4-21-2011

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Suggested Citation:

H.T. Fortune and R. Sherr. (2011). Structure of 2^+, $t = 2$ states in $A = 12$ Nuclei. *Physical Review C* **83**, 044313.

http://dx.doi.org/10.1103/PhysRevC.83.044313

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Abstract

Using a reasonable but simple model, properties of 2^+ states in ^{12}Be and ^{12}O are calculated and compared with results of experiments.

Structure of 2^+, $T = 2$ states in $A = 12$ nuclei

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(Received 29 December 2010; published 21 April 2011)

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DOI: 10.1103/PhysRevC.83.044313 PACS number(s): 21.10.Jx, 21.10.Sf, 21.10.Tg, 27.20.+n

I. INTRODUCTION

Excitations into the $2s1d$ shell are important at quite low excitation energies in ^{12}Be. Several different experiments have demonstrated a large $(sd)^2$ component in the ^{12}Be ground state (gs). In the past, we have used a simple model to describe low-lying states in nucleus $A + 2$ in terms of two neutrons in the *sd* shell coupled to a *p*-shell core *A*. This description has been successful for ^{14,16}C [1], ^{17}N [2], ^{15}C [3], ^{13}B [4], ^{11}Be [5], and the 0^+ states of ^{12}Be [6,7]. Here, we apply it to the 2^+ states of ^{12}Be (and, by isospin invariance, to ^{12}C and ^{12}O).

The model is not meant to be rigorous, but it does contain the principal elements of the nuclear structure. It uses "local" single-particle energies (spe's) and "global" two-body residual interaction matrix elements. In the present case, we take the spe's for $2s_{1/2}$ and $1d_{5/2}$ from the $1/2^+$ and $5/2^+$ states of ^{11}Be [8]. We know those are not pure single-particle (sp) states, but this represents the simplest approach. The $(sd)^2$ two-body matrix elements (listed in Ref. [7]) are the same as we have used throughout this mass region. They first arose in a description of two-particle (2p) and four-particle, two-hole (4p-2h) states in ^{18}O [9]. Here, for the *sd* shell, we allow only the $1d_{5/2}$ and $2s_{1/2}$ orbitals, abbreviated *d* and *s*, respectively. After diagonalizing the $(sd)^2$ Hamiltonians, the wave functions for the two 0^+ and two 2^+ states are as listed in Table I. The $(sd)^2$ states are then allowed to mix with the *p*-shell ones, for which we use the results of Cohen-Kurath [10].

In ^{10}Be(t,p) [11], the cross section of the first 2^+ state is about 20 times larger than that calculated for the *p*-shell 2^+ state, but is consistent with the state being predominantly of $(sd)^2$ character. In ^{14}C(p,t) [12], a peak at 2.06 MeV above the lowest 0^+ $T = 2$ state appears to contain contributions from both 0^+ and 2^+ states. Fitting the angular distribution to the sum of 0^+ and 2^+ suggests [7] that the 2^+ cross section is $19 \pm 9\%$ of that expected for the *p*-shell 2^+ state, using amplitudes from Cohen-Kurath [10]. Even with core excitation in ^{14}C(gs) [13], the $2n$ pickup is all from the *p* shell [7]. So, we take as given that 2_1^+ contains about $19 \pm 9\%$ of the *p*-shell 2^+ state.

