration as it improves its converter efficiency from 0.85 to 1. For the scenario shown in the figure, the beta distribution has $a = 0.65, b = 1$ and a positive skewness of 0.3872.

(ii) Negatively skewed Beta-distributions

Figure 11 shows that instabilities can arise even for negatively skewed (-0.3205) beta-distributions where $a = 2.2, b = 1.45$.

Figure 12 shows that Technology 1 can hurt itself as well as the overall market penetration as it improves its converter efficiency from 0.85 to 1 as shown in the figure with a beta distribution for parameters $a = 2, b = 1.5$ and a negative skewness of -0.2227.

2) Other Externality Functions:

$$U_1 = \theta q_1 + ({x_1}^{\rho} + \alpha_1 \beta {x_2}^{\rho}) - p_1 \tag{54}$$
$$U_2 = \theta q_2 + (\beta {x_2}^{\rho} + \alpha_2 {x_1}^{\rho}) - p_2 \tag{55}$$

(i) Sub-linear network benefits

The plot on the left in Figure 13 shows an example of instability in the adoption process when the externality benefits are of the form ${x_i}^{0.7},\ i = \{1, 2\}$. The plot on the right provides an example where Technology 1 on improving its converter efficiency from 0.85 to 0.9 hurts its own market as well as the overall market levels across the two technologies.

(ii) Super-linear network benefits

The plot on the left in Figure 14 shows an example of instability in the adoption process when the externality benefits are of the form ${x_i}^{1.4},\ i = \{1, 2\}$. Once again, even for the superlinear externalities, we find that the plot on the right shows that Technology 1 can potentially harm itself as well as the overall market. In this figure, the Technology 1 improves its converter efficiency from 0.87 to 0.91 leading to a drop in its own market by about 0.16.

(iii) Logarithmic network benefits

We also considered the case where the externality benefits are of the form $\log_2(x_i + 1),\ i = \{1, 2\}$.

$$U_1 = \theta q_1 + (\log_2(x_1 + 1) + \alpha_1 \beta \log_2(x_2 + 1)) - p_1 \tag{56}$$
$$U_2 = \theta q_2 + (\beta \log_2(x_2 + 1) + \alpha_2 \log_2(x_1 + 1)) - p_2 \tag{57}$$

The plot on the left in Figure 15 shows an example of the instabilities that arise in presence of converters in case of a logarithmic externality function. Technology 1 may again hurt itself as well as the overall market while improving its converter efficiency as shown on the plot on the right. Such a behavior is shown in this plot as Technology 1 improves its converter efficiency from 0.9 to 0.99.

3) Extending User Heterogeneity to Network Benefits: If the users have similar heterogeneous preferences over both the intrinsic (stand-alone) quality of the technology and the network externality benefit, then their utility from the two alternative technologies are:

$$U_1 = \theta(q_1 + x_1 + \alpha_1 \beta x_2) - p_1 \tag{58}$$
$$U_2 = \theta(q_2 + \beta x_2 + \alpha_2 x_1) - p_2 \tag{59}$$

For this utility form, we again identify that deployment and improvement in converters can lead to behaviors like drop in overall market penetration and adoption instability.

(i) Drop in overall market penetration

The plot on the right in Figure 16 considers a case where $p_1 = 0.6, p_2 = 3.9, q_1 = 0.5, q_2 = 4.2, \beta = 7, \alpha_2 = 0$ *i.e.,* Technology 1 is cheaper and lower in quality than Technology 2, which also provides larger network benefits. In this plot, when Technology 1 introduces a converter of 0.45 efficiency then the overall market penetration at equilibrium is about 0.54. However if the first technology introduces a converter, it improves its own market but the overall penetration drops to 0.3737. Therefore even for this utility form, the overall market penetration across the two technologies can be hurt by the converters.

(ii) Creation of instability in adoption process

We also find that instabilities may be arise in the adoption process when converters are present. The plot on the left in Figure 16 shows such a scenario for $\alpha_1 = 0.9, \alpha_2 = 0, p_1 =$

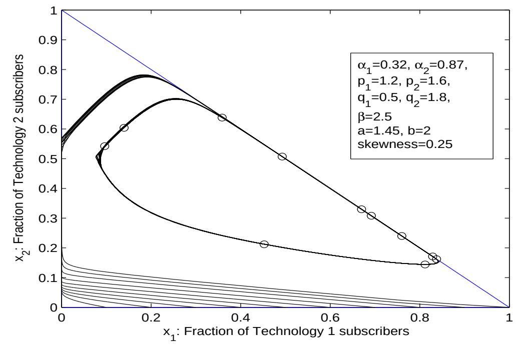

Fig. 9. Positively skewed Beta-distribution showing Instability.

