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Stable Levitation and Alignment of Compact Objects by Casimir Spring Forces

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Abstract

We investigate a stable Casimir force configuration consisting of an object contained inside a spherical or spheroidal cavity filled with a dielectric medium. The spring constant for displacements from the center of the cavity and the dependence of the energy on the relative orientations of the inner object and the cavity walls are computed. We find that the stability of the force equilibrium—unlike the direction of the torque—can be predicted based on the sign of the force between two slabs of the same material.

Disciplines

Physical Sciences and Mathematics | Physics

Comments

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Stable Levitation and Alignment of Compact Objects by Casimir Spring Forces

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We investigate a stable Casimir force configuration consisting of an object contained inside a spherical or spheroidal cavity filled with a dielectric medium. The spring constant for displacements from the center of the cavity and the dependence of the energy on the relative orientations of the inner object and the cavity walls are computed. We find that the stability of the force equilibrium—unlike the direction of the torque—can be predicted based on the sign of the force between two slabs of the same material.

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The Casimir force between atoms or macroscopic objects arises from quantum fluctuations of the electrodynamic field [[1\]](#page-5-0). Typically, it is found that the force is attractive, as long as the space between the objects is empty, and the magnetic susceptibility of the objects is negligible compared to their electric susceptibilities. But when space is filled with a medium with electric permittivity ϵ_M intermediate between that of two objects, ϵ_1 < $\epsilon_M < \epsilon_2$, the force between the two becomes repulsive [[2\]](#page-5-1). This effect has recently been verified experimentally in the large separation (retarded) regime [[3](#page-5-2)]. But while repulsive forces are nothing new, they become interesting for applications when they produce stable equilibria, which, for example, the Coulomb force cannot.

For an infinite cylinder enclosed in another, the Casimir force has recently been shown to have a stable equilibrium in the two directions perpendicular to the cylinder axes, when the material properties are chosen so that the force between two slabs of the same materials would be repulsive [\[4](#page-5-3)]. On the other hand, for a metal sphere or an electrically polarizable atom inside an otherwise empty spherical cavity there is no point of stable equilibrium [[5\]](#page-5-4).

In this Letter we investigate the first configuration, depicted in Fig. [1,](#page-2-0) in which a compact object levitates stably due to the Casimir force alone. We consider the following cases: a finite sphere or a small spheroid inside a spherical cavity, and a small spheroid inside a slightly deformed spherical cavity. The Casimir energy

$$
\mathcal{E} = \mathcal{E}_0 + \frac{1}{2}k\frac{a^2}{R^2} + \frac{1}{3!}k_3\frac{a^3}{R^3} + \frac{1}{4!}k_4\frac{a^4}{R^4} + \cdots
$$
 (1)

is characterized by the spring constant k and the coefficients k_n in a series expansion in a/R , where a is the magnitude of the displacement from the center of the cavity and R the radius of the (undeformed) spherical cavity. (k has units of energy here.) Unlike the case of infinite cylinders, our case exhibits, for appropriately chosen materials, true stability in all directions and applies to realistic situations. For example, we compute the force on a metal sphere in a spherical drop of liquid surrounded by air. By determining the mean square deviation from the center, $\langle a^2 \rangle = 3k_BTR^2/k$, the spring constant k can be measured experimentally. We can estimate that the size of the droplet R has to be smaller than $\approx 3 \mu m \frac{r^3}{R^3}$, where r is the typical dimension of the inner object, for the thermal motion to be confined near the center of the cavity. (This length scale is obtained by balancing $3k_BT$ for room temperature with a rough estimate of the spring constant, $k \sim$ $\frac{\hbar c}{R} \frac{r^3}{R^3}$.) To keep the two objects nearly concentric against the gravitational force, R has to be smaller than \approx 1 μ m for the typical metal or liquid densities considered here. Our the typical metal or liquid densities considered here. Our calculations show that, for example, a sphere of gold of radius $r = \frac{3}{4}R$ inside a spherical drop of ethanol of radius $R = 0.1 \mu$ m has an rms deviation $\sqrt{\langle a^2 \rangle} = 0.04R$ from the center due to thermal motion and a displacement from the center due to thermal motion and a displacement from the center by $a \approx 10^{-6}R$ due to gravity. A variety of technologies may benefit from our analysis, e.g., nanocarriers $(R \approx 0.1 \mu m)$ for drug particles [[6](#page-5-5)] or molecular cages for explosive molecules [7] for explosive molecules [\[7](#page-5-6)].