Our calculated energy of the lowest 2^+ state (3.63 MeV, Table I) is significantly higher than the experimental value of 2.1 MeV. This is also true of other calculations. Blanchon *et al.* [14] get the first two 2^+ states at 3.86 and 4.59 MeV. In Ref. [15], the lowest is at 3.8 MeV. The fact that the calculated energy of the 2_1^+ state is significantly higher than the experimental energy is perhaps an indication that some collective component has not been included. The most obvious candidate is ^{10}Be$(2^+) \times (sd)_0^2$. Nunes *et al.* [16] showed that including this configuration does indeed bring the 2_1^+ energy down. However, that configuration cannot be a major component because it has no direct one-step route in ^{10}Be(t,p) and (as noted above) the state is very strong there. We ignore this component for now, even though we expect it to be present at some level in all the 2^+ states. We will return to this point later. Hamamoto and Shimoura [17] reproduce the 2^+ energy with deformation. For ^{11}Be, they assume the lowest $1/2^+$, $5/2^+$, and (supposed) $3/2^+$ states are members of a decoupled $1/2^+$ rotational band built on the Nilsson deformed orbital $[220]1/2^+$. These energies allow them to compute the moment-of-inertia and decoupling parameters for ^{11}Be. They then scale the former to get a value for ^{12}Be, leading to a 2^+ energy of 2.09 MeV. So, fixing the 2^+ energy is not a problem, but the fixes are outside the present scope.

In our work, we assume isospin invariance, namely that the wave-function amplitudes are the same for different T_z members of an isospin multiplet. The effect of the Coulomb interaction is merely to change the radial-wave function. We note, however, that Grigorenko *et al.* [18] found significant isospin violation, namely an s^2 intensity in ^{12}O(gs) that is 1.5–2.0 times the value in ^{12}Be(gs). Even without isospin conservation, a value of about 50% s^2 in ^{12}O(gs) is necessary to explain its Coulomb energy.

We described the two lowest 0^+ states as linear combinations of the first $(sd)^2$ state and the *p*-shell one [6,7]. If we take the first 2^+ state to be a mixture of the lowest $(sd)^2$ 2^+ state and the *p*-shell 2^+ and use the ^{14}C(p,t) results of $19 \pm 9\%$ of the *p*-shell component in the first 2^+, then the wave function of this state is

$$2_1^+ = 0.84\,ds + 0.32\,dd + 0.44\,p \text{ shell},$$

where we temporarily ignore the uncertainty in the last term. In this simple description, the second and third 2^+ states then should be linear combinations of

$$0.41\,ds + 0.16\,dd - 0.90\,p \text{ shell, and } 0.41\,ds - 0.93\,dd.$$

Takashina [19] states that the lowest 0^+ and 2^+ states are mostly $(sd)^2$. Because the second $(sd)^2$ 2^+ and the

TABLE I. Energies and wave-function intensities in ^{12}Be.

J^π	Space	State	E_x (MeV)	s^2	d^2	p shell
0^+	$(sd)^2$	0_1^+	0.20	0.78	0.22	–
	$(sd)^2$	0_2^+	4.35	0.22	0.78	–
	$(sd)^2 + p$ shell	gs		0.53	0.15	0.32
				ds	d^2	
2^+	$(sd)^2$	2_1^+	3.63	0.87	0.13	–
	$(sd)^2$	2_2^+	5.42	0.13	0.87	–
	$(sd)^2 + p$ shell	2.11 MeV		0.71	0.10	0.19

p-shell 2^+ state are close together, the mixing of the two could be considerable. However, the lowest 2^+state should be reasonably stable to that mixing. And, of course, the ^{10}Be(2^+) $\times$ $(sd)_0^2$ configuration provides another 2^+ state, and this strength is probably spread among all the 2^+ levels.

II. ^{12}O

We now use this 2_1^+ wave function to calculate the expected energy and width in ^{12}O. Pure configuration energies are listed in Table II. With our admixture, the resulting ^{12}O(2^+) energy is 1.80 MeV. The $\pm 9\%$ uncertainty in the 19% p-shell intensity provides an uncertainty of ± 15 keV in this energy. From other work, we have found that our Coulomb energy calculations produce energies in mirror nuclei with deviations of $<$40–70 keV from experimental values. It is well known that a state with a large $s_{1/2}$ component will have much lower energy in the proton-rich member of a mirror pair (the so-called Thomas-Ehrman effect). Here, both the gs and first-excited states have large $s_{1/2}$ admixtures, so their energy difference in ^{12}O is not significantly less than in ^{12}Be.