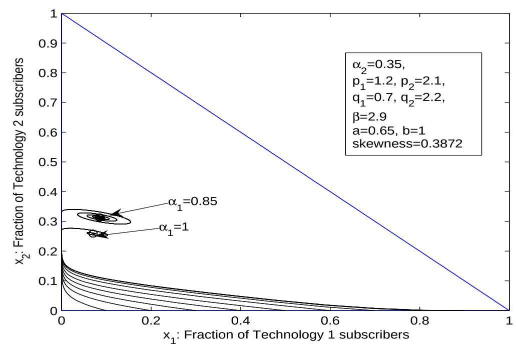

Fig. 10. Technology 1 hurts itself and overall market for a positively skewed Beta-distribution.

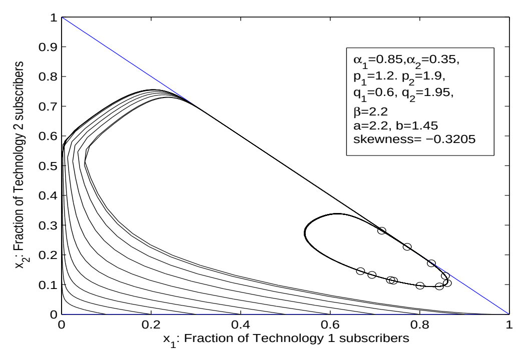

Fig. 11. Negatively skewed Beta-distribution showing Instability.

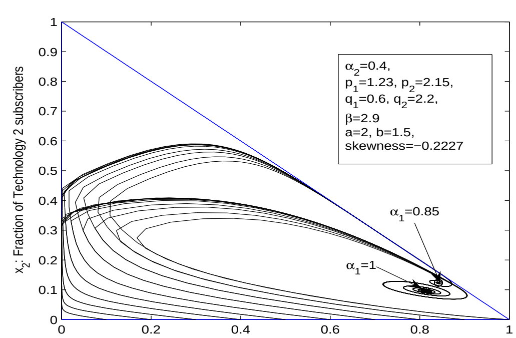

Fig. 12. Technology 1 hurts itself and overall market for a negatively skewed Beta-distribution.

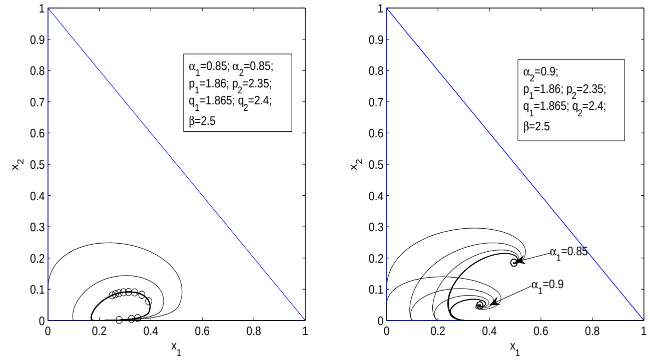

Fig. 13. Effects of sublinear network benefits.

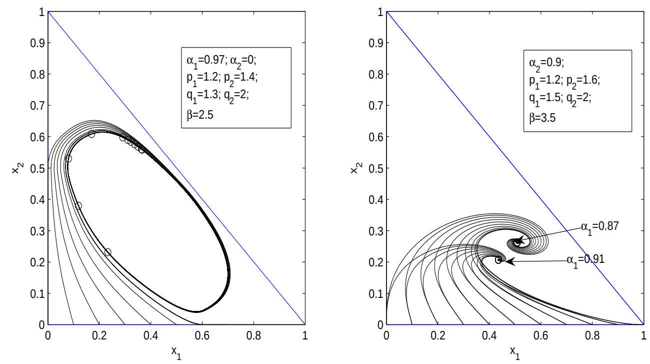

Fig. 14. Effects of superlinear network benefits.