Whether the center of the cavity is a point of stable or unstable equilibrium turns out to be correlated with

FIG. 1 (color online). Summary of the configurations we consider and of the results. We have assumed that the small spheroid's zero frequency permittivity satisfies $\epsilon_{I,0} > \epsilon_{M,0}$ and that it is larger in the body-fixed \hat{z} direction, so $\alpha_{zz}^E > \alpha_{\perp\perp}^E$.
Eurthermore, the magnetic permeabilities are all set to one. Furthermore, the magnetic permeabilities are all set to one. (a) Direction of the force \bf{F} on such a spheroid in a spherical cavity if $\epsilon_{M,0} > \epsilon_{O,0}$, and the direction of the torque τ when either $\epsilon_{M,0} > \epsilon_{O,0}$ or $\epsilon_{M,0} \ll \epsilon_{O,0}$. (b) A finite size sphere experiences a restoring force F for the various combinations of materials listed in Table [I.](#page-4-0) (c) Direction of the torque τ in the center of a slightly spheroidal cavity if either $\epsilon_{M,0} < \epsilon_{O,0}$ or $\epsilon_{M,0} \gg \epsilon_{O,0}.$

whether the Casimir force is repulsive or attractive for two parallel plates under those conditions. The direction of the torque, on the other hand, depends on the dielectric properties of the medium and the cavity walls in an unintuitive way, which cannot be predicted by the pairwise summation or proximity force approximations (PSA or PFA), see Fig. [1.](#page-2-0) In particular, this behavior is not due to dispersion effects, which explain similar phenomena reported in Ref. [\[4\]](#page-5-3). We calculate the torque on a small spheroid that is displaced from the center of a spherical cavity [Fig. $1(a)$] or concentric with a slightly spheroidal cavity [Fig. [1\(c\)\]](#page-2-1). In the former case, the orientation dependence manifests itself in k, which captures both the torque and the orientation dependence of the total force. In the latter case, \mathcal{E}_0 depends on the relative orientation of the spheroid inside the deformed cavity. (If the cavity is spherical, \mathcal{E}_0 is a finite constant that can be ignored.) By choosing appropriate materials, the inside object, e.g., a nanorod, can be made to align in different ways with the cavity shape, a situation which is reminiscent of a compass needle aligning with the magnetic field of the earth.

The starting point of the analysis is the scattering theory approach. The method is explained and derived in detail in Ref. [[8](#page-5-7)], where a partial overview over its precursors, e.g., [\[9](#page-5-8)], is provided. The Casimir energy $\mathcal{E} =$ $\frac{\hbar c}{2\pi} \int_0^\infty d\kappa \ln \det(I - \mathcal{F}_0^{ii} \mathcal{W}^{io} \mathcal{F}_1^{ee} \mathcal{V}^{io})$ is expressed in terms of the inner object's exterior T matrix, \mathcal{F}_I^{ee} , and the outer object's interior T matrix, \mathcal{F}_0^{ii} . The exterior T matrix describes the scattering of regular wave functions to outgoing waves when the source lies at infinity. The interior T matrix expresses the opposite, the amplitudes of the regular wave functions, which result from scattering outgoing waves from a source inside the object. The translation matrices \mathcal{W}^{io} and \mathcal{V}^{io} convert regular wave functions between the origins of the outer and the inner objects; they are related by complex transpose up to multiplication by (-1) of some matrix elements.

With uniform, isotropic, and frequency-dependent permittivity $\epsilon_x(i c \kappa)$ and permeability $\mu_x(i c \kappa)$ functions $(x = I : \text{inner object } x = O : \text{outer object } x = M : \text{medium})$ the *I*: inner object; $x = 0$: outer object; $x = M$: medium) the T matrix of the sphere is diagonal. It is given by

$$
\mathcal{F}_{I,lmE,lmE}^{ee}(ic\kappa) = \mathcal{F}_{I,lE}^{ee}(\xi)
$$
\n
$$
= -\frac{i_l(\xi)\partial_r(ri_l(z_I\xi)) - \frac{\epsilon_l}{\epsilon_M}i_l(z_I\xi)\partial_r(ri_l(\xi))}{k_l(\xi)\partial_r(ri_l(z_I\xi)) - \frac{\epsilon_l}{\epsilon_M}i_l(z_I\xi)\partial_r(rk_l(\xi))}
$$