In ^{12}O, the ds component in the first 2^+ state can decay to the ^{11}N(gs) via $\ell = 2$ emission, and the p-shell component can decay to the $1/2^-$ first-excited state via $\ell = 1$. The spectroscopic factor for the pure p-shell 2^+ state is very small—S = 0.0376 [20]. Thus, if the p-shell component of the physical state is only 19(9)%, then the value of S for p-wave decay is 0.0071(35). We have computed $\ell = 1$ and 2 single-particle widths $\Gamma_{\rm sp}$ in a potential well with r_0, a = 1.25, 0.65 fm. (The same potential was used to compute the Coulomb energies.) The well depth was adjusted to provide an energy of 1.80 MeV. We integrated over the natural width of the ^{11}N states. The expected widths are then obtained from

TABLE II. Excitation energy (MeV) in ^{12}O of the mirror of ^{12}Be (2^+, 2.1 MeV).

Configuration	E_x
ds	1.68[a]
dd	2.33
p shell	1.94
Mixed[b]	1.80

[a]$(5/2^+ \times s + 1/2^+ \times d)/2$.
[b]Configuration in last line of Table I.

TABLE III. Widths (keV) for decay of ^{12}O (2^+, 1.8 MeV).

^{11}N	ℓ	$\Gamma_{\rm sp}$	S	$\Gamma_{\rm calc}$
gs $1/2^+$	2	150	0.52	78
$1/2^-$	1	180	0.007	1.3

$\Gamma_{\rm calc} = S\Gamma_{\rm sp}$. They are listed in Table III. The upshot is that this 2^+ state near 1.8 MeV should be quite narrow. Earlier, we had predicted the ^{12}O energy of 0_2^+ to be 1.95 MeV [7]. A recent ^{14}O(p,t) experiment [21] observed a peak at 1.8(4) MeV, with a total width of 1.6(3) MeV, where the resolution width was 1.0(5) MeV. Because the ^{14}C(p,t) reaction populated both 0_2^+ and 2_1^+ states, the same should be true here. By isospin invariance, the $0_2^+/2_1^+$ cross-section ratio should be roughly equal in the two reactions. Suzuki *et al.* [21] analyzed their peak as a single state, but we expect it contains both states. Even though narrow, the 2^+ peak would have been about 1 MeV wide from the resolution, making it very difficult to resolve the two states.

III. ^{12}Be

We return now to the case of ^{12}Be. In ^{10}Be(t,p), a candidate for a second 2^+ state was observed at an excitation energy of 4.56 MeV. Millener [20] has suggested this might instead be a 3^- state, or a $2^+/3^-$ doublet, because it is too strong to be 2^+. Indeed, given the observed (t,p) cross section for the first 2^+ state, we find that the 4.56-MeV cross section is significantly larger than the remaining 2^+ strength expected for the entire $d_{5/2}$, $s_{1/2}$, p-shell space. At these negative Q values, 2^+ and 3^- angular distributions are very similar [22], making them difficult to distinguish. However, the cross section appears to be slightly too large for a single 3^- state, even if this state had a pure $(1p_{1/2})(1d_{5/2})$ configuration. If it is a doublet, then the two states are quite close together and have about the same width [107(17) keV], or one of them has most of the strength. (The 3^- could be strong and the 2^+ weak.) If it is all 3^-, then the other 2^+ state(s) are too weak to observe or are above 6 MeV. Fortune, Liu, and Alburger [11] placed an upper limit of 30 $\mu b/sr$ for an unobserved *narrow* state below 6 MeV. However, a broad state could have had a significantly larger cross section and have been missed. One possible candidate is near 5.4 MeV, and another is on the low-energy side of the 5.70-MeV 4^+ state. If one 2^+ state contains the bulk of the remaining p-shell configuration, it should be quite strong in ^{14}C(p,t), but no candidate was observed. At this time, we are unable to say anything further about other possible 2^+ states.