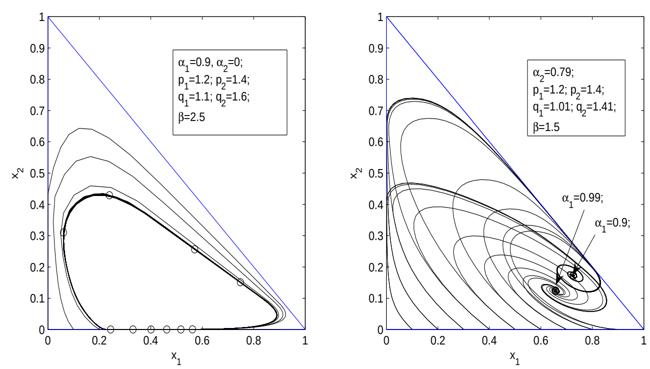

Fig. 15. Instability for logarithmic network benefits.

$0.5, p_2 = 1.3, q_1 = 0.3, q_2 = 1.4, \beta = 10$. Thus the behavior for instability in adoption dynamics may also arise for this alternative utility form.

(iii) Special Case: User's only value network benefits

$$U_1 = \theta q_1(x_1 + \alpha_1 \beta x_2) - p_1 \tag{60}$$

$$U_2 = \theta q_2(\beta x_2 + \alpha_2 x_1) - p_2 \tag{61}$$

In this case as well we find instances where deployment of converters can hurt the overall market. Figure 17 illustrates such a scenario.

G. Switching Costs

Network technologies often try to introduce switching costs by implementing contracts with penalty and by developing 'lock-in' strategies. For example, ISPs such as AOL practice 'lock-in' by restricting their users to send instant messages only to other fellow subscribers, thus preventing a user who switches to another ISP from messaging to his/her previous network. Another such strategy is the lowering of provider specific email addresses (e.g., Comcast, AOL). In spite of such efforts, the annual customer turnover in the ISP market remain very high at above 72%, suggesting that consumers in the ISP market still have sufficiently low switching costs [4], [14]. In other online markets, the use of 'lock-in' strategies based on

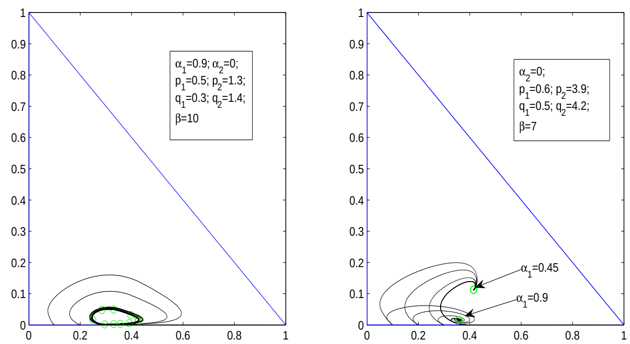

Fig. 16. Drop in overall market penetration.

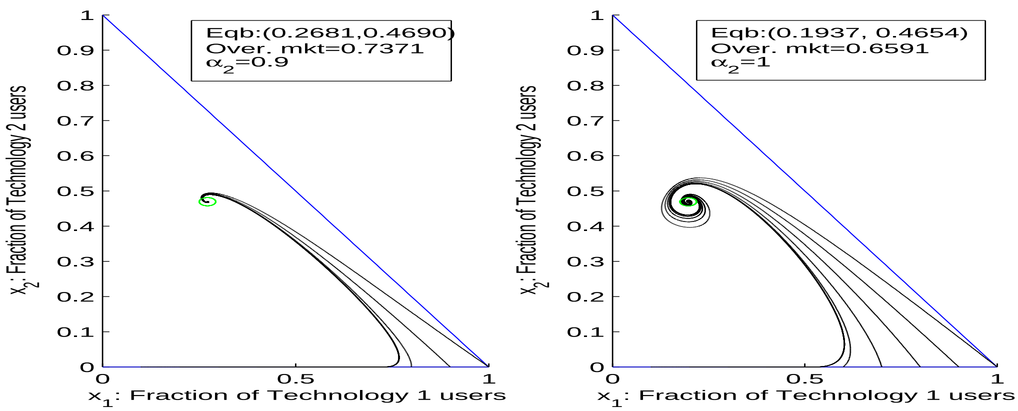

Fig. 17. Drop in overall market penetration.$(\alpha_1 = 0.185, p_1 = 0.4, p_2 = 1.8, q_1 = 4.65, q_2 = 7.3, \beta = 0.7)$

proprietary IT are also on the decline due to the emergence of web browsers and technologies like XML that lower switching costs by allowing interoperability between disparate systems [4]. However, some amount of switching costs may indeed continue to exist through contractual commitments, learning curve, specialized formats and customer loyalty programs.