for E (electric) polarization and by the same expression with $\frac{\epsilon_I}{\epsilon_M}$ replaced by $\frac{\mu_I}{\mu_M}$ for M (magnetic) polarization (not to be confused with subscript M indicating the medium's response functions). In the above equation the frequency dependence of the response functions has been suppressed. The indices of refraction $n_x(i\epsilon\kappa) = \sqrt{\epsilon_x(i\epsilon\kappa)}\mu_x(i\epsilon\kappa)$ of the sphere and the medium appear in the ratio $z_i(i\epsilon\kappa) =$ the sphere and the medium appear in the ratio $z_I(i c \kappa) =$ $n_I(i c \kappa)/n_M(i c \kappa)$ and the argument $\xi = n_M(i c \kappa) \kappa r$. The interior T matrix of the spherical cavity is obtained from the exterior T matrix of the sphere by inserting the outside object's radius and response functions in place of those of the inside object and exchanging the modified spherical Bessel functions i_l and k_l .

However, the scattering approach is not limited to simple geometries. An array of techniques is available for calculating the scattering amplitudes of other shapes. We employ the perturbation approach to find the T matrix of a deformed spherical cavity [\[10](#page-5-9)[,11\]](#page-5-10) of radius $R + \delta(1 3/2\sin^2\theta$. The deformation, indicated in Fig. [1\(c\),](#page-2-1) is chosen so that the volume is unchanged to first order in δ . We find the $O(\delta)$ correction, $\mathcal{F}^{(1)}$, to the T matrix in a negligible property expansion. $\mathcal{F} = \mathcal{F}^{(0)} + \mathcal{F}^{(1)} + \cdots$ perturbation series expansion, $\mathcal{F} = \mathcal{F}^{(0)} + \mathcal{F}^{(1)} + \cdots$,
by matching the regular and outgoing fields according to by matching the regular and outgoing fields according to the Maxwell boundary conditions along the deformed object's surface [[12](#page-5-11)]. On the other hand, for a small object (compared to the wavelength of the radiation), we can approximate the T matrix to lowest order in κ using the static polarizability tensor, $\mathcal{F}_{I,1mP,1m^{\prime}P}^{ee} = 2/3(n_{M,0}\kappa)^3 \alpha_m^P$
 $Q(\omega^5)$, where the subscript 0 indicates the statio (i) potarizability cluster, $J_{I,1mP,1m/P}$ $\geq J_{I,M,(0)}$, $J_{mnn'}$
 $O(\kappa^5)$, where the subscript 0 indicates the static ($i c \kappa =$

(i) limit and P is the polarization label. The T-matrix 0) limit and P is the polarization label. The T -matrix elements involving higher angular momenta $l > 1$ are higher order in κ . For a small ellipsoid, in particular, the electric polarizability tensor $\boldsymbol{\alpha}^E$ is diagonal in a coordinate system aligned with the ellipsoid's body axes, $\alpha_{ii}^E = \frac{V}{4\pi}$
 $\epsilon_{i0} - \epsilon_{M,0}$ em angled with the empsoid's body axes, $\alpha_{ii} = \frac{z_{i0} - \epsilon_{M0}}{4\pi}$. $\frac{\epsilon_{f,0} - \epsilon_{M,0}}{\epsilon_{M,0} + (\epsilon_{f,0} - \epsilon_{M,0})n_i}$, where $i \in \{x, y, z\}$ [\[13](#page-5-12)]. The larger the semiaxis in direction i , the smaller the depolarization factor n_i (not to be confused with the index of refraction). The magnetic polarizability tensor α^M is obtained by exchanging $\mu_{x,0}$ for $\epsilon_{x,0}$ in the expression for α^E . In the small size limit the polarizability tensor of a perfect metal ellipsoid is obtained by taking both $\epsilon_{L,0} \to \infty$ and $\mu_{L,0} \to 0$.
For simplicity we specialize to a spheroid which h

For simplicity we specialize to a spheroid, which has two equal semiaxes. We choose the semiaxes along \hat{x} and \hat{y} to be equal; therefore, $\alpha_{xx}^P = \alpha_{yy}^P = \alpha_{\perp\perp}^P$. We fix the \hat{z} axis of the lab frame to be along the direction of displacement of the lab frame to be along the direction of displacement of the spheroid from the center of the cavity. θ denotes the angle between the spheroid's and the lab's \hat{z} axes. For such a small spheroid inside a spherical cavity of radius R [Fig. [1\(a\)\]](#page-2-1), the spring constant is obtained by expanding the log determinant in the expression for $\mathcal E$ to first order,