Earlier, we estimated the amount of s^2 in ^{12}Be (and ^{12}O) ground states by computing the ^{12}Be-^{12}O mass difference, which is quite sensitive to this component. Our result was 53% for the s^2 intensity [6]. With a reasonable, but simple, shell-model calculation, we suggested an s^2/d^2 ratio of 0.78/0.22, and hence 68% $(sd)^2$, 32% p shell for ^{12}Be(gs). Navin *et al.* [23], in a subsequent experiment, coincidentally suggested the identical configuration admixture—68% $(sd)^2$, 32% p shell. If ^{11}Be(gs) were pure $2s_{1/2}$, the spectroscopic factor for ^{12}Be(gs) would be just twice this s^2 intensity, and for 2^+, S would be equal to the ds intensity. However, ^{11}Be(gs) is only about 74%

TABLE IV. Spectroscopic factors in ^{11}Be(d,p) for lowest three states.

State	S_{exp} (Ref. [24])	Calculated (present)	
		Simple	Reduced
gs	$0.28^{+.03}_{-.07}$	1.06	0.78
0_2^+	$0.73^{+.27}_{-.40}$	0.50	0.37
2_1^+	$0.10^{+.09}_{-.07}$	0.70	0.52

^{10}Be $\times$ $2s_{1/2}$. So, the S's above need to be reduced by this factor. These numbers are listed in the Simple and Reduced columns in Table IV.

A very recent experiment [24] investigated the ^{11}Be(d,p) reaction in inverse kinematics, at a center-of-mass bombardment energy of 8.5 MeV. They measured S for the lowest three states of ^{12}Be. Because the $0_2^+/2_1^+$ states were not resolved, they used χ^2-squared minimization to fit the doublet angular distribution to a sum of $\ell = 0$ and 2 distorted-wave curves. Their spectroscopic factors are also listed in Table IV. We note that the experimental S's for the gs and 2^+ are significantly smaller than the calculated ones, while $S(0_2^+)$ is larger than calculated. All reasonable shell-model calculations predict $S(2_1^+)$ to be ~0.5, in rough agreement with our value of 0.52. Various theoretical values in Ref. [24] are 0.41, 0.50, and 0.55. It is extremely difficult to envision a scenario in which this spectroscopic factor could be as small as 0.10 (1σ upper limit 0.19), found in Ref. [24]. Part of the problem could be an incorrect separation of the $0_2^+/2_1^+$ components of the unresolved doublet. However, the authors state that at the 2σ level, all the doublet strength could be 2^+, and they arrive at $S = 0.25$—still a very small value. If isospin is not conserved and ^{12}Be(gs) has a smaller s^2 occupancy than ^{12}O(gs), the gs spectroscopic factor would be smaller than the calculated value in Table IV. However, the dominance of $(sd)^2$ over p-shell components is established from the ^{10}Be(t,p) reaction (and confirmed by other work). So, we would not expect a great reduction from the values in Table IV.

Kanungo *et al.* [24] state that "no experimental information exists on the detailed configurations of the excited states in ^{12}Be." Of course, the ^{10}Be(t,p) reaction does provide such information. The gs cross section is seven times as large as it would be if it were a pure p-shell state, and the 2^+ is 20 times as strong as the p-shell 2^+ should be. The absolute magnitude of the gs cross section requires the s^2 intensity to be significantly larger than d^2. The extreme weakness of 0_2^+ in (t,p) puts a rigorous constraint on its configuration. The 2_1^+ cross section requires significantly more ds than d^2 in its wave function. (For 2^+, the pure ds cross section is about four times that for pure d^2.) Also, the ^{14}C(p,t) reaction limits the p shell component of the first 2^+ state to about $19 \pm 9\%$. So, the (t,p) results clearly require more s^2 than d^2 in the gs and more ds than d^2 in 2_1^+. For both states, the $(sd)^2$ intensity is larger than the p-shell part. Takashina and Kanada-En'yo [19] agree.