Our model may be extended to include such switching costs, but it will introduce non-trivial complexity in the modeling effort because of the "memory" it adds to the individual users behavior. Additionally, there are different possible types of switching cost configurations one may need to consider, each requiring different utility forms for user decision. For example, the learning costs may significantly affect non-adopters when they join a technology, while contract- breaking cost affect only the users who disadopt or switch technologies. Also, the lack of a clear answer as to when and how many times such costs can affect switching behavior, adds complexity from a modeling standpoint. Therefore, a general analytical solution with switching costs quickly becomes intractable. However, it is possible in some cases to formulate generalized expressions for the indifference thresholds introduced in the paper and derive results in the way outlined next.

Assume that when a user switches from one technology to the other, or becomes a non-adopter, he/she incurs a symmetric switching cost of S due to prior contractual commitments, and that the learning costs for all users are negligible ($L = 0$). Then utilities U_1 and U_2 for the current non-adopters remain the same as before and so does the corresponding expressions for the indifference points (cite Eqns.). The utilities of the current adopters of Technology 1 become

$U_0 = -S$

$U_1 = \theta q_1 + (x_1 + \alpha_1 \beta x_2) - p_1$

$U_2 = \theta q_2 + (\beta x_2 + \alpha_2 x_1) - (p_2 + S)$

which give the indifference points as

$\theta_1^0(1) = \frac{p_1 - S - (x_1 + \alpha_1 \beta x_2)}{q_1}$

$\theta_2^0(1) = \frac{p_2 - (\beta x_2 + \alpha_2 x_1)}{q_2}$

$\theta_2^1(1) = \frac{(1-\alpha_2)x_1 - \beta(1-\alpha_1)x_2 + p_2 - p_1 + S}{q_2 - q_1}$

Similarly, the utilities of the current adopters of Technology 2 will be

$U_0 = -S$

$U_1 = \theta q_1 + (x_1 + \alpha_1 \beta x_2) - (p_1 + S)$

$U_2 = \theta q_2 + (\beta x_2 + \alpha_2 x_1) - p_2$

which give the indifference points as

$\theta_1^0(2) = \frac{p_1 - (x_1 + \alpha_1 \beta x_2)}{q_1}$

$\theta_2^0(2) = \frac{p_2 - S - (\beta x_2 + \alpha_2 x_1)}{q_2}$

$\theta_2^1(2) = \frac{(1-\alpha_2)x_1 - \beta(1-\alpha_1)x_2 + p_2 - p_1 - S}{q_2 - q_1}$

Note that the indifference points will now need to be represented as as $\theta_j^i(k)$, where the additional index $k,\ k = \{0, 1, 2\}$ will be used to represent the user category. Additionally, for any values of $x_1(t)$ and $x_2(t)$ and $S > 0$, the set of indifference points for the above three categories of users must satisfy the following relationships:

$$\begin{aligned}\theta_1^0(1) < \theta_1^0(0) = \theta_1^0(2) \\ \theta_2^0(2) < \theta_2^0(0) = \theta_2^0(1) \\ \theta_2^1(2) < \theta_2^1(0) < \theta_2^1(1)\end{aligned} \qquad (62)$$

Therefore if at time t, the set of indifference points are represented by $\theta_j^i(t)$, the new values at $t+1$ *i.e.,* $\theta_j^i(t+1)$ will have to calculated based on the relative positions of $\theta_j^i(k)(t+1)$ with respect to $\theta_j^i(t)$. In other words,

$\theta_j^i(t+1) = F(\theta_j^i(k)(t), \theta_j^i(t)), \; for \; k = \{0,1,2\}, \; i,j = \{0,1,2\}, \; i < j, \; i \neq j.$

where the function F needs to be carefully determined by considering the possible arrangements of these indifference points. For example, in the above case if we consider that the initial arrangement of the indifference point followed the order $\theta_1^0(t) < \theta_2^0(t) < \theta_2^1(t)$, then the function F for the new position of the indifference point $\theta_2^1(t+1)$ (based on relationships in eqn.(62)) will be given by:

$$\begin{aligned}\theta_2^1(t+1) &= \theta_2^1(2)(t+1) \; if \; \theta_2^1(2)(t+1) \geq \theta_2^1(t) \\ &= \theta_2^1(1)(t+1) \; if \; \theta_2^1(1)(t+1) \leq \theta_2^1(t) \\ &= \theta_2^1(t) \qquad if \; \theta_2^1(2)(t+1) < \theta_2^1(t) < \theta_2^1(1)(t+1)\end{aligned}$$