$$
k_{R\to\infty} = \frac{\hbar c}{R^4 n_{M,0}} \left[\text{Tr}\alpha^E f_1^E + (\alpha_{zz}^E - \alpha_{\perp\perp}^E) \frac{3\cos^2\theta - 1}{2} f_2^E + E \to M \right], \quad (2)
$$

where the material dependent functions

$$
f_1^E = \int_0^\infty \frac{\xi^5 d\xi}{9\pi} [\mathcal{F}_{0,1M}^{ii} - 2\mathcal{F}_{0,1E}^{ii} - \mathcal{F}_{0,2E}^{ii}],
$$

$$
f_2^E = \int_0^\infty \frac{\xi^5 d\xi}{9\pi} \left[\frac{4}{5} \mathcal{F}_{0,1E}^{ii} - \frac{1}{5} \mathcal{F}_{0,2E}^{ii} - \mathcal{F}_{0,1M}^{ii} \right]
$$
(3)

express the rotation invariant and the orientation dependent parts of the energy, respectively. f_1^M and f_2^M are obtained from f_1^E and f_2^E by exchanging E and M everywhere. This

FIG. 2 (color online). f_1^E and f_1^M describe the part of the spring constant $k_{R\to\infty}$, which is invariant under a rotation of the inside object. The vertical lines indicate the values pertaining to the configurations presented in Table [I](#page-4-0), ethanol-vacuum (0.16), bromobenzene-vacuum (0.30), and gold cavity walls (1). In this plot, $\mu_{M,0} = \mu_{O,0}$.

result is valid for asymptotically large R ; it involves only the zero frequency ($ic \kappa = 0$) response functions since R is greater than any material dependent length scales as well as the size of the inner object. Notice that only the $l = 1, 2$ scattering amplitudes of the cavity walls appear in Eqs. [\(2\)](#page-3-0) and ([3\)](#page-3-1).

The behavior of the functions f_1^P , depicted in Fig. [2](#page-4-1) for $\mu_{M,0} = \mu_{O,0}$, is as expected: f_1^E is monotonic, positive
when $\epsilon_{MS} > \epsilon_{SO}$ negative when $\epsilon_{MS} \leq \epsilon_{SO}$ and f^M alwhen $\epsilon_{M,0} > \epsilon_{O,0}$, negative when $\epsilon_{M,0} < \epsilon_{O,0}$, and f_1^M always has the opposite sign of f_1^E . When f_1^E is positive, a small object with $\epsilon_{I,0} > \epsilon_{M,0}$ is levitated stably, when f_1^E is negative, $\epsilon_{M,0} > \epsilon_{I,0}$ has to hold. Thus, stability occurs under the same conditions as repulsion for two half spaces. The opposite sign of f_1^M is expected from equivalent expressions for the two-infinite-slab geometry [[14\]](#page-5-13). When the dielectric contrast between the medium and the outer sphere is taken to small or large limits, stability or instability is maximized.

To verify whether stability is observable for realistic materials and object sizes, we evaluate the energy $\mathcal E$ numerically for a sphere of radius r inside a spherical cavity of radius R filled with various liquids [Fig. [1\(b\)\]](#page-2-1) [\[15\]](#page-5-14). The coefficients k , k_4 , and k_6 in the series expansion in Eq. [\(1\)](#page-2-2) are listed in Table [I](#page-4-0). For comparison, the asymptotic result $k_{R\rightarrow\infty}$ is also included. If both inside and outside object are spherically symmetric, the series in Eq. [\(1](#page-2-2)) does not contain terms $\sim \frac{a^n}{R^n}$ with *n* odd.
The three materials were

The three materials were chosen so that the sequence of permittivities ϵ_I , ϵ_M , ϵ_O either increases or decreases for the imaginary frequencies that contribute most to the energy. Contrary to the prediction of the PSA and PFA the force is not symmetric with respect to exchange of the inner and outer permittivities. In the same medium, a high dielectric sphere is held more stably in the center of a cavity with low dielectric walls than a low dielectric sphere inside a cavity with high dielectric walls.