IV. SUMMARY

For the first 2^+ state at 2.1 MeV in ^{12}Be, the large cross section observed in the ^{10}Be(t,p) reaction is totally incompatible with the small spectroscopic factor claimed for it in the ^{11}Be(d,p) reaction. As both the gs and 2_1^+ spectroscopic factors in Ref. [24] are smaller than expected in most models, it is conceivable that something is wrong with the absolute cross-section scale in Ref. [24]. We encourage another look at this reaction, difficult though it may be.

The supposed 2^+ state at 4.56 MeV has too much strength in (t,p) for another 2^+ state. It is more likely to be 3^-.

In ^{14}C(p,t), the data are consistent with the first 2^+ $T = 2$ state having about 20% of the strength expected for the pure p-shell 2^+. There is no evidence in that reaction for another 2^+ state with most of the remaining p-shell strength.

In ^{12}O, the first 2^+ state is expected near 1.8 MeV and should be narrow (width ~80 keV). The second 0^+ state should be near 1.95 MeV, with a width of about 800 keV. A better ^{14}O(p,t) experiment might be able to separate the two.

[1] H. T. Fortune, M. E. Cobern, S. Mordechai, G. E. Moore, S. LaFrance, and R. Middleton, Phys. Rev. Lett. **40**, 1236 (1978).
[2] H. T. Fortune, G. E. Moore, L. Bland, M. E. Cobern, S. Mordechai, R. Middleton, and R. D. Lawson, Phys. Rev. C **20**, 1228 (1979).
[3] S. Truong and H. T. Fortune, Phys. Rev. C **28**, 977 (1983).
[4] H. T. Fortune and R. Sherr, Phys. Rev. C **68**, 024301 (2003).
[5] G.-B. Liu and H. T. Fortune, Phys. Rev. C **42**, 167 (1990).
[6] R. Sherr and H. T. Fortune, Phys. Rev. C **60**, 064323 (1999).
[7] H. T. Fortune and R. Sherr Phys. Rev. C **74**, 024301 (2006).
[8] F. Ajzenberg-Selove, Nucl. Phys. A **506**, 1 (1990).
[9] R. D. Lawson, F. J. D. Serduke, and H. T. Fortune, Phys. Rev. C **14**, 1245 (1976).
[10] S. Cohen and D. Kurath, Nucl. Phys. A **141**, 145 (1970).
[11] H. T. Fortune, G.-B. Liu, and D. E. Alburger, Phys. Rev. C **50**, 1355 (1994).
[12] D. Ashery *et al.*, Phys. Rev. C **13**, 1345 (1976).
[13] H. T. Fortune and G. S. Stephans, Phys. Rev. C **25**, 1 (1982).
[14] G. Blanchon, N. V. Mau, A. Bonaccorso, M. Dupuis, and N. Pillet, Phys. Rev. C **82**, 034313 (2010).
[15] C. Romero-Redondo, E. Garrido, D. V. Fedorov, and A. S. Jensen, Phys. Rev. C **77**, 054313 (2008).
[16] F. M. Nunes, I. J. Thompson, and J. A. Tostevin, Nucl. Phys. A **703**, 593 (2002).
[17] I. Hamamoto and S. Shimoura, J. Phys. G **34**, 2715 (2007).
[18] L. V. Grigorenko, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, Phys. Rev. Lett. **88**, 042502 (2002).
[19] M. Takashina and Y. Kanada-En'yo, Phys. Rev. C **77**, 014604 (2008).
[20] J. D. Millener (private communication).
[21] D. Suzuki *et al.*, Phys. Rev. Lett. **103**, 152503 (2009).
[22] S. Mordechai, H. T. Fortune, G. E. Moore, M. E. Coben, R. V. Kollarits, and R. Middleton, Nucl. Phys. A **301**, 463 (1978).
[23] A. Navin *et al.*, Phys. Rev. Lett. **85**, 266 (2000).
[24] R. Kanungo *et al.*, Phys. Lett. B **682**, 391 (2010).