Given the complexity of the cases, solutions that account for switching costs need to resort to either numerical solutions (when it is possible to generalize equations for indifference thresholds) or simulations. We have investigated using both approaches to demonstrate that the results that our analytically tractable simplified model allow us to explicitly identify, namely the possible presence of instability in technology adoption and that better converters can hurt the incumbent as well as the overall market level etc., remain present across different switching cost configurations. Figure 18 and Figure 19 show these behaviors using the numerical solution. For clarity of the plots only the adoption paths for initial penetration levels of $x_1 = 0.5, x_2 = 0$ in Fig. 18 and $x_1 = 1, x_2 = 0$ in Fig. 19 are shown.

In the next section we provide more evidence of these behaviors under different cost configurations (learning cost, contract-termination cost etc.) through simulation results to establish that the results are robust to the introduction of a broad range of switching costs.

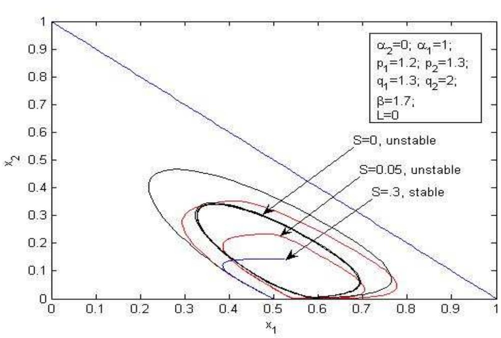

Fig. 18. Instability in presence of Switching Costs

1) Simulation Results: We consider three types of configurations for the purpose of our simulation to show the robustness of the observed behavior under different switching

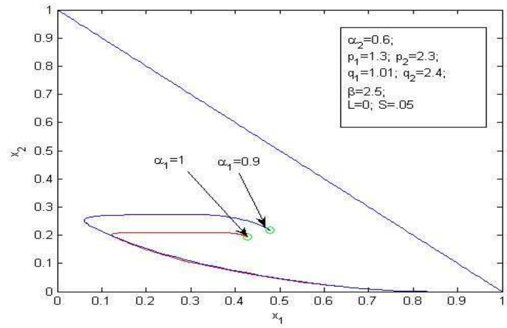

Fig. 19. Market Level drops in presence of Switching Costs

cost configurations. The simulations consider a population size of $N = 500$, each with a type value θ that is uniformly distributed between 0 and 1. The plots for that (i) presence of instability and (ii) incumbent's converter hurting itself as well as the overall market, are shown only for initial penetration levels of $x_1 = 0.3, x_2 = 0$ and $x_1 = 1, x_2 = 0$ respectively for the purpose of clarity.

Case (A): Switching Cost due to Contract breaking

In this scenario we consider that a user of either technology who decides to become a non-adopter or switches to the other technology incurs a certain cost as penalty for breaking a contract. It is assumed in this case that there is no learning cost for the users. The instability plot in Fig 20 shows the sample diffusion trajectories for switching cost of $S = 0.05$. But as the switching cost increases, the outcome stabilizes (*e.g.,* , $S = 0.3$) since the high switching cost makes it difficult for users to infinitely switch back and forth between the two technologies. Fig 21 shows the drop in the overall market and incumbent's market as its converter efficiency is increased from 0.9 to 1.

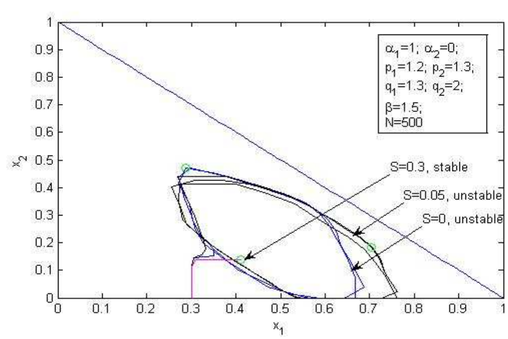

Fig. 20. Instability in presence of Switching Costs

Case (B): Switching Cost due to Contract breaking & Learning Costs for new adopters