The asymptotic result $k_{R\rightarrow\infty}$ yields a good approximation of k for $\frac{r}{R} = \frac{1}{4}$. But from Eq. ([2\)](#page-3-0) one would expect
k to grow linearly with the volume of the inner sphere $k_{R\rightarrow\infty}$ to grow linearly with the volume of the inner sphere, since polarizability is proportional to the volume. In fact, k

TABLE I. $k, k_{R\rightarrow\infty}, k_4$, and k_6 are listed for various combinations of materials for the case of a spherical inner object inside a spherical cavity, depicted in Fig. [1\(b\).](#page-2-1) The dimensionless numbers in the table have to be multiplied by $\frac{hc}{R}$. R is given in microns $[\mu m]$. $k_{R\rightarrow\infty}$ depends on R only through the ratios $\frac{\hbar c}{R}$ and $\frac{L}{R}$ so its numerical prefector is the same for all R. The highest $\frac{r}{R}$, so its numerical prefactor is the same for all R. The highest cutoff used was $l_{\text{max}} = 30$. (The asymptotic result $k_{R\rightarrow\infty}$ only requires $l = 1, 2$.)

Inside-medium-outside	$R \thinspace r/R \thinspace k$		$k_{R\rightarrow\infty}$	k_4	k ₆
Gold-bromobenzene-vacuum 0.1 1/4 0.040 0.054 1.2					8.2×10^{1}
		$3/4$ 22 1.4		4200	1.8×10^{6}
			1.0 $1/4$ 0.069 0.054 2.6		2.0×10^{2}
					$3/4$ 70 1.4 18000 1.0×10^7
Gold-ethanol-vacuum			$0.1 \t1/4 \t0.050 \t0.037 \t1.6$		1.0×10^{2}
		$3/4$ 27 0.99		5200	2.2×10^6
				$1.0 \t1/4 \t0.045 \t0.037 \t1.7$	1.4×10^{2}
		$3/4 \quad 60 \quad 0.99$			$18\,000$ 1.2×10^7
Silica-ethanol-gold				$0.1 \quad 1/4 \quad 0.015 \quad 0.012 \quad 0.46$	3.1×10^{1}
		$3/4$ 10 0.33		1900	8.4×10^5
					1.0 $1/4$ 0.019 0.012 0.82 7.2 \times 10 ¹
					$3/4$ 41 0.33 12000 7.6 \times 10 ⁶

for $\frac{r}{R} = \frac{3}{4}$ is about 1000 times larger than for $\frac{r}{R} = \frac{1}{4}$, instead of just 27 times. This means that for a gold sphere in a liquid drop with $\frac{r}{R} = \frac{3}{4}$ and $R = 1$ μ m at room temperature, indeed, the Casimir spring holds the particle near the center effectively, $\sqrt{\langle a^2 \rangle}/R < 0.1$.
Although k k, and k, increase l

Although k , k_4 , and k_6 increase by 3 orders of magnitude in some cases, the prefactor $\frac{1}{n!}$ ensures that the coefficients in the Taylor expansion in Eq. [\(1\)](#page-2-2) increase only by 1 order of magnitude. Thus, for small excursions from the center, e.g., $a/R < 0.1$, the higher corrections $n \ge 4$ can be neglected.

Compared to the stability conditions studied thus far, the orientation dependence of the energy is more varied. f_2^E and f_2^M , plotted in Fig. [3](#page-4-2), have the same sign for most ratios of medium to outside permittivities, unlike f_1^E and f_1^M , which always have opposite signs. In these ranges of values, the contributions to the torque from electric and magnetic polarizability are opposite for a small perfect metal spheroid, for which $\epsilon_{I,0} > \epsilon_{M,0}$ and $\mu_{I,0} < \mu_{M,0}$. Unlike f_1^E and f_1^M , also, f_2^E and f_2^M change sign again at $\epsilon_{0.0} \approx 80$ and at $\epsilon_{0.0} \approx 2000$ respectively. So, while the $\frac{\epsilon_{0.0}}{\epsilon_{M,0}} \approx 80$ and at $\frac{\epsilon_{0.0}}{\epsilon_{M,0}} \approx 2000$, respectively. So, while the

FIG. 3 (color online). f_2^E and f_2^M describe the part of the spring constant $k_{R\to\infty}$, which changes with the orientation of the inside spheroid. In this plot, $\mu_{M,0} = \mu_{O,0}$.

FIG. 4 (color online). g^E and g^M describe the dependence of the energy \mathcal{E}_0 on the relative orientation of the inside spheroid and the deformed cavity walls.

direction of the total force and the stability of the equilibrium can be determined based on the relative magnitudes of the permittivities, the torque cannot be. The PSA and PFA predict the orientation with the lowest energy in the vicinity of $\frac{\epsilon_{0,0}}{\epsilon_{M,0}} = 1$ correctly but not when $\frac{\epsilon_{0,0}}{\epsilon_{M,0}} \gg 1$. Furthermore, the second sign change of f_2^E and f_2^M raises the question whether calculations of the Casimir torque for infinite conductivity metals are ''universal'' in the sense that they produce the correct qualitative results for real materials.