This scenario considers both the presence of learning and switching costs. A non-adopter incurs a learning cost of L on joining either technology while an existing user of a technology incurs a switching cost of S for either becoming a non-adopter or switching over to the competing technology. As before, Figures 22 and 23 demonstrate the presence of the

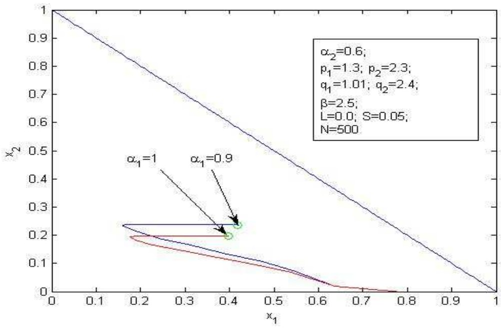

Fig. 21. Market Level drops in presence of Switching Costs

interesting behaviors.

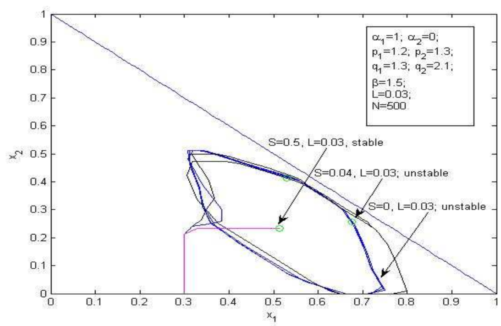

Fig. 22. Instability in presence of Learning & Switching Costs

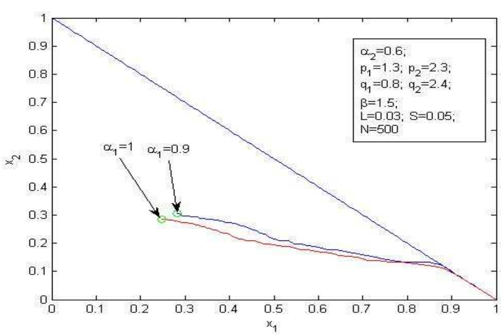

Fig. 23. Market Level drops in presence of Learning & Switching Costs

Case (C): Switching Cost due to 'Lock-In' but no contract breaking costs

In this scenario we consider a case where a user only incurs a switching cost when he/she has to move from one technology to its competitor but not if he/she becomes a non-adopter. This situation arises mainly when the switching cost is not in the form of a contract but due to 'lock-in' strategies. For example, if a user of a online music service with customization options decides to migrate to a competing site, he/she incurs a switching cost due to 'lock-in', but however if the person gets disinterested in the technology and becomes a non-adopter he/she does not incur this cost. Again, Figure 24 and 25 provide examples of the noticed behaviors for this scenario.

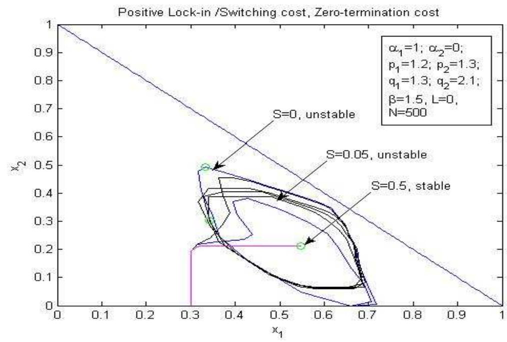

Fig. 24. Instability in presence of only 'Lock-in' Costs

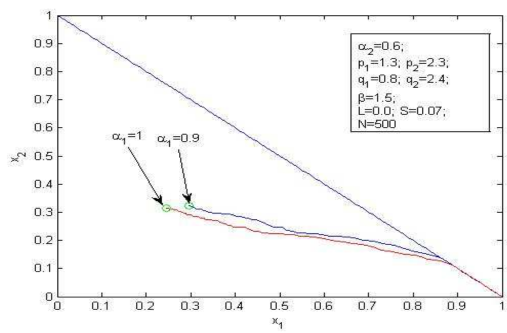

Fig. 25. Market Level drop in presence of only 'Lock-in' Costs

These simulation results therefore demonstrate that the results we identified with the help of our simplified model are quite robust, and they provide insights into the possible interesting adoption behaviors that can arise in a wide variety of switching cost configurations.