The orientation dependent part of the energy for the configurations discussed thus far vanishes, of course, when the small spheroid is in the center of the spherical cavity. If the cavity is slightly deformed, however, the energy, \mathcal{E}_0 , depends on the relative orientation of the spheroid and the cavity [Fig. [1\(c\)](#page-2-1)]. We deform the spherical cavity as described earlier and obtain to first order in $\delta/R,$

$$
\mathcal{E}_0 = \frac{\hbar c \cos^2 \theta}{R^4 n_{M,0}} \frac{\delta}{R} [(\alpha_{zz}^E - \alpha_{\perp \perp}^E) g^E + E \to M], \quad (4)
$$

where the orientation-independent part of the energy has been dropped. For $\mu_{O,0} = \mu_{M,0}$, g^E and g^M are given by

$$
g^{E} = \int_{0}^{\infty} \frac{\xi^{4} d\xi}{10\pi} \frac{i_{1}k_{0} + i_{0}k_{1}}{(\tilde{k}_{0}i_{1} + \tilde{k}_{1}i_{0}z_{0} + i_{1}\tilde{k}_{1}/(z_{0}\xi)(1 - z_{0}^{2}))^{2}} \times (z_{0}^{2} - 1)[4\tilde{k}_{1}^{2}/\xi^{2} - (\tilde{k}_{0} + \tilde{k}_{1}/(z_{0}\xi))^{2}],
$$

$$
g^{M} = \int_{0}^{\infty} \frac{\xi^{4} d\xi}{10\pi} \frac{i_{1}k_{0} + i_{0}k_{1}}{(\tilde{k}_{1}i_{0} + \tilde{k}_{0}i_{1}z_{0})^{2}} (z_{0}^{2} - 1)\tilde{k}_{1}^{2},
$$
(5)

where the arguments of the modified spherical Bessel functions and of $z_O(0)$ are suppressed. i_l and k_l are functions of ξ , and \tilde{k}_l stands for $k_l(z_0(0)\xi)$, where $z_0(i c \kappa) =$
 $n_o(i c \kappa)/n_v(i c \kappa)$ is the ratio of the permittivities of the $n_o(ic\kappa)/n_M(ic\kappa)$ is the ratio of the permittivities of the cavity walls and the medium. The material dependent functions g^E and g^M are plotted in Fig. [4.](#page-5-15)

Again, g^E has the same sign for $\frac{\epsilon_{0.0}}{\epsilon_{M,0}} \to 0$ (left in Fig. [4\)](#page-5-15)
and $\frac{\epsilon_{0.0}}{\epsilon_{M,0}} \to \infty$ (right). In addition to the root at $\frac{\epsilon_{0.0}}{\epsilon_{M,0}} = 1$, g^E
also vanishes at $\frac{\epsilon_{0.0}}{\epsilon_{M,0}} \approx 0.46$.

The rich orientation dependence of the energy is expected to collapse as the size of the inside object grows to fill the cavity and the PFA becomes applicable. Based on the stability analysis for finite inside spheres, though, we expect the asymptotic results, f_2^P and g^P , to predict the orientation dependence for reasonably small inside spheroids. For comparison with real experiments, of course, various corrections to the idealized shapes considered here have to be taken into account.

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materials we use an oscillator model $\varepsilon(i\alpha)$ = materials we use an oscillator model, $\epsilon(i c \kappa)$ = $1 + \sum_{n=1}^{N} \frac{C_n}{1 + (c\kappa/\omega_n)}$
10¹⁴ H_z and [*C*] with $[\omega_n]_{n=1,2} = [6.6, 114] \times$ 10^{14} Hz and $[C_n]_{n=1,2} = [23.84, 0.852]$ for ethanol [\[17\]](#page-5-17) as used in Ref. [\[4\]](#page-5-3), with $[\omega_n]_{n=1,2} = [5.47, 128.6] \times$ 10^{14} Hz and $[C_n]_{n=1,2} = [2.967, 1.335]$ for bromobenzene [\[17](#page-5-17)] as used in Ref. [[3](#page-5-2)], and with $[\omega_n]_{n=1-4}$ = $[0.867, 1.508, 2.026, 203.4] \times 10^{14}$ Hz and $[C_n]_{n=1-4}$ = $[0.829, 0.095, 0.798, 1.098]$ for silica (SiO₂) [\[18\]](#page-5-18).
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